University of Illinois at Urbana-Champaign Department of Physics

Physics 401 Classical Physics Laboratory

Experiment 34

QUALITATIVE STUDIES WITH MICROWAVES

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Introduction

The purpose of this experiment is to acquaint the student with the properties of electromagnetic waves. Microwaves are well suited for this purpose because the wavelength and the dimensions of the apparatus are convenient for bench use. Properties of the radiation, such as its polarization and its reflection by various materials, can also be demonstrated directly and simply.

The Lectronic Research Labs Microwave Training Kit is a convenient source of microwaves with a wavelength of about 3.5 cm. The radio–frequency power generated by a Klystron is fed into a waveguide and is radiated into the room through a small horn. We will not be concerned with the generation of these waves but will study some of their properties.

Propagation of electromagnetic waves

We will choose experimental situations which are closely analogous to the transmission and reflection of waves on lines. This is the case for plane–dielectric boundary.

Electromagnetic waves in free space or in an ideal dielectric satisfy Maxwell's equations with the density of free charge and the conduction current set equal to zero. In SI units

$$\nabla \cdot \vec{D} = 0 \tag{1}$$

$$\nabla \cdot \vec{B} = 0 \tag{2}$$

$$\nabla \times \vec{E} = -\frac{\partial \vec{B}}{\partial t} \tag{3}$$

$$\nabla \times \vec{H} = \frac{\partial \vec{D}}{\partial t} \tag{4}$$

The constants μ and ε are given by $\mu = \kappa_m \mu_o$ and $\varepsilon = \kappa_e \varepsilon_o$. κ_m and κ_e are the relative permeability and permittivity of the medium. It will be assumed that κ_m and κ_e are constants. If we consider a uniform plane wave traveling in the z direction, there can be no variation of $\vec{D}$ or $\vec{H}$ with x and y. Since, from Eq. (1)

$$\nabla \cdot \vec{D} = \varepsilon \left[\frac{\partial E_x}{\partial x} + \frac{\partial E_y}{\partial y} + \frac{\partial E_z}{\partial z} \right] = 0$$

and $\vec{E}$ does not vary with x or y, E_z must either be constant or zero. In the same way we can show that H_z is a constant or zero. If we disregard constant fields, we have a purely transverse wave. The two curl equations (3) and (4) require that $\vec{E}$ and $\vec{H}$ are perpendicular. If we choose the x direction to lie along $\vec{E}$, $\vec{H}$ will be in the y direction, as in Fig. 1.

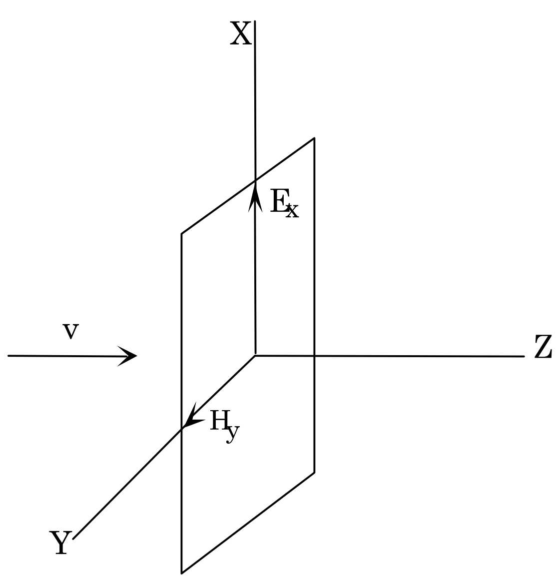


Fig. 1. Plane wave propagating in z direction

Eq. (4) then reduces to

$$\frac{\partial H_y}{\partial z} = -\varepsilon \frac{\partial E_x}{\partial t} \tag{5}$$

$$\frac{\partial E_x}{\partial z} = -\mu \frac{\partial H_y}{\partial t} \tag{6}$$

Differentiating (6) with respect to z, utilizing the relation

$$\frac{\partial^2}{\partial t\,\partial z} = \frac{\partial^2}{\partial z\,\partial t}$$

and substituting Eq. (5), we obtain

$$\frac{\partial^2 E_x}{\partial z^2} = \varepsilon\,\mu \frac{\partial^2 E_x}{\partial t^2}. \tag{7}$$

This is the equation for the propagation of the field E_x in a plane wave, and has the general solution

$$E_x(z,t) = f\left(t - \frac{z}{v}\right) + g\left(t + \frac{z}{v}\right), \tag{8}$$

where $v = 1/\sqrt{\varepsilon\,\mu}$ is the velocity of propagation of the wave. Similar equations can be obtained for H_y.

