Millikan Oil Drop Experiment

Physics 401, Spring 2014

Eugene V. Colla

Agenda

1. **Measuring of the charge of electron.**
2. **Robert Millikan and his oil drop experiment**
3. **Theory of the experiment**
4. **Laboratory setup**
5. **Data analysis**

Measuring of the charge of the electron

1. **Oil drop experiment. Robert A. Millikan.. (1909). e=1.5924(17)×10^{-19} C**
2. **Shot noise experiment. First proposed by Walter H. Schottky**
3. **In terms of the Avogadro constant and Faraday constant $e = \frac{F}{N_A}$; F- Faraday constant, N_A - Avagadro constant. Best uncertainty ~1.6 ppm.**
4. **From Josephson ($K_J = \frac{2e}{h}$) and von Klitzing $\left(R_K = \frac{h}{e^2}\right)$ constants**
5. **Recommended by NIST value 1.602 176 565(35) 10^{-19} C**

Robert Millikan. Oil drop experiment

ROBERT ANDREWS MILLIKAN
1868-1953

The Nobel Prize in Physics 1923.
Robert A. Millikan "for his work on the elementary charge of electricity and on the photoelectric effect".

Vol. II. No. 2.] *ELECTRICAL CHARGE AND AVOGADRO CONSTANT.* 109

ON THE ELEMENTARY ELECTRICAL CHARGE AND THE AVOGADRO CONSTANT.

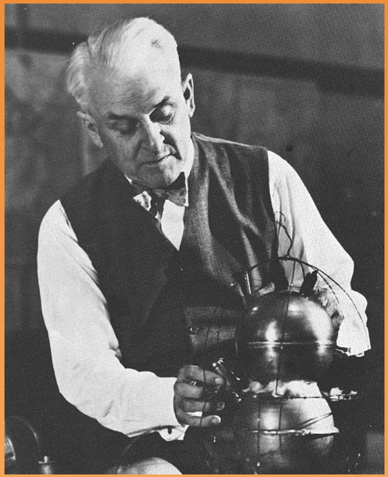
By R. A. Millikan.

1. Introductory.

University of Chicago

22nd of March, 1868, Morrison, Ill

Robert Millikan. Oil drop experiment



ROBERT ANDREWS MILLIKAN
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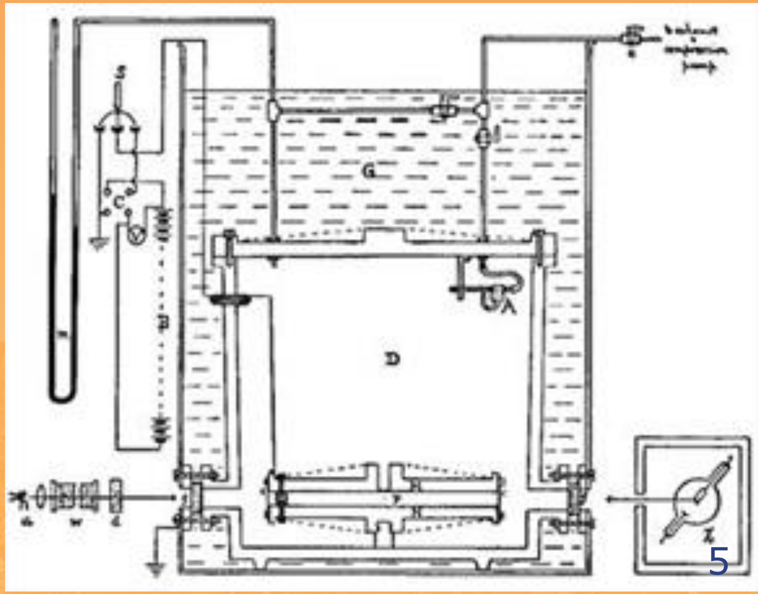


Diagram and picture of apparatus

Oil drop experiment.

Motivation:

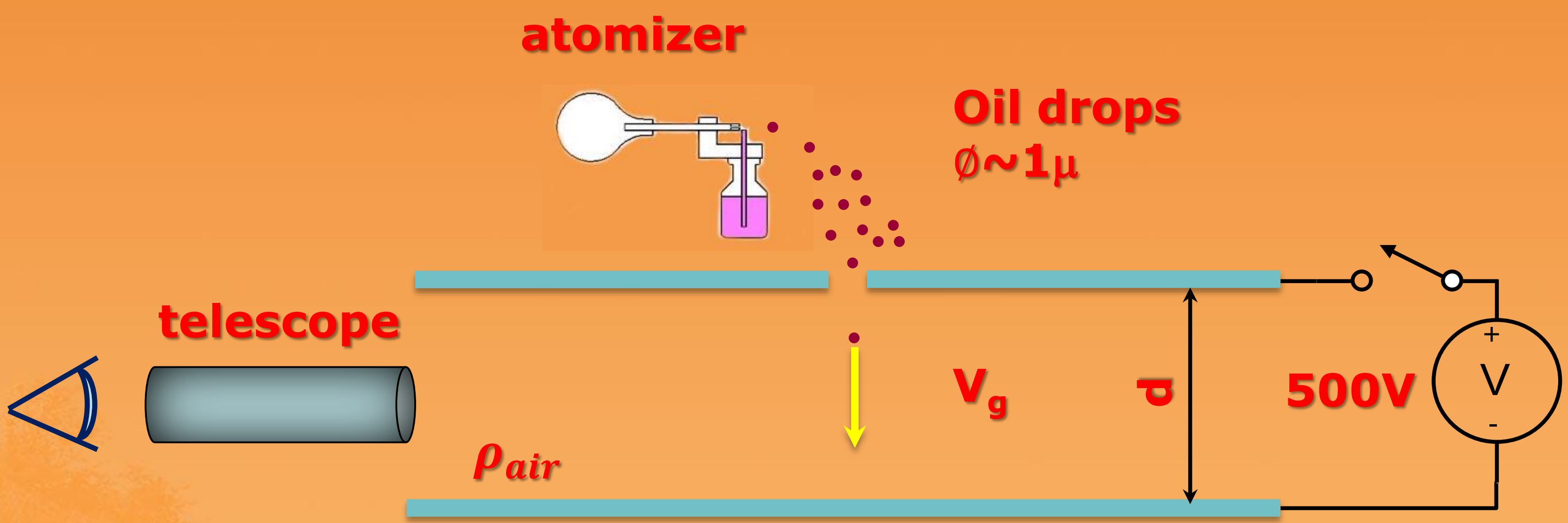
Measurement of the magnitude of the electron charge!

Demonstrate that the electron charge is quantized!

Measure the charge of an electron to ±3%

Picture of the PASCO setup

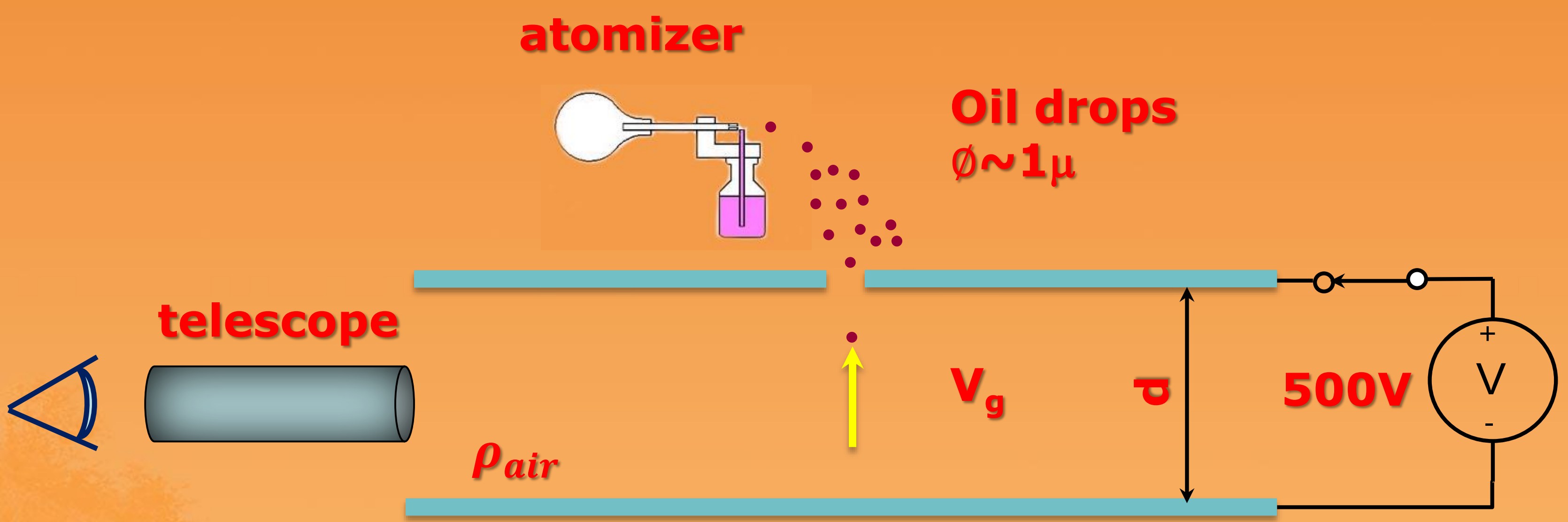
Oil drop experiment.



