University of Illinois at Urbana-Champaign   Department of Physics

Physics 401 Classical Physics Laboratory

Experiment 67

HALL PROBE MEASUREMENT OF MAGNETIC FIELDS

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Introduction

Whereas no convenient technique exists for measuring arbitrary electric fields $\vec{E}$, several techniques are available for the practical measurement of magnetic fields $\vec{B}$. These include the observation of the force exerted on a current-carrying wire, the emf induced in a rotating coil, the frequency at which certain atomic or nuclear systems exhibit resonant absorption, and the Hall voltage induced in a current-carrying conductor. The latter technique utilizing the Hall effect has the advantage of requiring only a very small probe and very simple instrumentation. During this laboratory, you will use a Hall probe to study the magnetic field distributions produced by both a Helmholtz coil and a solenoid. The physical mechanism of the Hall effect is discussed in Appendix I.

Magnetic fields from single and multiple current loops

The magnetic field due to a single circular current loop at an arbitrary point in space is shown in Fig. 1. It is a rather complicated function of the coordinates.
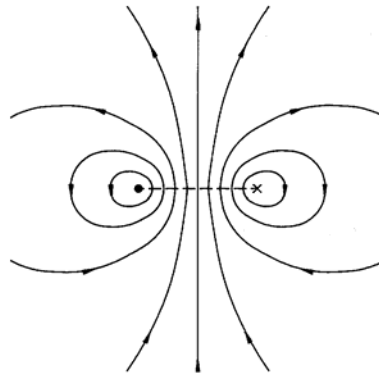

Figure 1. Field lines for a single, circular current loop

On the axis of the loop, however, a simple expression may be found for the field due to the rotational symmetry of the system.

Consider a loop of radius a whose axis lies along the z axis and whose center is at the origin, as shown in Fig. 2 below.
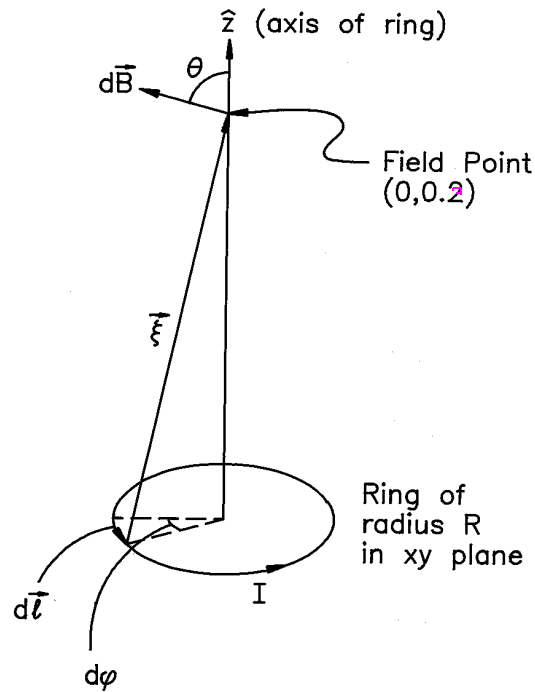
Fig. 2. Geometry for the calculation of the B field on the axis of a circular current loop

We apply the Biot-Savart Law

$$\vec{B} = \frac{\mu_0}{4\pi} I \oint \frac{\overrightarrow{d\ell} \times \vec{\xi}}{\xi^3}$$

where $\vec{\xi} = \vec{r} - \vec{r}'$, and $\vec{r}$ and $\vec{r}'$ specifiy the field and source points, respectively. Due to rotational symmetry, only the z component of $\vec{B}$ survives the integration. In cylindrical coordinates, $\overrightarrow{d\ell} = a\, d\phi\, \hat{\phi}$ and $\vec{\xi} = z'\hat{z} - a\hat{\rho}$, so $\left(\overrightarrow{d\ell} \times \vec{\xi} \right) \cdot \hat{z} = a^2\, d\phi$. Thus

$$d\mathrm{B}_z = \frac{\mu_o}{4\pi} \frac{I\, a^2\, d\phi}{\xi^3}.$$

As ξ remains constant as we integrate around the loop, we find

$$\vec{B} = \frac{\mu_o I a^2}{2} \frac{1}{\left(z^2 + a^2 \right)^{\frac{3}{2}}} \hat{z}.$$

Using this result, we may now find the expression for the field on the axis of both a Helmholtz coil and a solenoid. A Helmholtz coil consists of two identical circular current loops of radius a, each having N turns. The coils are parallel and coaxial, and separated by distance a, as shown in Fig. 3.
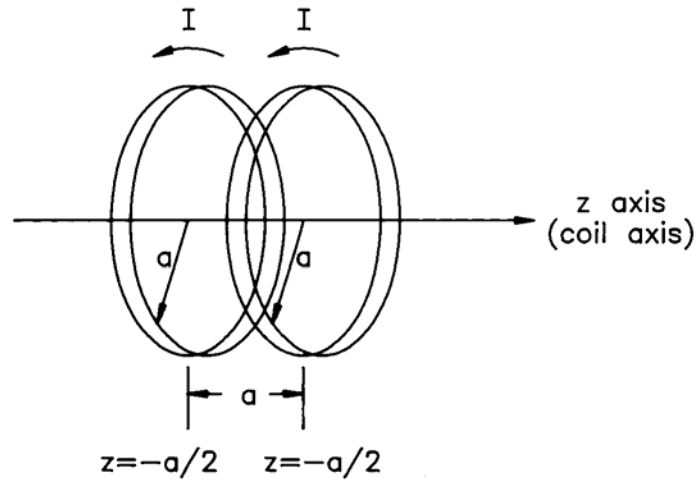

Figure 3. Geometry of the Helmholtz coil

The same current I flows through both coils. As was the case for a single current loop, the field from a Helmholtz coil for an arbitrary point in space is shown in Fig. 4, a complicated
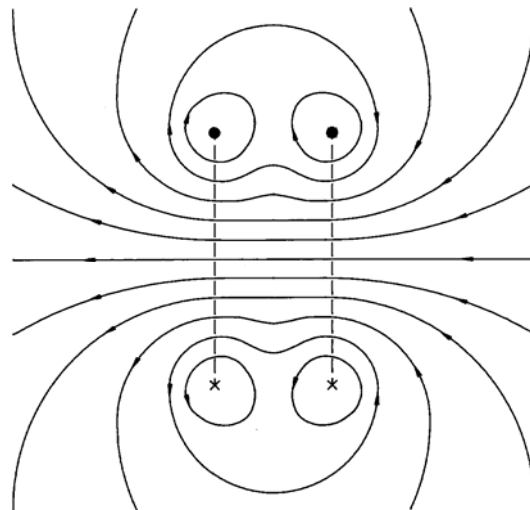

Figure 4. Field lines for a Helmholtz Coil

function of the coordinates, but rotational symmetry simplifies the calculation on the field of the axis. To find the axial magnetic field due to a Helmholtz coil, we first note that, by superposition, the field from each loop of N turns is simply N times the field due to a coil of a single turn.

We choose the z axis along the axis of the coils, and choose the origin at the midpoint between the coils, as shown in Fig. 3. By superposition, the total field from the Helmholtz coil is the sum of the fields from each coil may be obtained from our expression for the single loop by applying the coordinate transformation $z \rightarrow z - \dfrac{a}{2}$ for the right hand coil, and $z \rightarrow z + \dfrac{a}{2}$ for the left hand coil. Thus the axial field is given by

$$\vec{B} = \frac{\mu_0 \, N \, I a^2}{2} \left\{ \frac{1}{\left[\left(z + \dfrac{a}{2} \right)^2 + a^2 \right]^{3/2}} + \frac{1}{\left[\left(z - \dfrac{a}{2} \right)^2 + a^2 \right]^{3/2}} \right\} \hat{z}$$

$$= \frac{\mu_0 \, I N}{2 a} \left\{ \frac{1}{\left[\left(\dfrac{z}{a} + \dfrac{1}{2} \right)^2 + 1 \right]^{3/2}} + \frac{1}{\left[\left(\dfrac{z}{a} - \dfrac{1}{2} \right)^2 + 1 \right]^{3/2}} \right\} \hat{z}$$

(1)

and shown in Figure 5 below. N is the number of turns per coil.
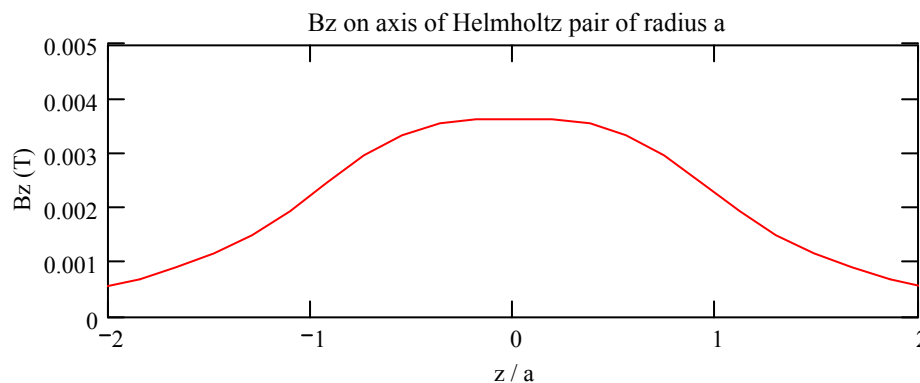

Figure 5. Magnetic field along the axis of a Helmholtz coil.

Helmholtz coils are frequently used in the laboratory because they provide a reasonably large region free of material in which there is a uniform magnetic field (the region midway between the coils and along the axis). The calculated field on the axis of a Helmholtz coil with 145 turns per coil and with a radius a = 10.8 cm and at a current of 3.0 A is shown in Fig. 5 below. On the horizontal axis the coils are at $z = \pm a/2$ ($z/a = \pm 1/2$).

The solenoid is another current distribution which provides a region of uniform field. The magnetic field distribution for a solenoid whose length is equal to twice its diameter is shown in Fig. 6.
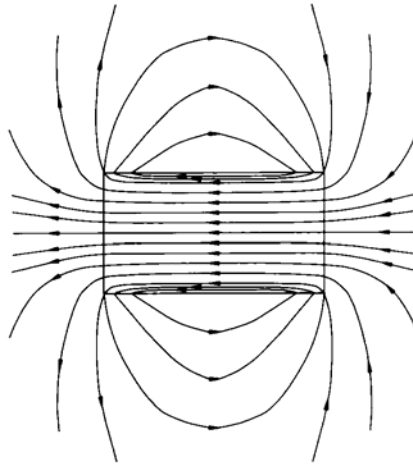

Figure 6.   Lines of B for a solenoid whose length is equal to twice its diameter.

