

Millikan Oil Drop Experiment

Physics 401, Spring 2013

Eugene V. Colla



Agenda

- 1. Measuring of the charge of electron.**
- 2. Robert Millikan and his oil drop experiment**
- 3. Theory of the experiment**
- 4. Laboratory setup**
- 5. Data analysis**
- 6. Errors**



Measuring of the charge of the electron

- 1. Oil drop experiment. Robert A. Millikan.. (1909).
 $q=1.5924(17) \times 10^{-19} \text{ C}$**
- 2. Shot noise experiment. First proposed by Walter H. Schottky**
- 3. In terms of the Avogadro constant and Faraday constant
 $e = \frac{F}{N_A}$; F- Faraday constant, N_A - Avagadro constant. Best uncertainty ~ 1.6 ppm.**
- 4. From Josephson ($K_J = \frac{2e}{h}$) and von Klitzing ($R_K = \frac{h}{e^2}$) constants**
- 5. Recommended by NIST value $1.602\ 176\ 565(35) 10^{-19} \text{ C}$**



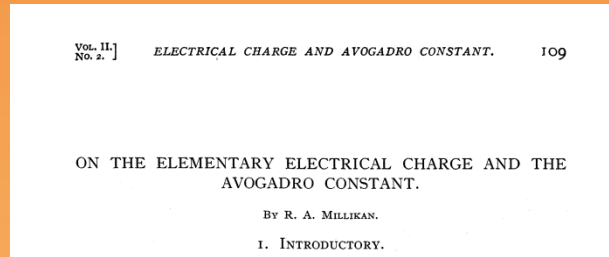
Robert Millikan. Oil drop experiment



**ROBERT ANDREWS
MILLIKAN
1868-1953**



**The Nobel Prize in Physics 1923.
Robert A. Millikan "for his work on the
elementary charge of electricity and on
the photoelectric effect".**

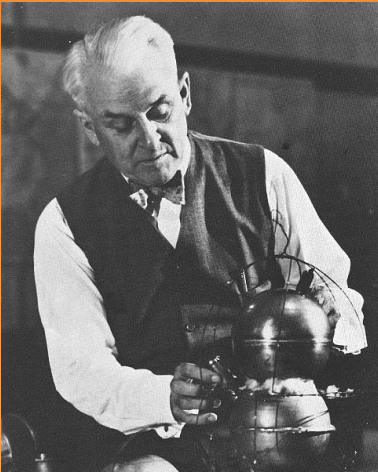


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2/11/2013



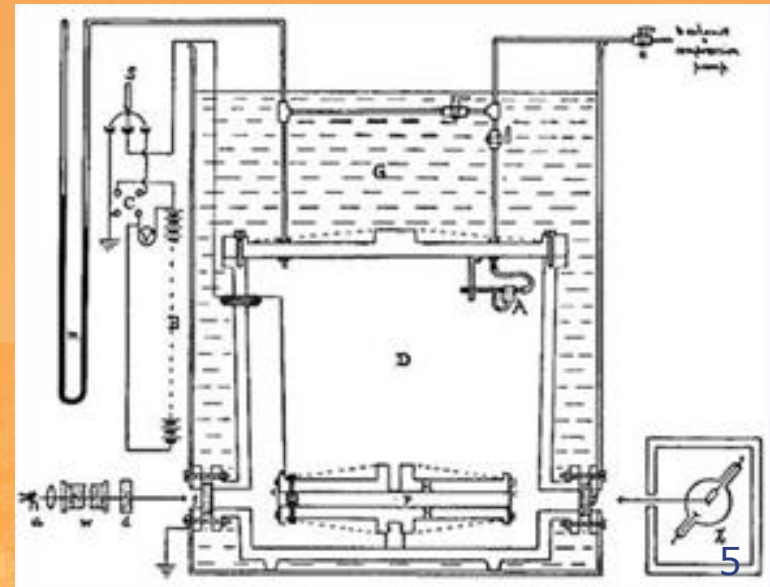
Robert Millikan. Oil drop experiment



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Diagram and picture of apparatus



Oil drop experiment.

Motivation:

Measurement of the magnitude of the electron charge!

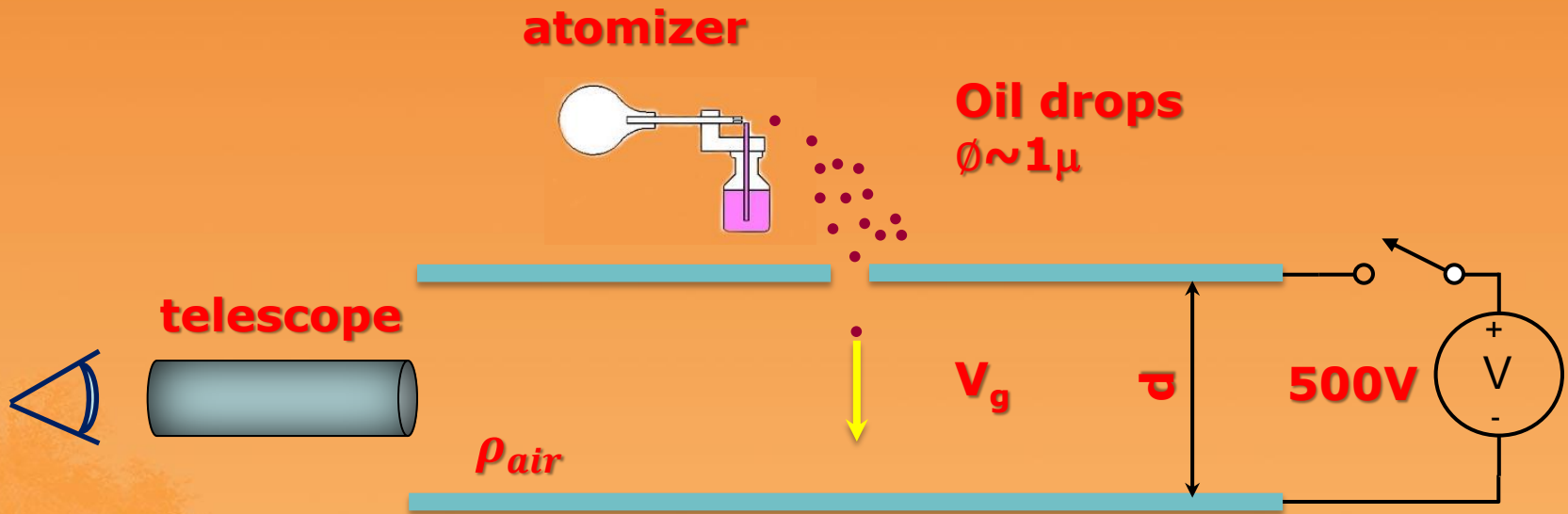
Demonstrate that the electron charge is quantized!