If we let $u = \left(t - \frac{z}{v}\right)$, and express the arbitrary function $f(u)$ as $E_x(u)$ and the arbitrary function representing the magnetic field as $H_y(u)$, we can write Eq. (5) in the form

$$\frac{dH_y}{du}\frac{\partial u}{\partial z} = -\varepsilon \frac{dE_x}{du}\frac{\partial u}{\partial t}, \tag{9}$$

where $\dfrac{\partial u}{\partial t} = 1$ and $\dfrac{\partial u}{\partial z} = -\dfrac{1}{v} = -\sqrt{\varepsilon\mu}$. When these substitutions are made, Eq. (9) becomes

$$dH_y = \sqrt{\frac{\varepsilon}{\mu}} dE_x \tag{10}$$

and after integration we have

$$H_y = \sqrt{\frac{\varepsilon}{\mu}} E_x \tag{11}$$

where we again have discarded the constant of integration. This simple relation between E_x and H_y indicates that these are identical functions of $\left(t - \dfrac{z}{v}\right)$ except for a proportionality factor. When (11) is written in the form $E_x = Z\,H_y$ we can see that, since $\dfrac{E_x}{H_y}$ has the dimension of $\dfrac{volts}{amperes}$, $Z = \sqrt{\dfrac{\mu}{\varepsilon}}$ has the units of ohms. Z is sometimes referred to as the characteristic impedence of the medium. For the backward–going wave described by a function of $\left(t + \dfrac{z}{v}\right)$, the same ratio $\dfrac{E_x}{H_y}$ is obtained except for a negative sign.

$$\left(\frac{E_x}{H_y}\right)_{forward} = +\sqrt{\frac{\mu}{\varepsilon}} \qquad \left(\frac{E_x}{H_y}\right)_{backward} = -\sqrt{\frac{\mu}{\varepsilon}} \tag{12}$$

The impedaence of the medium is always taken to be positive. For free space it takes on the value

$$Z_{free\ space} = \sqrt{\frac{\mu_o}{\varepsilon_o}} \approx 377\,ohms\ ,$$

and for a medium with relative dielectric constant $\kappa_e = \varepsilon/\varepsilon_o$ it is

$$Z_{dielectric} = \frac{377\,ohms}{\sqrt{\kappa_e}} \tag{13}$$

If waves from free space impinge on a medium with a characteristic impedence of 377 ohms, there is no reflection of the incident radiation. If, on the other hand, the radiation impinges normally on a dielectric medium with characteristic impedence different from 377 ohms, there is reflection at the interface.

Reflection of waves at a boundary

The electric intensity vector has a phase shift of 180° when the wave is reflected from a boundary having lower impedance, but it has no phase change if the medium beyond has a higher impedance. The magnitude of the reflection and transmission coefficients can be determined by using the boundary conditions for $\vec{E}$ and $\vec{H}$ as outlined below. A reflecting power can also be defined to represent the fraction of the power which is reflected.

The tangential components of $\vec{E}$ and $\vec{H}$ must be continuous across the interface,

$$E_{T1} = E_{T2}, \text{ and } \qquad H_{T1} = H_{T2} \tag{14}$$

Let the subscripts *i*, *r*, and *t* refer to the incident, reflected, and transmitted waves at a plane boundary between two media. For normal incidence, if we use the directions of the waves as shown in Fig. 2.

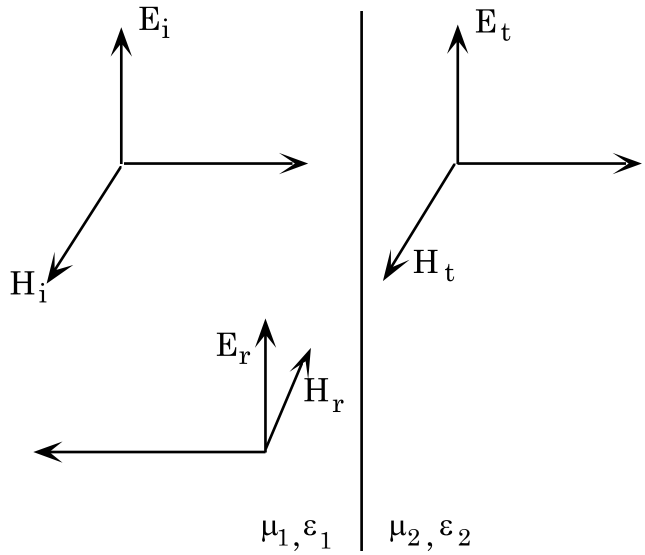


Fig. 2. Reflection at a plane boundary

$$E_i + E_r = E_t \tag{15}$$

$$H_i - H_r = H_t \tag{16}$$

If the two media are uniform, linear, isotropic dielectrics, one has

$$\varepsilon_1 = \kappa_{e1}\varepsilon_o, \qquad \varepsilon_2 = \kappa_{e2}\varepsilon_o, \qquad \text{and } \mu_1 = \mu_2 = \mu_o.$$

From Eq. (12), Eq. (16) can be written for this case

$$\sqrt{\kappa_{e1}}E_i - \sqrt{\kappa_{e1}}E_r = \sqrt{\kappa_{e2}}E_t \tag{17}$$

Eqs. (15) and (17) can be solved for the reflected and transmitted fields in terms of the incident field. The results are

$$\frac{E_r}{E_i} = \frac{\sqrt{\kappa_{e1}} - \sqrt{\kappa_{e2}}}{\sqrt{\kappa_{e1}} + \sqrt{\kappa_{e2}}} = \frac{Z_2 - Z_1}{Z_2 + Z_1} \tag{18}$$

$$\frac{E_t}{E_i} = 1 + \frac{E_r}{E_i} = \frac{2\sqrt{\kappa_{e1}}}{\sqrt{\kappa_{e1}} + \sqrt{\kappa_{e2}}} = \frac{2Z_2}{Z_2 + Z_1} \tag{19}$$