Forces on the oil drop:

1) Gravity + buoyant force (air displaced by oil drop)
2) Drag force of the oil drop in the air

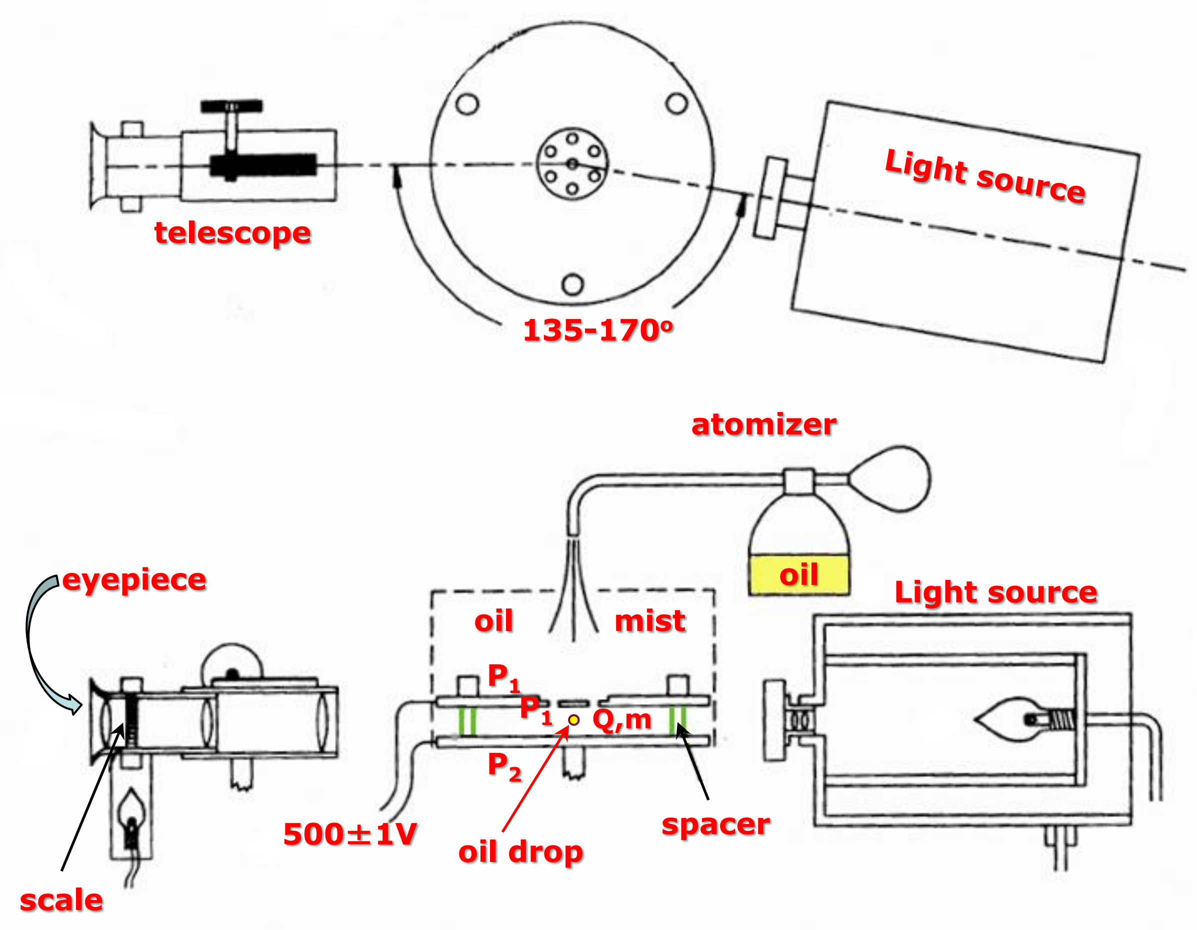
Oil drop experiment.



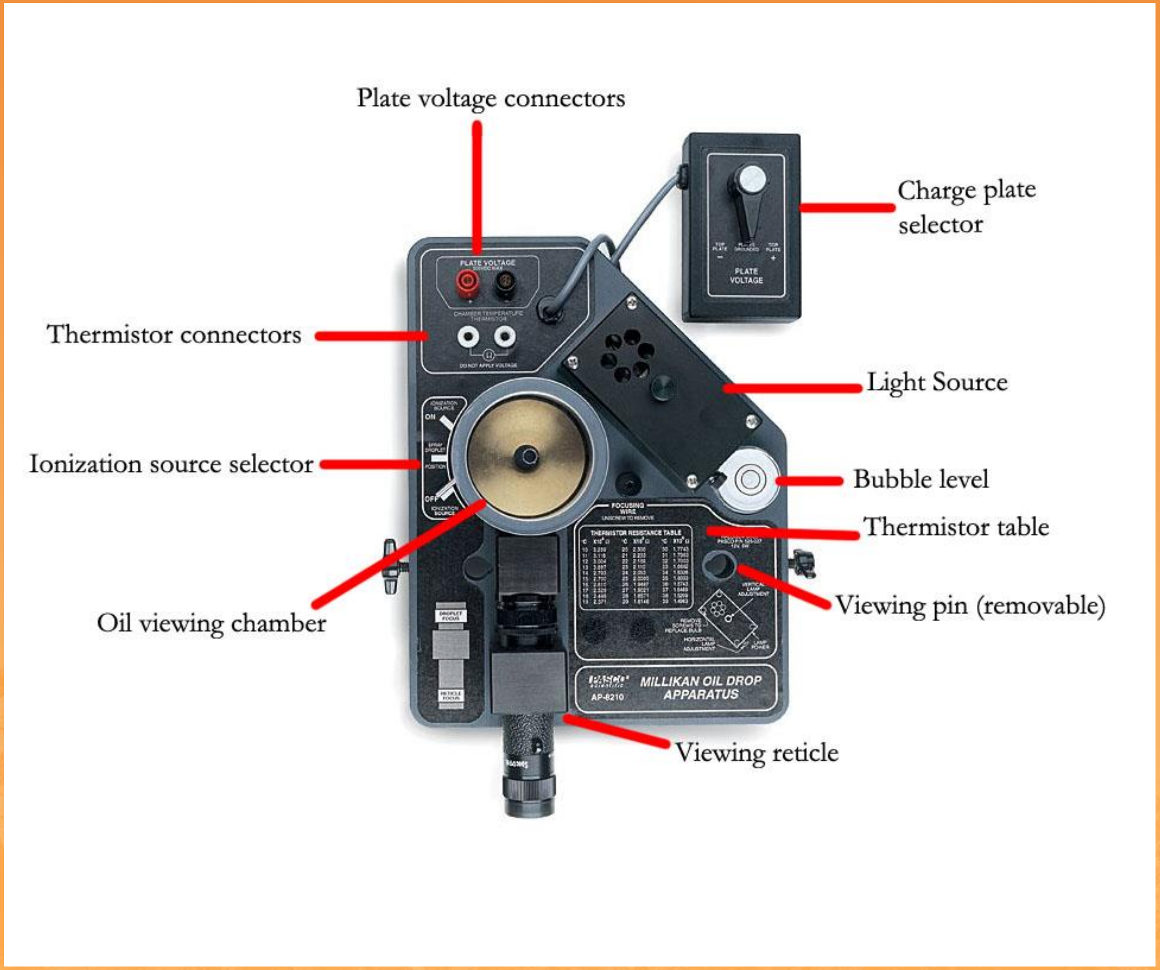
Forces on the oil drop:

1) Gravity + buoyant force (air displaced by oil drop)
2) Drag force of the oil drop in the air
3) Electric force on oil drops which carry charge Q

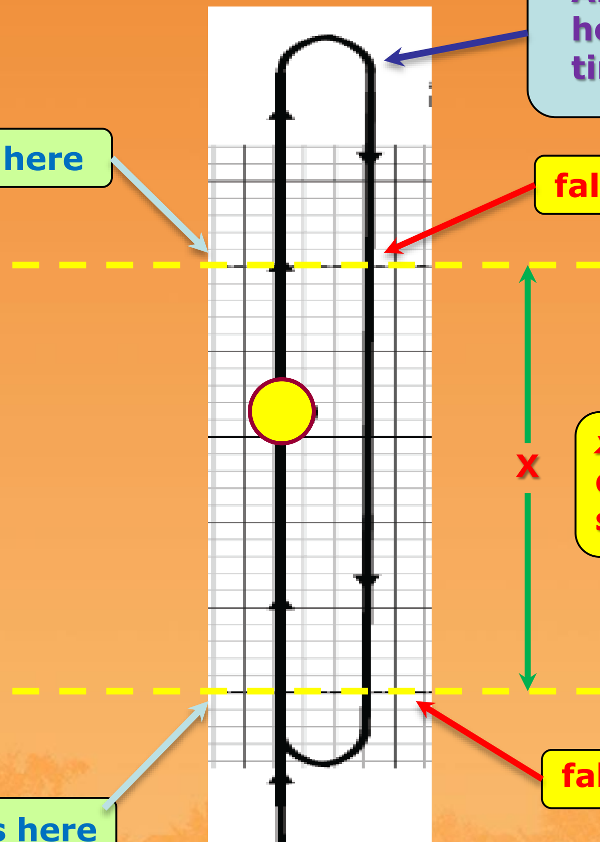
Apparatus. Schematic Layout



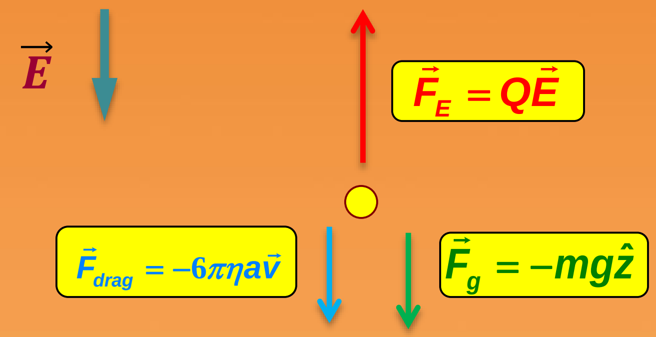
Apparatus: actual setup



What is measured



Balance of Forces: Newton's Law



a : radius of drop
ρ : density $\rho = \rho_{oil} - \rho_{air}$
v : velocity of oil drop
Q : charge of oil drop
E : electric field $E=V/d$
V : Voltage across plates
η : viscosity of air
g : gravitational const.