Measure the charge of
an electron to $\pm 3\%$

Picture of the PASCO setup

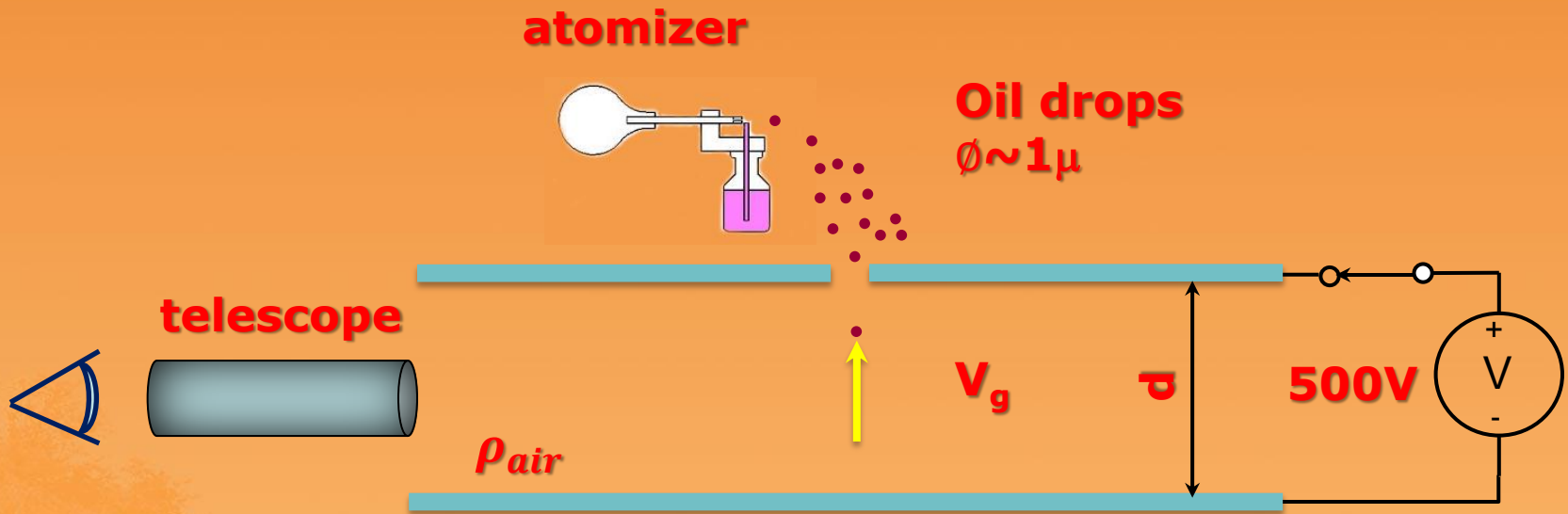
Oil drop experiment.



Forces on the oil drop:

- 1) Gravity + buoyant force (air displaced by oil drop)
- 2) Drag force of the oil drop in the air

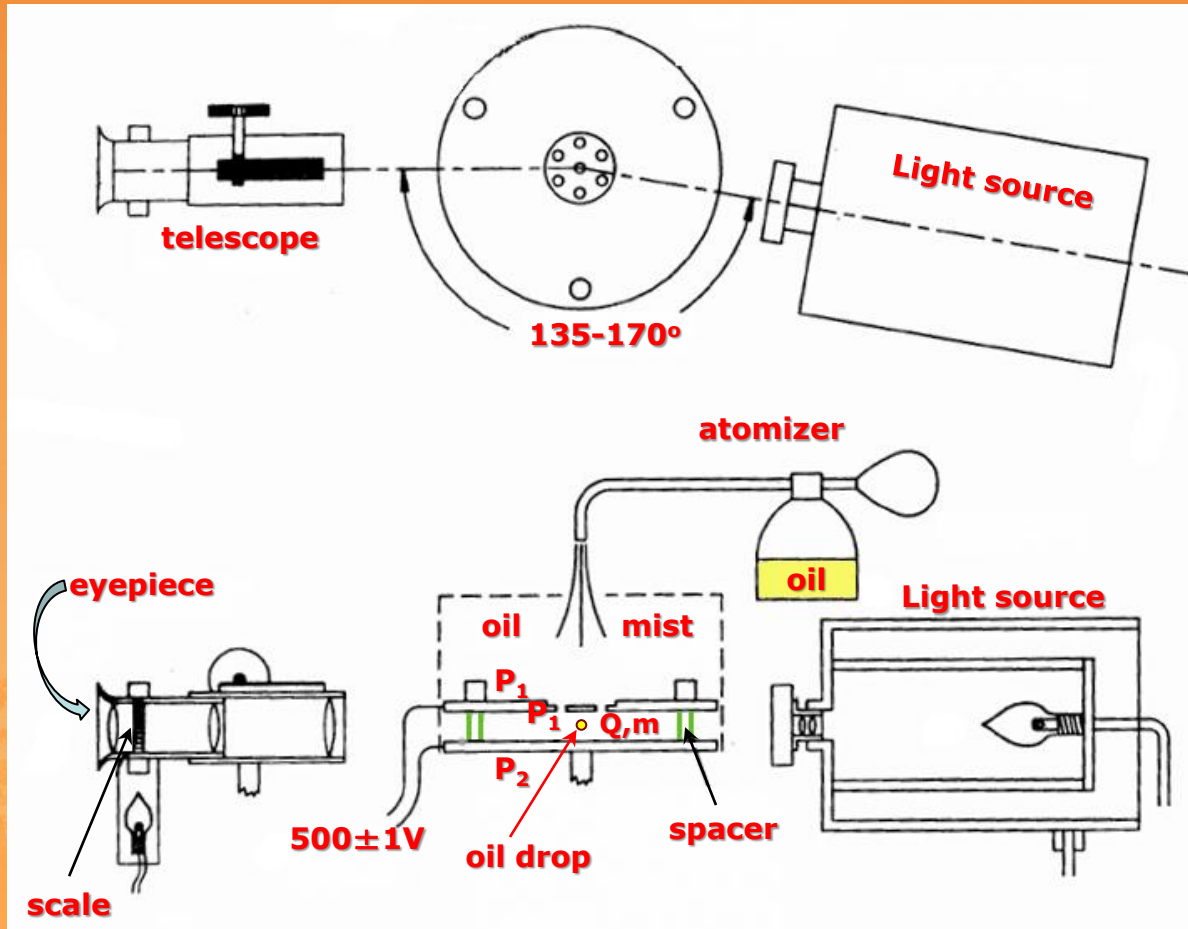
Oil drop experiment.



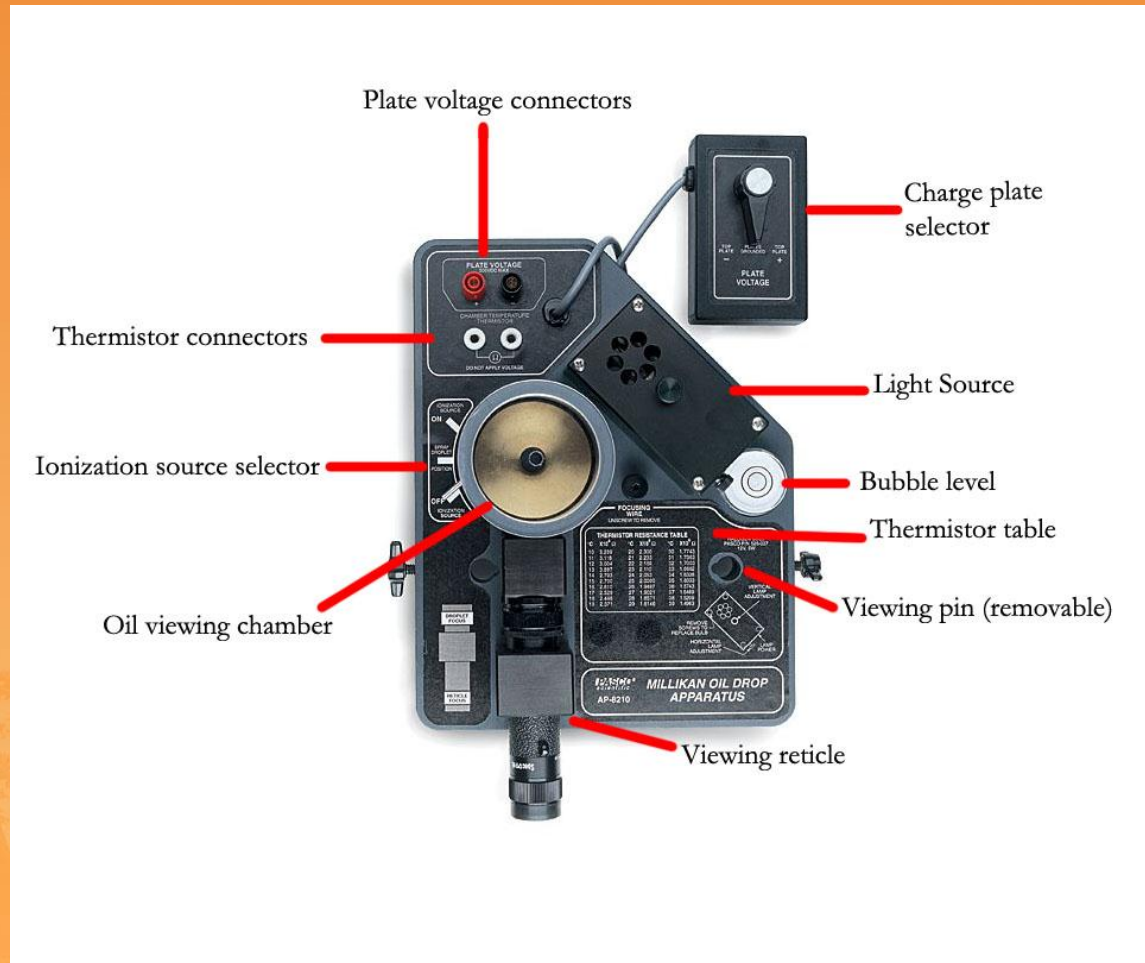
Forces on the oil drop:

- 1) Gravity + buoyant force (air displaced by oil drop)
- 2) Drag force of the oil drop in the air
- 3) Electric force on oil drops which carry charge Q

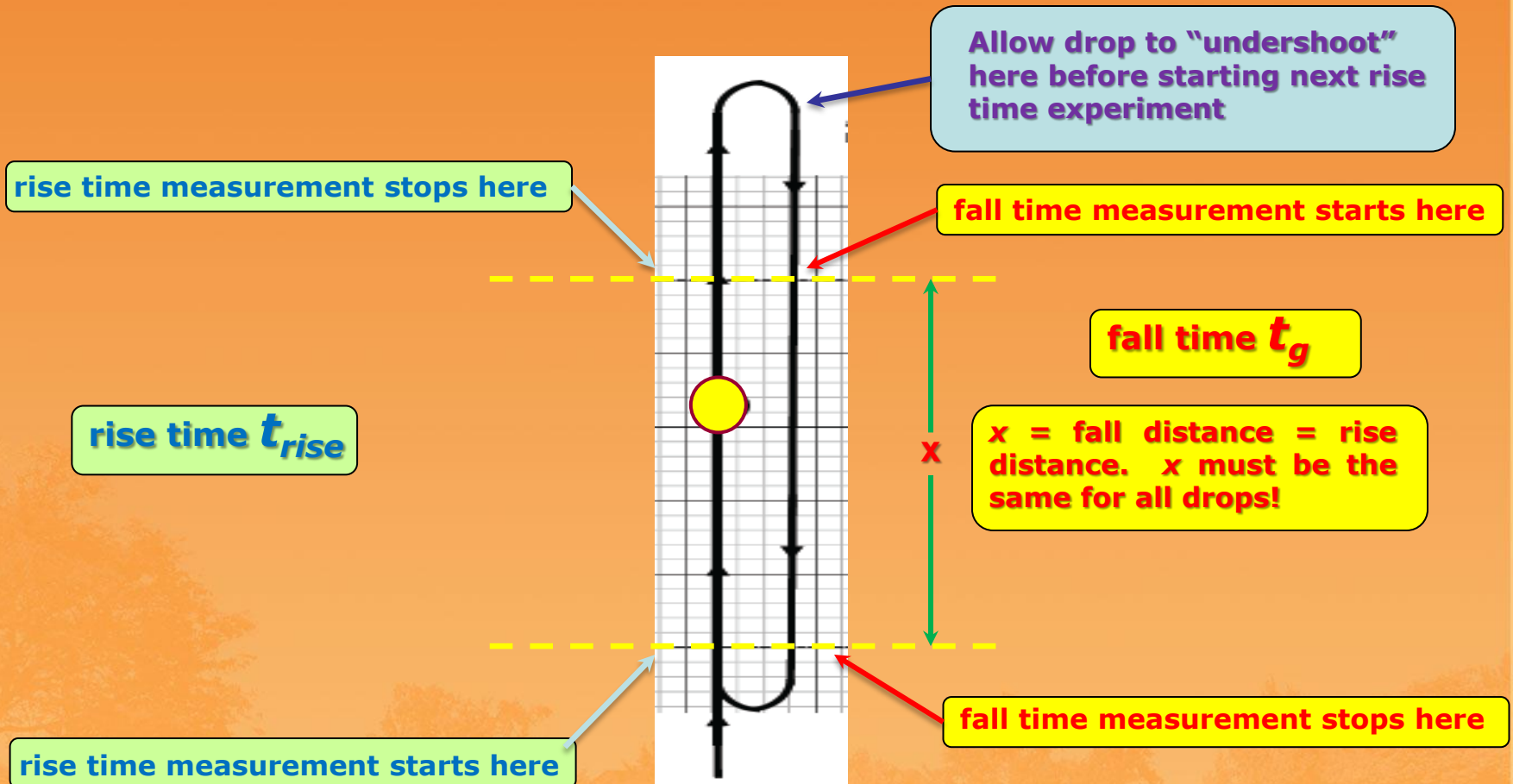
Apparatus. Schematic Layout



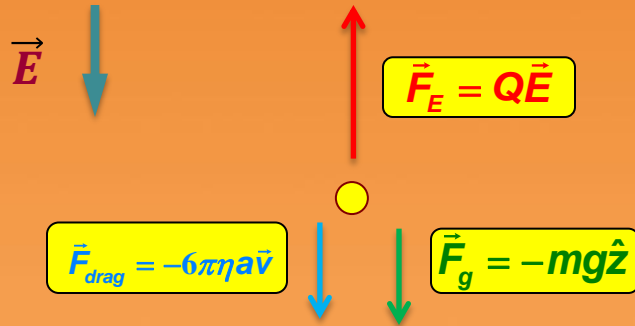
Apparatus: actual setup



What is measured



Balance of Forces: Newton's Law



a : radius of drop
 ρ : density $\rho = \rho_{oil} - \rho_{air}$
 v : velocity of oil drop
 Q : charge of oil drop
 E : electric field $E=V/d$
 V : Voltage across plates
 η : viscosity of air
 g : gravitational const.

Particle reached terminal velocity $\frac{d\vec{v}}{dt} = 0$

$$\vec{F}_g + \vec{F}_{drag} + \vec{F}_E = 0$$

$$\vec{F}_g = -mg\hat{z} \quad (1)$$

$$\vec{F}_{drag} = -6\pi\eta a\vec{v} \quad (2)$$

$$\vec{F}_E = Q\vec{E} \quad (3)$$

$$\vec{F} = m \frac{d\vec{v}}{dt} = \vec{F}_g + \vec{F}_{drag} + \vec{F}_E$$

Forces on the oil drop:

- (1) Gravity + buoyant force (air displaced by oil drop)
- (2) Drag force of the oil drop in the air
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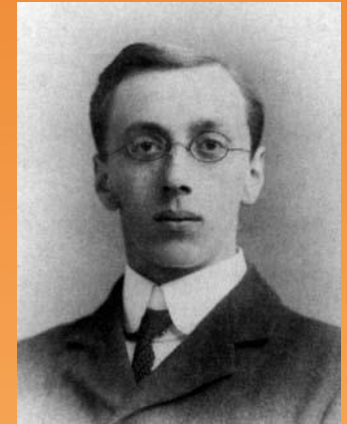
Modification to Stokes Law



George Gabriel Stokes
(1819-1903)

$$\vec{F}_{drag} = -6\pi\eta a\vec{v}$$

For small particle radius ($a < 15\mu$) Stokes law need to be corrected. This correction was derived by E. Cunningham.



