AC Measurement of Magnetic Susceptibility

Ferromagnetic materials such as iron, cobalt and nickel are made up of microscopic domains in which the magnetization of each domain has a well defined orientation. A macroscopic sample contains a large number of domains all with random orientation and consequently does not possess a magnetization. Ferromagnetic samples can be magnetized by applying an external field large enough to cause a majority of the domains to align in the direction of the external field.

Microscopically, an individual domain experiences a local magnetic field from the neighboring domains. In order to re-orient a given domain, the magnetic torque $\vec{\tau} = \vec{m} \times \vec{B}$ produced by the external field must be large enough to overcome the forces which keep the domain pinned. As the external field is increased, the magnetization of the material grows until all of the domains align. The maximum magnetization is referred to as the saturation magnetization. The dynamics involved in re-orienting the spins are quite complex and often involve dissipation of energy.

A common experimental procedure used to characterize the magnetic susceptibility is to apply an external DC field to the sample and measure the induced field within the sample. An external field $\vec{H}$ applied to the sample will produce a magnetic field within the sample given by

$$\vec{B} = \mu_0(\vec{H} + \vec{M}(H)) \tag{1}$$

Here, μ_0 is the vacuum permeability and $\vec{M}$ is the magnetization of the material $\vec{M}(\vec{H}) = \chi(H)\vec{H}$ and $\chi(H)$ is the magnetic susceptibility which depends on the magnitude of the applied field. Equation (1) may be written as $\vec{B} = \mu\vec{H}$, where $\mu = \mu_0(1 + \chi(H))$.

An alternative way to measure the magnetic susceptibility of a material is to superimpose a small time varying or AC excitation $H_1 \cos \omega t$ on top of the DC external field H_0 and measure the AC response of the material.

(2) $$H = H_0 + H_1 \cos \omega t$$

A picture of this procedure is demonstrated on a typical B-H curve shown in Figure 1.

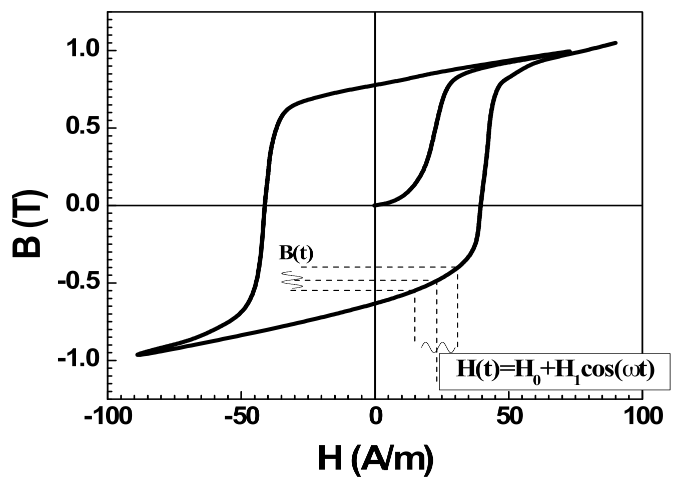

Fig.1 Typical hysteresis loop of the magnetic material

The dissipation caused by rearranging the magnetic domains gives rise to a complex magnetic susceptibility $\chi = \chi' - i\chi''$. The real part of the susceptibility is proportional to the component of the magnetization that is induced in-phase with the applied modulation while the imaginary part is proportional to the $\pi/2$ out of phase or quadrature component of the magnetization. It is this latter part which is directly proportional to the dissipation in the material.

In this experiment, the DC field H_0 will be slowly varied while keeping the magnitude of the modulation H_1 constant. The time varying AC excitation $\delta H = H_1 \cos \omega t$ will produce a small time varying response $\delta B = B_1 \cos(\omega t + \theta)$. The response need not have the same phase as the drive. In fact, if the material possess a non-zero imaginary susceptibility, θ will also be non-zero. At a given H_0, the AC response at frequency ω measures the derivative of the B-H curve.

$$B(H) = \mu(H)H$$
$$B_0 + \delta B = \left[\mu(H_0) + \left(\frac{\partial \mu}{\partial H}\right)_{H_0} \delta H + \frac{1}{2}\left(\frac{\partial^2 \mu}{\partial H^2}\right)_{H_0} (\delta H)^2 + \ldots \right](H_0 + \delta H)$$

where $\mu(H_0) = \mu_0 \left(1 + \chi(H_0)\right)$. We seek to find δB to lowest order in δH. We are justified in making this approximation provided that the term linear in δH is much larger than the higher order terms. Furthermore, if we assume that the variation in μ over the range δH is also small, then we find

