Transients and Oscillations in RLC Circuits

Physics 401, Fall 2018.

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Transients and Oscillations in RLC Circuits. Outline

- Transients. Definition.
- Transients in RLC
- Resonance in RLC
- Data analysis. Origin. Fitting.
Main goals of this week Lab:

- To understand what are the transients in general
- Transients in RLC circuits. Different regimes of dumping
- Data analysis using Origin software
Transient (physics) a short-lived oscillation in a system caused by a sudden change of voltage or current or load.

A transient response or natural response is the response of a system to a change from equilibrium.
Transients in RLC circuit.

Resistance $R$ [Ohm]
Capacitance $C$ [$\mu$F] ($10^{-6}$ F)
Inductance $L$ [$m$H] ($10^{-3}$ H)
According the Kirchhoff’s law

\[ V_R + V_L + V_C = V(t) \]

\[ L \frac{d^2}{dt^2} q(t) + R \frac{d}{dt} q(t) + \frac{q(t)}{C} = 0 \]  

(1)

\*See Lab write-up for details
The solution of this differential equation can be found in the form

\[ q(t') = Ae^{st} \]

This will convert (1) in quadratic equation

\[ s^2 + \left( \frac{R}{L} \right)s + \frac{1}{LC} = 0 \]

with solutions:

\[ s_{1,2} = -\frac{R}{2L} \pm \sqrt{\left( \frac{R}{2L} \right)^2 - \left( \frac{1}{LC} \right)} \equiv -a \pm b \]

\[ a = \frac{R}{2L}, \quad b = \sqrt{\left( \frac{R}{2L} \right)^2 - \left( \frac{1}{LC} \right)} \]

- \( b^2 > 0 \) over-damped solution
- \( b^2 = 0 \) critically damped solution
- \( b^2 < 0 \) under-damped solution
Transients in RLC circuit. Over-damped solution: 

\[ b^2 > 0 \]

In this case the solution will be aperiodic exponential decay function with no oscillations:

\[ q(t) = e^{-at} \left( A_1 e^{bt} + B_1 e^{-bt} \right) \]

\[ i(t) = \frac{dq}{dt} = -ae^{-at} (A_1 e^{bt} + B_1 e^{-bt}) + be^{-at} (A_1 e^{bt} - B_1 e^{-bt}) \]

\[ b^2 > 0 \rightarrow R^2 > \frac{4L}{C} \]
Transients in RLC circuit. Over-damped solution: $b^2 > 0$

Taken in account the initial conditions: $q(0) = q_0$ and $i(0) = 0$

\[ q(t) = q_0 e^{-at} \left( \cosh bt + \frac{a}{b} \sinh bt \right) \]

\[ \frac{q_0}{2} \left( 1 + \frac{a}{b} \right) e^{-(a-b)t} \]

\[ i(t) = -\frac{q_0}{2} \left( \frac{a^2 - b^2}{b} \right) e^{-(a-b)t} \]

This is exponential decay function
Transients in RLC circuit. Critically-damped solution: \( b^2 = 0 \)

For this case the general solution can be found as

\[ q(t) = (A_2 + B_2 t)e^{-at}. \]

Applying the same initial condition, the current can be written as

\[ i = -a^2 q_0 t e^{-at}. \]

\[ b^2 = 0 \rightarrow R^2 = \frac{4L}{C} \quad \text{and} \quad a = \frac{R}{2L} \]

Critical damped case shows the fastest decay with no oscillations.

Critically-damped conditions for our network.
Transients in RLC circuit. Critically-damped solution: $b^2=0$. Real data analysis.

In this experiment $R=300$ ohms, $C=1\mu F$, $L=33.43\text{mH}$.

The output resistance of Wavetek is 50 ohms and resistance of coil was measured as 8.7 ohms, so actual resistance of the network is $R_a=300+50+8.7=358.7$ ohms.

Decay coefficient:

$$\alpha = \frac{R}{2L} = \frac{358.7}{2\times33.43\times10^{-3}} \approx 5365$$
Now the experimental results:

Calculated decay coefficient $\sim 5385$,

Obtained from fitting $\sim 5820$.

Possible reason – it is still slightly over damped

Calculated $b^2$ is $b^2=2.99e7-2.90e7>0$

Transients in RLC circuit. Critically-damped solution: $b^2=0$. Real data analysis.