$$\vec{F}_g = -mg\hat{z} \quad (1)$$

$$\vec{F}_{drag} = -6\pi\eta a\vec{v} \quad (2)$$

$$\vec{F}_E = Q\vec{E} \quad (3)$$

$$\vec{F} = m\frac{d\vec{v}}{dt} = \vec{F}_g + \vec{F}_{drag} + \vec{F}_E$$

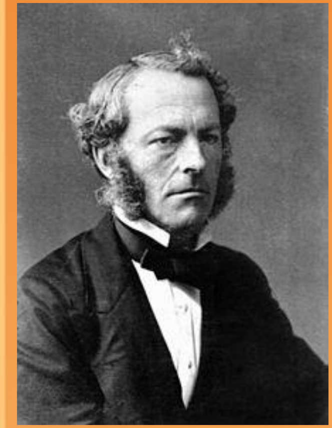
Forces on the oil drop:

(1) Gravity + buoyant force (air displaced by oil drop)
(2) Drag force of the oil drop in the air
(3) Electric force on oil drops which carry charge Q

Particle reached terminal velocity $\frac{d\vec{v}}{dt} = 0$

$$\vec{F}_g + \vec{F}_{drag} + \vec{F}_E = 0$$

Modification to Stokes Law



George Gabriel Stokes
(1819-1903)

$$\vec{F}_{drag} = -6\pi\eta a\vec{v}$$

For small particle radius (a<15μ) Stokes law need to be corrected. This correction was derived by E. Cunningham.

Ebenezer Cunningham
(1881-1977)

$$\vec{F}_{drag} = -6\pi\eta \frac{a}{f_c}\vec{v}$$

$$f_c = 1 + A\frac{\lambda}{a} + B\frac{\lambda}{a}e^{-c\frac{a}{\lambda}} \;,\; A = 1.246,\ B = 0.42,\ C = 0.78$$

$$f_c \approx 1 + A\frac{\lambda}{a} = 1 + \frac{r_c}{a} \approx 1.1 \;,\; \text{for } a \approx 10^{-6}\text{m} \;,\; r_c = \frac{6.18\times10^{-5}}{p[\text{mmHg}]}$$

Here **a** – particle radius; **λ** – mean free path of the gas molecules

negligible term

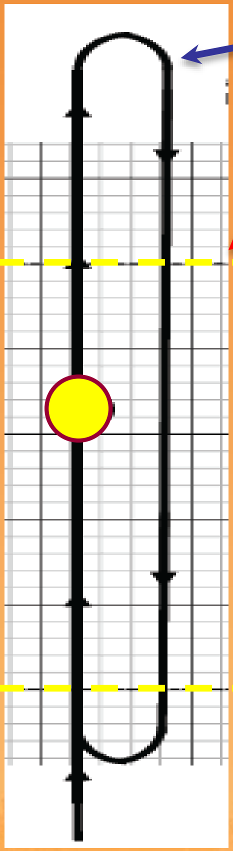
$$\lambda\,[\text{m}] = 6.53\times10^{-8}\frac{760}{p[\text{mmHg}]}$$

We measure: t_g and t_{rise}

Allow drop to "undershoot" here before starting next rise time experiment

rise time measurement stops here

fall time measurement starts here



fall time t_g

rise time t_{rise}

x

x = fall distance = rise distance. x must be the same for all drops!

rise time measurement starts here

fall time measurement stops here

Solving Newton's law: $Q(t_g, t_{rise})$

f_c can be found from Newton law equation in the case of V=0 (falling drop)

$$\vec{F}_g + \vec{F}_{drag} = \frac{4}{3}a^3\rho g - 6\pi\eta\frac{a}{f_c}\vec{v} = 0$$

(see write-up)

$$\frac{1}{f_c^{\frac{2}{3}}} \approx 1 - \left(\frac{t_g}{\tau_g}\right)^{\frac{1}{2}} \;;\quad \tau_g = \frac{2\eta x}{\rho g r_c^2};\quad r_c[m] = \frac{6.18\times 10^{-5}}{p[mmHg]}$$

Solving Newton's law: $Q(t_g, t_{rise})$

$$Q = n \times e = \frac{1}{f_c^{3/2}} \frac{9\pi d}{V} \sqrt{\frac{2\eta^3 x^3}{g\rho}} \sqrt{\frac{1}{t_g}} \left[\frac{1}{t_g} + \frac{1}{t_{rise}} \right]$$

- Q : charge of oil drop
- n : number of unpaired electrons in drop
- e : elementary charge
- d : plate separation
- V : Voltage across plates

- ρ : density $\rho = \rho_{oil} - \rho_{air}$
- η : viscosity of air
- g : gravitational constant
- x : drift distance for oil drop
- t_g : fall time
- t_{rise} : rise time

Route of charge calculation Q(t_g, t_{rise}).

$$\frac{1}{f_c^{\frac{3}{2}}} \approx 1 - \left(\frac{t_g}{\tau_g}\right)^{\frac{1}{2}} \; ; \quad \tau_g = \frac{2\eta x}{\rho g r_c^2}; \quad r_c[m] = \frac{6.18\times10^{-5}}{p[mmHg]}$$

$$Q = F \bullet S \bullet T = \left(\frac{1}{f_c^{3/2}}\right)\frac{9\pi d}{V}\sqrt{\frac{2\eta^3 x^3}{g\rho}}\frac{1}{\sqrt{t_g}}\left(\frac{1}{t_g}+\frac{1}{t_{rise}}\right)$$

$$F = \frac{1}{f_c^{3/2}} \approx 1 - \left(\frac{t_g}{\tau_g}\right)^{\frac{1}{2}}$$

$$S = \frac{9\pi d}{V}\sqrt{\frac{2\eta^3 x^3}{g\rho}}$$

$$T = \frac{1}{\sqrt{t_g}}\left(\frac{1}{t_g}+\frac{1}{t_{rise}}\right)$$

Route of charge calculation. Origin projects. Data collecting.

Projects ***Section L1.opj … Section L4.opg***

Locations:

\\engr-file-03\PHYINST\APL Courses\PHYCS401\Common\Origin templates\Oil drop experiment

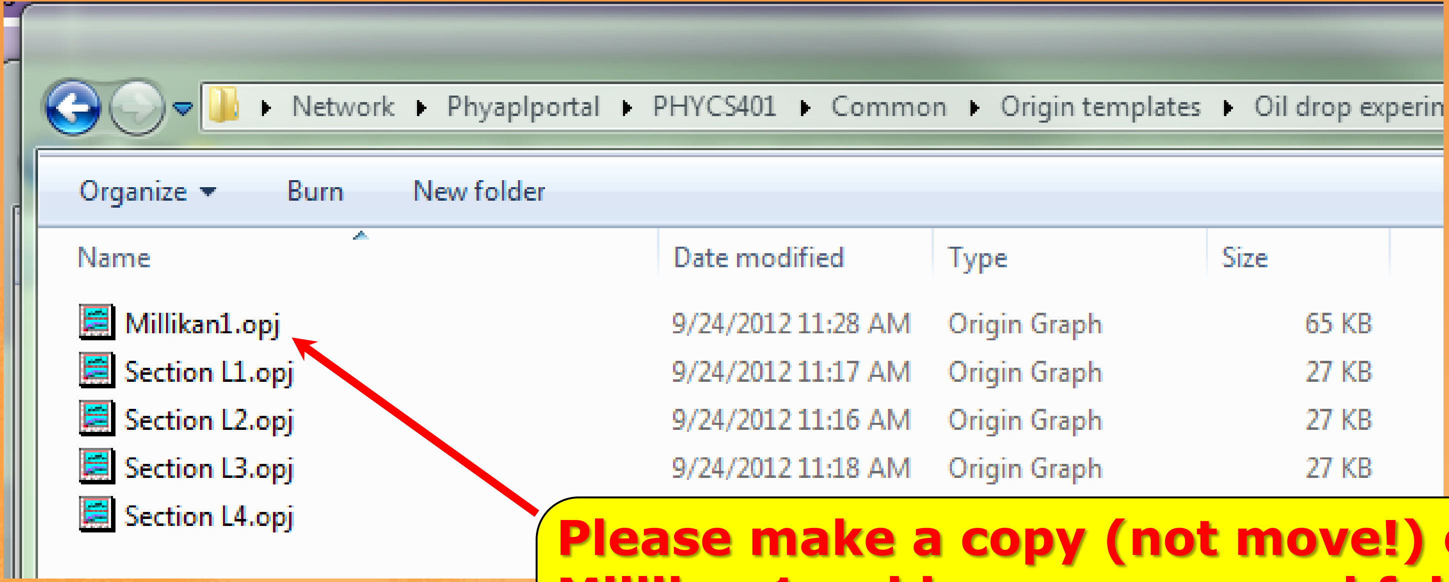
\\engr-file-03\PHYINST\APL Courses\PHYCS401\Students\1.Millikan Oil Drop experiment

	A(L)	B(Y)	C(Y)	D(Y)	E(Y)	F(Y)	G(Y)	H(Y)	
Long Name	parameter label	Par	tg	tr	parameter label	Par	tg	tr	pa
Units									
Comments	*student1, student2*	*student1, student2*	*student1, student2*	*student1, student2*	*student3, student4*	*student3, student4*	*student3, student4*	*student3, student4*	*s*
1	η				η				η
2	Δη/ΔT				Δη/ΔT				Δη
3	ρ1				ρ1				ρ1
4	ρ2				ρ2				ρ2
5	ρ1−ρ2				ρ1−ρ2				ρ1
6	g				g				g
7	p				p				p
8	x				x				x
9	d				d				d
10	V				V				V
11	Ta				Ta				Ta
12									
13									
14									
15									
16									
17									

Route of charge calculation. Origin projects. Data analysis.