(4a) $$\delta B = \mu(H_0)\delta H$$

(4b) $$\mu(H_0) = \left.\frac{dB}{dH}\right|_{H=H_0}.$$

By writing $\delta H = \mathrm{Re}\left\{H_1 e^{i\omega t}\right\}$, $\mu(H_0) = \mu_0 \left(1 + \left[\chi'(H_0) - i\chi''(H_0)\right]\right)$ and $\delta B = \mathrm{Re}\left\{\mu(H_0)\delta H\right\}$, we find

(5a) $$\delta B(t) = H_1 \mu_0 \left[\left(1 + \chi'(H_0)\right)\cos \omega t + \chi''(H_0)\sin \omega t\right]$$

(5b) $$\delta B(t) = H_1 \left[\mu'(H_0)\cos \omega t + \mu''(H_0)\sin \omega t\right]$$

(5c) $$\theta = -\tan^{-1}\left(\frac{\mu''}{\mu'}\right).$$

B-H curve shown in Fig.1 is obtained by integrating the magnitude of the permeability as a function of H_0.

(6) $$B(H) = \int_0^H \left|\mu(H')\right| dH'$$

The simplest geometry for magnetic field measurements is the toroid (Fig.2). High permeability ferromagnetic materials confine the externally applied magnetic flux to the interior of the ferromagnet. By making the material into a toroid, the flux lines are totally confined. The magnetic field created by a coil that is wound on toroid is given by

$$H = \frac{N_p I_p}{2\pi r} \tag{7}$$

where N_p is the number of the turns in the primary coil, I_p is the current in the primary coil and r is a radius in the range $r_1 < r < r_2$. The toroid geometry was chosen because it allows us to simply relate the magnitude of the H field to the applied current.

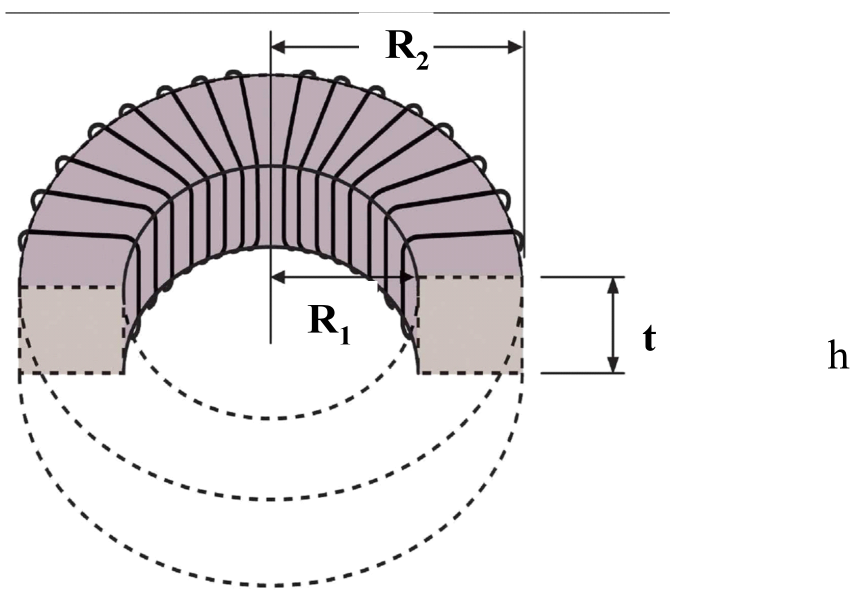

Fig.2. Schematic of toroid geometry with primary coil.

The magnetic flux through each turn of the pickup coil is given by

$$d\Phi = \mu \int \vec{H} \cdot d\vec{a} = \frac{\mu\, I_p N_p h}{2\pi} \int_{r_1}^{r_2} \frac{dr}{r} = \frac{\mu I_p N_p h}{2\pi} \ln \frac{r_2}{r_1} \tag{8}$$

where μ is the magnetic permeability of the core. The total flux through the solenoid is

(9) $$\Phi = N_{pickup} d\Phi = \frac{\mu N_{pickup} N_p I_p h}{2\pi} \ln \frac{r_2}{r_1}$$

The inductance of the solenoid is given by $L = \Phi / I$. Since the permeability of the material is complex, we can express the coil inductance as a complex quantity.

(10) $$L = \frac{\mu L_0}{\mu_0} = (\mu' - i\mu'')L_0$$

where L_0 is the air filled coil inductance.