Ebenezer Cunningham
(1881-1977)

$$\vec{F}_{drag} = -6\pi\eta \frac{a}{f_c} \vec{v}$$

$$f_c = 1 + A\frac{\lambda}{a} + B\frac{\lambda}{a}e^{-C\frac{a}{\lambda}}, \quad A = 1.246, \quad B = 0.42, \quad C = 0.78$$

$$f_c \approx 1 + A\frac{\lambda}{a} = 1 + \frac{r_c}{a} \approx 1.1, \quad \text{for } a \approx 10^{-6}\text{m}, \quad r_c = \frac{6.18 \times 10^{-5}}{\rho[\text{mmHg}]}$$

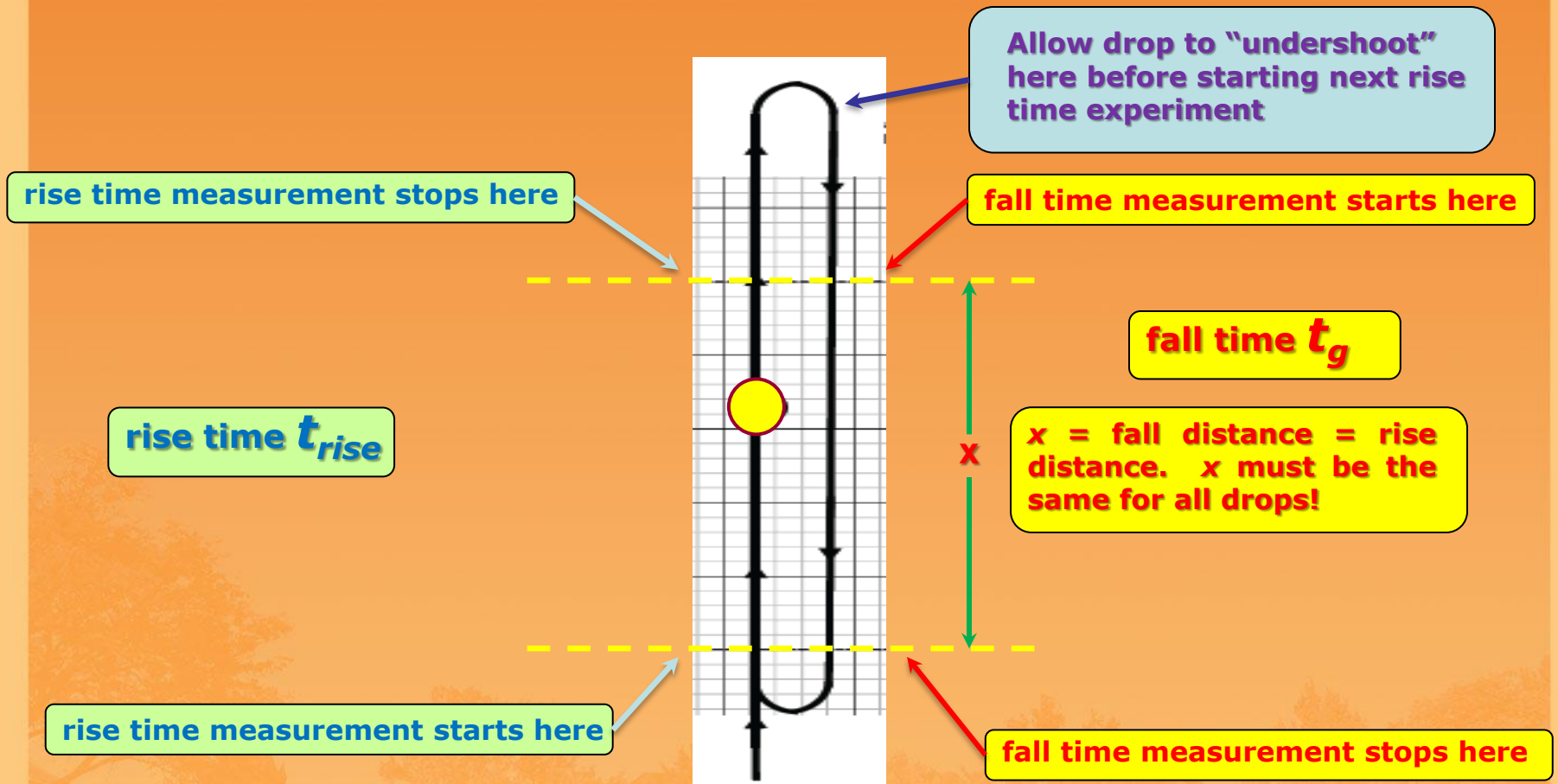
Here **a** – particle radius; **λ** – mean free path of the gas molecules

negligible term

$$\lambda [\text{m}] = 6.53 \times 10^{-8} \frac{760}{\rho[\text{mmHg}]}$$



We measure: t_g and t_{rise}



Solving Newton's law: $Q(t_g, t_{rise})$

f_c can be found from Newton law equation in the case of $V=0$ (falling drop)

$$\vec{F}_g + \vec{F}_{drag} = \frac{4}{3} a^3 \rho g - 6\pi\eta \frac{a}{f_c} \vec{v} = 0$$



(see write-up)

$$\frac{1}{f_c^3} \approx 1 - \left(\frac{t_g}{\tau_g} \right)^2; \quad \tau_g = \frac{2\eta x}{\rho g r_c^2}; \quad r_c[m] = \frac{6.18 \times 10^{-5}}{\rho[mmHg]}$$



Solving Newton's law: $Q(t_g, t_{rise})$

$$Q = n \times e = \frac{1}{f_c^{3/2}} \frac{9\pi d}{V} \sqrt{\frac{2\eta^3 x^3}{g\rho}} \sqrt{\frac{1}{t_g} \left[\frac{1}{t_g} + \frac{1}{t_{rise}} \right]}$$

Q : charge of oil drop
 n : number of unpaired electrons in drop
 e : elementary charge
 d : plate separation
 V : Voltage across plates

ρ : density $\rho = \rho_{oil} - \rho_{air}$
 η : viscosity of air
 g : gravitational constant
 x : drift distance for oil drop
 t_g : fall time
 t_{rise} : rise time



Route of charge calculation $Q(t_g, t_{rise})$.

$$\frac{1}{f_c^2} \approx 1 - \left(\frac{t_g}{\tau_g}\right)^2; \quad \tau_g = \frac{2\eta x}{\rho g r_c^2}; \quad r_c[m] = \frac{6.18 \times 10^{-5}}{\rho[mmHg]}$$

$$Q = F \cdot S \cdot T = \left(\frac{1}{f_c^{3/2}}\right) \frac{9\pi d}{V} \sqrt{\frac{2\eta^3 x^3}{g\rho}} \frac{1}{\sqrt{t_g}} \left(\frac{1}{t_g} + \frac{1}{t_{rise}}\right)$$

$$F = \frac{1}{f_c^{3/2}} \approx 1 - \left(\frac{t_g}{\tau_g}\right)^2$$

$$S = \frac{9\pi d}{V} \sqrt{\frac{2\eta^3 x^3}{g\rho}}$$

$$T = \frac{1}{\sqrt{t_g}} \left(\frac{1}{t_g} + \frac{1}{t_{rise}}\right)$$



Route of charge calculation. Origin projects. Data collecting.

