University of Illinois at Urbana-Champaign Department of Physics

Physics 401 Classical Physics Laboratory

Experiment 5

Transients and Oscillations in RLC Circuits

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I. Introduction

In this experiment we first study the behavior of transients in a series RLC circuit as we vary the resistance and the capacitance. We find that two qualitatively different transients are possible, a damped oscillation and an exponential decay. We then drive the RLC circuit with an external sinusoidal voltage and find that the response of the circuit depends on the driving frequency. We find that the response of the circuit shows maximum at some particular frequency. We compare our observations to a simple model.

A wide variety of physical systems are understood as examples of oscillating systems: the simple pendulum, the mass on a spring, the charged particle in a storage ring, and the series RLC circuit. In each of these physical systems we determine how a single variable, for example, the position of the mass on the spring or the charge on the capacitor, changes with time. There are many elements common to the description of all oscillating systems. For example, all of these systems are, in fact, described by equations of the same functional form. Consider the two systems shown in Figure 1, below.

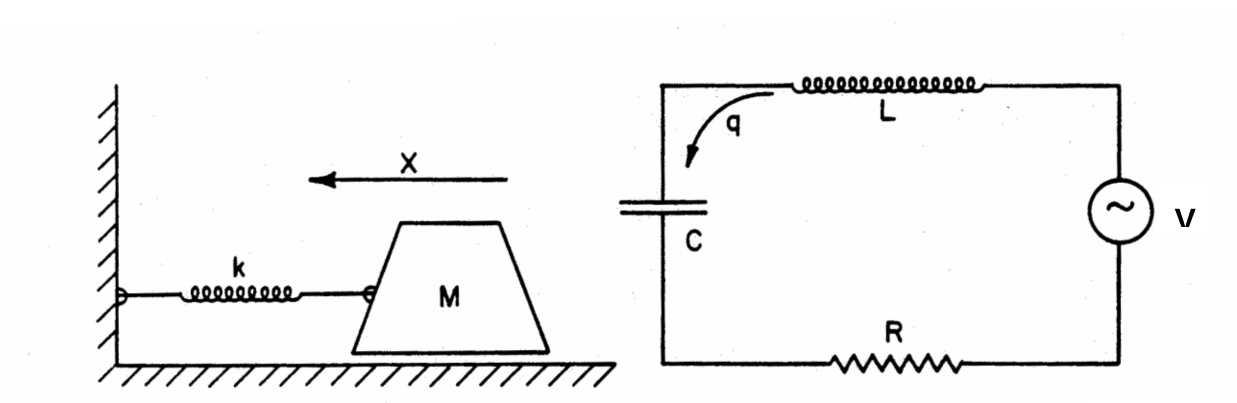

Figure 1 Two equivalent dynamical systems

For the mechanical system, $x(t)$ is the displacement of the mass and the equation of motion is

$$M\frac{d}{dt}\left(\frac{dx}{dt}\right)+r\frac{dx}{dt}+kx=F \tag{0.1}$$

On the left hand side of this equation, M is the mass and the first term is then mass times acceleration; r is the coefficient of viscous friction (the frictional force is proportional to velocity[1]); and k is the spring constant (the spring force is $-kx$). On the right-hand side of the equation, F represents an external driving force (not shown in the figure).

For the RLC circuit, $q(t)$ is the charge on the capacitor, and Kirchoff's voltage law (see Appendix I for a very brief exposition of Kirchoff's laws) gives the equation

$$L\frac{d}{dt}\left(\frac{dq}{dt}\right)+R\frac{dq}{dt}+\frac{1}{C}q=V(t) \tag{0.2}$$

On the left-hand side of the equation, the first term is the voltage drop across the inductor. We have used the derivative of the charge on the capacitor for the current through the inductor. The second term is the voltage drop across the resistor, and the third term is the voltage drop across the capacitor. On the right-hand side $V(t)$ is an externally applied voltage. We find in comparing Equation (0.1) and Equation (0.2) that the mass behaves the same as an inductor, and the spring the same as an inverse capacitance. Displacement becomes charge, and the viscous friction is replaced by a resistor. Such correspondences can be found in other systems.

The theory below first treats the case in which there is no externally applied voltage. The solution has two qualitatively different forms. Several important quantities are defined: natural (angular) frequency for oscillation, ω_o, the quality factor, Q, and the logarithmic decrement, δ, which is also called the log decrement. The logarithm has the base e, but no one ever calls δ the *ln* decrement. The concept of critical damping is also introduced. The case in which there is an externally applied sinusoidal voltage is discussed only briefly. The concept of resonance is introduced.

II. Theory

The simplest RLC circuit where there is no externally applied voltage is shown schematically in Figure 2, below. At $t=0$, the switch is closed to permit the initially charged capacitor to discharge through the inductor and resistor. (In the mechanical system the spring would be stretched initially, and the mass would be released at $t=0$.)

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[1] The figure shows a block sliding on a surface. Unfortunately, this figure is not a good example of viscous friction, since the friction between the block and the surface is better described as constant, dependent on direction but independent of velocity. A better example would have the block move through a fluid.

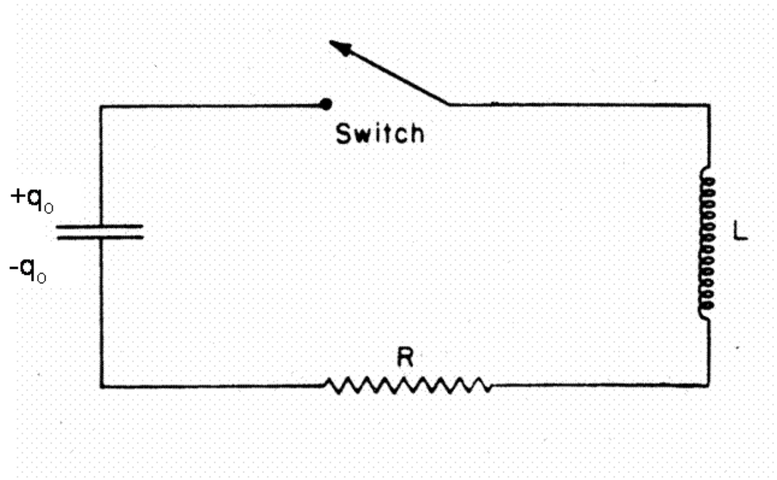

Figure 2 RLC circuit with voltage on capacitor initially

Application of Kirchhoff's law gives

$$L\frac{d}{dt}\left(\frac{dq}{dt}\right)+R\frac{dq}{dt}+\frac{q}{C}=0, \qquad (0.3)$$

where $\frac{dq}{dt}$ is the current. This equation is a homogeneous, linear, second-order differential equation and has solutions of the form $q(t)=Ae^{st}$, where A is an arbitrary constant. Substituting this function into Equation (0.2) produces a quadratic equation for s:

$$s^2+\left(\frac{R}{L}\right)s+\frac{1}{LC}=0, \qquad (0.4)$$

which must be satisfied for $q(t)=Ae^{st}$ to be a solution. The two roots of Equation (0.4) are

$$s_1, s_2=-\left(\frac{R}{2L}\right)\pm\sqrt{\left(\frac{R}{2L}\right)^2-\left(\frac{1}{LC}\right)}=-a\pm b \qquad (0.5)$$

where $$a=\left(\frac{R}{2L}\right) \text{ and } b=\sqrt{\left(\frac{R}{2L}\right)^2-\left(\frac{1}{LC}\right)} \qquad (0.6)$$

The nature of the solution depends on whether the under radical equation is positive ($b^2 > 0$, over-damped solution), equal to 0 ($b^2 = 0$; critically damped solution), or negative ($b^2 < 0$, under-damped, or oscillatory solution).

