Transients and Oscillations in RLC Circuits

Physics 401, Spring 2014.

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Transients and Oscillations in RLC Circuits. Outline

Transients. Definition.

- •Transients in RLC

- •Resonance in RLC

Data analysis. Origin. Fitting.

Main goals of this week Lab:

➢ **To understand what are the transients in general**

➢ **Transients in RLC circuits. Different regimes of dumping**

➢ **Data analysis using Origin software**

Transients. Definition.

transient (physics) a short-lived oscillation in a system caused by a sudden change of voltage or current or load

a transient response or natural response is the response of a system to a change from equilibrium.
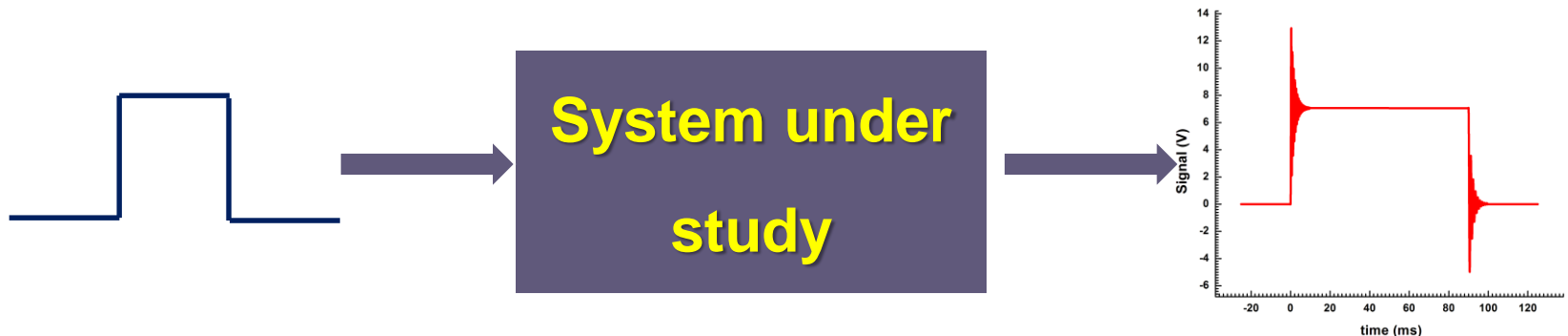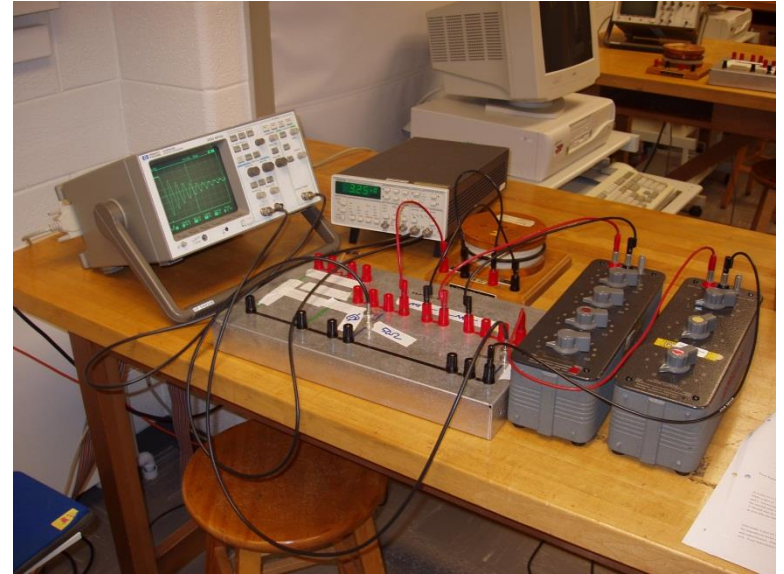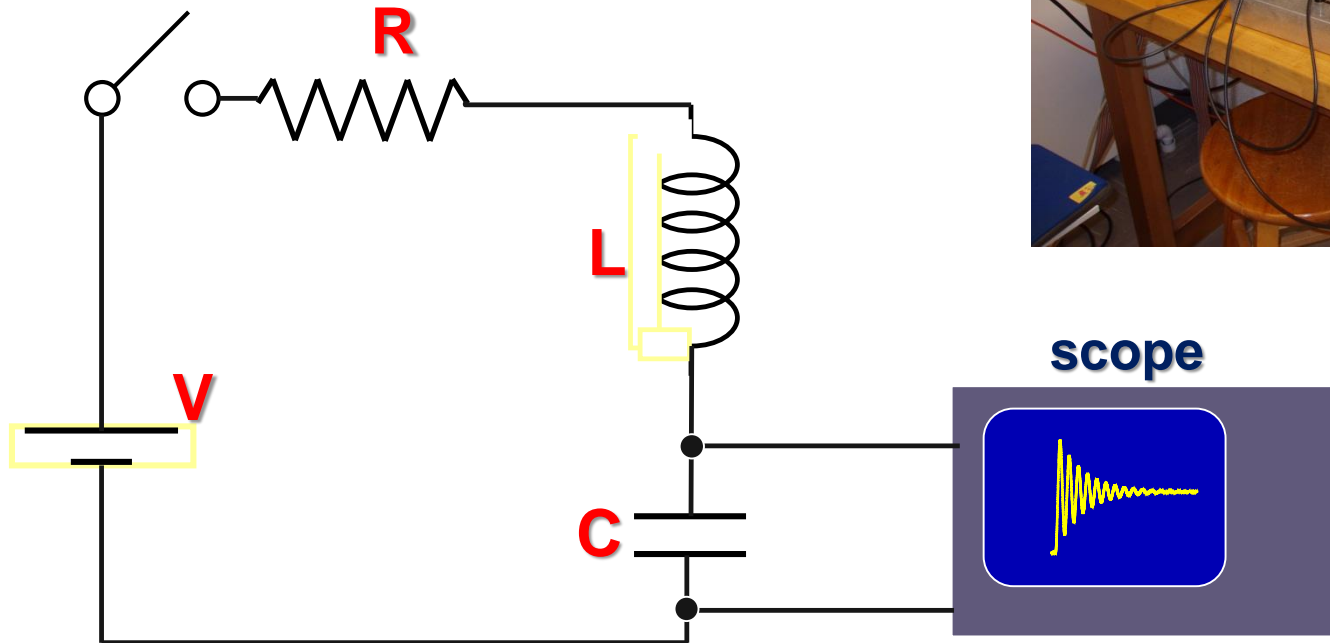
Transients in RLC circuit.