Project: ***Millikan1.opj***

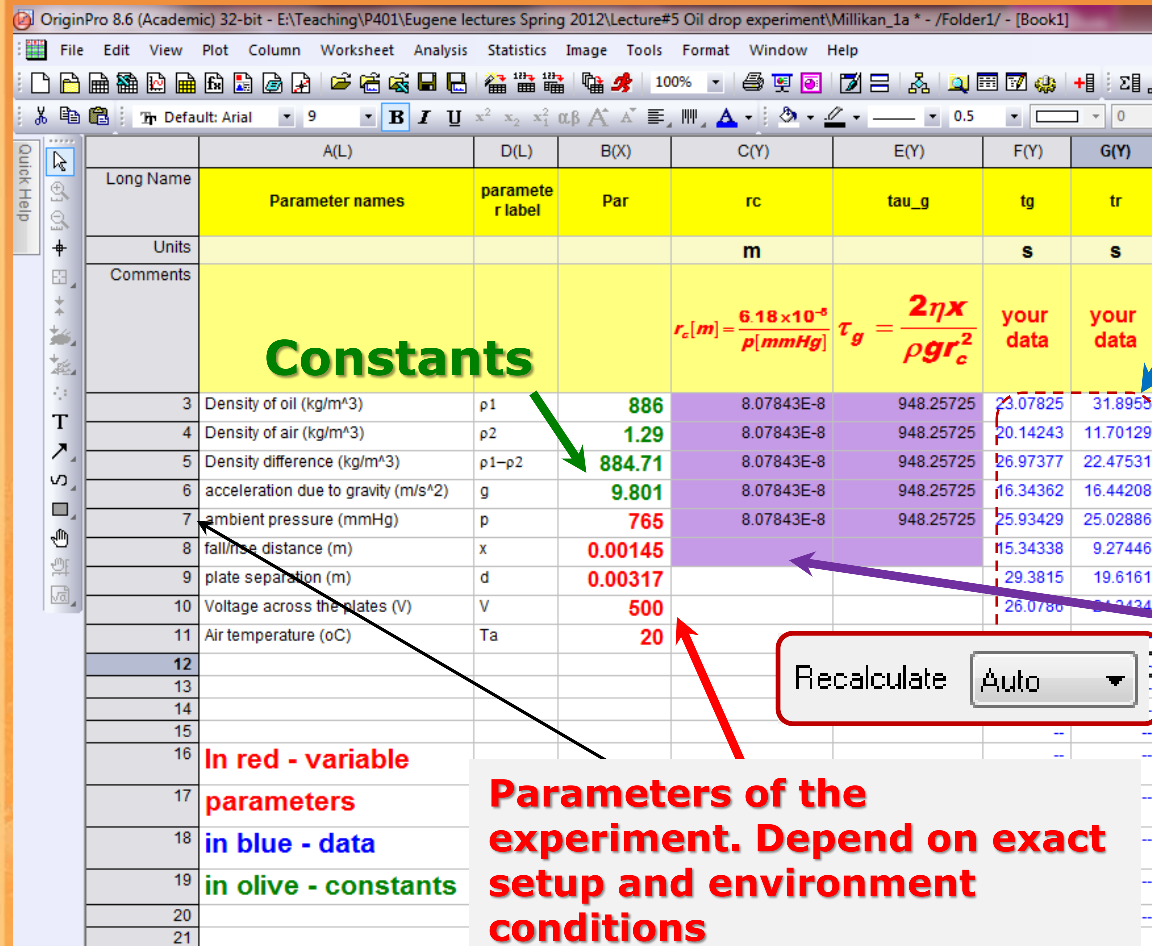
Locations: **\\Phyaplportal\PHYCS401\Common\Origin templates\Oil drop experiment**
\\Phyaplportal\PHYCS401\Students\1. Millikan Oil Drop experiment



Please make a copy (not move!) of Millikan1.opj in your personal folder and start to work with your personal copy of the project

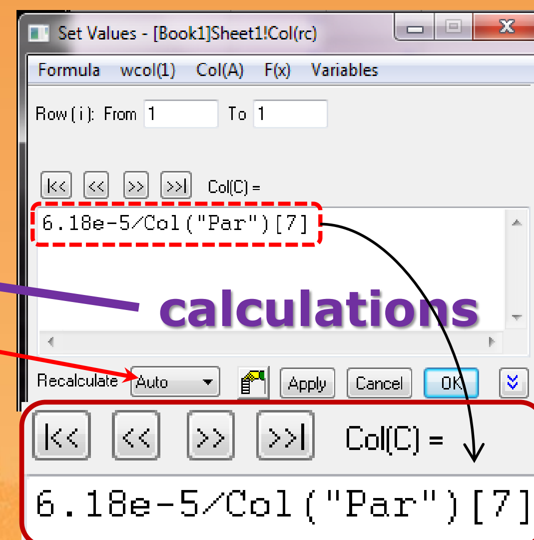
Route of charge calculation. Origin project. Data analysis.

Project *Millikan1.opj*



	A(L)	D(L)	B(X)	C(Y)	E(Y)	F(Y)	G(Y)
Long Name	Parameter names	parameter label	Par	rc	tau_g	tg	tr
Units				m		s	s
Comments				$r_c[m]=\frac{6.18\times10^{-5}}{p[mmHg]}$	$\tau_g=\frac{2\eta x}{\rho g r_c^2}$	your data	your data
3	Density of oil (kg/m^3)	ρ1	886	8.07843E-8	948.25725	23.07825	31.8955
4	Density of air (kg/m^3)	ρ2	1.29	8.07843E-8	948.25725	20.14243	11.70129
5	Density difference (kg/m^3)	ρ1−ρ2	884.71	8.07843E-8	948.25725	26.97377	22.47531
6	acceleration due to gravity (m/s^2)	g	9.801	8.07843E-8	948.25725	16.34362	16.44208
7	ambient pressure (mmHg)	p	765	8.07843E-8	948.25725	25.93429	25.02886
8	fall/rise distance (m)	x	0.00145			15.34338	9.27446
9	plate separation (m)	d	0.00317			29.3815	19.6161
10	Voltage across the plates (V)	V	500			26.0786	[illegible]
11	Air temperature (oC)	Ta	20				

Plugin your data here



calculations

Parameters of the experiment. Depend on exact setup and environment conditions

$$r_c[m]=\frac{6.18\times10^{-5}}{p[mmHg]}$$

P(mmHg)→Col("Par")[7]

index

Route of charge calculation. Origin project. Data analysis.