For the same total number of ampere-turns of a given of a given radius and occupying the same overall length, the uniformly wound solenoid gives a somewhat higher field at the center than the Helmholtz coils. The field of the Helmholtz coils at its center is, however, more uniform than the field for the solenoid at its center. The solenoid has the experimental difficulty that the uniform field region is accessible only from the ends of the solenoid while for the Helmholtz coil the uniform field region is accessible from the side as well. To calculate the B field for a point P on the axis we establish the coordinate system shown in Fig. 7. The origin is chosen at point P, and the solenoid extends from $z = z_1$ to $z = z_2$. If the solenoid has n turns per unit length, then in a length dz there is a current $n\,I\,dz$ flowing, and the field at the point P due to the length dz is given by

$$d\vec{B} = \frac{\mu_o n \, I dz}{2} \frac{a^2}{\left(z^2 + a^2\right)^{\frac{3}{2}}} \hat{z},$$
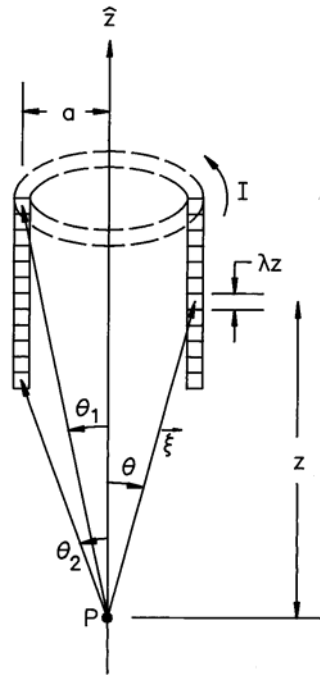

Figure 7.   Geometry for the calculation of the B field on the axis of a solenoid.

and the axial field due to the whole solenoid is given by the integral

$$\vec{B} = \frac{\mu_o n \, I a^2}{2} \int_{z_1}^{z_2} \frac{dz}{\left(z^2 + a^2\right)^{\frac{3}{2}}} \hat{z}.$$

Making the changing variables $z = a \tan \theta$ gives

$$\vec{B} = -\frac{\mu_o n I}{2} \int_{\theta_1}^{\theta_2} \sin \theta \, d\theta \hat{z} = \frac{\mu_o n I}{2} \left[\cos \theta_1 - \cos \theta_2\right] \hat{z}, \tag{2}$$

where $\cos \theta_1 = z_1 / \sqrt{a^2 + z_1^2}$ and similarly for $\cos \theta_2$. Hence, the on-axis field of a solenoid is proportional to the difference in cosines of the angles subtended by the ends. The angles for field points both exterior and interior to the solenoid are shown in Figure 8 below.
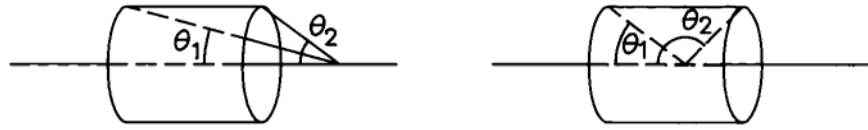
Figure 8. Angles subtended by the ends of the solenoid for exterior and interior field points.

The calculated field from a solenoid with 245 turns, a radius of $a = 5.1$ cm and length $2b = 20.3$ cm with a current of 3.1 A is shown in Figure 9 below. On the horizontal axis the points ` $z/b = \pm 1$ represent the ends of the solenoid.
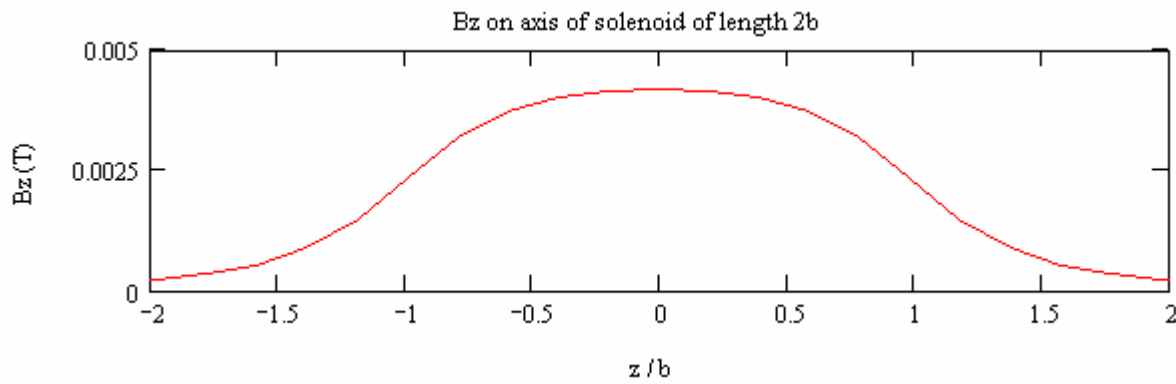

Figure 9. Magnetic field along the axis of a solenoid.

One can obtain a little more information about the magnetic field of a solenoid without solving for the field everywhere. If one applies Amperes circuital law to a path containing an edge of the solenoid, as shown in the Figure 10 below, one obtains: $\oint \vec{B} \cdot d\vec{\ell} = \mu_o N I$.
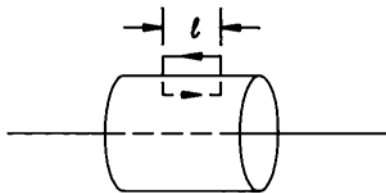

Figure 10.　　　Integration path for Ampere's law.

Here, N is the number of turns of wire contained by the integration loop. We suppose that the contribution to the line integral along the radial paths is negligible. Then evaluating the integral gives

$$\left[B_{z,inside} - B_{z,outside} \right] \ell = \mu_o N I$$

or

$$\Delta B_z \equiv \left[B_{z,inside} - B_{z,outside} \right] = \mu_o n I . \quad \text{(Recall that } n = N/\ell .) \qquad (3)$$

Thus the difference in the z-components of the magnetic field is proportional to the current density. In fact, this result is true for any shape current sheet; it is only necessary to remain close to the sheet.

For our last consideration we want to relate the rates of change of a transverse-component and the z-component of the magnetic field of a solenoid. To this end we use the fact that any magnetic field has zero divergence, i.e.

$$\vec{\nabla} \cdot \vec{B} = \frac{\partial B_x}{\partial x} + \frac{\partial B_y}{\partial y} + \frac{\partial B_z}{\partial z} = 0 .$$

Utilizing the symmetry of the solenoid we have, on-axis, $\dfrac{\partial B_y}{\partial y} = \dfrac{\partial B_x}{\partial x}$ hence

$$\frac{\partial B_x}{\partial x} = -\frac{1}{2} \frac{\partial B_z}{\partial z} . \qquad (4)$$

On the axis of the solenoid the transverse component is zero, but it has a rate of change with distance off-axis that is given by one-half the rate of change of the axial component.

Experimental Procedure

A.   Measurement of the Helmholtz Coil Field

You will use a commercial gaussmeter to measure the magnetic fields. There are two types of probes available: axial and transverse. The axial probe has a round cross section and measures the component of the $\vec{B}$ parallel to the cylinder axis. The end of transverse probe has a rectangular cross section and measures the component of $\vec{B}$ perpendicular to its broad face.

The AlphaLab Hall DC magnetometer and probes have been calibrated by the manufacture. No other calibration should be necessary. There is a calibrated permanent magnet available to check the calibration. There is an OFFSET control which is used to cancel out an existing field by adding or subtracting a certain value from the field strength. Fields of up to ± 10 G can be cancelled with this control. The AC/DC switch selects measurement of time varying and steady magnetic fields. The fields in this experiment are constant in time. Insert the probe into the zero-gauss chamber and adjust the control to obtain zero gauss on the 200 G scale. Note that there is a filter in the electronics for the 200 G scale which requires at least 3 s to obtain a reading. The Magnetometer must be zeroed if the probe is changed from axial to transverse, or vice versa.

1. Record the necessary parameters of the Helmholtz coil in your notebook. Connect the Helmholtz coil as shown in Figure 11. This is a split coil having a separation equal to the radius of one of the windings. Adjust the current until 3.0 A flows in the Helmholtz coil. Use a DMM
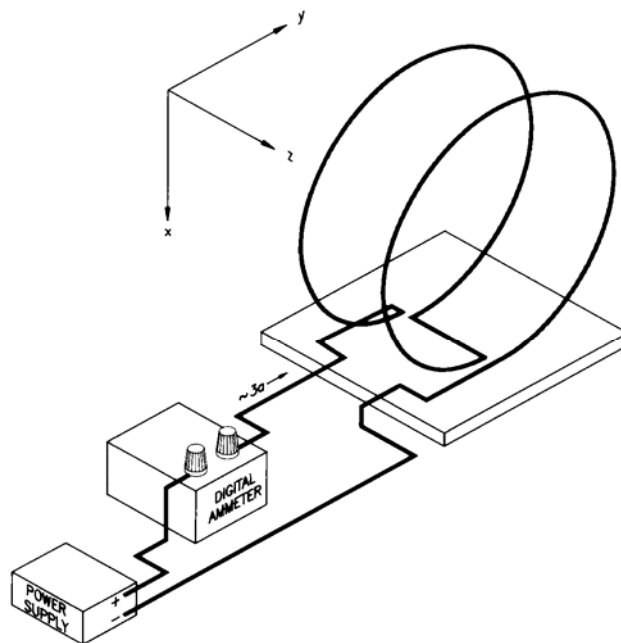

Figure 11. Basic circuit for field measurement with Helmholtz coil.

to measure the current. The meter in the power supply only has one digit precision. (Be sure

your power supply is operating in the current regulating mode, and <u>not</u> in the voltage-regulating mode. Check the power supply manual for current regulating mode operation.) Before making measurements, zero the probe.

2. In this part we demonstrate that the field at the center of the Helmholtz coil is axial. Measure the field at the center of the pair on the median plane for various angles of placement of the transverse Hall probe, see Figure 12 below. Measure the field while rotating the transverse probe at the angular intervals of 30° over a 180° range.

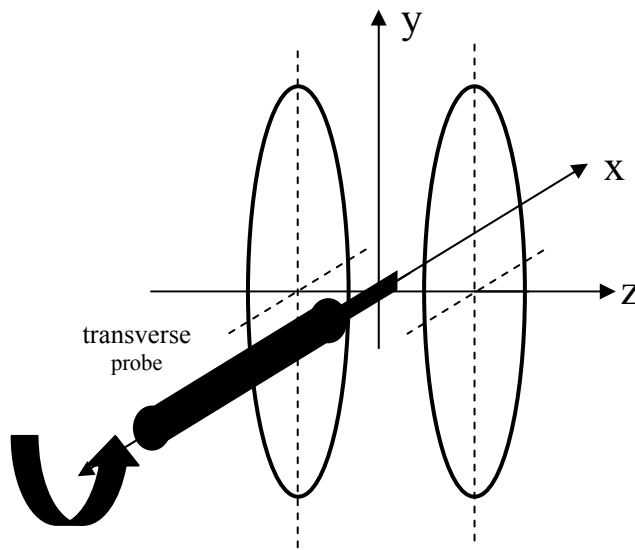Part A2 Direction of field at center of Helmholtz coils

Figure 12. Procedure for finding direction of field at center of Helmholtz coil.