Resistance   R [Ohm]
Capacitance C [uF] (10^{-6}F)
Inductance   L [mH] (10^{-3}H)

R

L

V

C

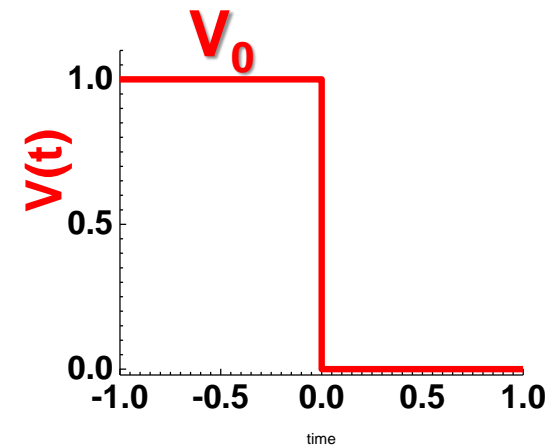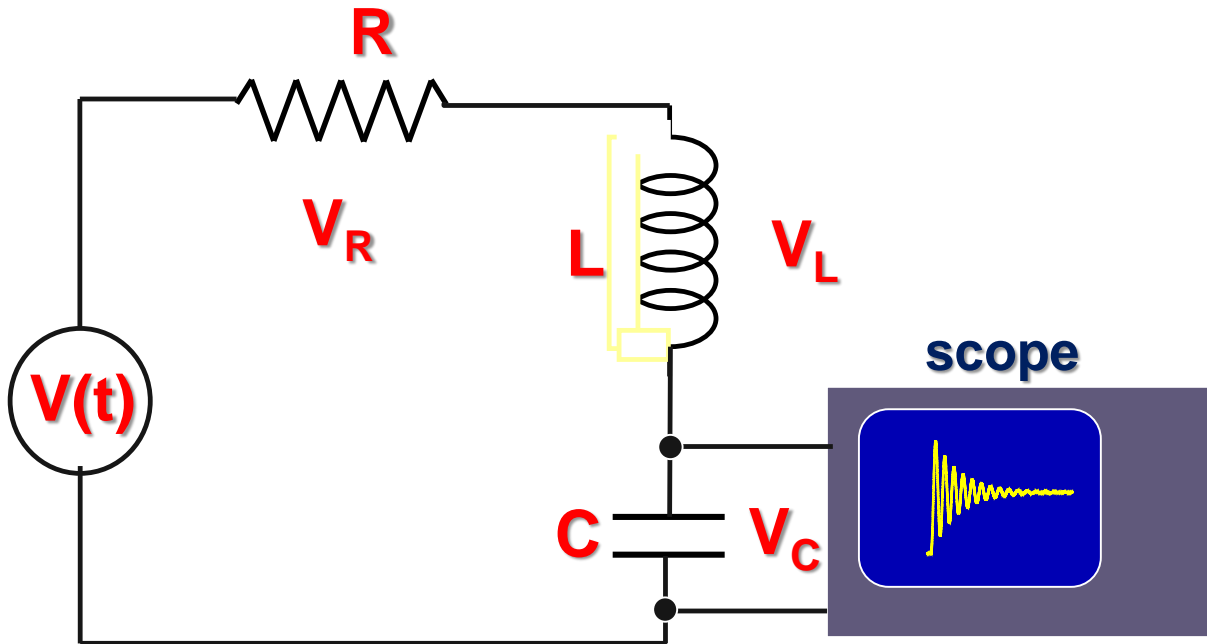**scope**

Transients in RLC circuit.

According the Kirchhoff's law $V_R + V_L + V_C = V(t)$

*

$$L\frac{d^2}{dt^2}q(t) + R\frac{d}{dt}q(t) + \frac{q(t)}{C} = 0 \qquad (1)$$

*See Lab write-up for details

R

V_R

L

V_L

scope

$V(t)$

C

V_C

V_0

V(t)

1.0

0.5

0.0

-1.0   -0.5   0.0   0.5   1.0

time

Transients in RLC circuit. Three solutions

The solution of this differential equation can be found in the form

$$q(t) = A e^{st}$$

This will convert (1) in quadratic equation

$$s^2 + \left(\frac{R}{L}\right) s + \frac{1}{LC} = 0$$

with solutions:

$$s_{1,2} = -\frac{R}{2L} \pm \sqrt{\left(\frac{R}{2L}\right)^2 - \left(\frac{1}{LC}\right)} \equiv -a \pm b$$

$$a = \frac{R}{2L} \quad , \quad b = \sqrt{\left(\frac{R}{2L}\right)^2 - \left(\frac{1}{LC}\right)}$$

$b^2 > 0$ over-damped solution
$b^2 = 0$ critically damped solution
$b^2 < 0$ under-damped solution

Transients in RLC circuit. Over-damped solution: $b^2>0$

In this case the solution will be aperiodic

exponential decay function with no

oscillations:

$$q(t) = e^{-at}\left(A_1 e^{bt} + B_1 e^{-bt}\right)$$

$$i(t) = \frac{dq}{dt} = -ae^{-at}(A_1 e^{bt} + B_1 e^{-bt}) + be^{-at}(A_1 e^{bt} - B_1 e^{-bt})$$

Transients in RLC circuit. Over-damped solution: $b^2 > 0$

Taken in account the initial conditions: $q(0) = q_0$ and $i(0) = 0$

$$q(t) = q_0 e^{-at}\left(\cosh bt + \frac{a}{b}\sinh bt\right)$$

$$\xrightarrow[(a-b)t \gg 1]{} \quad \frac{q_0}{2}\left(1 + \frac{a}{b}\right)e^{-(a-b)t}$$

$$i(t) = -\frac{q_0}{2}\left(\frac{a^2 - b^2}{b}\right)e^{-(a-b)t}$$

This is exponential decay function

Transients in RLC circuit. Critically-damped solution $b^2=0$

For this case the general solution can be found as

$q(t)=(A_2+B_2t)e^{-at.}$ Applying the same initial condition

the current can be written as $i=-a^2q_0te^{-at}$

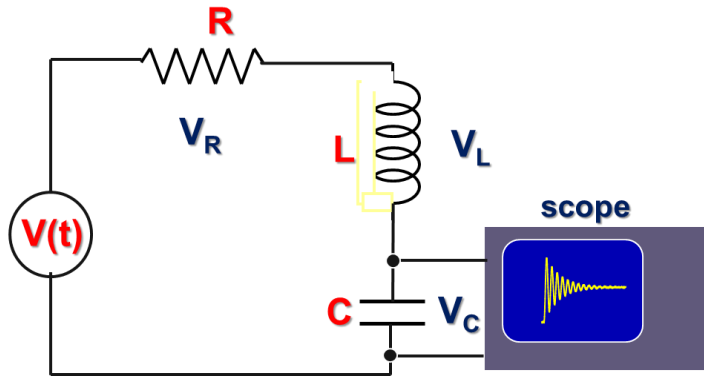$$b^2 = 0 \rightarrow R^2 = \frac{4L}{C}$$ and $$a = \frac{R}{2L}$$ Critically-damped conditions for our network

Critical damped case shows the fastest decay with no oscillations

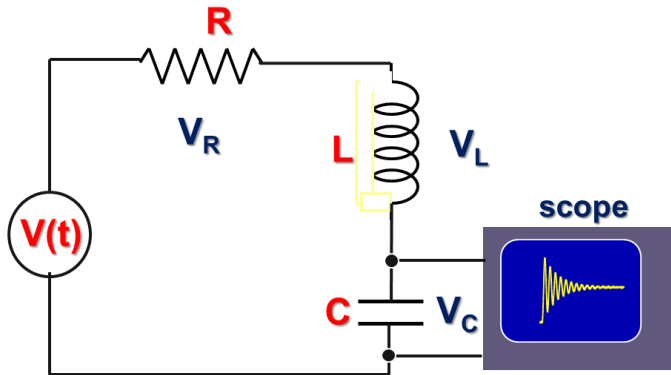# Transients in RLC circuit. Critically-damped solution. Real data analysis.

