Agenda

- Waves in waveguides
- Standing waves and resonance
- Setup
- Experiment with microwave cavity
- Comments on Bragg diffraction experiment
Maxwell’s Equations

Uniform plane wave traveling in z-direction $\rightarrow H \perp E$

$\nabla D = 0$

$\nabla B = 0$

$\nabla \times \vec{E} = -\frac{\partial \vec{B}}{\partial t}$

$\nabla \times \vec{H} = \frac{\partial \vec{D}}{\partial t}$

Wave equation

$\frac{\partial^2 E_x}{\partial z^2} = \frac{1}{v^2} \frac{\partial^2 E_x}{\partial t^2}$

General form of solution

$E_z(z,t) = f \left( t - \frac{z}{v} \right) + g \left( t + \frac{z}{v} \right)$

Propagation speed

$v = \frac{1}{\sqrt{\varepsilon \mu}}$

$Z = \sqrt{\frac{\mu}{\varepsilon}}$

$E$ vs $H$

$E_x = E_0 e^{i(\omega t - k z)}$

$H_y = \sqrt{\frac{\varepsilon}{\mu}} E_x$

$E_x = ZH_y$
Wave Propagation in Wave Guides

\[ E_y = E_0 \sin k_x x \cdot e^{i(\omega t - kz)} \]
Standing Waves in Cavities

\[ E_y = E_0 \sin k_x x \cdot e^{i(\omega t - k_z)} \]

\[ E_y = E_0 \sin k_x x \cdot e^{i(\omega t + k_z)} \]

\[ L = n^* \lambda / 2 \]
Standing Waves in Cavities

\[ Ey = Ey(z) \]

\[ Ey = Ey(x \text{ or } z) \]

H-field
Resonances for transverse Electric Waves

\[ \omega_{mnp}^2 = \nu_0^2 \left[ \left( \frac{m\pi}{a} \right)^2 + \left( \frac{n\pi}{b} \right)^2 + \left( \frac{p\pi}{c} \right)^2 \right] \]

\[ \nu_0^2 \text{ - phase velocity} \]

**TE_{101} mode**: m=1, n=0, p=1

\[ \omega_{101}^2 = \nu_0^2 \pi^2 \left[ \left( \frac{1}{a} \right)^2 + \left( \frac{1}{c} \right)^2 \right] \]
Equivalent Circuit

Coaxial wave guide

Coupling loop

Cavity

Outer conductor

Inner conductor

Line

Impedance of wave guide

11/2/2015
In the coupling of a waveguide and a cavity, the impedance of the waveguide is

\( Z_0 \)

Maximum power transfer:

\[ Z_0 = R \rightarrow \beta = 1 \]

\[ Q_L = \frac{1}{2} Q_0 \]

\( Q_L = \frac{\omega L}{Z_0 (1 + \frac{R}{Z_0})} = \frac{Q_0}{(1 + \beta)} \)

\( \beta \): coupling coefficient
Microwaves in Cavities. Overview of the Experiment.
Microwaves in Cavities.
The Setup of the Experiment.
Experiment. Wavelength measurement.

Use detector to find distance between minimums in the slotted line (wave guide)
Use detector to find distance between minimums in the slotted line (wave guide). Distance between consequent minima correspond $\lambda/2$.  

$\lambda/2$
Use plunger to change the dimension of the cavity in z-direction and search for maxima in power stored using the cavity detector. Identify $\text{TE}_{101}$ and $\text{TE}_{102}$. 
Experiment. Cavity resonance.

\[ \omega_{102}^2 = \nu_0^2 \pi^2 \left[ \left( \frac{1}{a} \right)^2 + \left( \frac{2}{c} \right)^2 \right] \]

\[ f_{102} = \frac{\nu_0}{2} \sqrt{\left[ \left( \frac{1}{a} \right)^2 + \left( \frac{2}{c} \right)^2 \right]} \]

**Graphs:**

- **Graph 1:**
  - Title: TE_{102}
  - Parameters:
    - \( c_0 = 13.88 \)
    - \( c_1 = 13.86 \)
    - \( c_2 = 13.92 \)
  - X-axis: \( I \) (cm)
  - Y-axis: \( I \) (mA)

- **Graph 2:**
  - Title: TE_{102}
  - Parameters:
    - \( Q = \frac{f_0}{\Delta f} \sim 450 \)
  - X-axis: \( f \) (GHz)
  - Y-axis: \( I \) (mA)
By moving the plunger we changing the resonance frequency of the cavity.

