

UNIVERSITY OF ILLINOIS  
AT URBANA-CHAMPAIGN

# Investigation of the Properties of RLC Circuits

Physics 401, Fall 2015.

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illinois.edu

# **This Lecture Covers Two Labs:**

- 1. Transients and Oscillations in RLC Circuits.**
- 2. Frequency Domain Analysis of Linear Circuits Using Synchronous Detection**

## **This week:**

- 1. Transients. Definition.**
- 2. Transients in RLC**
- 3. Resonance in RLC**
- 4. Data analysis. Origin. Fitting.**

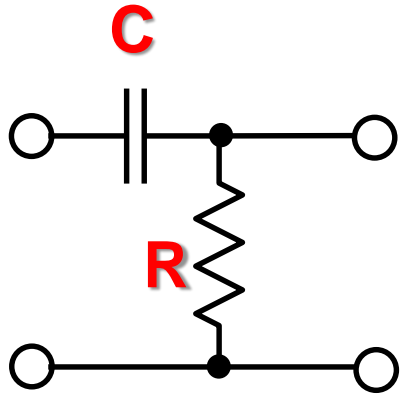
# **This Lecture Covers Two Labs:**

- 1. Transients and Oscillations in RLC Circuits.**
- 2. Frequency Domain Analysis of Linear Circuits Using Synchronous Detection**

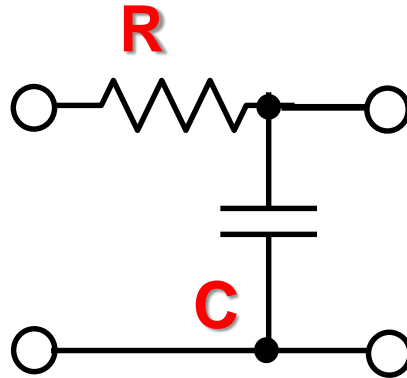
## **Next week:**

- 1. Time domain and Frequency domain representation of the data.**
- 2. Frequency domain spectroscopy (FDS)**
- 3. Lock-in amplifiers**
- 4. Practical application of lock-in's in FDS**
- 5. Taking data and simple data analysis using OriginPro**

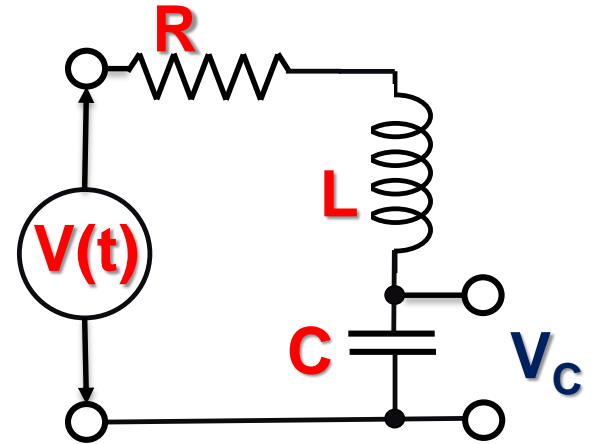
# RLC Circuits:



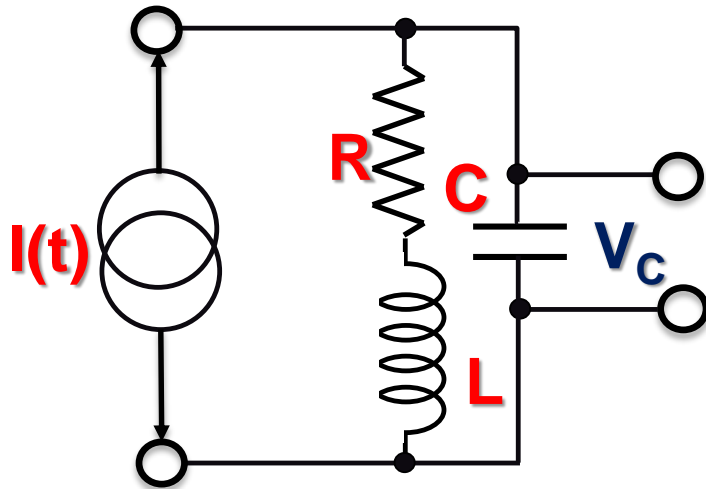
Hi-pass filter



Low-pass filter



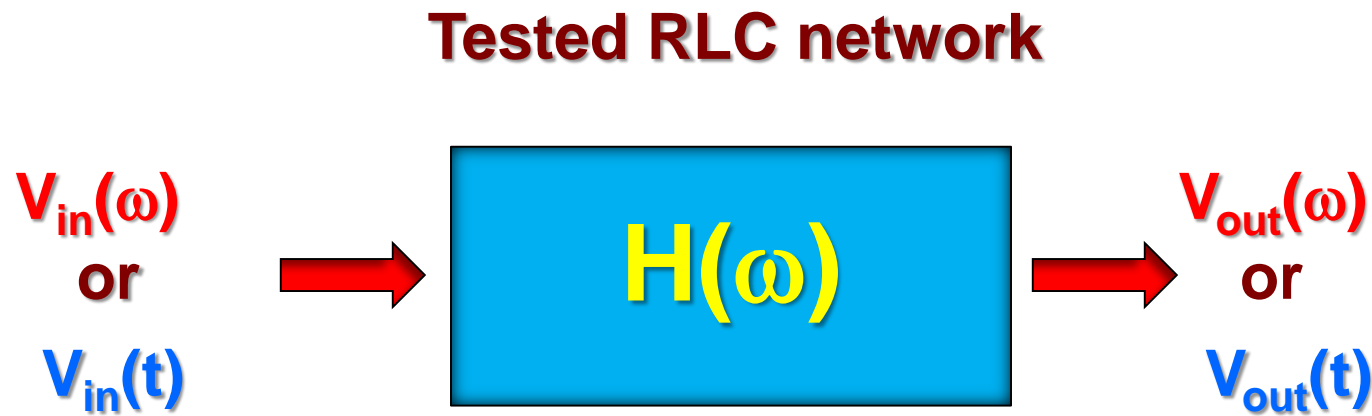
Series resonance circuit



Parallel resonance circuit



# Investigation of the Properties of the RLC Circuits. General Idea.



$$\check{H}(\omega) = \frac{\check{V}_{out}(\omega)}{\check{V}_{in}(\omega)}$$

**Frequency domain technique**

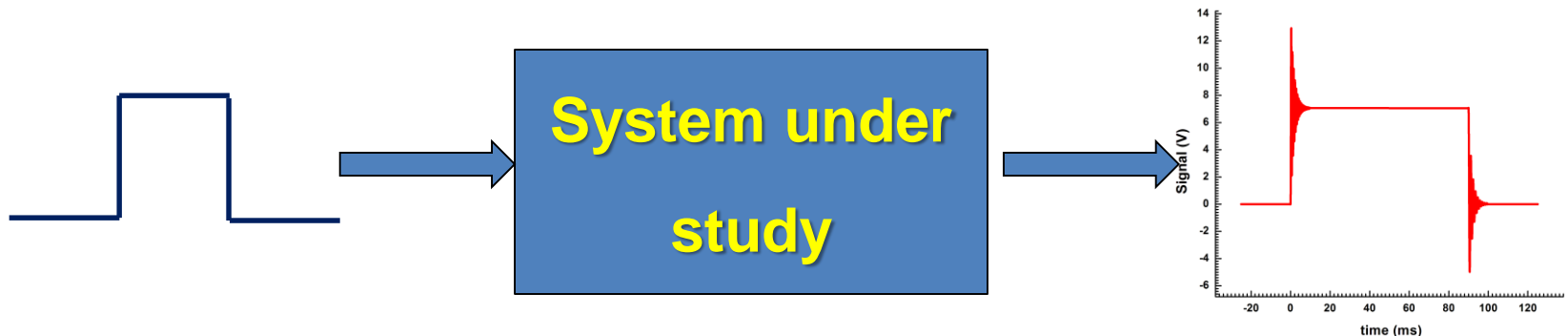
**Time domain technique**



# Time Domain Technique. Transients. Definition.

*transient ( physics ) a short-lived oscillation in a system caused by a sudden change of voltage or current or load*

*a transient response or natural response is the response of a system to a change from equilibrium.*

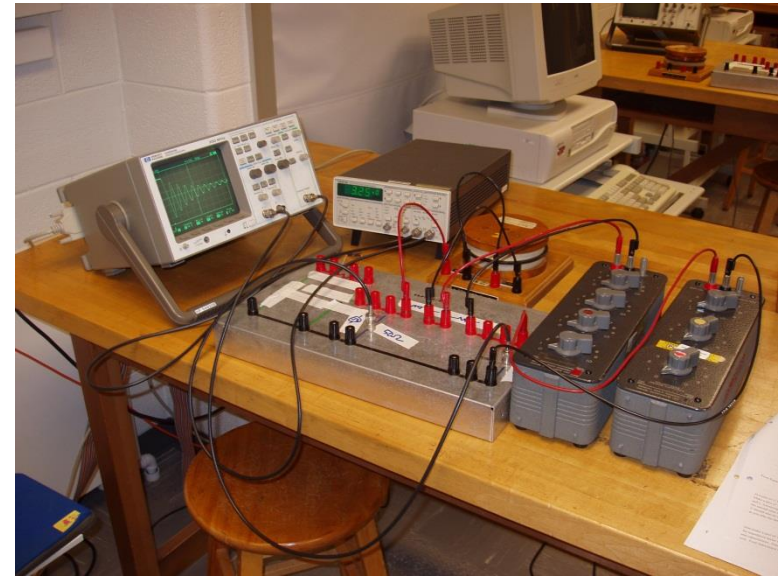
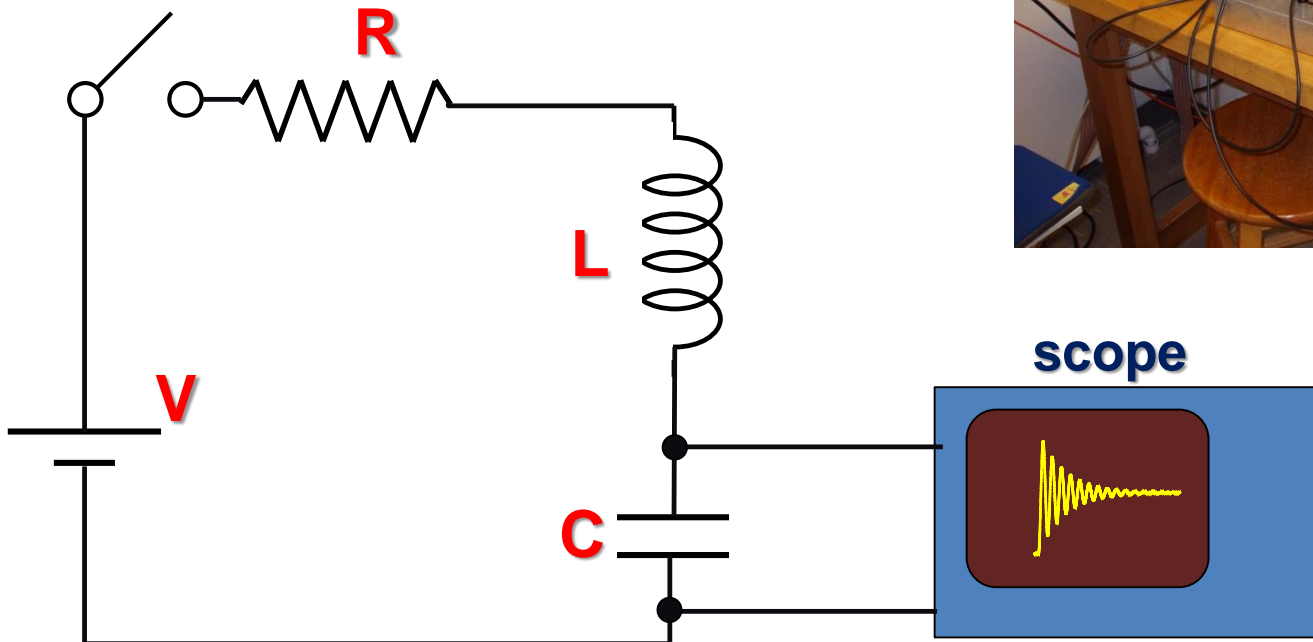


# Transients in RLC Circuit.

**Resistance**  $R$  [Ohm]

**Capacitance**  $C$  [ $\mu\text{F}$ ] ( $10^{-6}\text{F}$ )

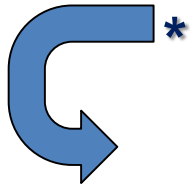
**Inductance**  $L$  [mH] ( $10^{-3}\text{H}$ )



# Transients in RLC Circuit.

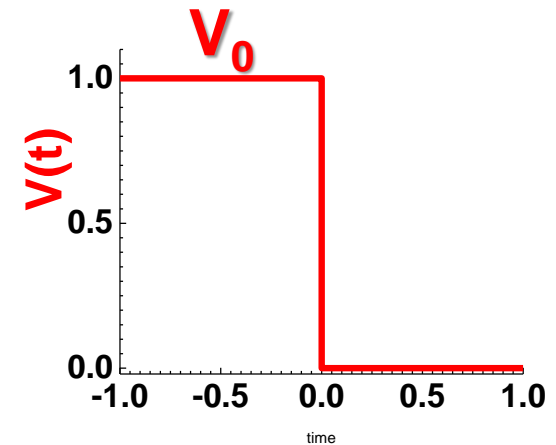
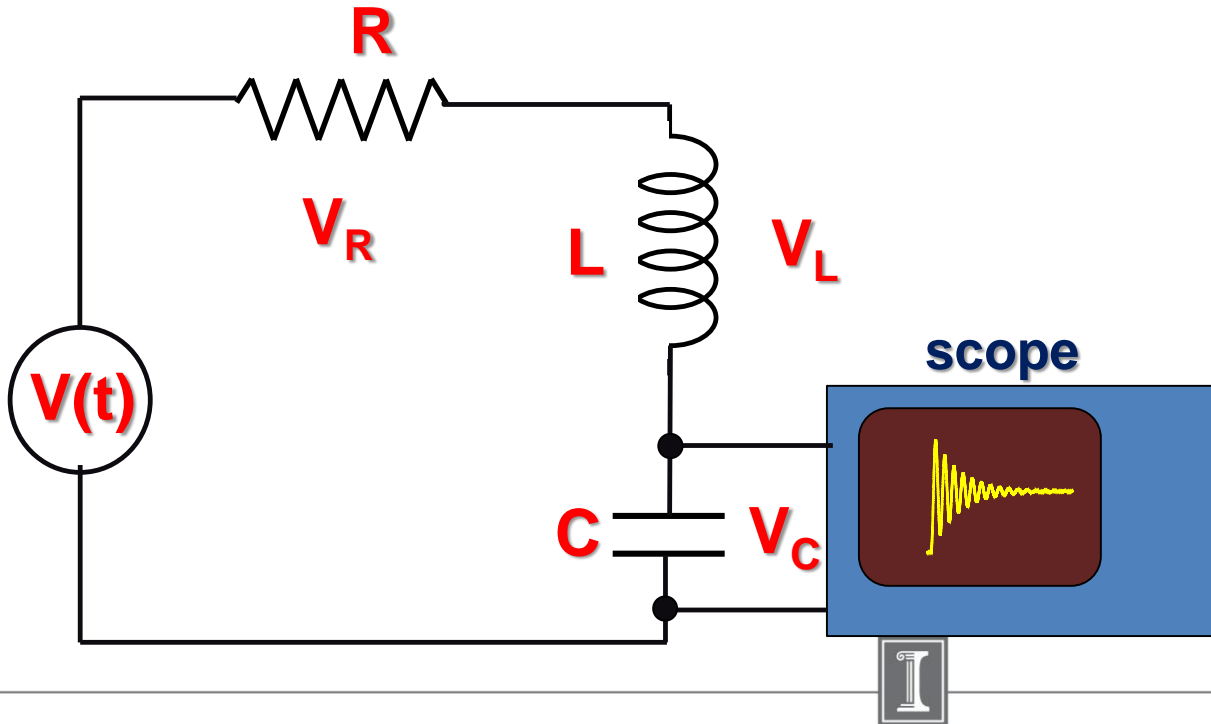
According to Kirchhoff's law

$$V_R + V_L + V_C = V(t)$$



$$L \frac{d^2 q(t)}{dt^2} + R \frac{dq(t)}{dt} + \frac{q(t)}{C} = 0 \quad (1)$$

\*See Lab write-up for details



# Transients in RLC Circuit. Three Solutions.

The solution of this differential equation can be found in the form

$$q(t) = Ae^{st}$$

This will convert (1) in quadratic equation

$$s^2 + \left(\frac{R}{L}\right)s + \frac{1}{LC} = 0$$

with solutions:

$$s_{1,2} = -\frac{R}{2L} \pm \sqrt{\left(\frac{R}{2L}\right)^2 - \left(\frac{1}{LC}\right)} \equiv -a \pm b$$

$$a = \frac{R}{2L}, \quad b = \sqrt{\left(\frac{R}{2L}\right)^2 - \left(\frac{1}{LC}\right)}$$

$b^2 > 0$  over-damped solution  
 $b^2 = 0$  critically damped solution  
 $b^2 < 0$  under-damped solution



# Transients in RLC Circuit. Over-damped Solution:

$$b^2 > 0$$

In this case the solution will be aperiodic exponential decay function with no

oscillations:

$$q(t) = e^{-at} (A_1 e^{bt} + B_1 e^{-bt})$$

$$i(t) = \frac{dq}{dt} = -ae^{-at} (A_1 e^{bt} + B_1 e^{-bt}) + be^{-at} (A_1 e^{bt} - B_1 e^{-bt})$$



# Transients in RLC Circuit. Over-damped Solution:

$$b^2 > 0$$

Taken in account the initial conditions:  $q(0)=q_0$  and  $i(0)=0$

$$q(t) = q_0 e^{-at} \left( \cosh bt + \frac{a}{b} \sinh bt \right)$$

$$\xrightarrow{(a-b)t \gg 1} \frac{q_0}{2} \left( 1 + \frac{a}{b} \right) e^{-(a-b)t}$$

$$i(t) = -\frac{q_0}{2} \left( \frac{a^2 - b^2}{b} \right) e^{-(a-b)t}$$

This is exponential decay function



# Transients in RLC Circuit. Critically-damped Solution: $b^2=0$

For this case the general solution can be found as

$q(t)=(A_2+B_2t)e^{-at}$ . Applying the same initial condition the

current can be written as  $i=-a^2q_0te^{-at}$

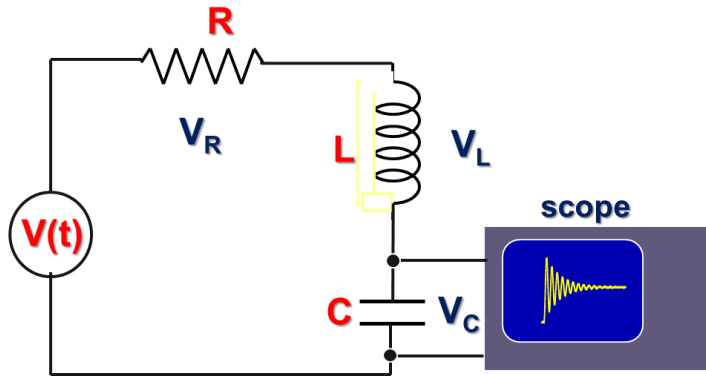
$$b^2 = 0 \rightarrow R^2 = \frac{4L}{C} \quad \text{and} \quad a = \frac{R}{2L}$$

Critically-damped  
conditions for our  
network

**Critical damped case shows the  
fastest decay with no oscillations**



# Transients in RLC Circuit. Critically-damped Solution: $b^2=0$ . Real Data Analysis.



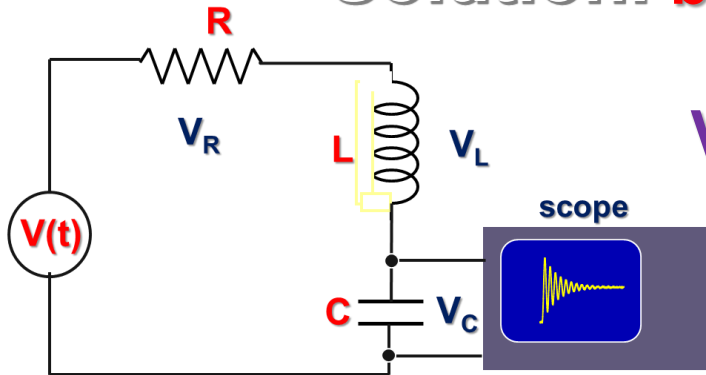
In this experiment  $R=300$  ohms,  $C=1\mu\text{F}$ ,  $L=33.43\text{mH}$ .

