



# Qualitative Studies with Microwaves

**Physics 401, Spring 2013**

**Eugene V. Colla**

UNIVERSITY OF ILLINOIS

AT URBANA-CHAMPAIGN



3/25/2013

illinois.edu

1

# Qualitative Studies with Microwaves

The main goals of the Lab:

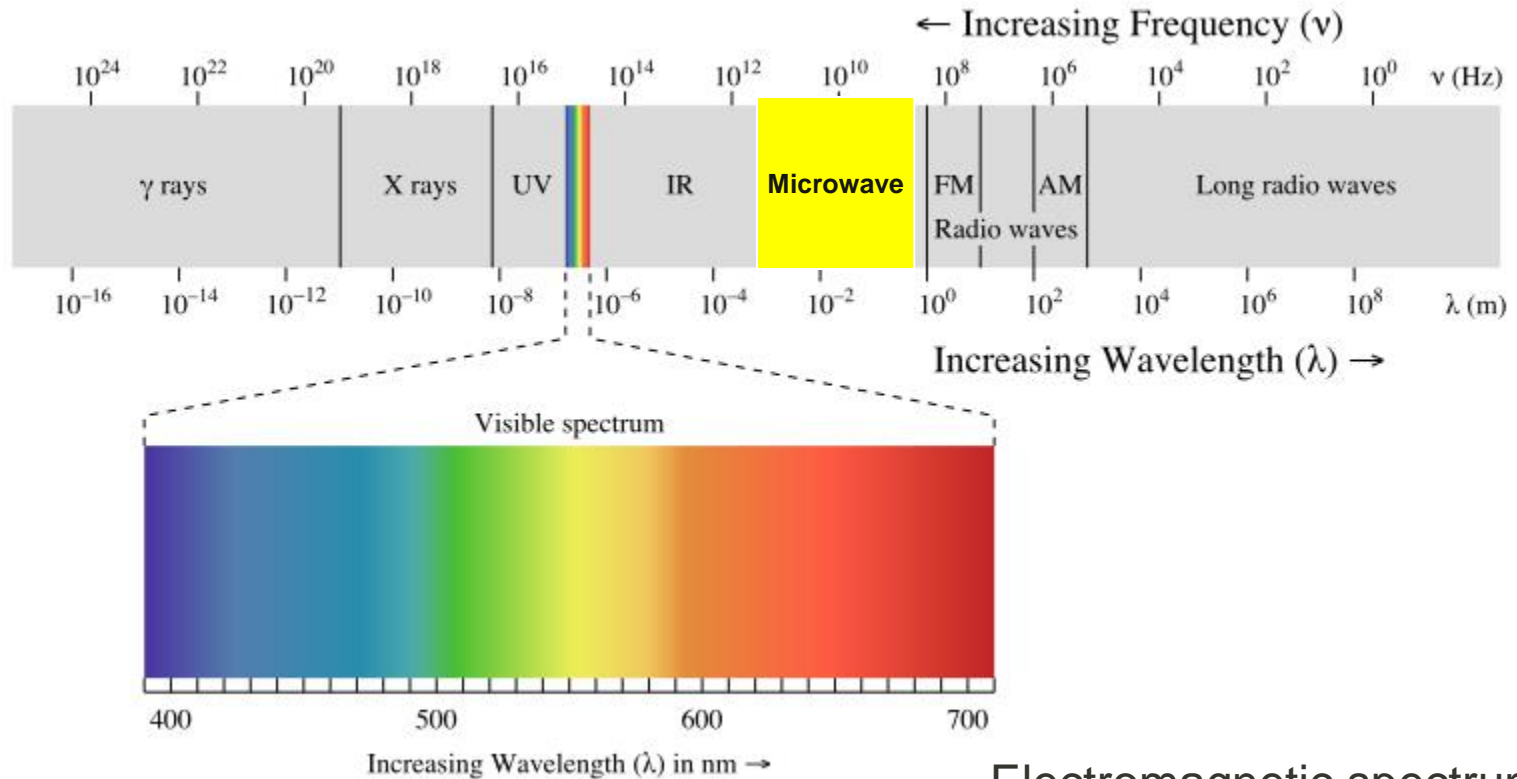
- ✓ Refreshing the memory about the electromagnetic waves propagation
- ✓ Microwaves. Generating and detecting of the microwaves
- ✓ Microwaves optic experiments

This is two weeks Lab



# Microwaves place in the electromagnetic spectrum

The microwave range includes ultra-high frequency (UHF) (0.3–3 GHz), super high frequency (SHF) (3–30 GHz), and extremely high frequency (EHF) (30–300 GHz) signals.



Electromagnetic spectrum\*

# Application of the microwaves



Microwave oven (2.45GHz)



Communication (0.8-2.69GHz)



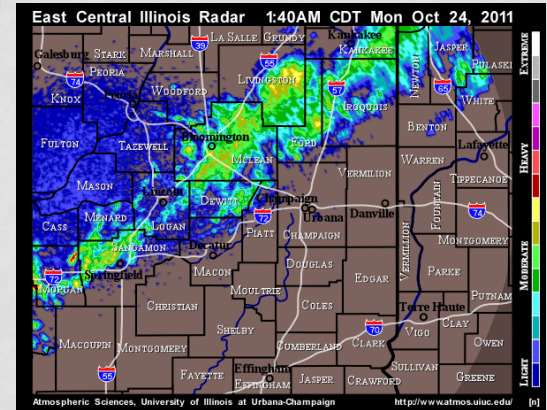
Satellite TV (4-18GHz)



Radar (up to 110GHz)



Motion detector (10.4GHz)



Weather radar (8-12GHz)

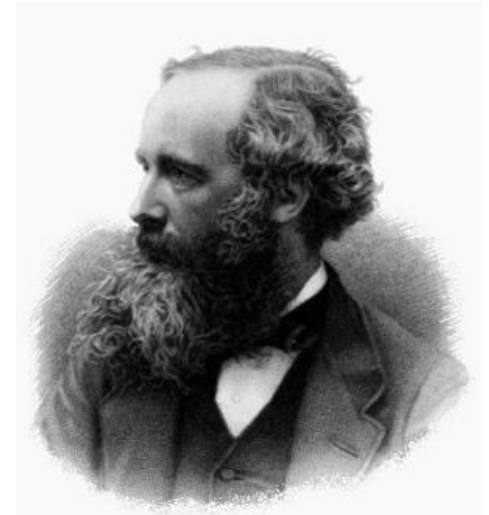
\*by courtesy Wikipedia

# Maxwell equations

$$\nabla \vec{D} = \rho \quad (1) \quad \nabla \times \vec{E} = -\frac{\partial \vec{B}}{\partial t} \quad (3)$$
$$\nabla \vec{B} = 0 \quad (2) \quad \nabla \times \vec{H} = \vec{J} + \frac{\partial \vec{D}}{\partial t} \quad (4)$$

If  $\rho = 0$  and  $\vec{J} = 0$  and taking in account that  $\vec{D} = \epsilon \vec{E}$   
 $\vec{B} = \mu \vec{H}$  (1) and (4) can be rewritten as

$$\nabla \vec{D} = \epsilon \left[ \frac{\partial E_x}{\partial x} + \frac{\partial E_y}{\partial y} + \frac{\partial E_z}{\partial z} \right] = 0 \quad \nabla \times \vec{H} = \frac{\partial \vec{D}}{\partial t}$$

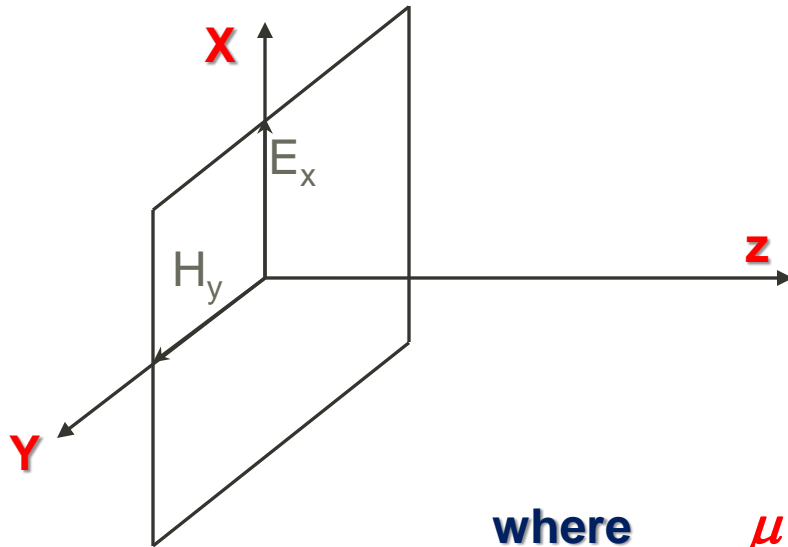


James Clerk Maxwell  
(1831–1879)

# Plane wave

Now assuming that plane wave propagate in z direction and what leads to  $E_y=E_z=0$  and  $H_x=H_z=0$

Now (3) and (4) could be simplified as



$$\frac{\partial E_x}{\partial z} = -\mu \frac{\partial H_y}{\partial t} \quad (5)$$

$$\frac{\partial H_y}{\partial z} = -\epsilon \frac{\partial E_x}{\partial t} \quad (6)$$

where  $\mu = \mu_0 \mu_r$        $\epsilon = \epsilon_0 \epsilon_r$

$\mu_0$  is the free space permeability,  $\epsilon_0$  is the free space permittivity  
 $\mu_r$  is permeability of a specific medium,  $\epsilon_r$  is permittivity of a specific medium