(11) $$L_0 = \frac{\mu_0 N_{pickup} N_p h}{2\pi} \ln \frac{r_2}{r_1}$$

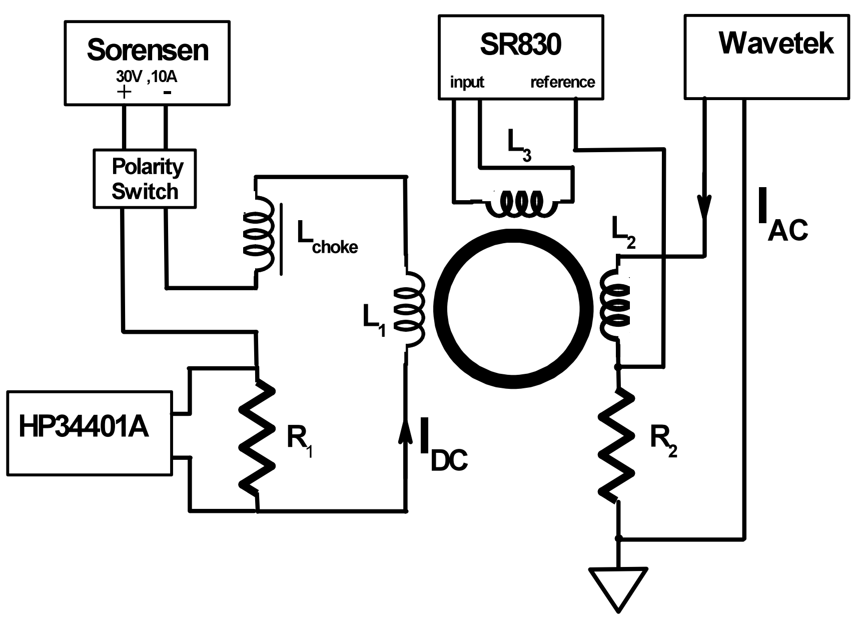

Fig.3. Block diagram of the setup for investigation of the hysteresis loops.

For measurement of the magnetic permeability we will use the setup shown in Figure 3. The bias magnetic field in the toroid is generated by driving a DC current through the primary coil L_1. The current is supplied by the Sorensen power supply, which can provide a maximum current of 10A into a 1.2 Ω load. The AC modulation is provided by the Wavetek function generator. In all of our measurements, the impedance of the AC modulation coil L_2 is much smaller than series resistor R_2. We can, therefore, assume with reasonable precision that the AC current I_{AC} is determined by the voltage on the output of the function generator V_{AC} divided by the resistance R_2.

The voltage induced in the pickup coil L_3 can be related to the induced magnetic field $B_1(t)$ in the toroid.

$$V_{lock-in} = -L_3 \frac{dI}{dt} = -\frac{d\Phi}{dt} \tag{12}$$

Here, $V_{lock-in}$ is the voltage supplied to the input of the lock-in, i.e. the voltage across L_3. It will be up to you to derive an expression for $V_{lock-in}$ in terms of the experimental parameters $N_2, N_3, R_2, V_{AC}, \omega$ and the dimensions of the toroid, where V_{AC} is the voltage amplitude of the excitation to the coil N_2. (Hint: in this calculation, the coil L_2 provides the field H via I_{AC}) The voltage across L_3 is measured using the SR830 lock-in amplifier. The X and Y output channels of the lock-in amplifier measure the in-phase and quadrature response respectively. The reference for the lock-in is derived from the voltage across R_2 which is directly proportional to the amplitude of the applied modulation H_1. For this experiment, the lock-in amplifier works in the external reference mode locked to phase of the AC current. In order to allow large DC currents, R_1 was chosen to be a small value of ~0.05Ω. The choke (L_{choke}) serves to minimize any AC contribution to the current experienced at L_1 by effectively increasing the AC impedance without changing the DC impedance (remember, coils look like straight wires in DC circuits). By changing the DC current through L_1, we can measure the magnetic permeability of the material as function of H_0. Note, a full description of the lock-in amplifier can be found the Stanford Research Systems manual for the SR830. A pdf version of the manual will be posted on the course website (This manual is an excellent source for understanding the theory of a lock-in amplifier). A hard copy of the manual will also be placed next to each amplifier in the lab.