The output resistance of Wavetek is 50 ohms and resistance of coil was measured as 8.7 ohms, so actual resistance of the network is  $R_a=300+50+8.7=358.7$

Decay coefficient 
$$a = \frac{R}{2L} = \frac{358.7}{2 * 33.43E-3} \approx 5365$$



# Transients in RLC Circuit. Critically-damped Solution: $b^2=0$ . Real Data Analysis.



$V_C \sim q$ , fitting function:  $V_C = V_{co}(1+at)e^{-at}$   
Now the experimental results:

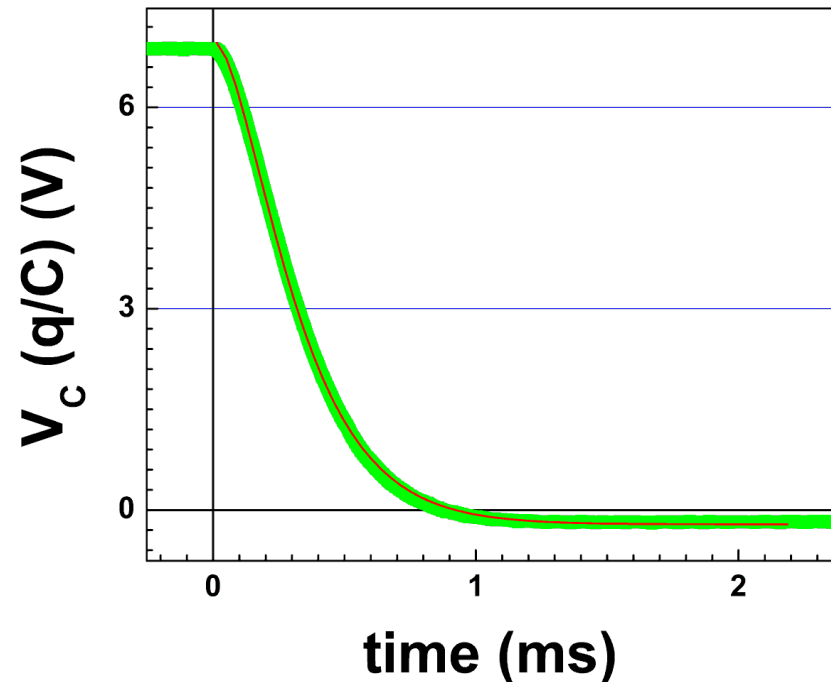
Calculated decay coefficient  
 $\sim 5385$ ,

Obtained from fitting -  $\sim 5820$ .

Possible reason – it is still  
slightly over damped

Calculated  $b^2$  is

$b^2 = 2.99e7 - 2.90e7 > 0$



# Transients in RLC Circuit. Under-damped Solution.

If  $b^2 < 0$  we will have oscillating solution. Omitting the details (see Lab write-up) we have the equations for charge and current as:

$$q(t) = q_0 e^{-at} \left( \cos bt + \frac{a}{b} \sin bt \right) = q_0 e^{-at} \sqrt{1 + \frac{a^2}{b^2}} \sin(bt + \varphi)$$

$$i(t) = q_0 e^{-at} \left( \frac{a^2 + b^2}{b} \right) \sin bt$$

$$a = \frac{R}{2L}, \quad b = \sqrt{\left(\frac{R}{2L}\right)^2 - \left(\frac{1}{LC}\right)}; \quad f = \frac{1}{2\pi} \sqrt{\left(\frac{1}{LC}\right) - \left(\frac{R}{2L}\right)^2}$$



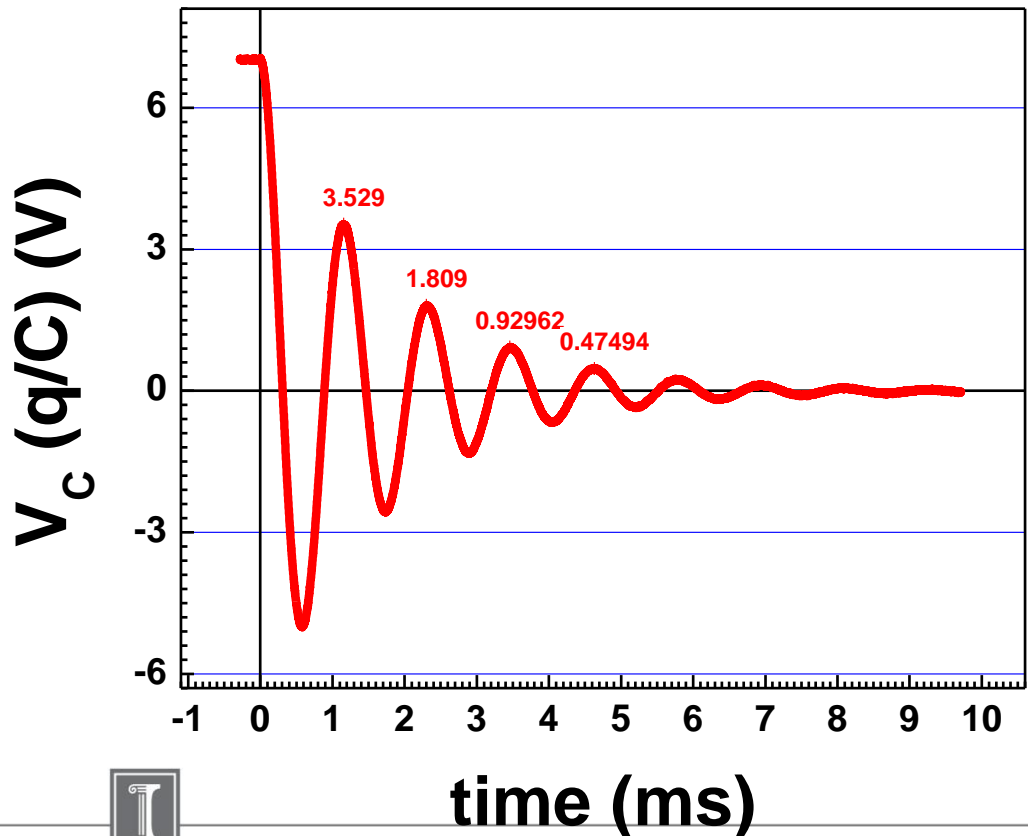
# Transients in RLC Circuit. Under-damped Solution. Log Decrement. Quality Factor.

Log decrement can be defined as  $\delta = \ln \left( \frac{q(t_{max})}{q(t_{max}+T_1)} \right) =$   
 $\ln \left( \frac{e^{-at_{max}}}{e^{-at_{max}+T_1}} \right) = aT_1$ , where  $T_1=1/f$

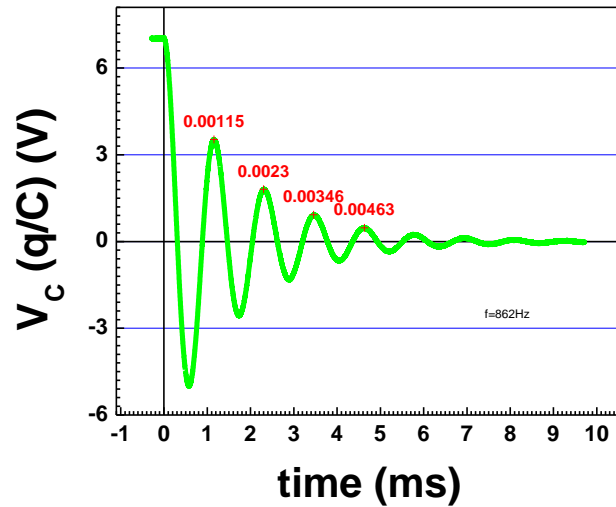
Quality factor can be defined as  $Q = 2\pi \frac{E}{\Delta E}$ ,

For RLC  $Q = \frac{\omega_1 L}{R} = \frac{\pi}{\delta}$

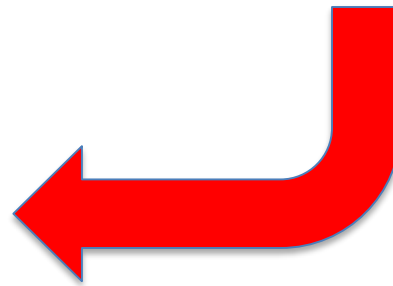
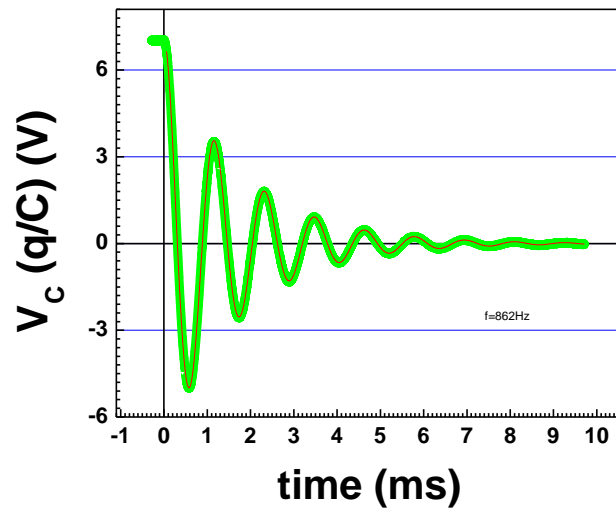
From this plot  $\delta \approx 0.67$   
 $Q \approx 4.7$



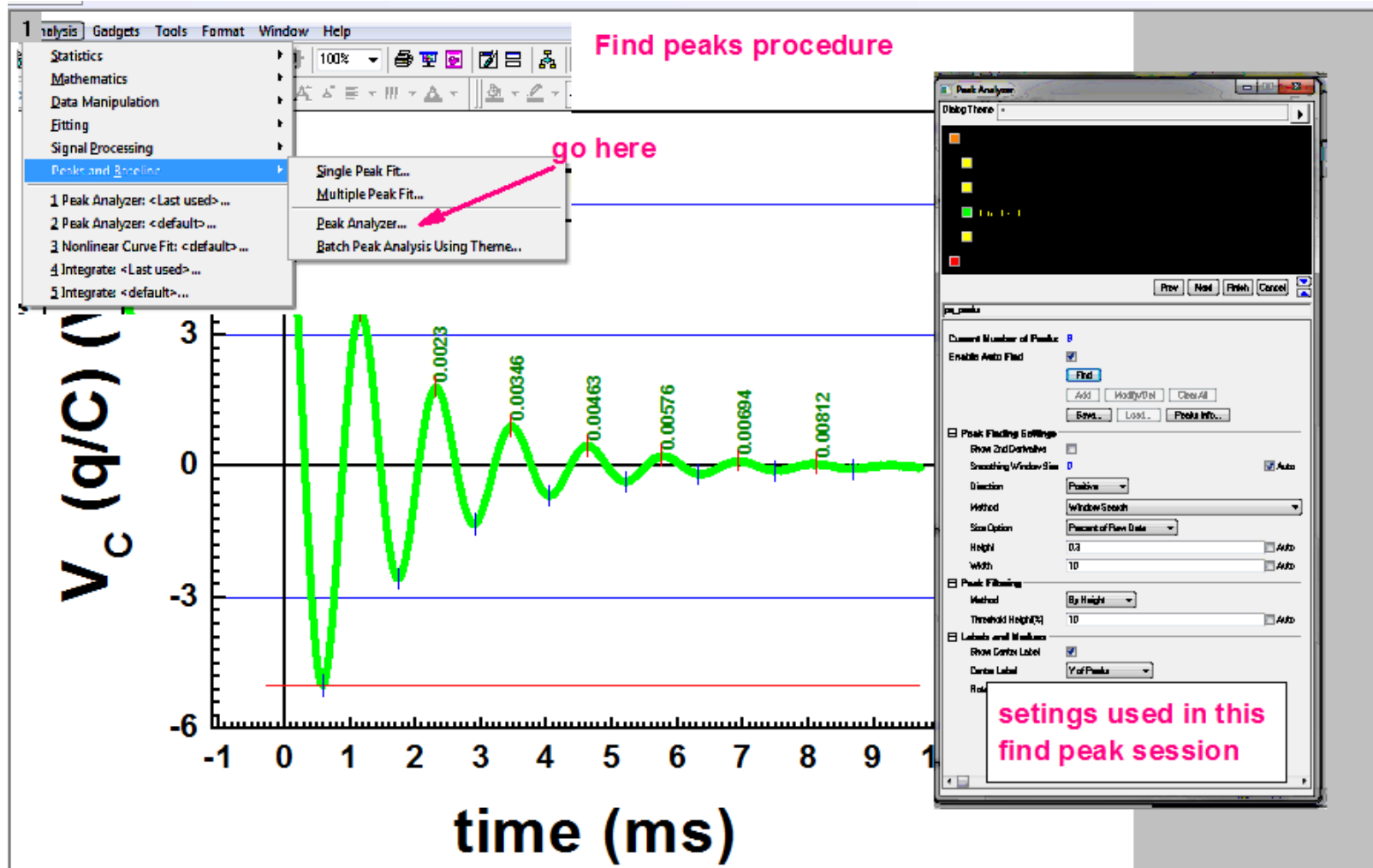
# Transients in RLC Circuit. Data Analysis. Using Origin Software.



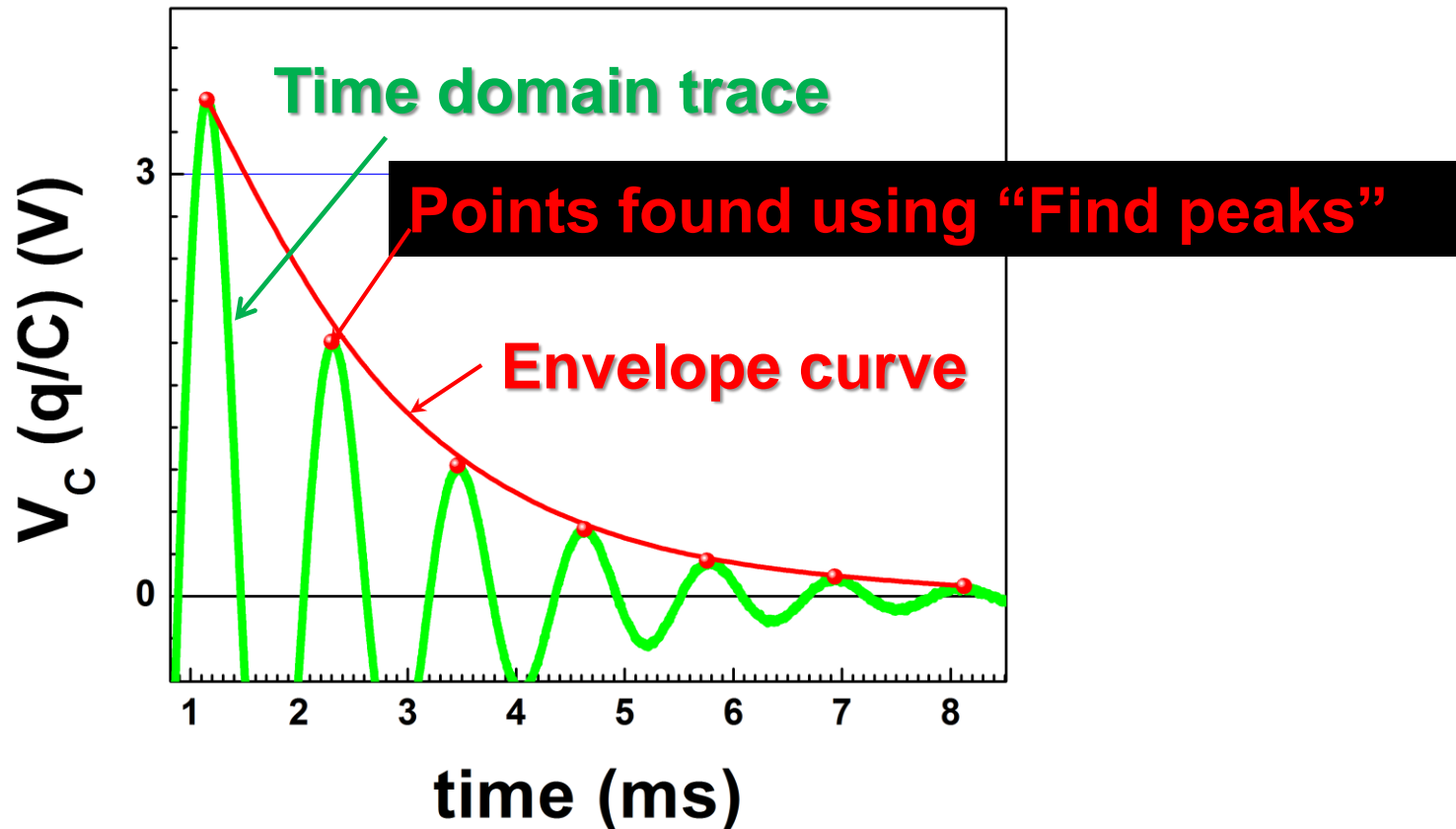
1. Pick peaks
2. Envelope
3. Nonlinear fitting



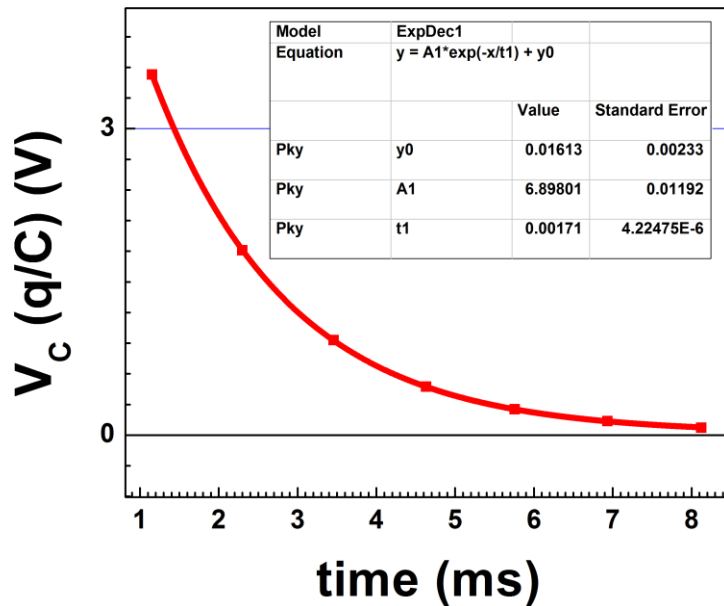
# Transients in RLC Circuit. Under-damped Solution. Log Decrement. Quality Factor.



# Transients in RLC Circuit. Data Analysis. Log Decrement. Using Origin Software. Results.



# Transients in RLC Circuit. Data Analysis. Log Decrement. **Using Origin Software.** Results.



The screenshot shows the NLFit (ExpDec1) dialog box in Origin Software. The 'Function Selection' tab is active, showing the 'ExpDec1' function selected. The 'Category' is 'Exponential' and the 'Function' is 'ExpDec1'. The 'Description' is 'Exponential Decay 1' and the 'File Name(.FDF)' is 'C:\Program Files\OriginLab\Origin\fitfunc\expdec1.fdf'. The 'Fit' button is highlighted.