Where $Z_1 = \frac{1}{\sqrt{\kappa_{e1}}}\sqrt{\frac{\mu_o}{\varepsilon_o}}$ and $Z_2 = \frac{1}{\sqrt{\kappa_{e2}}}\sqrt{\frac{\mu_o}{\varepsilon_o}}$. These relations are the same as those for the reflection and transmission of signals at the junction between two cables having characteristic impedances Z_1 and Z_2. Other relations, however, are needed in discussing waves incident at arbitrary angles.

Experimental apparatus

The basic features of the apparatus are shown in Figure 7.

The transmitter arm (T) consists of a microwave oscillator (a klystron tube or a solid state generator which generates the microwaves), a frequency meter, an attenuator, and a diode detector. The oscillator is powered from the LRL 510A power supply. Turn on the power with the switch at the lower right on the control box (“A/C power on/off ”). A solid state microwave generator will generate microwaves immediately. For the klystron, wait for the apparatus to warm up for a few minutes (typically 1-3 minutes). Then turn on the switch “RF on/off”, a switch located at the lower left on the control box to generate the microwaves.

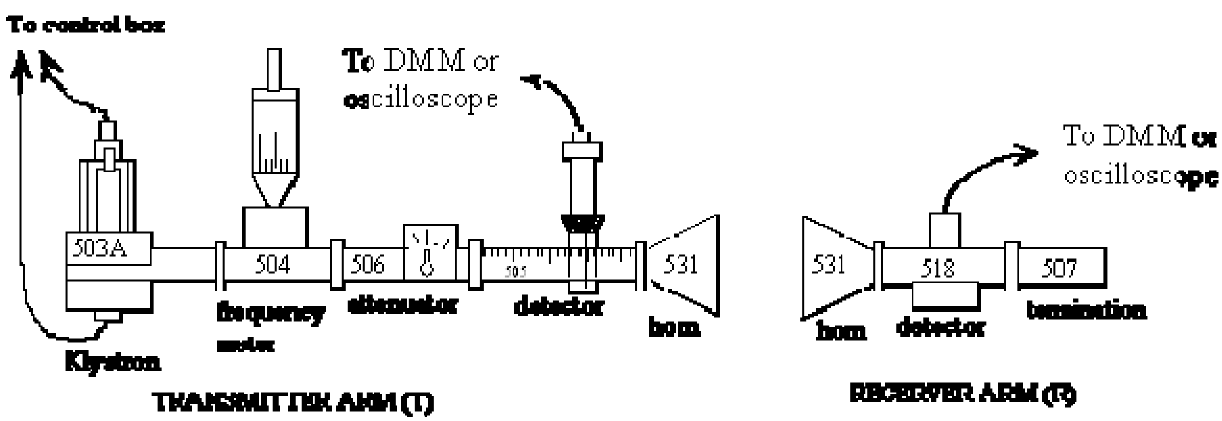


Figure 7. Basic experimental apparatus setup.

Please read cautions carefully before continuing.

Caution 1:

The klystrons get extremely hot when in use and must not be handled while hot! Serious burns can result.

Caution 2:

The klystron mount, power supply, and klystron tube plate caps have high voltages present when in use. Exercise extreme caution! Shock or death can result.

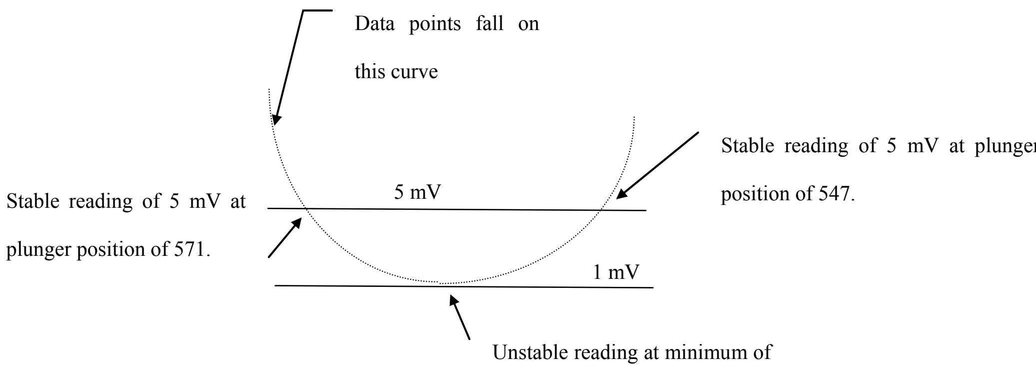
Caution 3:

The microwave power levels in this kit are not harmful, but a human eye can be damaged by low levels of radiation. Do not look into any waveguide at any time when units are on.

