UNIVERSITY OF ILLINOIS
AT URBANA-CHAMPAIGN

Pulses in transmission lines

Physics 401, Spring 2014

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illinois.edu

Transmission lines. Agenda.

- Definition
- Distributed parameters network
- Pulses in transmission line
- Wave equation and wave propagation
- Reflections. Resistive load
- Thévenin's theorem
- Reflection. Non resistive load
- Appendix. Error propagation

Transmission lines. Distributed parameters network.

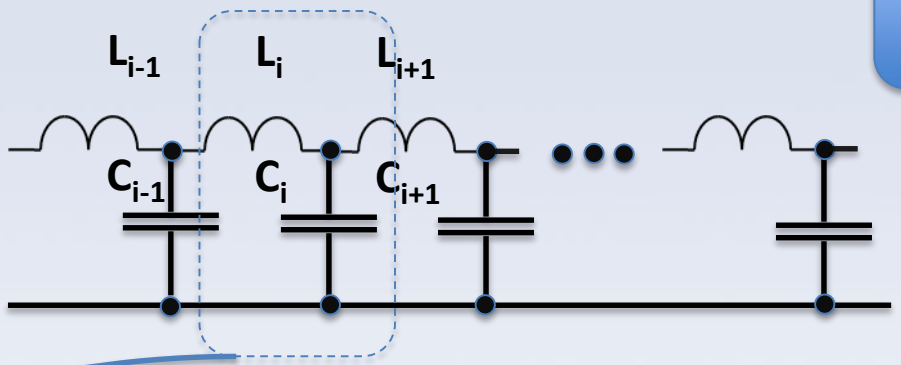
- Transmission line is a specialized cable designed to carry alternating current of radio frequency, that is, currents with a frequency high enough that its wave nature must be taken into account.

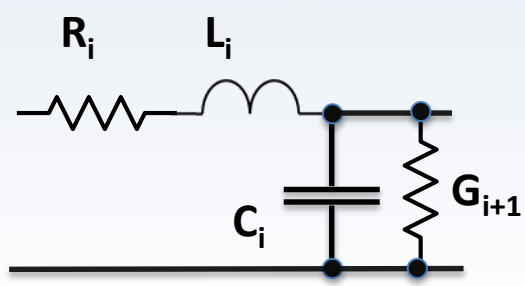
Courtesy Wikipedia

Transmission lines. Distributed parameters network

Simplified equalent circuit

Ideal case

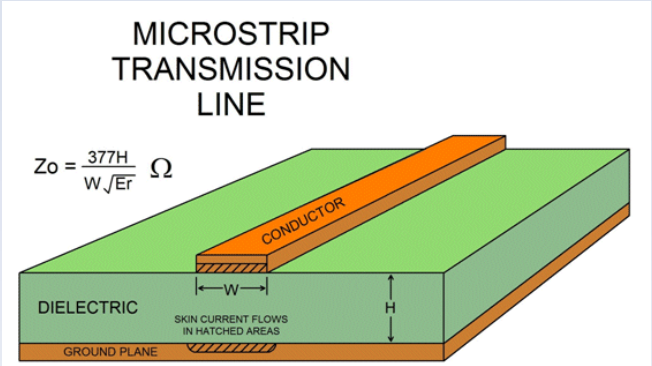




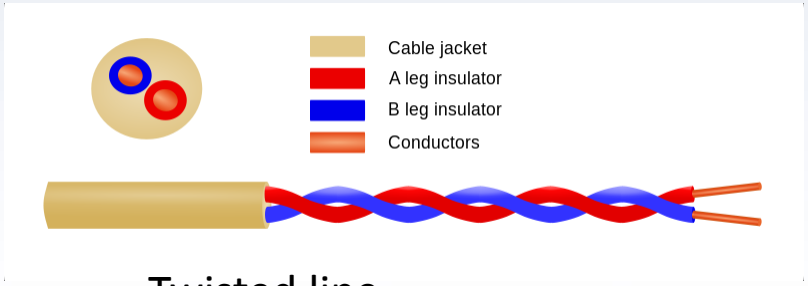
Real situation

Transmission lines. Different types.