$b^2=0$

In this experiment R=300 ohms, C=1μF, L=33.43mH.

The output resistance of Wavetek is 50 ohms and resistance of coil was measured as 8.7 ohms, so actual resistance of the network is R_a=300+50+8.7=358.7

Decay coefficient

$$a = \frac{R}{2L} = \frac{358.7}{2*33.43E-3} \approx 5365$$

Transients in RLC circuit. Critically-damped solution. Real data analysis. $b^2=0$
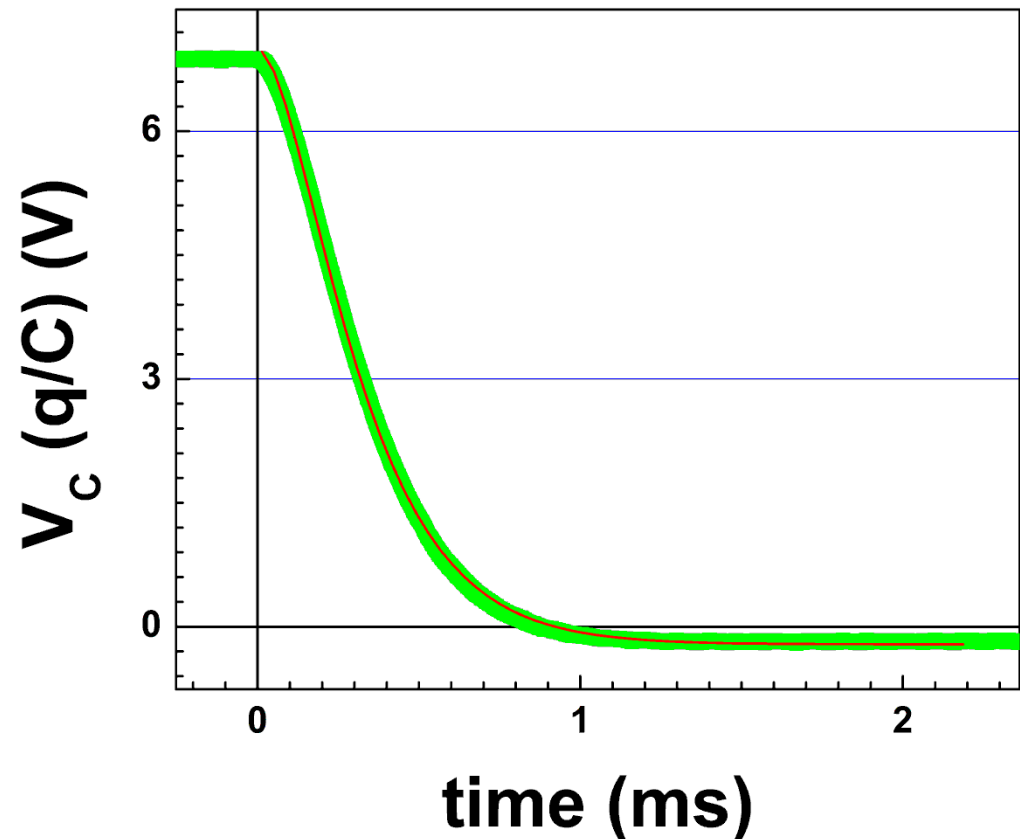

$V_c \sim q$, fiiting function: $V_c=V_{co}(1+at)e^{-at}$

Calculated decay coefficient ~5385,

Obtained from fitting - ~5820.
Possible reason – it is still slightly over damped
Calculated b^2 is
$b^2=2.99e7-2.90e7>0$

Transients in RLC circuit. Under-damped solution.

If **b²<0** we will have oscillating solution. Omitting the details (see Lab write-up) we have the equations for charge and current as:

$$q(t) = q_0 e^{-at}\left(\cos bt + \frac{a}{b}\sin bt\right) = q_0 e^{-at}\sqrt{1 + \frac{a^2}{b^2}}\ \sin(bt + \varphi)$$

$$i(t) = q_0 e^{-at}\left(\frac{a^2 + b^2}{b}\right)\sin bt$$

$$a = \frac{R}{2L}\ ,\ \ b = \sqrt{\left(\frac{R}{2L}\right)^2 - \left(\frac{1}{LC}\right)}; f = \frac{1}{2\pi}\sqrt{\left(\frac{1}{LC}\right) - \left(\frac{R}{2L}\right)^2}$$

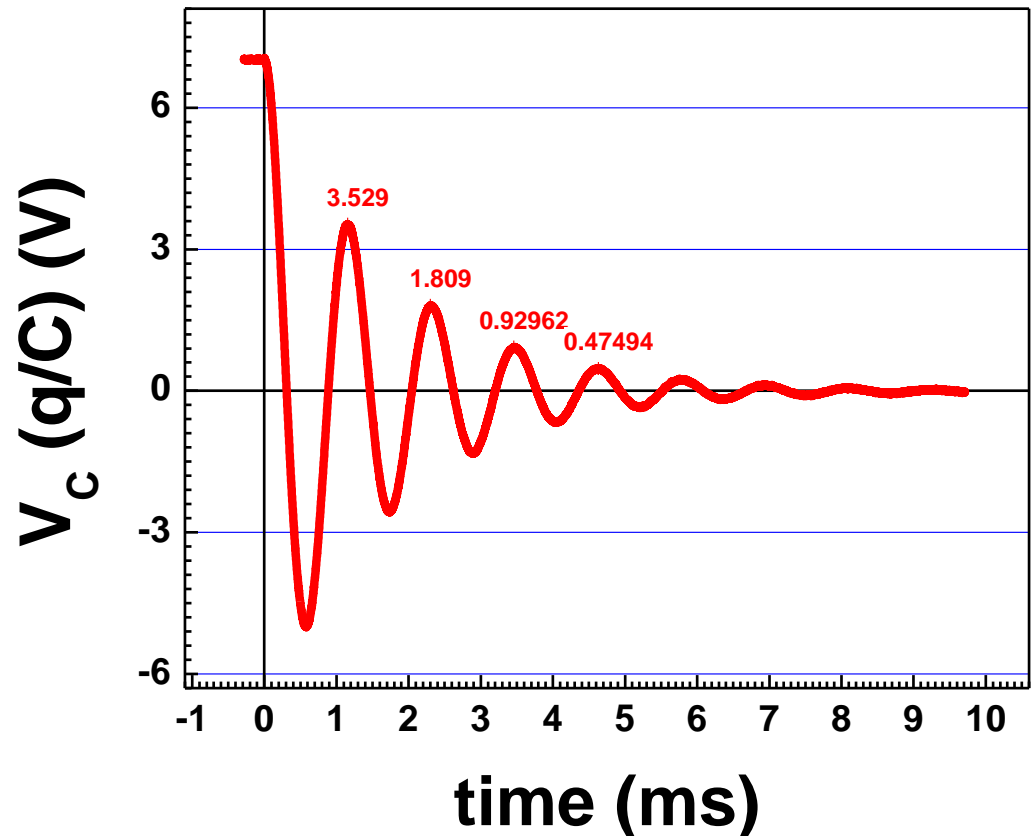# Transients in RLC circuit. Under-damped solution. Log decrement. Quality factor.