The frequency and amplitude of the microwaves are adjusted by the repeller voltage. The “Klystron repeller” should be adjusted occasionally to maximize the signal at the detector, but do not adjust it in the midst of a series of measurements.

Calibrating the Klystron Frequency

The frequency meter is an absorption cavity whose size is adjusted by the micrometer dial. When the cavity size is adjusted for resonance, the current at the receiving detector (or oscilloscope) will be at a minimum. Occasionally, it may be difficult to locate the signal minimum accurately. In this case, better results usually can be obtained by measuring the plunger positions of points on both sides of a voltage minimum and find the mean value of it as shown in Figure 8. Use the calibration formula found on the meter barrel to convert the micrometer reading into a frequency. The micrometer dial should start from the top. This usually will lift up the plunger to the top-most position and the dial should read over 800 counts (usually 812 or 820). Each smallest count of the wavemeter dial is equal 0.001 inch of plunger travel. Maximum travel of the plunger for a typical wavemeter is about 0.810 in. The attenuator can be used to control the amplitude of the transmitted microwave power. **Note, after calibrating the frequency, the micrometer must be moved far off resonance in order for the microwaves to exit the horn.**



Plunger position of at minimum = (571 + 547) / 2 = 559

Fig. 8. Accurate measurement of minimum position. The numbers correspond to a frequency meter dial reading, shown as an example.

The detector responds to the component of the electric field parallel to the shortest dimension of the waveguide in which it is located. The dc detector current is obtained by using a diode which generates a current approximately proportional to the square of the voltage across it. A cable carries the signal from the detector on the transmitter arm to either a DMM or a digital oscilloscope. The voltage measurement option of the oscilloscope can be very useful.

The receiver arm (R) consists of another diode detector followed by a matched load that prevents reflections. The current generated by this detector is directly read by the DMM or digital oscilloscope, typically set to the low mV range.

Both the transmitter arm and the receiver arm are fitted with horns, which act as antennas for sending signals through free space.

Procedure

There are six separate setups, and laboratory partners will rotate from one table to another.

I. Standing Waves

A. Refer to Figure 9. Use the detector in the transmitter arm. Set the attenuator in the range 6–12 dB. Use a metal plate to reflect microwaves back toward the detector. Keep the metal plate perpendicular to the direction of propagation of the microwaves. Record the positions of five to ten minima.

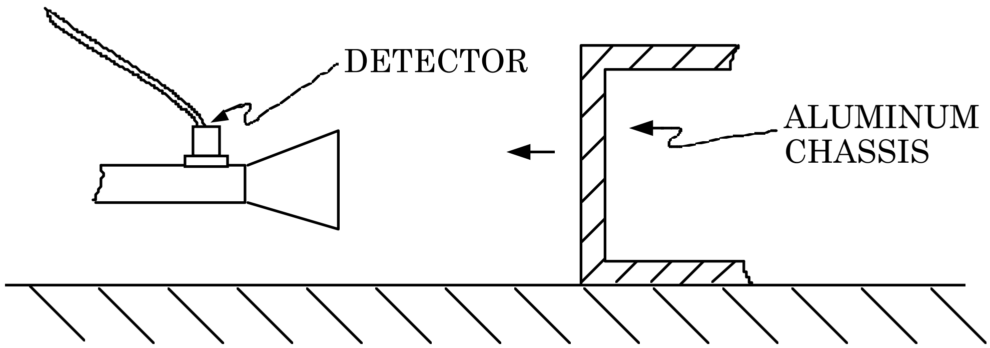


Figure 9. Schematic diagram of standing waves measurement.

The distance between successive minima (or maxima) is the half–wavelength of the radiation in air. Calculate this wavelength from your data.

B. Refer to Figure 10 for this exercise. Place the detector horn directly in front of the transmitter at distance of about 20 cm. Find the point between the two horns, and find a line through this point and perpendicular to the line between the two horns. Position a metal plate on this line, and move the plate away from the line between the two horns with the reflecting surface parallel to the line between the two horns. Microwaves radiated from the transmitter horn will be reflected by this plate into the detector horn. These waves will interfere with the waves that propagate directly from the transmitter horn to the reflector horn.

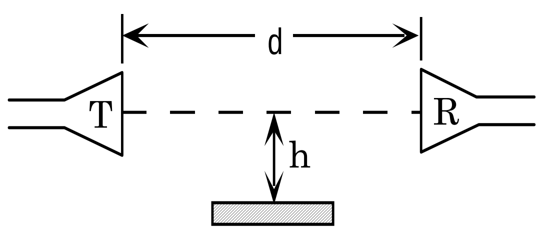


Figure 10. Standing waves measurement using a vertical reflector.

(a) Record the separation between the two horns, *d*, and several successive values of *h* at which the detector current is at a minimum.

(b) Calculate the phase difference between the direct and reflected waves due to the difference in path lengths and the 180° phase change upon reflection at the conductor in terms of d, h and λ. Find an expression for h when the detector current is a minimum.

(c) Use this expression to calculate values for h at which the detector current is a minimum using the wavelength obtained from part (A).

(d) Tabulate your calculated and experimental h values and compare.