A. Over-damped solution $b^2 > 0$

If b^2 is positive, then the solution is aperiodic and $q(t)$ falls to zero smoothly with no oscillations. The solution is of the form

$$q(t) = e^{-at}\left(A_1 e^{bt} + B_1 e^{-bt}\right). \tag{0.7}$$

The constants A_1 and B_1 are determined from the initial conditions. For the circuit in Figure 2, the charge on the capacitor at $t = 0$ is q_o. Evaluating Equation (0.7) at $t = 0$ gives $q_o = \left(A_1 + B_1\right)$. At $t = 0$, the current is zero. Differentiating Equation (0.7) gives the current

$$i(t) = -ae^{-at}\left(A_1 e^{bt} + B_1 e^{-bt}\right) + be^{-at}\left(A_1 e^{bt} - B_1 e^{-bt}\right). \tag{0.8}$$

Evaluating Equation (0.8) at $t = 0$ gives $-a\left(A_1 + B_1\right) + b\left(A_1 - B_1\right) = 0$. After some algebra we obtain

$$q(t) = q_o e^{-at}\left(\cosh bt + \frac{a}{b}\sinh bt\right) \to \frac{q_o}{2}\left(1 + \frac{a}{b}\right)e^{-(a-b)t} \quad (a-b)t \gg 1, \tag{0.9}$$

$$i(t) = -q_o e^{-at}\left(\frac{a^2 - b^2}{b}\right)\sinh bt \to -\frac{q_o}{2}\left(\frac{a^2 - b^2}{b}\right)e^{-(a-b)t} \quad (a-b)t \gg 1. \tag{0.10}$$

Since $b^2 > 0$, from Equation (0.6) we find that $a > b$. Thus, at large times the charge and current decay with an exponential $a - b < a$.

B. Critically damped solution $b^2 = 0$

For $b^2 = 0$, the solution is critically damped and $q(t)$ will fall to zero in the minimum time without oscillation. There are no longer two distinct solutions to Equation (0.4), and the form of the solution is now

$$q(t) = (A_2 + B_2 t)e^{-at} \tag{0.11}$$

Evaluating Equation (0.11) at $t = 0$ gives $A_2 = q_o$. Differentiating Equation (0.11) gives the current.

$$i(t) = B_2 e^{-at} - a(A_2 + B_2 t)e^{-at} \tag{0.12}$$

Evaluation Equation (0.12) at $t = 0$ gives $B_2 - a\,A_2 = 0$. After some algebra we obtain

$$q(t) = q_o\,(1 + at)e^{-at}\text{, and} \tag{0.13}$$

$$i(t) = -a^2 q_o\, t\, e^{-at}. \tag{0.14}$$

The charge decays with an exponential a. The current starts at zero, goes through a maximum, and then also decays with an exponential a.

<u>C. Under-damped solution</u> $b^2 < 0$

The most interesting case is for $b^2 < 0$. Then the two roots of Equation (0.4) are complex. It is convenient to make b real (change the signs of the factors under the radical in Equation (0.4)) and use $j = \sqrt{-1}$. (We use j because i is the current.) Then the solution becomes

$$q(t) = e^{-at}\left(A_3 e^{jbt} + B_3 e^{-jbt}\right) \tag{0.15}$$

where in the above

$$a = \left(\frac{R}{2L}\right) \text{ and } b = \sqrt{\left(\frac{1}{LC}\right) - \left(\frac{R}{2L}\right)^2} \tag{0.16}$$

Evaluating Equation (0.15) at $t = 0$ gives $q_o = (A_3 + B_3)$. Differentiating Equation (0.15) gives the current.

$$i(t) = -ae^{-at}\left(A_3 e^{jbt} + B_3 e^{-jbt}\right) + jbe^{-at}\left(A_3 e^{jbt} - B_3 e^{-jbt}\right) \tag{0.17}$$

Evaluating Equation (0.17) at $t=0$ gives $-a\left(A_3+B_3\right)+jb\left(A_3-B_3\right)=0$. After some algebra we obtain

$$q(t)=q_o\,\frac{\sqrt{a^2+b^2}}{b}\,e^{-at}\left(\frac{e^{j(bt-\phi)}+e^{-j(bt-\phi)}}{2}\right)=q_o\,\frac{1}{\cos\phi}\,e^{-at}\cos\left(bt-\phi\right) \tag{0.18}$$

where $\tan\phi=a/b$. Alternatively,

$$q(t)=q_oe^{-at}\left(\cos bt+\frac{a}{b}\sin bt\right), \tag{0.19}$$

$$i(t)=-q_oe^{-at}\left(\frac{a^2+b^2}{b}\right)\sin bt\,. \tag{0.20}$$

The angular frequency, ω_1, of the oscillation is b, and the frequency, f_1, is

$$f_1=\frac{\omega_1}{2\pi}=\left(\frac{1}{2\pi}\right)\sqrt{\left(\frac{1}{LC}\right)-\left(\frac{R}{2L}\right)^2}\,. \tag{0.21}$$

The solution oscillates with frequency, f_1, and the amplitude of the oscillation decays with exponential a. Even in the presence of damping, the frequency can easily be determined by measuring the time between zeroes of charge. Note that with no damping, $R=0$; this frequency of oscillation is the natural frequency of the LC oscillator.

$$f_o=\frac{\omega_o}{2\pi}=\frac{1}{2\pi}\sqrt{\frac{1}{LC}} \tag{0.22}$$

With damping, the frequency of oscillation is shifted to a smaller value.

The logarithmic decrement, δ, is the natural log of the ratio of the charge (see equation 0.19) or current (see equation 0.20) between two successive maxima, which are separated in time by the damped period, $T_1=1/\,f_1$.

$$\delta=\ln\left(\frac{q\left(t_{\max}\right)}{q\left(t_{\max}+T_1\right)}\right)=\ln\left(\frac{e^{-at_{\max}}}{e^{-a\left(t_{\max}+T_1\right)}}\right)=aT_1 \tag{0.23}$$

Magnetic energy is stored in the inductor, and electric energy is stored in the capacitor. The energy is dissipated in the resistor. The Q of the circuit, or quality factor, is defined as

$$Q = 2\pi \frac{\text{total stored energy}}{\text{decrease in energy per period}} . \tag{0.24}$$

(To be very clear, Q here is not the charge.) For an RLC circuit Q is found to be

$$Q = \frac{\omega_1 L}{R} = \frac{\pi}{\delta} . \tag{0.25}$$

(It takes some effort to derive this result.) For small R, the damping or logarithmic decrement is small, and the Q of the circuit is large. Note also for small damping (large Q) that the damped frequency, f_1, and the natural frequency, f_o, are approximately equal. (We introduce a third frequency, f, into the experiment when we drive the RLC circuit with a sinusoidal voltage. We also introduce another expression for Q.)

III. Practical capacitors and inductors

A catalog of an electronics supply company lists dozens and dozens of pages of passive components, *i.e.* resistors, capacitors, and inductors. Practical considerations in the use of these components are many, but they are of little importance for this laboratory exercise. We may assume that the real resistor has the behavior of the ideal resistor and that the real capacitor has the behavior of the ideal capacitor. The real inductor, however, ***has resistance in addition to inductance.*** The inductors are coils of copper wire, and copper has some resistivity. In the circuit of Figure 2, we simply add an additional series resistance.