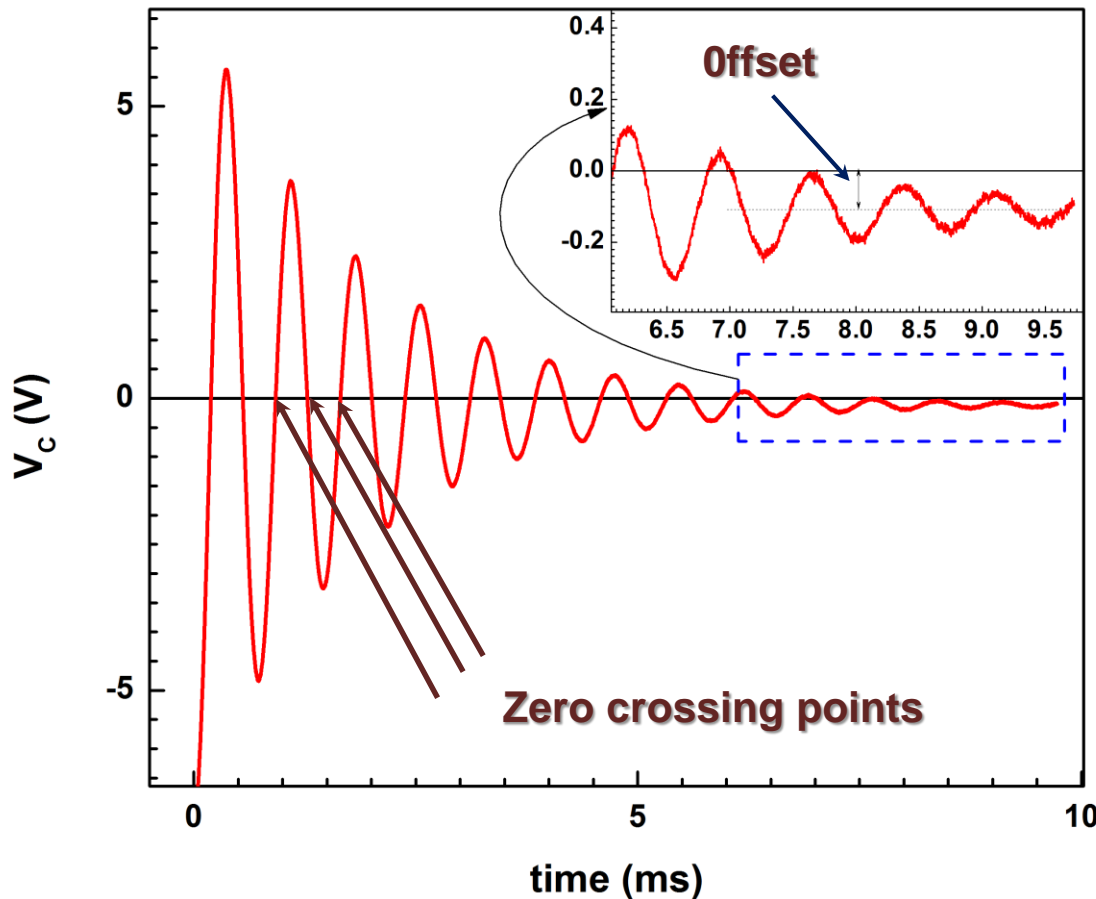
$$y = y_0 + Ae^{-x/t}$$

**Fitting the “envelope data” to exponential decay function**



# Transients in RLC Circuit. Data Analysis. (1/T)<sup>2</sup> vs 1/C Experiment.

$$q(t) = Ae^{-at} \sin(\omega t + \varphi) + \text{offset}$$



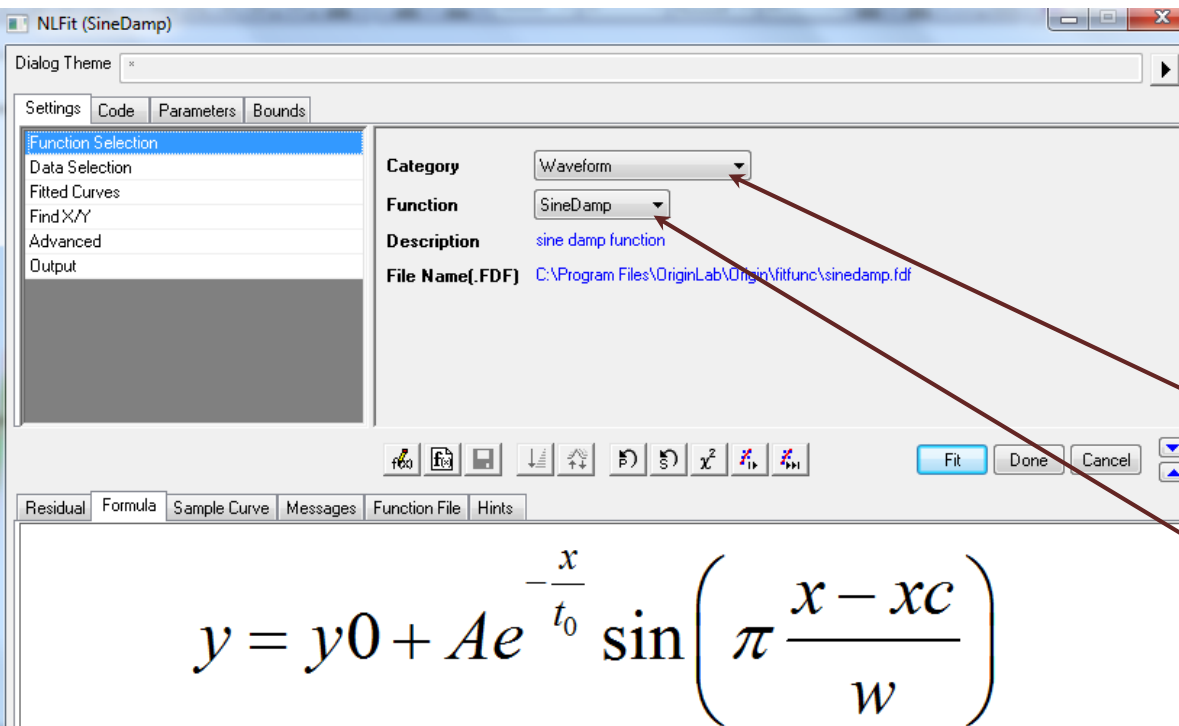
Manual evaluation of the period of the oscillations

Limited accuracy

Results can be effected by DC offset



# Transients in RLC Circuit. Data Analysis. (1/T)<sup>2</sup> vs 1/C Experiment. **Using Origin Software.**



$$q(t) = Ae^{-at} \sin(\omega t + \varphi)$$

**Use Origin standard function**

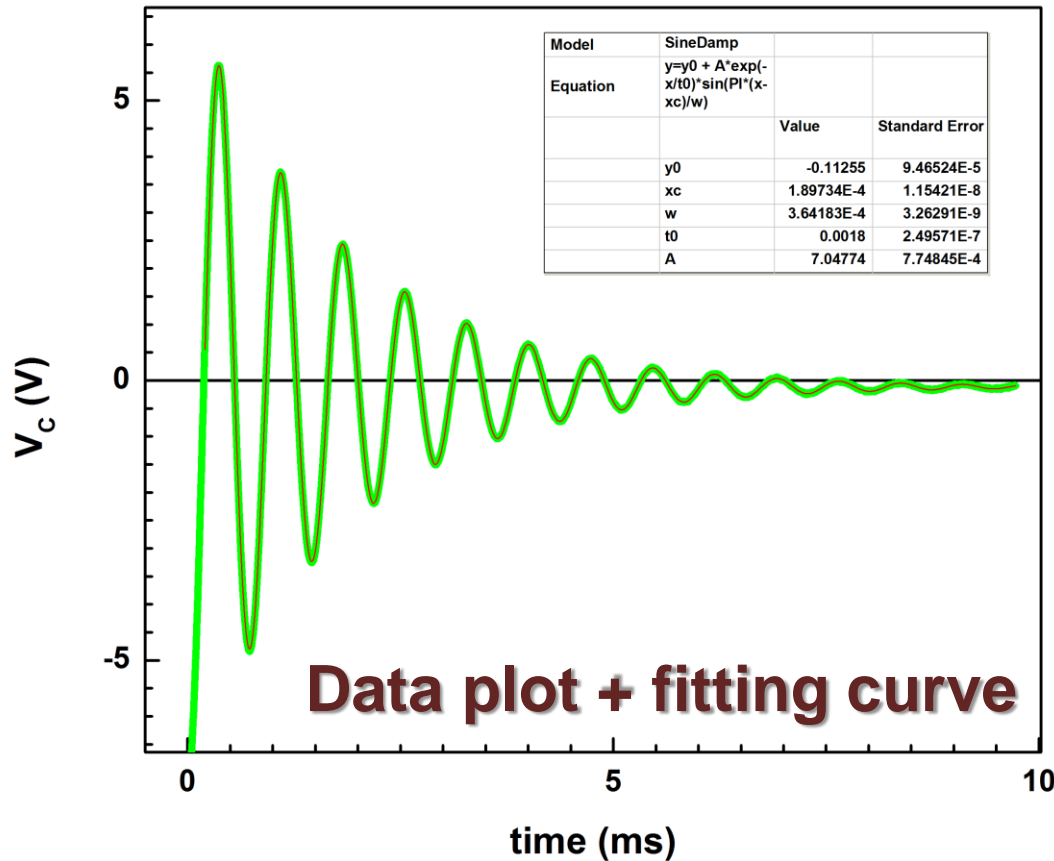
**Category: Waveform  
Function: SineDamp**

**Fitting function ;  $y_0, A, t_0, x_c, w$  – fitting parameters**

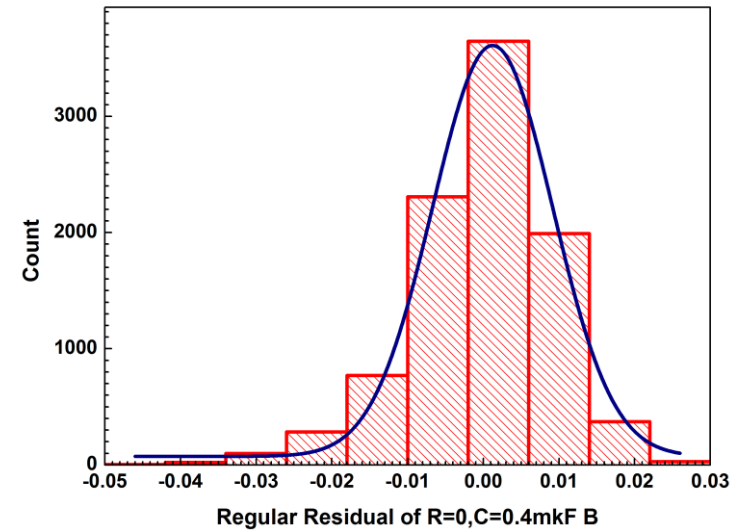


# Transients in RLC Circuit. Data Analysis.

## $(1/T)^2$ vs $1/C$ Experiment. Using Origin Software.

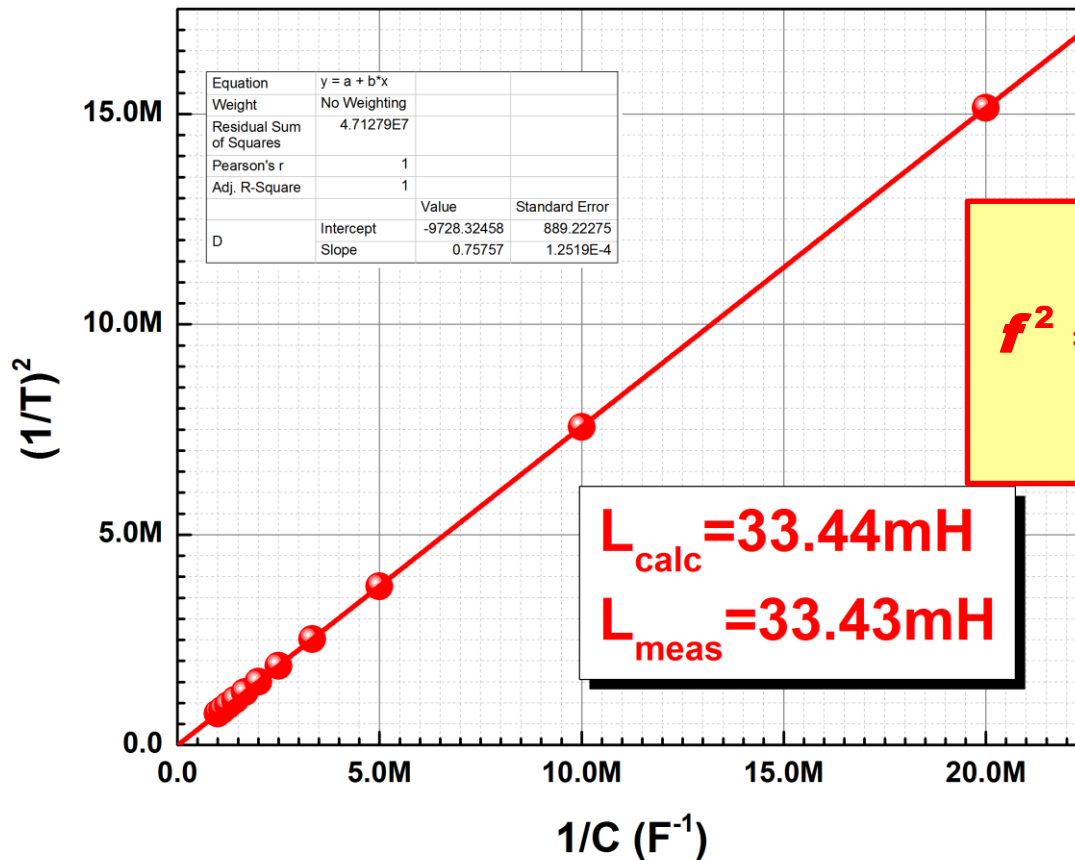


$$q(t) = Ae^{-at} \sin(\omega t + \varphi)$$



# Transients in RLC Circuit. Data Analysis.

## $(1/T)^2$ vs $1/C$ Experiment. Using Origin Software.



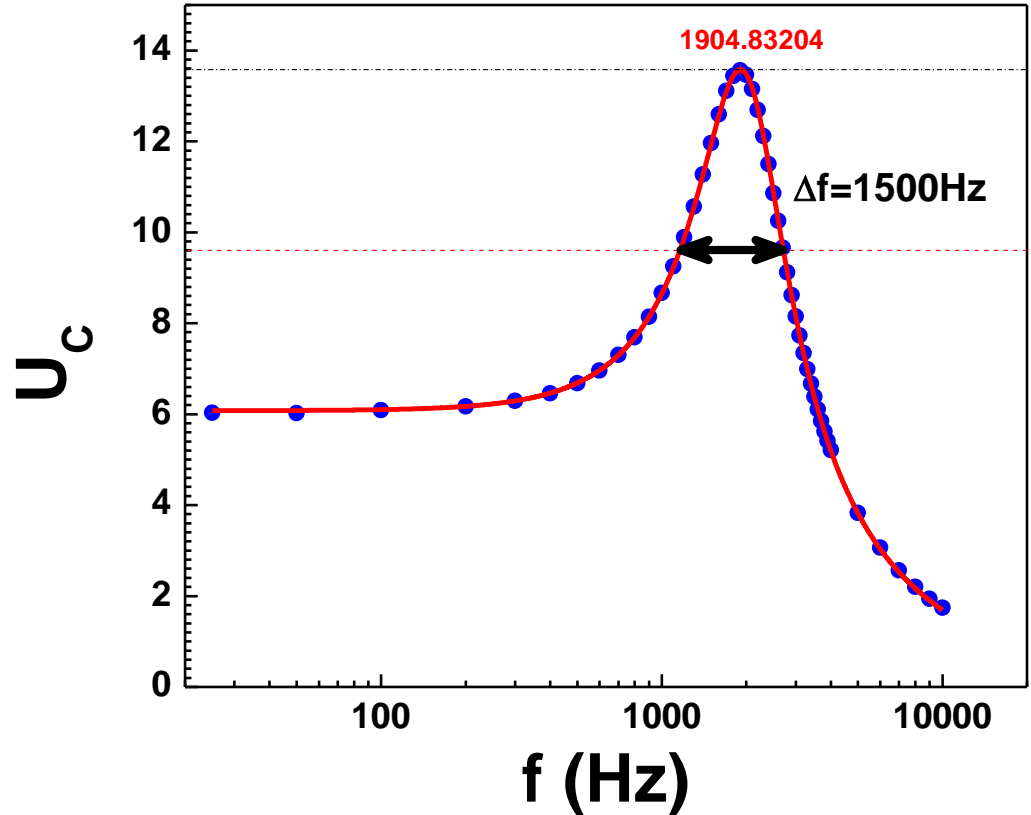
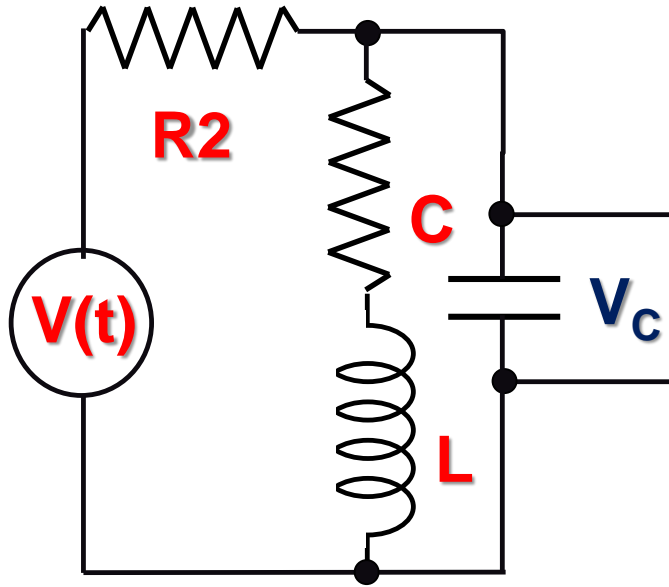
$$q(t) = Ae^{-at} \sin(\omega t + \varphi)$$

$$f^2 = \left(\frac{1}{T}\right)^2 = \frac{1}{2\pi} \left( \left(\frac{1}{LC}\right) - \left(\frac{R}{2L}\right)^2 \right)$$

**Final results**



# Resonance in RLC Circuit.



$$Q = \frac{f}{\Delta f} = \frac{1904}{1500} = 1.26$$



*now*

# Frequency Domain Analysis of Linear Circuits Using Synchronous Detection



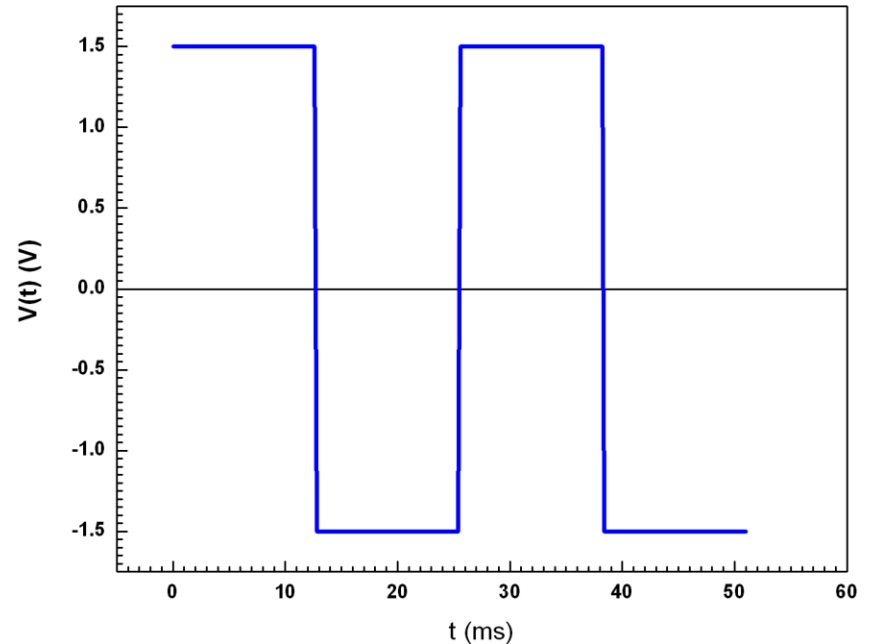
in 1822, Jean Baptiste Fourier developed the theory that shows that any real waveform can be represented by the sum of sinusoidal waves.

Let us try to create the square wave as a sum of sine waves of different frequencies

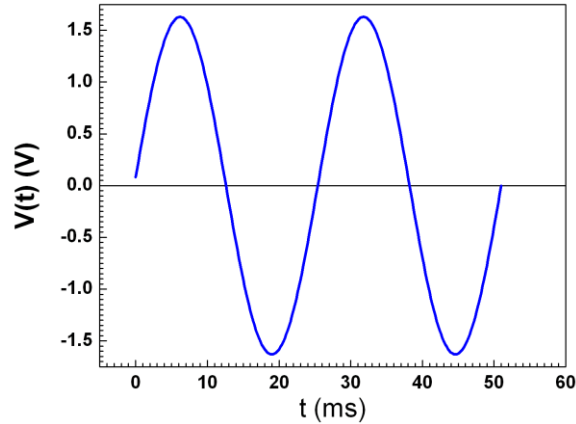


**Jean Baptiste Joseph  
Fourier  
(1768 – 1830)**

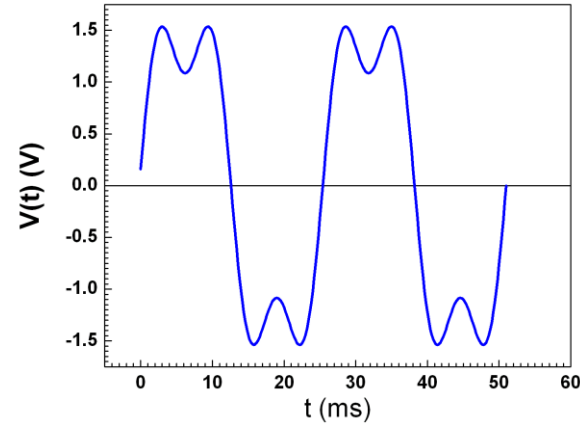
**Square wave.  
 $F=40\text{Hz}$ ,  $A=1.5\text{V}$**



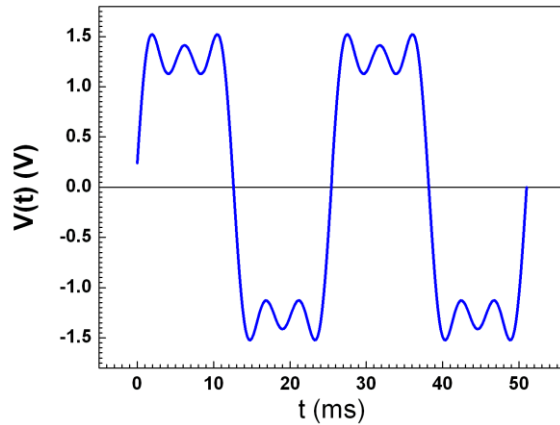
$$A_1 \sin(2\pi\omega t)$$



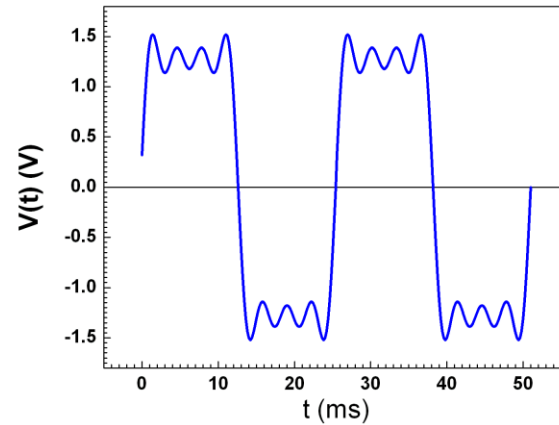
$$A_1 \sin(2\pi\omega t) + A_3 \sin(2\pi 3\omega t + \varphi_3)$$



$$A_1 \sin(2\pi\omega t) + A_3 \sin(2\pi 3\omega t + \varphi_3) + A_5 \sin(2\pi 5\omega t + \varphi_5)$$



$$A_1 \sin(2\pi\omega t) + A_3 \sin(2\pi 3\omega t + \varphi_3) + A_5 \sin(2\pi 5\omega t + \varphi_5) + A_7 \sin(2\pi 7\omega t + \varphi_7)$$



# Fourier Transform

The continuous Fourier transformation of the signal  $h(t)$  can be written as:

$$H(f) = \int_{-\infty}^{+\infty} h(t) e^{2\pi jft} dt; \quad j = \sqrt{-1}$$

$H(f)$  represents in frequency domain mode the time domain signal  $h(t)$

Equation for inverse Fourier transform gives the correspondence of the infinite continuous frequency spectra to the corresponding time domain signal.

$$h(t) = \int_{-\infty}^{+\infty} H(f) e^{-2\pi jft} df$$

In real life we working with discrete representation of the time domain signal recorded during a finite time.