Log decrement can be defined as $\delta = ln\left(\frac{q(t_{max}}{q(t_{max}+T_1)}\right) =$

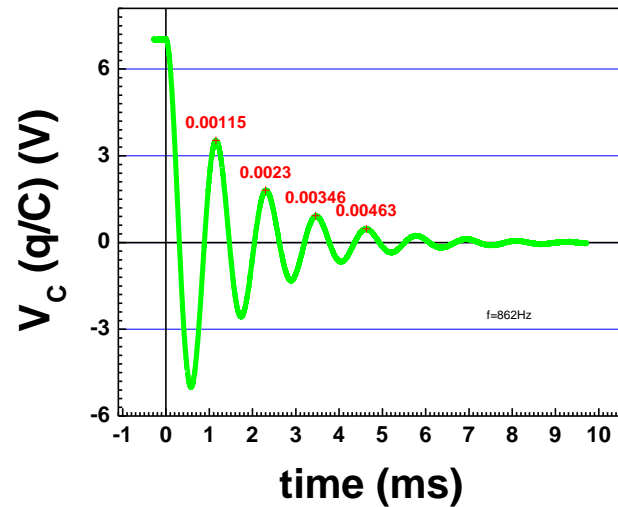$ln\left(\frac{e^{-at_{max}}}{e^{-at_{max}+T_1}}\right) = aT_1$, where T$_1$=1/f1

Quality factor can be defined as $Q = 2\pi\frac{E}{\Delta E}$,

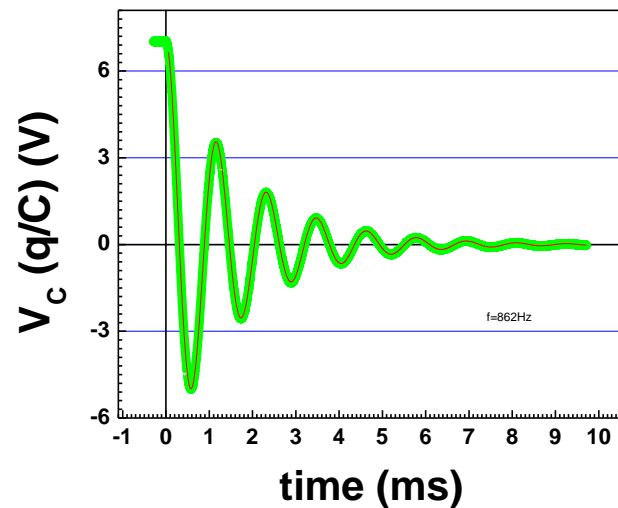For RLC $\quad Q = \frac{\omega_1 L}{R} = \frac{\pi}{\delta}$

From this plot δ≈0.67
Q≈4.7

Transients in RLC circuit. Data analysis. Using Origin software.