Coaxial cable



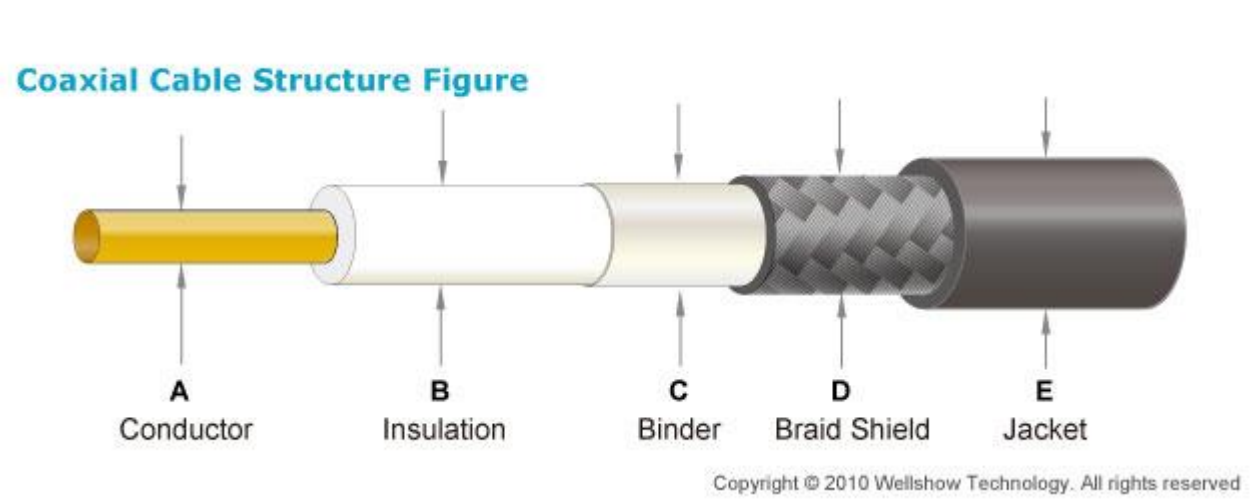
Courtesy ***Analog Devices***



Twisted line
Courtesy ***Wikipedia***

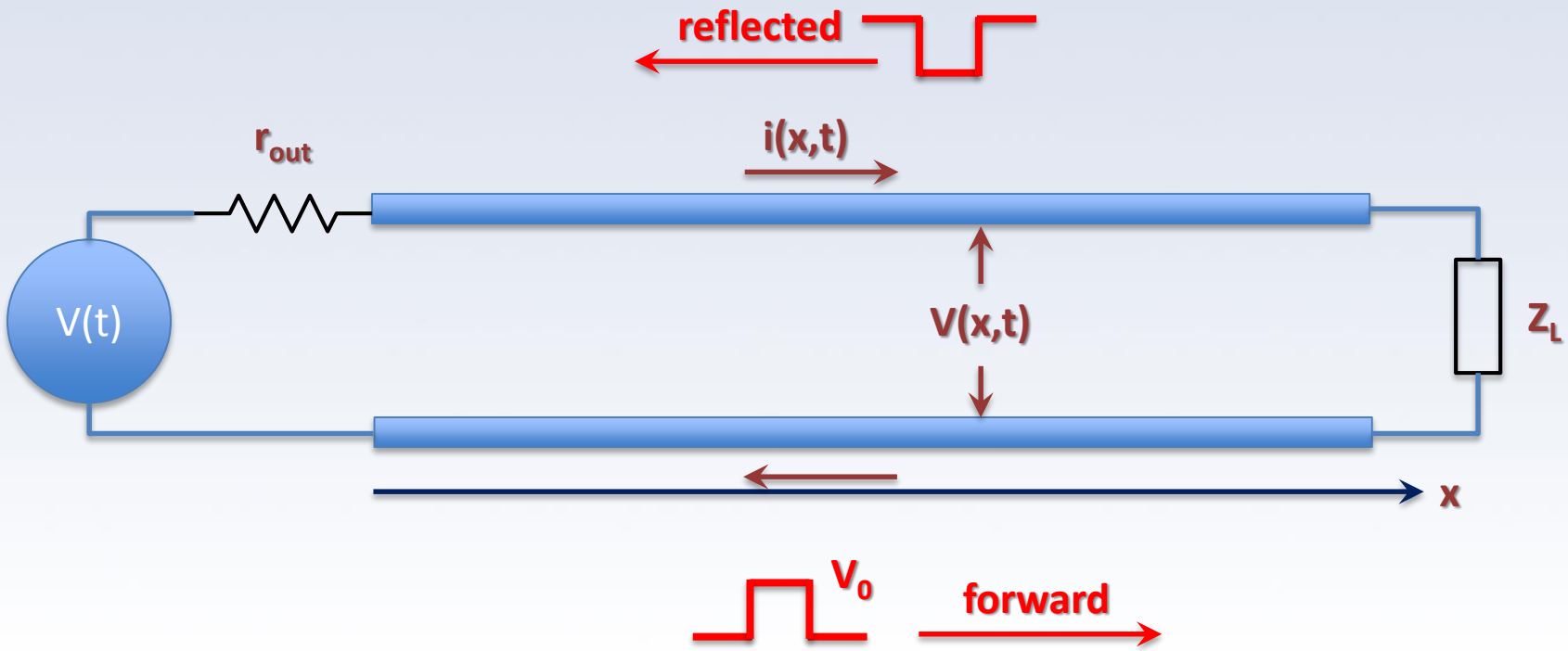
Twin lead
Courtesy ***Wikipedia***

Coaxial cable

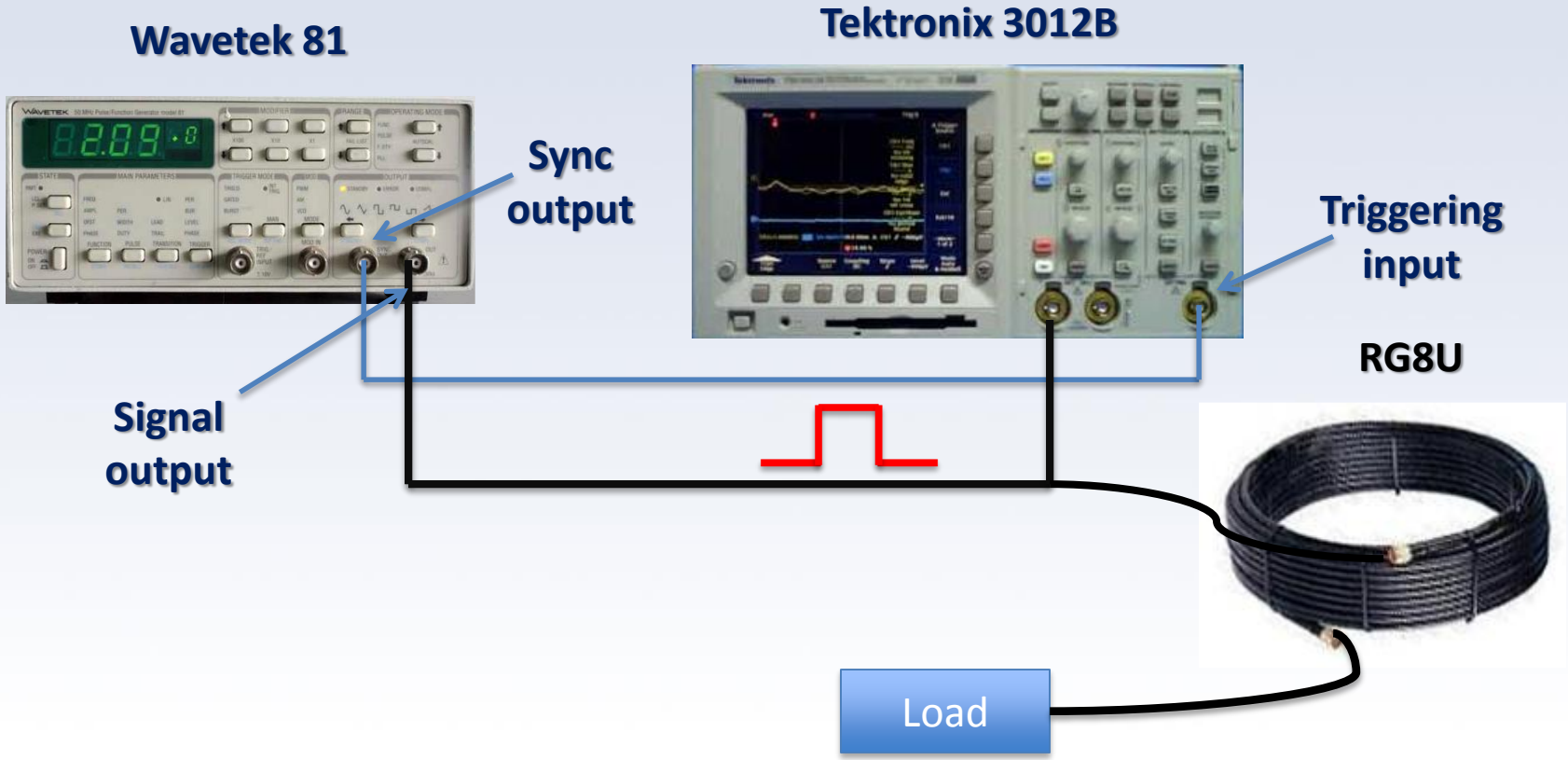


Specification:
Impedance: 53 Ω
Capacitance: 83 pF/m
Conductor: Bare Copper Wire (1/1.02mm)

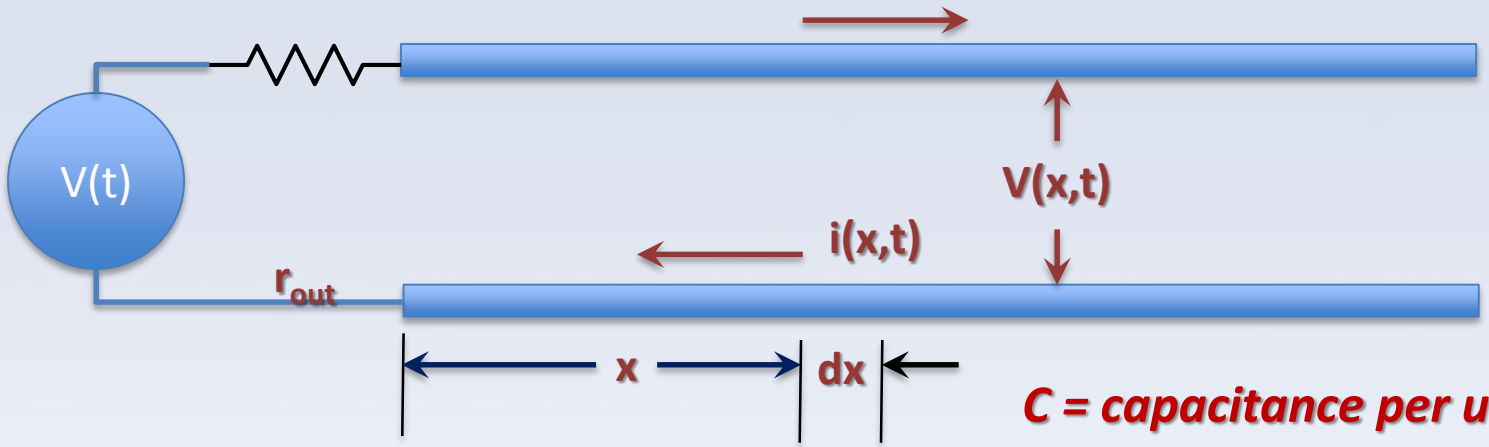
Pulses in transmission line



Setup


Wavetek 81
Tektronix 3012B
Sync output
Triggering input
RG8U
Signal output
Load

The Wave Equation



C = capacitance per unit length

L = inductance per unit length

$$CdxV = -dq;$$

$$C\frac{\partial V}{\partial t} = -\frac{\partial q}{\partial t} = i;$$

$$\frac{\partial i}{\partial x} = -C\frac{\partial V}{\partial t}$$

$$dV = -(Ldx)\frac{di}{dt};$$

$$\frac{\partial V}{\partial x} = -L\frac{\partial i}{\partial t}$$

The Wave Equation

$$\frac{\partial i}{\partial x} = -C\frac{\partial V}{\partial t}$$

$$\frac{\partial V}{\partial x} = -L\frac{\partial i}{dt}$$

$$\frac{\partial}{\partial t}$$

$$\frac{\partial}{\partial x}$$

$$\frac{\partial^2 i}{\partial t \partial x} = -C\frac{\partial^2 V}{\partial t^2} \quad (1)$$

$$\frac{\partial^2 V}{\partial x^2} = -L\frac{\partial^2 i}{\partial x \partial t} \quad (2)$$

Combining (1) and (2)

$$\frac{\partial^2 i}{\partial x^2} = LC\frac{\partial^2 i}{\partial t^2} \qquad \frac{\partial^2 V}{\partial x^2} = LC\frac{\partial^2 V}{\partial t^2}$$

The Wave Equation. Voltage and current waves.

$$\frac{\partial^2 i}{\partial x^2} = LC\frac{\partial^2 i}{\partial t^2} \qquad \frac{\partial^2 V}{\partial x^2} = LC\frac{\partial^2 V}{\partial t^2}$$

Looking for solution

$$V(x,t) = V_0 \sin\omega\left(t - \frac{x}{v}\right)$$

$$i(x,t) = i_0 \sin\omega\left(t - \frac{x}{v}\right)$$

Now substituting *V(x,t)* and *i(x,t)* in

$$\frac{\partial V}{\partial x} = -L\frac{\partial i}{dt} \qquad \frac{\partial i}{\partial x} = -C\frac{\partial V}{\partial t}$$

We can find $V_0 = i_0\sqrt{\frac{L}{C}}$ or

$$v = \frac{1}{\sqrt{LC}}$$

Speed of wave propagation

$$V(x,t) = \sqrt{\frac{L}{C}}\, i(x,t) = Z_k\, i(x,t)$$

Z_k - characteristic Impedance

Equivalent to Ohm's law equation

Characteristic impedance

$$Z_k = \sqrt{\frac{L}{C}}$$

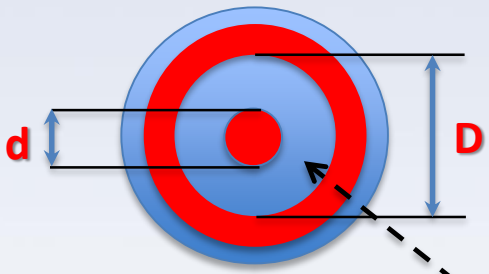
C = capacitance per unit length

L = inductance per unit length

$$\varepsilon_0 = 8.854 \times 10^{-12} \text{ (F/m)}$$

$$\mu_0 = 4\pi \times 10^{-7} \text{ (H/m)}$$

Cross-section of the coaxial cable



$$C = \frac{2\pi\varepsilon_0\varepsilon_r}{\ln\left(\frac{D}{d}\right)} \text{ (F/m)}$$

$$L = \frac{\mu_0\mu_r}{2\pi}\ln\left(\frac{D}{d}\right) \text{ (H/m)}$$

ε_r – dielectric permittivity

μ_r-magnetic permeability ≈1

Finally for coaxial cable: $$Z_k = \frac{138}{\sqrt{\varepsilon_r}}\log\left(\frac{D}{d}\right)(Ohms)$$