IV. Procedures for studying transient and steady-state response

Figure 3 below is the circuit diagram that closely approximates the idealized circuit of Figure 2. The significant difference is that the voltage source, the Wavetek™, is coupled to the RLC

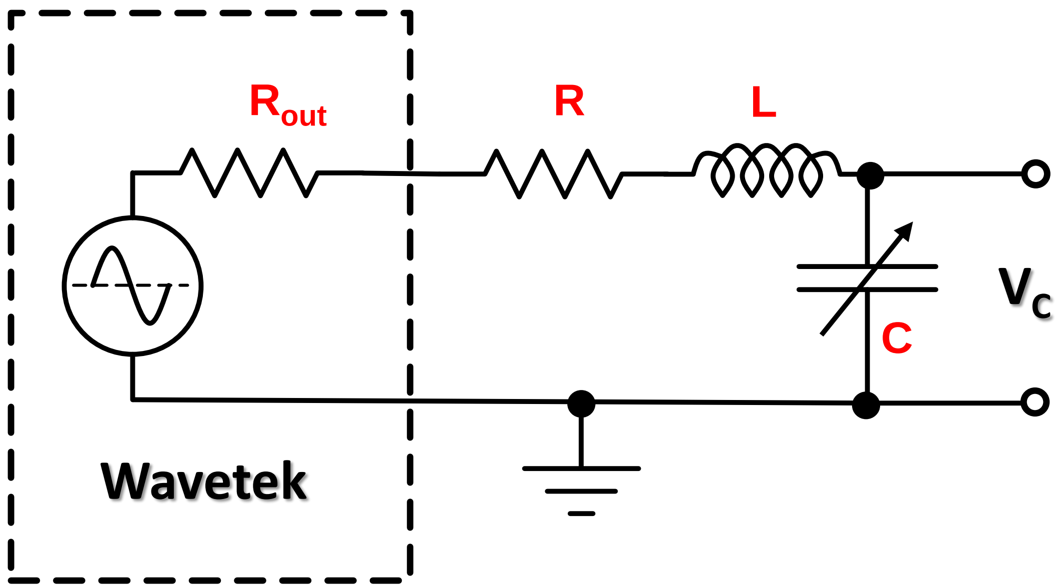

Figure 3 Connection of Wavetek™ to RLC circuit and oscilloscope

circuit through a resistance R_{out} which is the output resistance of the function generator and is of about 50 Ω. This 50 Ω is an additional series resistance in the circuit.
The Wavetek™ is used to generate a 8.0 V unipolar square wave of a period of 0.10 s (frequency of 10 Hz) and a duty factor of 50%, as shown in the Figure 4 below. Duty factor is defined as the ratio of the duration of the pulse (0.05 s here) to the period (0.10 s).

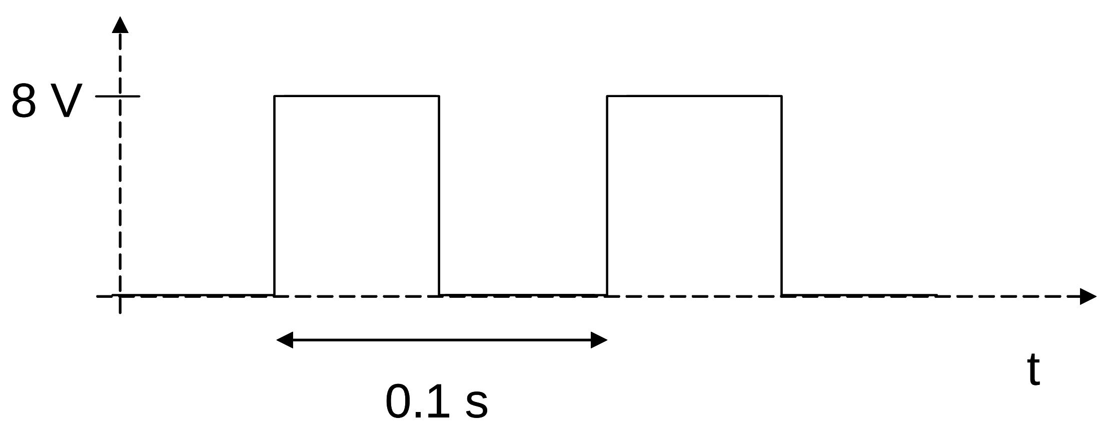

Figure 4 Wavetek™ signal to excite RLC circuit

On the leading edge of the pulse, the capacitor charges. On the trailing edge of the pulse, the capacitor discharges. The capacitor discharges on the trailing edge of the pulse. The transitions occur every 50 ms, and this time is long compared to the time constants of the transient behavior of the circuit. The voltage across the capacitor is observed with the TDS3012B oscilloscope. It is very convenient to use this circuit to see changes in the response of the circuit as the resistance and capacitance values are changed. The inductor is actually two coils wound on the same bobbin. The two inner terminals of the double coil should be connected to obtain a larger inductance (by roughly a factor of 4) than one coil. The capacitor is a decade capacitor box, and the resistor is decade resistor box. Make a note of the number of the coil, and measure its series resistance with a DMM. In the exercises below you will change the values of the capacitance box and the resistance box. Your laboratory bench has an aluminum utility box with terminal posts that are set up for easy connection to the circuit elements. The box was built for an earlier version of the experiment which did not use the Wavetek™. Even so, there should be little difficulty in making the circuit shown in Figure 3.

The ***SYNC OUT*** of the Wavetek™ could be used to trigger the oscilloscope as an external trigger. By selecting ***Trigger >> Slope*** it is possible to see if the oscilloscope is triggering on the leading or the trailing edge of the pulse
Recall that the accuracy of the time and voltage readings of the oscilloscope depend on the scale settings of the oscilloscope so recording the time per division and voltage per division are essential.

A. Determine dependence of the resonance frequency on capacitance

The goal of this experiment is to observe the damped, oscillatory behavior of the circuit and measure how the frequency of the oscillation depends on capacitance. Set the decade resistance box to zero, and set the decade capacitance box to $C = 1.0\ \mu\text{F}$. A sweep of 2 ms/div and a vertical sensitivity of 5 V/div should be reasonable, but you should make adjustments to get an optimal view of the transient. It should be possible to see about 10 periods of the oscillation. The time cursors allow you to measure accurately the period of the oscillation. First you should verify that, indeed, there is a definite period in the oscillation. Verify that the time between zero crossings in the transients is constant. Then record the period of the oscillation for $C = 1.0, 0.9, 0.8, 0.7, 0.6, 0.5, 0.4, 0.3, 0.2, 0.1, 0.05\ \mu\text{F}$. You may need to adjust the vertical sensitivity and the time base to maintain an optimal view. You will get a better value for the period if you measure the period over a number of oscillations. Over the entire range of capacitance values, it should be possible to see ten oscillations. Use ten oscillations when you measure the period. It is useful to transfer a picture of the oscilloscope display for documentation. Figure 5 below shows a typical oscillation.