Projects *Section L1.opj ... Section L4.opg*

Locations: [\\Phyapportal\PHYCS401\Common\Origin templates\Oil drop experiment](#)
[\\Phyapportal\PHYCS401\Students\1. Oil drop experiment - Origin templates](#)

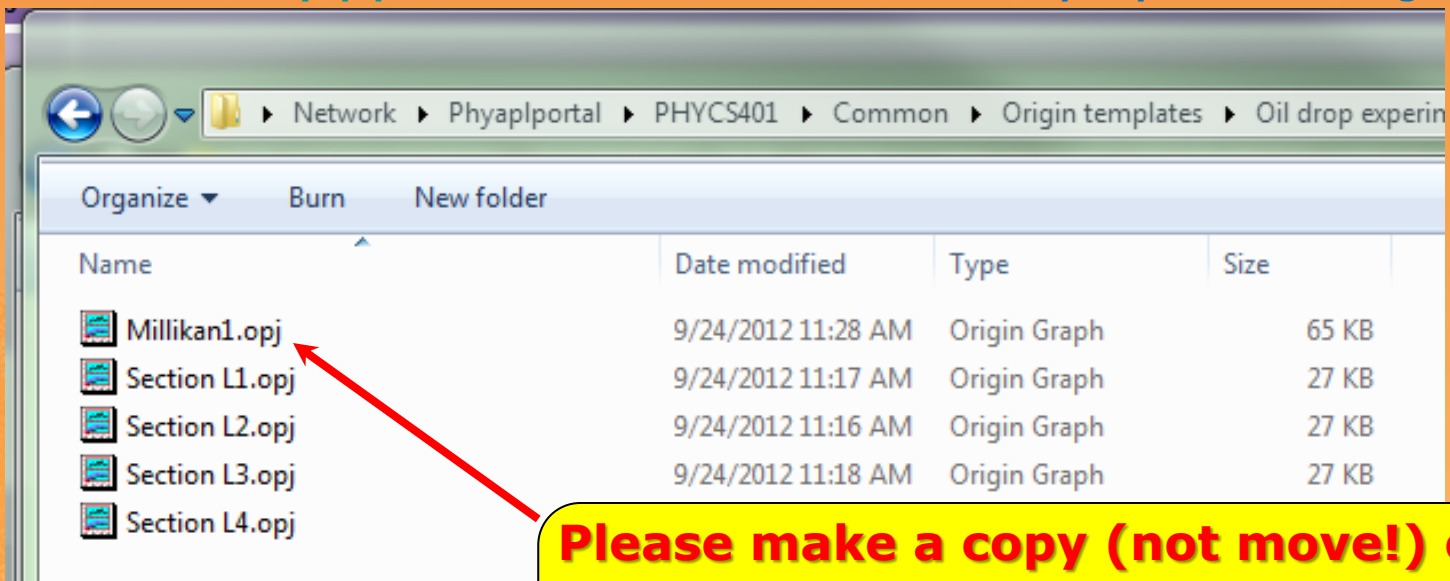
	A(L)	B(Y)	C(Y)	D(Y)	E(Y)	F(Y)	G(Y)	H(Y)	I(Y)
Long Name	parameter label	Par	tg	tr	parameter label	Par	tg	tr	pa
Units									
Comments	<i>student1, student2</i>	<i>student1, student2</i>	<i>student1, student2</i>	<i>student1, student2</i>	<i>student3, student4</i>	<i>student3, student4</i>	<i>student3, student4</i>	<i>student3, student4</i>	s
1	η				η				η
2	$\Delta\eta/\Delta T$				$\Delta\eta/\Delta T$				$\Delta\eta$
3	ρ_1				ρ_1				ρ_1
4	ρ_2				ρ_2				ρ_2
5	$\rho_1 - \rho_2$				$\rho_1 - \rho_2$				ρ_1
6	g				g				g
7	p				p				p
8	x				x				x
9	d				d				d
10	V				V				V
11	T_a				T_a				T_a
12									
13									
14									
15									
16									
17									



Route of charge calculation. Origin projects. Data analysis.

Project: *Millikan1.opj*

Locations: [\\Phyaplportal\PHYCS401\Common\Origin templates\Oil drop experiment](#)
[\\Phyaplportal\PHYCS401\Students\1. Oil drop experiment - Origin templates](#)



Please make a copy (not move!) of Millikan1.opj in your personal folder and start to work with your personal copy of the project

Route of charge calculation. Origin project. Data analysis.

Project *Millikan1.opj*

Long Name	Parameter names	parameter label	Par	rc	tau_g	tg	tr
Units				m		s	s
Comments							
				$r_c[m] = \frac{6.18 \times 10^{-5}}{p[mmHg]}$	$\tau_g = \frac{2\eta x}{\rho g r_c^2}$	your data	your data
3	Density of oil (kg/m ³)	ρ1	886	8.07843E-8	948.25725	23.07825	31.8955
4	Density of air (kg/m ³)	ρ2	1.29	8.07843E-8	948.25725	20.14243	11.70129
5	Density difference (kg/m ³)	ρ1-ρ2	884.71	8.07843E-8	948.25725	26.97377	22.47531
6	acceleration due to gravity (m/s ²)	g	9.801	8.07843E-8	948.25725	16.34362	16.44208
7	ambient pressure (mmHg)	p	765	8.07843E-8	948.25725	25.93429	25.02886
8	fall/rise distance (m)	x	0.00145			15.34338	9.27446
9	plate separation (m)	d	0.00317			29.3815	19.6161
10	Voltage across the plates (V)	V	500			26.0786	21.3434
11	Air temperature (oC)	Ta	20				

Constants

- In red - variable parameters
- in blue - data
- in olive - constants

Parameters of the experiment. Depend on exact setup and environment conditions

Recalculate Auto

Plugin your data here

Set Values - [Book1]Sheet1!Col(rc)

Formula wcol(1) Col(A) F(x) Variables

Row (i): From 1 To 1

Col(C) =

6.18e-5/Col("Par")[7]

calculations

Recalculate Auto Apply Cancel OK

Col(C) =

6.18e-5/Col("Par")[7]

$$r_c[m] = \frac{6.18 \times 10^{-5}}{p[mmHg]}$$

P(mmHg) → Col("Par")[7]

index

20



Route of charge calculation. Origin project. Data analysis.

Project *Millikan1.opj*

H(Y)	I(Y)	J(Y)	K(Y)	L(Y)
F	S	T	Q=FST	n=Q/1.602e-19
$F = \frac{1}{f_c^{3/2}} \approx 1 - \left(\frac{t_g}{\tau_g}\right)^2 \quad S = \frac{9\pi d}{V} \sqrt{\frac{2\eta^3 x^3}{g\rho}} \quad T = \frac{1}{\sqrt{t_g}} \left(\frac{1}{t_g} + \frac{1}{t_{rise}}\right) \quad Q = F \cdot S \cdot T$				
$Q = F \cdot S \cdot T = \left(\frac{1}{f_c^{3/2}}\right) \frac{9\pi d}{V} \sqrt{\frac{2\eta^3 x^3}{g\rho}} \frac{1}{\sqrt{t_g}} \left(\frac{1}{t_g} + \frac{1}{t_{rise}}\right)$				
Follow correct order of calculations: $r_c \rightarrow \tau_g \rightarrow (F, S, T) \rightarrow Q \rightarrow n$				

Route of charge calculation. Origin project. Data analysis.