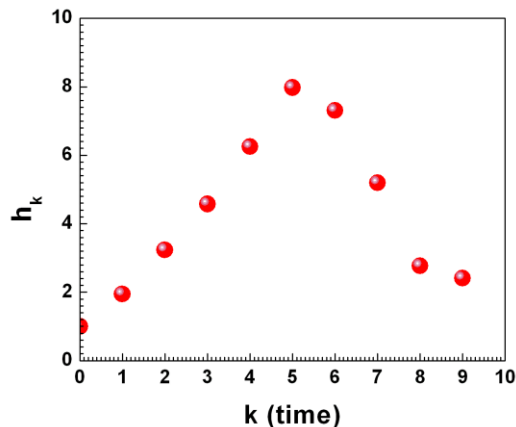


# Discrete Fourier Transform

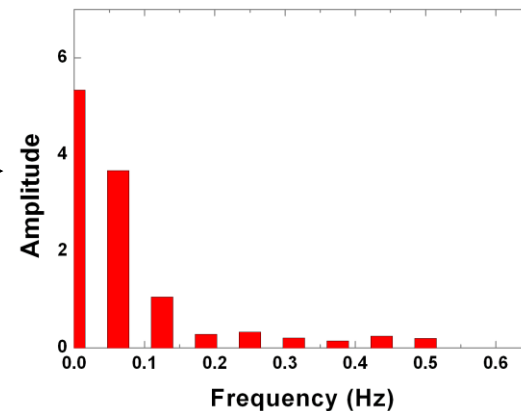
It comes out that in practice more useful is the representation the frequency domain pattern of the time domain signal  $h_k$  as sum of the frequency harmonic calculated as:

$$H_n = H(f_n) = \frac{1}{N} \sum_{k=0}^{N-1} h_k e^{2\pi kn/N}$$

$\Delta$  is the sampling interval,  $N$  – number of collected points



Time domain



Frequency domain



# Discrete Fourier Transform

For periodic signals with period  $T_0$ :

$$F(t) = \frac{a_0}{2} + \sum_{n=1}^{\infty} a_n \cos\left(\frac{2\pi n t}{T_0}\right) + \sum_{n=1}^{\infty} b_n \sin\left(\frac{2\pi n t}{T_0}\right)$$

$$a_n = \frac{2}{T_0} \int_0^{T_0} F(t) \cos\left(\frac{2\pi n t}{T_0}\right) dt; \quad b_n = \frac{2}{T_0} \int_0^{T_0} F(t) \sin\left(\frac{2\pi n t}{T_0}\right) dt;$$

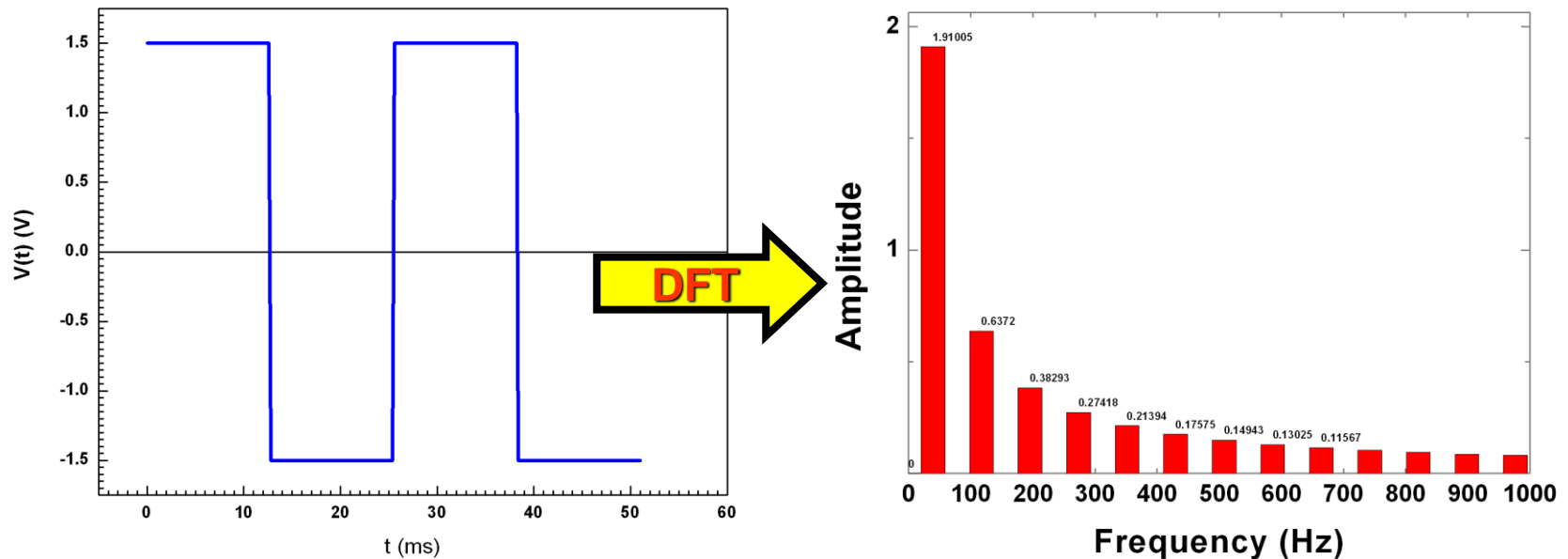
$$a_0 = \frac{2}{T_0} \int_0^{T_0} F(t) dt;$$



# Discrete Fourier Transform

Now how I found the amplitudes of the harmonics to compose the square wave signal from sine waves of different frequencies.

## Time domain signal



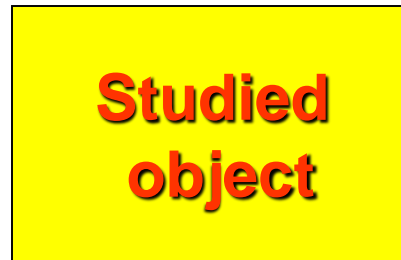
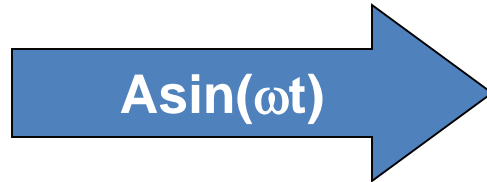
Decomposition the signal into the sine wave harmonics. The only modulus's of the harmonics amplitudes are presented in this picture.



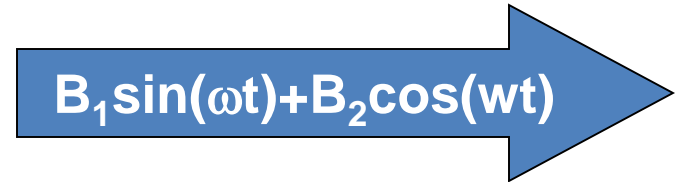
# Frequency Domain Spectroscopy

(linear system)

Applied test signal



Response of the studied system

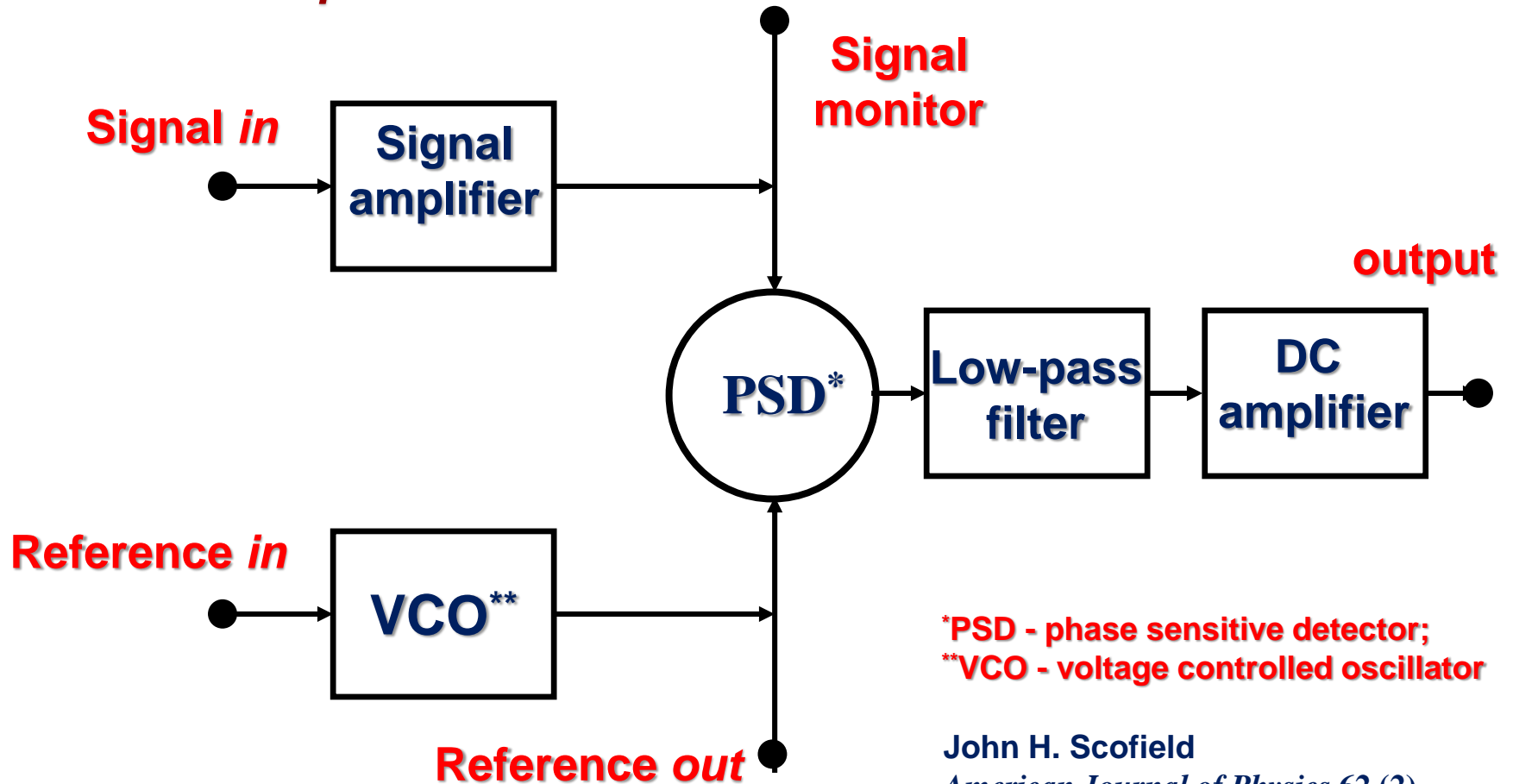


We applying the sine wave signal to the tested object and measuring the response. Varying the frequency we can study the frequency properties of the system.



# Lock-in amplifier

*Now about the most powerful tool which can be used in frequency domain technique.*

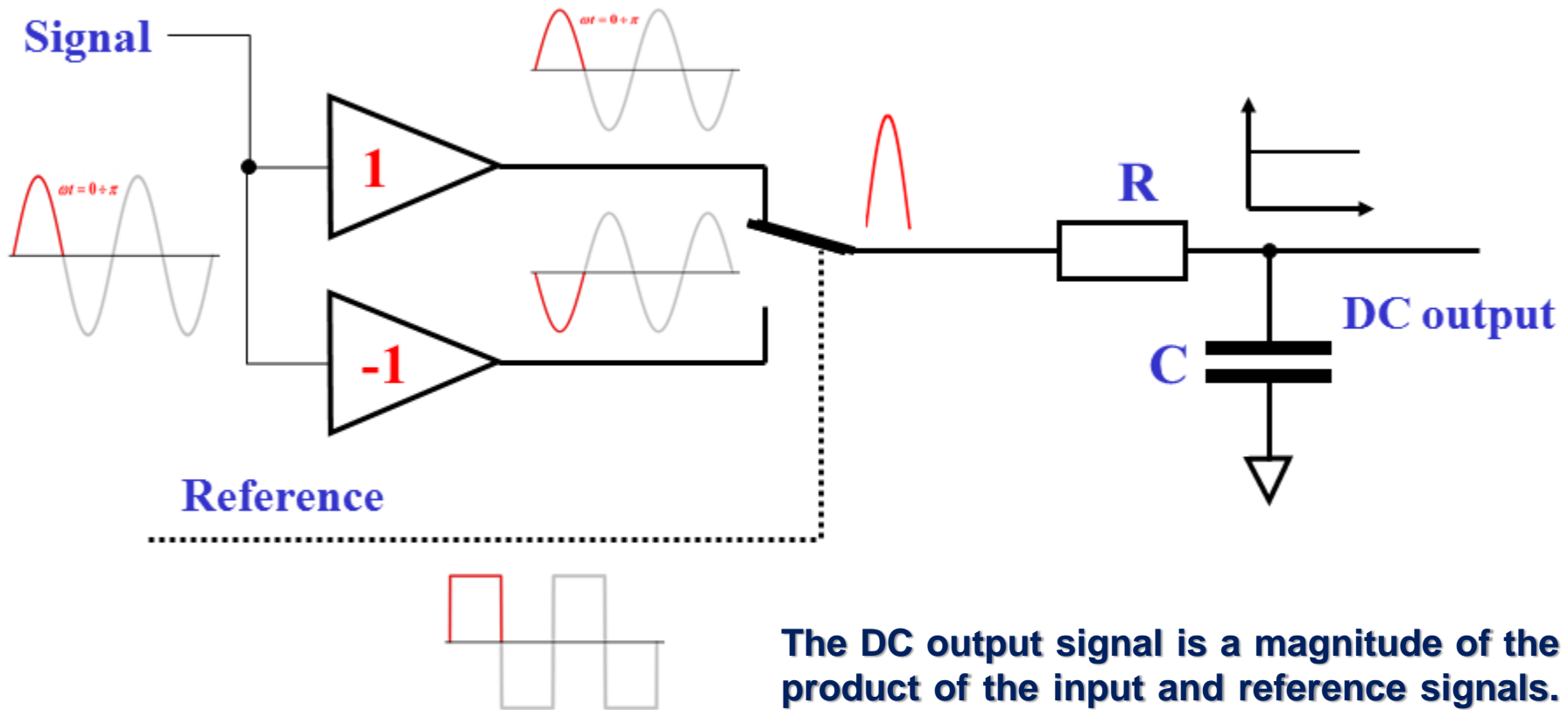


\*PSD - phase sensitive detector;  
\*\*VCO - voltage controlled oscillator

John H. Scofield  
*American Journal of Physics* 62 (2)  
129-133 (Feb. 1994).



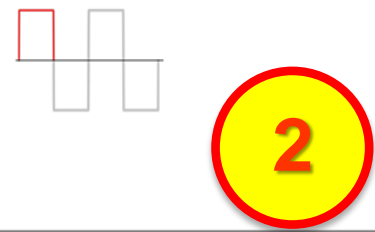
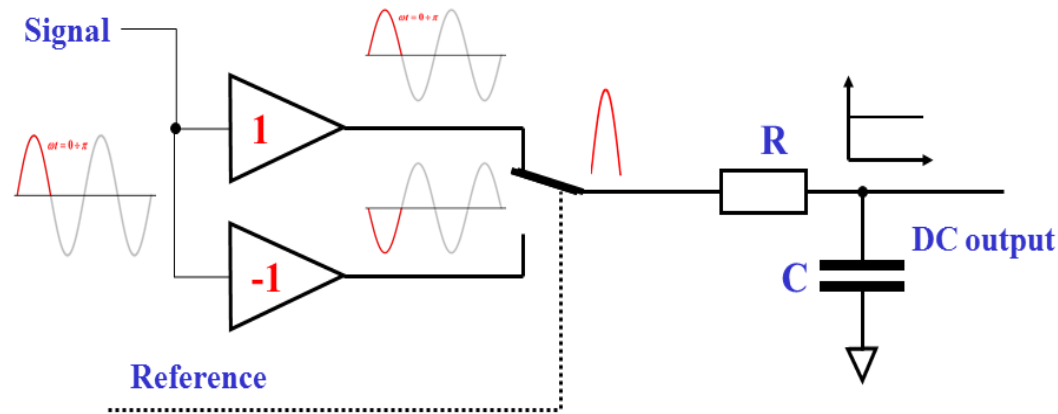
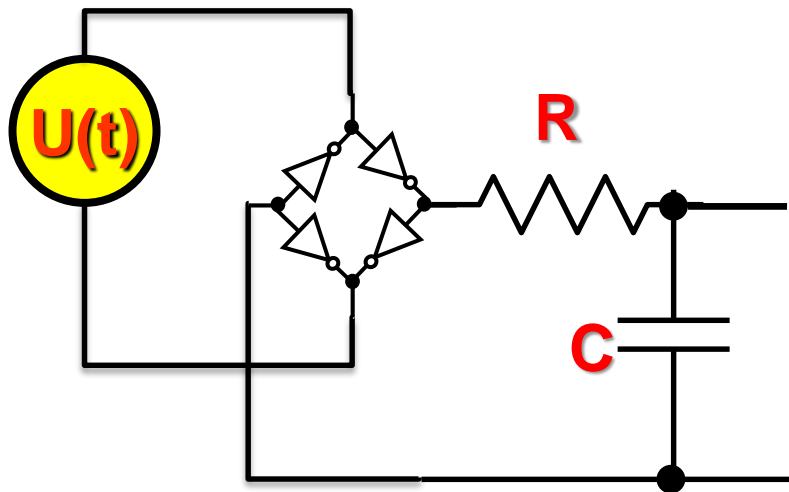
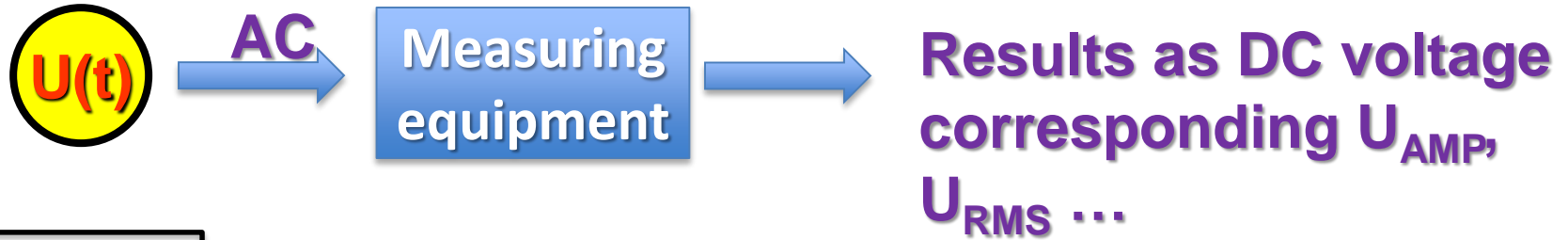
# Lock-in amplifier. How it works.