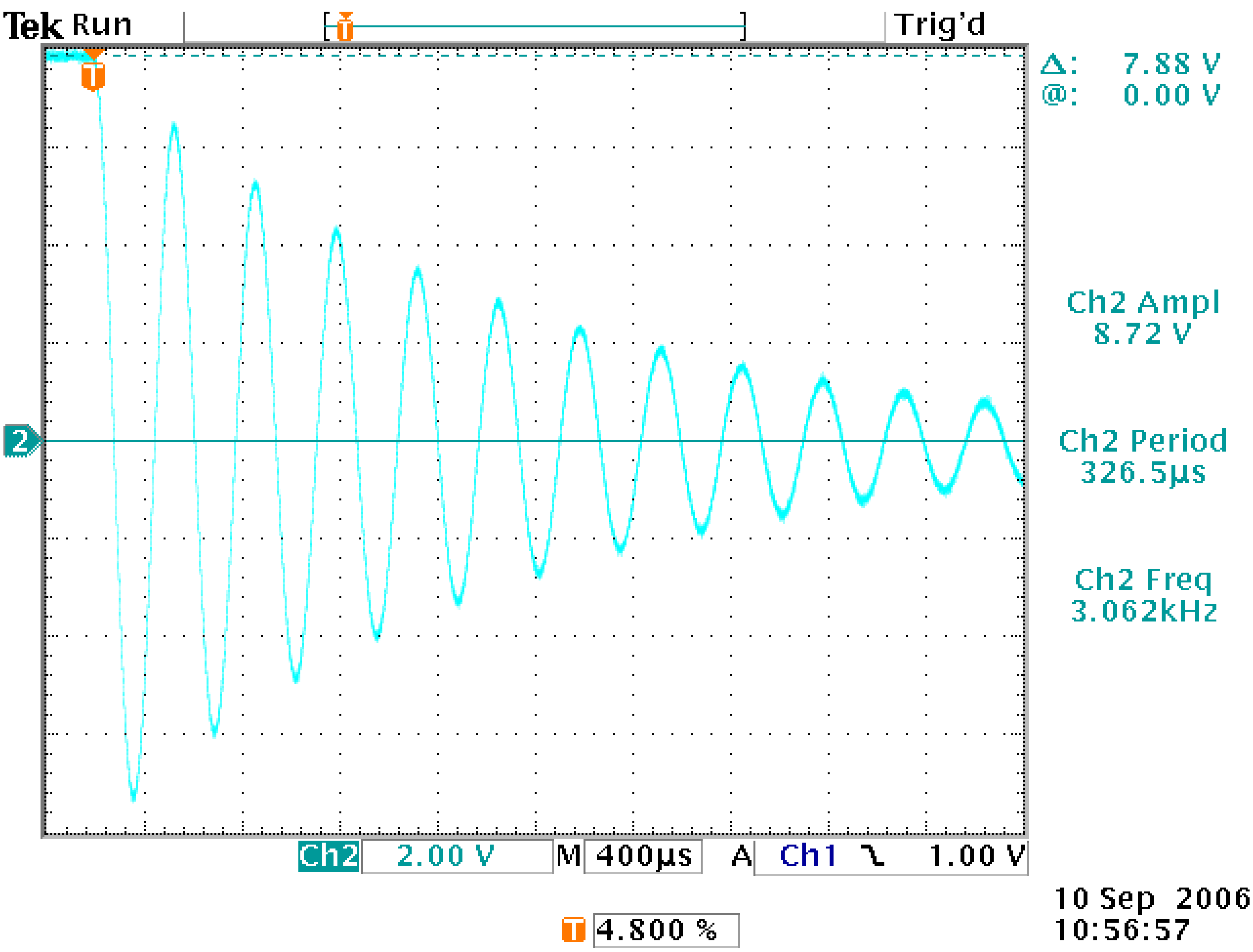

Figure 5 Transient response of RLC circuit

From Equation (0.21) above,

$$f_1^2 = \frac{1}{T^2} = \left(\frac{1}{2\pi}\right)^2 \left[\left(\frac{1}{LC}\right) - \left(\frac{R}{2L}\right)^2\right] \tag{0.26}$$

so a plot of $1/T^2$ versus $1/C$ should be a straight line. From slope you can calculate the inductance. Make a plot of $1/T^2$ versus $1/C$, and find the inductance of your coil. (It should be about 50 mH.) Assuming that R in Equation (0.26) above is the series resistance of the coil plus output resistance of the Wavetek™ (~ 50 Ω). Simple calculation can show that $\frac{1}{LC} >> \left(\frac{R}{2L}\right)^2$

Since the second term is negligible, a plot of T^2 versus C should also be a straight line. Make this plot (and the other plots described below) with Excel as you take the data. Print a copy to paste in your notebook.

$$T^2 = (2\pi)^2 LC \tag{0.27}$$

An impedance meter is available in the laboratory, the Z-meter, which can measure inductances and capacitances. Find the meter, read the brief instructions, and measure the inductance of your coil. Compare the measured inductance with those obtained from slope.

B. Determine the dependence of log decrement on resistance

The goal of this part of the exercise is to find how the rate of damping of the transient depends on the resistance. With $\mathrm{C} = 1.0\ \mu\mathrm{F}$ increase the resistance of the decade resistance box, R, from zero to $300\,\Omega$ in large steps $(50\ \Omega)$ while you observe the oscillating transient. Again you will need to adjust the horizontal sweep and vertical sensitivity to get an optimal view. At what resistance do you find it difficult to see more than one or two strongly damped oscillations? Use the H Bar cursors of the oscilloscope to measure the amplitudes of the peaks of the transient. If the amplitude of the first positive peak is V1 and the second positive peak is V2, the log decrement is $\delta = \log(V1/V2)$. For $R = 0\ \Omega$, verify that the log decrement is, indeed, a constant by measuring the voltage ratio for a few adjacent positive peaks, say, V1/V2, V2/V3, V3/V4, V4/V5. Since these ratios are constant, you will get a better measurement for the log decrement if you measure it between several oscillations. However, it is also possible to adjust the vertical sensitivity of the oscilloscope to get a good measurement of the log decrement just between adjacent peaks. When changing the value of R you should also note that the period changes, but only slowly. For R=100, 90, 80, 70, 60, 50, 40, 30, 10, 0 Ω, measure the log decrement of the transient.

From (0.24).

$$\delta = aT = \left(\frac{R}{2L}\right)T = \left(\frac{R}{2L}\right)\frac{2\pi}{\sqrt{\frac{1}{LC} - \left(\frac{R}{2L}\right)^2}} = \pi\frac{R}{L}\sqrt{LC}\frac{1}{\sqrt{1 - \frac{R^2C}{4L}}} \approx \pi\sqrt{\frac{C}{L}}R\,. \tag{0.28}$$

If the period does not change much (equivalently, if $R^2C/4L \ll 1$), a plot of log decrement versus R then should be a straight line. Make a plot log decrement versus R and note that the line does not pass through the origin. The offset is due to additional resistance in the circuit, for example, the resistance of the coil plus output resistance of the Wavetek™ (~50Ω). Determine the effective additional resistance in the circuit from the zero intercept of your plot.