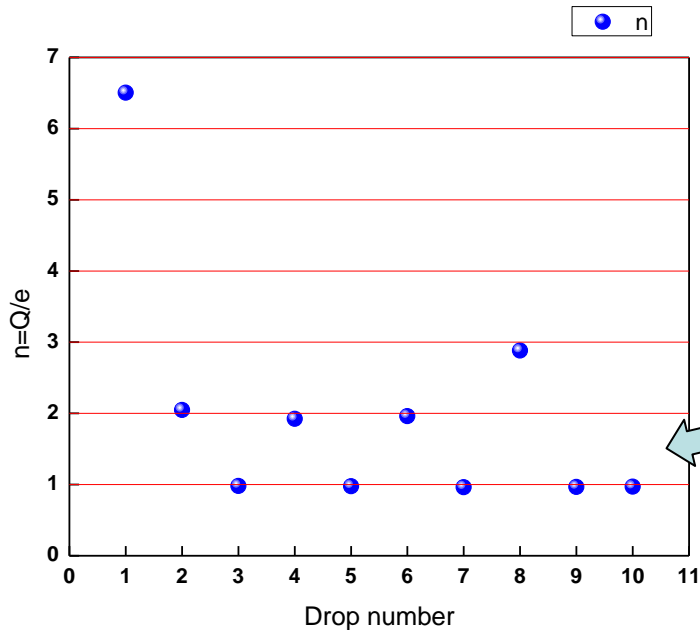
Project *Millikan1.opj*

$$F = \frac{1}{f_c^{3/2}}$$

Follow

**Indexes for parameters in Col("Par")
Actual air viscosity should be calculated manually
before any other calculation**

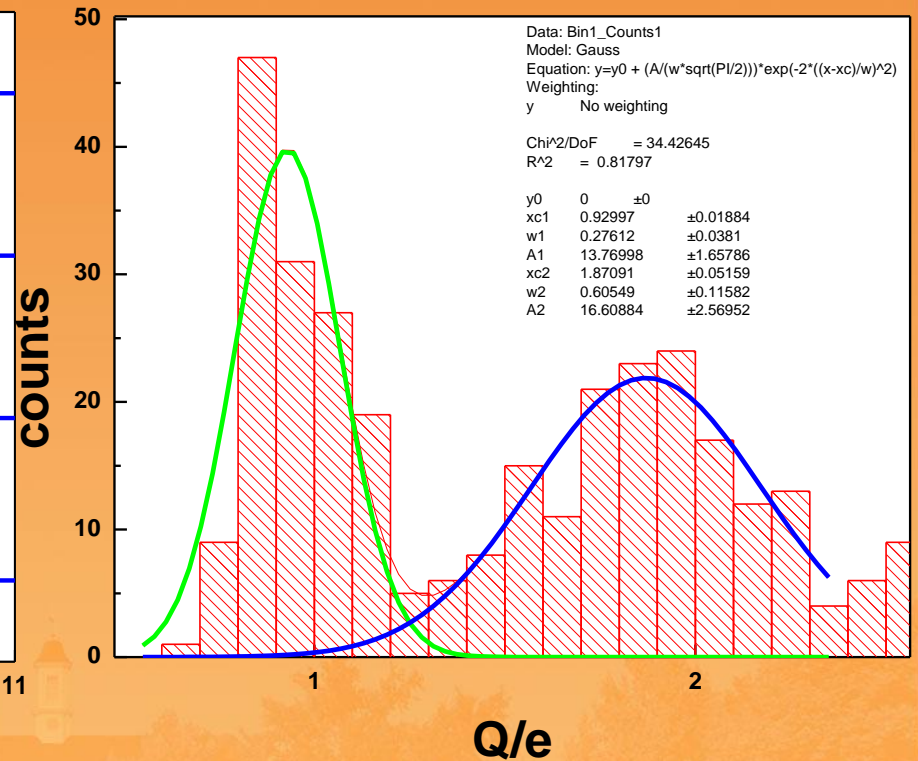
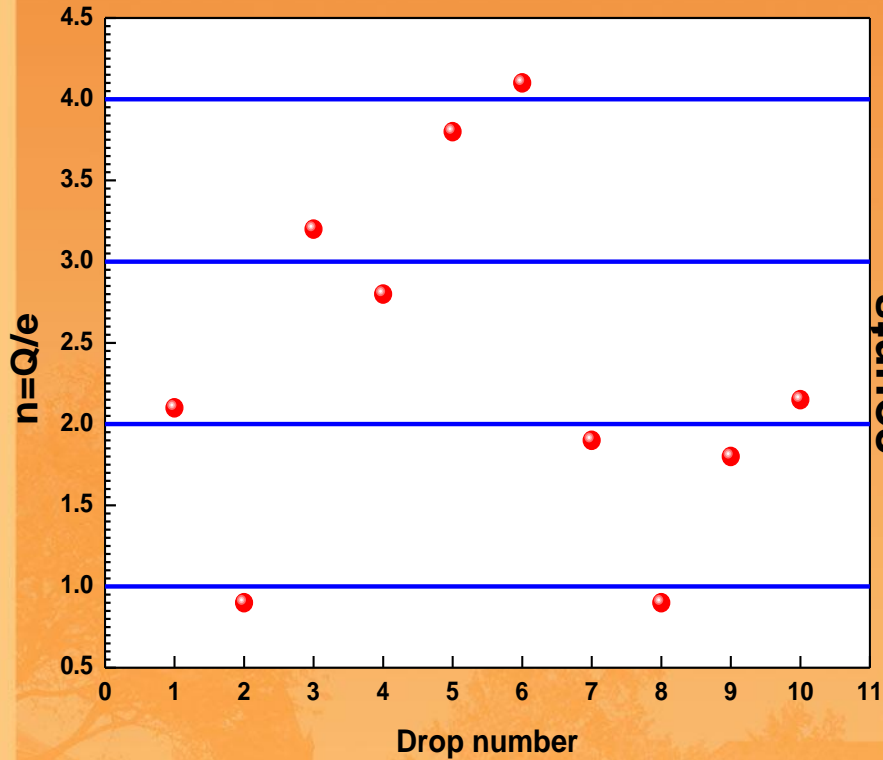
Charge calculation. Origin project.