II. Polarization

A. Place the detector horn on the rotatable ring, directly in front of the transmitter at about 20 cm away. Rotate the detector horn in steps of 15°, and record the detector current. Cover the range ϕ = 0° to 180°. Recall that the polarization of the microwaves radiated from the transmitter horn is parallel to the short dimension of the waveguide. This direction is vertical for the normal orientation of the waveguide.

Plot the detector current as a function of ϕ, and compare the plot to $I = K\cos^2\phi$, which is expected if the detector current is proportional to the square of the electric field component parallel to the axis of the diode.

B. Place the detector horn at 90° with respect to the transmitter. Record the variation in detector current as a grid mounted just in front of the detector is rotated through various azimuthal angles, from vertical to horizontal. Explain the results qualitatively in terms of current induced in the grid. Record the maximum current obtained, and the approximate angle. (You will need to record the detector current for the case of no grid, and at the angle 90° between detector and transmitter.)

C. Qualitatively observe reflections from the grid as its azimuthal angle is varied.

III. Michelson interferometer

Use two horns, two metal reflecting surfaces, and the partially transmitting mirror to build the Michelson interferometer, as shown in Figure 11. Align components carefully, with mirror C at 45°.

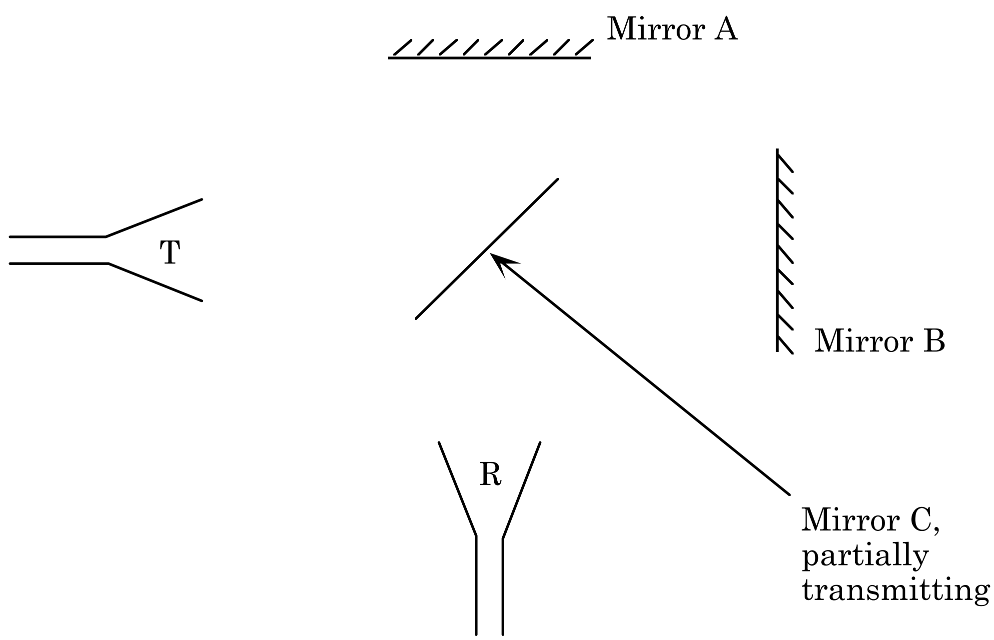


Figure 11. Michelson Interferometer setup.

A. Leaving mirror A fixed, move mirror B, recording its position for several minima and maxima to measure λ_{air}.

B. Adjust mirror B to give a minimum detector current. Record its position. Measure the thickness of the provided Lucite sheet (about 6 mm thickness). Insert that Lucite sheet between mirrors A and C, and readjust the position of B to again give a minimum (B will need to be moved away from C). Using the relationship that the optical path length is the product of the index of refraction and the thickness of the medium, calculate the increase in path length due to the addition of the Lucite sheet. From this, calculate the index of refraction of Lucite at these microwave frequencies.

IV. Total internal reflection

The relation between the angle of incidence and the angle of refraction is given by Snell's law. $n_1 \sin\theta_1 = n_2 \sin\theta_2$. If the wave propagates from medium 1 to medium 2 with $n_1 > n_2$, then there is no refracted wave for θ_1 beyond a critical angle, $\theta_c = \sin^{-1}(n_2/n_1)$. Beyond the critical-angle incidence, there is a total internal reflection at the interface.

Using two horns and two paraffin prisms, assemble the configuration as shown in Fig. 12. Vary the angle θ between the prism by adjusting the turntable starting from θ=0° and going beyond the critical angle $\theta_c = \sin^{-1}(n_2/n_1)$. Record and plot the receiver current as a function of θ. In carrying out the measurements it is important make sure that you only turn the turntable, keeping the geometry fixed between the receiver horn and the prism. Determine the index of refraction of the plastic prism.