3. In this part we map the axial field along the axis of the coils, i.e. $B_z(0,0,z)$ vs. z. Switch to the axial probe and zero it again. Measure the field along the axis of the Helmholtz coil from z = -20 cm to z = +20 cm in steps of 2 cm. See Figure 13 below. It may be easier to move the coil and leave the probe fixed.

Part A3 B_z on axis of Helmholtz coils
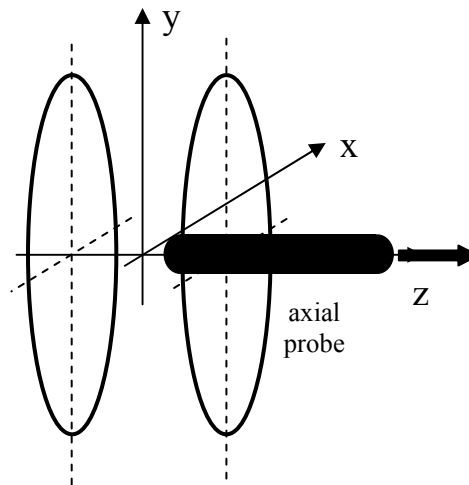$B_z(0,0,z)$ vs z

Figure 13. Procedure for measuring the field on the axis of Helmholtz coil.

4.   In this part we map the field on the median plane of the Helmholtz coil using the
transverse probe, i.e. Bz(x,0,0) vs x. (We switch to the transverse probe to avoid running into
the coils.) Measure the field from x = -20 cm to x = +20 cm in steps of 2 cm, see Figure 14
below.

Part A4 B_z on median plane of Helmholz coils
$B_z(x,0,0)$ vs x
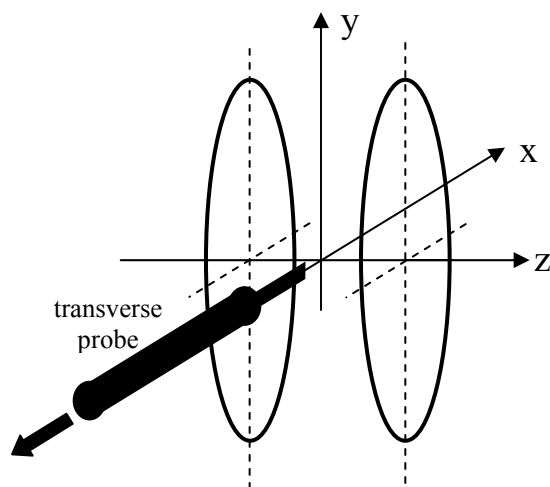

Figure 14. Procedure for measuring the field on the median plans of the Helmholtz coil.

B.   Measurement of the Solenoid Field

1. Record the necessary parameters of the solenoid coil in your notebook. Connect the solenoid in place of the Helmholtz coil and adjust the current to 3.0 A.

2. In this part we map the axial field of the solenoid on the axis of the solenoid. See Figure 15 below. Take at least twenty (20) measurements at intervals of 1 cm from the center of the solenoid. Near the end of the solenoid make measurements in 0.5 cm steps. Measure only in one direction from the center of the coil.

Part B2 B_z on axis of solenoid
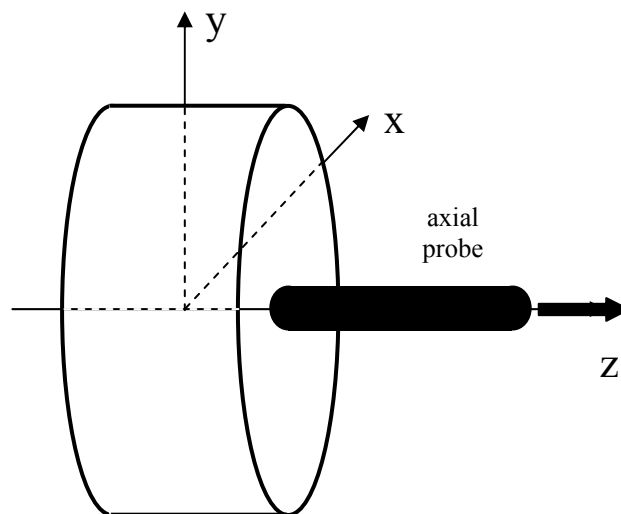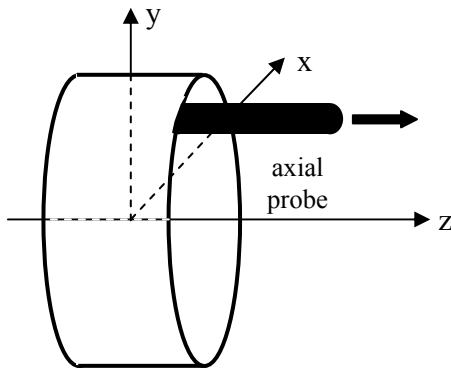$B_z(0,0,z)$ vs z

Figure 15. Procedure for measuring the field on the axis of the solenoid.

3. In this step we measure the component of field parallel to the axis, i.e. Bz, just inside and just outside of the coil.   With these data we will compare to Ampere's Law. See Figures 16a and 16b below. Take at least twenty (20) measurements at intervals of 1 cm from the center of the solenoid.

Part B3a B_z inside solenoid near coil

Part B3b B_z outside solenoid near coil $B_z(x_o,0,z)$ vs z
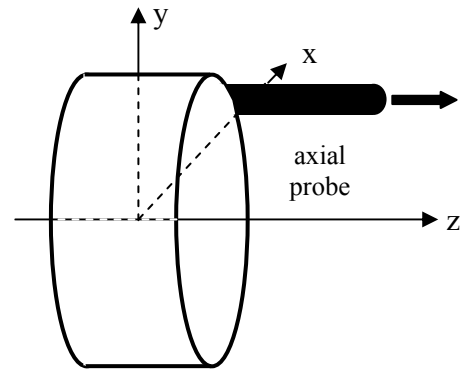
Figs 16a and 16b. Procedure for measuring the axial field off the axis inside and outside of the solenoid.

4. In this part we measure the component of the field perpendicular to the axis near the end of the solenoid. With these data we will compare to the divergance law. Switch to the transverse probe. Map field along the transverse direction at the end of the solenoid. See Figure 17 below. Make measurements from -5 cm < x < 5 cm. Use steps of 0.5 cm.

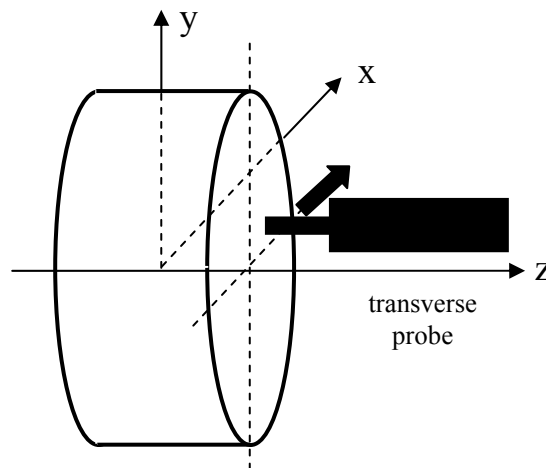Part B4 B_x outside solenoid near coil $B_x(x,0,b)$ vs x

Figure 17. Procedure for measuring the transverse field outside the solenoid.

Calculations and Discussion

Use proper units consistently. Note: 1 Tesla $= 10^4$ gauss.

A.   Make a plot of the measured B field vs. angle for the Helmholtz coil using the results obtained in procedure A2. Comment on the dependence of the measured A on angle.

B.   Make a plot of B_Z vs. z for the Helmholtz coil, using the results obtained in procedure A3. Compare the measurements with the field calculated with Eq 2.

C.   Make a plot of B_Z vs. x for the Helmholtz coil, using the results obtained in procedure A4. Comment on the results.

D.   Utilizing the plots obtained in (C) and (D) above, estimate the radius of a spherical volume centered within the Helmholtz coil within which the field is constant to 5%.

E.   Make a plot of B_Z vs. z for the solenoid, using the results obtained in procedure B2. Compare the measurements with the field calculated with Eq. 2. to your values of B_Z for the middle and the end of the solenoid with the theoretical values.

F.   Make a plot of $\Delta B_z \equiv \left[B_{z,inside} - B_{z,outside} \right]$ vs. z for the solenoid as obtained in procedure B3. Compare your maximum value of ΔB_z with the value calculated with Eq 3.

G.   Make a plot of B_X vs x for the field at the end of the solenoid as obtained in procedure B4.

H.   Determine the slopes $\dfrac{\partial B_z}{\partial z}$ and $\dfrac{\partial B_x}{\partial x}$ from the data of part F and part H. Compare their ratio to the expected value from Eq 4. (Note they must both be evaluated at the same point in space.)

Report

In the report must include:

1.   The specifications of the solenoid and Helmholtz coil.

2.   The radius of the sphere in part E above.

3.   Theoretical and experimental values for the central fields of the Helmholtz coil and solenoid.

4.   Answers to all questions and all plots mentioned under Calculations and Discussion. No detailed error analysis is required.

Appendix I

Theory of the Hall effect

Let us begin by considering the motion of charge carriers, each of charge q, in a conductor of thickness b and width a as shown in Fig. A1. We note that q could be either positive or negative. This conductor is referred to as the Hall element in this experiment.
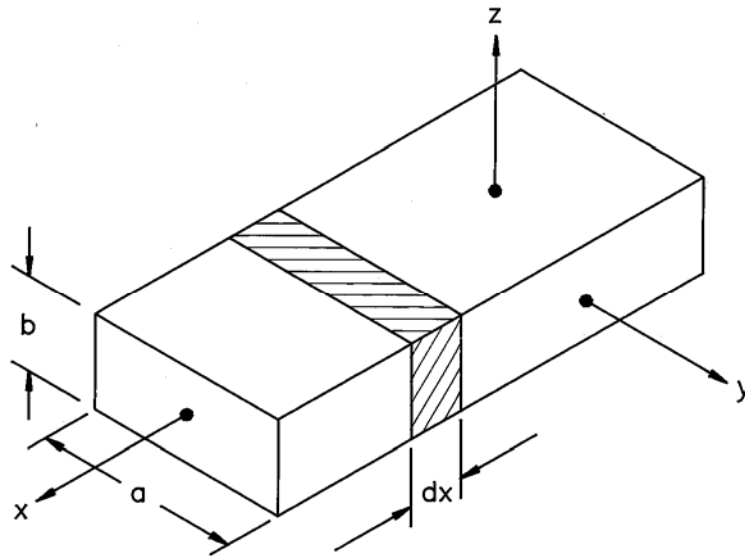
Figure A1. The Hall element.

