University of Illinois at Urbana-Champaign Department of Physics

Physics 401 Classical Physics Laboratory

Experiment 44

MICROWAVE CAVITIES

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Resonant frequency

The resonant (angular) frequencies, ω_{mnp}, of the TE modes in a rectangular cavity of dimensions ; *a*, *b*, and *c* are

$$\omega_{mnp}{}^2 = v_o{}^2\left[\left(\frac{m\pi}{a}\right)^2 + \left(\frac{n\pi}{b}\right)^2 + \left(\frac{p\pi}{c}\right)^2\right]. \qquad (1)$$

In the above expression *m*, *n*, and *p* are integers, no more than one of which can be zero at the same time; and v_o is the phase velocity in infinite space of the medium filling the cavity. v_o is the speed of light in vacuum for an empty cavity. (Since we use *c* for one of the cavity dimensions, we cannot also use it for the speed of light in vacuum.)

If *b* is taken as the shortest dimension, the lowest mode (lowest resonant frequency) is obtained for $m=1$, $n=0$, and $p=1$. This mode is called the TE_{101} mode. For this mode, Eq. 1 takes the simpler form

$$\omega_{101}{}^2 = v_o{}^2\pi^2\left[\left(\frac{1}{a}\right)^2 + \left(\frac{1}{c}\right)^2\right]. \qquad (2)$$

The electric and magnetic field configurations in the TE_{101} mode can be represented schematically as in Fig. 1 where *a*, *b*, and *c* are in the *x*, *y*, and *z* directions, respectively.

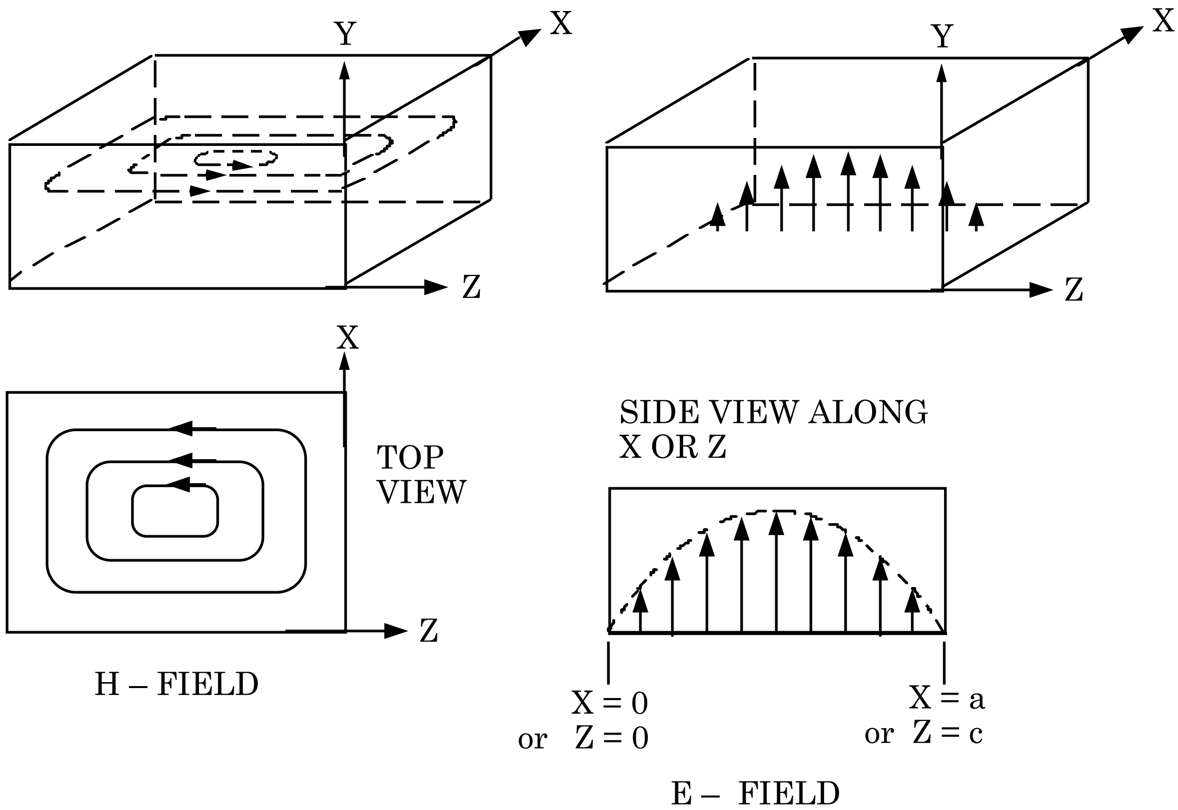


Fig. 1. Field configurations in the TE_{101} mode

The *Q* of a cavity

Resonant cavities will lose energy by power dissipation in the walls and by radiation. The losses are usually discussed in terms of the quality factor *Q* defined in the same way as for a lumped RLC resonant circuit:

$$Q = 2\pi \frac{\text{total stored energy}}{\text{decrease in energy per period}} . \qquad (3)$$