Speed of wave propagation, delay.

$$v = \frac{1}{\sqrt{LC}}$$

Speed of wave propagation

$$v = \frac{1}{\sqrt{\mu_0 \mu_r \varepsilon_0 \varepsilon_r}} = \frac{c}{\sqrt{\mu_r \varepsilon_r}} \approx \frac{c}{\sqrt{\varepsilon_r}}$$

$\mu_r \cong 1$

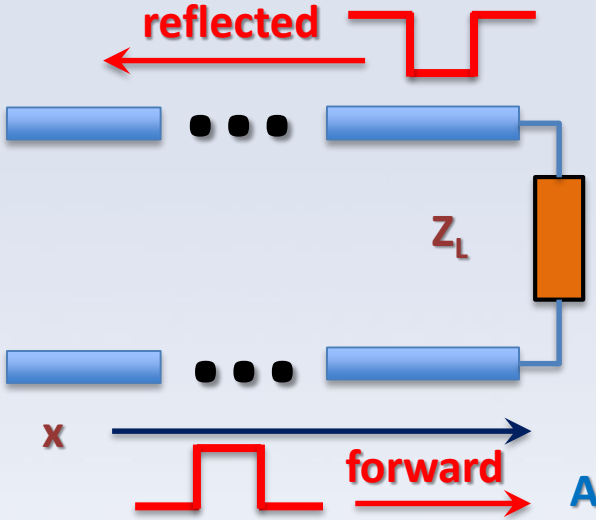
For polyethylene ε_r~2.25(up to 1GHz)

Delay time $\tau = \frac{1}{v}$ **(s/m)** $\approx$ $3.336 \bullet 10^{-9} \sqrt{\varepsilon_r}\,(s/m) = 3.336\sqrt{\varepsilon_r}\,(ns/m)$

RG-8/U, RG58U:

Inner Insulation Materials: Polyethylene
Nominal Impedance: 52 ohm
Delay time ~5ns/m

Reflection in transmission line



$$\frac{\partial^2 V}{\partial x^2} = LC\frac{\partial^2 V}{\partial t^2}$$

solution for the traveling in opposite direction

$$V(x,t) = V_0 \sin\omega\left(t - \frac{x}{v}\right)$$

$$i(x,t) = i_0 \sin\omega\left(t - \frac{x}{v}\right)$$

For reflected wave $V_r = -Z_k i_r$

At any point of the transmission line:

$$V = V_r + V_i$$

$$i = i_r + i_i = \frac{V_i}{Z_k} - \frac{V_r}{Z_k}$$

1. Resistive load $Z_L = R_L$

$$\frac{V}{i} = R_L$$

$$\frac{V_i + V_r}{V_i - V_r} = \frac{R_L}{Z_k} \quad \text{or} \quad V_r = \frac{R_L - Z_k}{R_L + Z_k} V_i$$

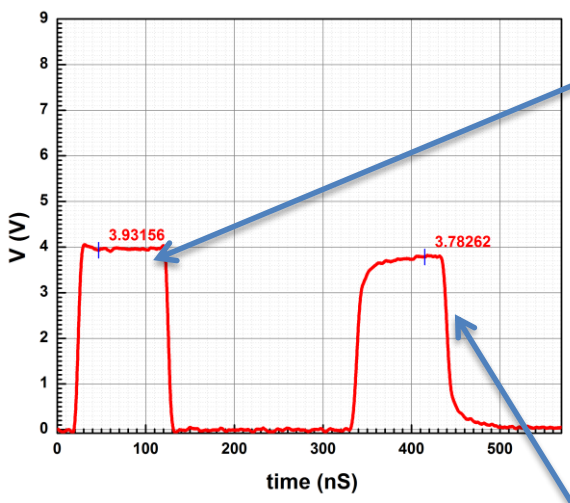
Reflection in transmission line

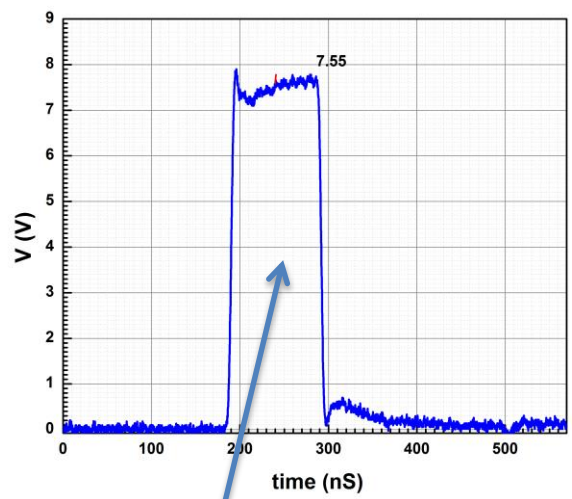
Resistive load $Z_L=R_L$

$$\frac{V_i+V_r}{V_i-V_r}=\frac{R_L}{Z_k} \quad \text{or} \quad V_r=\frac{R_L-Z_k}{R_L+Z_k}V_i$$

Open line $R_L=\infty$ → $V_r = V_i$ and $V=V_i + V_r$

Incident pulse





Reflected pulse

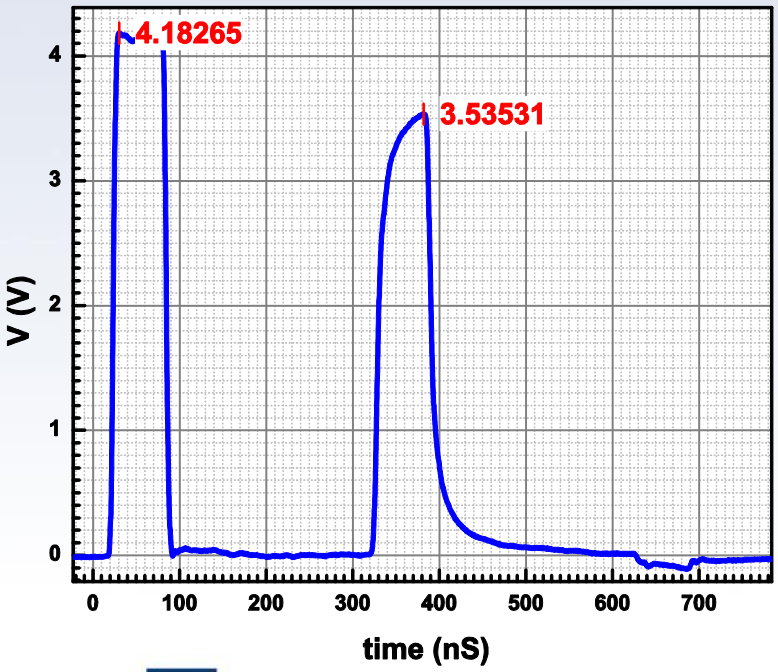
End of the line

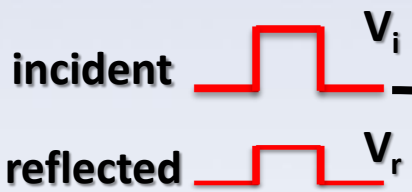
Reflection in transmission line. Loses.

Theory: $R_L=\infty \rightarrow V_r = V_i$

Attenuation (dB per 100 feet)					
MHz	30	50	100	146	150
RG-58U	2.5	4.1	5.3	6.1	6.1

Experiment RG 58U

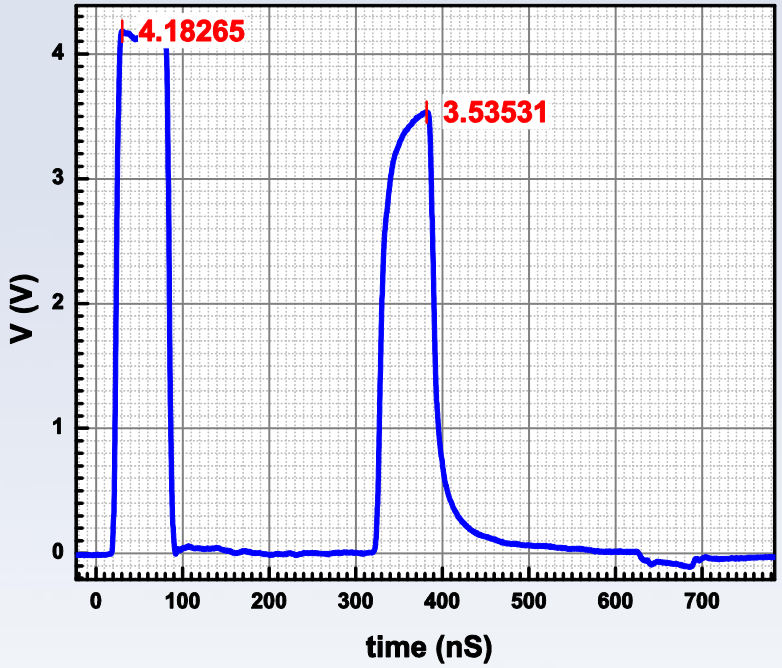




$$ATTN(db) = 20\log\left(\frac{V_i}{V_r}\right)$$

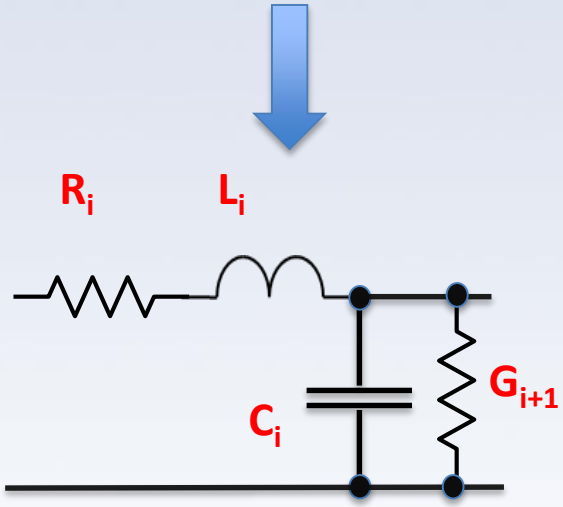
Important parameter for cable is attenuation per length

Reflection in transmission line. Loses.