If there are N charge carriers per unit volume, each small element of length dx contains a charge

$$dQ = N\,q\,a\,b\,dx \qquad\qquad\qquad (A1)$$

associated with the carriers. A current is made to flow in the $+x$ direction if a battery is connected to the ends of the element as shown in the figure below.
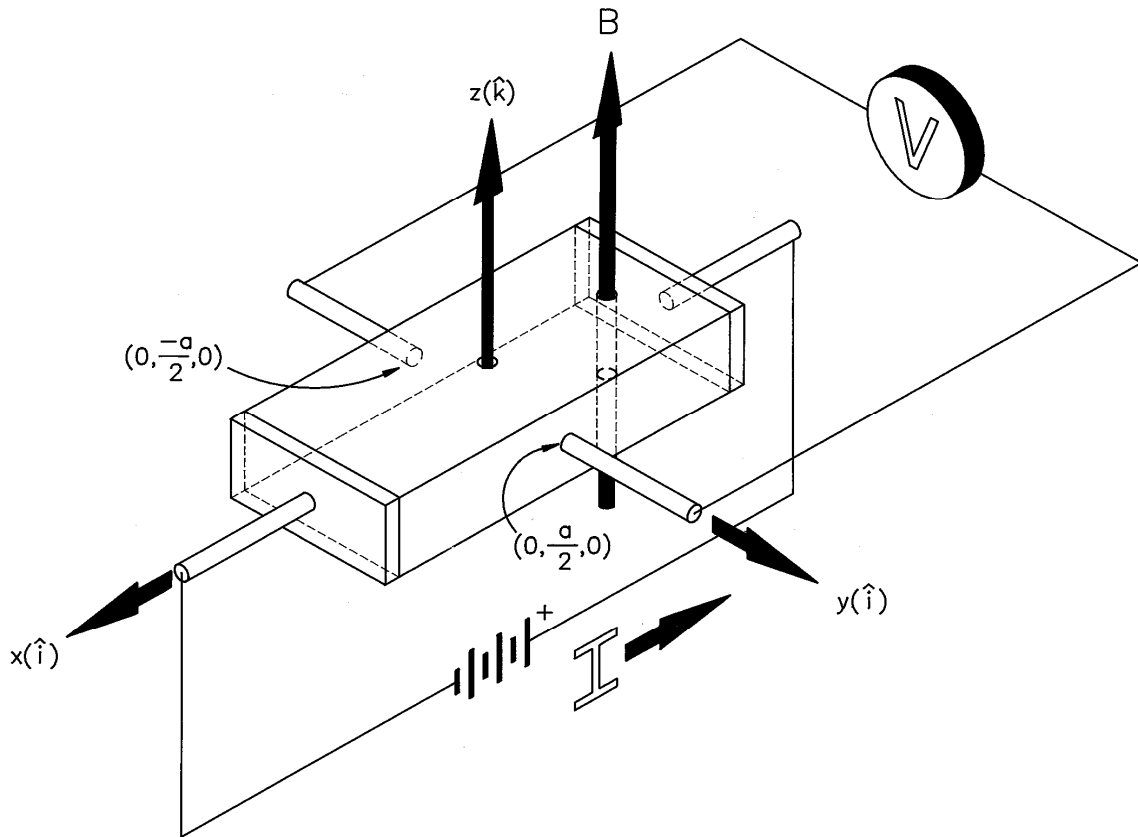
Figure A2. Electrical connections to the Hall element.

The carriers are accelerated but, because of collisions within the conductor, they attain an average velocity v_x called the drift velocity. In a time dt the carriers move an average distance $dx = v_x\, dt$. Hence a charge

$$dQ = N\, q\, a\, b\, v_x\, dt \tag{A2}$$

passes out of each volume element into the next during a time interval dt. This motion just describes a current $I_c = dQ/dt$, which we will call the control current. We can write

$$\vec{v}_x = \frac{I_c}{N\, q\, a\, b}\, \hat{i}\ . \tag{A3}$$

(Note that the sign of q determines the direction of $\vec{v}_x$. I_c is in the +x direction with our choice of battery orientation.) If we now turn on a magnetic field, $\vec{B} = B_z \hat{k}$, in the +z direction as shown in Fig. A2 above, the drifting carriers experience a force

$$\vec{F} = q\vec{v} \times \vec{B} = q\left(\frac{I_c}{Nqab}\hat{i}\right) \times \left(B_z \hat{k}\right) = -\frac{I_c B_z}{Nab}\hat{j} \tag{A4}$$

in the –y direction. As a result of this force the charge carriers move to the -y edge of the conductor. This motion produces an excess of charge on the -y edge and a deficit of charge on the +y edge. On the -y edge there is then a net charge density +σ, which has the same sign as the charge carriers, and on the +y edge , there is a net charge density of -σ, which has the opposite sign as the charge carriers. These charge densities give rise to an electric field, E_y, in the +y direction, if the charge carriers are positive, or in the -y direction, if the charge carriers are negative. This electric field exerts a force on the carriers in the +y direction which opposes the magnetic force of Eq. A4. Carriers continue to flow toward the -y edge until the electric force and the magnetic force balance, that is, until

$$q E_y \hat{j} - \frac{I_c B_z}{Nab}\hat{j} = 0.$$

Substituting the value of v_x from Eq. A3, we find

$$E_y = \frac{I_c B_z}{Nqab}. \tag{A5}$$

This equilibrium field is determined by measuring the potential difference across the sample using the voltmeter, V, shown in Fig. A2 above. The potential difference, V_H, is

$$V_H \equiv V(0,+\frac{a}{2},0) - V(0,-\frac{a}{2},0) = -\int_{-a/2}^{+a/2} E_y \, dy = -E_y a$$

Using Eq. A5 for E_y, we obtain for V_H , the Hall voltage,

$$V_H = -\frac{I_c B_z}{N q b}.$$

$$(A6)$$

Note that the Hall voltage is negative if $q > 0$ and positive if $q < 0$, given our choices for the directions of the current and the magnetic field. We define a quantity $R_H = \frac{1}{N q}$, the Hall coefficient, which depends only the sign and density of the charge carriers. We can rearrange Eq. A6 into the form

$$B_z = -\left(\frac{b}{R_H}\right)\frac{V_H}{I_c}.$$

$$(A7)$$

The Hall voltage then determines the component of $\vec{B}$ which is perpendicular to the face of the Hall element, namely B_z. We would need to change the orientation of the Hall element to determine field components in other directions. In general, we would need three orthogonal directions to determine all components of $\vec{B}$.

Table 1 gives Hall coefficients for various materials. The value of R_H for Bi and InAs are given with the proper sign. The magnitude is only approximate. The charge carriers for the materials in which $R_H < 0$ are electrons for which $q = -e$. The charge carriers for the materials in which $R_H > 0$ have $q = +e$ and are known as holes.

Table 1

Material	R_H (m^3/ C)
Cu	-5.3×10^{-11}
Na	-21.0×10^{-11}
Cr	$+35.0 \times 10^{-11}$
Bi	$-10^3 \times 10^{-11}$
InAs (approx.)	$-10^7 \times 10^{-11}$

If the actual Hall probe were perfect, it would be possible to determine the constant b/R_H in Eq. A7 by simply placing the Hall probe in a known (reference) magnetic field and measuring V_H and I_c. In a perfect probe, the transverse connections are exactly opposite each

other, and the Hall voltage V_H is zero in the absence of a magnetic field. In practice there is always some misalignment, δ, as shown in Fig. A3. Therefore the voltmeter does not measure just V_H.
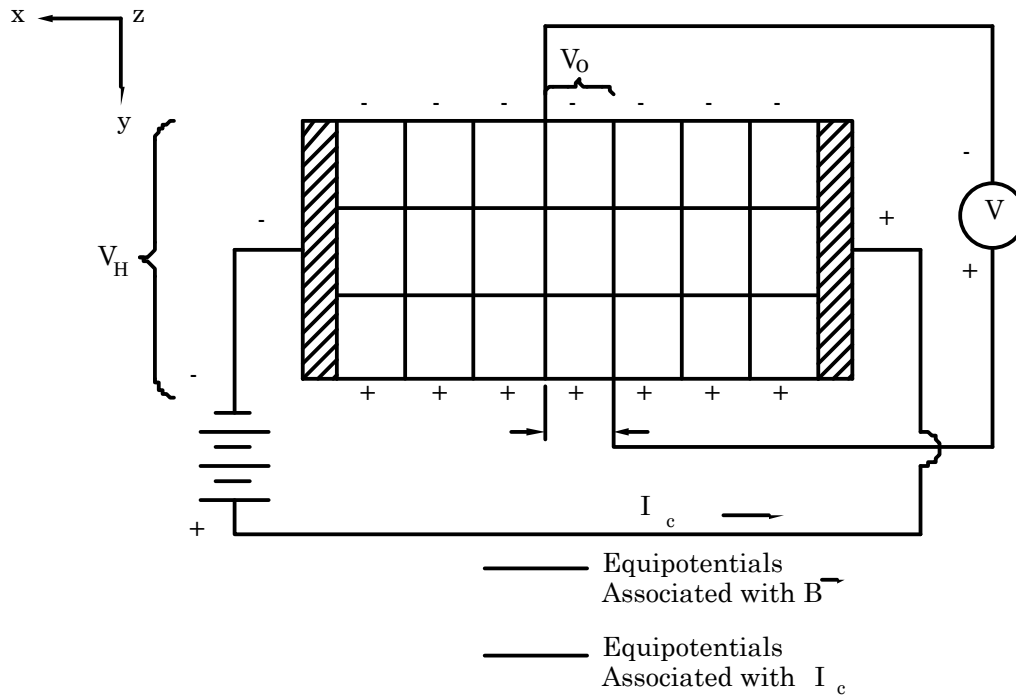

Figure A3. The Offset voltage.

Fig. A3 is a view of the Hall element looking down the z axis toward the origin. We can imagine the actual potential field to be separated into two sets of equipotentials. The set designated by horizontal lines is due to the Hall effect considered in the last section. The set of equipotentials designated by vertical lines is related to the current I_c flowing through the Hall element. From Fig. A3 we see that the voltmeter measures the sum of two voltages,

$$V_{meas} = V_H + V_O , \qquad\qquad (A8)$$

where $V_H = -\dfrac{I_c B_z}{N q b}$ is the Hall voltage previously discussed and $V_O = I_c \dfrac{\rho \delta}{a b}$ is the offset

voltage. $\dfrac{\rho \delta}{a b}$ is recognized as the resistance of a length δ of Hall element. It is easy to

determine V_O. In the absence of a magnetic field, Eq. A6 shows that there is no Hall voltage.