The DC output signal is a magnitude of the product of the input and reference signals. AC components of output signal are filtered out by the low-pass filter with time constant  $\tau$  (her  $\tau=RC$ )

# Lock-in Amplifier. What is the Advantage of Using Synchronous Detecting?

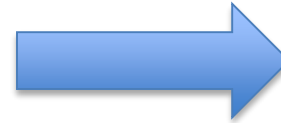
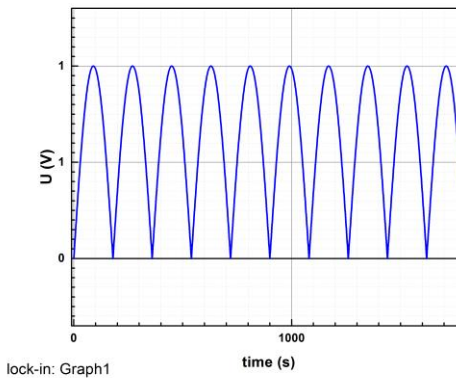
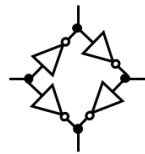
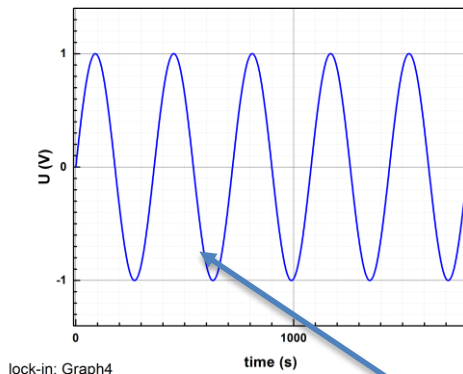
**DMM, lock-in etc.**



# Lock-in Amplifier. What is the Advantage of Using Synchronous Detecting?

1

Clean sine wave – no “noise”



$$U_{DC} = 0.63643V$$

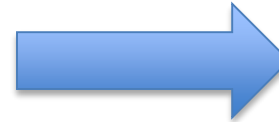
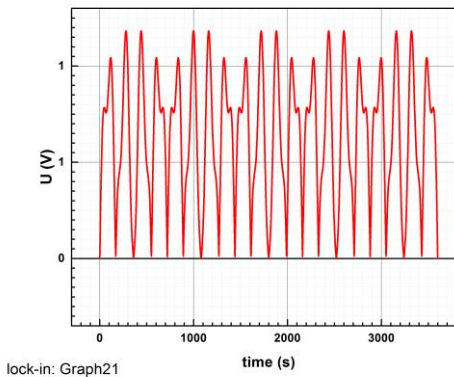
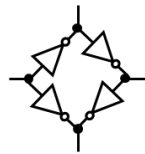
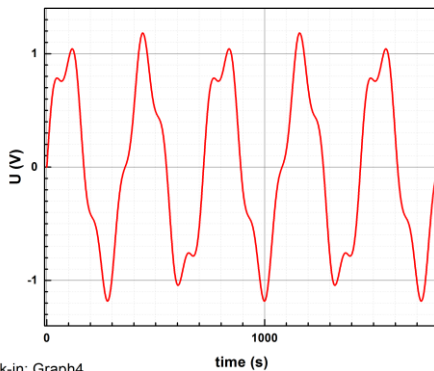
We need to measure the amplitude/rms value of the sine wave



# Lock-in Amplifier. What is the Advantage of Using Synchronous Detecting?

1

“Noisy” sine wave



$U_{DC} = 0.64208V$   
compare to

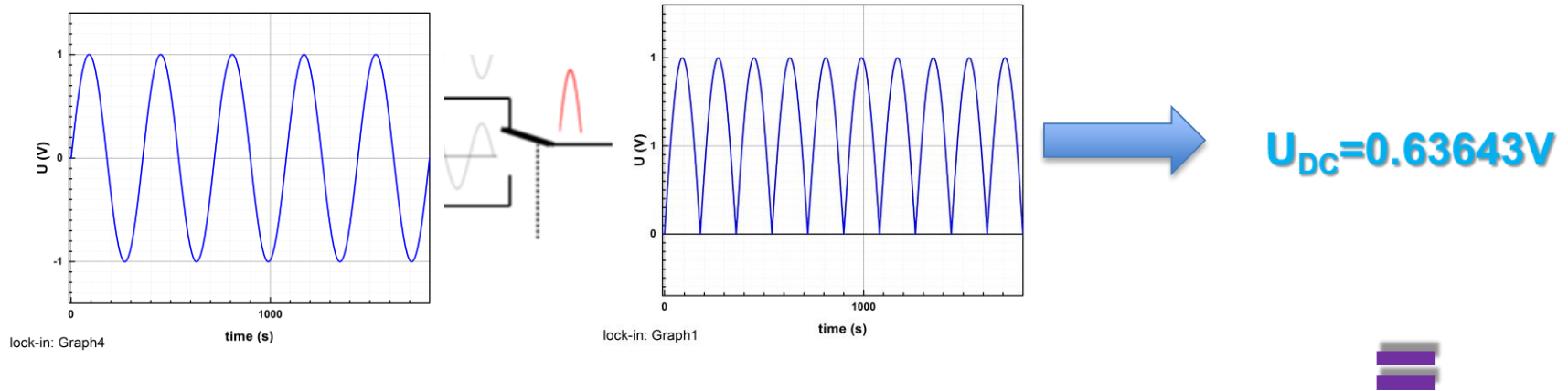
$U_{DC} = 0.63643V$



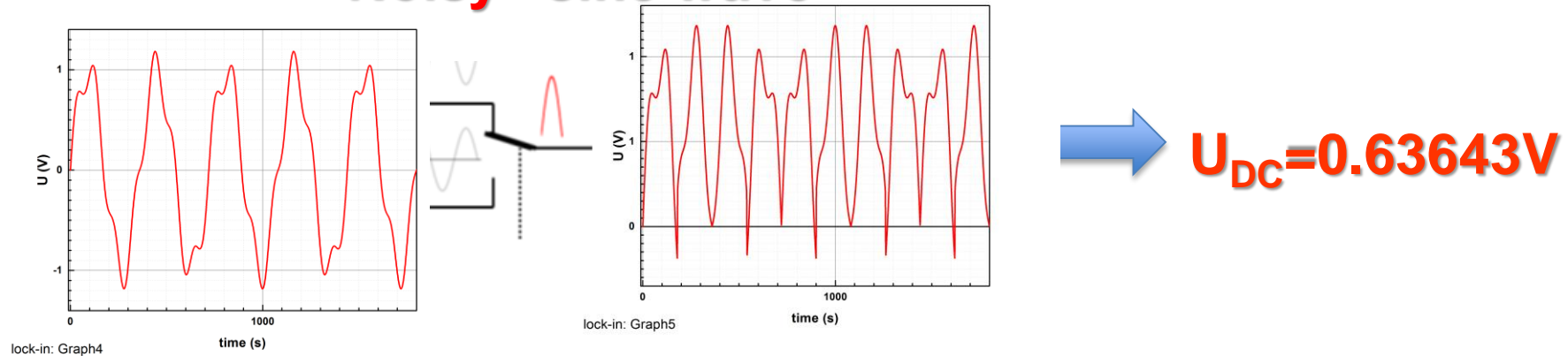
# Lock-in Amplifier. What is the Advantage of Using Synchronous Detecting?

2

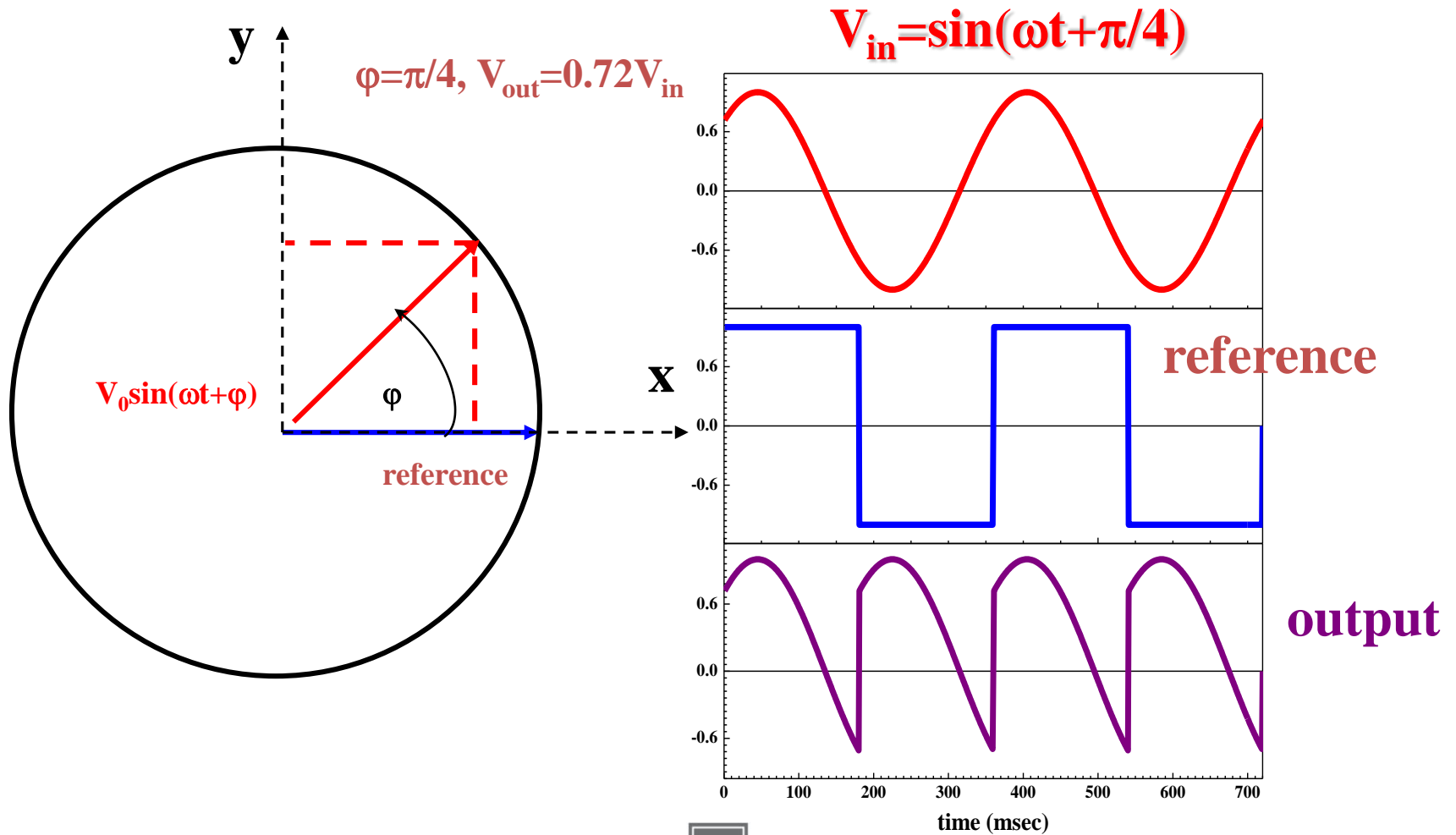
Clear sine wave – no “noise”



“Noisy” sine wave

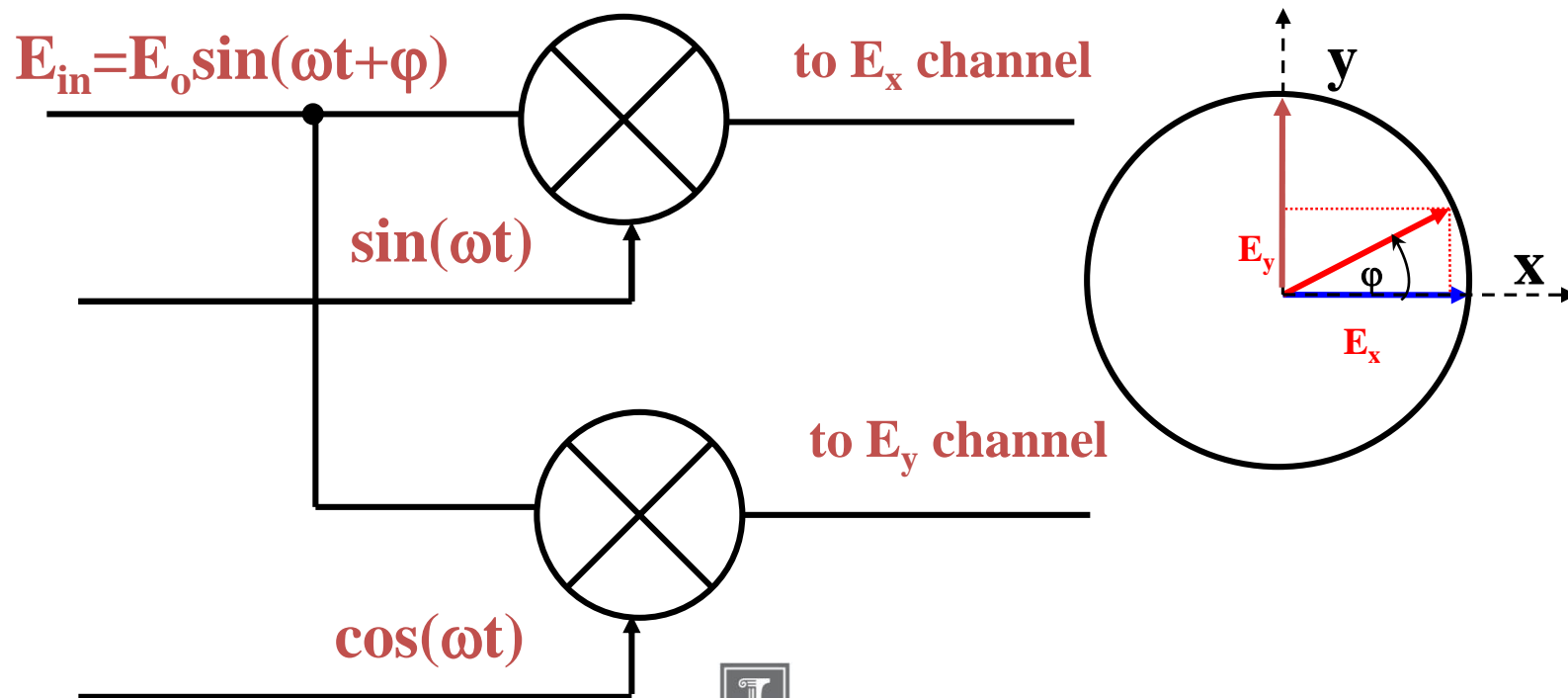


# Lock-in Amplifier. Phase shift.

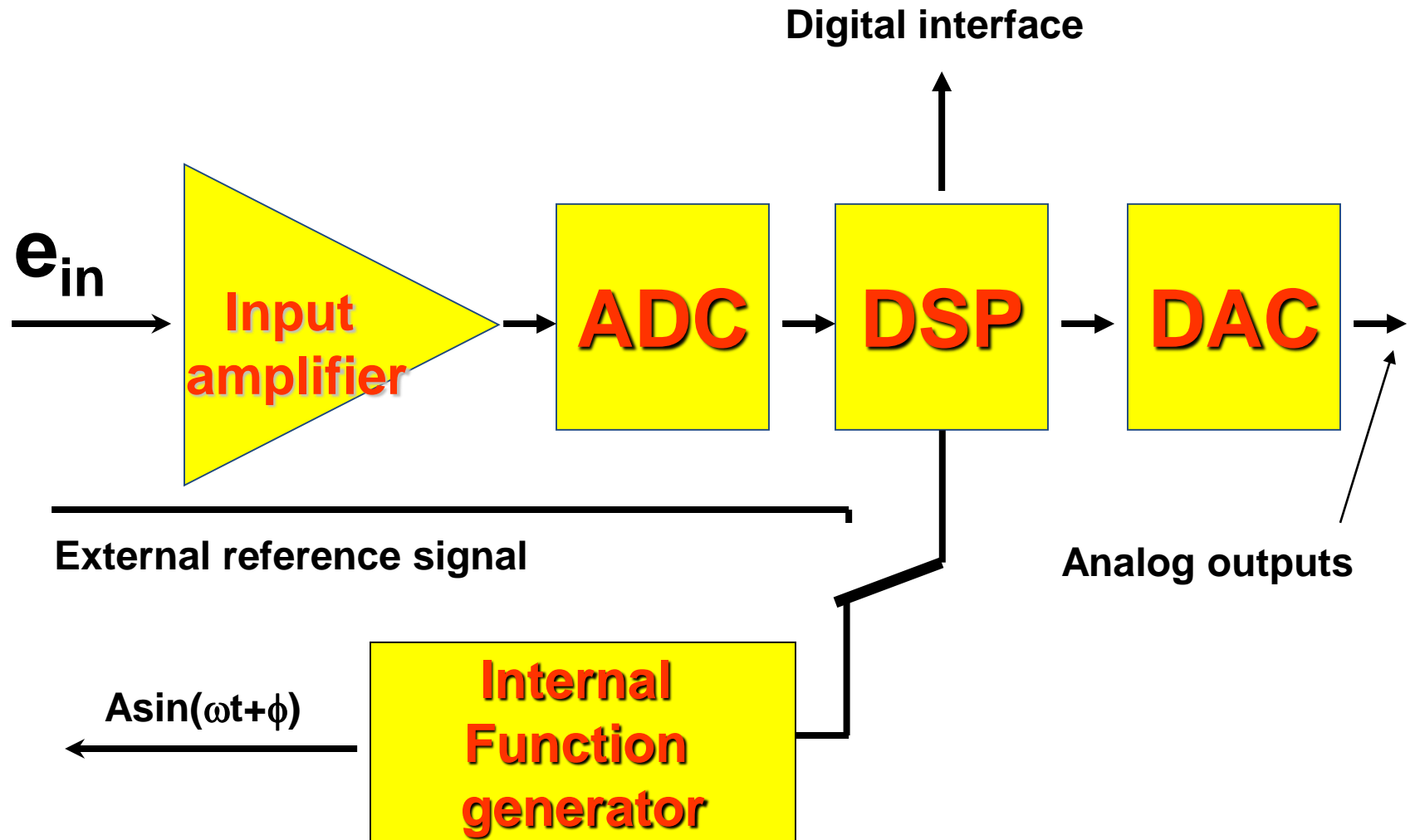


# Lock-in Amplifier. Two Channels Demodulation.

In many scientific applications it is a great advantage to measure both components ( $E_x$ ,  $E_y$ ) of the input signal. We can use two lock-ins to do this or we can measure these value in two steps providing the phase shift of reference signal  $0$  and  $\pi/2$ . Much better solution is to use the lock-in amplifier equipped by two demodulators.



# Digital Lock-in Amplifier



# SR830. Digital Lock-in Amplifier



In SR830 manual you can find the chapter dedicated to general description of the lock-in amplifier idea

## SR830 BASICS

### WHAT IS A LOCK-IN AMPLIFIER?

Lock-in amplifiers are used to detect and measure very small AC signals - all the way down to a few nanovolts! Accurate measurements may be made even when the small signal is obscured by noise

experiment at the reference frequency. In the diagram below, the reference signal is a square wave at frequency  $\omega_r$ . This might be the sync output from a function generator. If the sine output from

[\\engr-file-03\PHYINST\APL Courses\PHYCS401\Common\EquipmentManuals](#)



# Digital Lock-in amplifier. SR830

Time constant

And output filter sensitivity

Auto functions

Channel#1

Channel#2



Inputs

Notch filter settings

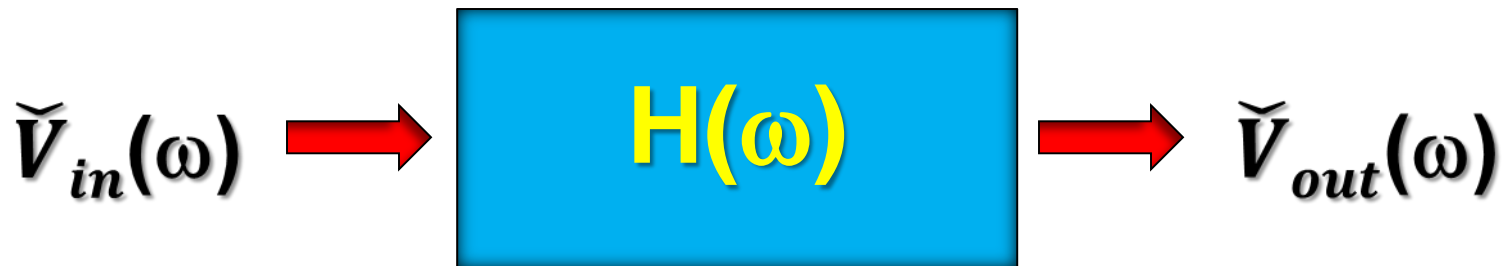
Analog outputs

Interface settings

Function generator



# Experiments. Main idea. Investigating the frequency response of circuit.



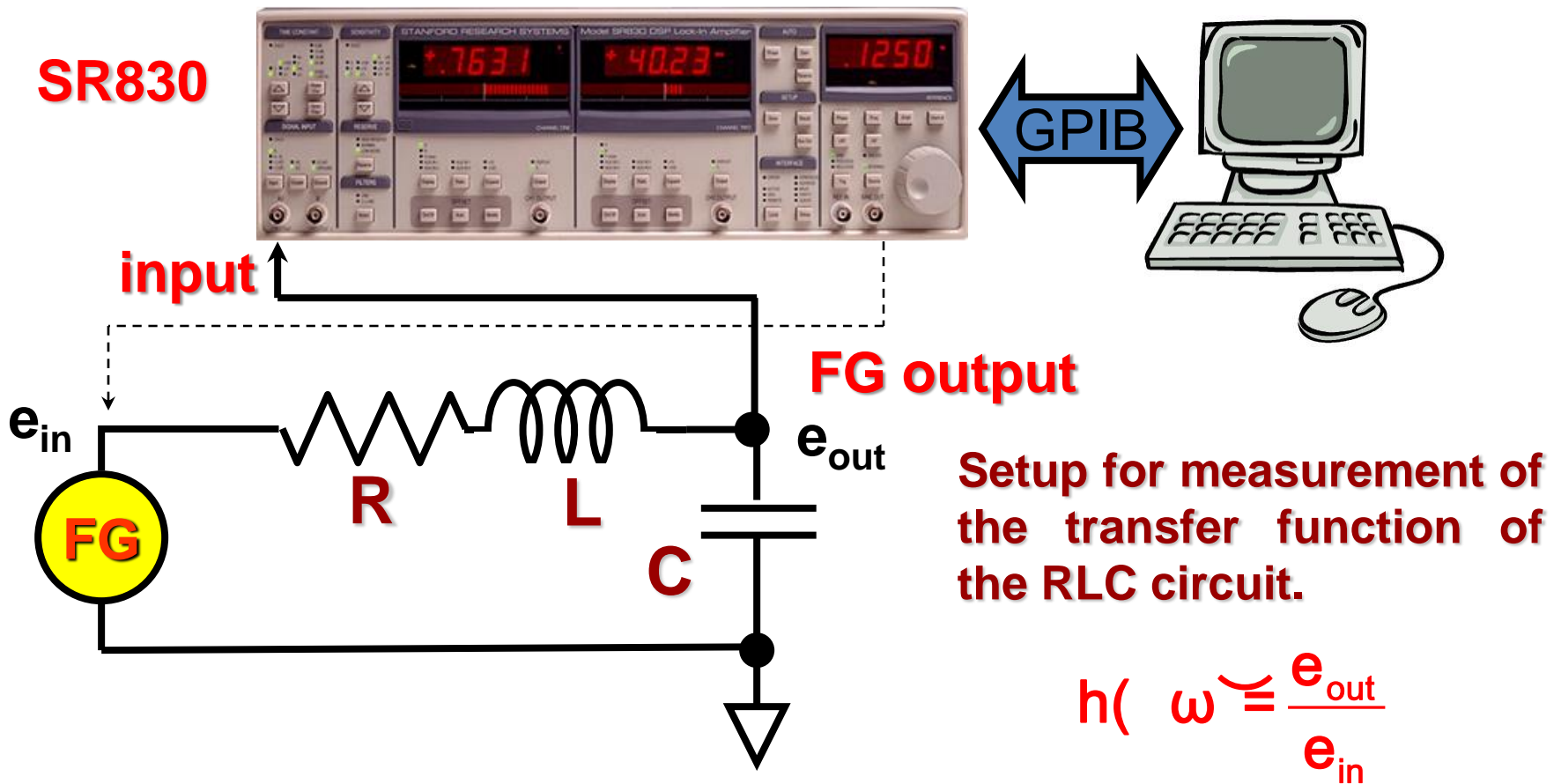
Frequency domain representation of the system

Response function  $\rightarrow \check{H}(\omega) = \frac{\check{V}_{out}(\omega)}{\check{V}_{in}(\omega)}$

Linear systems are those that can be modeled by linear differential equations.