Recall that the *Q* of a RLC circuit is

$$Q = \frac{\omega L}{R} . \qquad (4)$$

where the resonant frequency, ω, is $\omega = \sqrt{\left(\frac{1}{LC}\right) - \left(\frac{R}{2L}\right)^2}$. For small damping (high Q) the resonant frequency $\omega \approx \omega_o = \sqrt{\frac{1}{LC}}$. It can be shown (see for example, Berkeley Physics Course) that for a high-Q lumped circuit, the Q is also given by

$$Q = \frac{\omega_o}{\Delta\omega} \tag{5}$$

where $\Delta\omega = (\omega_2 - \omega_1)$ is the difference between the two frequencies, ω_1 and ω_2, at which the absorbed power is reduced by a factor of two, or the current is reduced by a factor of root 2 from their maximum values. The same relation is valid for a resonant cavity. Thus, Q can be determined from the frequency response curve. Other methods of determining Q include standing wave measurements as a function of frequency, and the measurement of the decay of the fields as a function of time.

Coupling to a cavity

In order to excite fields in a cavity and to make measurements on it, one or more transmissions lines must be coupled to the cavity. This coupling may be done with an opening, known as an iris diaphragm, an electric-dipole antenna, or a magnetic-dipole antenna. The latter is called a coupling loop. The coupling is usually discussed in terms of the equivalent circuit of Fig. 2.

The coupling coefficient β describes the strength of coupling. In defining β, it is convenient to use the circuit of Fig. 3, in which the cavity is fed by a line of impedance Z_o. Z_o is the impedance of the real line transformed to the cavity terminals by the ideal transformer represented by the inductances, (L, L_o), in Fig. 2. Z_o is essentially a pure resistance. Here β is defined as $\beta = Z_o/R$, and may have any value between 0 and ∞.

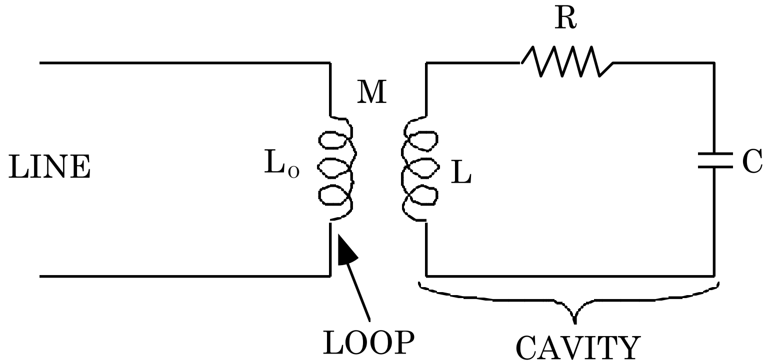


Fig. 2. Equivalent circuit of a cavity and coupling loop

Fig. 3. Transformed Cavity–Coupling System

The Power Transfer Theorem tells us that the greatest power is transferred to a load when the load impedance equals the source impedance. If we apply this theorem to the circuit of Fig. 3 at resonance where the cavity is resistive, we see that maximum power transfer to the cavity occurs when $R = Z_o$, or $\beta = 1$. This condition is called critical coupling. At critical coupling the standing wave ratio at resonance takes on a minimum value, which will be close to unity if the coupling loop is small enough to be non-inductive. The coupling coefficient can be changed by rotating the loop about the axis of the line.

Loaded and unloaded *Q*

When the cavity is coupled to an external power source and detector in order to measure the characteristics of the cavity, some of the losses in the external circuit are added to those in the cavity. The *Q* measured is therefore less than the value characteristic of the cavity alone. The intrinsic *Q* of the cavity is called the unloaded *Q*, often designated as Q_o, and the measured value is called the loaded *Q*, often designated as Q_L. At resonance, the total losses in the system correspond to a series resistance $R + Z_o$ in Fig. 3, and the loaded *Q* is therefore given by

$$Q_L = \frac{\omega L}{R + Z_o}.$$

This may be written as

$$Q_L = \frac{\omega L}{R\left(1 + \frac{Z_o}{R}\right)} = \frac{Q_o}{1+\beta}. \tag{6}$$

If the coupling is adjusted for maximum power transfer ($\beta = 1$), then

$$Q_L = \frac{1}{2} Q_o. \tag{7}$$

Apparatus

To study such a cavity it will be necessary to excite electromagnetic waves inside the cavity. To do this we will set up the apparatus shown in Fig. 4.