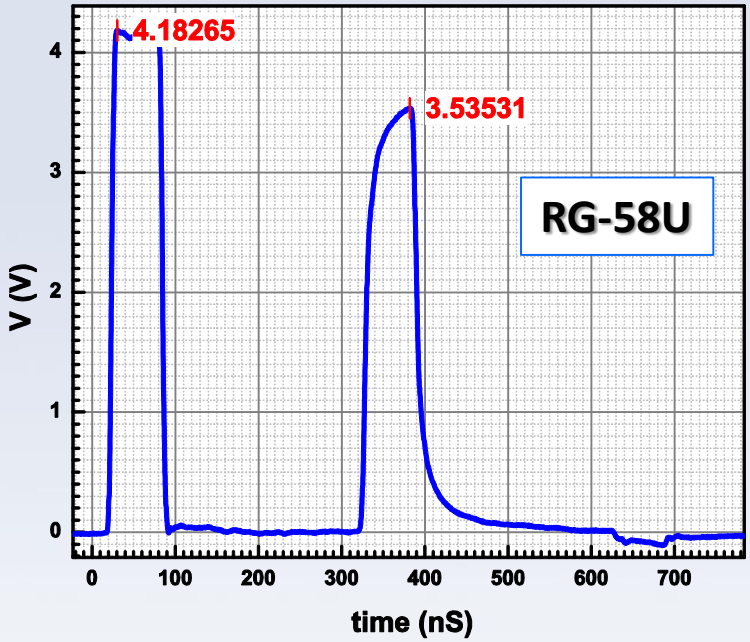


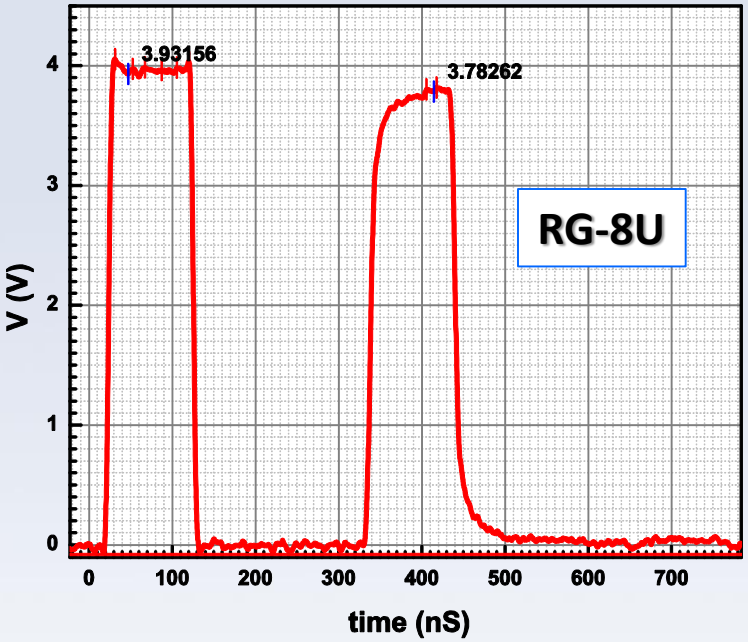
In our case: $Attn(200\,ft) = 20\log\left(\frac{4.18}{3.54}\right) \approx 1.46dB$

Where it is coming from?



Different cables loses.

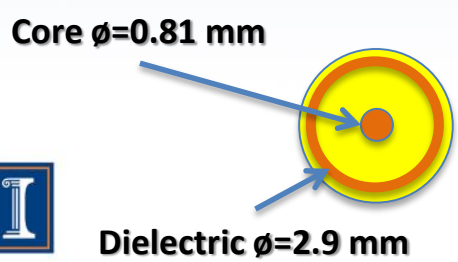


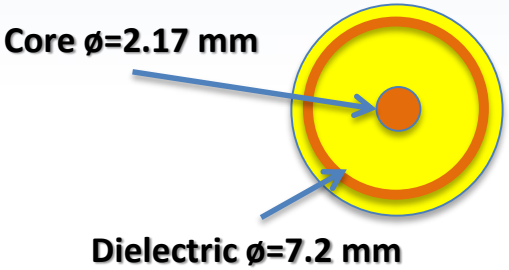


$$Attn(200\ ft) = 20\log\left(\frac{4.18}{3.54}\right) \approx 1.46dB$$

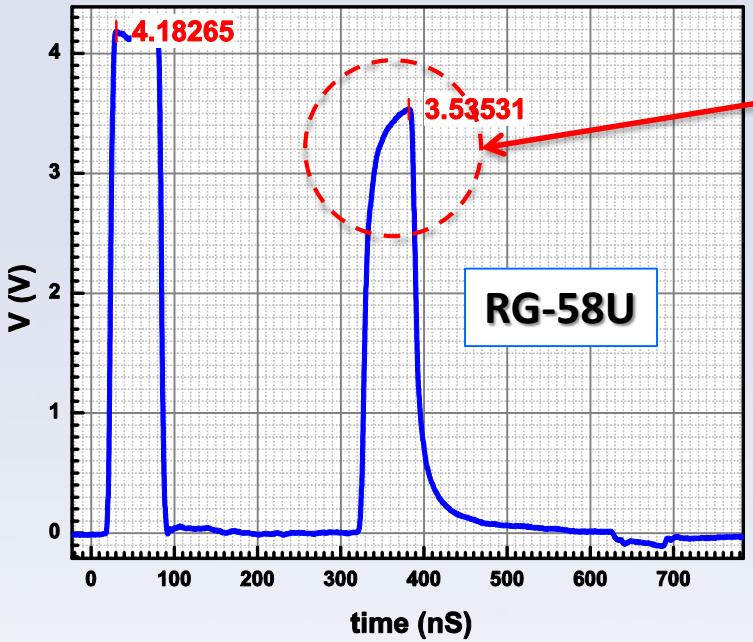
>

$$Attn(200\ ft) = 20\log\left(\frac{3.932}{3.78}\right) \approx 0.335dB$$

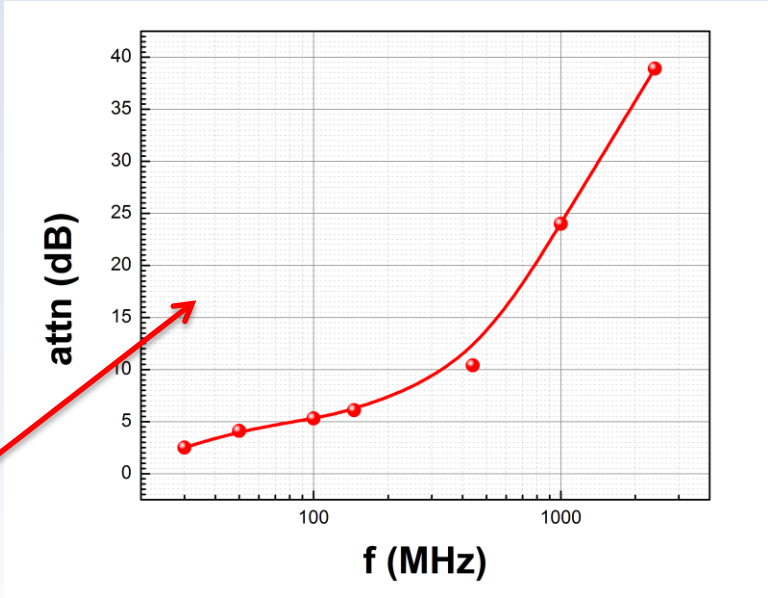




Loses. Frequency dispersion.