1. **Pick peaks**
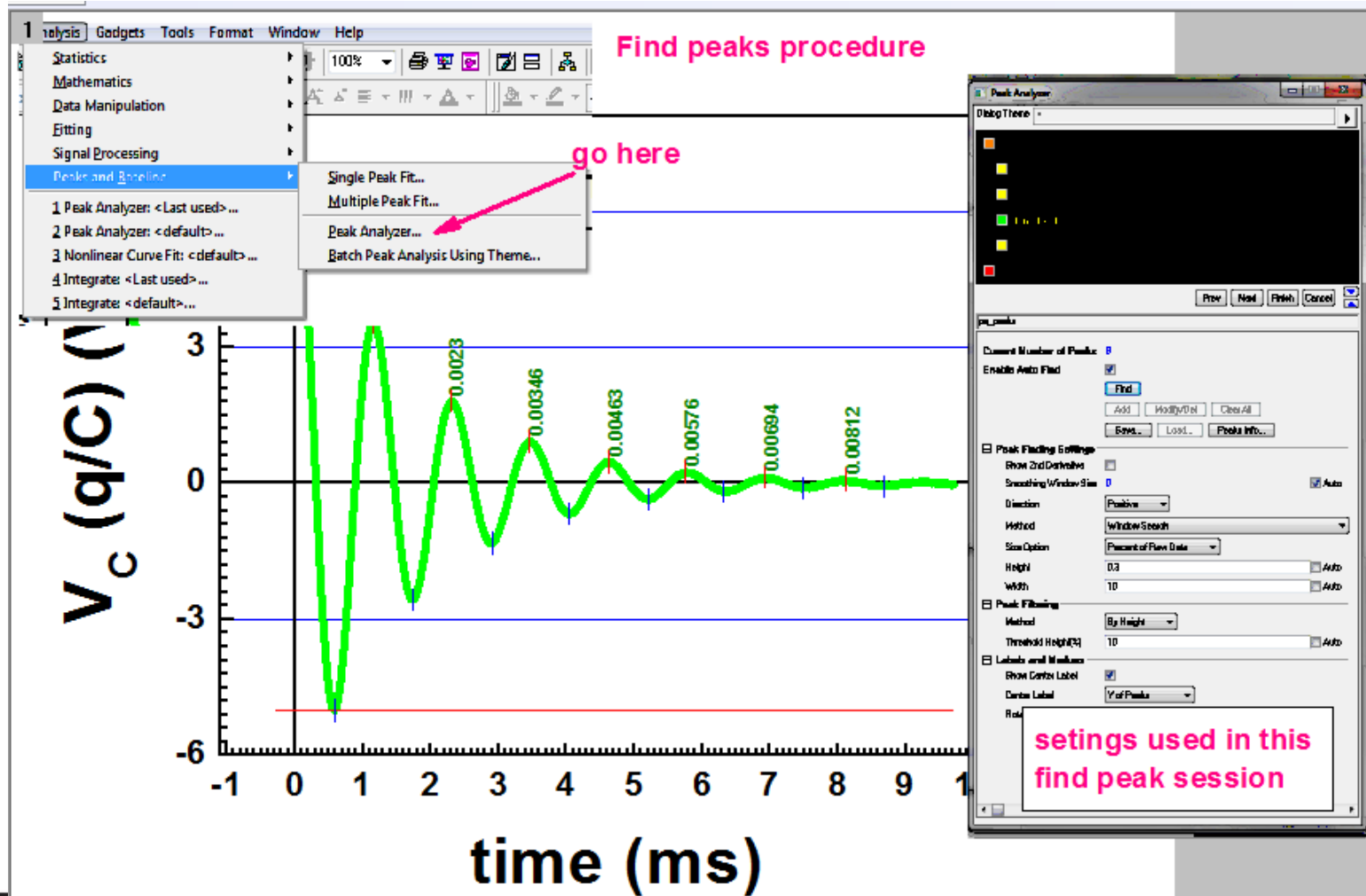2. **Envelope**
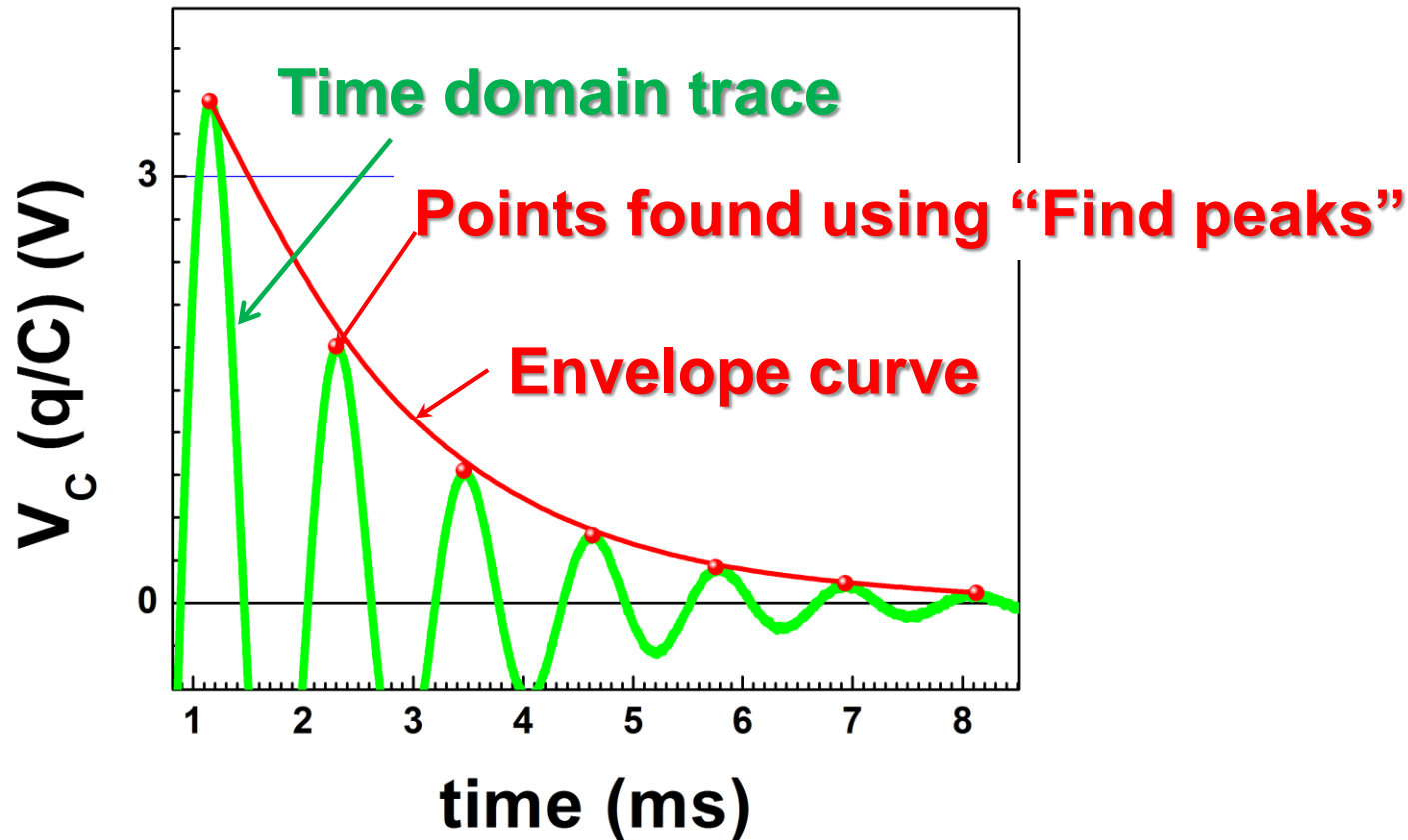3. **Exponential term**
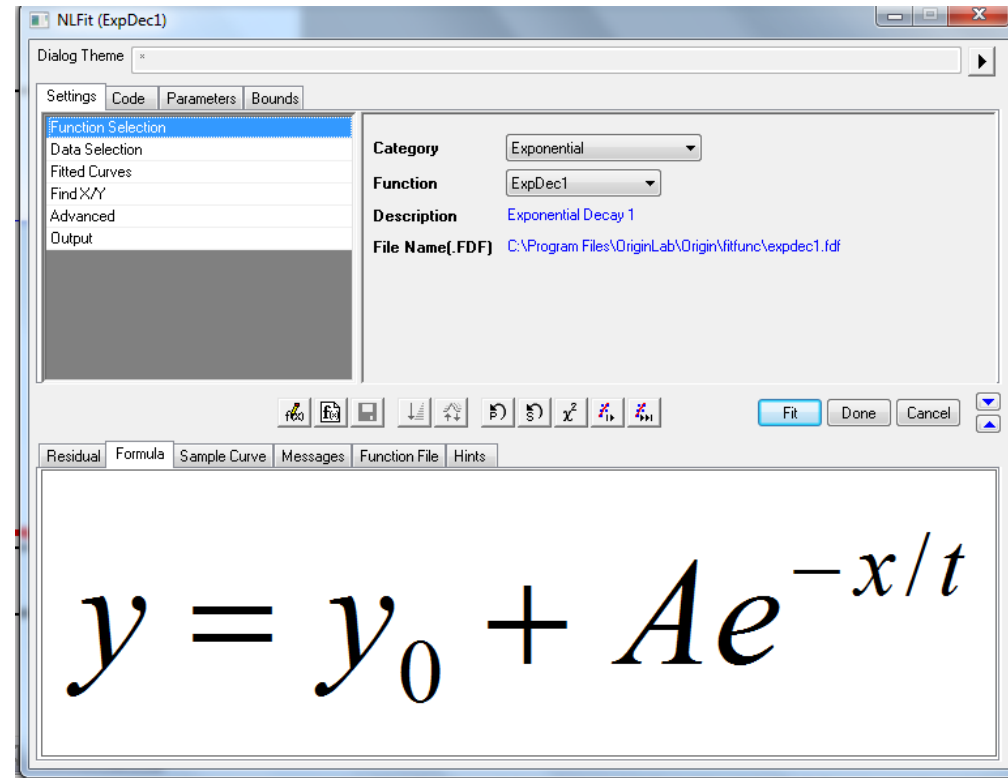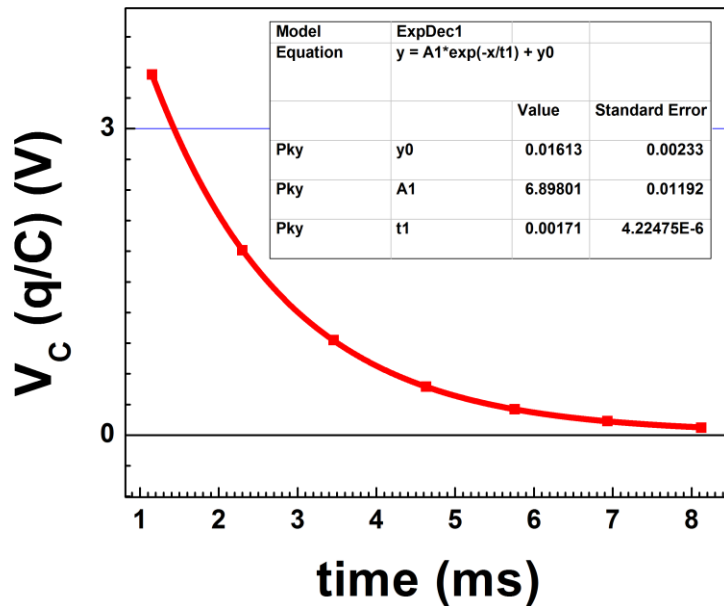4. **Nonlinear fitting**