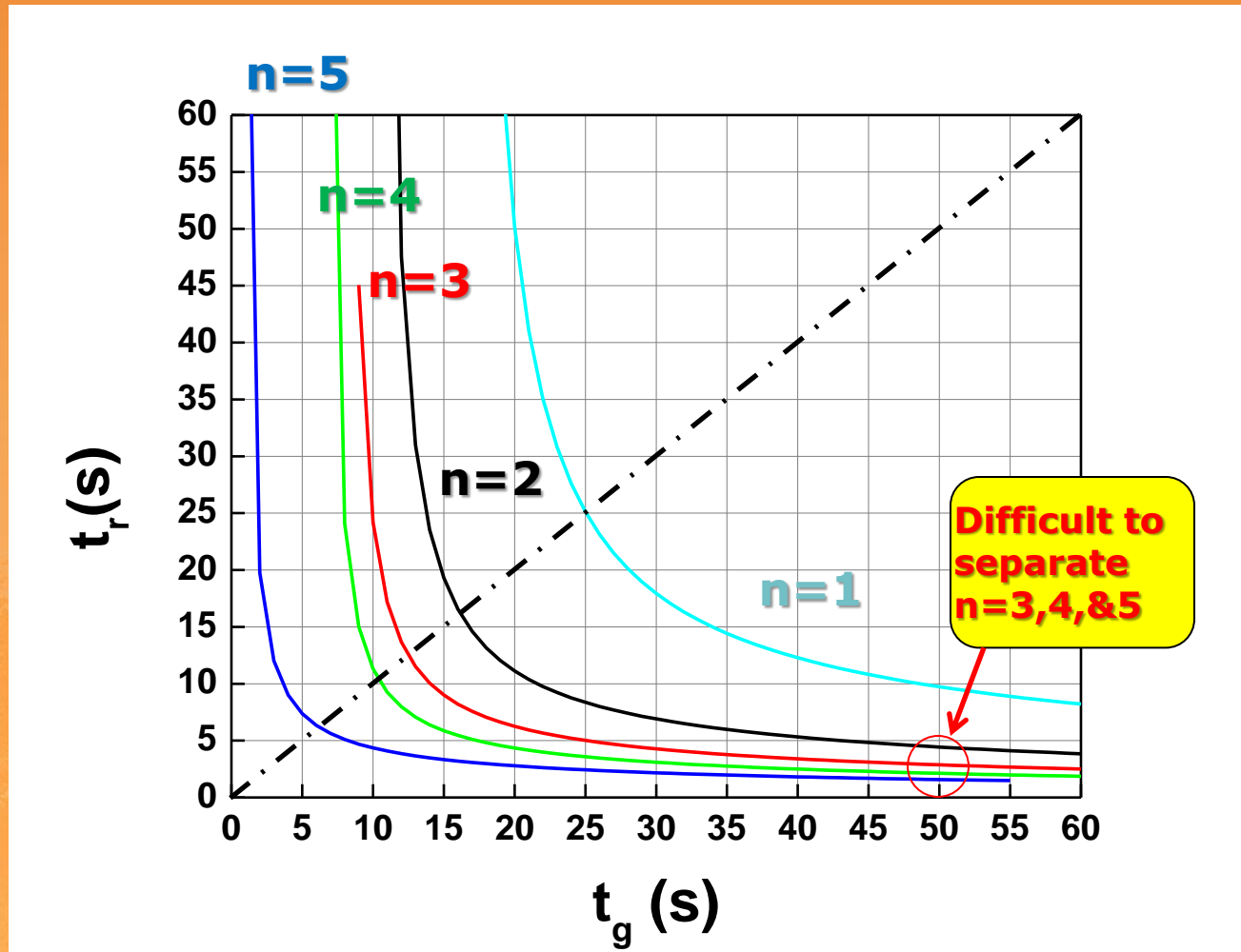
I(Y)	J(Y)	K(Y)	L(Y)
S	T	Q=FST	n=Q/1.602e-19
			c
$S = \frac{9 \pi d}{V} \sqrt{\frac{2 \eta^3 x^3}{g \rho}}$	$T = \frac{1}{\sqrt{t_g}} \left(\frac{1}{t_g} + \frac{1}{t_{rise}} \right)$	Q=F*S*T	number of elementary charges
--	--	1.04196E-19	6.50414
--	--	1.57231E-19	2.0471
--	--	3.08168E-19	0.98147
--	--	1.56458E-19	1.92365
--	--	3.14167E-19	0.97664
--	--	1.54196E-19	1.9611
--	--	4.61924E-19	0.96252
--	--	1.54861E-19	2.88342
--	--	1.55468E-19	0.96668
--	--		0.97046
--	--		
--	--		
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--	--		
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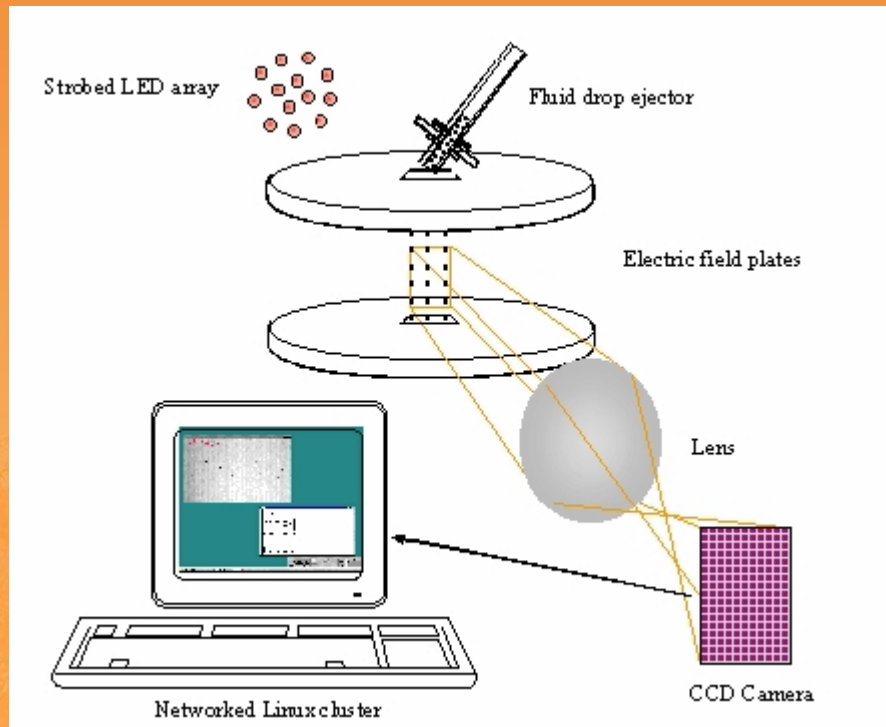
Expected results



Choice of Oil Drops for the Analysis: rise and fall times



Modern experiments at



- **Drop generation rate 1 Hz**
- **Fluid - Dow Corning silicon oil**
- **Number of drops - 17 million**
- **Mass - 70.1 milligrams**
- **Duration - 8 months**

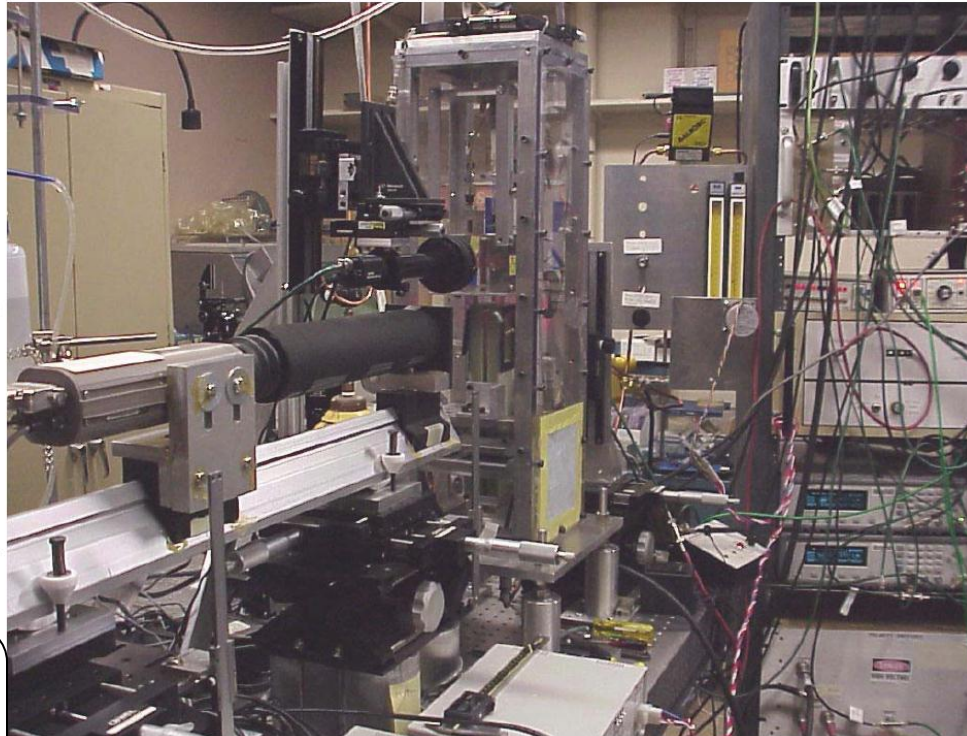
Modern experiments at



Machine vision mediated auto-control of: the average charges of the drops, the fall path of the drops, the upward laminar air flow, and the electronic drive to the drop ejector are new features of this fluid drop charge measurement system.

The experiment is ran from 2004 - 2007.