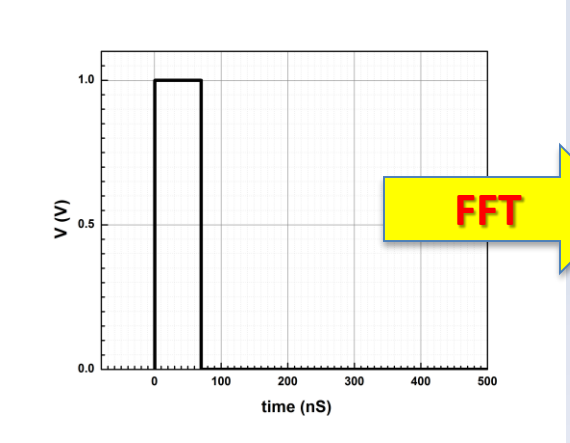


Reflected pulse does not follow the shape of the incident pulse

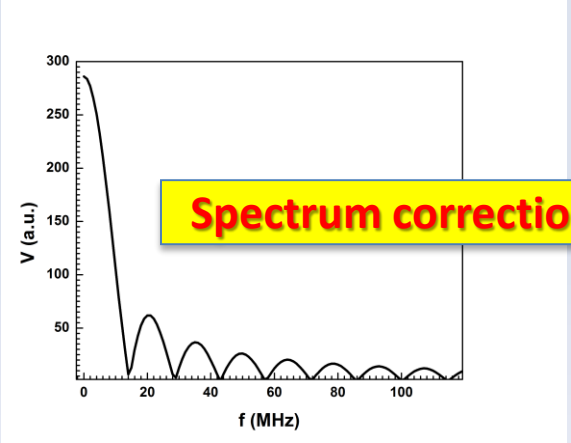


Frequency dependence of the attenuation RG-58U cable

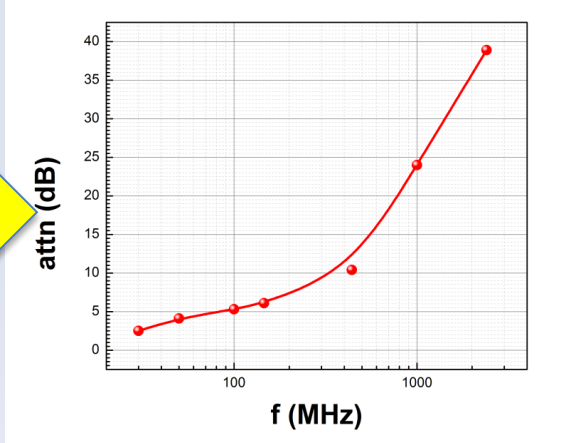
Loses. Frequency dispersion.

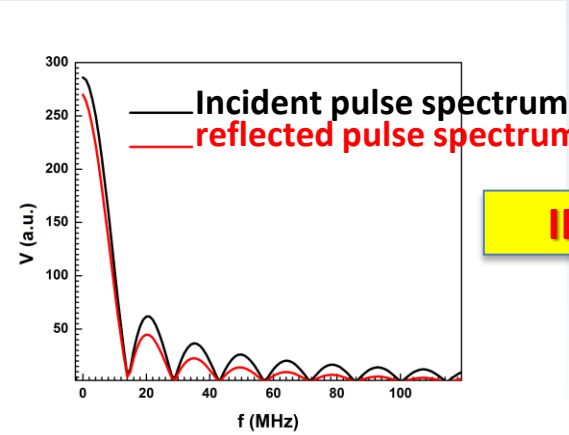


FFT

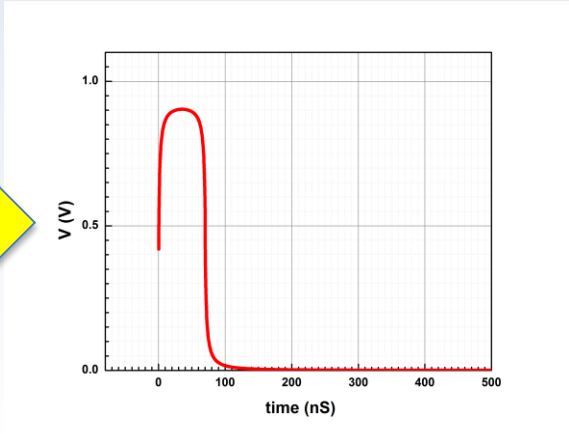


Spectrum correction





IFFT

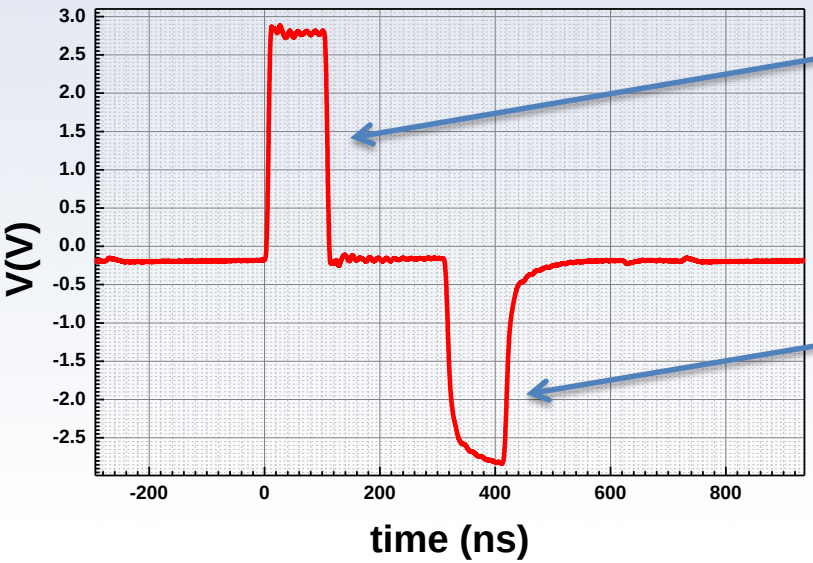


Reflection in transmission line

Resistive load $Z_L = R_L$

$$\frac{V_i + V_r}{V_i - V_r} = \frac{R_L}{Z_k} \quad \text{or} \quad V_r = \frac{R_L - Z_k}{R_L + Z_k} V_i$$

Shorted line $R_L = 0$ → $V_r = -V_i$



Incident pulse

Reflected pulse

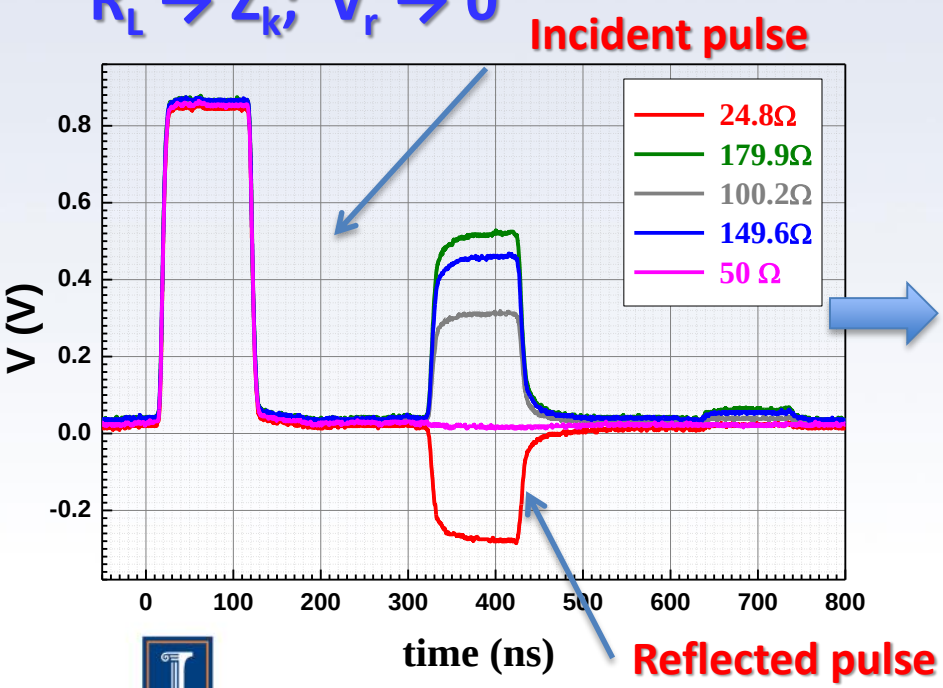
Reflection in transmission line.

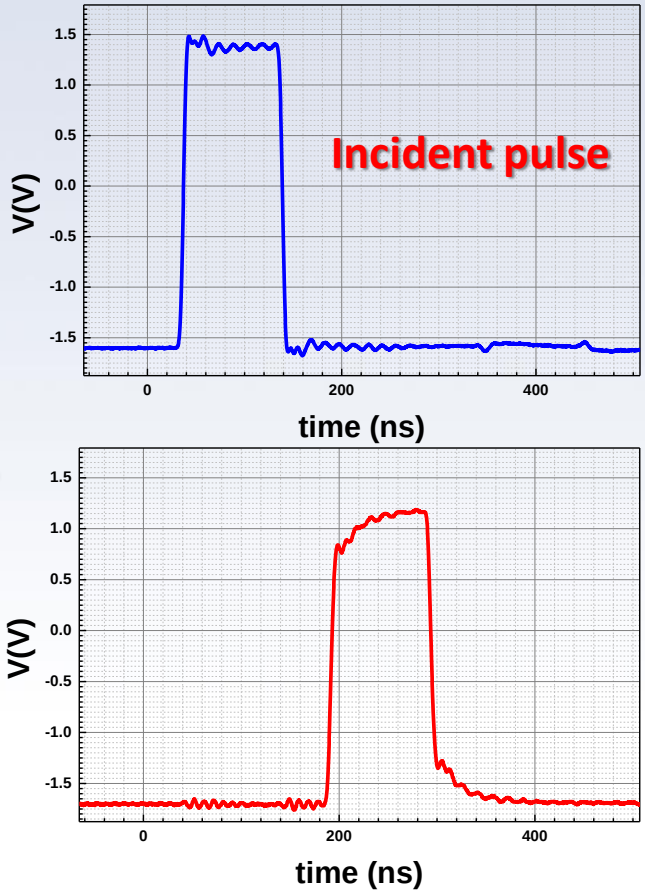
Resistive load $Z_L = R_L$

$$\frac{V_i + V_r}{V_i - V_r} = \frac{R_L}{Z_k} \quad \text{or} \quad V_r = \frac{R_L - Z_k}{R_L + Z_k} V_i$$

