Millikan Oil Drop Experiment

Physics 401, Fall 2015

Eugene V. Colla
1. Measuring of the charge of electron.
2. Robert Millikan and his oil drop experiment
3. Theory of the experiment
4. Laboratory setup
5. Data analysis
Measuring of the charge of the electron

   \[ e = 1.5924(17) \times 10^{-19} \text{ C} \]

2. Shot noise experiment. First proposed by Walter H. Schottky

3. In terms of the Avogadro constant and Faraday constant \( e = \frac{F}{N_A} \); F- Faraday constant, \( N_A \) - Avagadro constant. Best uncertainty \( \sim 1.6 \text{ ppm} \).

4. From Josephson \( (K_J = \frac{2e}{h}) \) and von Klitzing \( (R_K = \frac{h}{e^2}) \) constants

5. Recommended by NIST value \( 1.602\ 176\ 565(35) \times 10^{-19} \text{ C} \)
The Nobel Prize in Physics 1923. Robert A. Millikan "for his work on the elementary charge of electricity and on the photoelectric effect".

ROBERT ANDREWS MILLIKAN 1868-1953

University of Chicago

22nd of March, 1868, Morrison, Ill
Robert Millikan. Oil drop experiment

Diagram and picture of apparatus
**Oil Drop Experiment.**

**Motivation:**
Measurement of the magnitude of the electron charge!
Demonstrate that the electron charge is quantized!

Measure the charge of an electron to ±3%
Oil Drop Experiment.

Forces on the oil drop:
1) Gravity + buoyant force (air displaced by oil drop)
2) Drag force of the oil drop in the air
Forces on the oil drop:

1) Gravity + buoyant force (air displaced by oil drop)
2) Drag force of the oil drop in the air
3) Electric force on oil drops which carry charge $Q$
Apparatus. Schematic Layout

- Telescope
- Atomizer
- Oil
- Eyepiece
- Scale
- Oil drop
- Mist
- Spacer
- Light source

Parameters:
- \( P_1 \) and \( P_2 \)
- \( Q, m \)
- 500 ± 1 V

Date: 10/12/2016
Apparatus: Actual Setup

- Plate voltage connectors
- Charge plate selector
- Thermistor connectors
- Light Source
- Ionization source selector
- Bubble level
- Thermistor table
- Oil viewing chamber
- Viewing pin (removable)
- Viewing reticle
What is Measured

**rise time measurement stops here**

**rise time** \( t_{\text{rise}} \)

**fall time** \( t_g \)

\[ x = \text{fall distance} = \text{rise distance} \]

\( x \) must be the same for all drops!

**allow drop to “undershoot” here before starting next rise time experiment**

**fall time measurement starts here**

**fall time measurement stops here**
Balance of Forces: Newton’s Law

Forces on the oil drop:
(1) Gravity + buoyant force (air displaced by oil drop)
(2) Drag force of the oil drop in the air
(3) Electric force on oil drops which carry charge Q

Particle reached terminal velocity
\[ \frac{d\vec{v}}{dt} = 0 \]

1 µ size particle reaches the terminal velocity in \( \sim 10^{-5}\) s

\[
\vec{F}_g + \vec{F}_{\text{drag}} + \vec{F}_E = 0
\]

\[
\vec{F}_g = -mg\hat{z} \quad (1)
\]

\[
\vec{F}_{\text{drag}} = -6\pi \eta a\vec{v} \quad (2)
\]

\[
\vec{F}_E = Q\vec{E} \quad (3)
\]

\[
\vec{F} = m \frac{d\vec{v}}{dt} = \vec{F}_g + \vec{F}_{\text{drag}} + \vec{F}_E
\]

\( a \) : radius of drop
\( \rho \) : density \( \rho = \rho_{\text{oil}} - \rho_{\text{air}} \)
\( \vec{v} \) : velocity of oil drop
\( Q \) : charge of oil drop
\( E \) : electric field \( E = V/d \)
\( V \) : Voltage across plates
\( \eta \) : viscosity of air
\( g \) : gravitational const.
Modification to Stokes Law

\[ \vec{F}_{\text{drag}} = -6\pi\eta a \vec{v} \]

For small particle radius \((a<15\mu)\) Stokes law need to be corrected. This correction was derived by E. Cunningham.