Transients in RLC circuit. Under-damped solution. Log decrement. Quality factor.

Transients in RLC circuit. Data analysis. Log decrement. Using Origin software. Results.

Time domain trace

Points found using "Find peaks"

Envelope curve

Transients in RLC circuit. Data analysis. Log decrement. Using Origin software. Results.

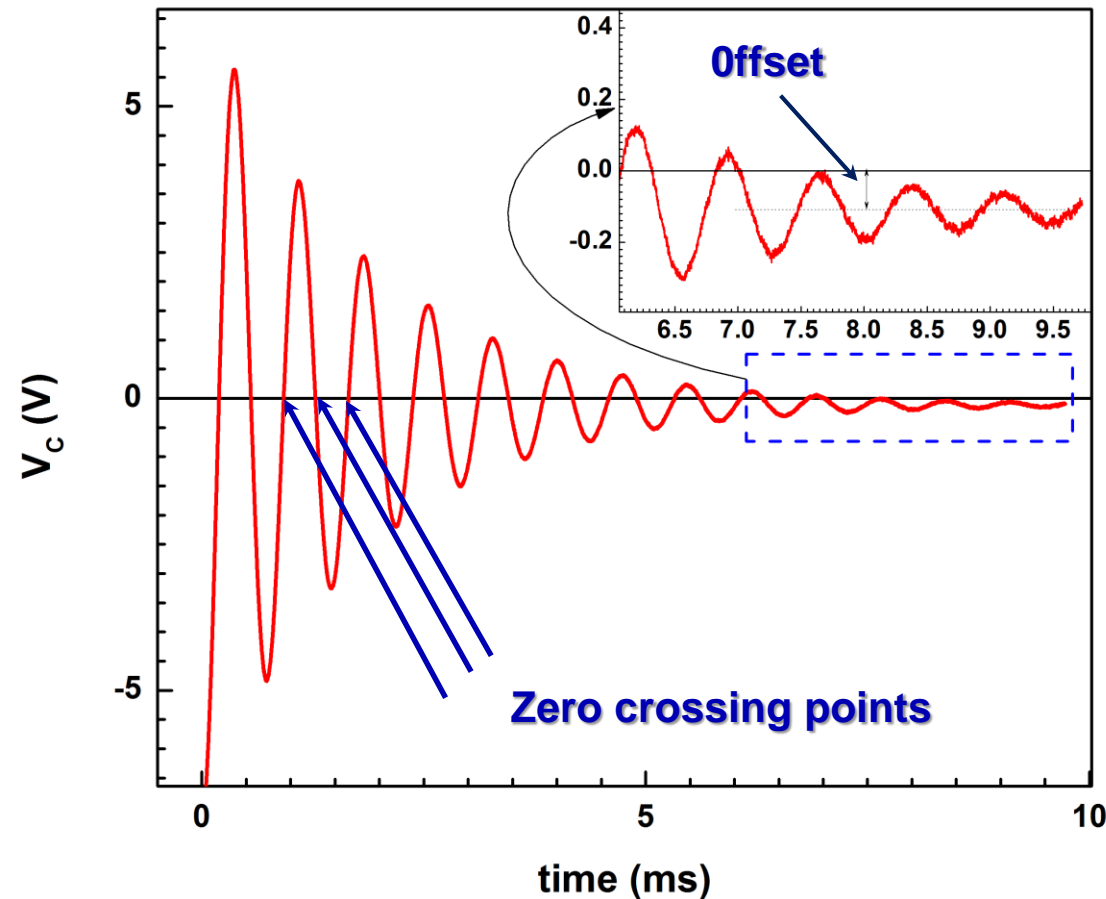| Model	ExpDec1		
Equation	y = A1*exp(-x/t1) + y0		
		Value	Standard Error
Pky	y0	0.01613	0.00233
Pky	A1	6.89801	0.01192
Pky	t1	0.00171	4.22475E-6

$$y = y_0 + Ae^{-x/t}$$

Fitting the "envelope data" to exponential decay function

Transients in RLC circuit. Data analysis. (1/T)² vs 1/C experiment.