Matching the load impedance

$R_L \rightarrow Z_k$; $V_r \rightarrow 0$





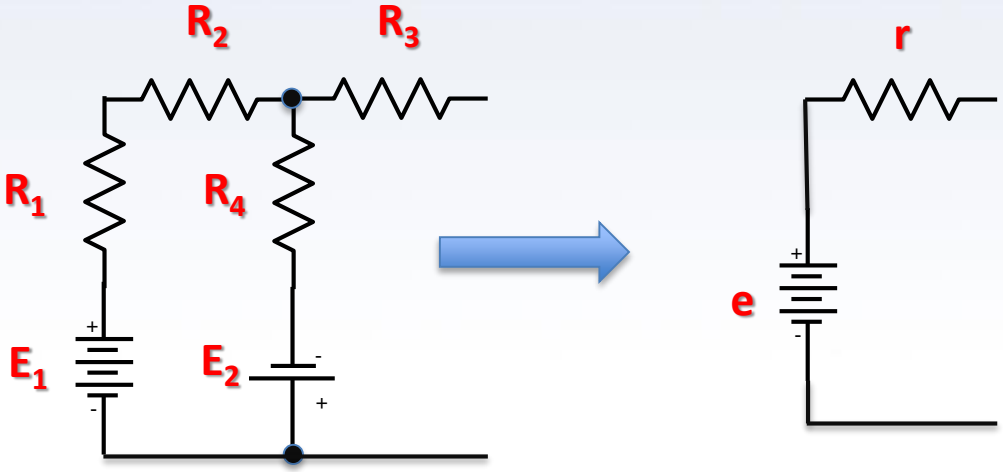
End of the line

Thévenin's theorem

Hermann Ludwig Ferdinand von Helmholtz (1821-1894)

Any combination of batteries and resistances with two terminals can be replaced by a single voltage source e and a single series resistor r

Léon Charles Thévenin (1857–1926)

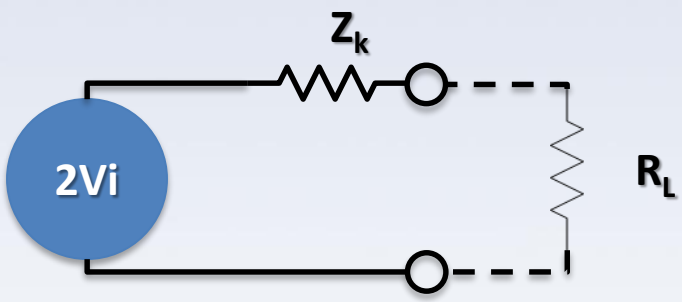


Thévenin's theorem. Transmission line.

$$V = V_r + V_i = i \bullet R_L$$

$$i = i_r + i_i = \frac{V_i}{Z_k} - \frac{V_r}{Z_k}$$

$$i = \frac{2V_i}{R_L + Z_k}$$

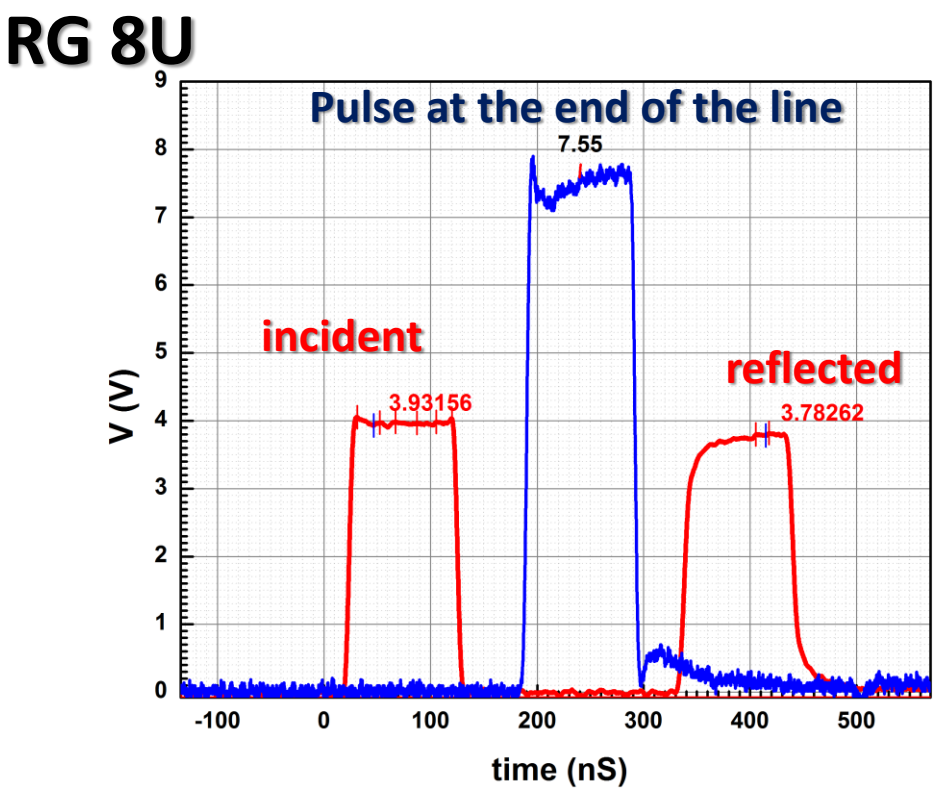


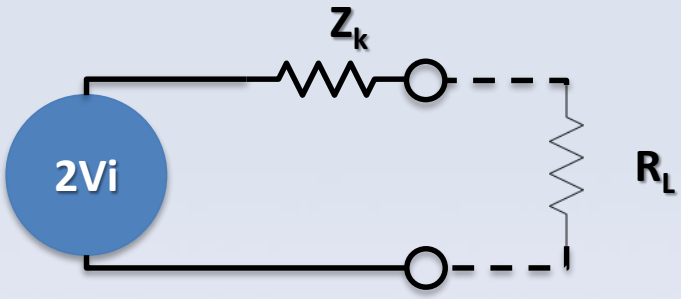
From this equivalent equation we can find the maximum possible power delivered to R_L

$$P = i^2 R_L = \frac{(2V_i)^2}{(R_L + Z)^2} R_L$$

$P = P_{max}$ if $R_L = Z_k$ (no reflection)

Thévenin's theorem. Experiment.



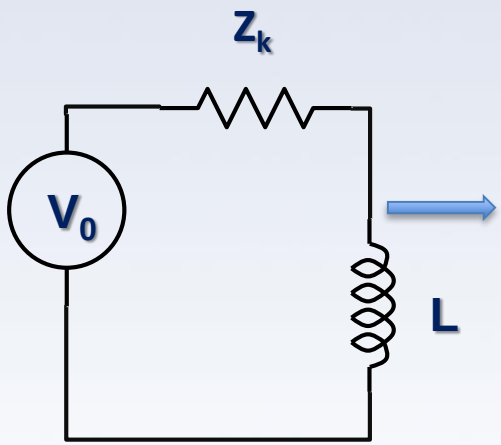


This experiment better to perform on RG 8U cable because of lower attenuation

RL=∞, amplitude of the pulse at the end of line is expected to be 2Vi, where Vi is the amplitude of the incident pulse

Reflection. Inductive load.