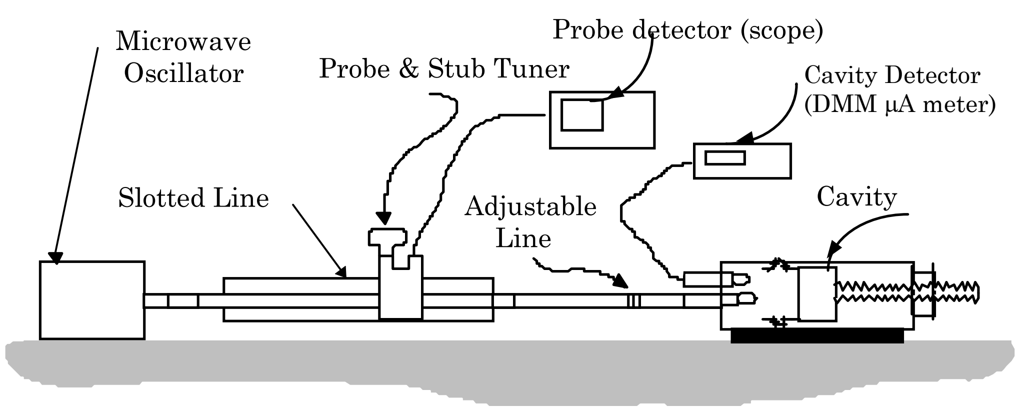


Fig. 4. Apparatus

Microwaves are generated by the solid state oscillator and propagate down the coaxial line and adjustable line to the termination loop in the cavity. An expanded view of the cavity and waveguide termination is shown in a side cross section in Fig. 5.

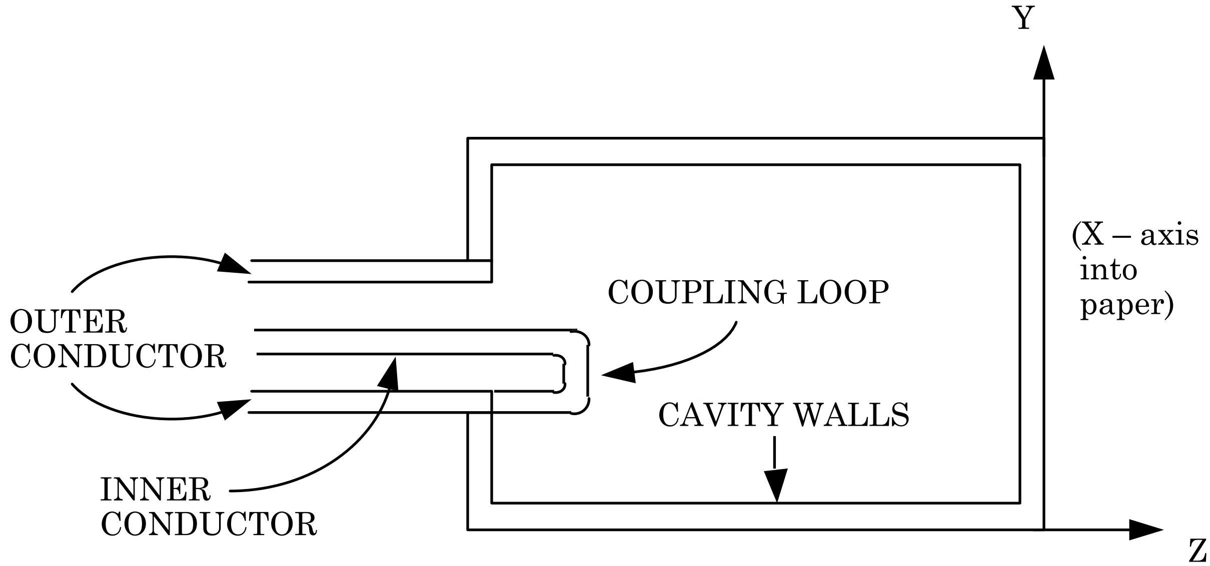


Fig. 5. Side view of the cavity

Notice that currents in the coaxial line will produce magnetic fields in the cavity in planes parallel to the *xz* plane. If these fields are oscillating at the proper frequency, they will satisfy the boundary conditions and standing waves will be observed.

<u>Experimental procedure</u>

I. Wavelength and frequency measurement

Record the model number, serial number and the frequency range of the microwave oscillator. Observing the correct polarity, plug the microwave oscillator into the 20 VDC power source on the terminals near the bench. The shorted tuning stub on the probe should have already been adjusted. In case it is necessary, it should be inserted approximately half way into its mount. Its position should be adjusted to provide maximum signal on the scope.