**Experiment. Cavity resonance.**

By moving the plunger we changing the resonance frequency of the cavity.
Experiment. Cavity resonance. Oscillator tuning.

Scope is in X-Y mode.

- Wavetek FG
- Sync out
- $V_m$
- $10V$
- $0$
- $V_{\text{Tune IN}}$
- Microwave Oscillator
- $\mu$Wave OUTPUT
- $V_{\text{Tune OUT}}$
- Cavity
1. Oscilloscope should run in X-Y mode
2. To plot the $I(f)$ dependence you have to download both Ch1 and Ch2 data
3. Use triangular waveform as a voltage applied to modulation input of the oscillator
4. Use a proper time scale setting on the scope which could estimated from scanning frequency
5. Apply the calibration equation to calculate the frequency of the oscillator from the modulation voltage

$$f = 0.03706V_{\text{mod}} + 2.9349$$
Experiment. Cavity resonance. Oscillator tuning.

Voltage tunable oscillator ZX95-3250a-S+ from Mini-Circuits®
FM Calibration for microwave oscillator

ZX95-3250a-S+

<table>
<thead>
<tr>
<th>Equation</th>
<th>y = Intercept + B1<em>x^1 + B2</em>x^2</th>
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<tr>
<td>Weight</td>
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<tr>
<td>Residual Sum of Squares</td>
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<td>Adj. R-Square</td>
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<table>
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<th>frequency</th>
<th>Value</th>
<th>Standard Error</th>
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<tr>
<td>Intercept</td>
<td>2.91874</td>
<td>8.0905E-4</td>
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<tr>
<td>B1</td>
<td>0.03588</td>
<td>3.43148E-4</td>
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<tr>
<td>B2</td>
<td>-4.41E-5</td>
<td>3.20212E-5</td>
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</table>
Experiment. Cavity resonance.

\[ Q = 340 \]

\[ Q = 133 \]

TE101

\[ f \text{ (GHz)} \]

\[ I \text{ (a.u.)} \]
By changing the coupling between oscillator and cavity, we can control the quality factor of the cavity resonance, but at the same time, we change the power delivered to the cavity.
Experiment. Coupling: Detecting of the Magnetic field.

While in resonance: turn orientation of the input loop from the vertical direction in 10° steps to 360°. Read cavity detector.
Experiment. Coupling: Detecting of the Magnetic field.

Experimental result. Fitted to $A|\cos(\alpha + \phi)|^n + A_0$
Presence of dielectric reduces length of cavity at a given resonance frequency $\omega_0$.
This effect grows with the electric field strength $E_y$.

(0) Without dielectric the cavity length at resonance is $c_0$.
(1) Place dielectric into cavity and move in 0.5cm steps, $l_i$.
(2) At each place tune plunger to resonance and record $c_i$.
(3) Plot $\Delta c_i = |c_0 - c_i|$ versus $l_i$ : this measures now $E_y$ vs $l_i$!
Electric Field Distribution.

Courtesy of P. Debevec
Quality factor $(TE_{101}$ mode) of unloaded cavity can be calculated as:

$$Q_0 = \frac{abc(a^2 + c^2)}{\delta \left[ 2b(a^3 + c^3) + ac(a^2 + c^2) \right]}$$

$\delta$ is the skin depth at frequency $\omega_0$

$$\delta = \sqrt{\frac{2\rho}{\mu \omega}}$$

$\rho$ – resistivity of the cavity material

$\mu = \mu_t \mu_0 \approx \mu_0 = 4\pi \times 10^{-7}$
For red brass  \( \rho = 6 \times 10^{-8} \Omega \text{m} \)
\( \mu \approx 4\pi \times 10^{-7} \)
\( \delta = 2.25 \times 10^{-6} \text{m} \)

\[ \delta = \sqrt{\frac{2 \rho}{\mu \omega}} \]

\( a = 7.22 \text{ cm}, \ b = 3.42 \text{ cm}, \ c = 6.91 \text{ cm} \) (TE\(_{101}\))

\[ Q_0 = \frac{abc \left( a^2 + c^2 \right)}{\delta \left[ 2b \left( a^3 + c^3 \right) + ac \left( a^2 + c^2 \right) \right]} \]

\( Q_0 \approx 7700 \)
Bragg diffraction.

\[ n\lambda = 2d \sin \theta \]

\text{Bragg's Law}

\[ \theta' = 90^\circ - \theta \]
Bragg diffraction. Possible origin of the $\sim 10^\circ$ peak

Second order reflection

Lloyd’s mirror effect