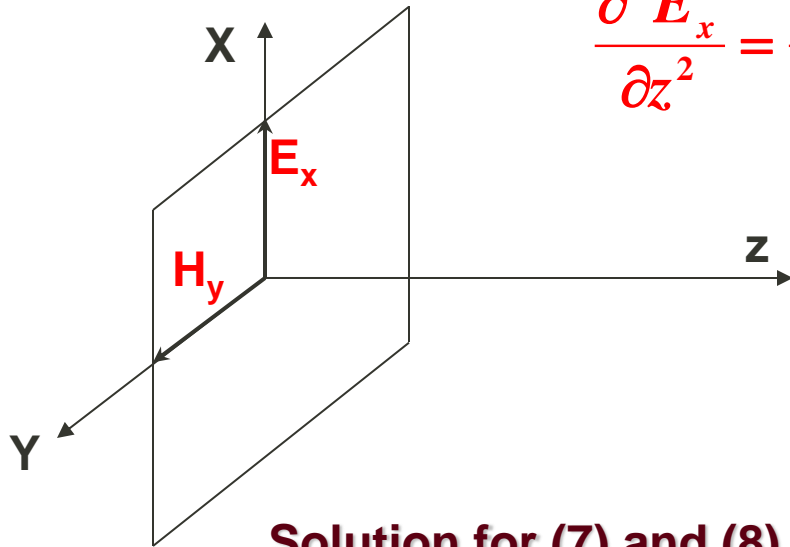


# Plane wave

Combining (5) and (6) (see Lab write-up for more details) we finally can get the equations of propagation of the plane wave:

$$\frac{\partial^2 E_x}{\partial z^2} = \frac{1}{v^2} \frac{\partial^2 E_x}{\partial t^2} \quad (7) \quad \frac{\partial^2 H_y}{\partial z^2} = \frac{1}{v^2} \frac{\partial^2 H_y}{\partial t^2} \quad (8)$$

where  $v = \frac{1}{\sqrt{\epsilon\mu}}$



$$E_x = E_{x0} \cos(\omega t - kx)$$

$$H_y = H_{y0} \cos(\omega t - kx)$$

Solution for (7) and (8) can found as  $H_y = \sqrt{\frac{\epsilon}{\mu}} E_x$  or  $E_x = Z H_y$

where  $Z = \sqrt{\frac{\mu}{\epsilon}}$  known as characteristic impedance of medium

$k$  is wave vector and is defined as  $k = \frac{2\pi}{\lambda}$  or  $k = \frac{\omega}{v}$

For free space ( $\epsilon_r=1$  and  $\mu_r=1$ )  $Z_{fs} = \sqrt{\frac{\mu_0}{\epsilon_0}} \approx 377 \text{ohms}$



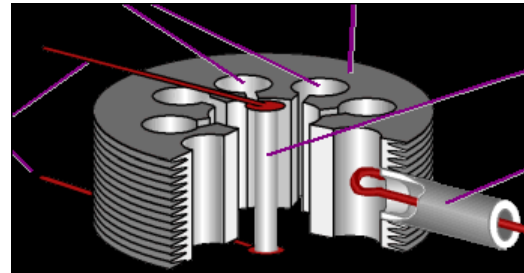
# Generating of the microwaves

Vacuum tubes: klystron, magnetron, traveling wave tube

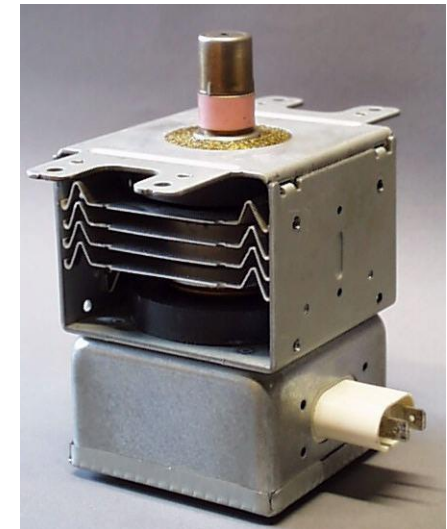
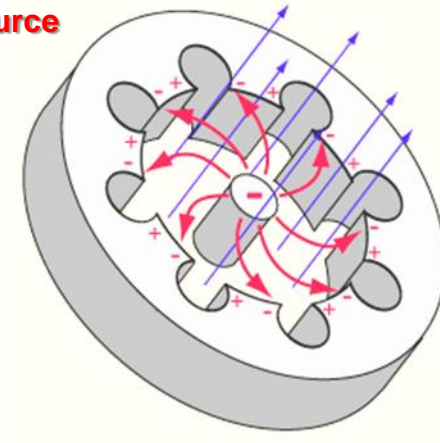
Solid state devices: FET, tunneling diodes, Gunn diodes



Tunable frequency from 9 to 10GHz; maximum output power 20mW



Heated cathode as electron source



Microwave oven magnetron; typical power 0.7-1.5kW

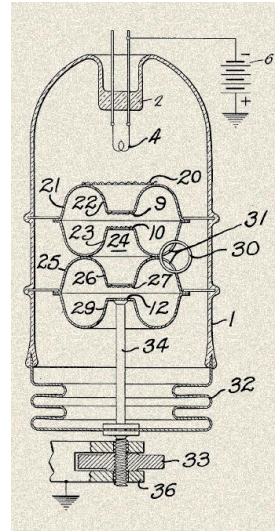


# Klystron. A piece of history.



**Russell Harrison  
Varian** (April 24, 1898  
– July 28, 1959)

**Sigurd Fergus  
Varian** (May 4, 1901  
– October 18, 1961)



Patented May 20, 1941

2,242,275

## UNITED STATES PATENT OFFICE

2,242,275

### ELECTRICAL TRANSLATING SYSTEM AND METHOD

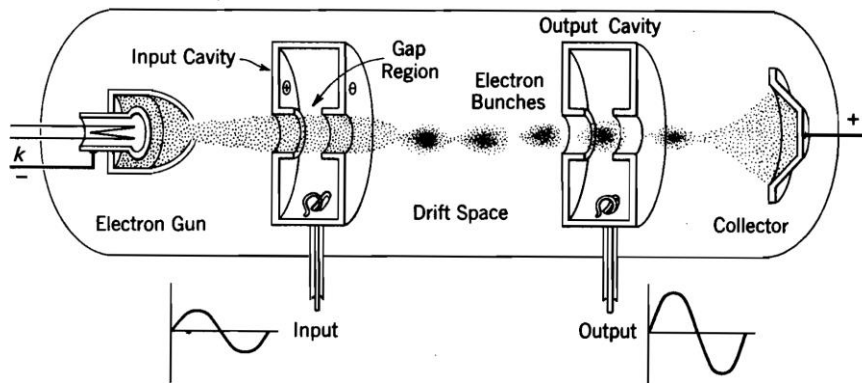
Russell H. Varian, Stanford University, Calif., as-  
signor to The Board of Trustees of The Leland  
Stanford Junior University, Stanford Uni-  
versity, Calif., a corporation of California

Application October 11, 1937 Serial No. 162,955

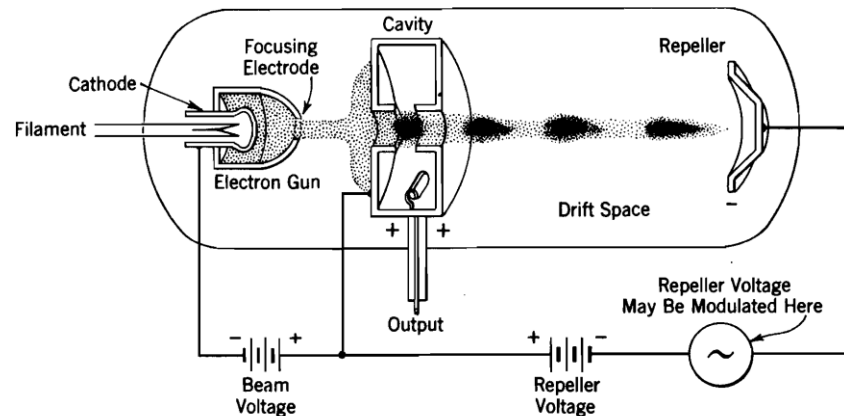
Varian Brothers...Klystron Tube (1940)



# Generating of the microwaves. Klystron.



**Single transit klystron**



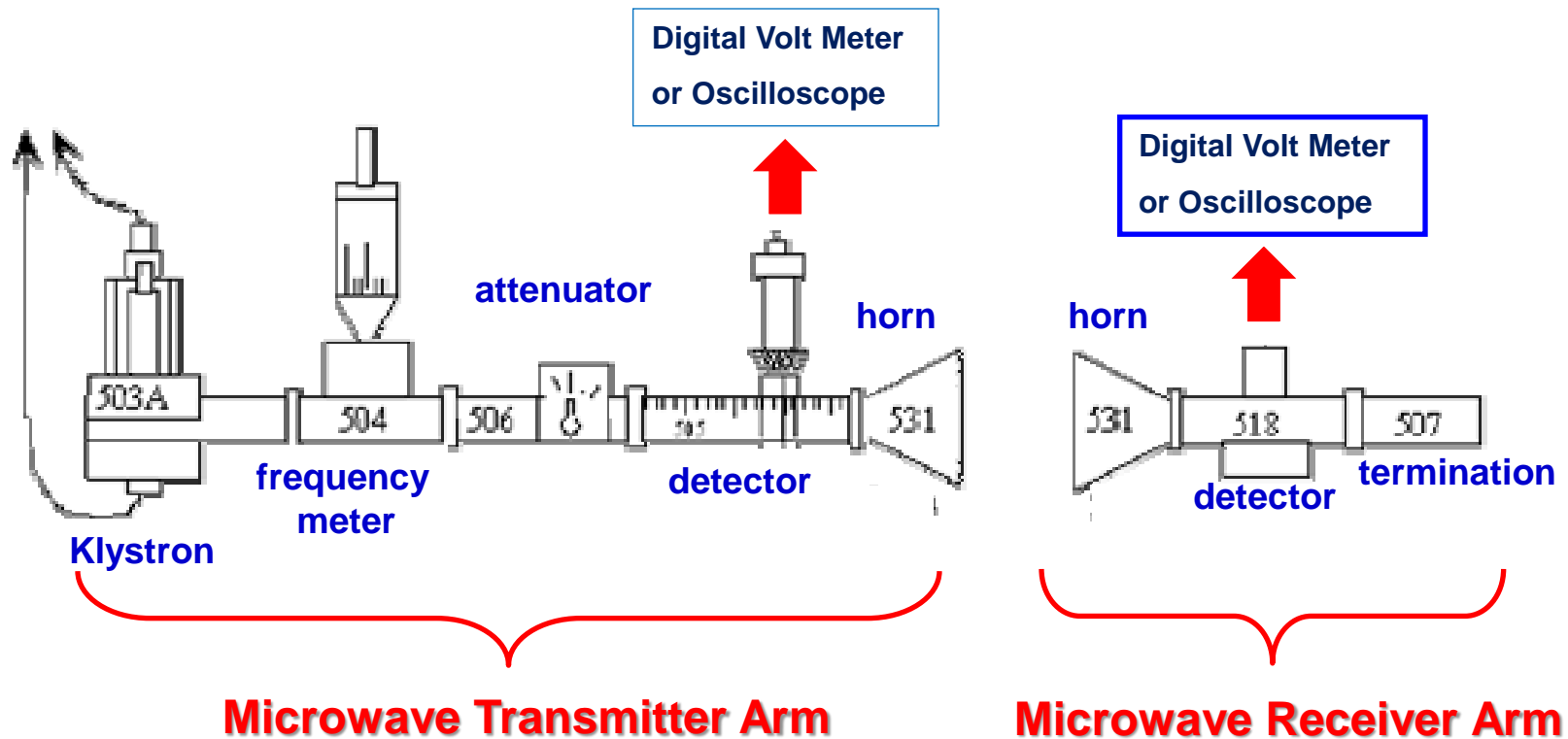
**Reflection klystron**

**Advantages:** well defined frequencies, high power output

High power klystron used in Canberra Deep Space Communications Complex



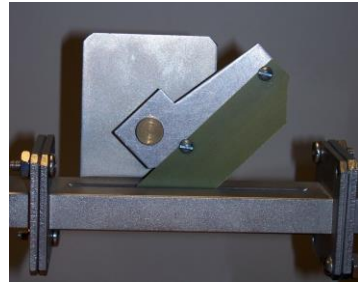
# Experimental setup. Main components.