The scope is connected to the diode detector which is positioned inside the coaxial slotted line to detect the electric field. The diode rectifies the voltage and provides a DC current. Since the scope has large input impedance, you should be able to see the corresponding voltage on the screen. The minimum is defined as the voltage closest to the ground level and the opposite is true about the maximum. Adjust the horizontal and vertical sensitivities appropriately. It is convenient to use the measurement option to read the voltage. You may move the probe along the slotted line to a maximum of the detector current to adjust for the optimum scope setting. If it is necessary, reduce the sensitivity of the detector by shifting the position of the tuning stub on the probe.

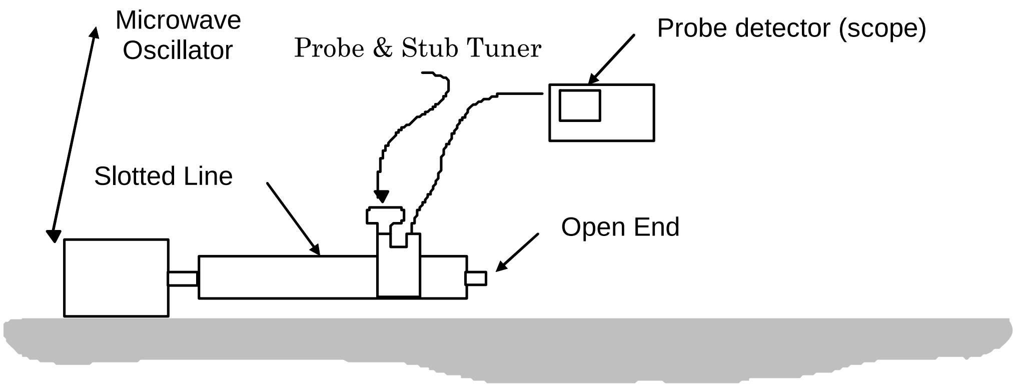


Fig. 6. Wavelength measurement with the slotted line.

Determine the oscillator frequency, f_o, by measuring the wavelength in the slotted line. Remember that the wavelength in a coaxial line is equal to the wavelength in air and is equal to twice the distance between successive minima. Find the minima on either end of the slotted line, and count the number of minima in between them to find the number of half wavelengths in the line. Make the measurement of wavelength carefully, since an accurate value of the frequency is needed. It should be possible to measure wavelength to 0.1%. Calculate the frequency and angular frequency of the oscillator and their uncertainties.

II. Cavity Resonance

Examining Eq. 1, we see that the cavity resonance may be observed by varying the frequency for a cavity of fixed dimensions, or (as we do here) it can be observed for a fixed frequency by changing the physical dimensions of the cavity. (We obtain resonance by varying the right-hand side of Eq. 1, keeping the left-hand side fixed.)

To the open end of the slotted line add the adjustable line with rotating joint, the coupling loop, the cavity and the detector loop, as shown in Fig. 7.

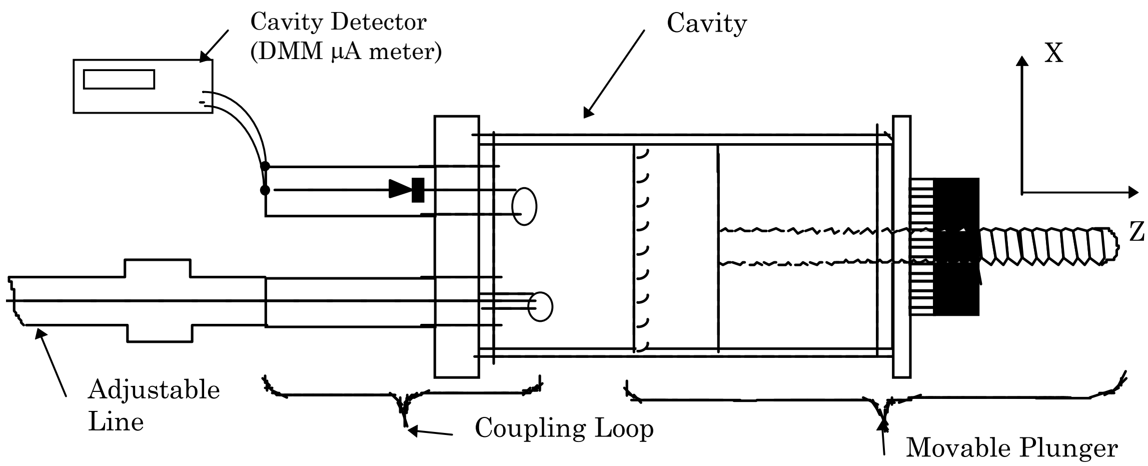


Fig. 7. Top view of the cavity

The plane of the coupling loop should be vertical to produce maximum coupling to the direction of the *H* fields we expect at resonance. The detector loop should be parallel to the coupling loop.