Here \(a\) – particle radius; \(\lambda\) – mean free path of the gas molecules

\[
f_c = 1 + A \frac{\lambda}{a} + B \frac{\lambda}{a} e^{-\frac{a}{\lambda}}, \quad A = 1.246, \quad B = 0.42, \quad C = 0.78
\]

\[
f_c \approx 1 + A \frac{\lambda}{a} = 1 + \frac{r_c}{a} \approx 1.1, \quad \text{for} \ a \approx 10^{-6} \text{m}, \quad r_c = \frac{6.18 \times 10^{-5}}{\rho[\text{mmHg}]}
\]

\(\lambda [\text{m}] = 6.53 \times 10^{-8} \frac{760}{\rho[\text{mmHg}]}

10/12/2016
We Measure: $t_g$ and $t_{rise}$

$x = \text{fall distance} = \text{rise distance}$. $x$ must be the same for all drops!

Allow drop to “undershoot” here before starting next rise time experiment.

$\text{fall time measurement stops here}$

$\text{fall time measurement starts here}$

$\text{rise time measurement stops here}$

$\text{rise time measurement starts here}$

$\text{rise time} t_{rise}$

$\text{fall time} t_g$
Solving Newton’s Law: $Q(t_g, t_{rise})$

$f_c$ can be found from Newton law equation in the case of $V=0$ (falling drop)

$$F_g + F_{drag} = \frac{4}{3} \rho g a^3 - 6\pi \eta \frac{a}{f_c} \ddot{v} = 0$$

(see write-up)

$$1 - \left( \frac{t_g}{\tau_g} \right)^2 \approx 1; \tau_g = \frac{2\eta x}{\rho gr_c^2}; r_c[m] = \frac{6.18 \times 10^{-5}}{p[mmHg]}$$
Solving Newton’s law: $Q(t_{\text{g}}, t_{\text{rise}})$

$$Q = n \times e = \frac{1}{f_c^{3/2}} \frac{9\pi d}{V} \sqrt{\frac{2\eta^3 x^3}{g\rho}} \sqrt{\frac{1}{t_g}} \left[ \frac{1}{t_g} + \frac{1}{t_{\text{rise}}} \right]$$

- $Q$: charge of oil drop
- $n$: number of unpaired electrons in drop
- $e$: elementary charge
- $d$: plate separation
- $V$: Voltage across plates
- $\rho$: density $\rho = \rho_{\text{oil}} - \rho_{\text{air}}$
- $\eta$: viscosity of air
- $g$: gravitational constant
- $x$: drift distance for oil drop
- $t_g$: fall time
- $t_{\text{rise}}$: rise time
Route of Charge Calculation $Q(t_g, t_{rise})$.

$$\frac{1}{3} \approx 1 - \left( \frac{t_g}{\tau_g} \right)^{\frac{1}{2}}; \quad \tau_g = \frac{2 \eta x}{\rho gr_c}; \quad r_c[m] = \frac{6.18 \times 10^{-5}}{p[mmHg]}$$

$$Q = F \cdot S \cdot T = \left( \frac{1}{f_c^{3/2}} \right) \frac{9 \pi d}{V} \sqrt{\frac{2 \eta^3 x^3}{g \rho}} \frac{1}{\sqrt{t_g}} \left( \frac{1}{t_g} + \frac{1}{t_{rise}} \right)$$

$$F = \frac{1}{f_c^{3/2}} \approx 1 - \left( \frac{t_g}{\tau_g} \right)^{\frac{1}{2}}$$

$$S = \frac{9 \pi d}{V} \sqrt{\frac{2 \eta^3 x^3}{g \rho}}$$

$$T = \frac{1}{\sqrt{t_g}} \left( \frac{1}{t_g} + \frac{1}{t_{rise}} \right)$$

**Projects** *Section L1.opj ... Section L4.opg*

**Locations:**
\`
C:\\engr-file-03\\PHYINST\\APL Courses\\PHYCS401\\Common\\Origin templates\\Oil drop experiment
C:\\engr-file-03\\PHYINST\\APL Courses\\PHYCS401\\Students\\1.Millikan Oil Drop experiment
\`

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Please make a copy (not move!) of Millikan1.opj in your personal folder and start to work with your personal copy of the project.
### Route of Charge Calculation. Origin Project. Data Analysis.