C. Determine the value of resistance for critical damping

The goal of this part of the exercise is to observe critical damping of the circuit. From Equation (0.6), critical damping occurs when

$$b=\sqrt{\left(\frac{R_{critical}}{2L}\right)^2-\left(\frac{1}{LC}\right)}=0\rightarrow R_{critical}=2\sqrt{\frac{L}{C}}. \quad (0.29)$$

Thus the critical resistance is proportional to 1 over the square root of the capacitance. Change the capacitance by a factor of 100, and the critical resistance changes by only a factor of 10. For C=1.0 μF, increase R until no oscillations are observed in the transient. Adjust the vertical sensitivity of the oscilloscope to see a possible undershoot of the transient. The oscilloscope has a feature, ***Save/Recall* >> *Save Waveform*** that allows easy comparison of up to 4 oscilloscope traces in ***Ref1*** to ***Ref4***. After the traces are stored, use ***Recall Waveform*>> *Ref1*** etc to superimpose on the existing display. It is interesting to see a series of traces for a range of values of R. The transient should go from damped oscillations to no oscillations. To remove the Reference waveforms from display, press **REF** button and select the waveform to erase. Press **OFF** button (under VERTICAL area) to remove it. Note that the waveforms can be recalled later from the volatile memory. A picture of the oscilloscope display can be transferred to the PC. Figure 6 below shows the response for three resistance values above, below and at the critical value.

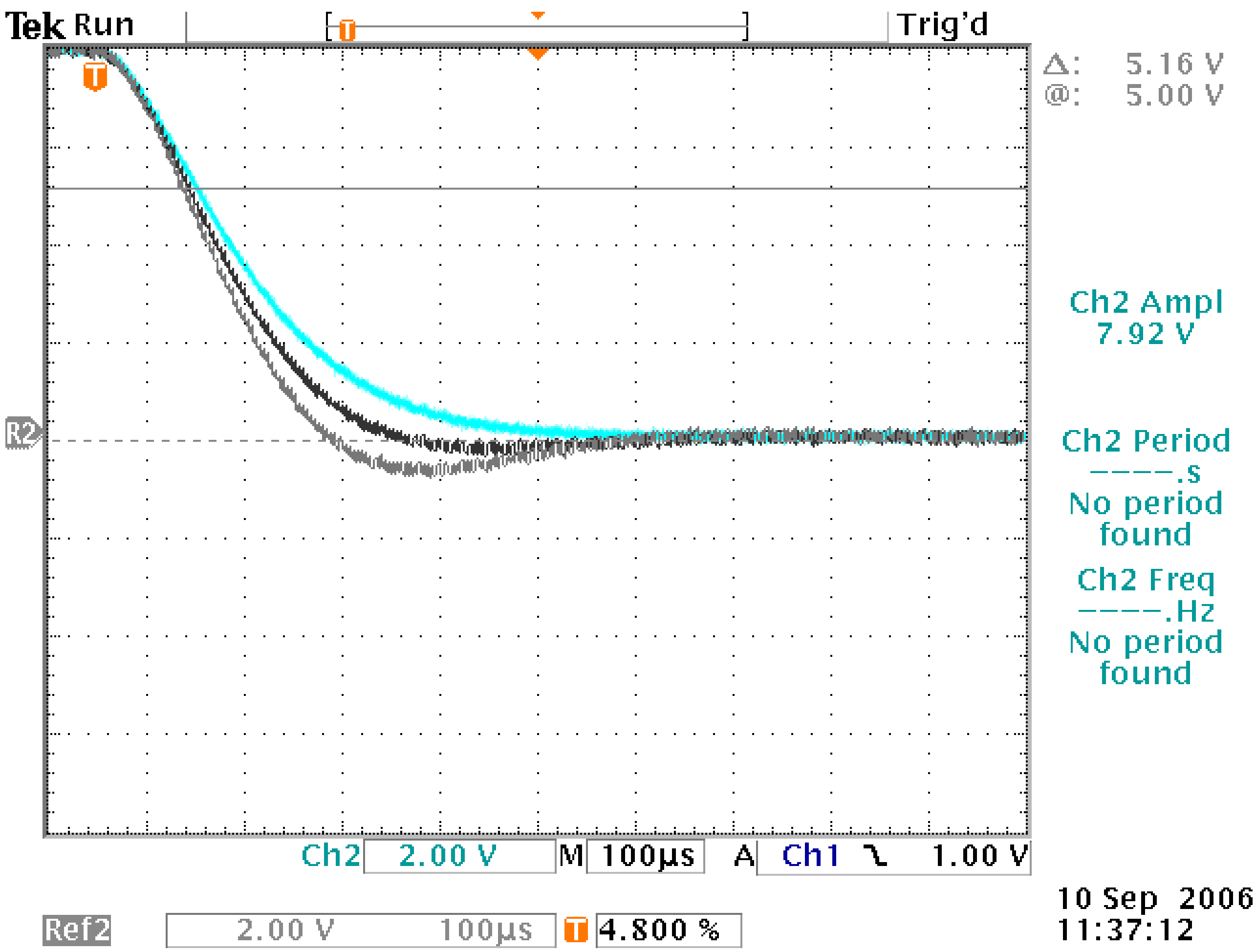

Figure 6 Critically damped response of RLC circuit

Equation (0.9) shows that an over-damped RLC decays with exponent $(a-b)$, and Equation (0.12) shows that a critically damped RLC circuit decays with exponent a. Thus the decay time for the over-damped circuit is longer! The digital oscilloscope is able to measure fall time directly when a signal passes from one level to another level. In ***Measure*** mean you should find the fall time option. This feature is useful when trying to find the critical resistance. This measurement takes some effort. If the resistance is too small, there will be an undershoot. If the resistance is too large, the fall time will be too long. For $C = 1.0, 0.5, 0.1, \quad 0.05, 0.01\ \mu\text{F}$, find the value of $R0$ at which critical damping occurs. Estimate the uncertainty in your $R0$ measurement. It should be possible to determine the resistance at which critical damping occurs to ~20% for each value of C. Plot $R_{critical}$ versus $\sqrt{1/C}$. You should see a straight line. This exercise completes your investigation of the transient response of the RLC circuit. Set the utility box aside. Next you will study the response to a sinusoidal external voltage.

D. Measure the response of an RLC circuit to a sinusoidal signal

The goal of this part of the exercise is to measure the response of the RLC circuit to a sinusoidal signal. When the series RLC circuit is driven by a voltage source, $V_0 \cos \omega t$, the voltage across a circuit element (for example, the capacitor) is at the same frequency, shifted in phase, with a certain amplitude, $V_c \cos(\omega t - \phi)$. Connect the circuit shown in Figure 7 below. The decade resistance box, R_B, is set to $50\,\Omega$, and the decade capacitance box is set to $C = 1.0\ \mu\text{F}$. The $0.1\ \text{M}\Omega$ resistor is from the Megohm box. Note the position of the common ground for the Wavetek™ and the oscilloscope in the circuit. The Sync Out of the Wavetek™ is used for the external trigger of the scope. The voltage across the capacitor is observed on the oscilloscope.