# Experimental setup. Main components.



Klystron



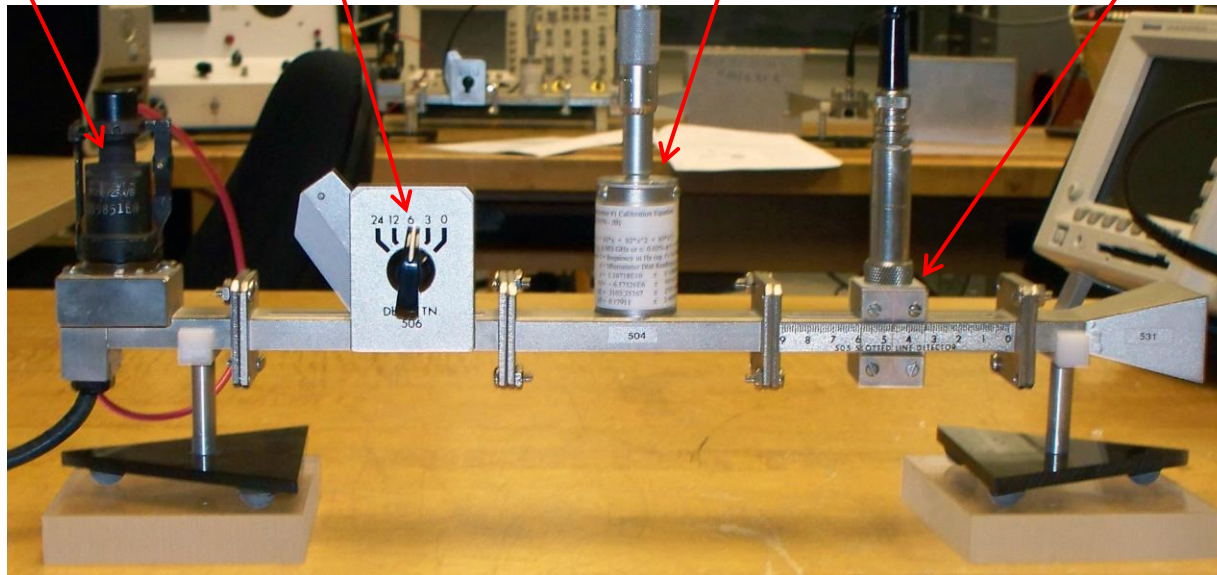
Attenuator



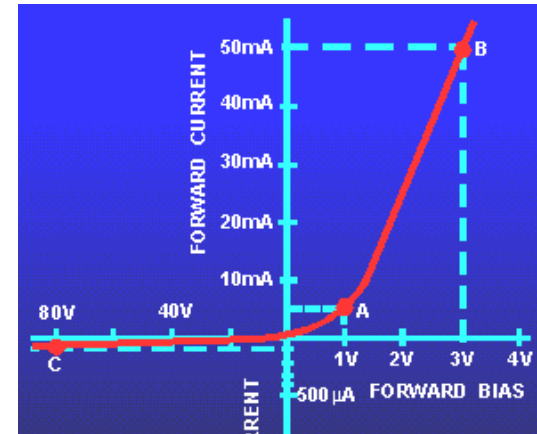
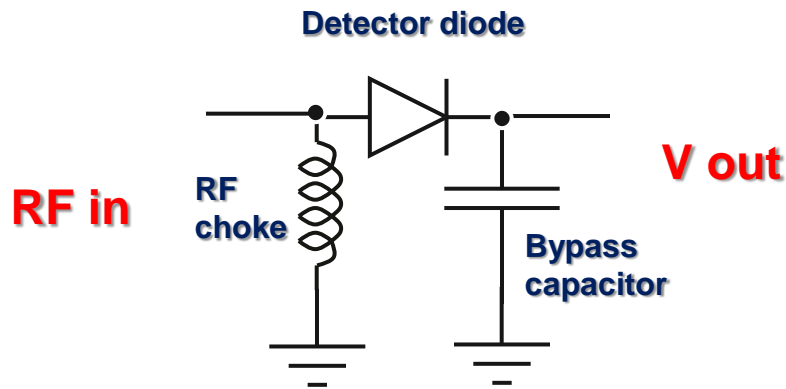
Frequency meter



detector



# Detecting of the microwaves



Typical I-V dependence for p-n diode

$$I = I_0 \left[ \exp\left(\frac{eV}{kT}\right) - 1 \right]$$

Taylor expansion for exp function will give

$$\exp(x) = 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \dots$$

$$I \propto aV + bV^2 + \dots$$

0(DC)

If  $V = V_0 \sin \omega t$

$$b * \frac{V_0^2}{2} (1 - \cos 2\omega t)$$

$$I_{DC} \propto b \frac{V_0^2}{2} + \dots$$

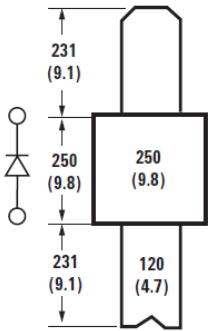
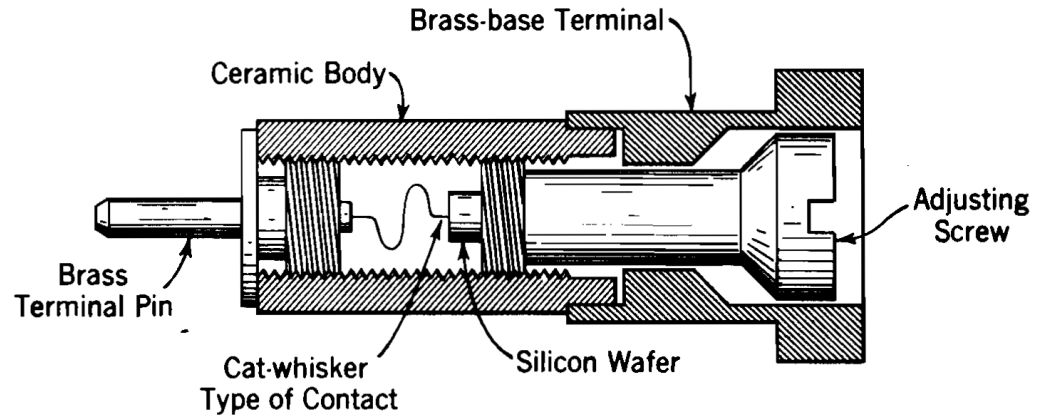
And finally



# Detecting of the microwaves

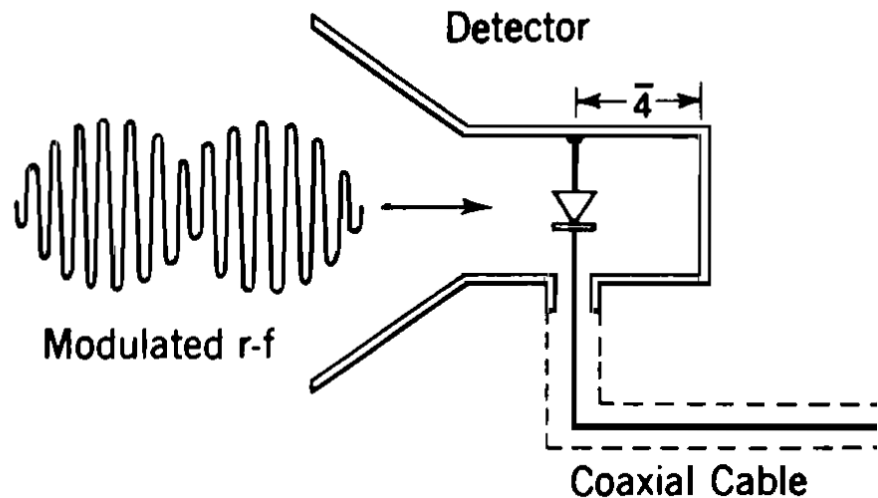
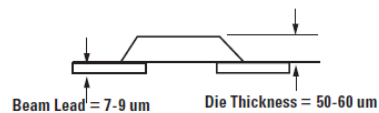


HSCH-9161  
HSCH-9162  
GaAs Detector Diode

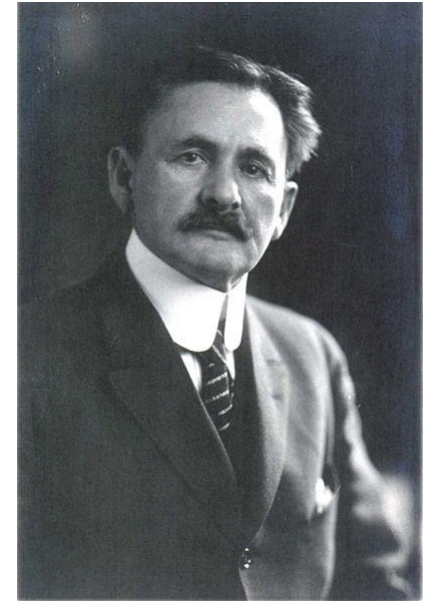


$f_c \sim 200\text{GHz}$

Note: All dimensions in microns (mils)