The loop of the cavity field detector acts as a small inductive loop which produces an EMF when placed in an alternating magnetic field. The EMF is rectified by the diode detector, and the current is measured with a sensitive ammeter. This loop is sufficiently far from the input coupling loop so that, except near resonance where large fields are produced in the cavity, the mutual inductance between them is very small.

First locate the position of the TE_{101} resonance approximately by moving the plunger out from the coupling loops in large steps. Use Eq. 1 to show that the TE_{101} mode will resonate when the length of the cavity, c, is about 7.5 cm and the TE_{102} mode will resonate when c is twice this value. After the position of the resonance is located approximately, move the plunger in a few millimeters. Devise an algorithm which gives the length of the cavity, c, in terms of the position of the plunger. The dimensions of the cavity and plunger are printed on the outside of the cavity. Then the plunger should be slowly and smoothly withdrawn by turning the knurled knob in very small steps. Resonance will be observed as a very sharp peak in the response of the cavity field detector. The steps must be small to determine accurately the width of the resonance. Note that near resonance the fields are very strong and this produces large EMFs. If the meter goes off scale, decouple the detector by rotating the plane of the detector loop away from the vertical to decrease the effective area of loop.

Determine the length of the cavity, c, at two resonances, TE_{101} and TE_{102}, and compare your measurements to the value calculated using Eq. 1, and the oscillator frequency f_o measured above.

III. Magnetic Field Direction and Coupling

As was discussed above, the H field of the TE_{101} and TE_{102} modes lies entirely in planes parallel to the xz plane. The magnetic field produced by the current in the loop will have some component in this direction and will therefore tend to excite that mode in the cavity. With the cavity in resonance for the TE_{101} mode, measure the current in the cavity field detector while changing the angle of the input coupling loop plane from the vertical, 0°, through 360° in steps of 10°. Explain the shape of the curve. See part V for a discussion of the response of the field detector to the field.

IV. Electric Field Distribution

The electric field, E_y, in the TE_{101} and TE_{102} modes varies in the z direction is $\sin(\pi z/c)$ as shown in Fig. 1. We will demonstrate this dependence with the following technique. (A more sophisticated version of this same method is used in practice for accurate mapping of the fields of a cavity.)

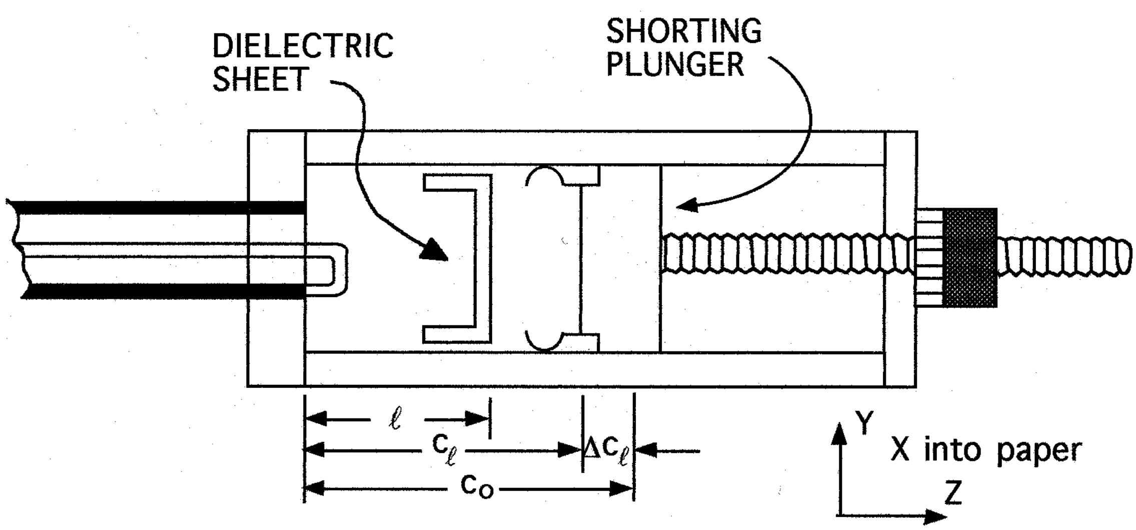


Fig. 8 Side view of cavity

A thin dielectric sheet is placed in the cavity with the plane of the sheet parallel to the *xy* plane, see Fig. 8 above. The sheet is placed a distance ℓ from the end of the cavity at $z = 0$, the end with the coupling loops. The sheet is affected only by the electric field in the *y* direction, E_y, at that position ℓ, since $E_x = E_z = 0$ in the TE_{101} and TE_{102} modes. It can be shown that the dielectric shortens the effective guide wavelength in the *z* direction, and the amount of shortening is proportional to the strength of the electric field at the dielectric position. The full expression contains an integral of the electric field strength squared over the entire cavity volume, so that a simple change in the overall magnitude of the driving microwaves does not also change the effective length of the cavity. The dielectric may be viewed as an addition of a lumped capacitance in parallel. Hence, the *c* dimension of the cavity when in resonance will be shorter as the sheet is placed in progressively stronger fields. Let c_o be the length of the cavity at resonance when no dielectric is present, and let c_ℓ be the length of the cavity at resonance when the dielectric sheet is at ℓ. Let the difference, $|c_o - c_\ell| = \Delta c_\ell$. This is difference in the length of the cavity at resonance due to the presence of the dielectric. If we measure Δc_ℓ as a function of ℓ, we will get an idea of the variation in the field in the *z* direction.