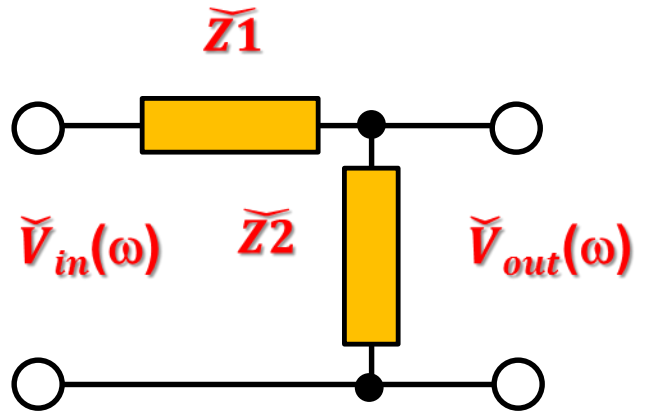
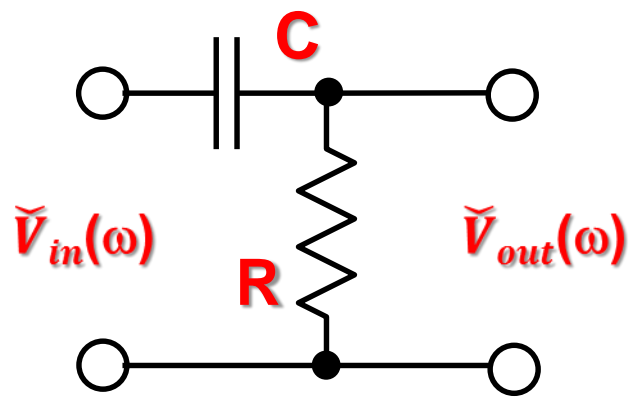
# Application of the Lock-in Amplifier for Study of the Transfer Function of the RLC Circuit



# Experiments. Main Idea.

## Calculation of the Response Function in Frequency Domain Mode.

### Example 1. High-pass filter.

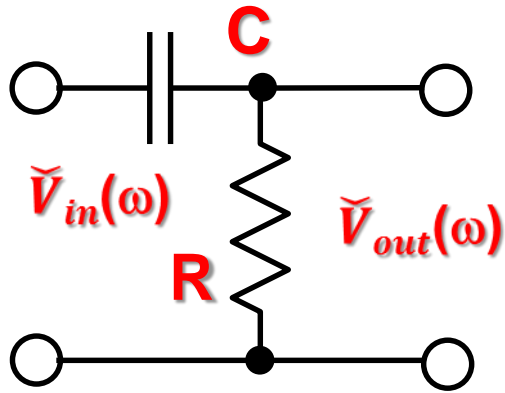


Applying the Kirchhoff Law to this simple network

$$\tilde{V}_{out}(\omega) = \tilde{H}(\omega) * \tilde{V}_{in}(\omega) = \tilde{V}_{in}(\omega) \frac{\tilde{Z}_2(\omega)}{\tilde{Z}_1(\omega) + \tilde{Z}_2(\omega)}$$



# Experiments. Calculation of the Response Function in Frequency Domain Mode. High-pass Filter



**Ideal case**

$$\tilde{Z}_R = R$$

$$\tilde{Z}_L = j\omega L$$

$$\tilde{Z}_C = \frac{1}{j\omega C} = -\frac{j}{\omega C}$$

**More realistic**

$$\tilde{Z}_R = R + \dots$$

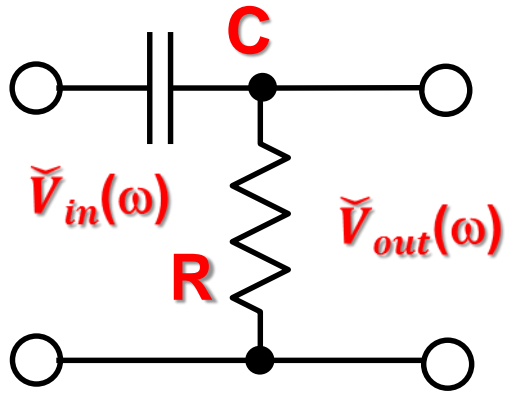
$$\tilde{Z}_L = j\omega L + R_L$$

$$\tilde{Z}_C = \frac{1}{j\omega C} = \frac{1}{j\omega C + R_C^{-1}}$$

$$\tilde{V}_{out}(\omega) = \tilde{H}(\omega) * \tilde{V}_{in}(\omega) = \tilde{V}_{in}(\omega) \frac{\tilde{Z}_2(\omega)}{\tilde{Z}_1(\omega) + \tilde{Z}_2(\omega)}$$



# Experiments. Calculation of the Response Function in Frequency Domain Mode. High-pass Filter



$\tau$  – time constant of the filter

$\omega_c$  - cutoff frequency

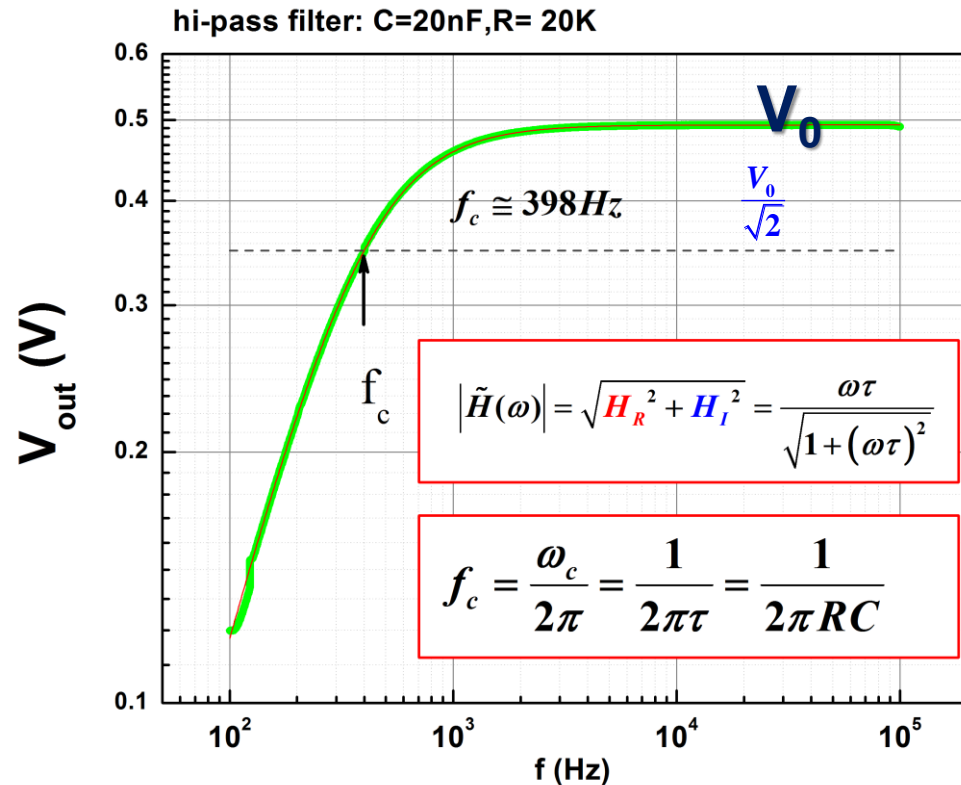
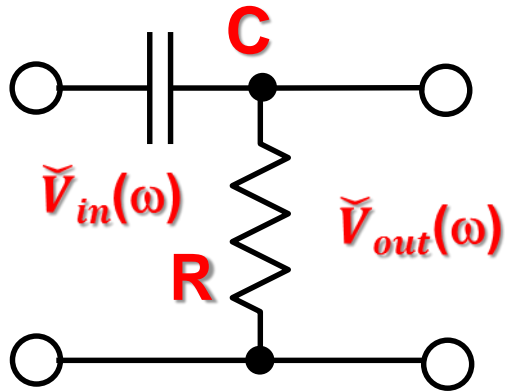
$$\tilde{H}(\omega) = H_R(\omega) + jH_I(\omega) = \frac{R}{R + \frac{1}{j\omega C}} = \frac{j\omega RC}{1 + j\omega RC} = \frac{j\omega\tau}{1 + j\omega\tau} = \frac{\omega\tau}{(1 + \omega^2\tau^2)}(\omega\tau + j);$$

where  $\tau = RC = \omega_c^{-1}$ ;

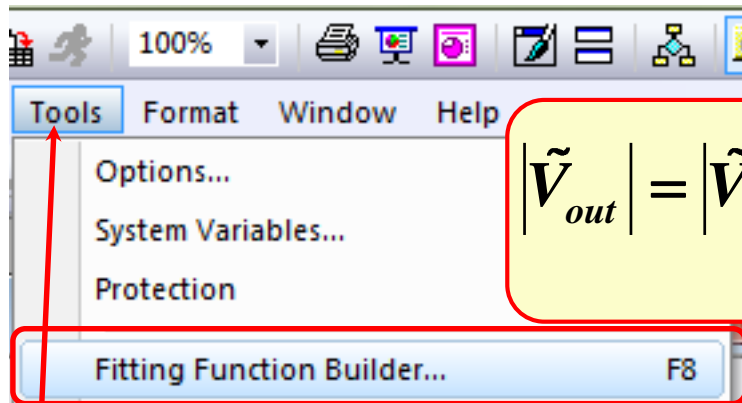
$$|\tilde{H}(\omega)| = \sqrt{H_R^2 + H_I^2} = \frac{\omega\tau}{\sqrt{1 + (\omega\tau)^2}}; \quad \theta(\omega) = \arctan\left(\frac{H_I(\omega)}{H_R(\omega)}\right) = \arctan\left(\frac{1}{\omega\tau}\right)$$



# Experiments. Calculation of the Response Function in Frequency Domain Mode. High-pass Filter



# High-pass Filter. Fitting.



$$|\tilde{V}_{out}| = |\tilde{V}_{in}| * |\tilde{H}(\omega)| = V_0 * \frac{\omega\tau}{\sqrt{1 + (\omega\tau)^2}}; \quad \tau = RC$$

**Fitting parameters:  $V_0$ ,  $\tau$ ,  $V_{off}$**

V0,tau,Voff

Param	Unit	Meaning	Fixed	Initial Value	Significant Digits
V0		?	<input type="checkbox"/>	1	System
tau		?	<input type="checkbox"/>	1	System
Voff		?	<input type="checkbox"/>	1	System

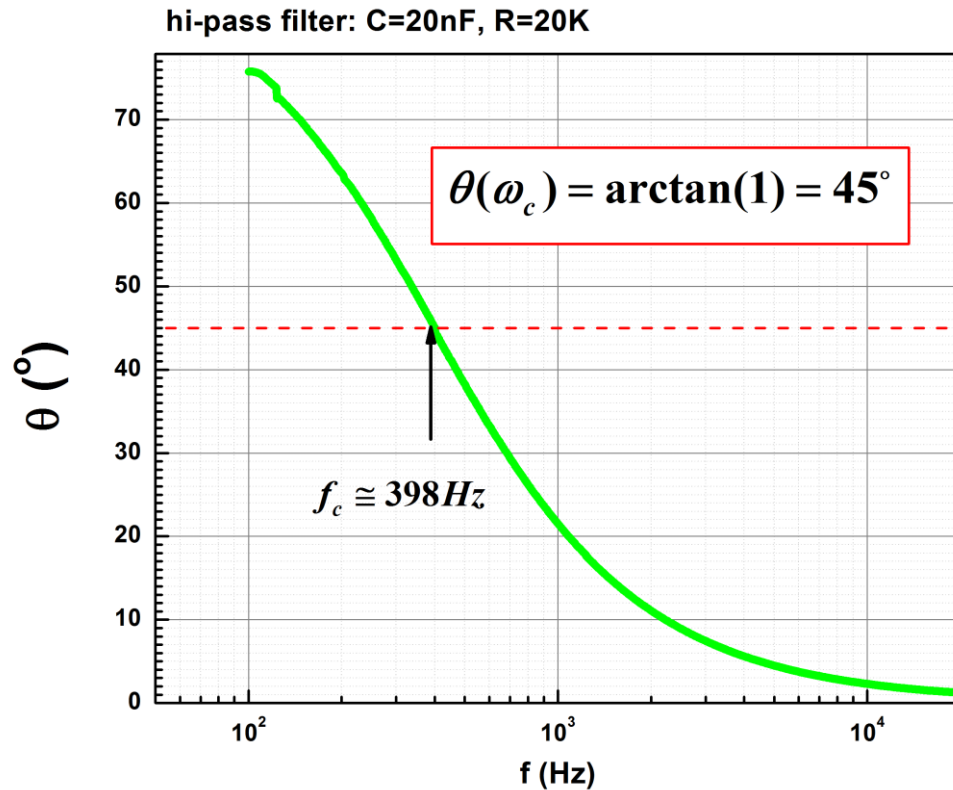
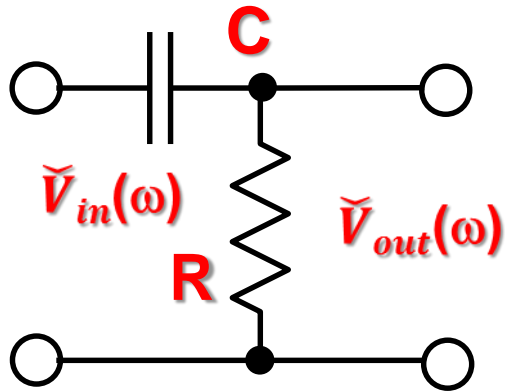
Function Body (Dependent Variables : y)

```
y=V0*2*pi*x*tau/sqrt(1+(2*pi*tau)^2)+Voff
```

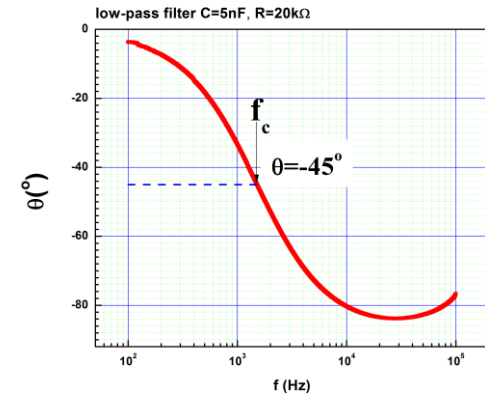
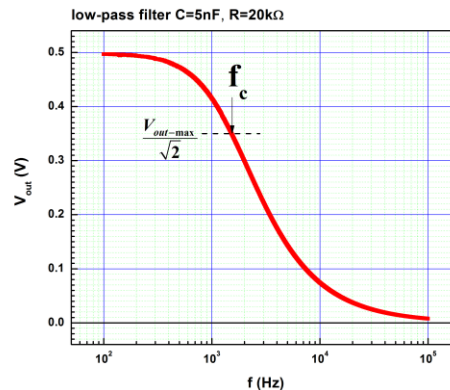
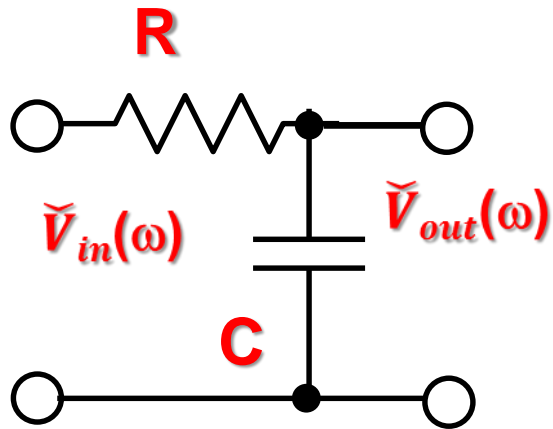
**Fitting function** →



# Experiments. Calculation of the Response Function in Frequency Domain Mode. High-pass Filter.



# Experiments. Calculation of the Response Function in Frequency Domain Mode. Low-pass Filter



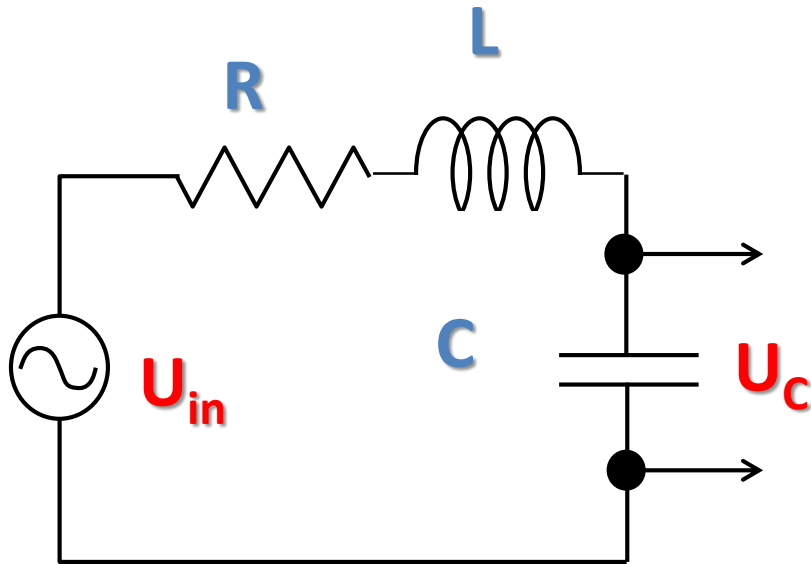
$$\tilde{H}(\omega) = H_R(\omega) + jH_I(\omega) = \frac{1}{R + \frac{1}{j\omega C}} = \frac{1}{1 + j\omega RC} = \frac{1}{1 + j\omega\tau} = \frac{(1 - j\omega\tau)}{(1 + \omega^2\tau^2)};$$

where  $\tau = RC = \omega_c^{-1}$ ;

$$|\tilde{H}(\omega)| = \sqrt{H_R^2 + H_I^2} = \frac{1}{\sqrt{1 + (\omega\tau)^2}}; \quad \theta(\omega) = \arctan\left(\frac{H_I(\omega)}{H_R(\omega)}\right) = -\arctan(\omega\tau)$$



# Application of the Lock-in Amplifier for Study of the Transfer Function of the RLC Circuit .



$$U_C = U_{in} \cdot \frac{Z_C}{Z_C + Z_L + R} =$$
$$= \frac{1}{\frac{1}{j\omega C} + j\omega L + R}$$