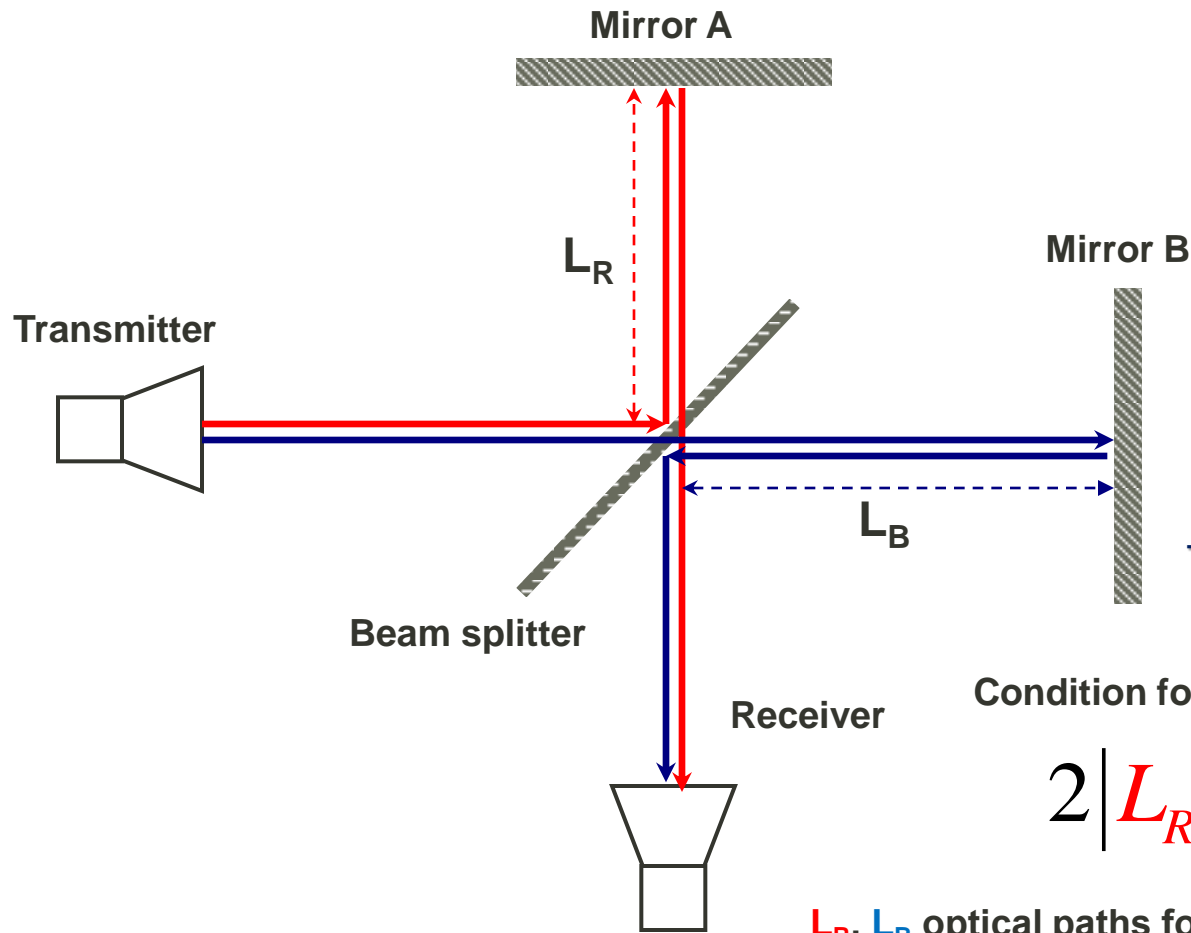


# Experiments: Michelson interferometer



Albert Abraham Michelson  
(1852 - 1931)

The Nobel Prize in Physics 1907



Condition for constructive interference

$$2|L_R - L_B| = n\lambda$$

$L_R, L_B$  optical paths for “red” and “blue” rays



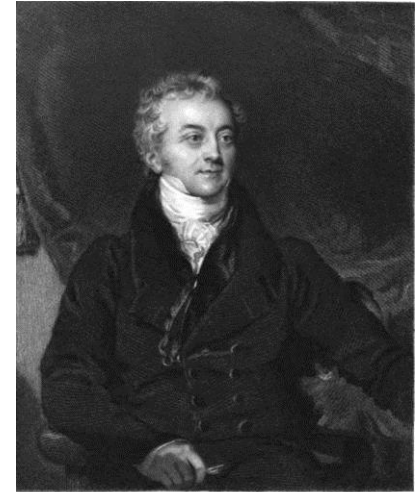
# Experiments: Michelson interferometer



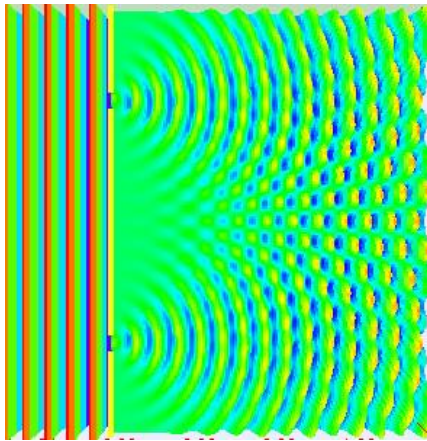
Physics 403 Lab Michelson interferometer setup



# Experiments: Double slits Interference. T. Young 1801



Thomas Young  
(1773 – 1829)



For constructive interference  
 $\Delta r = n\lambda$  or  $d \sin \theta = n\lambda$

The measured envelope of the diffraction pattern can be defined as:

$$|\psi_{ss}|^2 = |\psi_0|^2 \left( \frac{\sin x}{x} \right)^2 \times \cos^2 [ (kd \sin(\theta/2)) ]$$

where  $x = kb \sin(\theta/2)$  and

$$k = \frac{2\pi}{\lambda}$$

is wave vector of the plane wave

$$\Delta r = r_1 - r_2 = d \sin \theta$$

$\Delta r$

$r_1$

$r_2$

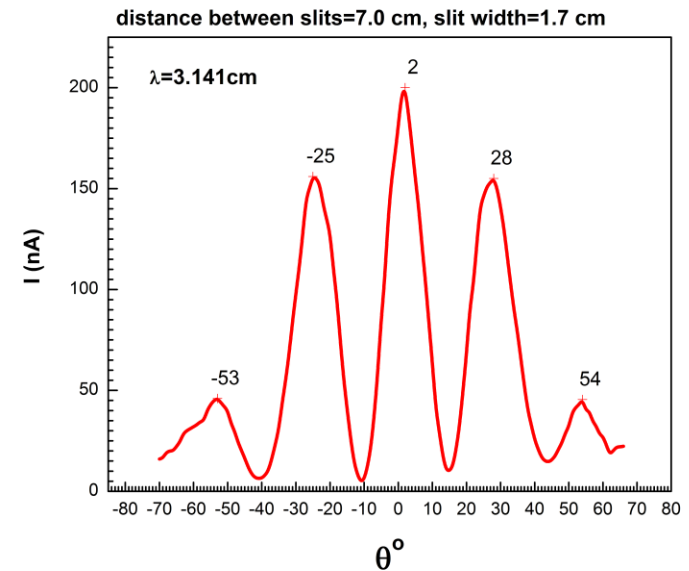
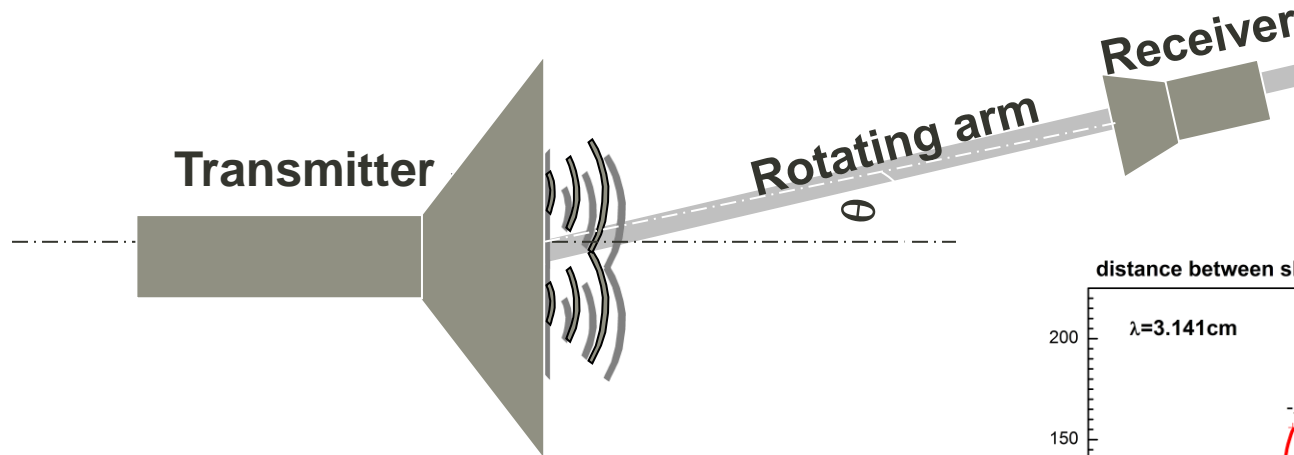
$\theta$

$d$

$b$



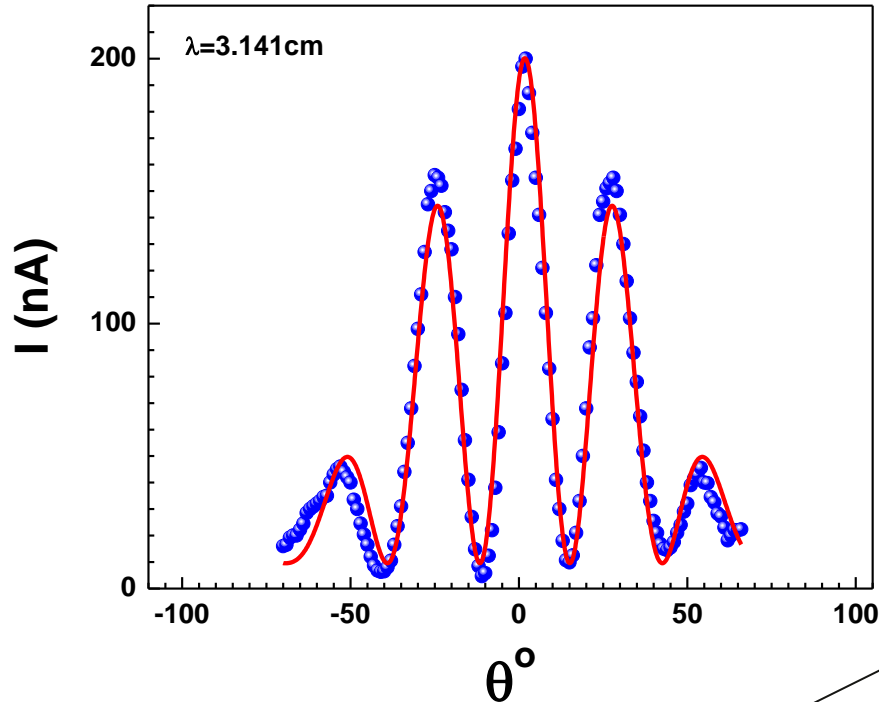
# Experiments: Double slits interference



**Physics 403 Lab setup and example of the data**

# Experiments: Double slits interference. Fitting

$$|\psi_{ss}|^2 = |\psi_0|^2 \left( \frac{\sin x}{x} \right)^2 \times \cos^2 [ (kd \sin(\theta/2)) ] \quad x = kb \sin(\theta/2)$$