The dependence of the electric field on position in the cavity is more easily seen in the TE_{102} mode than in the TE_{101} mode.

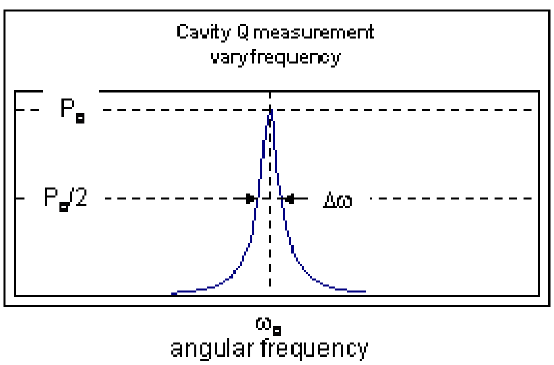
For each position of the dielectric sheet we must open the cavity, position the sheet, close the cavity, and the move the plunger to find the resonance. The dielectric sheet can be conveniently positioned with a simple jig. Open the cavity, insert the dielectric with the feet of the sheet toward the coupling loops, see Fig. 8, and then use the jig to push the dielectric sheet into the cavity to a position ℓ_1 which is 0.5 cm less than c_o. Close the cavity and move the plunger until resonance is observed. Record the position of the plunger, c_{ℓ_1}, and calculate the difference in the length of the cavity due to the presence of the dielectric, Δc_{ℓ_1}. Since the dielectric is in a weak field, this difference will be small. Next open the cavity, and use the jig to push the sheet an additional 0.5 cm toward the coupling loops. Record this position, ℓ_2. Close the cavity, and again move the plunger until resonance is observed. Record position c_{ℓ_2}, and calculate Δc_{ℓ_2}. Always moving the dielectric toward the coupling loops, repeat the process, moving the dielectric by approximately 0.5 cm in each step, until the dielectric is at the coupling loops. Then plot Δc_ℓ versus ℓ.

V. Cavity Q

As was discussed earlier, the Q of the cavity can be measured as $Q = f_o/\Delta f$, where f_o is the resonant frequency and Δf is the difference between the (angular) frequencies at which the fields fall to $1/\sqrt{2}$ of their maximum resonance values. (Equivalently, the power falls to 1/2 of its maximum resonance value.)

A resonance curve obtained from varying the frequency (at fixed cavity length) is shown in the figure on the left below. Power is plotted versus (angular) frequency. The resonant frequency f_o is identified, and the frequency difference at half power Δf is identified.

In our experiment we measure the loaded Q, Q_L. By sweeping the frequency of the oscillator, we identify Q_L with the measured $f_o/\Delta f$ as in the figure above on the left.



resonance curve P versus ω

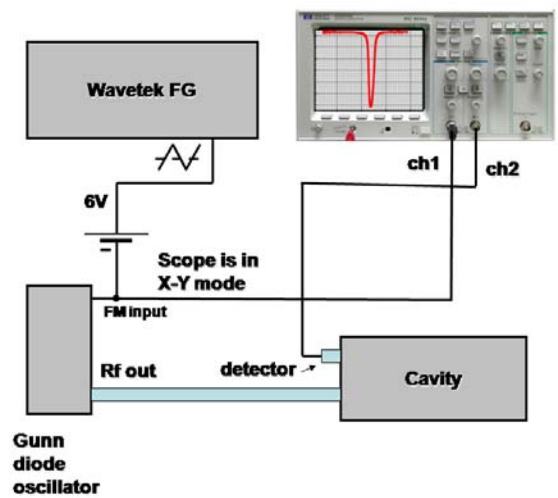
Recall that $Q_L = \dfrac{Q_o}{1+\beta}$. Then $Q_o = (1+\beta)Q_L = (1+\beta)(f_0/\Delta f)$. It is clear from these expressions that as the coupling of the cavity to external loads is decreased, i.e. as $\beta \to 0$, the measured loaded Q, Q_L, approaches the Q of the cavity, i.e. $Q_L \to Q_o$. The measured loaded Q is always smaller than the Q of the cavity. In the case of critical coupling, i.e. $\beta = 1$, $Q_o = (1+1)Q_L = (1+1)\, f_0/\Delta f = 2\, f_o/\Delta f$; however, we cannot assume critical coupling in our experiment.