**Project Millikan1.opj**

**Constants**

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<th>Par</th>
<th>$r_c[m]$</th>
<th>$\tau_g = \frac{2\eta x}{pgr_c^2}$</th>
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<th>your data</th>
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**Calculations**

- $r_c[m] = \frac{6.18 \times 10^{-5}}{p[mmHg]}$
- $P(mmHg) \rightarrow Col("Par")[7]$

**Parameters of the experiment. Depend on exact setup and environment conditions**

- In red - variable parameters
- In blue - data
- In olive - constants

10/12/2016
### Route of Charge Calculation. Origin Project. Data Analysis.

**Project** Millikan1.opj

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**Long Name**

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<td>4.8E-8</td>
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\[ \eta(T) = \eta(25\degree C) - \frac{d\eta}{dT}(T - 25) \]

\[ \eta = 1.8478E-5 \]

Follow correct order of calculations: \( r_c \rightarrow \tau_g \rightarrow (F,S,T) \rightarrow Q \rightarrow n \)

Project Millikan1.opj
Follow correct order of calculations:

\[ r \rightarrow \tau \rightarrow g \rightarrow (F, S, T) \rightarrow Q \rightarrow n \]

Indexes for parameters in Col("Par")
Actual air viscosity should be calculated manually before any other calculation
Charge calculation. Origin project.

\[ S = \frac{9 \pi d^4}{V} \sqrt{\frac{2 \eta^2 \lambda^3}{g \rho}} \sqrt{\frac{1}{t_g} \left( \frac{1}{t_e} + \frac{1}{t_m} \right)} \]

\[ Q = FST \]

\[ n = \frac{Q}{1.602 \times 10^{-19}} \]

\[ \text{number of elementary charges} \]

\[
\begin{array}{cccc}
S & T & Q = FST & n = \frac{Q}{1.602 \times 10^{-19}} \\
\hline
\text{---} & \text{---} & \text{---} & \text{---} \\
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\end{array}
\]
Expected results

Data: Bin1_Counts1
Model: Gauss
Equation: \( y = y_0 + \left(\frac{A}{w(\sqrt{\pi/2})}\right) \exp\left(-2\left(\frac{(x-x_c)}{w}\right)^2\right) \)

Weighting:
\( y \) No weighting

\( \chi^2/\text{DoF} = 34.42645 \)
\( R^2 = 0.81797 \)

- \( y_0 = 0 \pm 0 \)
- \( x_1 = 0.92997 \pm 0.01884 \)
- \( w_1 = 0.27612 \pm 0.0381 \)
- \( A_1 = 13.76998 \pm 1.65786 \)
- \( x_2 = 1.87091 \pm 0.05159 \)
- \( w_2 = 0.60549 \pm 0.11582 \)
- \( A_2 = 16.60884 \pm 2.56952 \)
Choice of Oil Drops for the Analysis: rise and fall times

Difficult to separate $n=3,4,\&5$

Choice of Oil Drops for the Analysis:
- rise and fall times

- $n=1$
- $n=2$
- $n=3$
- $n=4$
- $n=5$
Modern experiments at SLAC

- Drop generation rate 1 Hz
- Fluid - Dow Corning silicon oil
- Number of drops - 17 million
- Mass - 70.1 milligrams
- Duration - 8 months
Modern experiments at SLAC

Machine vision mediated auto-control of: the average charges of the drops, the fall path of the drops, the upward laminar air flow, and the electronic drive to the drop ejector are new features of this fluid drop charge measurement system.

The experiment is ran from 2004 - 2007.

Charge measurement accuracy achieved is better than 1/24 e for drops of up to 26 microns in diameter.
Modern experiments at SLAC

Summary as of January 2007.
Total mass throughput for all experiments - 351.4 milligrams of fluid
Total drops measured in all experiments - 105.6 million
No evidence for fractional charge particles was found.
Appendix #1

- **Traditional reminder:**
  - L1_Lab3_student name.pdf
  - Report-exp2.pdf

Please upload the files in proper folder!

This week folders:
- Pulses in transmission lines_L1
- Pulses in transmission lines_L3
- Pulses in transmission lines_L4
- Pulses in transmission lines_L5

This week you have the last chance to submit "*Transients in RLC*" report
Appendix #2

- Transmission line. Unknown load simulation

Location: \Phyapplportal\PHYCS401\Common\Transmission line software

10/12/2016
Appendix #2

- Transmission line. Unknown load simulation

Location: \PhyapInportal\PHYCS401\Common\Transmission line software