Model		Two_slit (User)	
Equation	$y = I_0 \cdot \left( \frac{\sin(K1 \cdot \sin(\pi \cdot x / 360 + f))}{K1 \cdot \sin(\pi \cdot x / 360 + f)} \right)^2 \cdot \cos^2(K2 \cdot \sin(\pi \cdot x / 360 + f)) + I_{00}$		
Reduced Chi-Sqr	94.62111		
Adj. R-Square	<b>0.96659</b>	Value	Standard Error
	<b>I0</b>	<b>190.6014</b>	<b>3.042882</b>
	<b>K1</b>	<b>4.384042</b>	<b>0.074754</b>
	<b>K2</b>	<b>13.51332</b>	<b>0.052244</b>
	<b>f</b>	<b>-0.01525</b>	<b>7.19E-04</b>
	<b>I00</b>	<b>9.572049</b>	<b>1.440409</b>

Fitting equation

$$y = I_0 \cdot \left( \frac{\sin(K1 \sin(\frac{\pi x}{360} + f))}{K1 \sin(\frac{\pi x}{360} + f)} \right)^2 \cos^2 \left( K2 \sin \left( \frac{\pi x}{360} + f \right) \right) + I_{00}$$

Here in fitting expression:

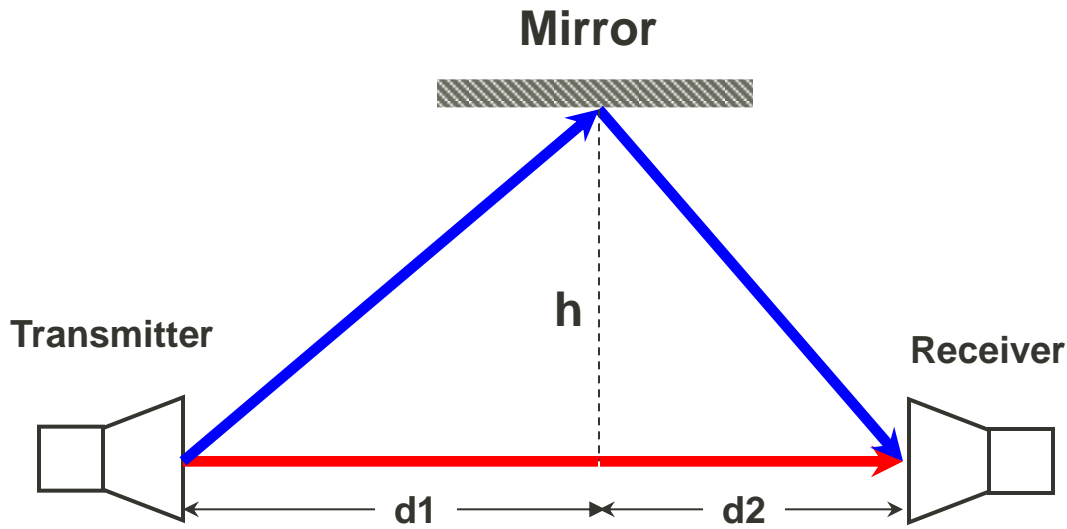
$$I_0 = |\psi_0|^2;$$

$$K1 = kb;$$

$$K2 = kd$$



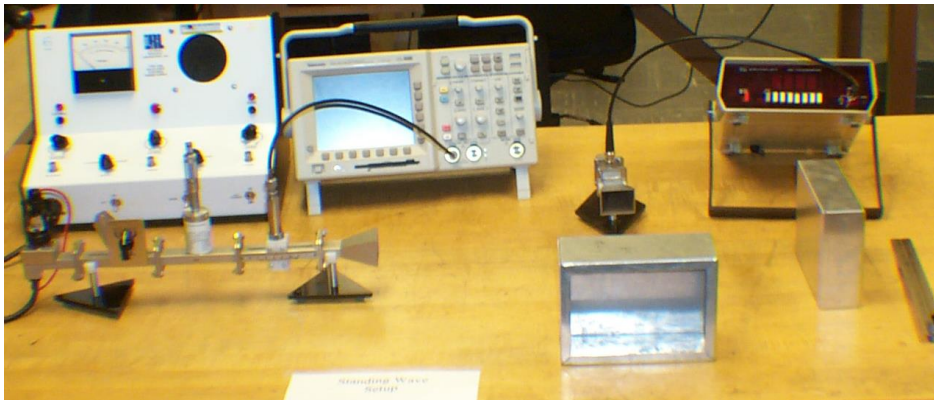
# Lloyd's Mirror experiment



Humphry Lloyd  
1802-1881

Difference of the wave paths of  
“red” and “blue” rays is:

$$\Delta S = \sqrt{h^2 + d1^2} + \sqrt{h^2 + d2^2} - (d1 + d2)$$



Lab setup picture

For constructive interference

$$\Delta S = n\lambda$$



# Total internal reflection experiment. Snell's law



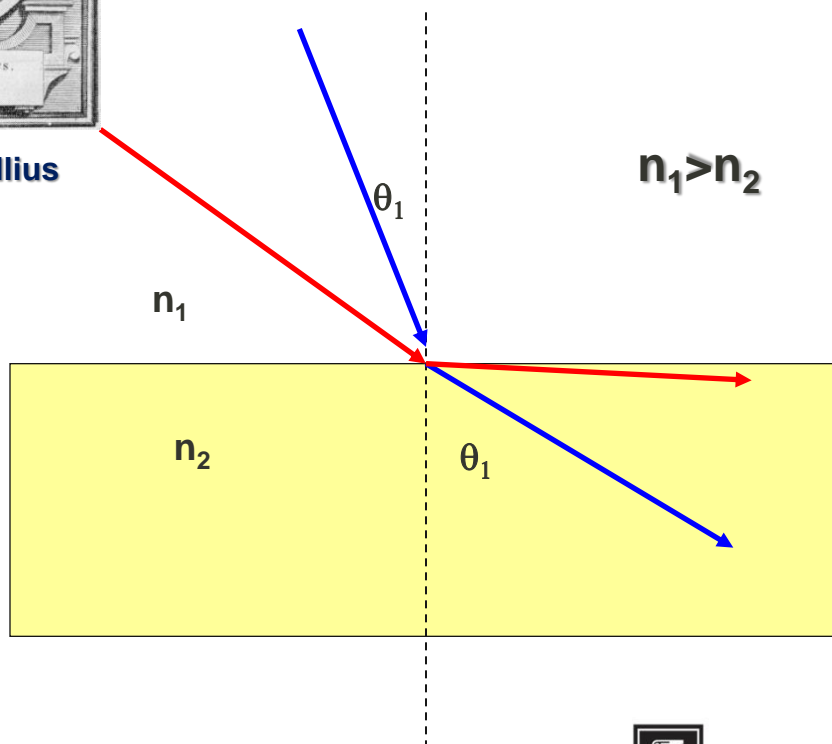
Willebrord Snellius  
1580-1626



Claudius Ptolemaeus  
after AD 83–c.168)

$$n_1 \sin \theta_1 = n_2 \sin \theta_2$$

Snell's law



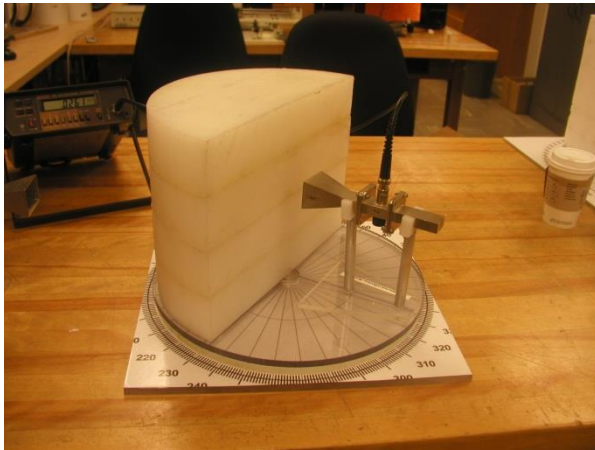
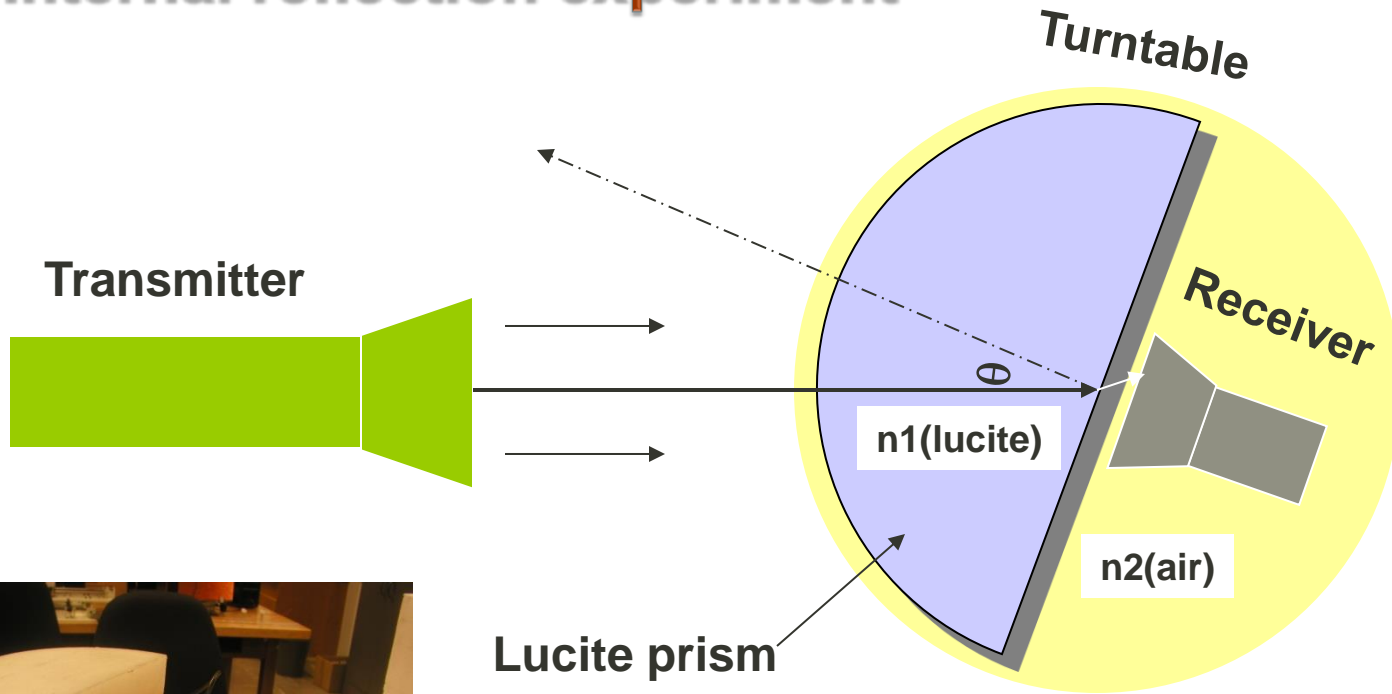
Equation for critical angle:

$$n_1 \sin \theta_c = n_2 \sin 90^\circ$$

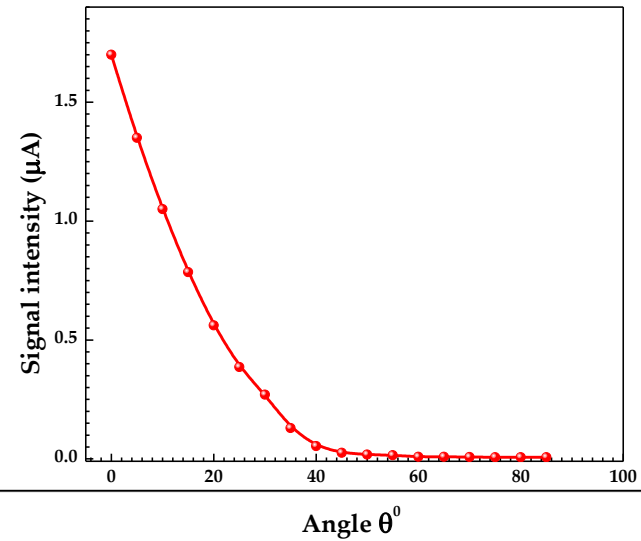
$$\theta_c = \sin^{-1}(n_2/n_1)$$



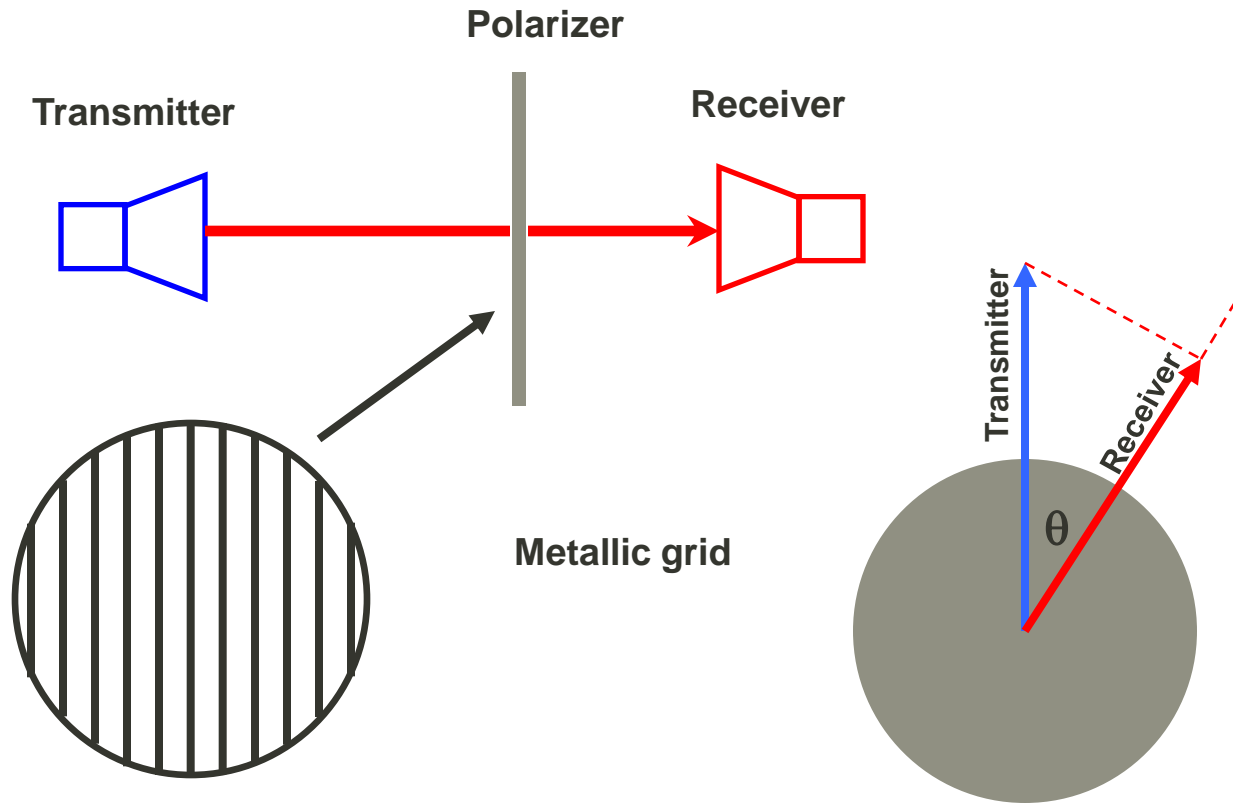
# Total internal reflection experiment



Experimental setup and the example of the data



# Microwave polarization

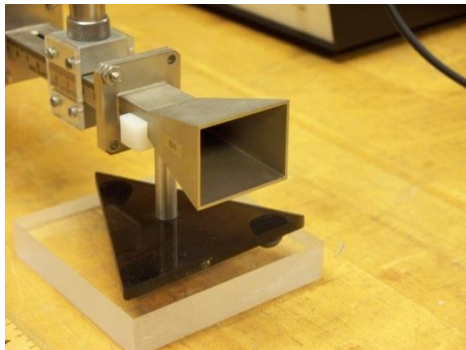


Etienne-Louis Malus  
1775 – 1812

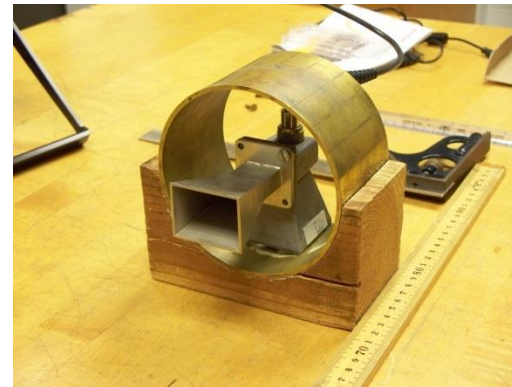
**Malus law**

$$I = I_0 \cos^2 \theta$$

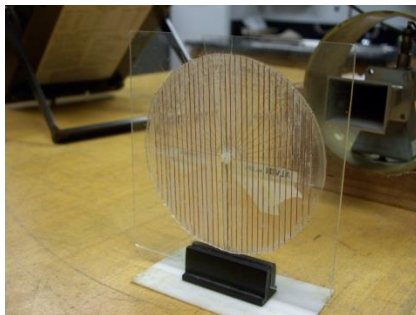
# Microwave polarization



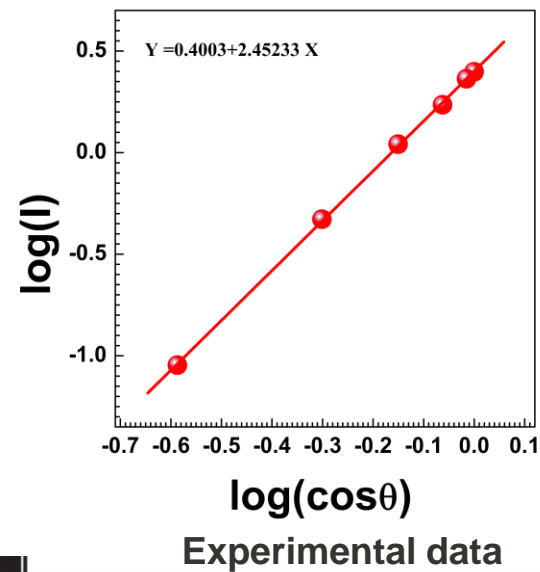
Transmitter



Rotatable receiver



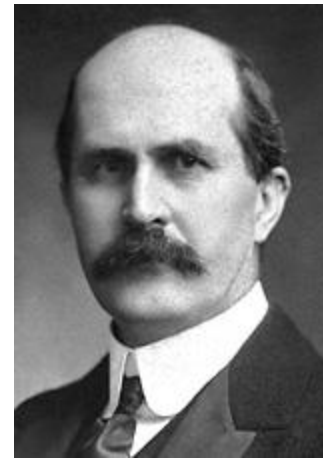
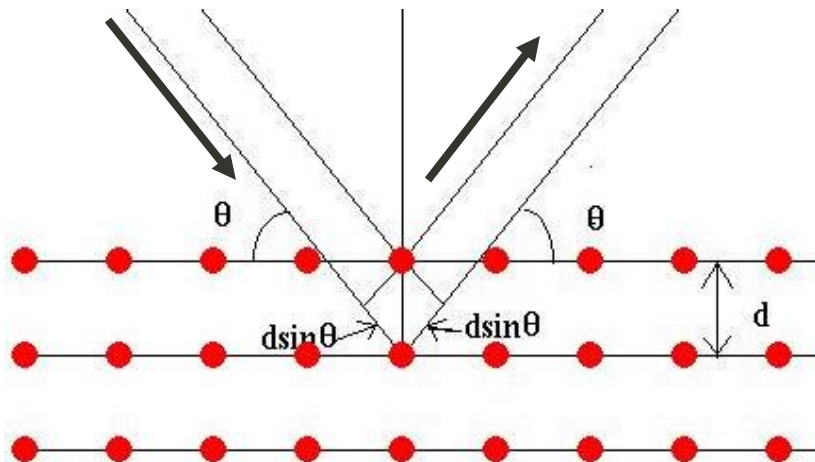
Polarizer