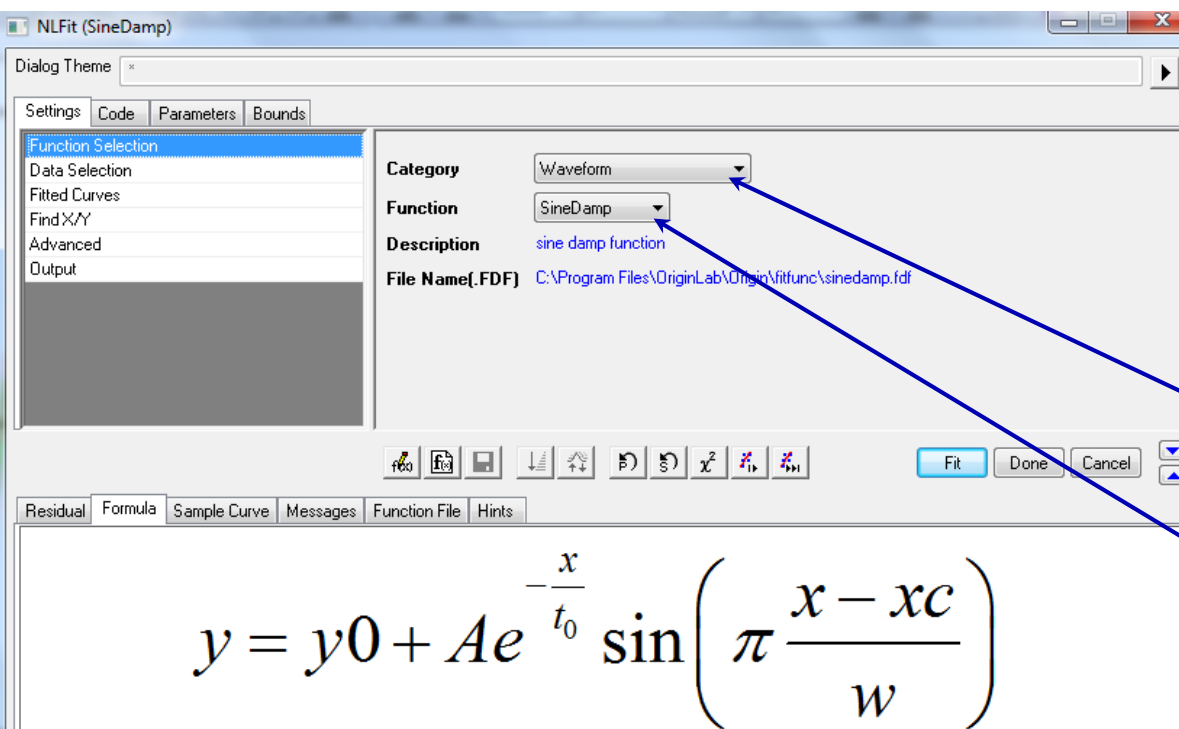$$q(t) = Ae^{-at}\sin(\omega t + \varphi) + \boxed{offset}$$

Manual evaluation of the period of the oscillations

Limited accuracy

Results can be effected by DC offset

Offset

Zero crossing points

Transients in RLC circuit. Data analysis. $(1/T)^2$ vs $1/C$ experiment. Using Origin software.

$$q(t) = Ae^{-at} s\text{in}(\omega t + \varphi)$$

Use Origin standard function

Category: Waveform

Function: SineDamp

$$y = y0 + Ae^{-\frac{x}{t_0}} \sin\left(\pi \frac{x - xc}{w}\right)$$

Fitting function ; y0, A, t_0 xc, w – fitting parameters

Transients in RLC circuit. Data analysis. (1/T)^2 vs 1/C experiment. Using Origin software.
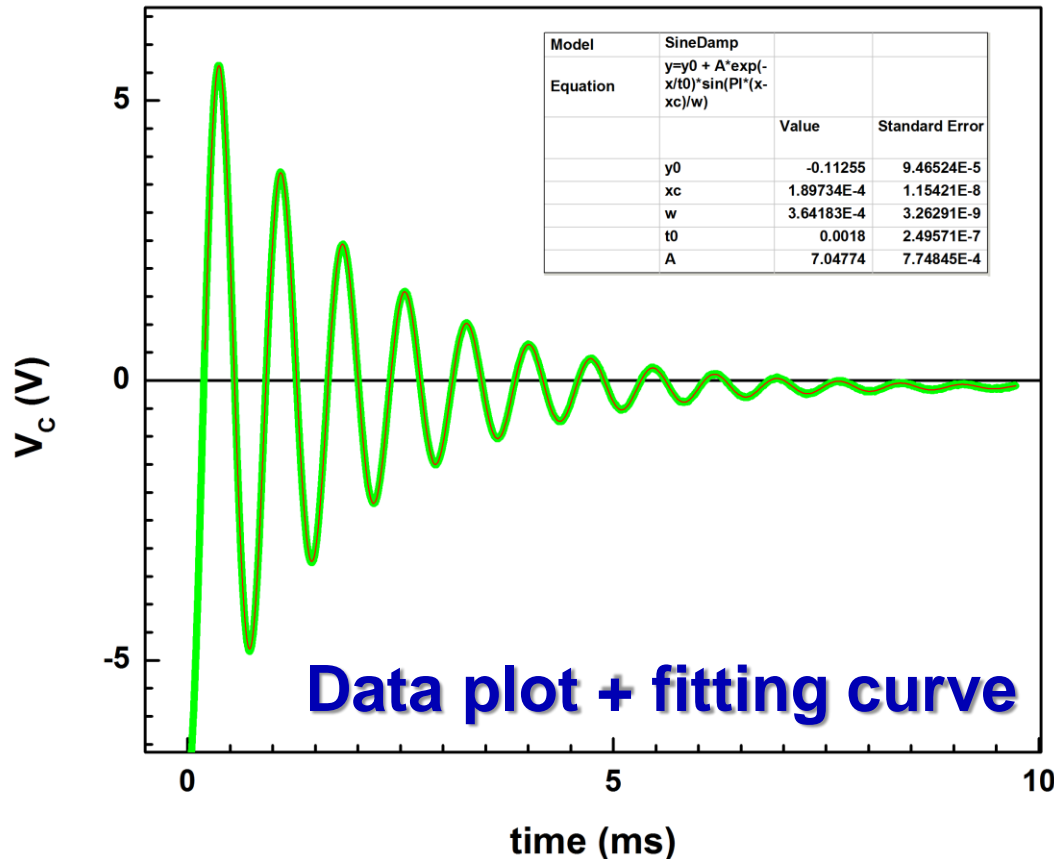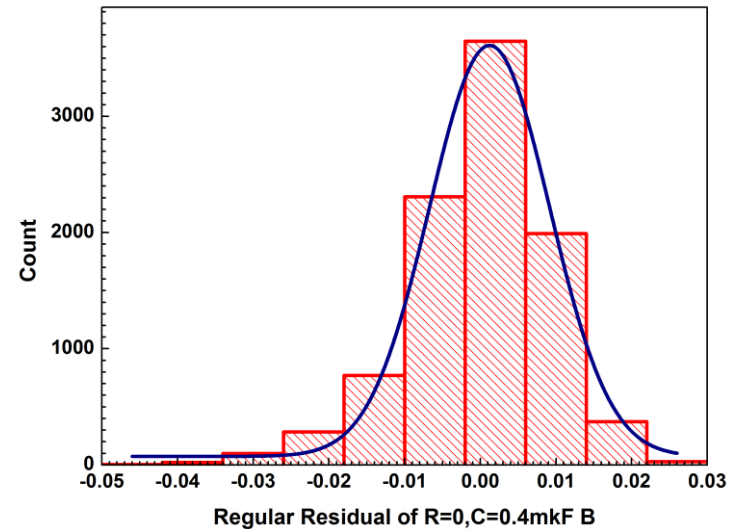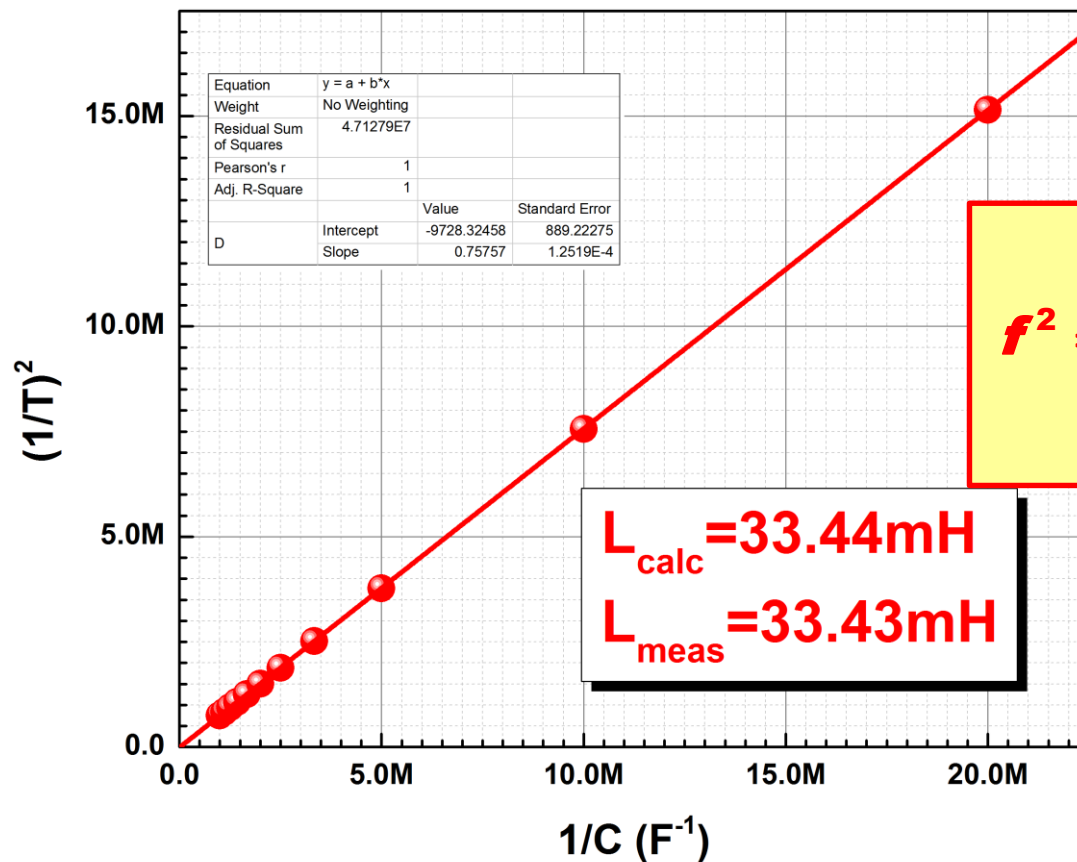