Thus $V_O = V_{meas}$. Because of the earth's magnetic field, it is necessary to insert the Hall element into a magnet shield to make this measurement.

A working equation is obtained by combining Eqs. A7 and A8,

$$B_z = -\left(\frac{b}{R_H}\right)\frac{V_{meas} - V_O}{I_c}.$$ (A9)

Remember that both V_{meas} and V_O can be positive or negative. The signs of V_{meas} and V_O affect the magnitude as well as the sign of B_z when using Eq. A9.

Note also that the voltmeter, V, in Figs. A2 and A3 must have a large internal resistance to prevent the charge buildup on the edges of the Hall element from being conducted away. The internal resistance of the DMM used in this laboratory, 10^{10} Ω, satisfies this requirement.

Experiment 67 using
Arrick Robotics MD-2 dual stepper motor system (XY table),
AlphaLab high sensitivity magnetometer (hall probe) and
HP34401 DMM

These instructions are a supplement to the Experiment 67 handout and should be used in conjunction with the handout. This supplement describes how to do the Experiment 67 exercises with the XY table. It includes one additional exercise, namely, a two-dimensional scan.

The apparatus for Experiment 67 is shown in Figure 1. It consists of an Arrick Robotics MD-2 dual stepper motor system XY table, an AlphaLab high sensitivity magnetometer (hall probe) with axial (identified by the white cylindrical plastic probe casing) and transverse (identified by rectangular brass probe casing) probes and an HP34401 DMM. The stepper motor system and the DMM are interfaced to a PC.
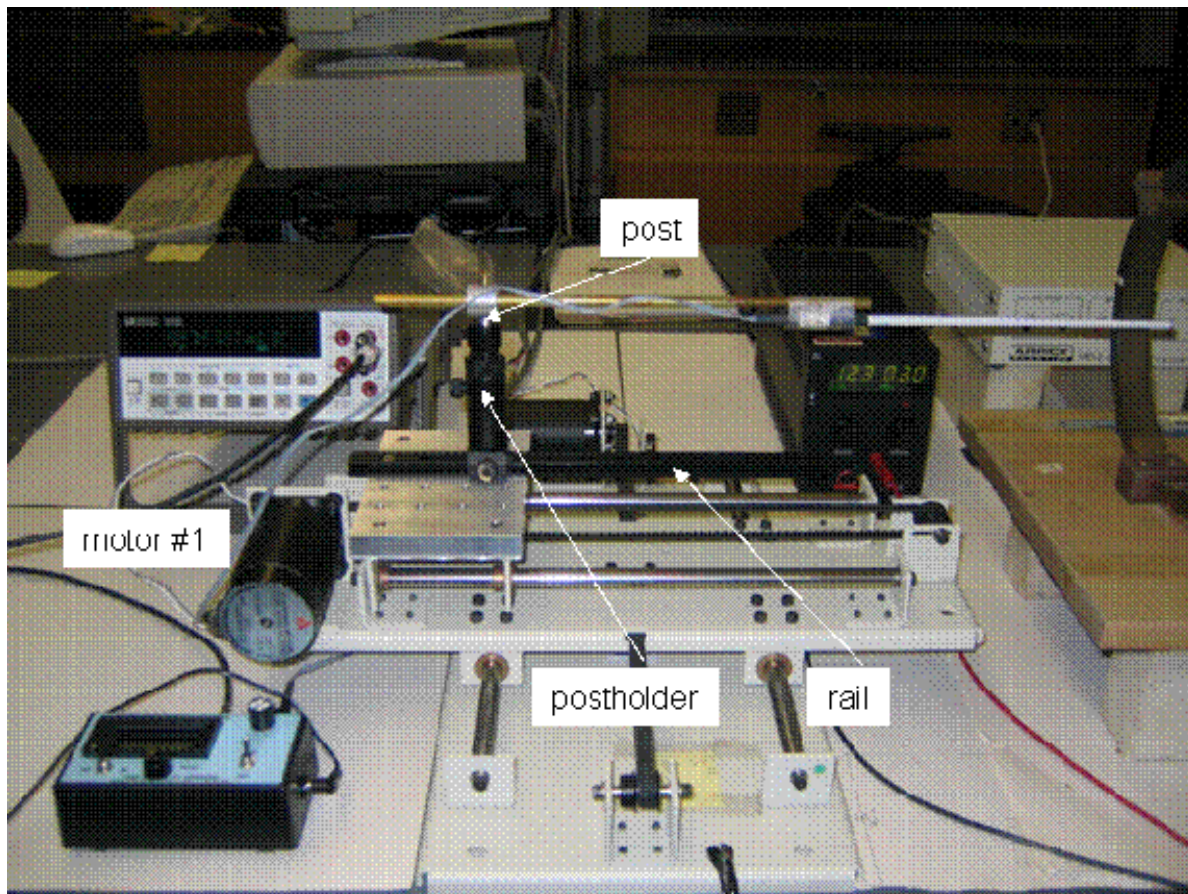

Figure 1 Experiment 67 apparatus

The bed of the XY table is fitted with a 12" ThorLab horizontal rail and a 3" ThorLab vertical translating postholder and post. The post carries an annular clamp that holds a brass rod onto which the hall probes are taped. The Thorlab rail and postholder allow for convenient, but accurate, movement of the hall probe. The thumbscrew on the base of the postholder secures the post the rail. By loosening one thumbscrew on the postholder, approximately 3" of free vertical

motion of the post is available. By loosening the other thumbscrew on the postholder, approximately 1/2" of vertical adjustment of the post by the knurled nut on the postholder is available. All thumbscrews should be tightened for mapping operations.

Setup of HP34401 DMM communication with the PC
The HP34401A DMM has both a RS-232 and a GPIB interface. Since the serial port (RS-232) is still common PC hardware, it is used to connect the DMM to the PC. The connection is to PC serial port COM1 through a standard cable. This connection is made by the 401 staff in preparation for the laboratory. The I/O parameters of the HP34401A must be setup before running the mapping program. The I/O parameters are stored in non-volatile memory, and it is possible that the correct parameters are in the DMM memory. You can test the communication link in the mapping program as described in the next section. Note that the magnetometer has the convenient calibration of 1 G = 1 mV. Full scale is 200 G (200 mV) and readings are stable at least 0.1 G (0.1 mV.)

Arrick dual stepper motor system and MagnetMapper program
The Arrick dual stepper motor system, MD-2, consists of an XY translation stage with a maximum travel of 9" in each direction. The stage is moved by two stepper motors with 1/200" per step. The two motors are connected to the MD-2 driver box with two stepper motor cables. The driver box is connected to the PC through the standard parallel port. The stepper motors and driver box are connected by 401 staff in preparation for the laboratory. It is not expect that the user will connect or remove cables, and the MD-2 instructions explicitly state: **DO NOT REMOVE MOTOR OR COMPUTER CABLES WHILE THE UNIT IS UNDER POWER.**
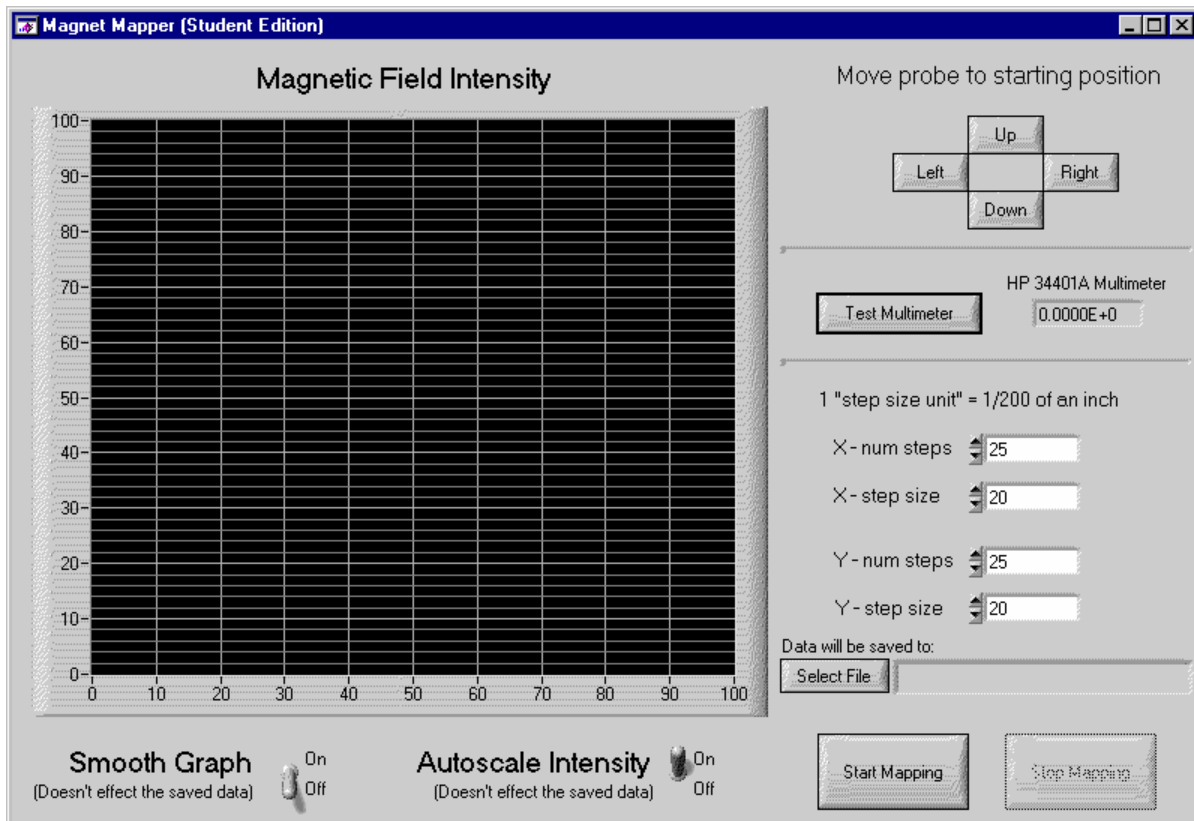

Figure 2 Magnet Mapper Instrument Panel

The stepper motor system is controlled by a LabWindows/CVI program, MagnetMapper_RS232, written by UIUC physics graduate student Kevin Mantey. Click on the program desktop icon to start the program. All program commands are entered through the program instrument panel. The program instrument panel is shown in Figure 2. From the instrument panel the user can (1) move the XY table, (2) test the connection to the DMM, (3) set the step size and number of steps, (4) see the data as it is acquired, (5) choose the file to which data is written, and (6) start and stop mapping. These options are accessed from buttons and boxes on the instrument panel. See Figure 2 for the instrument panel upon program start-up.