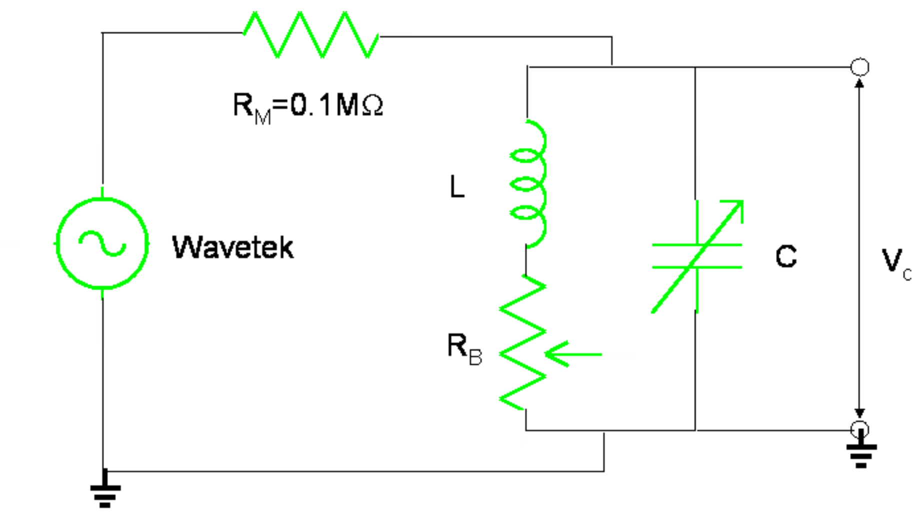

Figure 7 Circuit to study steady state RLC response

A simplified version of the circuit is shown in Figure 8. The box labeled with Z denotes the complex impedance of the RLC circuit.

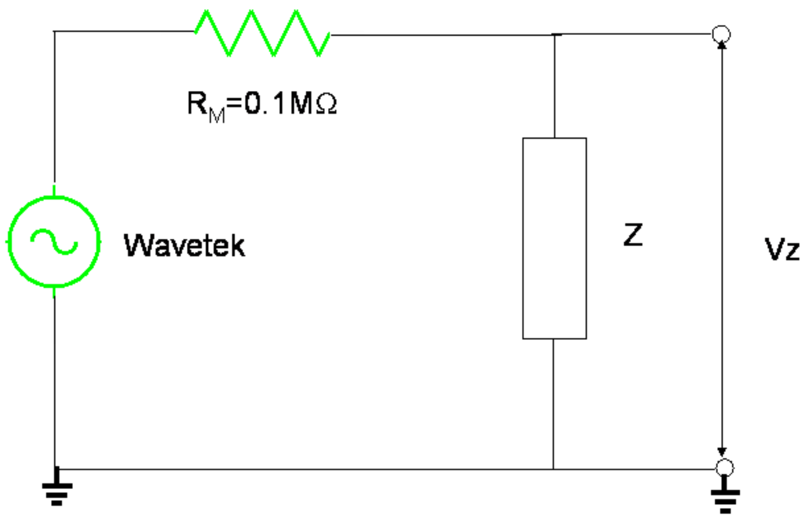

Figure 8 Simplified version of circuit to study steady state RLC response

The two elements, R_M and Z are a voltage divider, and the voltage across Z is

$$V_Z = V_0 \frac{Z}{R_M + Z}. \tag{0.30}$$

If $R_M >> Z$, this expression simplifies

$$V_Z \approx \frac{V_0}{R_M} Z; \tag{0.31}$$

Z is the complex impedance. From Figure 7 we see that we have the parallel combination of a capacitor and a resistance plus inductance in series. This complex impedance is

$$Z = \frac{\frac{1}{j\omega C}.\left[j\omega L + R\right]}{\frac{1}{j\omega C} + j\omega L + R}, \tag{0.32}$$

where R in the above expression is the resistance of the coil and the decade resistance box. Then

$$Z = \frac{\frac{1}{j\omega C}.\left[j\omega L + R\right]}{\frac{1}{j\omega C} + j\omega L + R} = \frac{R + j\omega L}{\left(1 - \frac{\omega^2}{\omega_o^2}\right) + j\omega RC} \tag{0.33}$$

$$|Z| = \frac{\sqrt{(R)^2 + (\omega L)^2}}{\sqrt{\left(1 - \frac{\omega^2}{\omega_o^2}\right)^2 + (\omega RC)^2}}, \qquad (0.34)$$

where we have introduced the natural frequency defined above in Equation (0.22). When the driving frequency is equal to the natural frequency, the magnitude of the complex impedance is a maximum, as can be seen from Equation (0.34), assuming that the term $\omega_o RC << 1$. This situation corresponds to small damping or large Q. The voltage across C is then also maximum. This is called “resonance”. Thus for small damping resonance occurs near the natural frequency.

To observe resonance in this circuit use, $C = 1.0\ \mu F$ and a resistance box resistance of 50Ω. Set the Wavetek™ amplitude to maximum, and observe the voltage across the capacitor with the digital oscilloscope. You will see a sine wave. The measured frequency will agree with the Wavetek™. Figure 9 below shows the response of the RLC circuit and the signal from the Wavetek™. Note that the average mode of the oscilloscope is used. The Wavetek is displayed on channel 2 and the RLC signal on channel 1. The two signals are in phase because the frequency is much below the resonant frequency of the circuit.