# Application of the Lock-in Amplifier for Study of the Transfer Function of the RLC Circuit .

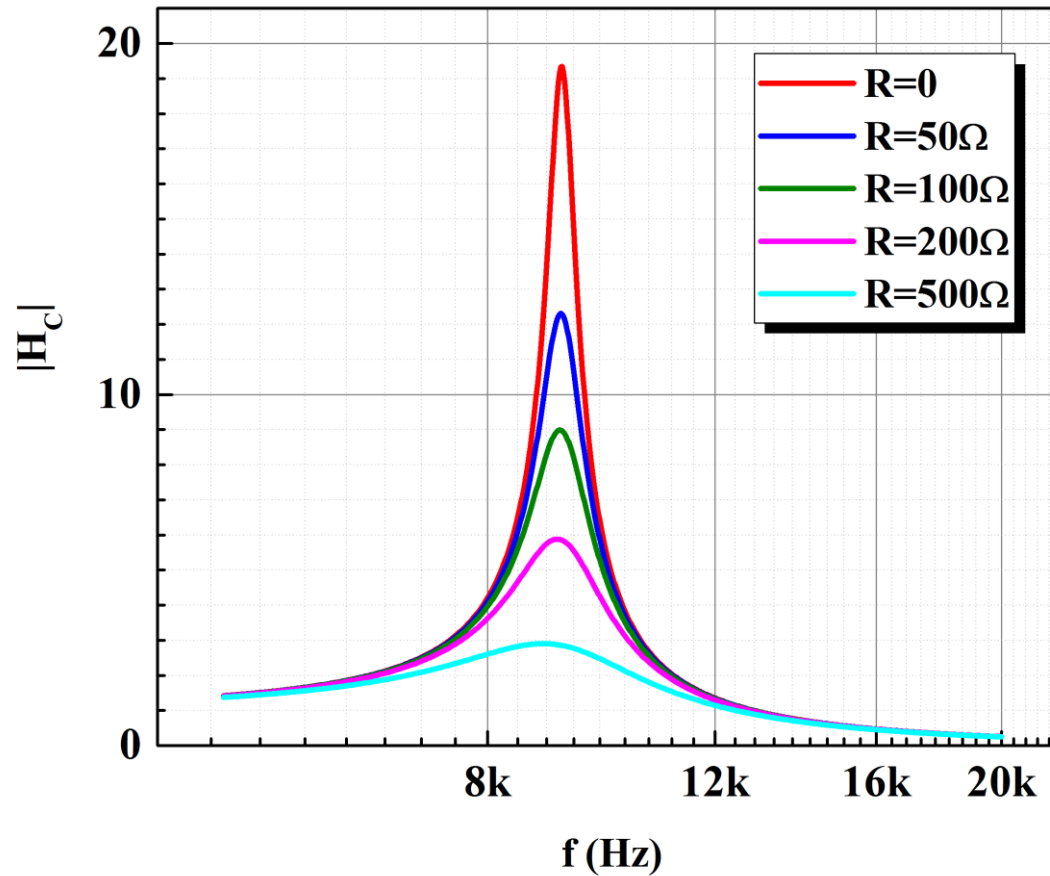
$$H = \frac{U_c}{U_{in}} = \frac{1}{(1 - \omega^2 LC) + j\omega CR} = \frac{\left(1 - \left(\frac{\omega}{\omega_0}\right)^2\right) - j\omega CR}{\left(1 - \left(\frac{\omega}{\omega_0}\right)^2\right)^2 + \omega^2 C^2 R^2} \times;$$

$$\omega_0 = \frac{1}{\sqrt{LC}}; \nu \equiv \frac{\omega}{\omega_0}; Q = \frac{1}{R} \sqrt{\frac{L}{C}};$$

$$H = \frac{(1 - \nu^2) - j\frac{\nu}{Q}}{(1 - \nu^2)^2 + \frac{\nu^2}{Q^2}}; \theta = -\tan^{-1}\left(\frac{\nu}{Q(1 - \nu^2)}\right)$$



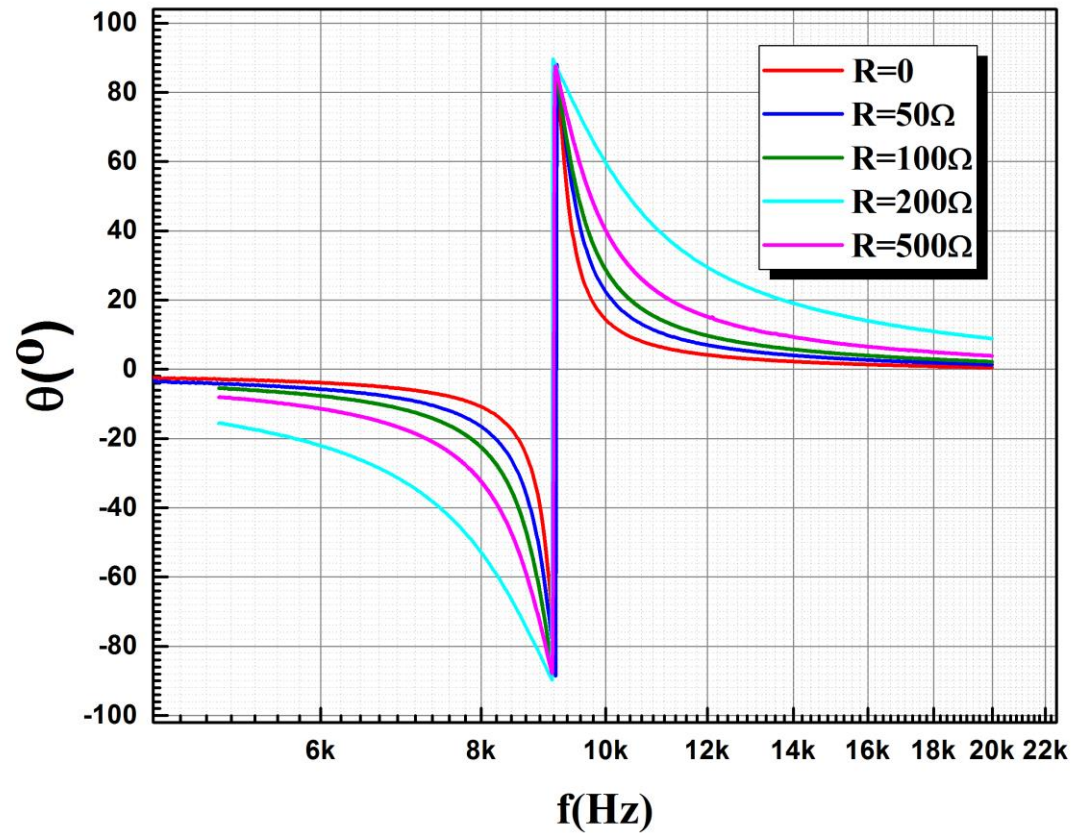
# Application of the Lock-in Amplifier for Study of the Transfer Function of the RLC Circuit .



**The resonance curves obtained on RLC circuits with different damping resistors.**



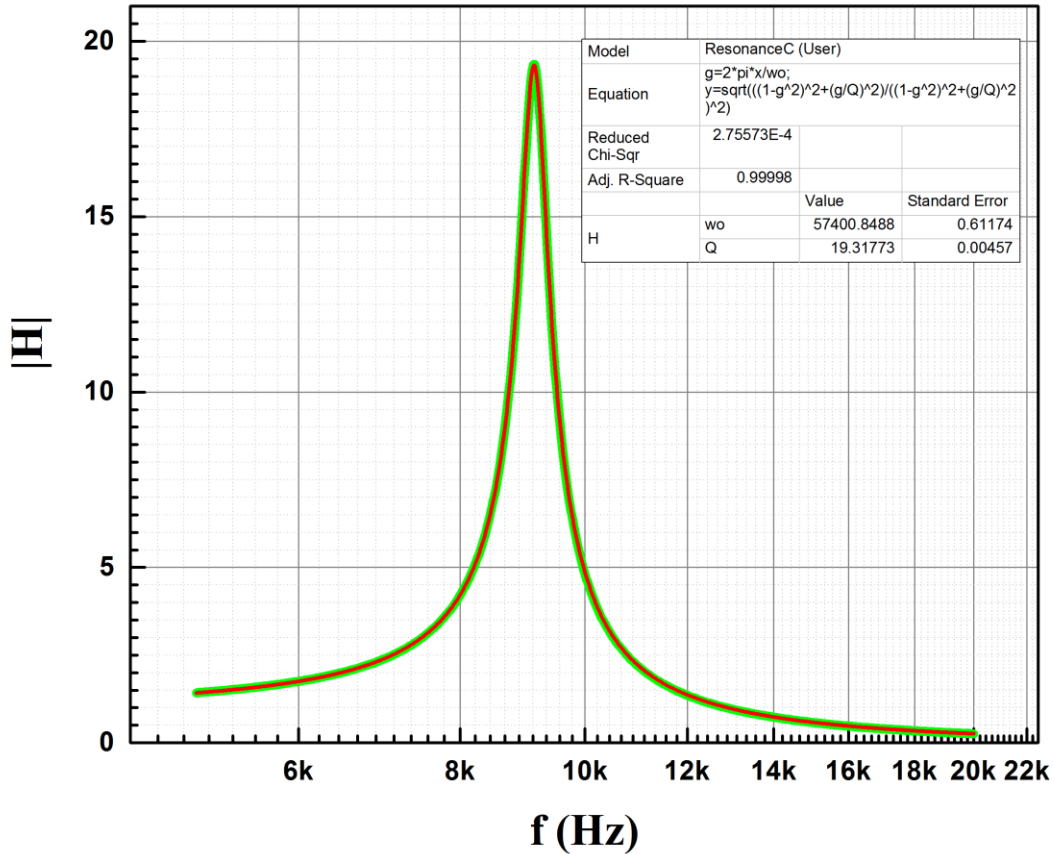
# Application of the Lock-in Amplifier for Study of the Transfer Function of the RLC Circuit



**The resonance curves obtained on RLC circuits with different damping resistors**



# Fitting. RLC Resonance Circuit.



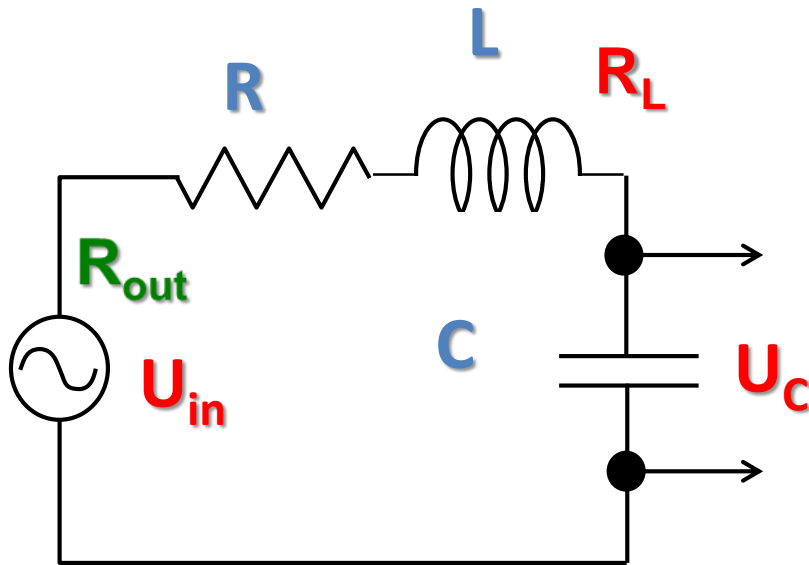
$$|H| = \frac{\sqrt{(1-\gamma^2)^2 + \left(\frac{\gamma}{Q}\right)^2}}{(1-\gamma^2)^2 + \left(\frac{\gamma}{Q}\right)^2}; \gamma = \frac{\omega}{\omega_0}$$

**fitting function for |H|**

**variable parameters:  
 $\omega_0$  and Q**



# Application of the Lock-in Amplifier for Study of the Transfer Function of the RLC Circuit



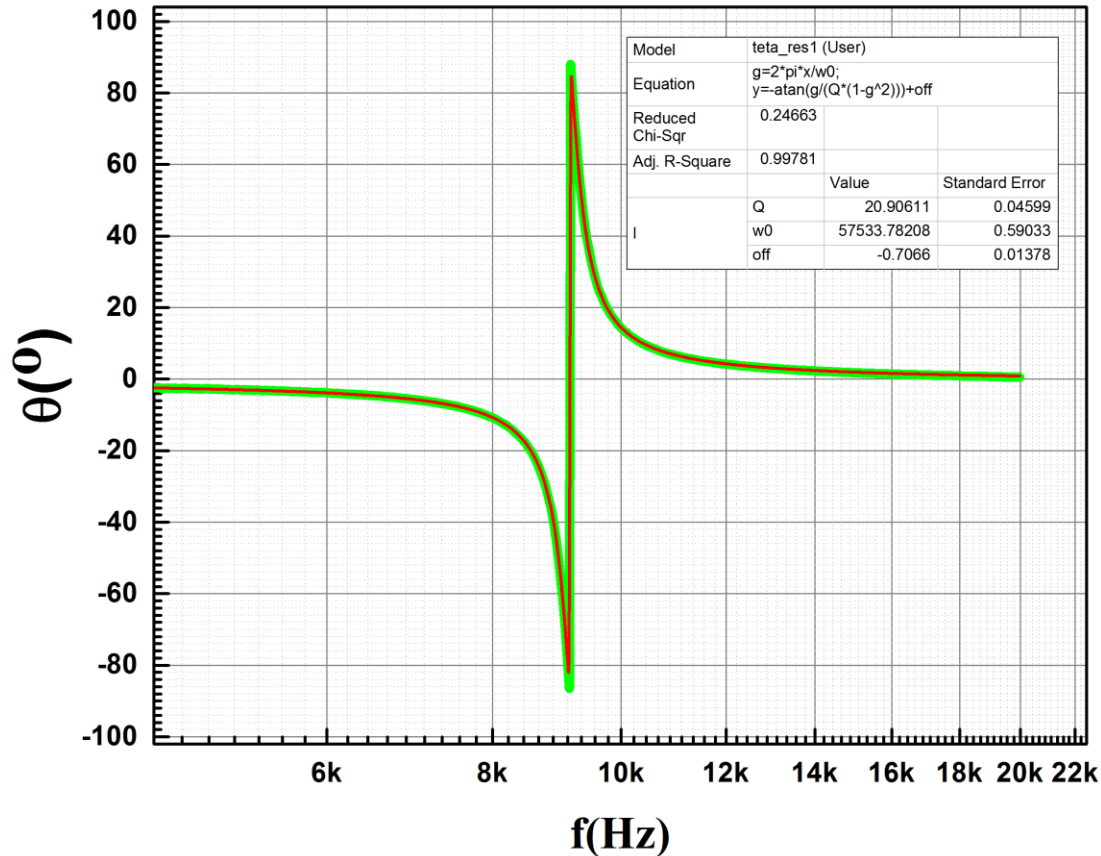
$$R=0; R_L=35.8\Omega; R_{out}=50\Omega$$

Actual damping resistance is a sum of  $R$ ,  $R_L$  (resistance of the coil) and  $R_{out}$  (output resistance of the function generator)

Actual  $R$  calculated from fitting pars is  $\sim 88.8\Omega$  what is reasonable close to  $85.8\Omega$



# Fitting. RLC Resonance Circuit.



$$\theta = \tan^{-1} \left( \frac{Y}{X} \right)$$

measured

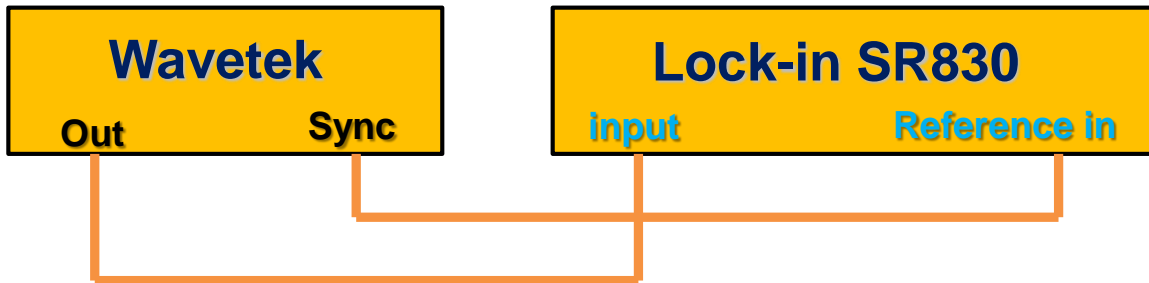
$$\theta = -\tan^{-1} \left( \frac{\gamma}{Q(1-\gamma^2)} \right); \gamma = \frac{\omega}{\omega_0}$$

fitting function

variable parameters:  
 $\omega_0$  and  $Q$



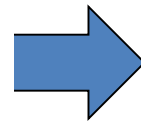
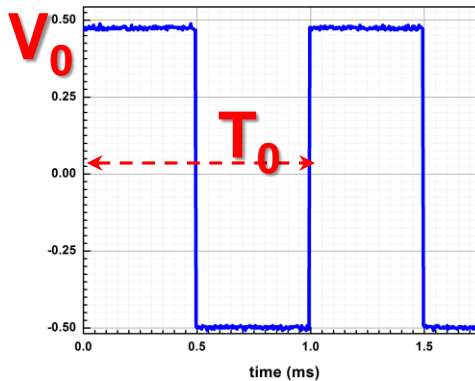
# From Time Domain to Frequency Domain. Experiment.



$$V = V_0 \left( 0 < t \leq \frac{T_0}{2} \right);$$

$$-V_0 \left( \frac{T_0}{2} < t \leq T_0 \right)$$

$F(t)$  – periodic function  $F(t)=F(t+T_0)$ :



**Frequency  
domain ?**

$$a_n = \frac{2}{T_0} \int_0^{T_0} F(t) \cos\left(\frac{2\pi nt}{T_0}\right) dt;$$

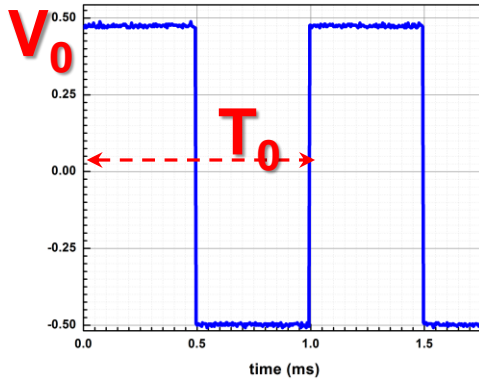
$$b_n = \frac{2}{T_0} \int_0^{T_0} F(t) \sin\left(\frac{2\pi nt}{T_0}\right) dt;$$

$$a_0 = \frac{2}{T_0} \int_0^{T_0} F(t) dt$$

**Time domain pattern**

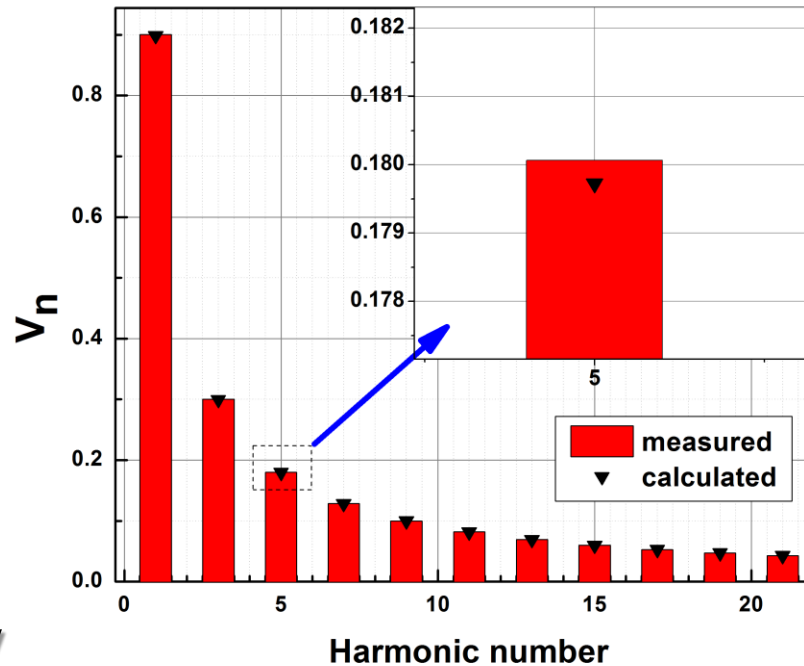


# From Time Domain to Frequency Domain. Experiment with SR830. Results.



**Time domain**

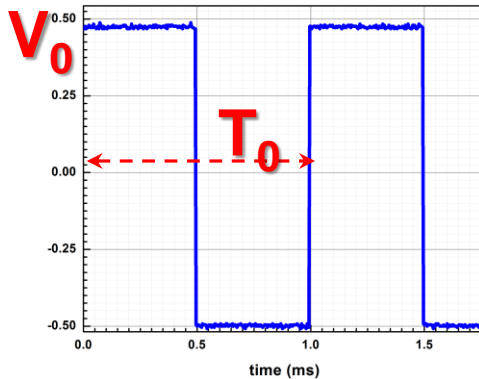
**Spectrum measured by  
SR 830 lock-in amplifier**