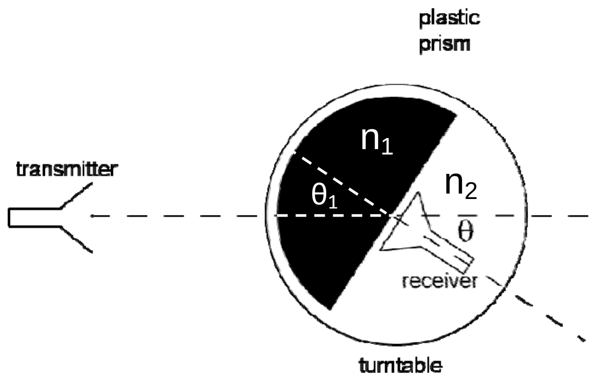


Figure 12: Schematic of total internal reflection experiment.

V. <u>Two Slit Interference</u>

Consider a uniform phase front of radiation, emitted from a point source, incident on two slits adjacent slits shown in Figure 13. We assume that the distances r_1 and r_2 are large compared to the slit width b and slit separation d. The field amplitude exiting each slit is given by $\psi(r,t)=\psi_0\cos(\omega t-kr)$. We can express the field amplitude at point P as the sum of the amplitudes from each slit.

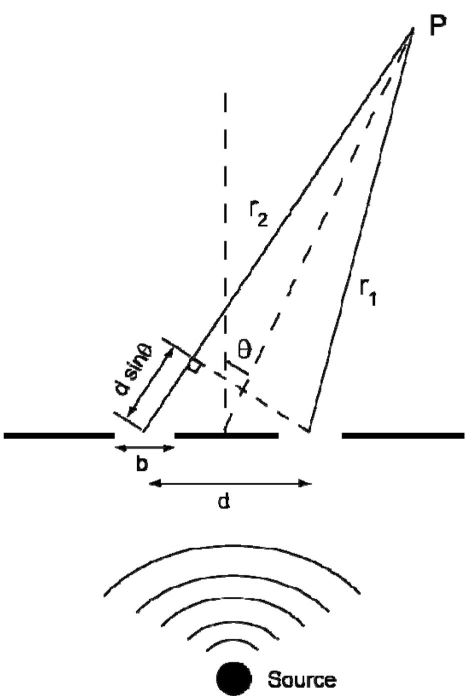
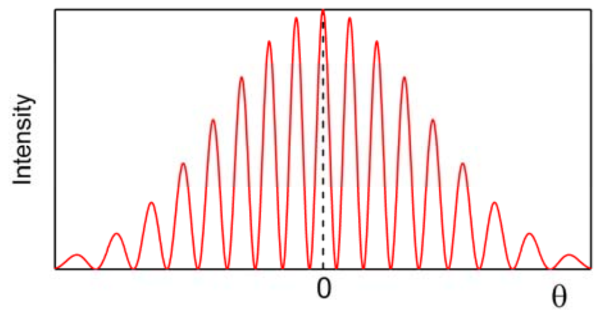


Figure 13: Schematic of 2-slit

$$\psi_p=\psi_0\left[\cos(\omega t-kr_1)+\cos(\omega t-kr_2)\right]$$
$$\psi_p=2\psi_0\cos\omega t\cos\left[k(r_2-r_1)/2\right]$$

where, $r_2-r_1\approx d\sin\theta$.

Thus, $\psi_p=2\psi_0\cos\omega t\cos\left[kd\sin\theta/2\right]$. The square law detector used in the experiment responds to the square of the amplitude, therefore, the measured signal will be proportional to $\left|\psi_p\right|^2=4\left|\psi_0\right|^2\cos^2\left[kd\sin\theta/2\right]$. Maxima occur when $d\sin\theta_n=n\lambda$, where n is an integer and θ_n is a particular value of θ which satisfies the above relationship. Experimentally, we observe the double slit pattern modulated by the single slit diffraction pattern given by $\left|\psi_{ss}\right|^2=\left|\psi_0\right|^2\left(\frac{\sin x}{x}\right)^2$, where $x=kb\sin\theta/2$.



2-slit diffraction pattern showing single-slit modulation.

The observed double slit patter is thus given by

$$\left|\psi_p\right|^2 = 4\left|\psi_0\right|^2 \left(\frac{\sin x}{x}\right)^2 \cos^2\left[kd\sin\theta/2\right].$$

For your measurement, vary the angle θ in steps of 5° over the angular range permitted by your setup (You may need to take finer steps near the maxima and minima in order to accurately determine the peak location). You should be able to observe approximately 5 peaks in the central portion of the interference pattern. In our experimental setup, it is very difficult quantitatively verify the single slit modulation pattern. From the location of the interference peaks, you should calculate the argument $kd\sin\theta/2$ using the measured values for k and d. Compare your findings with the theoretical angular values corresponding to the maxima and minima locations.

VI. <u>Bragg Diffraction of Microwaves</u>

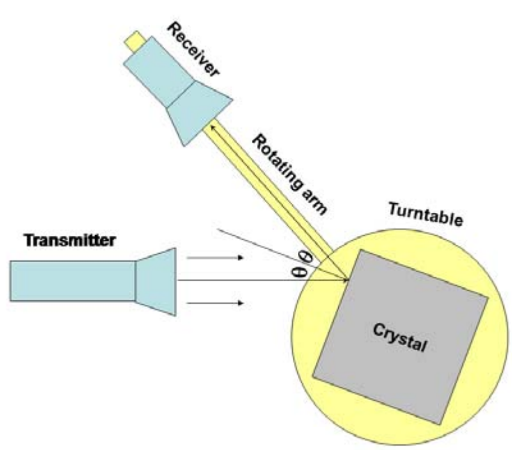


Figure 14: Schematic of Bragg diffraction experiment.