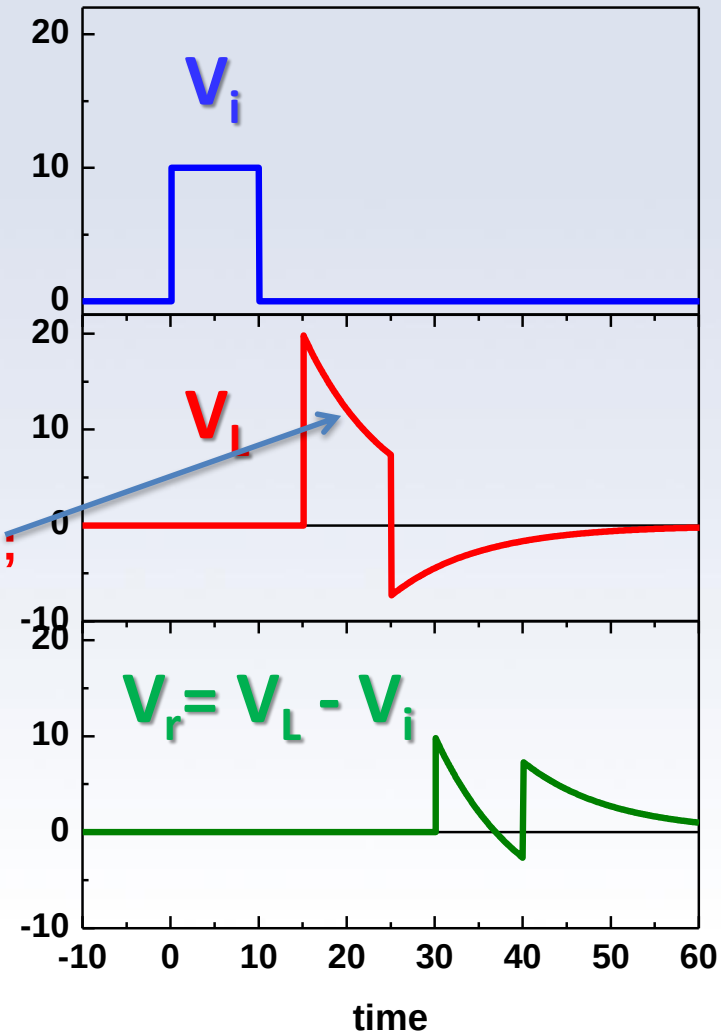
$$i = \frac{2V_i}{Z_L + Z_k}$$



$$2V_i = iZ_k - L\frac{di}{dt};$$

$$i = i_0\left(1 - \exp\left(\frac{-t}{\tau}\right)\right);$$

$$\tau = \frac{L}{Z_k}$$

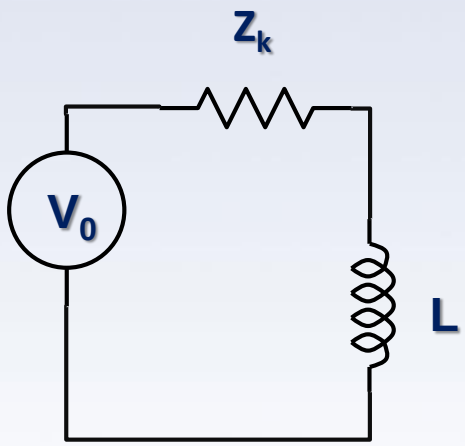


Reflection. Inductive load. Experiment

$$i = \frac{2V_i}{Z_L + Z_k}$$

$$2V_i = iZ_k - L\frac{di}{dt};$$

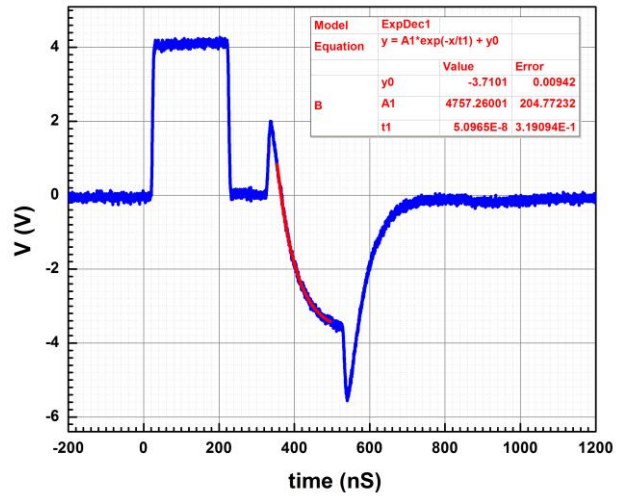
$$i = i_0\left(1 - \exp\left(\frac{-t}{\tau}\right)\right);$$

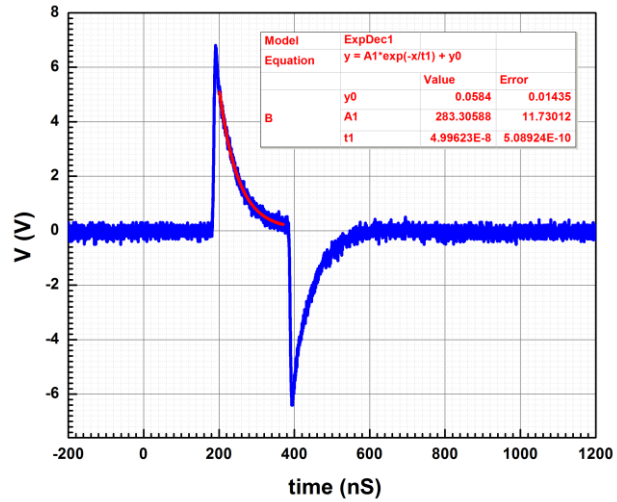


$$\tau = \frac{L}{Z_k}$$

$\tau \cong 50$ns,

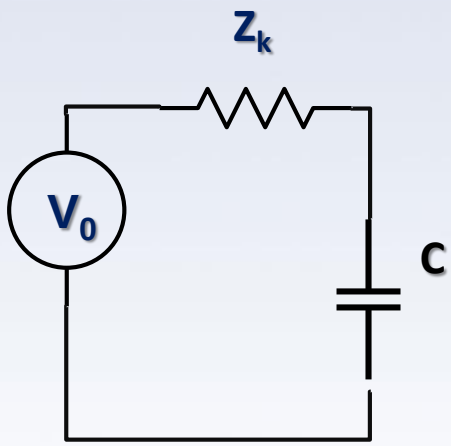
$L = \tau Zk \sim 2.5\mu H$





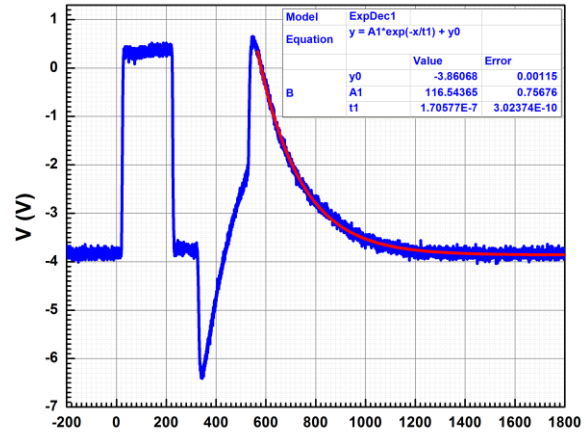
Reflection. Capacitive load. Experiment

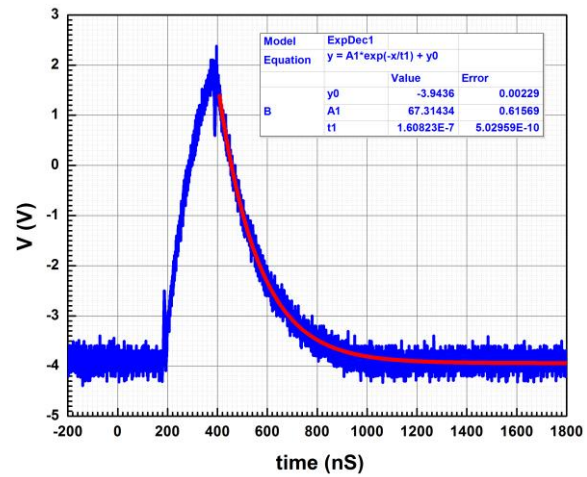
$$i = \frac{2V_i}{Z_L + Z_k}$$



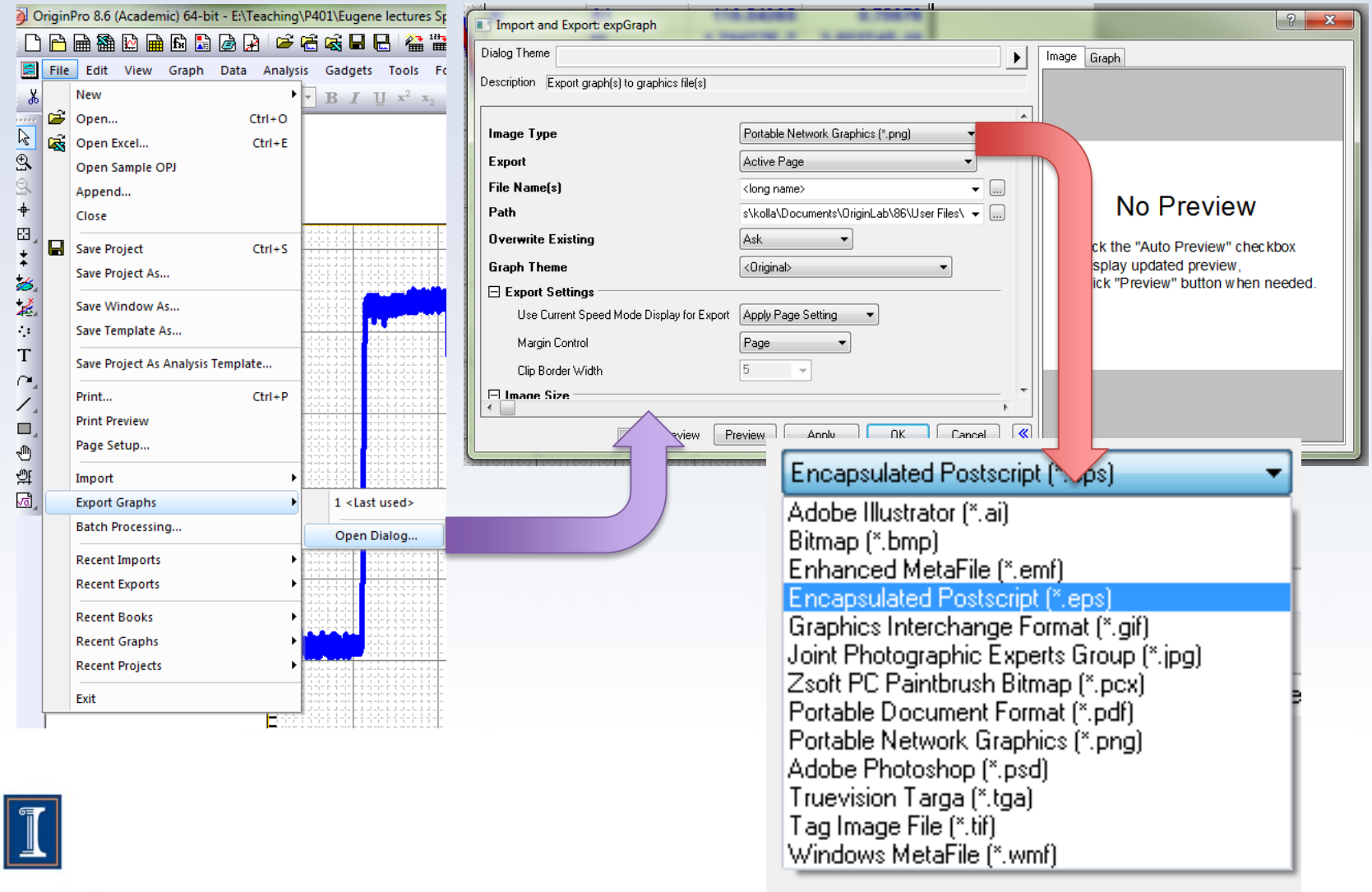
$$\tau = Z_k C$$

$$C = \frac{\tau}{Z_k} \approx 3.2nF$$





Appendix #1. Export graphs from Origin



Appendix #2. Reminders

1. The reports should be uploaded to the ***proper*** folder and ***only*** to the proper folder

For example folder ***RLC Lab Report_L1*** should used by students from L1 section only