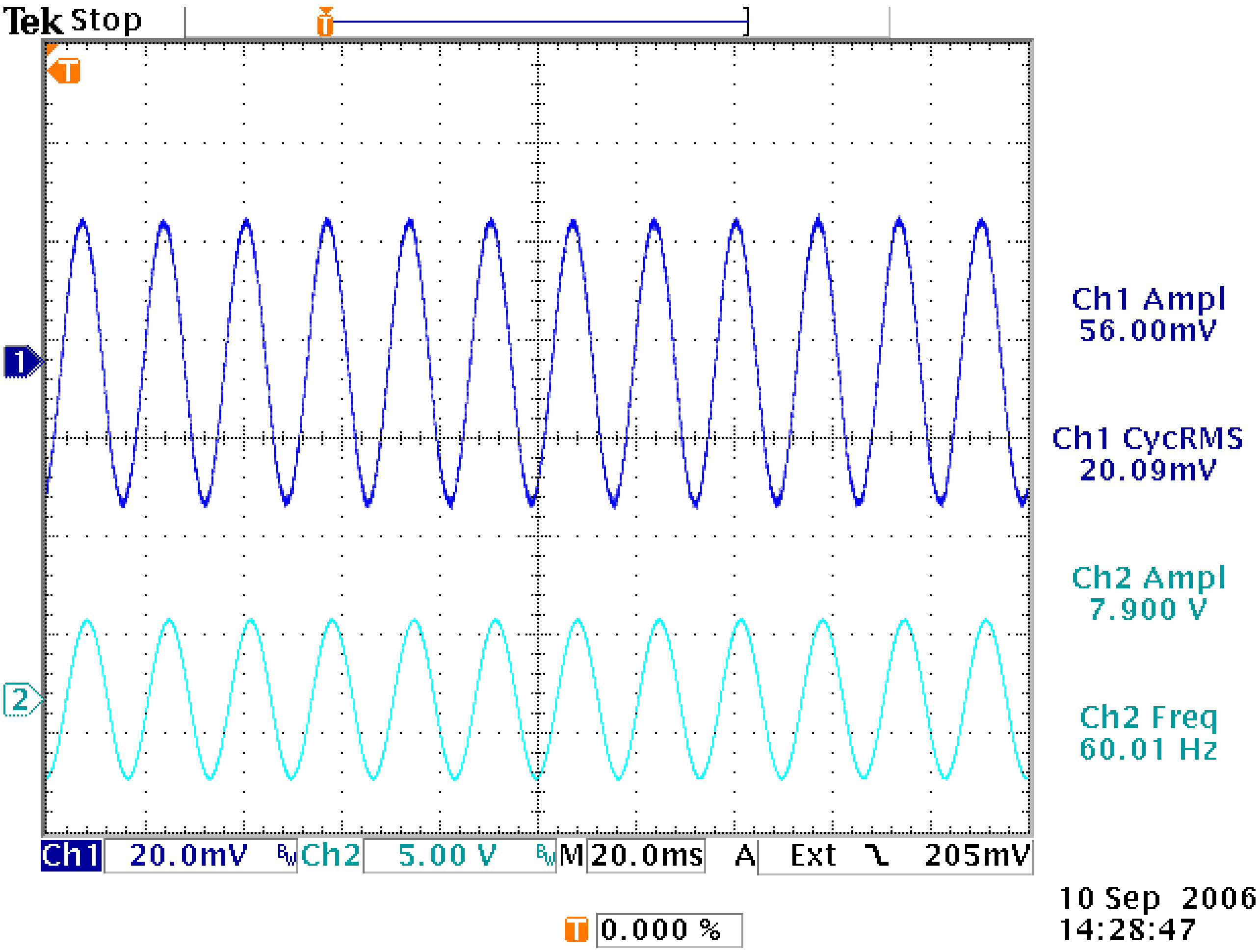

Figure 9 Steady state response of RLC circuit to sinusoidal signal

The signal is small. $Z/R_M << 1$ in Equation (0.31). If the signal collected from the capacitor is too noisy use the setting from ***Quick Menu >> Bandwidth >> 20 MHz***. If the noise is still present, you may use the signal averaging by setting ***Acquire >> Avg 512*** to obtain a better defined trace. The peak-to-peak voltage of the signal can be found either with the voltage cursors or with the ***Measurement > Amplitude*** feature of the oscilloscope. Vary the frequency, f, of the Wavetek™ from ~100 Hz to ~3 kHz and measure V_Z. Far from resonance steps of ~100 Hz are appropriate. Closer to resonance steps of ~10 Hz are needed. Typical resonance curve is shown in Figure 10.

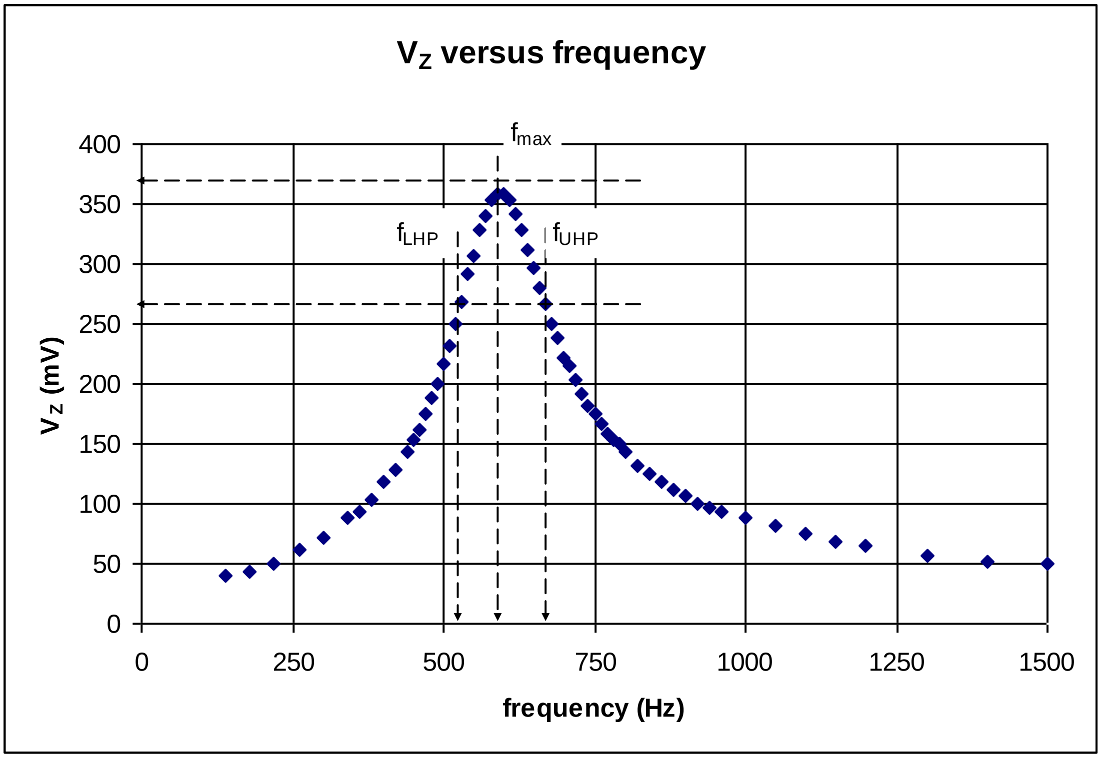

Figure 10 Typical resonance curve showing definitions of f_{LHP}, $f_{\max}$, and f_{UHP}.

Find the frequency, $f_{\max}$, for which V_Z is a maximum. Measure the magnitude of V_Z at $f_{\max}$. Then find the frequencies f_{LHP} and f_{UHP} for which the maximum value of V_Z is smaller by a factor of $1/\sqrt{2}$. The difference of frequencies, $\Delta f = \left(f_{UHP} - f_{LHP} \right)$, is called the bandwidth of the "tank" circuit. Determine the bandwidth. The bandwidth of an oscillator is related to its Q through the expression $Q = f_{\max} / \Delta f$. (It also takes some effort to show this result. Thus with equation 0.25 we have two ways to determine the Q of an RLC circuit.)

V. Report

The report consists of data, plots, and discussion from the different parts of the laboratory.

1. From part IV A above plot of $1/T^2$ versus $1/C$. Show from your data that neglect of the $(R/2L)^2$ term in Equation (0.20) is justified. Then plot T^2 versus C. Use the LINEST function of Excel to determine the slope and the error in the slope of this line.

2. From part 2 IV B above plot the logarithmic decrement, δ, versus R_B. Determine the extra "series" resistance needed to account for the negative intercept of this plot. Compare the extra resistance to the resistance of the winding and the parallel combination of the coupling resistor and input impedance of the Wavetek™.

3. From part IV C above use Excel to plot $(R_{\text{critical}})^2$ versus $1/C$ from your data. Also make a plot of the prediction for $(R_{\text{critical}})^2$ versus $1/C$ using Equation (0.29). Comment on agreement or disagreement between your data and the prediction.

4. From part IV D use Excel to plot the voltage across the capacitor versus frequency. On the same graph, plot Equation (0.34), the magnitude of the impedance versus frequency. Determine the resonant frequency, $f_{\max}$, and bandwidth, Δf from your data. Make a comparison between the theoretical expression and your experimental results.

5. In general discuss agreement and discrepancies of your measurements with expectations, and suggest possible improvements to the laboratory exercise.

Appendix I

Derivation of series RLC equation.

Elementary circuit analysis uses two laws: Kirchoff's current law and Kirchoff's voltage law. In Figure A1 below three currents enter the node denoted by the dot.