In this experiment, the diffraction of waves by a periodic lattice is studied using microwave radiation and model of a simple cubic lattice consisting of an array of steel balls. The microwave radiation is generated using a reflex klystron.

Bragg diffraction is generally associated with diffraction of X-rays from crystals where the wavelengths of the electromagnetic radiation is of the order of the inter-atomic distances in the solids, ie., of the order of 1 Å. In this experiment the wavelength of the microwave radiation is of the order of the spacing between the steel balls in the model cubic lattice. The basic experimental set-up is shown in Figure 14.

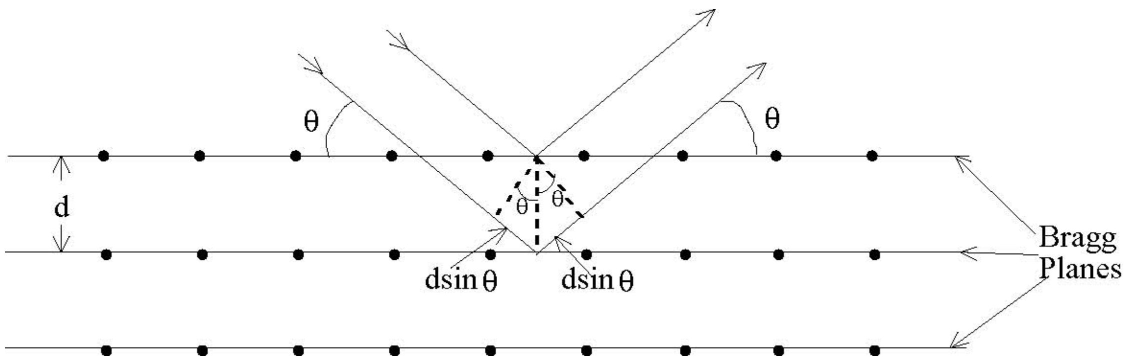


Figure 15: Diffraction from atomic planes.

Bragg Diffraction

In 1912, Max von Laue first suggested that crystals consisting of regularly spaced atoms might be used as diffraction gratings for X-rays. Shortly after, in 1913 Sir William Bragg, together with his son W. L. Bragg, developed a basic theory which explained X-ray diffraction in terms of reflections from families of atomic planes, called Bragg planes.

Consider the reflection of waves from the parallel rows of atoms forming a set of Bragg planes as shown in Figure 15. Suppose that the planes are a distance d apart. Then a portion of the incident beam is reflected from the first plane and a portion from the second plane, and so forth, such that the angle of reflection θ equals the angle of incidence. The reflected waves from successive layers of atoms then interfere constructively or destructively depending on the path difference between them. For most angles, the waves are out of phase and interfere destructively. However, if the path difference 2dsinθ is equal to an integral multiple n of the wavelength λ , constructive interference occurs for that particular angle. A strong reflection maximum occurs satisfying the equation $2d\sin\theta = n\lambda$, where n is an integer number indexing the order of the maximum. This equation is known as Bragg's Law and applies to the diffraction of any collimated, coherent waves from a periodic structure - whether the waves consist of X-rays, or wave-like particles such as electrons.

Miller Indices

Many different sets of parallel planes can be drawn through the atoms in a real crystal, each with a different value of d. Some examples for a simple cubic lattice are shown in Figure 16 where the various ordered numbers (100), (110), (210) etc. represent so-called "Miller indices" for the Bragg planes.

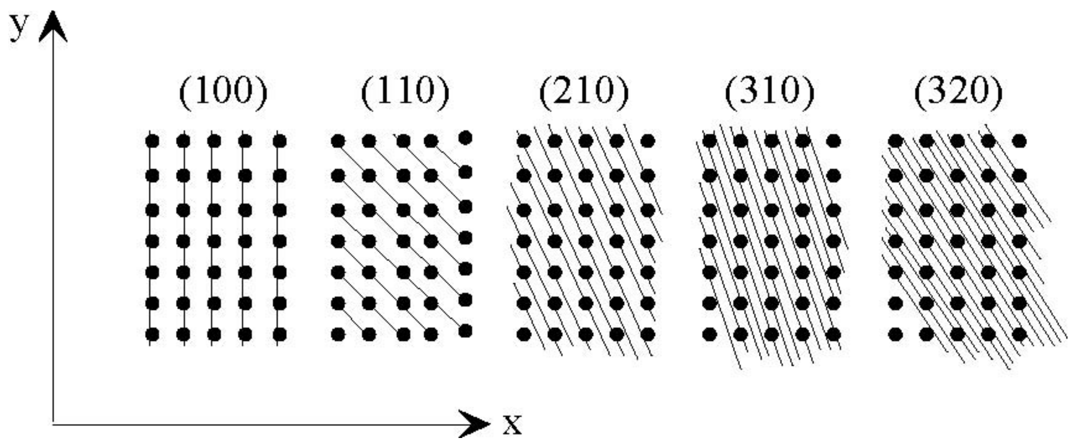


Figure 16: Various crystal orientations and the corresponding Miller indices.