To test the communication link between the PC and the DMM, on the Magnet Mapper program instrument panel, press the button "Test Multimeter." If the current DMM reading appears, then the DMM I/O interface is properly setup. If an error message appears, either on the computer screen or on the DMM display, setup the DMM and possibly the PC following the instructions in the Appendix.

XY Table and Coil Positioning
Since the stepping motor step size is specified in inches, it is also convenient to give many dimensions also in inches. Conversion of inches to metric units should be done in data analysis.

Each part of Experiment 67 will require the user to position the probe with respect to the coil before the start of mapping. The coils must be set on blocks, since the minimum height of the probe off the table is approximately 10". The images in the file, setupwithXYtable.ppt, show one possible arrangement of the probe and coils for both solenoid and Helmholtz coil scans. **Regardless of the exact height and position of the probe and coils, the user must insure that, while the probe is moving, the hall probe cable does not catch in the table movement mechanism, and that the hall probe does not run into the coil. The probe can be damaged if either occurs. Watch the probe as it is moving. If the probe would run into the coil, stop the scan with the "Stop Mapping" button.**
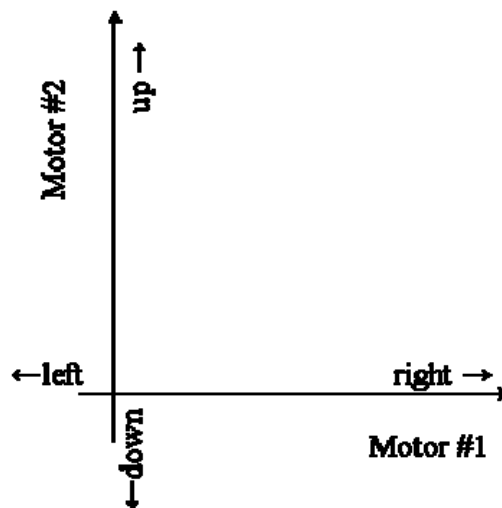

Figure 3 Table motion with standard table orientation

It is convenient to orient the XY table so that the motion indicated on the Magnet Mapper panel corresponds to the motion of the table. Then the "right" ("left") button will move the probe toward (away) from the coil. Also then the "up" ("down") button will move the probe toward and away from the user. The usual connection of the controller box will have motor #1 cable to the motor on the moving platform, and motor #2 cable to the motor on the fixed platform. Motor #1 moves the probe "up" and "down". Motor #2 moves the probe "right" and "left". See Figure 3.

Preparation to Take Data
The Magnet Mapper program writes the DMM readings out to a text file in Excel csv (comma separated value) format. The user must select the file to which the program writes. Dummy files must be prepared and then selected with the "Select File" button on the instrument panel. The dummy files are conveniently prepared using Excel to save a csv file with a convenient name, for example, E67_B2.csv. Several dummy files can be prepared in advance and kept in a folder.

The user must enter the step size and the number of steps in the appropriate windows on the Magnet Mapper instrument panel. Most of the maps are along one axis, and, assuming that the probe and coil are oriented so that motion in the y-direction moves the probe in the desired direction, the user would enter 0 (X-num steps), 0 (X-size steps), 19 (Y-num steps), 100 (Y-size steps) to move the probe $(19 – 1) \times 100/200" = 9"$ in 1/2" steps. The instrument panel after the DMM test and step number and size entries is shown in Figure 4. The fields of the solenoid and Helmholtz coils do not vary so rapidly that smaller steps are needed.
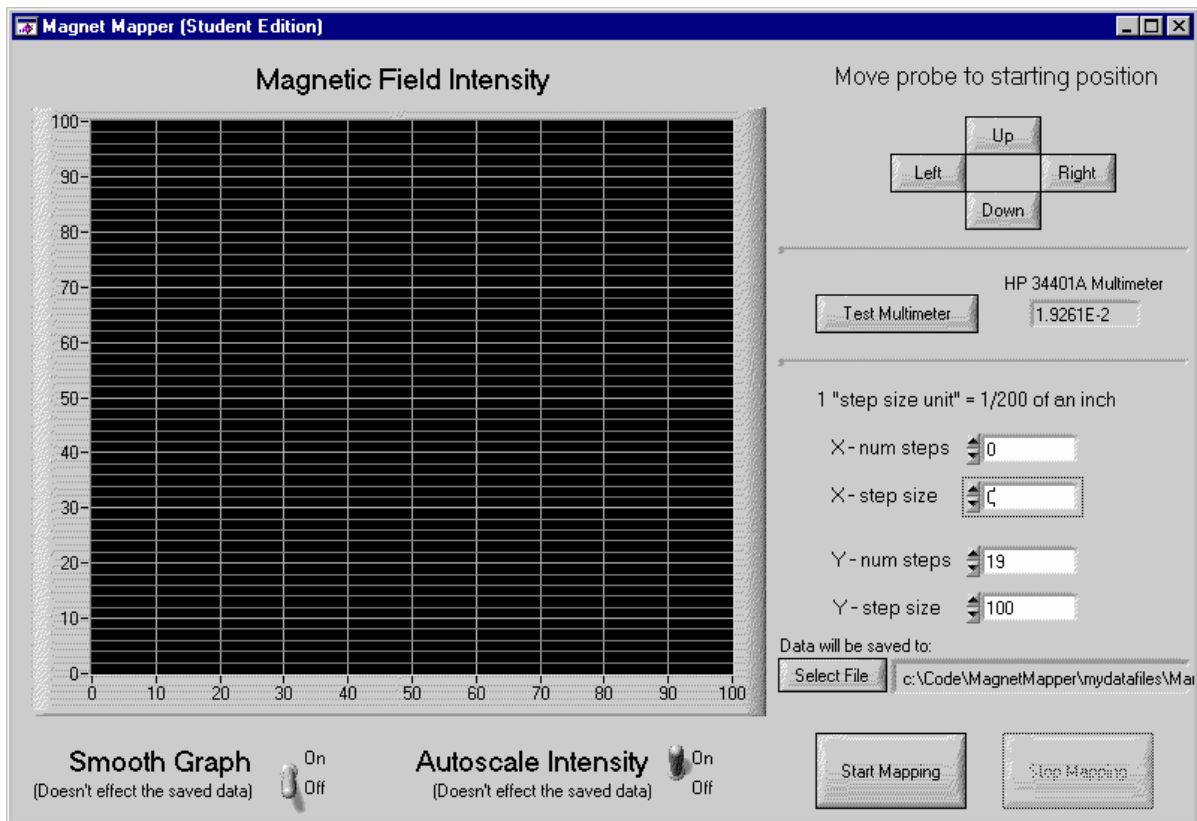

Figure 4 Magnet Mapper instrument panel after DMM test and step size and number entry

Typical sequence for mapping scan

Both the fields and the probes have a direction, and the direction should be noted. With appropriate orientation of the coil, the probe, and choice of the polarity of the DC supply, the readings of mapping scan will be positive. (The field can reverse direction outside the coil.) Mapping scans on the axis of the solenoid and the Helmholtz coil require positioning the height of the probe and the vector of its travel along the axis. The correct height is easy to find, but finding the correct line of travel is more of a challenge. It is convenient to use a scale to find the center of the coils and to move the probe with the UP/DOWN/RIGHT/LEFT buttons to achieve the correct line of travel.

For all of the mapping scans data should be taken on both sides of the center line to show the symmetry of the field. Also the length of the scan should be sufficient to show the complete field profile. A possible sequence for the axial scan of the Helmholtz coils (exercise A3) is as follows. See the file setupwithXYtable.ppt for pictures.

Helmholtz coil axial scan steps (A3)

1. Put the Helmholtz coils on blocks. 4" of height should be sufficient.
2. Choose the coil orientation to give a positive field reading with the axial probe.
3. Put the postholder at end of rail so the 11" of rail is exposed.
4. Move the probe with table control through the coil to the end of its travel. The motor stalls or chatters when it is at the end of its travel.
5. Position the coil so that probe is 4.5" from the median plane of the coils. At this position the end of the probe is about 2" beyond the coil. At this position the center of the coil is approximately at the midpoint of the 9" scan. (For an accurate positioning of the probe the sensitive volume of the probe must be determined empirically. The manufacture states that it is a few mm from the end. How would the sensitive volume of the probe be determined empirically?)
6. Move the probe with table control to its zero position. Again the motor will stall or chatter at the zero position.
7. Choose 19 steps of 100 step size, choose an appropriate dummy data file and make the scan. With 0" as the center of the coil, this scan is between -4.5" and +4.5".
8. At the completion of the scan, move the probe with table control to its zero position.
9. Verify from the instrument panel graph that the scan looks sensible. There should be no glitches. Repeat the scan if needed.
10. Move the postholder 8" on the rail so that 3" of the rail is exposed. Then the next 9" scan will have an overlap of three data points.
11. Continue with 19 steps of 100 step size, choose another dummy data file, and make the next scan. With 0" as the center of the coil, this scan is between -12.5" and -3.5".

Note that reversing the field by changing the current in the coils can test the zero of the probe (save for a possible offset from the ambient field of the earth.)

Helmholtz coil transverse scan (A4)

For the Helmholtz coil transverse scan (exercise A4) the coil must be rotated by 90°, and the transverse probe must be used. Follow the same steps as for the axial scan, and position the coil with respect to the probe so that the midpoint of the scan is at the center of the coil. See the file setupwithXYtable.ppt for pictures.

Solenoid axial scan (B2)
The diameter of the solenoid is smaller than that of the Helmholtz coils so 6" of blocks are needed to put the solenoid at the appropriate height. For the solenoid axial scan (exercise B2) with 0" as the center of the coil, the first scan should map between -7" and +2", and the second scan should map between -15" and -6". The axial probe must be used. The position of the probe with respect to the coil must be measured accurately, since data from exercise B2 is compared to data from exercise B3. Be certain to note which data point corresponds to the probe position just outside of the solenoid. (One or two centimeters outside the coil is convenient.) The transverse scan of the solenoid, exercise B4, must also make a measurement of the field with the probe at this position.

Solenoid inside and outside scan (B3)
In exercise B3 the solenoid is mapped along a line parallel to the axis just inside and just outside of the solenoid. The same position of the table and coil as in exercise B2 is used. The post holding the probe is raised to a vertical height just inside and just outside of the coil. For the probe at "inside" position and for the probe at the "outside" position, the first scan should again map between -7" and +2" from the centerline, and the second scan should map between -15" and -6" from the centerline. See the file setupwithXYtable.ppt for pictures.