Assuming that our calculation of the cavity Q from Eq. 11 is accurate (we only have a handbook value for the resistivity, and we neglect imperfections in the cavity), we can infer the coupling parameter for our experiment from the expression

$$\beta = \frac{Q_o}{Q_L} - 1 .$$

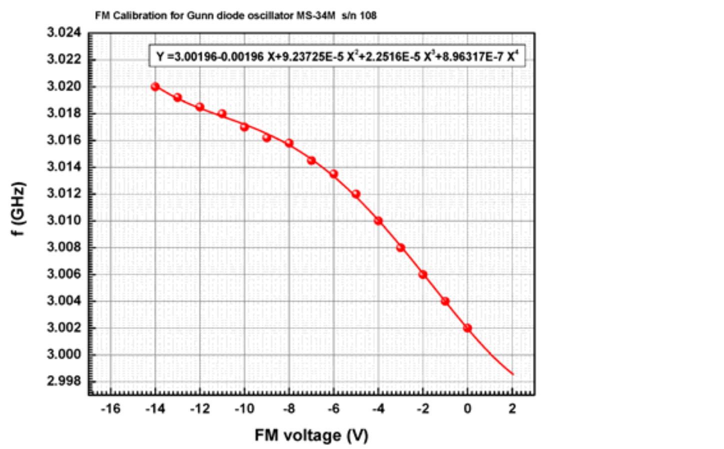
Using Eq 11, we find that Q_o is about 8000. Values for Q_L vary between 300 and 2000, depending on the orientation of the antenna and detector loops, the quality of the contact between the plunger fingers and the cavity walls, etc. For these values of Q_L, β then is between 6 and 27 – far from critical coupling.

In the cavity field detector, the voltage developed in the loop, V, is proportional to the magnetic field strength, $V \propto H$. However, the diode is very nearly a "square-law detector," i.e. to a good approximation, $I \propto V^2$. Thus the current developed in the diode, I, is proportional to the square of the field, $I \propto H^2$. At resonance where we observe maximum current, we have maximum field, $I_{\max} \propto H_{\max}{}^2$. At the half-power points where the field is reduced by a factor of $1/\sqrt{2}$, $I_{half-power} \propto \left(H_{\max}/\sqrt{2}\right)^2$. The current at half power is equal to half of the maximum current, $I_{half-power} = I_{\max}/2$. The frequencies, f_1 and f_2, at which the current falls to half of the maximum current, give the cavity bandwidth, $\Delta f = |f_2 - f_1|$, needed for our calculation of Q_L.



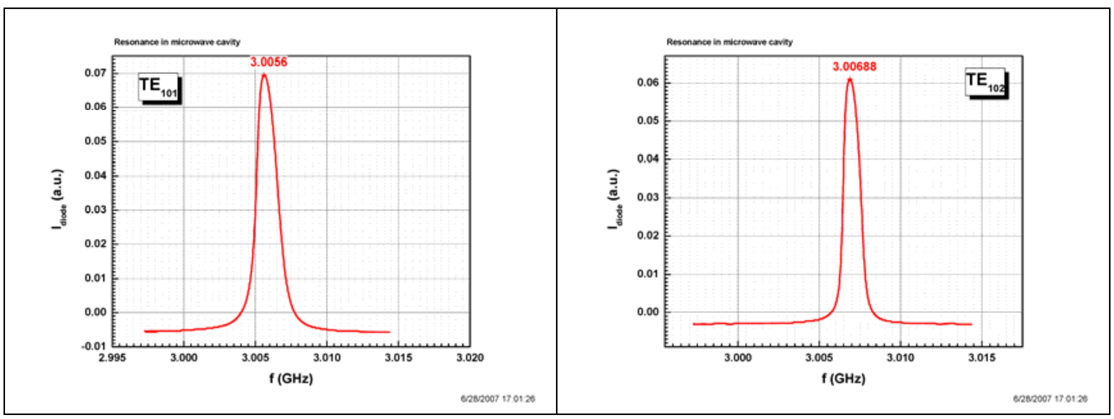
Frequency sweep of Gunn diode using the Wavetek function generator.

The resonance frequency will be swept by applying a voltage to the FM modulation input of the Gunn diode. The Gunn diode frequency can be tuned over an 18 MHz range about 3GHz by applying a voltage between $-2V$ to $-14V$ to the FM input (DO NOT EXCEED THIS RANGE). Start by removing the dielectric sheet from the cavity. The FM mod input is connected to the Wavetek function generator through a 6V battery. The battery is used to apply a negative voltage offset to the output of the function generator to ensure that the output voltage is within the specified range. Note, you should gradually increase the voltage output of the Wavetek and monitor the output on the oscilloscope. Set the Wavetek waveform to be sawtooth (see diagram) and set the frequency to be 50Hz – this will sweep the frequency of the Gunn diode at 50Hz with a modulation offset of up to 18MHz peak-to-peak. Adjust the offset of the Wavetek to keep the output voltage in the desired range. Remember that the battery adds $-6V$ to the output of the Wavetek.