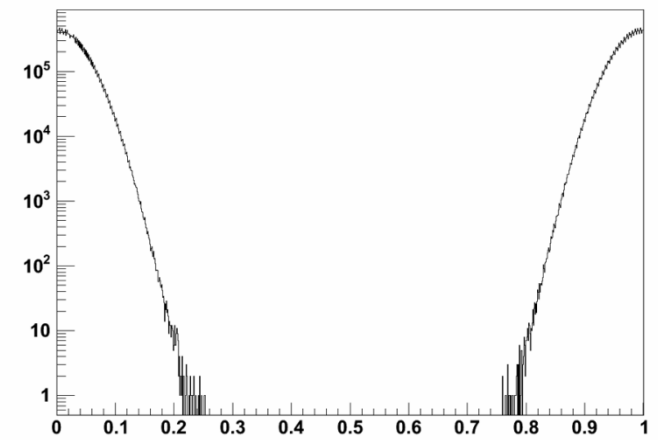
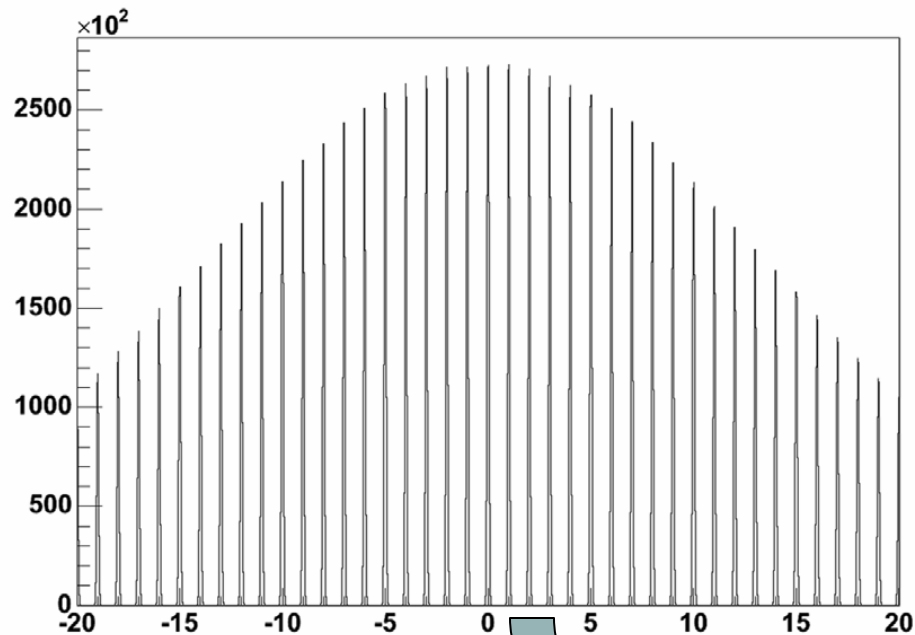
Charge measurement accuracy achieved is better than $1/24 e$ for drops of up to 26 microns in diameter.



Modern experiments at



"No electric charges were measured in the range of an integer charge $1/3 e$ or $2/3 e$." (SLAC-PUB-7357 August 1996)



Appendix #2

Transmission line. Unknown load simulation

The screenshot shows a simulation software interface for a transmission line with an unknown load. The interface is divided into several sections:

- Pulse Generator:** A sidebar on the left containing parameters such as Frequency (100k), Pulse Width (100n), Pulse Delay (0.3u), Rise Time (1p), Fall Time (1p), High (1), Low (0), Burst Mode (OFF), Burst Count (2), Burst Rep Rate (150), Time Span (10u), and Num Points (4096).
- Incident pulse:** A plot showing a pulse waveform. The Y-axis is labeled 'Y name' and ranges from 0.7 to 1.0.
- Pulse on the load:** A plot showing the pulse after reflection. The Y-axis is labeled 'Y name' and ranges from 1.2 to 1.8.
- Reflected pulse:** A plot showing the reflected pulse. The Y-axis is labeled 'Y name' and ranges from 1 to 2.
- Load parameters dialog:** A central dialog box titled 'Load parameters' with an 'EXIT' button. It contains two input fields: 'C (nF)' with the value '1' and 'R (ohms)' with the value '100'.
- Annotations:** Yellow text with arrows pointing to various parts of the interface:
 - 'Function generator parameters' points to the Pulse Generator sidebar.
 - 'Line characteristic impedance' points to the 'Zk (oHm)' field containing '50'.
 - 'Expected load' points to the 'C || R' button.
 - 'X-axes scaling' points to the 'Scale' dropdown menu in the bottom right, which is set to 'X'.
- Bottom Right Panel:** A panel with a 'Field name' table and a 'Value' column. The table has the following entries:

Field name	Value
Scale	X
Name	time
Min	0.2u
Max	0.5u
Mapping	Linear

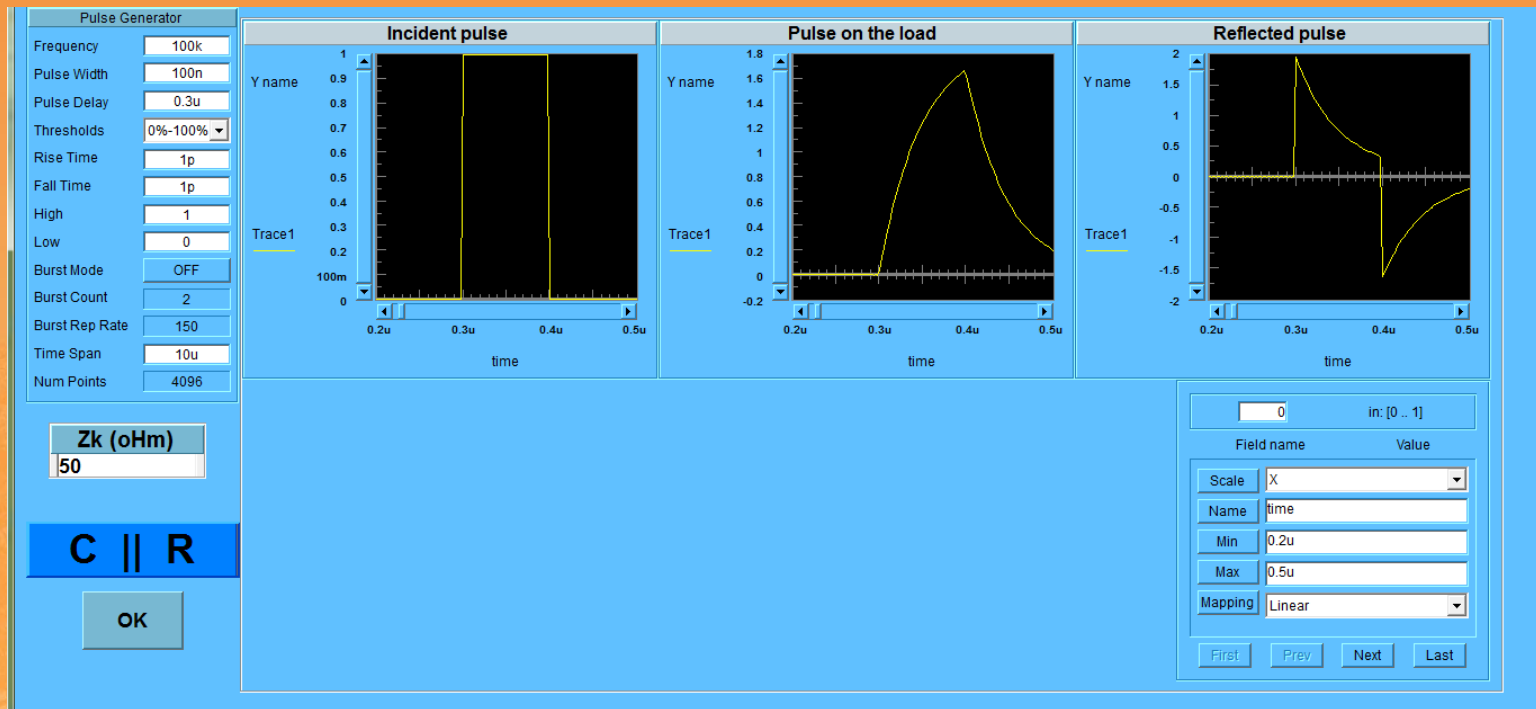
Below the table are buttons for 'First', 'Prev', 'Next', and 'Last'.

Location: \\Phyapportal\PHYCS401\Common\Transmission line software



Appendix #2

Transmission line. Unknown load simulation



Location: \\Phyap\portal\PHYCS401\Common\Transmission line software