I would recommend the file name style as:

L1_lab2_student1

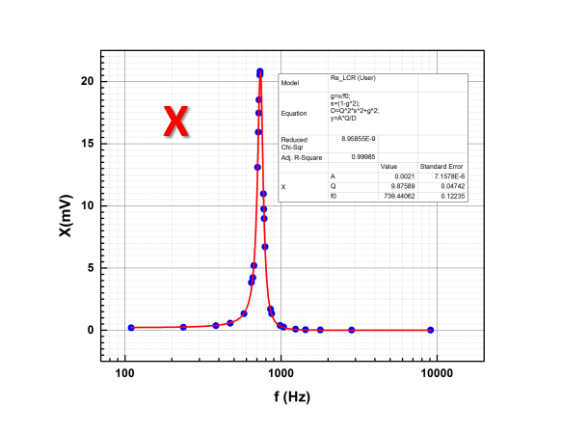
Lab section Lab number Your name

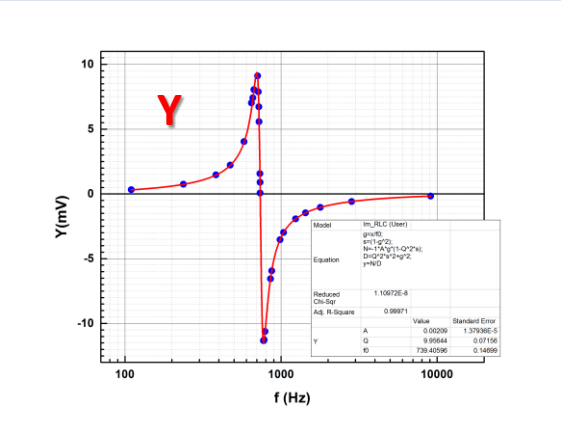
You do not need to submit two copies in pdf and in MsWord formats

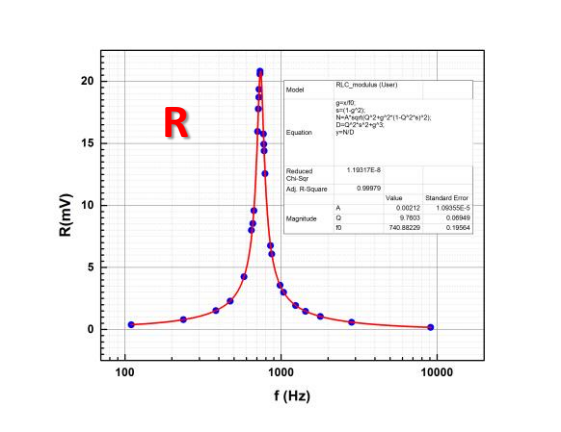
2. Origin template for this week Lab:

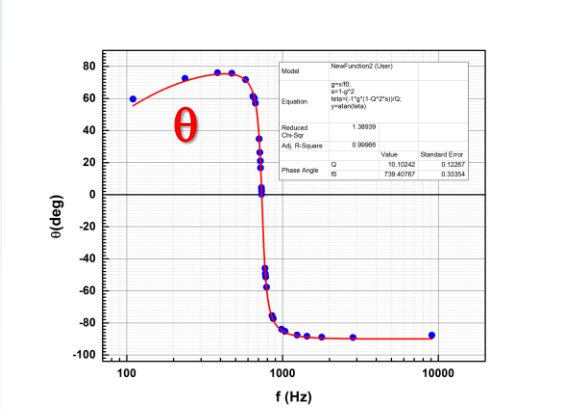
\\engr-file-03\phyinst\APL Courses\PHYCS401\Common\Origin templates\Transmission line\Time trace.otp

Appendix #3. LCR resonance circuit.Fitting results.







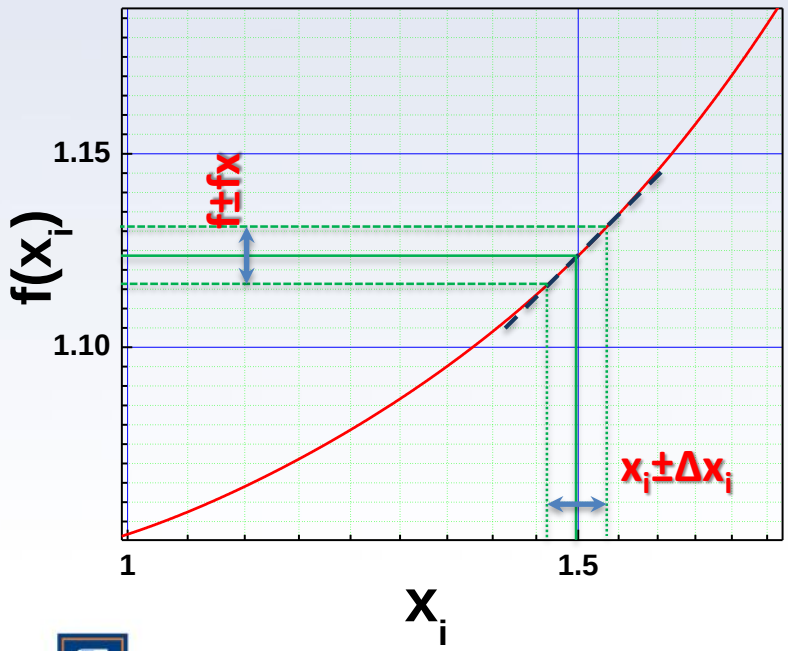


Data courtesy Tsung-Lin Hsieh, Physics 401, Fall 2011.

Appendix #4. Error propagation.

y = f(x1, x2 … xn)

$$\Delta f(x_i, \Delta x_i) = \sqrt{\sum_{i=1}^{n} \left[\frac{\partial f}{\partial x_i}\right]^2 \cdot \Delta x_i^2}$$



Error propagation. Example.

Derive resonance frequency *f* from measured inductance *L*±*ΔL* and capacitance *C*±*ΔC*

$$f_0(L,C) = \frac{1}{2\pi}\sqrt{\frac{1}{LC}}$$

$$L_1 = 10 \pm 1\text{mH}, \quad C_1 = 10 \pm 2\mu\text{F}$$

$$\Delta f(L,C,\Delta L,\Delta C) = \sqrt{\left[\frac{\partial f}{\partial L}\right]^2 \cdot \Delta L^2 + \left[\frac{\partial f}{\partial C}\right]^2 \Delta C^2}$$

$$\frac{\partial f}{\partial L} = \frac{-1}{4\pi} C^{-\frac{1}{2}} L^{-\frac{3}{2}};$$

$$\frac{\partial f}{\partial C} = \frac{-1}{4\pi} L^{-\frac{1}{2}} C^{-\frac{3}{2}}$$

Results:

$f(L_1,C_1)$=503.29212104487Hz

Δf=56.26977Hz

$f(L_1,C_1)$=503±56Hz

Error propagation. Example.

$L_1 = 10 \pm 1\text{mH}, \quad C_1 = 10 \pm 1\mu\text{F}$ **Where these numbers are coming from?**

1. Using commercial resistors, capacitors, inductances…

C=500pF±5%

L=35mH±10%

2. Measuring the parameters using standard equipment

SENCORE "Z" meter model LC53
Capacitance measuring accuracy ±5%
Inductance measuring accuracy ±2%

Agilent E4980A Precision LCR Meter

Basic accuracy ±0.05%

Appendix #5. Nonlinear fitting. Main idea

Origin uses the **Levenberg–Marquardt** algorithm for nonlinear fitting

From experiment you have the array **(x_i,y_i)** of independent and dependent variables: **x_i** (e.g. f- frequency) and **y_i** (e.g. magnitude of the signal) and you have optimize the vector of fitting parameters **β** of your model function ***f(x,β)*** in order to minimize the sum of squares of deviations:

$$S(\beta) = \sum_{i=1}^{m} \left[y_i - f(x_i, \beta) \right]^2$$

Important point is the choice of fitting parameters. In some cases the algorithm will work with **β=(1,1…1)** , but in many situations the choice of more realistic parameters will lead to solution

For details go to:

http://en.wikipedia.org/wiki/Levenberg%E2%80%93Marquardt_algorithm

K. Levenberg. "A Method for the Solution of Certain Non-Linear Problems in Least Squares".The Quarterly of Applied Mathematics, 2: 164-168 (1944).