Typical FM modulation calibration curve for the Gunn diode.

Set the scope to XY-mode and connect the diode to Chan1 and FM input to Chan2. Both channels should be set to 1MΩ termination. Trigger the scope using Chan 1. Measure the response from the diode detector using the HP 54603B oscillcope and download the resonance curve to the computer. Each station will have calibration curve for the Gunn diode frequency vs. modulation voltage. The curves have also been fitted with a polynomial which will allow you to convert the x-axis of your data to frequency. Plot your data as diode response as a function of frequency and include data for the TE101 and TE102 modes. Remember that the diode is a square law detector, thus your measured response is proportional to the square of the electric field amplitude.. Calculate $\Delta f = \left| f_2 - f_1 \right|$, and Q_L .



Sample data taken for the TE101 and TE102 modes.

In this measurement, the frequency of the drive is held nominally fixed (near 3GHz) and the length of the cavity is adjusted in order find the resonance. The procedure for finding these modes is as follows. First calculate using Eq. (1) the length of the cavity that corresponds to the TE101 and TE102 modes. Place the plunger to be close to the expected location for a given mode. Next, begin sweeping the frequency of the diode and monitor the response from the detector using the procedure described above. In order to quickly find the resonance, you can coarsely move the plunger by hand near the location that you expect the resonance to occur and observe the response. The resonance will go by quickly so it will takes a little practice – be patient. After you locate the resonance, you can fine tune its position by turning the thumb screw on the plunger. Measure the length of the cavity and compare your results with the predicted value.

Compare the measured value of Q_L with the value of the unloaded Q_o from the calculation carried out by M. Schwartz in the intermediate electricity and magnetism text, Principles of Electrodynamics, on pages 355–357 for the TE_{101} mode,

$$Q_o = \frac{abc\left(a^2+c^2\right)}{\delta\left[2b\left(a^3+c^3\right)+ac\left(a^2+c^2\right)\right]}. \qquad (11)$$

In this expression δ is the skin depth at angular frequency, ω_o. The skin depth is $\delta = \sqrt{2\rho/\mu\omega}$, where the resistivity of the cavity made of red brass, $\rho \approx 6\times10^{-8}\,\Omega\cdot\mathrm{m}$ and the permeability, $\mu \approx \mu_o = 4\pi\times10^{-7}\,\mathrm{H/m}$. (To be complete, note that Schwartz has several errors in his derivation, and his expression differs from Eq. 11 by a factor of 2. Eq. 11 is correct. See, for example, Technique of Microwave Measurements, Vol. 1, edited by C. G. Montgomery, section 5-4, for expressions for the Q of a rectangular cavity in an arbitrary TE_{lmn} mode.)

VERY IMPORTANT: The Q of the cavity depends critically on the coupling. In order to be able to observe the intrinsic Q of the cavity (the unloaded Q), you should minimize the coupling to the cavity for this part of the lab. You can do this easily by rotating **BOTH** the drive and the pickup loops so that the plane of the loop lies near the xy-plane. Note if it were to lie exactly in the xy-plane, there would be no coupling to the TE modes and you would not observe any signal. You will observe that as you increase your coupling by rotating the loop into the yz-plane, your signal grows but the Q decreases. You should optimize the coupling so that you have a nice measureable signal, but with as high a Q as possible.

REPORT

Include data and sample calculations for each part of the experiment. Include data analysis where necessary. Discuss briefly about your results and observations in each part.

1. Part I: (a) What is your wavelength in the slotted line? (b) What is the oscillator frequency? (c) What is the per cent uncertainty obtained from your measurement?

2. Part II: (a) What are the values of *c* for both the TE_{101} and TE_{102} modes? (b) Estimate the per cent uncertainties for the *c*.

3. Part III: (a) Plot the cavity field detector current versus the angle of input coupling loop plane. (b) Draw a sketch of the magnetic field lines inside the cavity and discuss how the current varies when the coupling loop is rotated. (c) From the shape of the plot, can you explain whether the detector current is directly proportional to the square of the magnetic field ?

4. Part IV: (a) Plot Δc_ℓ versus ℓ. (b) On the same graph, plot $\sin\left(\pi\ell/c\right)$ versus ℓ. (c) Discuss the nature of the two plots.

5. Part V: (a) From your measurements, calculate Q_L. Estimate its uncertainty. (b) Calculate the unloaded Q from Eq. 11. Compare the two values.