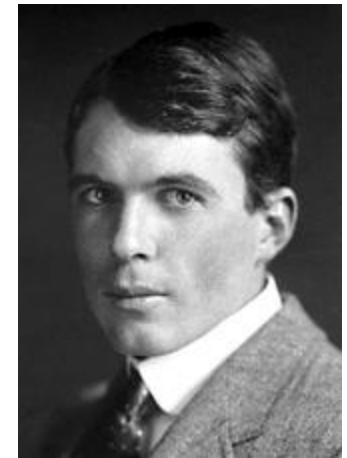
$$I = I_0 \cos^2\theta$$

# Bragg diffraction

Interference of the EM waves reflected from the crystalline layers



Sir William Henry Bragg  
1862-1942



William Lawrence Bragg  
1890-1971

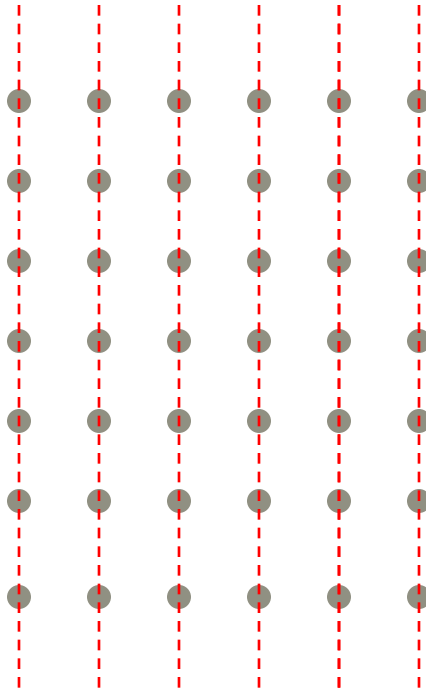


**The Nobel Prize in Physics 1915**  
"for their services in the analysis of  
crystal structure by means of X-rays"

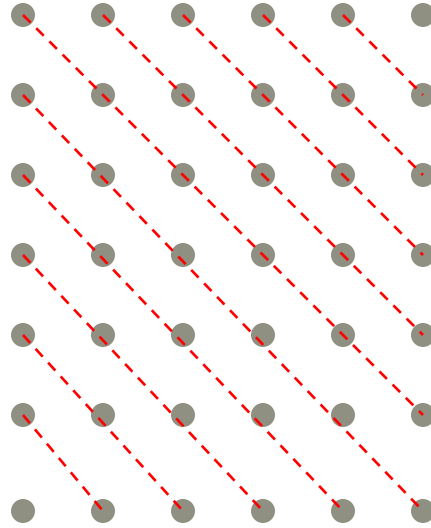
$$n\lambda = 2d \sin \theta \quad \text{Bragg's Law}$$



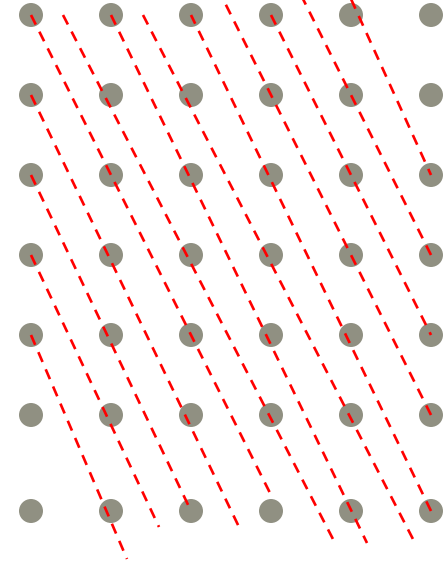
# Bragg diffraction



**(100)**



**(110)**



**(210)**

**Different orientations of the crystal**



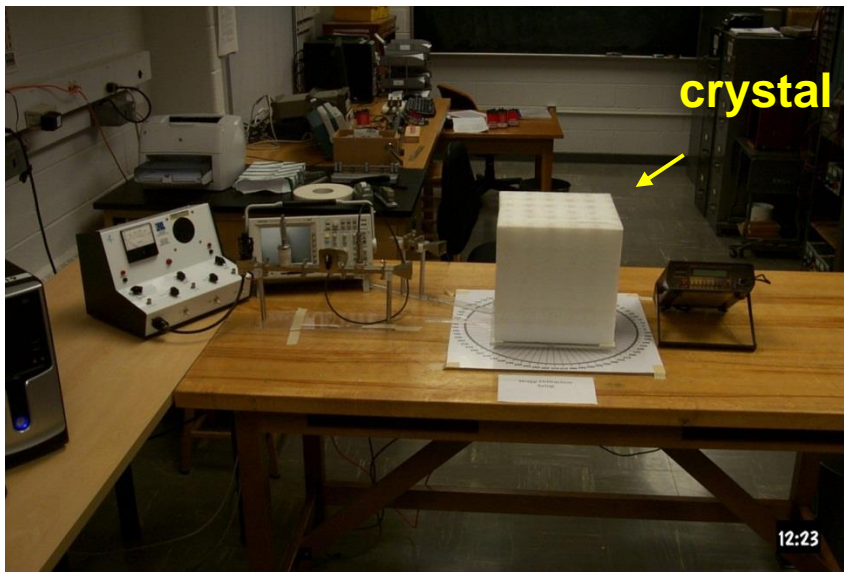
# Bragg diffraction

$$n\lambda = 2d \sin\theta$$

$$\lambda < 2d$$

In our experiment  $\lambda \sim 3\text{cm}$ ;  
For cubic symmetry the  
angles of Bragg peaks  
can be calculated from:

$$\left(\frac{\lambda}{2d}\right)^2 = \frac{\sin^2 \theta}{h^2 + k^2 + l^2}$$

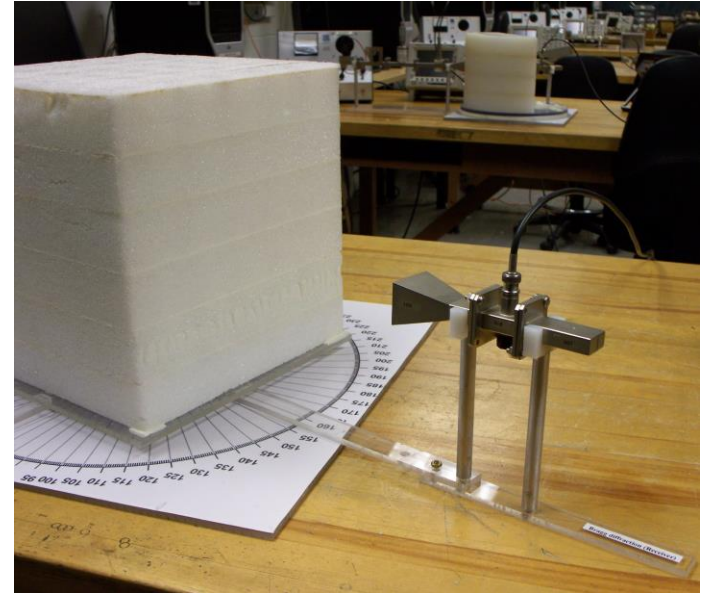
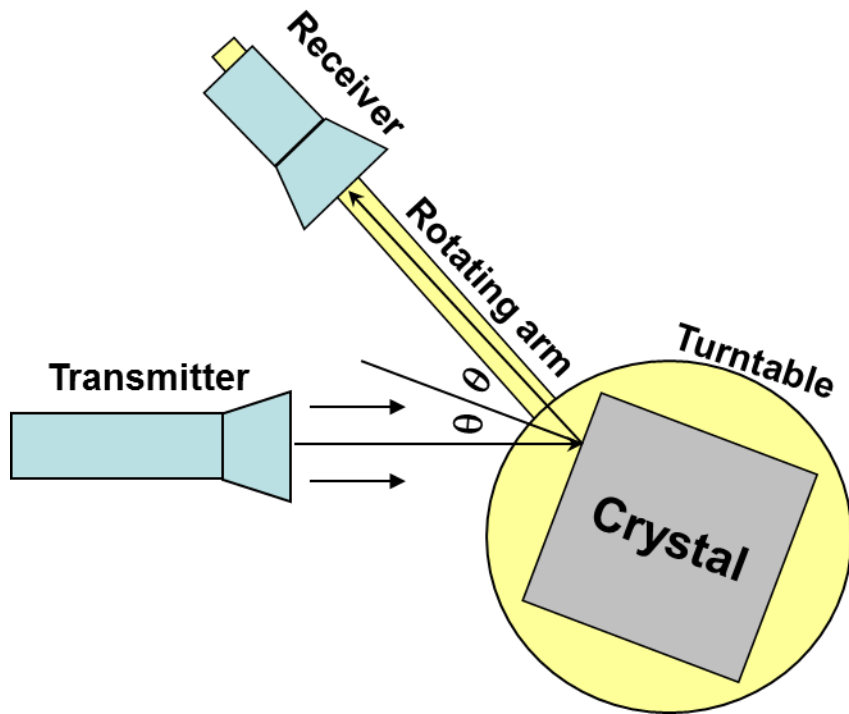


Experimental setup

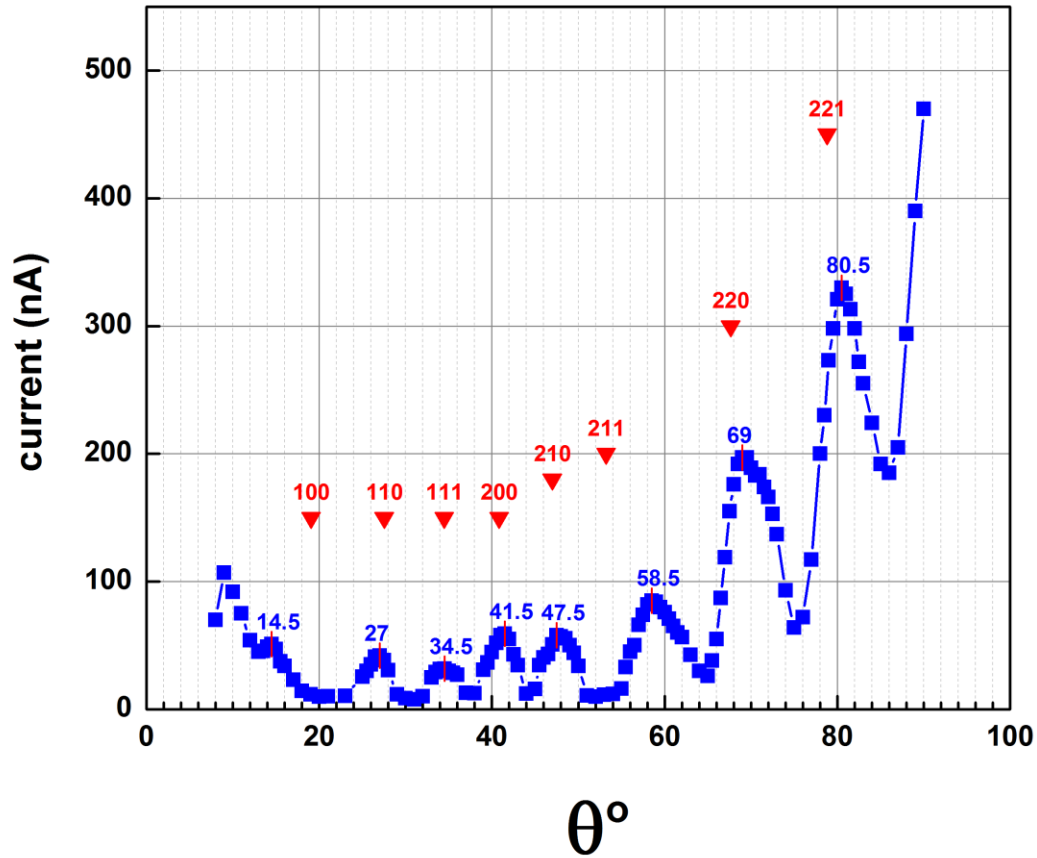
where  $h, k, l$  are the Miller Indices.  
For crystal with  $d=5\text{cm}$  and  $\lambda=3\text{cm}$   
the 3 first Bragg peaks for (100)  
orientation can be found at  
angles:  $\sim 17.5^\circ$ ;  $36.9^\circ$  and  $64.2^\circ$