Project *Millikan1.opj*

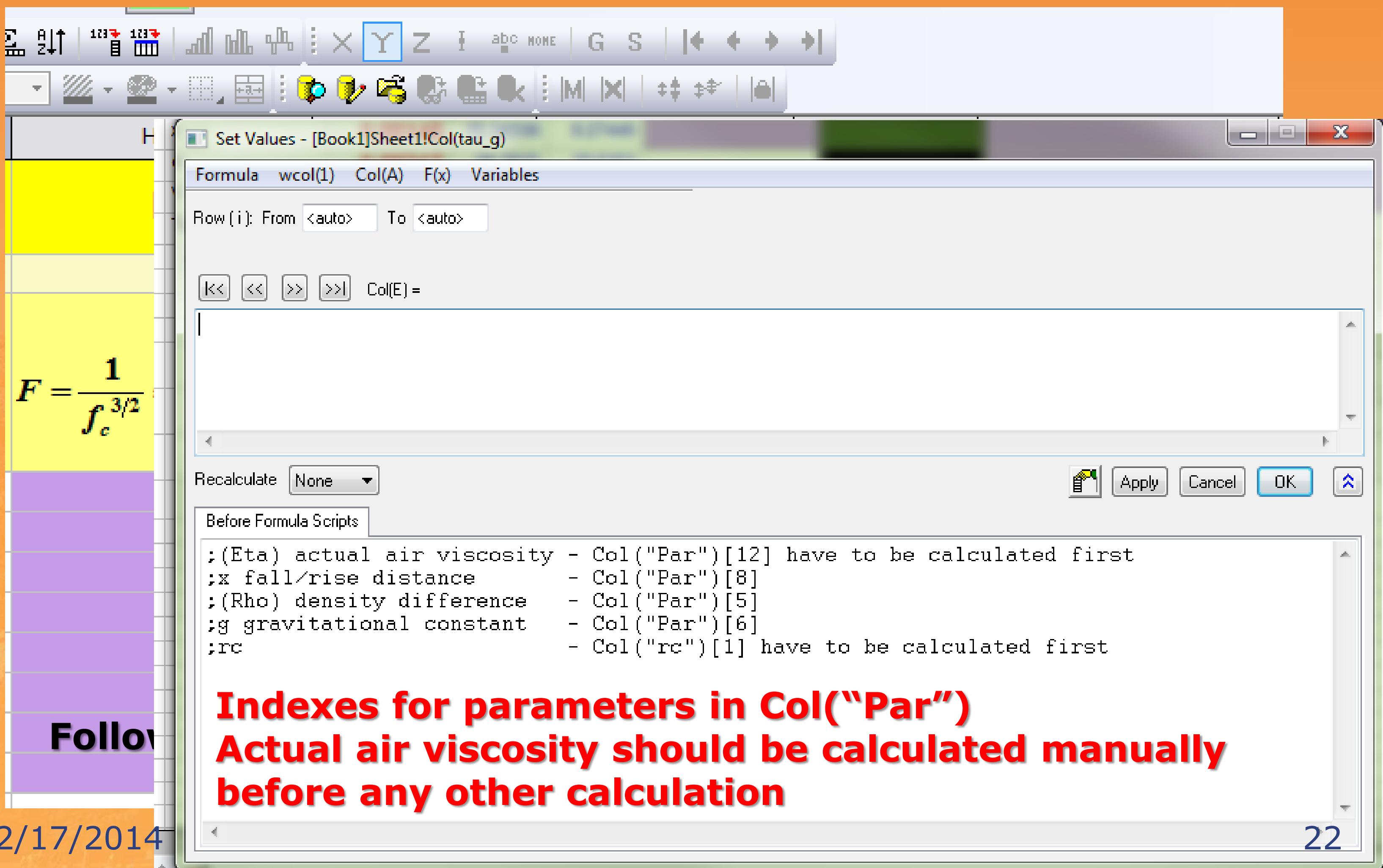
H(Y)	I(Y)	J(Y)	K(Y)	L(Y)
F	S	T	Q=FST	n=Q/1.602e-19
$F = \frac{1}{f_c^{3/2}} \approx 1 - \left(\frac{t_g}{\tau_g}\right)^{\frac{1}{2}}$	$S = \frac{9\pi d}{V}\sqrt{\frac{2\eta^3 x^3}{g\rho}}$	$T = \frac{1}{\sqrt{t_g}}\left(\frac{1}{t_g} + \frac{1}{t_{rise}}\right)$	Q=F*S*T	number of elementary charges

$$Q = F \bullet S \bullet T = \left(\frac{1}{f_c^{3/2}}\right)\frac{9\pi d}{V}\sqrt{\frac{2\eta^3 x^3}{g\rho}}\,\frac{1}{\sqrt{t_g}}\left(\frac{1}{t_g} + \frac{1}{t_{rise}}\right)$$

Follow correct order of calculations: $r_c \rightarrow \tau_g \rightarrow (F,S,T) \rightarrow Q \rightarrow n$

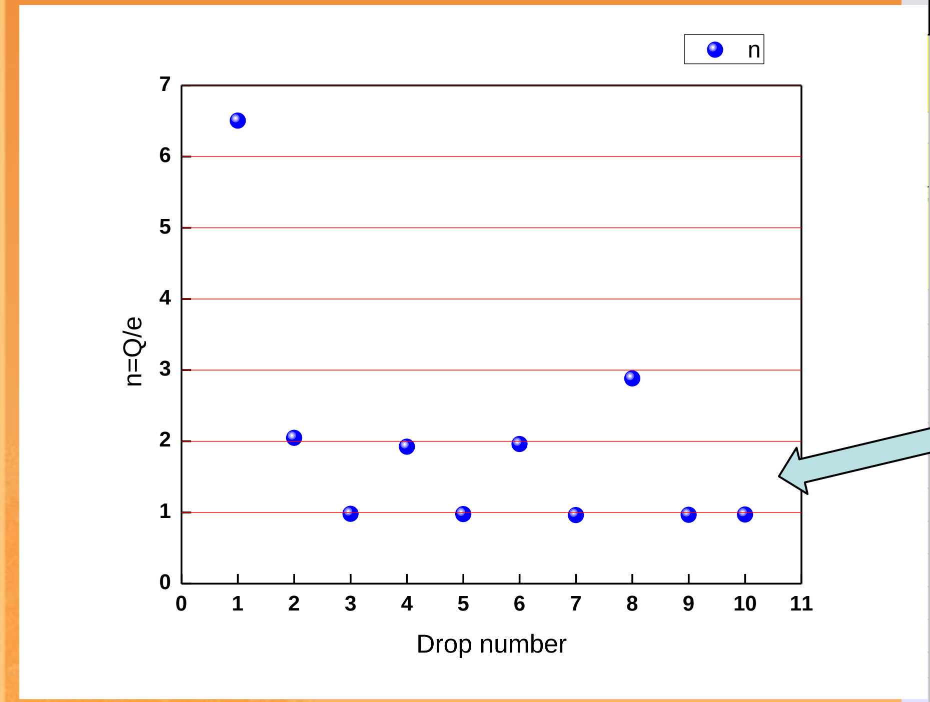
Route of charge calculation. Origin project. Data analysis.

Project *Millikan1.opj*



Indexes for parameters in Col("Par")
Actual air viscosity should be calculated manually before any other calculation

Charge calculation. Origin project.