Solenoid transverse scan (B4)
In exercise B4 the transverse field outside the solenoid is measured. This scan can be made with the transverse probe moving in the orthogonal direction, or with the axial probe and the coil rotated by 90°. The latter choice allows the repositioning of the postholder on the rail to obtain a longer scan. For the solenoid transverse scan with 0" on the axis of the coil, the first scan should map between -4.5" and +4.5", and the second scan should map between -12.5" and -3.5".

Helmholtz coil median plane scan
This exercise is in addition to the ones in the handout. The XY table can map the complete median plane of the Helmholtz coil. The transverse probe is used, and the axis of coil must be oriented in the vertical. The coil rests on blocks, see the file setupwithXYtable.ppt for pictures. The spacer clamps of the coil restrict the available opening to approximately a 5" by 5" scan. This scan is 36 points, and it takes some time. The data will show the striking uniformity of the median plane field. For this scan 6 steps of 200 steps (1") in both directions should be used. It is a little tricky to get the proper alignment of the probe and the coil. **Watch the probe as it is moving. If the probe would run into the coil, stop the scan with the "Stop Mapping" button.** Reposition the probe appropriately and begin the scan anew.

Appendix
HP34401A DMM PC communication setup

The communication parameters of the COM1 port of the PC are set in the Device Manager control panel. Find the control panel and set the five parameters: baud rate: 9600; data bits: 7; parity: even; stop bits: 2; glow control: Xon/Xoff.

The communication parameters at the DMM are set with the front panel buttoms. The pages below are copied from the HP 34401A manual. The user must select four parameters: interface: RS-232; baud rate: 9600; parity and data bits: even 7 bits; language: SCPI. These are the factory default parameters. Follow the four pages to select these parameters.

To Select the Remote Interface

The multimeter is shipped with both an HP-IB (IEEE-488) interface and an RS-232 interface. Only one interface can be enabled at a time.
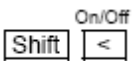
On/Off
Shift <

1 **Turn on the front-panel menu.**

A: MEAS MENU

< <

2 **Move across to the I/O MENU choice on this level.**

E: I/O MENU

∨ >

3 **Move down a level and then across to the INTERFACE command.**

2: INTERFACE

∨

4 **Move down to the "parameter" level to select the interface.**

Use the left/right arrow keys to see the interface choices. Choose from the following: HP-IB / 488 or RS-232.
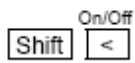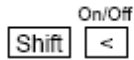
HP-IB / 488

Auto/Man
ENTER
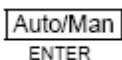
5 **Save the change and turn off the menu.**

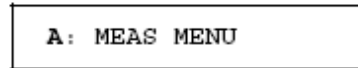The interface selection is stored in *non-volatile* memory, and *does not* change when power has been off or after a remote interface reset.

To Set the Baud Rate

You can select one of six baud rates for RS-232 operation. The rate is set to 9600 baud when the multimeter is shipped from the factory.
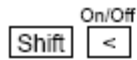
On/Off
Shift <

1 Turn on the front-panel menu.

```
A:  MEAS  MENU
```

< <

2 Move across to the I/O MENU choice on this level.

```
E:  I/O  MENU
```

∨   > >
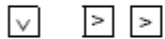
3 Move down a level and then across to the BAUD RATE command.

```
3:  BAUD  RATE
```

∨

4 Move down to the "parameter" level to select the baud rate.

Use the left/right arrow keys to see the baud rate choices. Choose from one of the following: 300, 600, 1200, 2400, 4800, or **9600** baud.

```
9600    BAUD
```

Auto/Man
ENTER

5 Save the change and exit the menu.

The baud rate selection is stored in *non-volatile* memory, and *does not* change when power has been off or after a remote interface reset.

To Set the Parity

You can select the parity for RS-232 operation. The multimeter is configured for even parity with 7 data bits when shipped from the factory.

On/Off
Shift | < |

1 **Turn on the front-panel menu.**

```
A: MEAS MENU
```

| < | | < |

2 **Move across to the I/O MENU choice on this level.**

```
E: I/O MENU
```

| v | | < | | < |
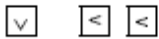
3 **Move down a level and then across to the PARITY command.**

```
4: PARITY
```

| v |

4 **Move down to the "parameter" level to select the parity.**

Use the left/right arrow keys to see the parity choices. Choose from one of the following: None (8 data bits), **Even** (7 data bits), or Odd (7 data bits). When you set parity, you are indirectly setting the number of data bits.

```
EVEN:    7 BITS
```

Auto/Man
ENTER
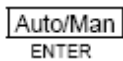
5 **Save the change and turn off the menu.**

The parity selection is stored in *non-volatile* memory, and *does not* change when power has been off or after a remote interface reset.

To Select the Programming Language

You can select one of three languages to program the multimeter from the selected remote interface. The language is SCPI when the multimeter is shipped from the factory.

On/Off

[Shift] [<]   **1 Turn on the front-panel menu.**

> ```
> A: MEAS MENU
> ```

[<] [<]   **2 Move across to the I/O MENU choice on this level.**

> ```
> E: I/O MENU
> ```

[v] [<]   **3 Move down a level and then across to the LANGUAGE command.**

> ```
> 5: LANGUAGE
> ```
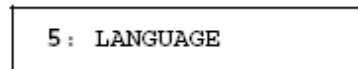
[v]   **4 Move down to the "parameter" level to select the language.**

> Choose from one of the following: **SCPI, HP 3478A,** or **Fluke 8840A.**

> ```
> SCPI
> ```

[Auto/Man]
  ENTER   **5 Save the change and turn off the menu.**

> The language selection is stored in *non-volatile* memory, and *does not* change when power has been off or after a remote interface reset.

➢ Halbach magnet geometry: An arrangement of permanent magnets which generates a desired field distribution within a confined volume while canceling the field outside.

➢ Applications include: uniform fields for nuclear magnetic resonance (NMR), particle beam steering (dipole, quadrupole, … lenses), undulators in free electron lasers (FEL), etc.

➢ Advantages: high fields exceeding 1T, no external power supplies, compact size.

➢ It's also pretty cool.

Dipole field distribution

$$B_x = \frac{3m}{r^3}\cos\theta\sin\theta\cos\phi$$

$$B_y = \frac{3m}{r^3}\cos\theta\sin\theta\sin\phi$$

$$B_z = \frac{m}{r^3}(3\cos^2\theta - 1)$$

In the zx-plane, $\phi = 0$:

$$B_x = \frac{3m}{r^3}\cos\theta\sin\theta$$

$$B_y = 0$$

$$B_z = \frac{m}{r^3}(3\cos^2\theta - 1)$$

z

x

θ $\vec{r}$

$\vec{m}$

$$\cos\theta = \frac{z_0 - z}{r}$$

$$\sin\theta = \frac{x_0 - x}{r}$$

$$\vec{r} = \vec{r}_0 - \vec{r}_p$$
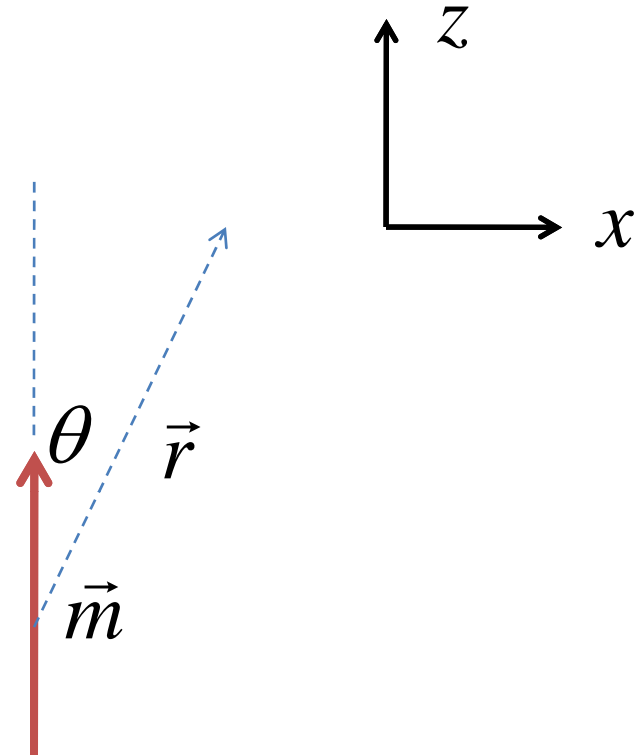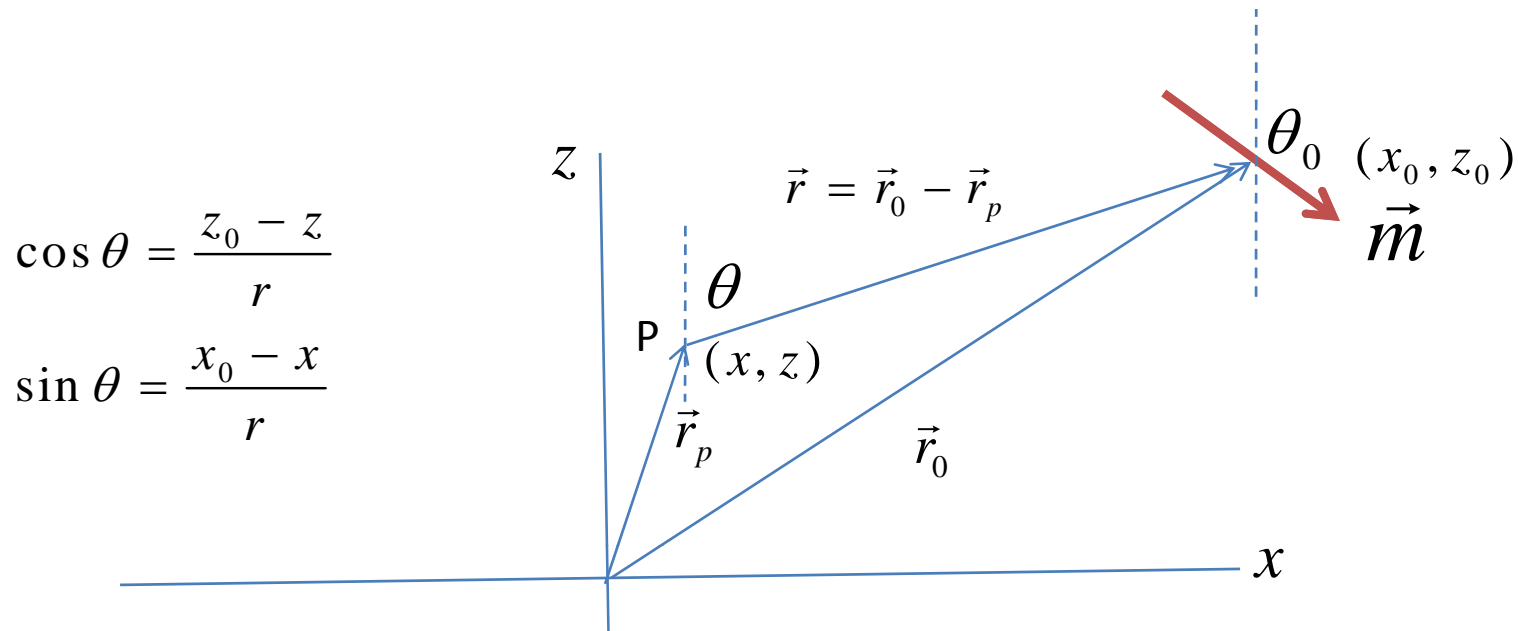
$$B_x' = \frac{3m}{r^3}\cos(\theta + \theta_0)\sin(\theta + \theta_0)$$