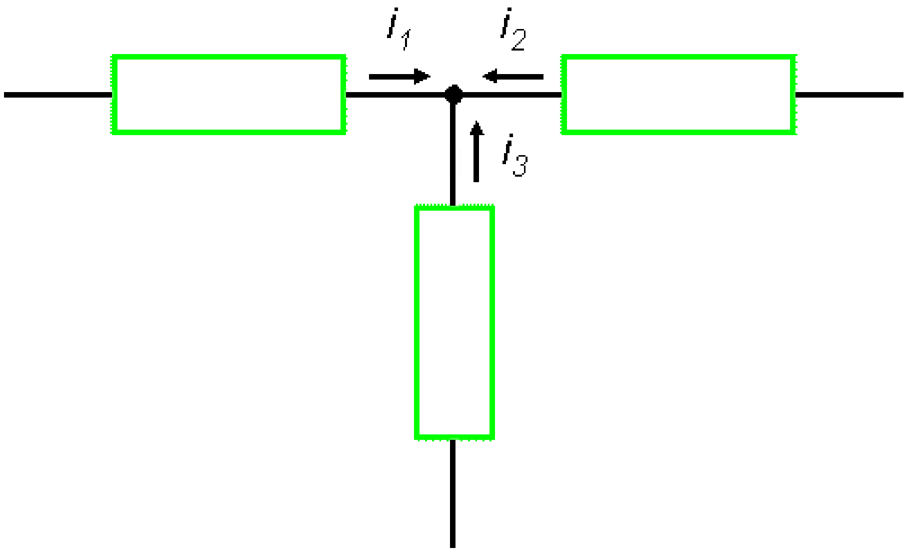

Figure A1 Example for Kirchoff's current law

Kirchoff's current law states that the sum of all currents entering a node is zero. For this example then, $i_1 + i_2 + i_3 = 0$. For each current the arrow indicates the direction of positive current. In Figure A2 below a circuit is formed by three elements.

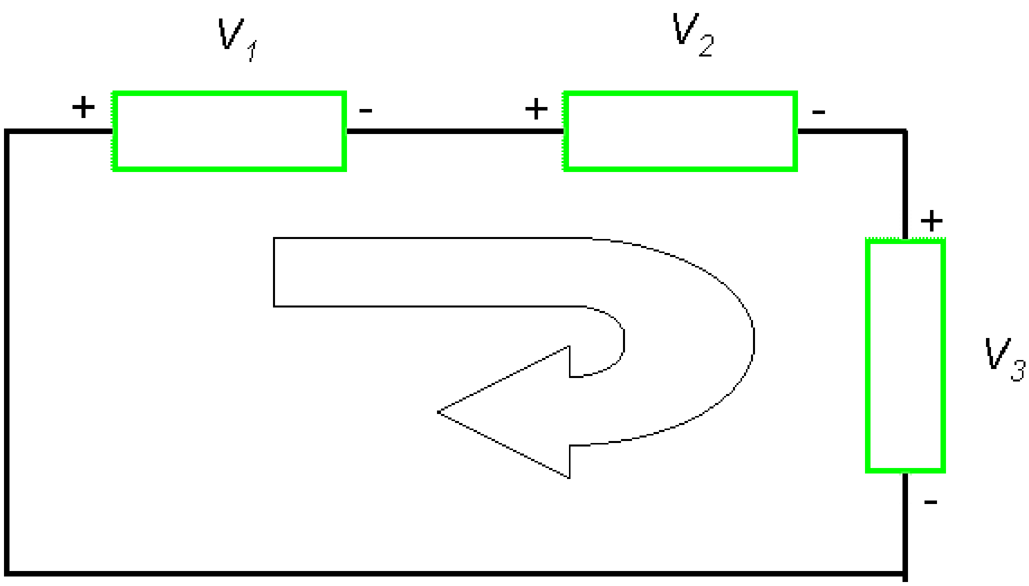

Figure A2 Example for Kirchoff's voltage law

Kirchoff's voltage law states that the sum of all voltage drops around a loop is zero. For this example then, $V_1 + V_2 + V_3 = 0$. For each voltage drop the + and – signs indicate the direction of the voltage drop.

The voltage-current relation for the three circuit elements, the resistor, capacitor and inductor are shown in Figure A3.

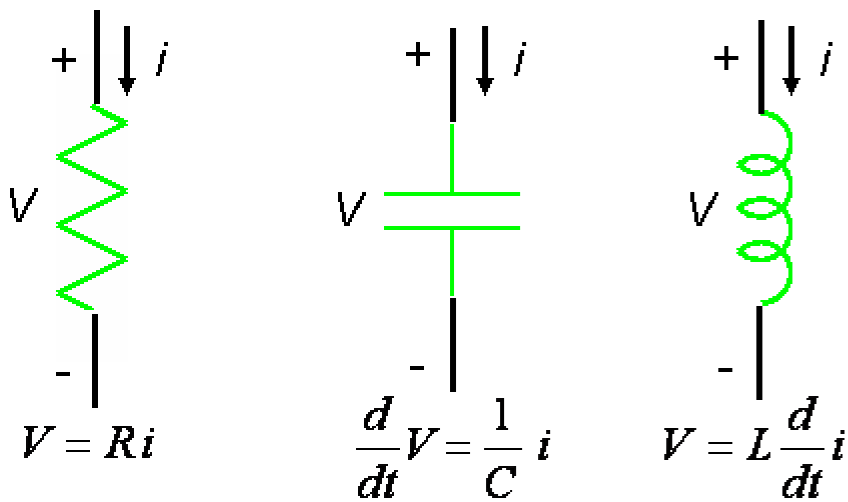

Figure A3 Voltage current relation for resistor, capacitor and inductor

The relation $V = Q/C$ for the capacitor, where $\pm Q$ is the charge on the capacitor plates, is also useful. Recall that $i = dQ/dt$.

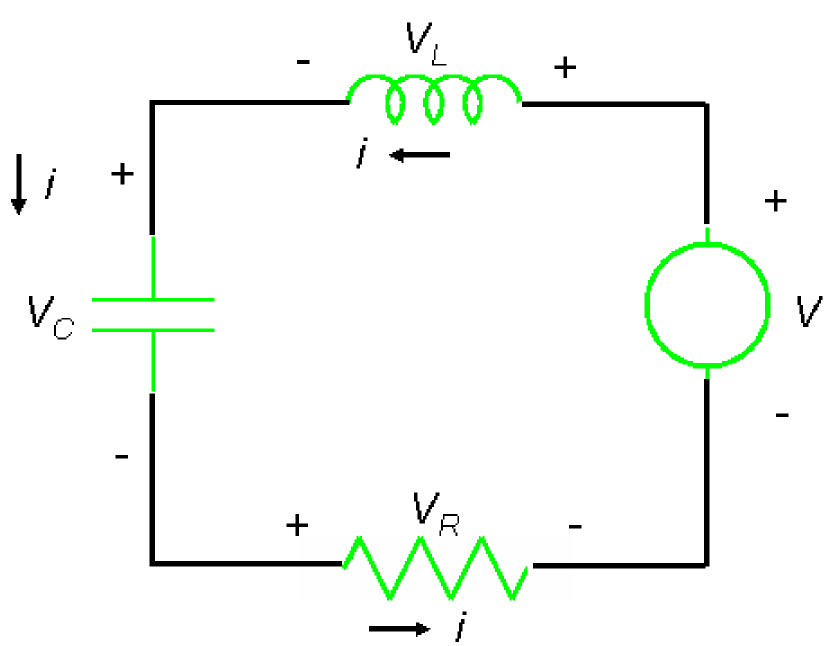

Figure A4 Series RLC circuit

Figure 4A shows a series RLC circuit with a voltage source. Going counter-clockwise around the loop, Kirchoff's voltage law gives $V_L + V_C + V_R - V = 0$. Using the voltage-current relations for the three circuit elements gives $L\frac{di}{dt} + \frac{1}{C}Q + R\,i = V$, and choosing Q as the dependent variable gives the equation, $L\frac{d}{dt}\left(\frac{dq}{dt}\right) + R\frac{dq}{dt} + \frac{1}{C}q = V(t)$.