I(Y)	J(Y)	K(Y)	L(Y)
S	T	Q=FST	n=Q/1.602e-19
			c
$S=\frac{9\pi d}{V}\sqrt{\frac{2\eta^3x^3}{g\rho}}$	$T=\frac{1}{\sqrt{t_g}}\left(\frac{1}{t_g}+\frac{1}{t_{rise}}\right)$	Q=F*S*T	number of elementary charges
--		1.04196E-[illegible]	6.50414
--		[illegible]945E-19	2.0471
--		1.57231E-19	0.98147
		3.08168E-19	1.92365
--		1.56458E-19	0.97664
--		3.14167E-19	1.9611
--		1.54196E-19	0.96252
--		4.61924E-19	2.88342
--		1.54861E-19	0.96668
--		1.55468E-19	0.97046
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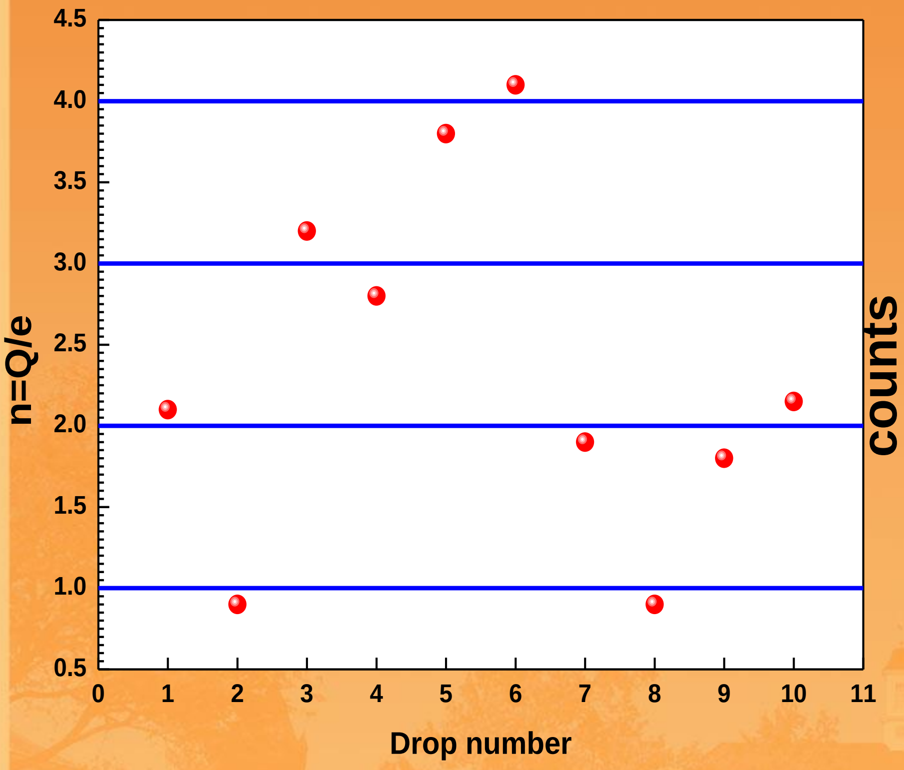
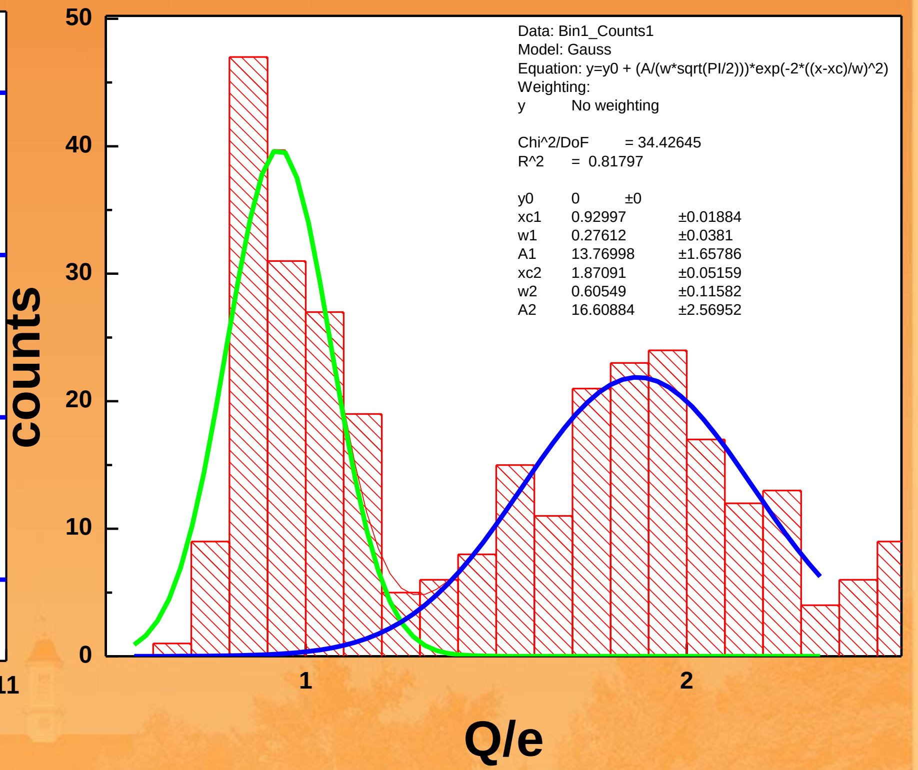
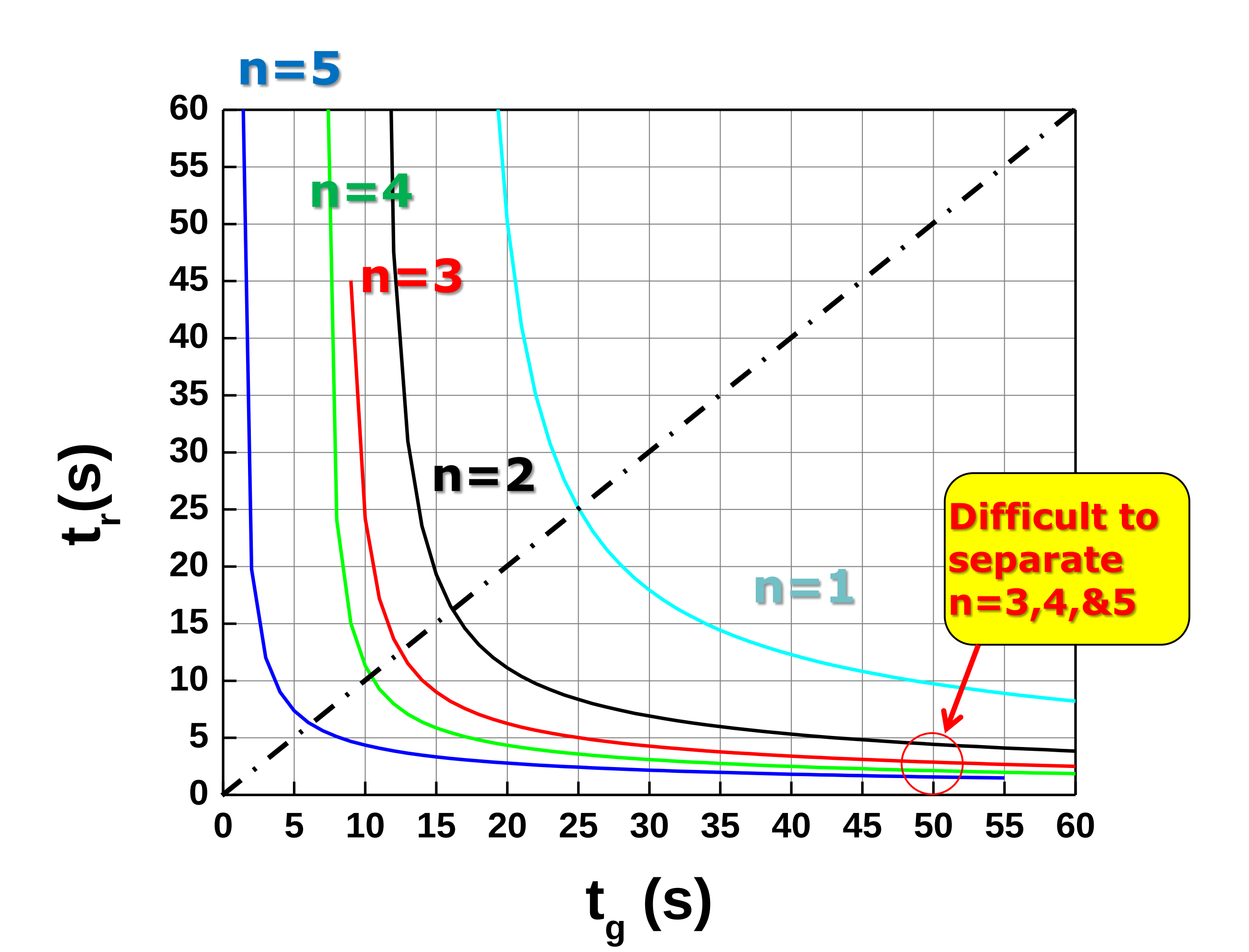
Expected results

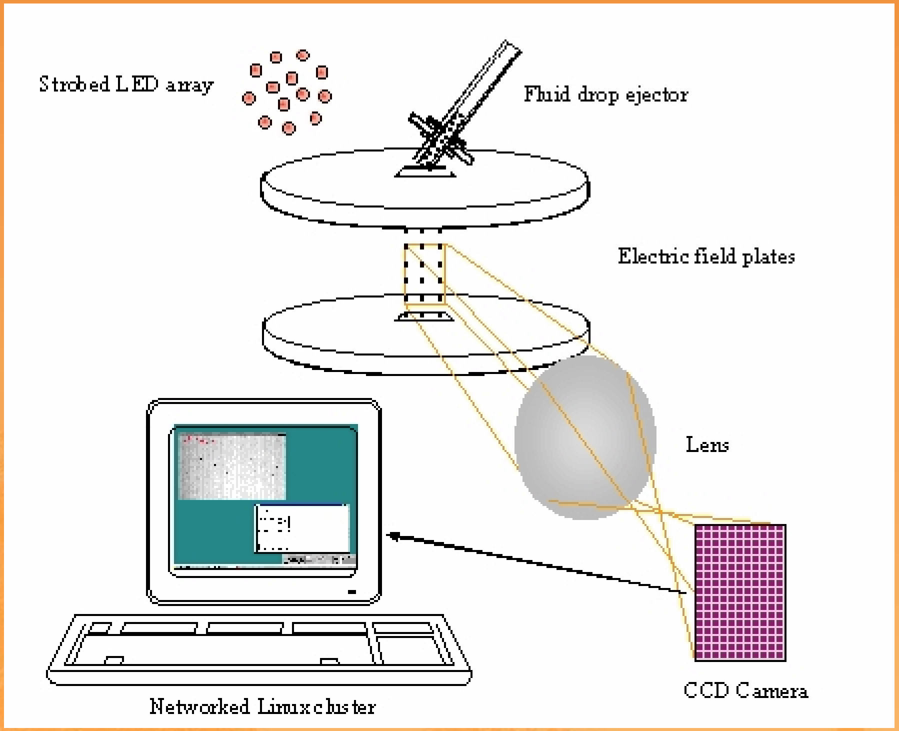




Choice of Oil Drops for the Analysis: rise and fall times



Modern experiments at SLAC



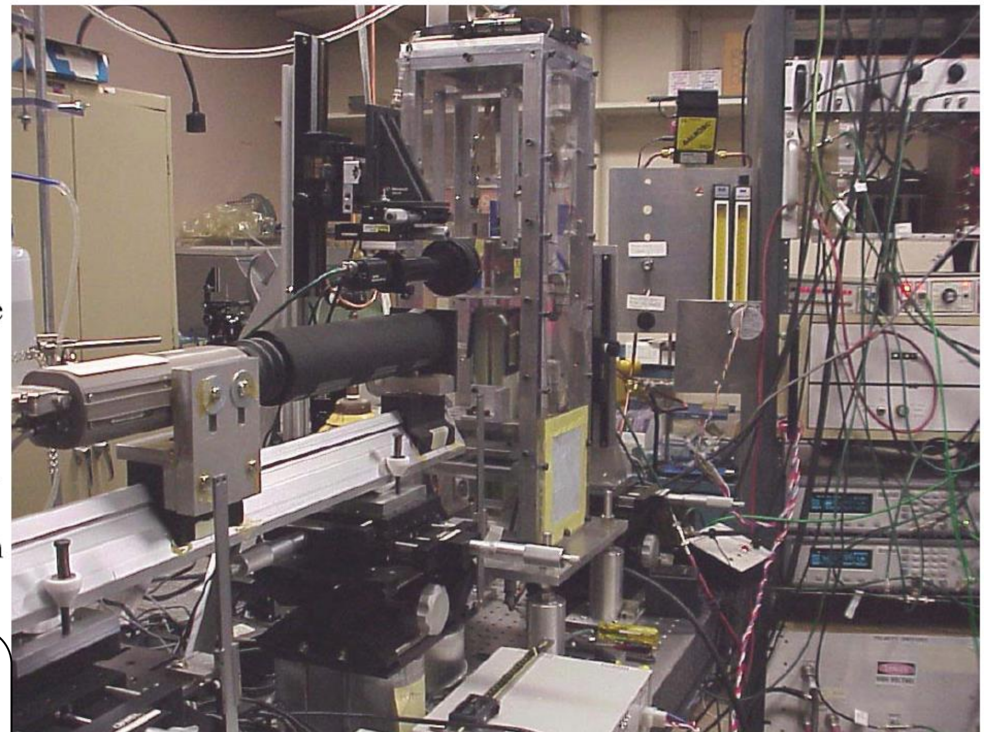
- Drop generation rate 1 Hz
- Fluid - Dow Corning silicon oil
- Number of drops - 17 million
- Mass - 70.1 milligrams
- Duration - 8 months

Modern experiments at SLAC

NATIONAL ACCELERATOR LABORATORY

Machine vision mediated auto-control of: the average charges of the drops, the fall path of the drops, the upward laminar air flow, and the electronic drive to the drop ejector are new features of this fluid drop charge measurement system.