$$B_x(x,z) = B_x'\cos\theta_0 + B_z'\sin\theta_0$$

$$B_z' = \frac{m}{r^3}(3\cos^2(\theta + \theta_0) - 1)$$

$$B_z(x,z) = -B_x'\sin\theta_0 + B_z'\cos\theta_0$$

Simple Halbach Array

Augmented

Canceled
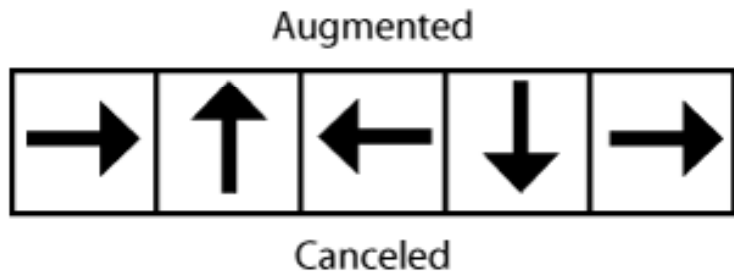
One-sided configuration

Free electron laser undulator
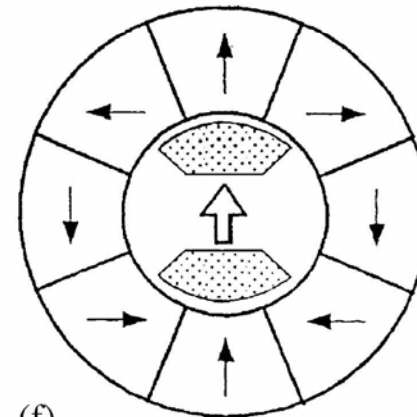

e beam

(a)

e beam

(b)
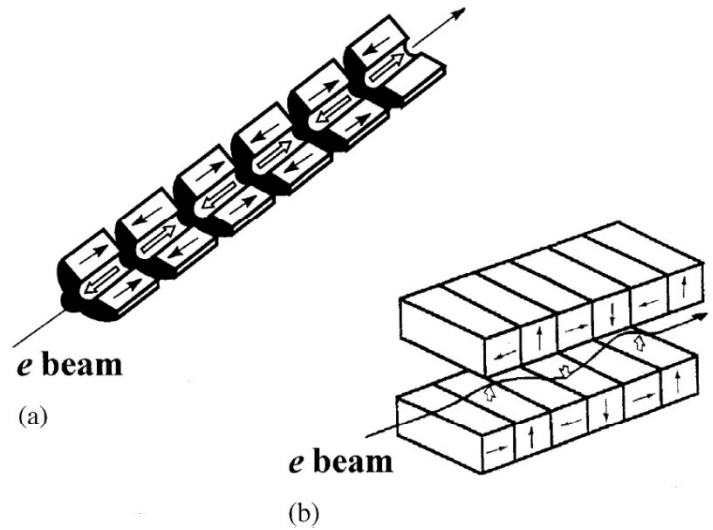
Dipole Field Configuration
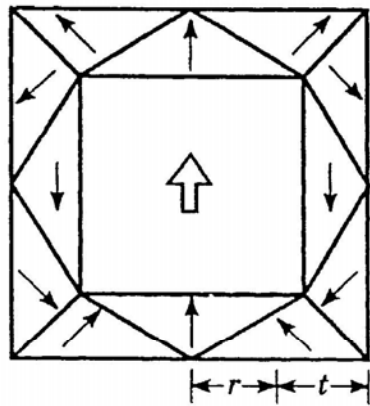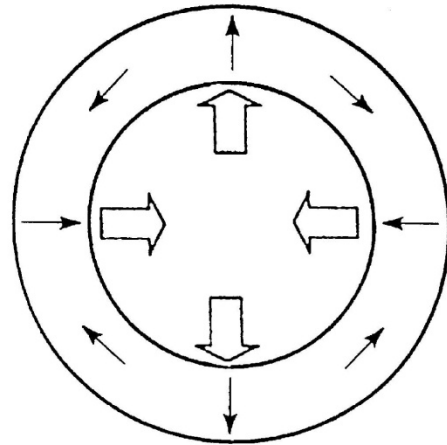

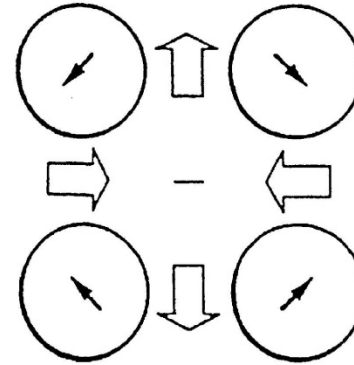(a)
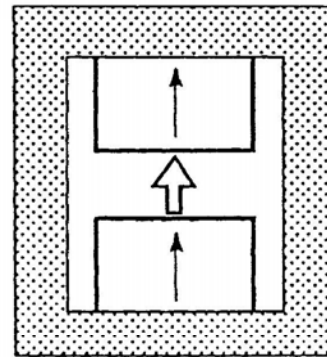
(b)
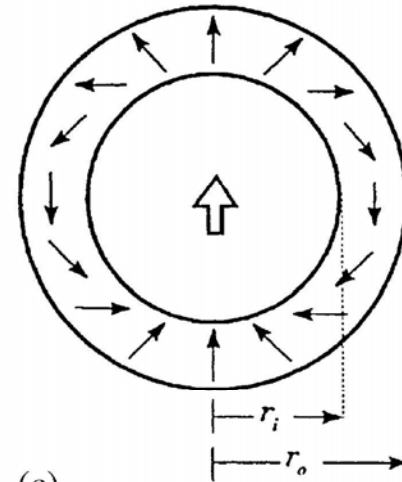
(c)

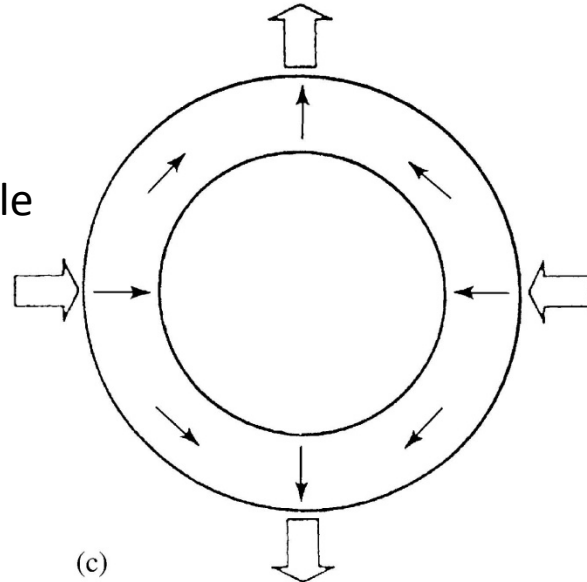**8-Pole Uniform Field**
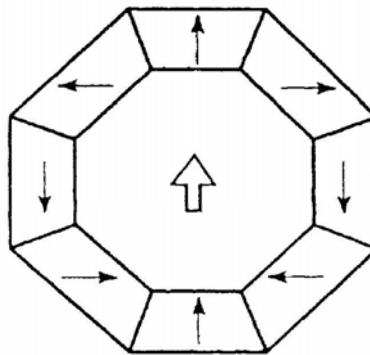
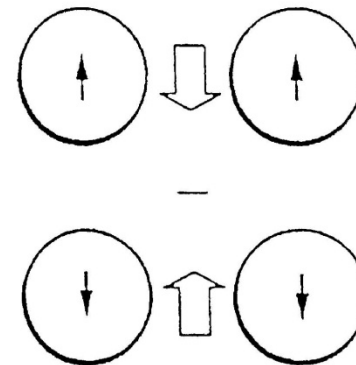(d)
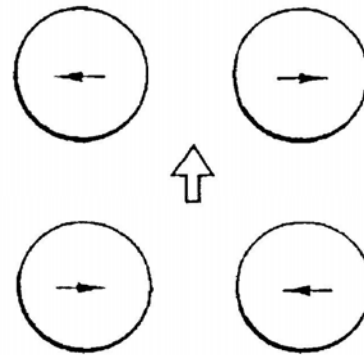
(e)

(f)

Quadrupole Field

Quadrupole fields

(a)

(b)

External quadruple

(c)

(d)

Field Gradient

Instructions for Halbach Measurements

• Map out Bz and Bx for **ONE** of the following: (1) quadrupole, (2) 8-pole uniform field, or (3) gradient field. Include your graphs in your report

• Use the transverse hall probe to determine the polling direction of each magnet cube.

• Construct a given Halbach geometry by using the ruler to indicate the location of each magnet on the Styrofoam block then press the magnets into the Styrofoam with your fingers. Keep in mind that the magnets are very strong so don't place the magnets closer than about 5 cm. The size of the geometry is up to you, but you should aim for each side to be between 5-10 cm.

• Use the transverse Hall probe to scan and map out Bz and Bx (the two components of the magnetic field in the plane of the magnets) as a function of x and z. You will need to make two separate scans to determine Bz and Bx. The probe will need to be rotated by 90° between scans.

• Keep the probe as close to the surface of the Styrofoam as possible in order to determine the in-plane magnetic filed.

• Scan the probe over an area that includes the magnets.

• The scans should include no fewer than 10X10 points. More points will allow you to resolve the spatial variation of the field more accurately.

- A Mathematica notebook is provided to calculate the field distribution. The file also generates text files containing the data for a given magnet geometry.

- The notebook has 4 pre-defined geometries that you will be measuring. To run a given configuration, remove the comments (* ... *) around the m-vector statement making sure that the other configurations are commented out. Set the number of elements Nm to indicate the number of magnets for the selected geometry. To execute the command, press Shift+Enter.

- The m-vector is organized as follows m={{x0,z0,θ0},{x1,z1,θ1},...}

- You can use the pre-defined definitions, or you can add/modify your own configuration - this is encouraged.

- You can import the data files generated by Mathematica into Origin to compare with your results. You will need to scale the calculated values in order to make this comparison.