The next step in the analysis is to obtain the B(H) curve by integrating $\mu(H)$ data obtained using the lock-in. In these measurements, you will be using low loss ferrites, thus the magnitude of the permeability will be dominated by the real part of the susceptibility. Data acquisition is fully automated. The parameters for the field sweep, such as the starting and ending DC current to apply to L_1, and magnitude and frequency of the modulation will be entered into the data acquisition program. The program will then generate a text file with column data that can be imported into Origin for analysis. This program has three main options: (i) preparation of the experiment – creating the H_0 vs. time profile to be used during the measurement, (ii) measurement of μ(H), and (iii) demagnetization of the core – this is necessary because the magnetic prehistory of the core is unknown and the core could have a nonzero magnetization.

Temperature Dependence of the Magnetic Susceptibility

In order to get a more complete understanding of these materials, the temperature dependence of the susceptibility is very useful. As mentioned previously, it is possible to align all of the magnetic domains in a ferromagnet by applying an external filed large enough to overcome the pinning force experienced by each domain. For sufficiently strong pinning forces, the domains, in principle, can stay aligned even after the external field is removed. Thermal energy, however, is capable of slowly unpinning the domains, which will weaken the strength of the magnet. As the temperature increases, thermal energy is able to randomize the domains more quickly up until the Curie temperature is reached where the sample is completely demagnetized. The goal of this measurement is to determine the Curie temperature of some samples.

The experimental setup is identical to the AC measurement described in Figure 3. Using a heater driven by a 500W Variac power supply, the temperature of the sample can be set at values exceeding 100°C courtesy of the vegetable oil. It is important to note that it will take some time for the sample to equilibrate at a constant temperature after changing the Variac output. Monitor the thermocouple voltage for a few minutes and determine it is steady before running the experiment. Also, make sure to record the thermocouple voltage before and after each field sweep measurement in order to help you determine the error associated with the temperature.

Report

Your report should be structured into the following sections:

Abstract (briefly describe the goal and results of the measurement)

Theory (give a theoretical background to the measurement)

Experimental Apparatus (Your description of the experiment should indicate a thorough understanding of the apparatus used in the measurement as well as the setup of the experiment. Include all the expressions that relate measured voltages to experimental parameters. These include for example the voltage applied to the primary coil and how it relates to the H field induced in the material and how the voltage measured from the lock-in amplifier relates to the physical quantity being observed, e.g. B field or μ.)

Discussion and Results (Discuss the data you obtained explaining the observed features. Here it would be helpful to look through the literature for references to previously published work relating to the measurement being discussed. Include a bibliography of all cited works.)

Conclusion (What did you learn in these measurements? Are there any improvements you can come up with to make the experiments better?)

- For measurements of susceptibility as a function of applied field you should obtain data for four samples. Your plots should display the magnetic field B induced in the sample as a function of the applied H field at a single frequency ω. Due to the high inductance of the pickup coil, your measurements for this section should be taken at a frequency below about 5kHz. At higher frequencies, spurious resonance could result from the cable capacitance and the inductance of the pickup loop.
- Compare the B-H curves obtained with the published literature for the material. Most materials are provided with data sheets that contain BH-curves or comparable data tables. Whenever possible, make a comparison between the data obtained in your experiment and published data.
- Derive an expression that quantitatively relates χ'' to P_c, the power dissipated per cycle. Plot P_c vs. H for all four samples.
- Obtain the temperature dependence for $\mu(H)$ the Ferroxcube ferrite. Compare with tabulated data.

Suggestions

- The report should be intelligible to someone who has not done the experiment.
- It is your responsibility to understand the theory, the purpose and the procedure of the experiment, and the apparatus before performing the experiment.
- It is not necessary to include detailed derivations in the report but only an outline of the theory. The procedure as well as a description of the apparatus in your own words should also be included in the report.
- Include error estimates whenever appropriate.

- Before coming to class, derive the expressions that relate the measured voltages to the physical quantities B and H.
- The data acquisition for the lab is fully automated. It is highly suggested that you spend your lab section analyzing the data as it comes in. **DON'T WAIT UNTIL THE LAST MINUTE TO ANALYZE YOUR DATA.** Your computers are capable of multitasking – that is you can be acquiring and analyzing data simultaneously.

Suggested References

1. *Magnetism Principles and Applications*, Derek Craik
2. *Introduction to Solid State Physics*, Charles Kittel
3. *Electromagnetic Fields*, Roald K. Wangsness

Most upper division E&M texts have a rudimentary discussion of magnetism. This is often a good place to start. I have included several sections from *Magnetism Principles and Applications* by D. Craik. This is a more advanced book that gives a very thorough treatise of magnetism. You are **NOT** expected to understand all of the material that is presented. It is there for those of you who may wish to understand the topic at a deeper level. Another excellent reference is *Introduction to Solid State Physics* by Charles Kittel.