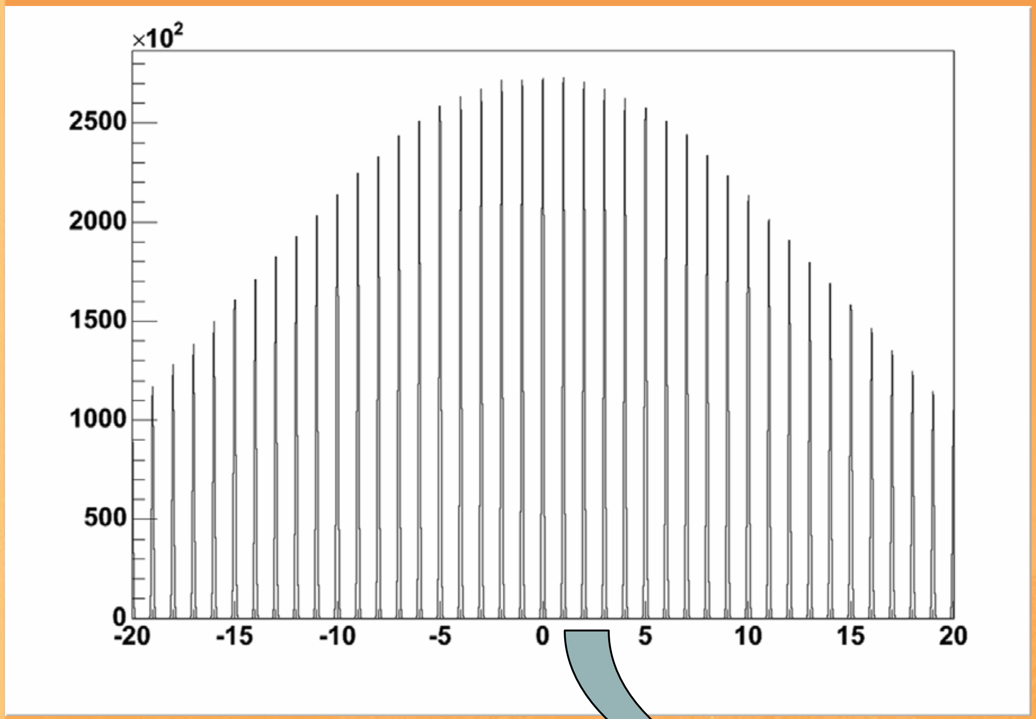
The experiment is ran from 2004 - 2007.

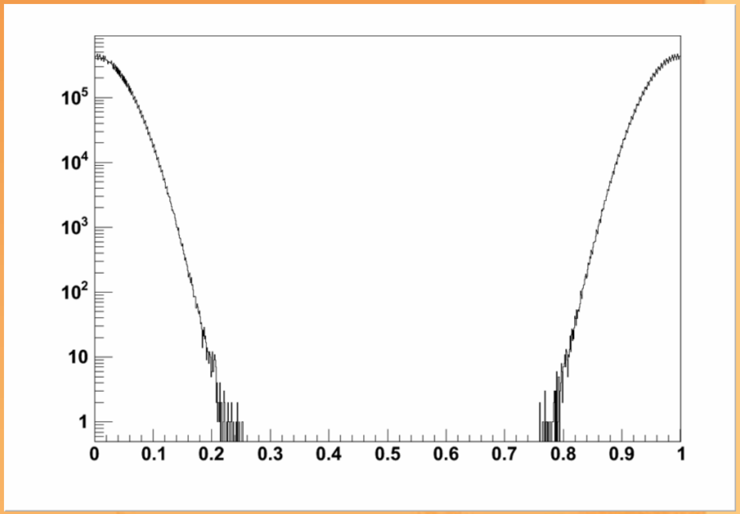


Charge measurement accuracy achieved is better than 1/24 e for drops of up to 26 microns in diameter.

Modern experiments at SLAC

"*No electric charges were measured in the range of an integer charge 1/3 e or 2/3 e.*" **(SLAC–PUB–7357 August 1996)**





Appendix #1

- **Traditional reminder:**

L1_Lab3_student name.pdf ~~***Report-exp2.pdf***~~

Please upload the files in proper folder!

This week folders:

Pulses in transmission lines_L1

Pulses in transmission lines_L3

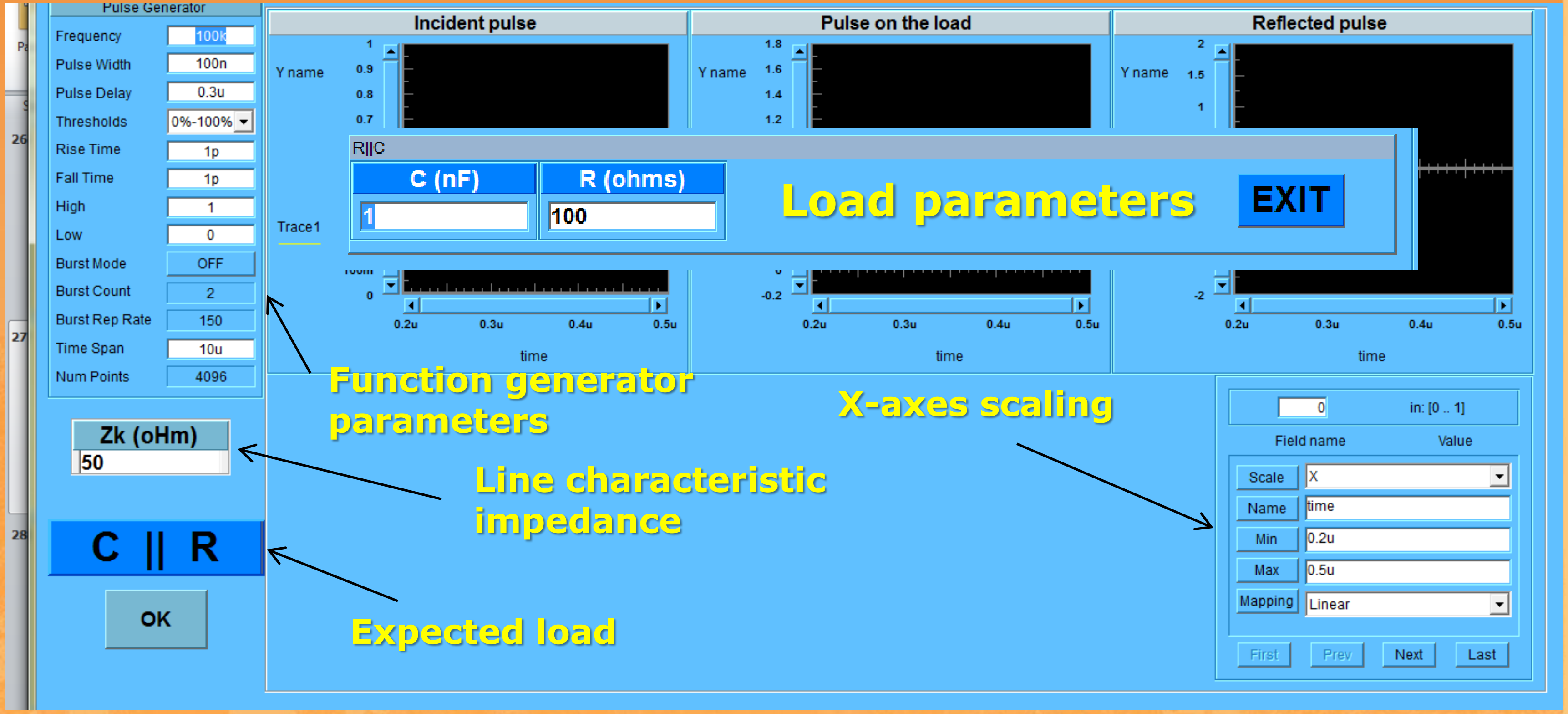
Pulses in transmission lines_L4

Pulses in transmission lines_L5

This week you have the last chance to submit "*Transients in RLC*" report

Appendix #2

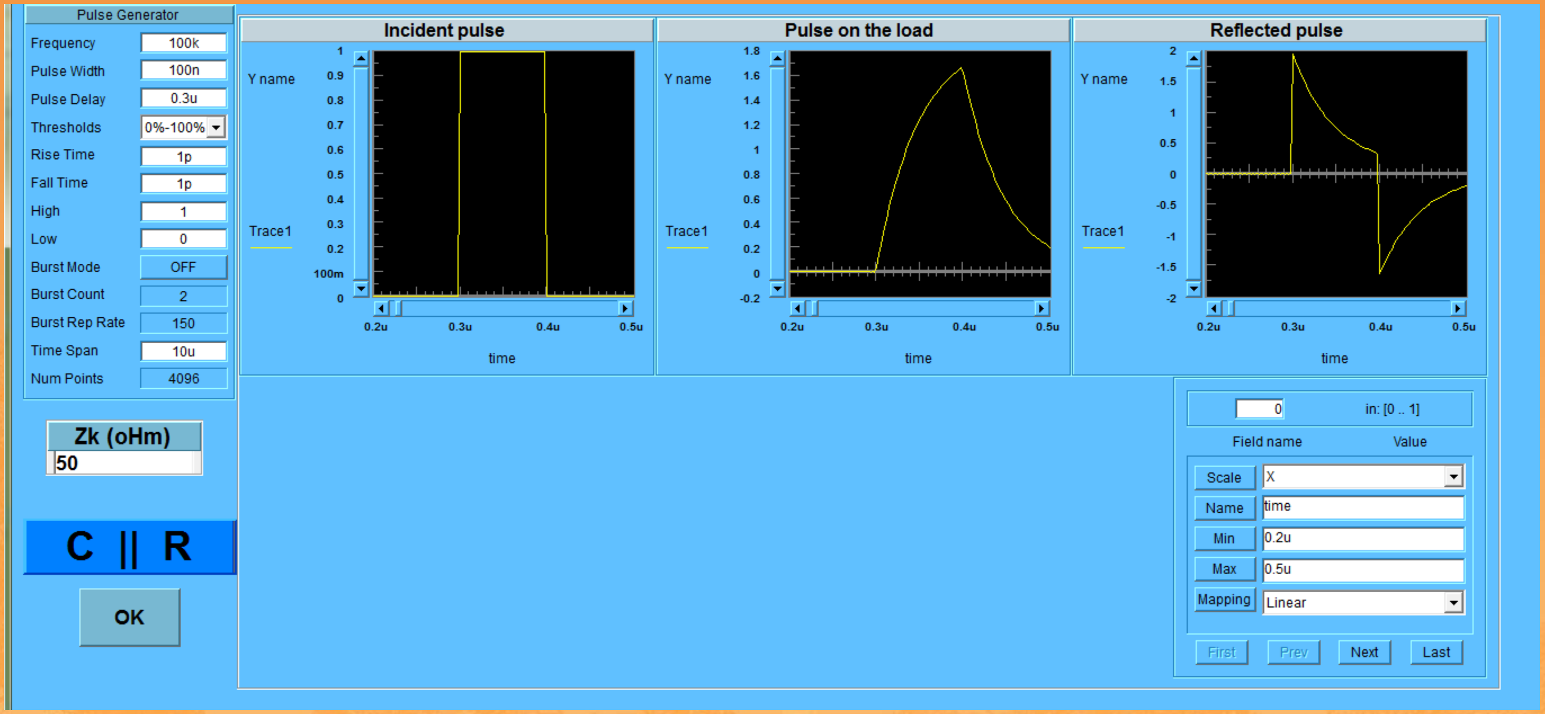
- **Transmission line. Unknown load simulation**



Location: \\Phyaplportal\PHYCS401\Common\Transmission line software

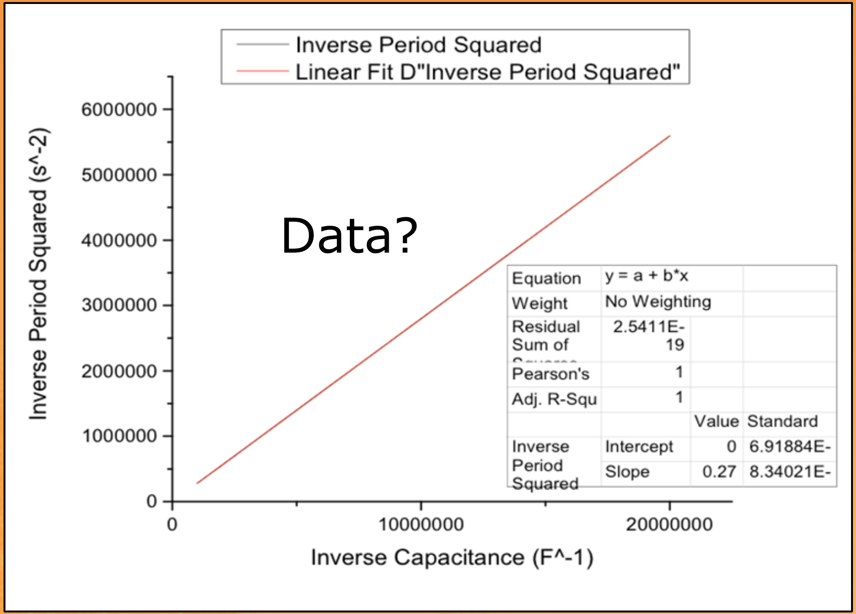
Appendix #2

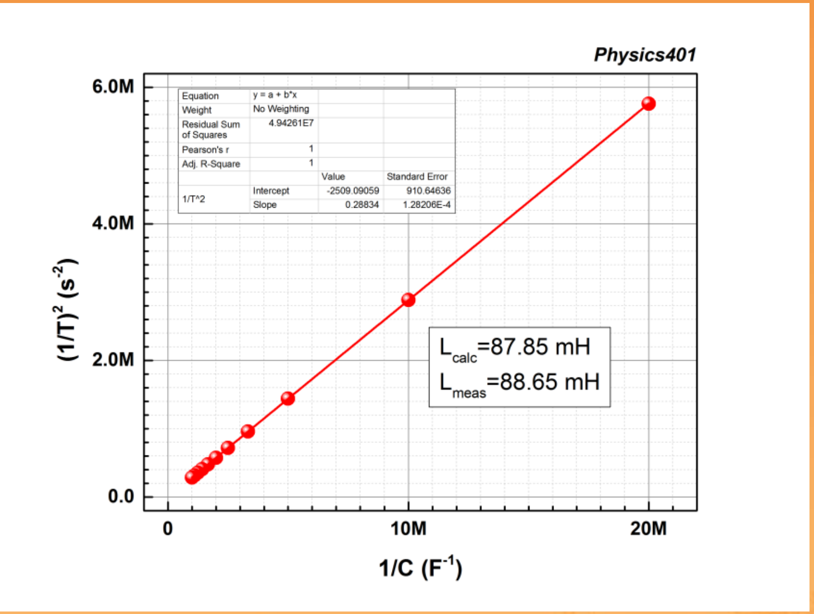
- **Transmission line. Unknown load simulation**



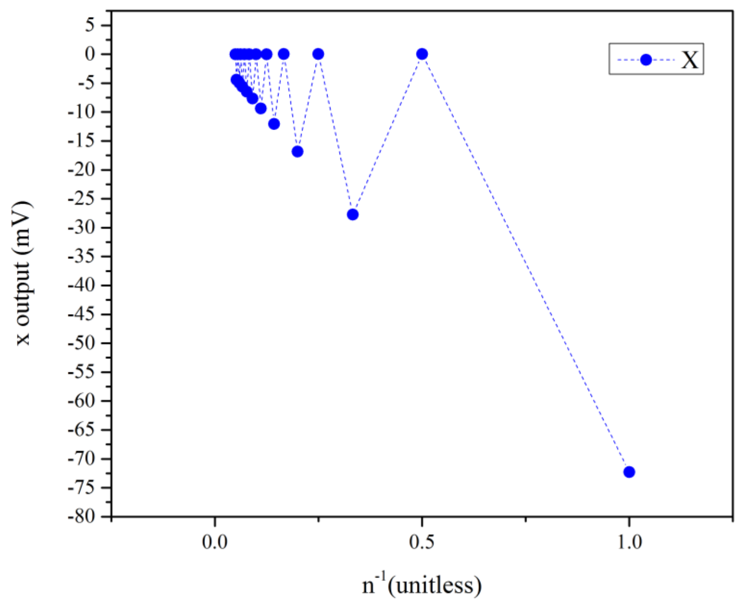
Location: **\\Phyaplportal\PHYCS401\Common\Transmission line software**

Appendix #3 Graphs





Appendix #3 Graphs



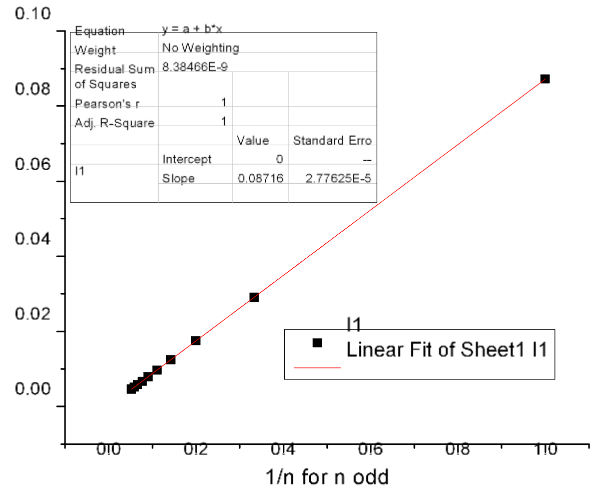


Figure 15: Odd harmonic amplitudes plotted against 1/*n*