Vc $\sim q$, fitting function: $V_c=V_{co}(1+at)e^{-at}$

$V_c \sim q$, fitting function: $V_c=V_{co}(1+at)e^{-at}$
If $b^2 < 0$ we will have oscillating solution. Omitting the details (see Lab write-up) we have the equations for charge and current as:

\[
q(t) = q_0 e^{-at} \left( \cos bt + \frac{a}{b} \sin bt \right) = q_0 e^{-at} \sqrt{1 + \frac{a^2}{b^2}} \sin(bt + \varphi)
\]

\[
i(t) = q_0 e^{-at} \left( \frac{a^2 + b^2}{b} \right) \sin bt
\]

\[
a = \frac{R}{2L}, \quad b = \sqrt{\left( \frac{R}{2L} \right)^2 - \left( \frac{1}{LC} \right)}; f = \frac{1}{2\pi} \sqrt{\left( \frac{1}{LC} \right) - \left( \frac{R}{2L} \right)^2}
\]

Log decrement can be defined as \( \delta = \ln \left( \frac{q(t_{\text{max}})}{q(t_{\text{max}}+T_1)} \right) = \ln \left( \frac{e^{-at_{\text{max}}}}{e^{-a(t_{\text{max}}+T_1)}} \right) = aT_1 \), where \( T_1 = 1/f_1 \)

Quality factor can be defined as \( Q = 2\pi \frac{E}{\Delta E} \),

For RLC \( Q = \frac{\omega_1 L}{R} = \frac{\pi}{\delta} \)

From this plot \( \delta \approx 0.67 \)
\( Q \approx 4.7 \)

1. Pick peaks
2. Envelope
3. Nonlinear fitting

Find peaks procedure

settings used in this find peak session

go here

Time domain trace

Points found using “Find peaks”

Envelope curve

\[ V_C (q/C) (V) \]

\[ \text{time (ms)} \]

\[ y = y_0 + Ae^{-x/t} \]

Fitting the “envelope data” to exponential decay function.
Transients in RLC circuit. Data analysis. 
(1/T)^2 vs 1/C experiment.

\[ q(t) = Ae^{-at} \sin(\omega t + \phi) + \text{offset} \]

Manual evaluation of the period of the oscillations

Limited accuracy

Results can be effected by DC offset

Zero crossing points
Transients in RLC circuit. Data analysis. 
$(1/T)^2$ vs $1/C$ experiment. Using Origin software.

$q(t) = Ae^{-at}\sin(\omega t + \varphi)$

$U_c = \frac{q(t)}{C}$

Use Origin standard function

Category: Waveform

Function: SineDamp

Fitting function ; $y0, A, t_0, xc, w$ – fitting parameters

From fitting you can get: $a = \frac{I}{t_0}$ and $T = \frac{I}{f} = 2w$
Transients in RLC circuit. Data analysis. 
$(1/T)^2$ vs $1/C$ experiment. Using Origin software.

$$q(t) = Ae^{-at}\sin(\omega t + \varphi)$$

Data plot + fitting curve

Residuals - criteria of quality of fitting

\[ q(t) = A e^{-at} \sin(\omega t + \varphi) \]

\[ f^2 = \left( \frac{1}{T} \right)^2 = \left( \frac{1}{2\pi} \right)^2 \left( \frac{1}{LC} - \left( \frac{R}{2L} \right)^2 \right) \]

**Final results**

\[ L_{\text{calc}} = 33.44 \text{mH} \]

\[ L_{\text{meas}} = 33.43 \text{mH} \]
Resonance in RLC circuit.

\[ Q = \frac{f}{\Delta f} = \frac{1904}{1500} = 1.26 \]
Origin templates for this week Lab.

\engr\file-03\phyinst\APL Courses\PHYCS401\Common\Origin templates

Open template button

\begin{itemize}
\item\begin{itemize}
\item Spectrum.otp
\item T^(-2) vs C^(-1).otp
\item Time trace1.otp
\end{itemize}
\end{itemize}

\begin{itemize}
\item $\Delta f = \ldots$ Hz
\item $L_{\text{calc}} = \ldots \text{mH}$
\item $L_{\text{meas}} = \ldots \text{mH}$
\end{itemize}
Very short and simple manual which covers only main general operations with Origin. Document located on server and there is a link from P401 WEB page

There are also manuals from OriginLab.

Do not forget about Origin Help

Video Tutorials at the site of the company