**Frequency domain**

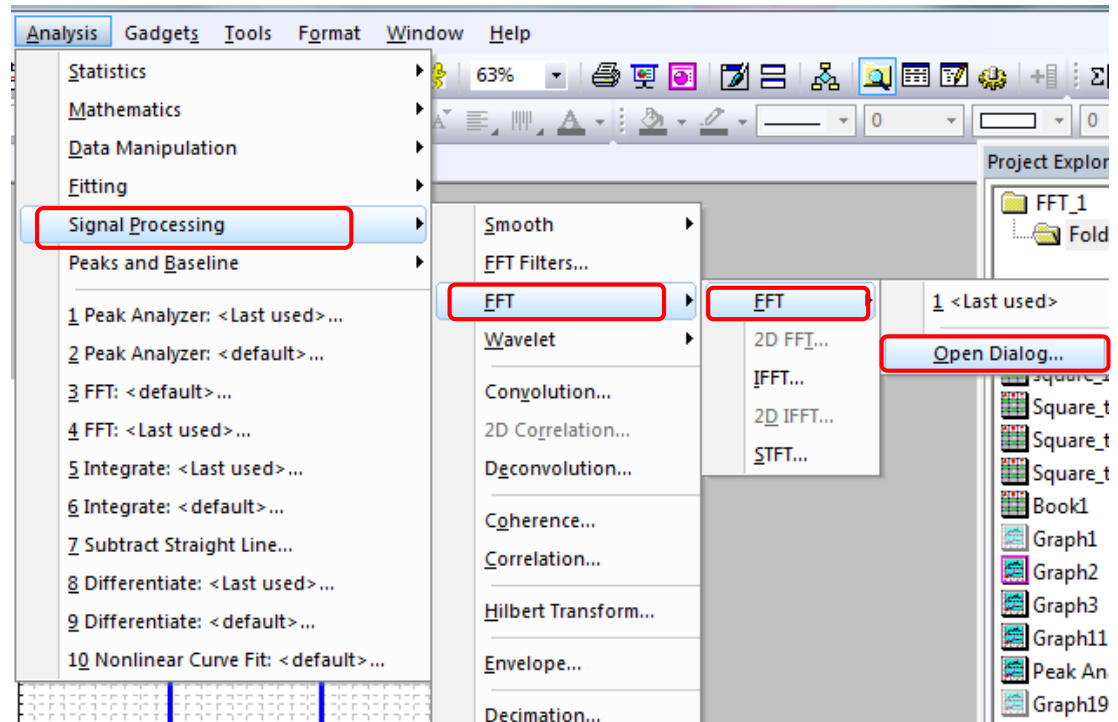


# From Time Domain To Frequency Domain. FFT using Origin. Results.

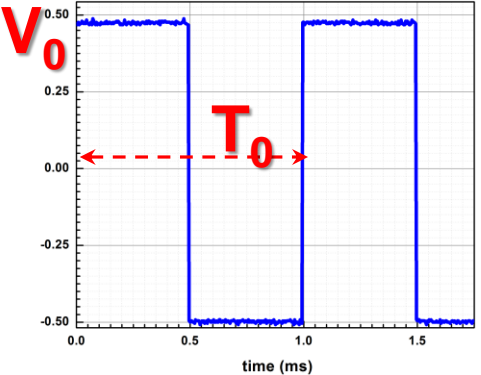


**Data file can be used to convert time domain to frequency domain**

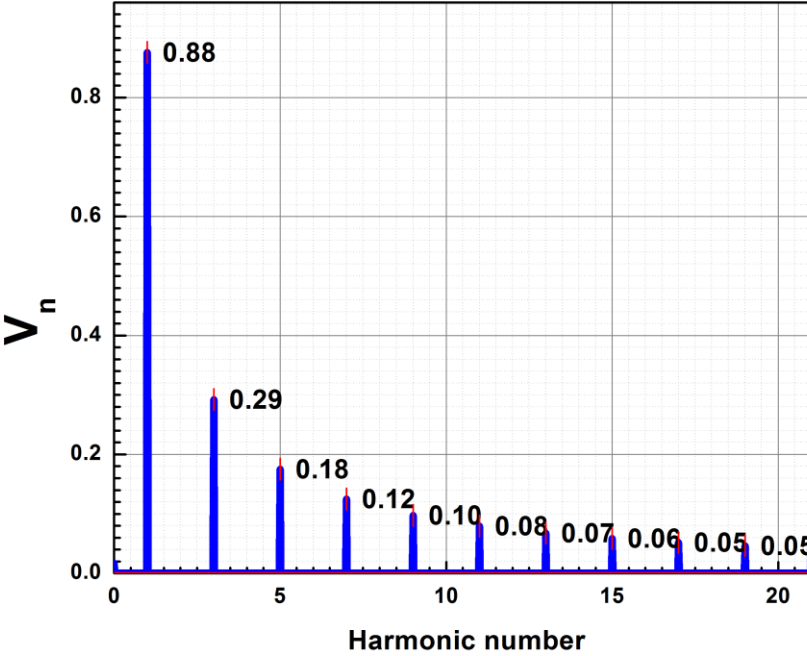
**Time domain taken by Tektronix scope**



# From Time Domain to Frequency Domain. FFT using Origin. Results.



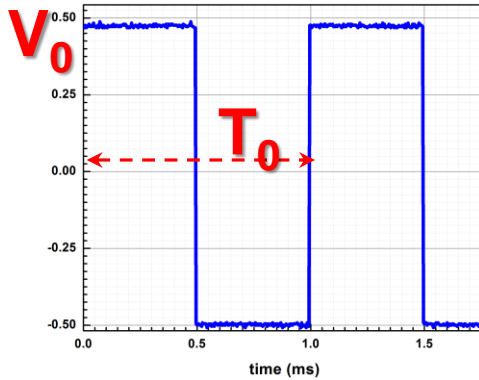
**Time domain taken by  
Tektronix scope**



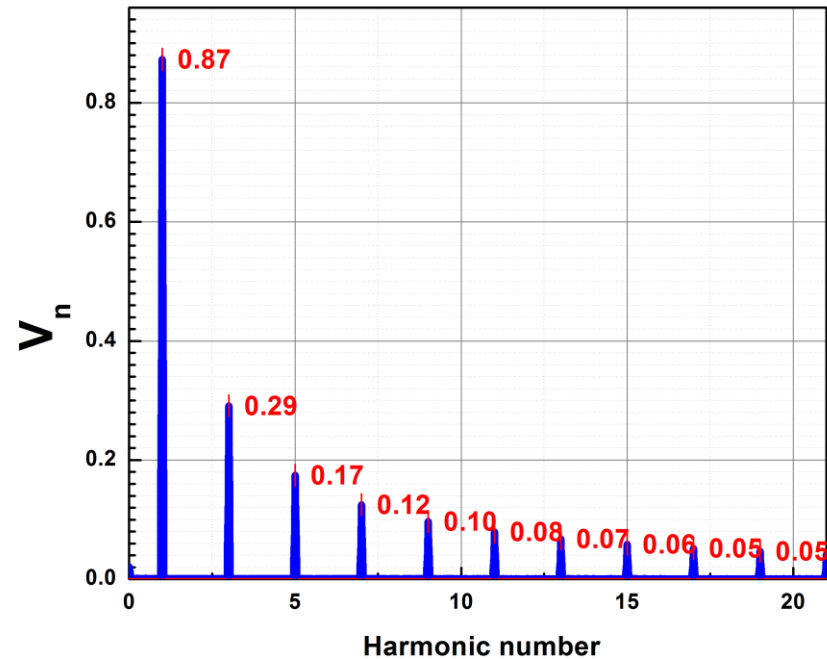
**Spectrum calculated by Origin. Accuracy is limited because of the limited resolution of the scope**



# From Time Domain to Frequency Domain. Using of the Math Option of the Scope.



**Time domain taken by  
Tektronix scope**

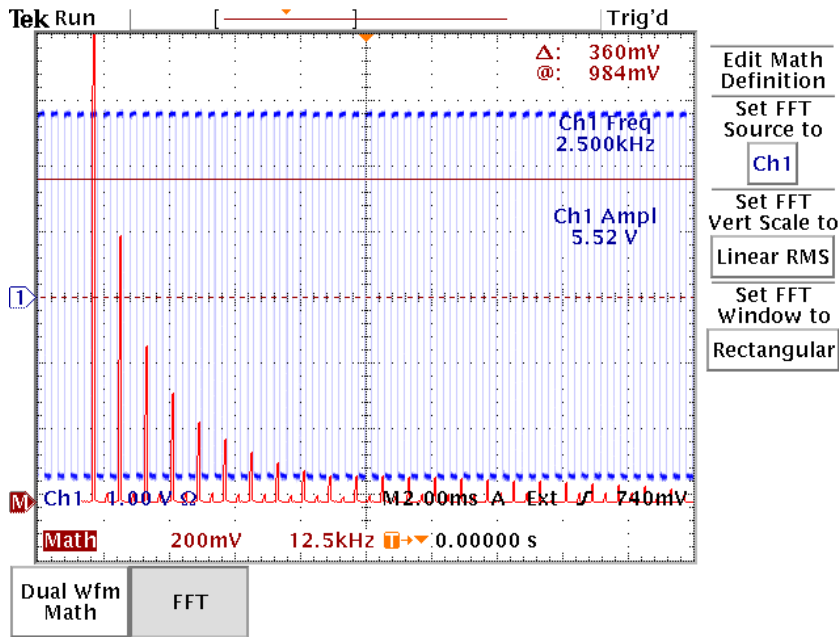


**Spectrum calculated by  
Tektronix scope.**

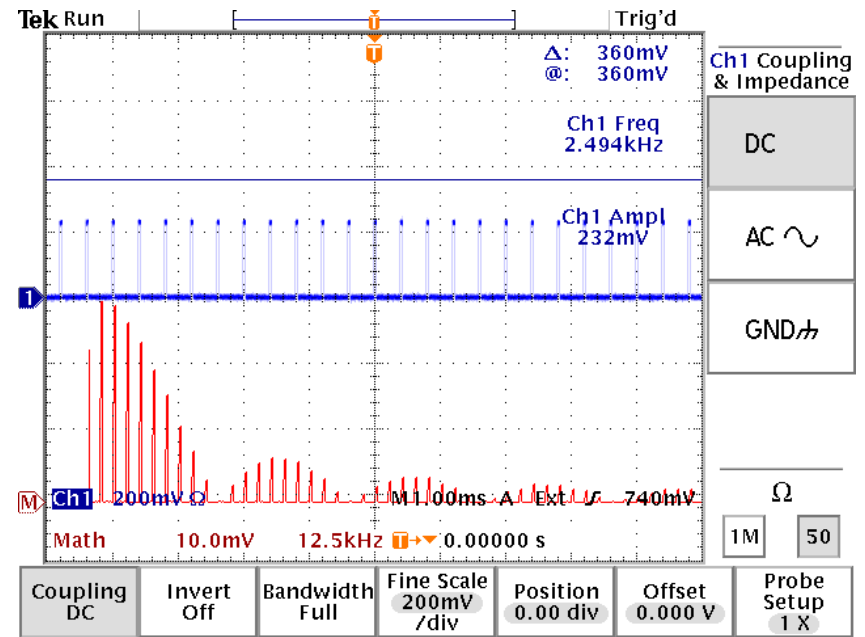
**Accuracy is limited because of the  
limited resolution of the scope**



# From Time Domain to Frequency Domain. Using of the Math Option of the Scope.



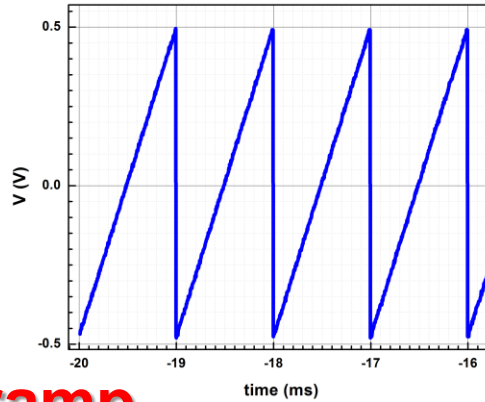
**Spectrum of the square wave signal**



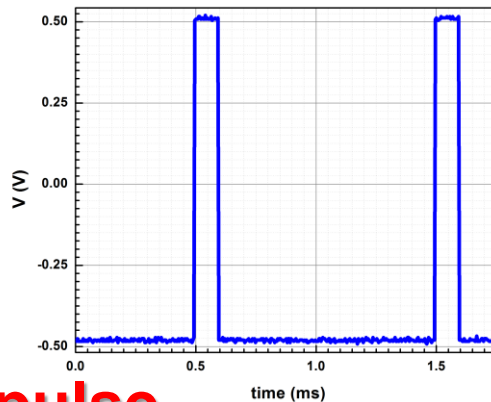
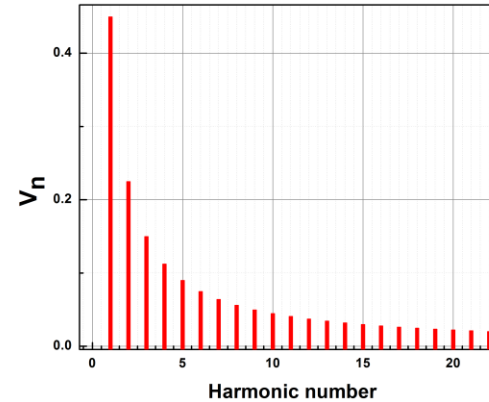
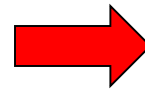
**Spectrum of the pulse signal**



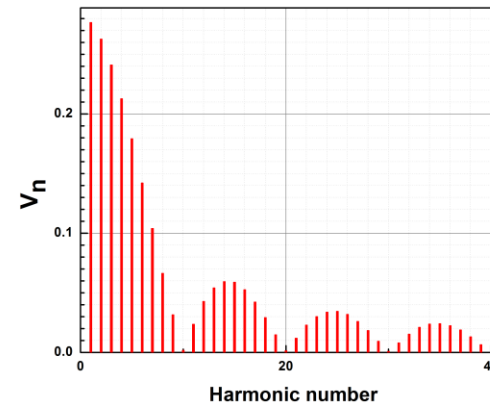
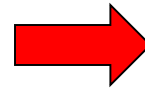
# From Time Domain to Frequency Domain. Different Waveforms. Using Lock-in.



**ramp**



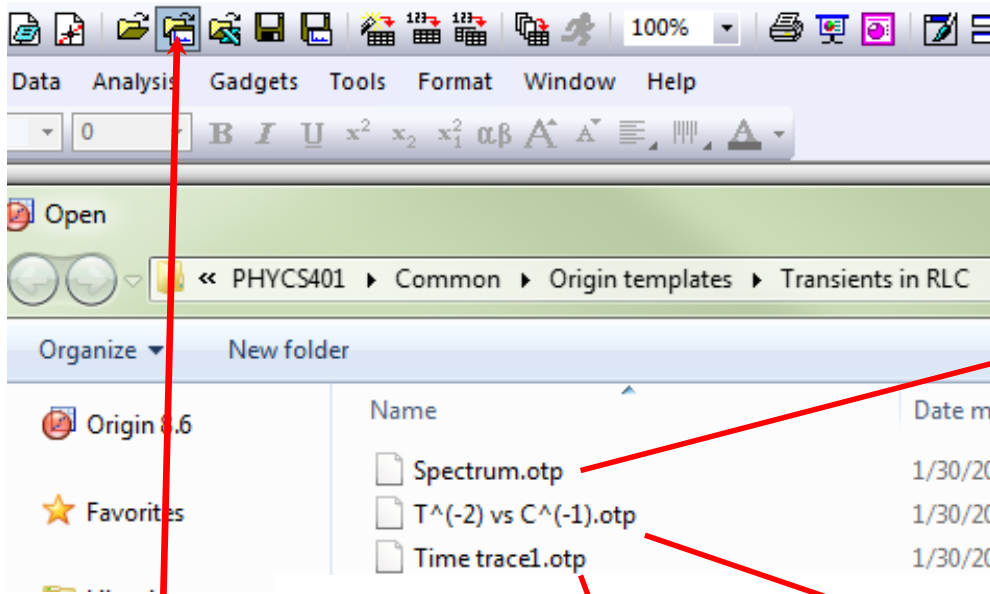
**pulse**



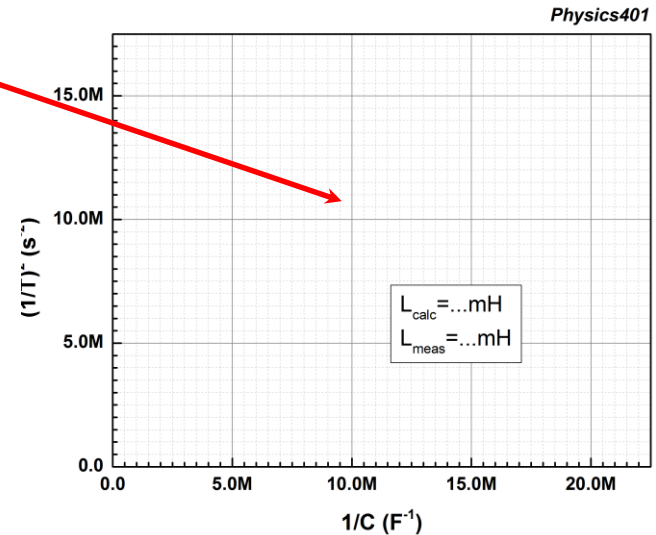
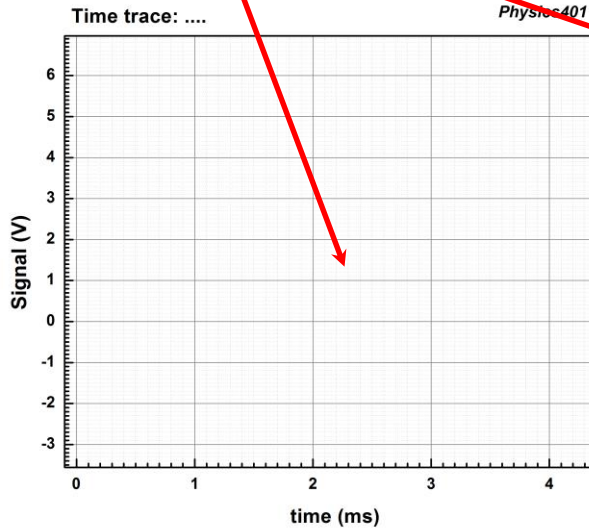
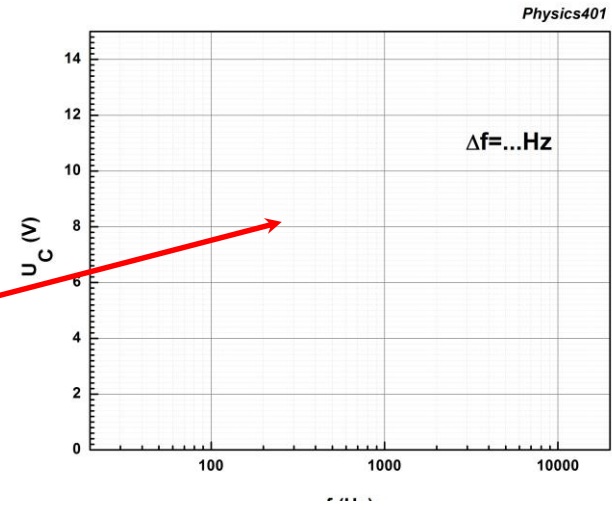
# Appendix #1

# Origin templates for this week Lab.

\\engr-file-03\phyinst\APL Courses\PHYCS401\Common\Origin templates

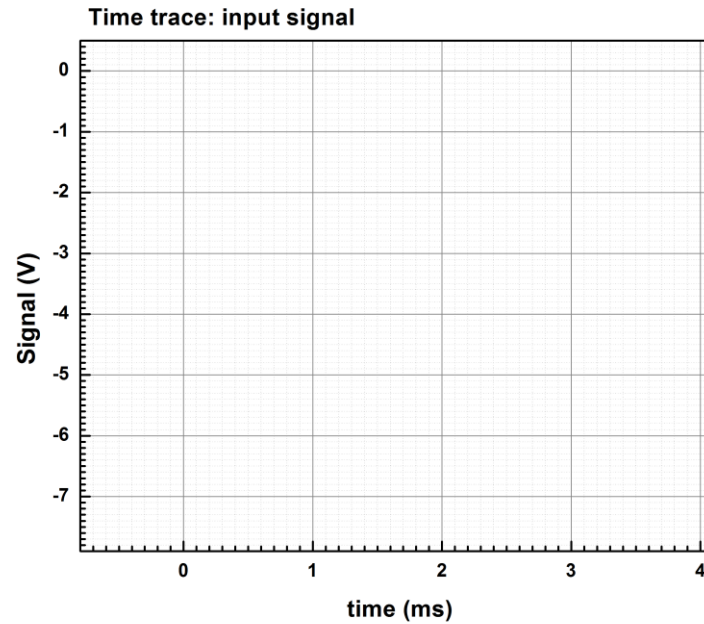
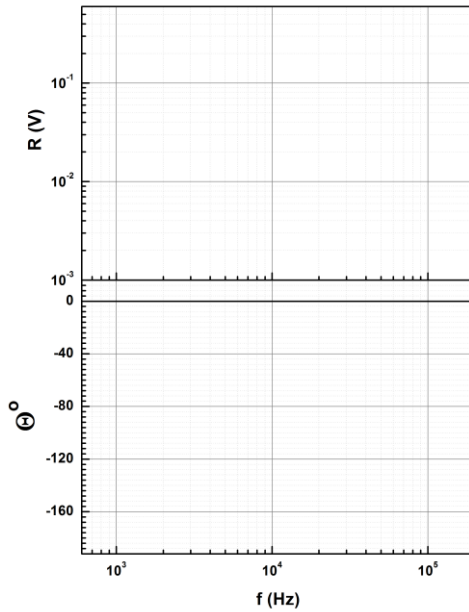


**Open template button**



# Appendix #2

## Origin templates for the next week Lab:



Physics401



# References:

1. John H. Scofield, “A Frequency-Domain Description of a Lock-in Amplifier” *American Journal of Physics* 62 (2) 129-133 (Feb. 1994).
  2. Steve Smith “The Scientist and Engineer's Guide to Digital Signal Processing” copyright ©1997-1998 by Steven W. Smith. For more information visit the book's website at: [www.DSPguide.com](http://www.DSPguide.com)”\*
- You can find a soft copy of this book in:
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