Miller indices, named after the British crystallographer W.H. Miller, are used to describe the orientation of a plane within a crystal lattice. Each type of lattice (eg. cubic, tetragonal, hexagonal, etc.) is characterized by a "unit cell" which repeats itself throughout the crystal. The size and shape of the unit cell is represented by three vectors $\vec{a}$, $\vec{b}$, $\vec{c}$ and these, in turn, define the crystallographic axes (Figure 17). In general these vectors are not necessarily of equal length nor need they be at right angles to each other, though they will be for the simple cubic lattice shown in Figure 16.

Miller indices (*hkl*) for a particular plane are obtained from the intercepts that the plane makes relative to the a,b,c axes. Referring to Figure 18, consider the plane passing through the three axes with intercepts (3a,0,0), (0,4b,0), (0,0,2c) where the values are multiples of $|\vec{a}|$, $|\vec{b}|$, and $|\vec{c}|$ respectively. To obtain the Miller indices, take the reciprocal of each intercept, clear the fractions by bringing them to a common denominator, and enclose the numerators in parentheses as shown:

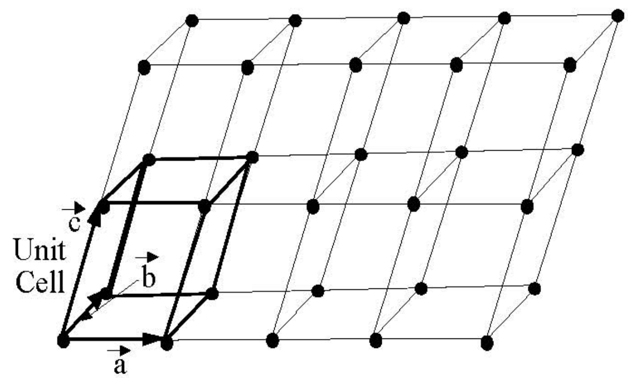


Figure 17

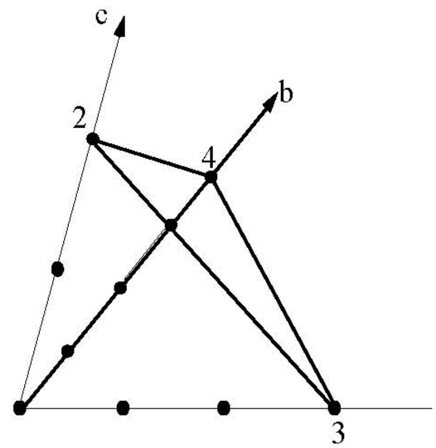


Figure 18

$$3a, 4b, 2c \rightarrow 3, 4, 2 \rightarrow \frac{1}{3}, \frac{1}{4}, \frac{1}{2} \rightarrow \frac{4}{12}, \frac{3}{12}, \frac{6}{12} \rightarrow (436)$$

The Miller indices of the plane above are (436). The plane with intercepts a = 1, b = 1, c = ∞ has Miller indices (110). For a simple cubic lattice with unit cell dimension a in each direction, it can be shown that the spacing d between successive planes (*hkl*) is given by

$$d = \frac{a}{\sqrt{h^2 + k^2 + l^2}}.$$

Wavelength of the Microwave Radiation

Prior to making measurements, you need to determine the wavelength of the microwave source. Determine the wavelength by calibrating the frequency of the klystron using the procedure described in the beginning of the lab manual. The wavelength is related to the frequency by $\lambda \nu = c$, where $c = 3.0 \times 10^8 m/s$ is the speed of light in air.

Bragg Reflections from the Model Crystal Lattice

Carefully align the transmitter and receiver horns so they face each other and are directed towards the center of the crystal. Since spurious reflections are a nuisance in this experiment, students should stand away from the apparatus, preferably behind the transmitter and receiver. In the setup, the transmitter arm is fixed and the receiver arm and crystal can be independently rotated with respect to the transmitter. By changing the relative angle between the transmitter and receiver arms and the normal to the crystal surface, different crystallographic planes can be measured. In this experiment, we will measure the (100) orientation which requires the transmitter and the detector to be placed at equal angles with respect to the crystal normal (see Figure 14.) For this measurement, you will vary θ by rotating both the goniometer platform which rotates the crystal and the receiver arm. Take closer data eg. (every 0.5°) when a noticeable Bragg reflection peak occurs. Measure the first three orders in the diffraction and determine the spacing between the crystal planes for the (100) orientation.

After the Bragg diffraction measurements are completed, measure the spacing between the steel balls and obtain an average value of the lattice constant for the cubic array.

Using the values of θ from your graph, calculate the interplanar spacing d for the (100) orientation. Identify the Bragg diffraction peaks on your graphs and label the corresponding values of the diffraction order n. Compare your findings to the actual value of the lattice spacing.

Report

Answer all questions in each part. Provide necessary plots. No error analysis is required.