# Bragg diffraction

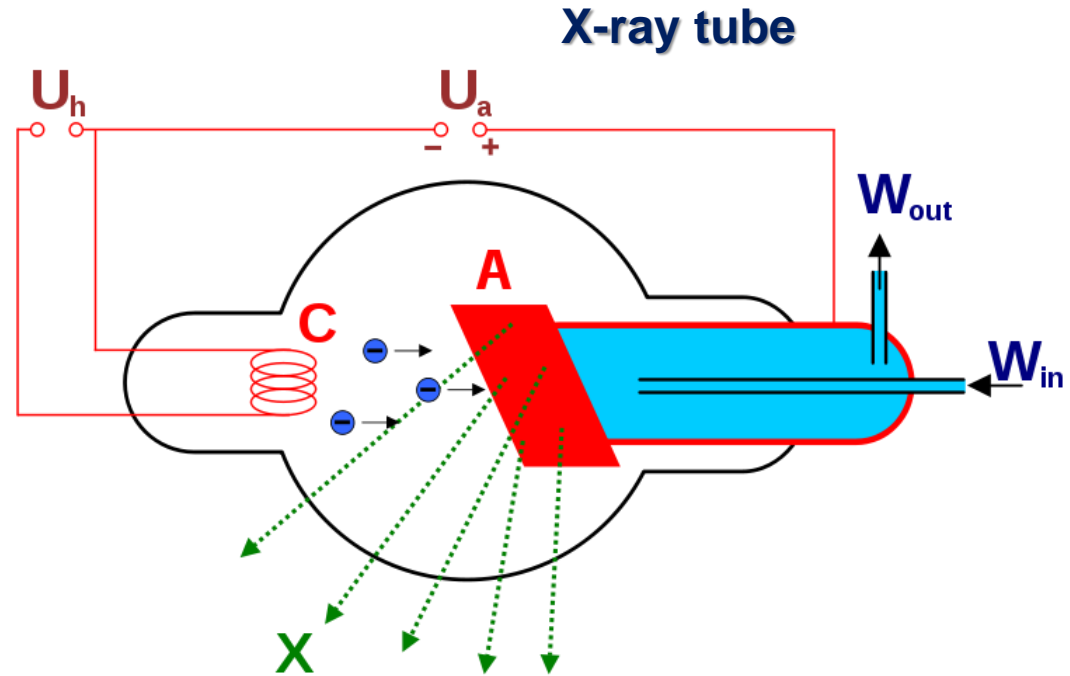
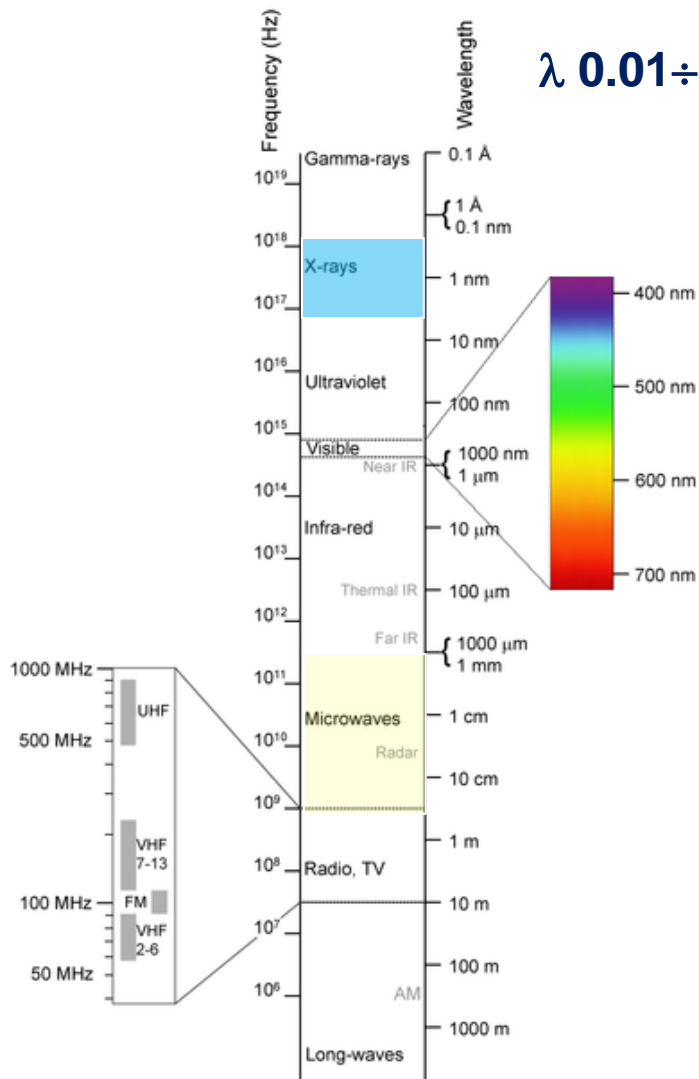


# Bragg diffraction. Results.\*



# Bragg diffraction. X-rays.

$\lambda$  0.01 ÷ 10 nm



# Bragg diffraction. X-rays.

X-ray K-series spectral line wavelengths (nm) for some common target materials

Target	$K\beta_1$	$K\beta_2$	$K\alpha_1$	$K\alpha_2$
Fe	0.17566	0.17442	0.193604	0.193998
Co	0.162079	0.160891	0.178897	0.179285
Ni	0.15001	0.14886	0.165791	0.166175
Cu	0.139222	0.138109	0.154056	0.154439
Zr	0.70173	0.68993	0.78593	0.79015
Mo	0.63229	0.62099	0.70930	0.71359

David R. Lide, ed. (1994). *CRC Handbook of Chemistry and Physics 75th edition*. CRC Press. pp. 10–227



# Bragg diffraction. X-rays.

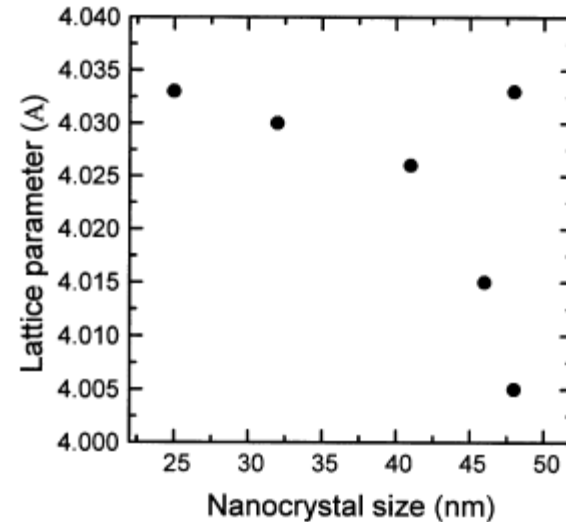
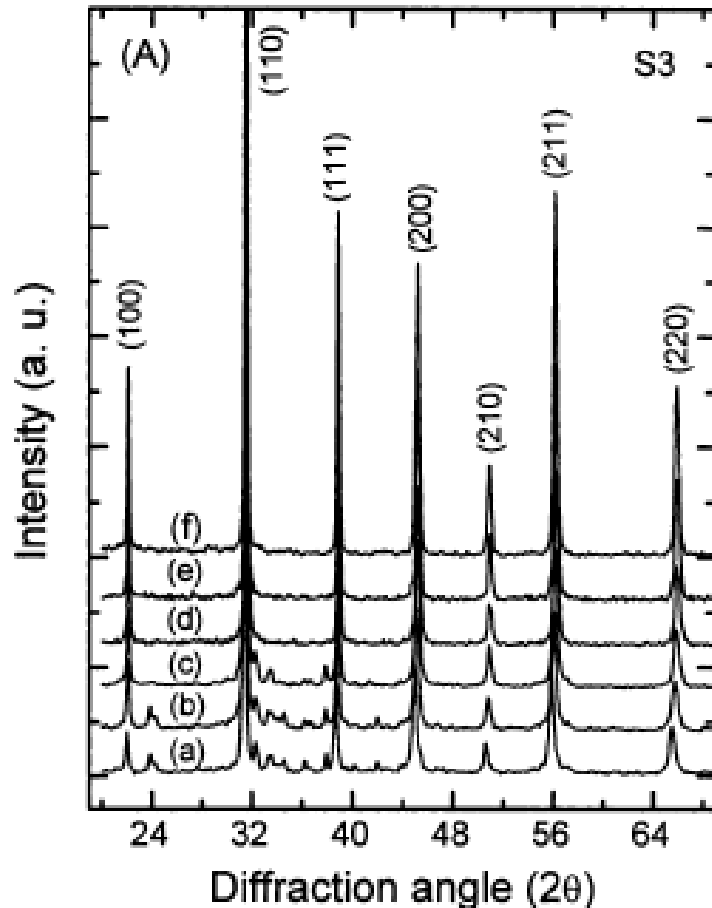


Fig. 4. Lattice parameter *c* versus the grain size in the BaTiO<sub>3</sub> nanocrystal.

F  
a  
fi

**Solid State Communications 119 (2001) 659–663**

Study of structural and photoluminescent properties in barium titanate nanocrystals synthesized by hydrothermal process

Ming-Sheng Zhang<sup>a,\*</sup>, Zhen Yin<sup>a</sup>, Qiang Chen<sup>a</sup>, Weifeng Zhang<sup>b</sup>, Wanchun Chen<sup>c</sup>



# Comments and suggestions

1. Klystron is very hot and the high voltage ( $\sim 300\text{V}$ ) is applied to repeller.
2. You have to do 6 (!) experiment in one Lab session – take care about time management. The most time consuming experiment is the “Bragg diffraction”.
3. Do not put on the tables any extra stuff – all this will causes extra reflections of microwaves and this could result in smearing of the data.
4. This is two weeks experiment but the equipment for the week 2 will be different. Please finish all week 1 measurements until the end of this week (Friday 28<sup>th</sup>)

**Good luck !**