$$q(t) = Ae^{-at}\sin(\omega t + \varphi)$$

Model	SineDamp		
Equation	y=y0 + A*exp(-x/t0)*sin(PI*(x-xc)/w)		
		Value	Standard Error
	y0	-0.11255	9.46524E-5
	xc	1.89734E-4	1.15421E-8
	w	3.64183E-4	3.26291E-9
	t0	0.0018	2.49571E-7
	A	7.04774	7.74845E-4

Data plot + fitting curve

Residuals - criteria of quality of fitting

Transients in RLC circuit. Data analysis. $(1/T)^2$ vs $1/C$ experiment. Using Origin software.

$$q(t) = Ae^{-at}\sin(\omega t + \varphi)$$

Equation	y = a + b*x		
Weight	No Weighting		
Residual Sum of Squares	4.71279E7		
Pearson's r	1		
Adj. R-Square	1		
		Value	Standard Error
D	Intercept	-9728.32458	889.22275
	Slope	0.75757	1.2519E-4

$$f^2 = \left(\frac{1}{T}\right)^2 = \frac{1}{2\pi}\left(\left(\frac{1}{LC}\right) - \left(\frac{R}{2L}\right)^2\right)$$

L_{calc}=33.44mH

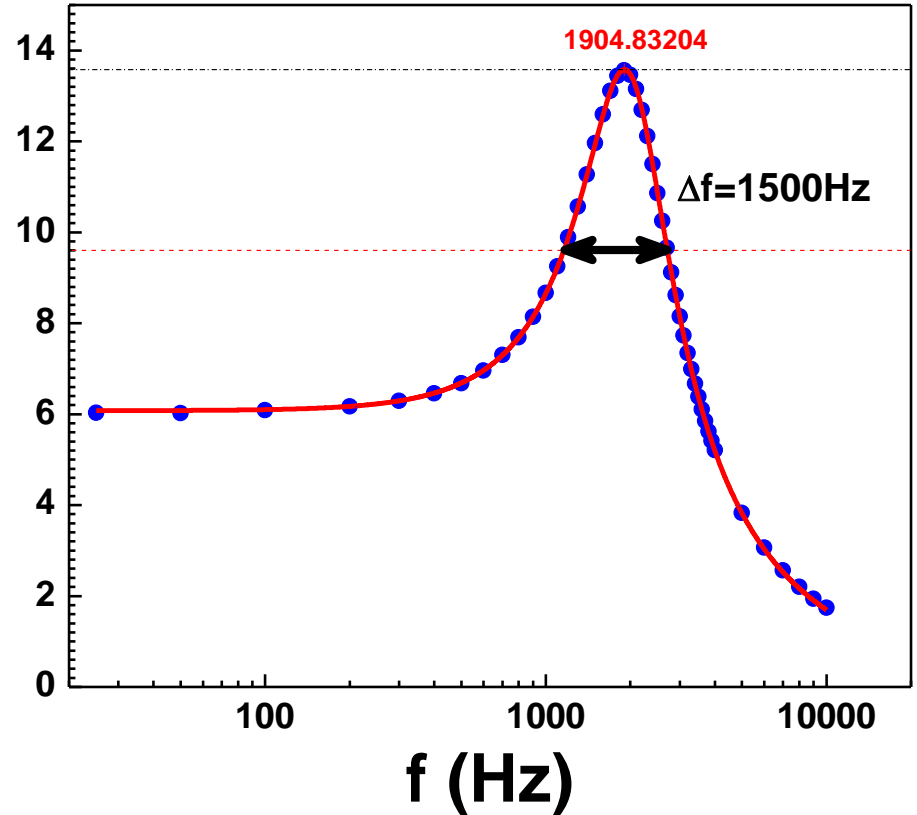L_{meas}=33.43mH

Final results

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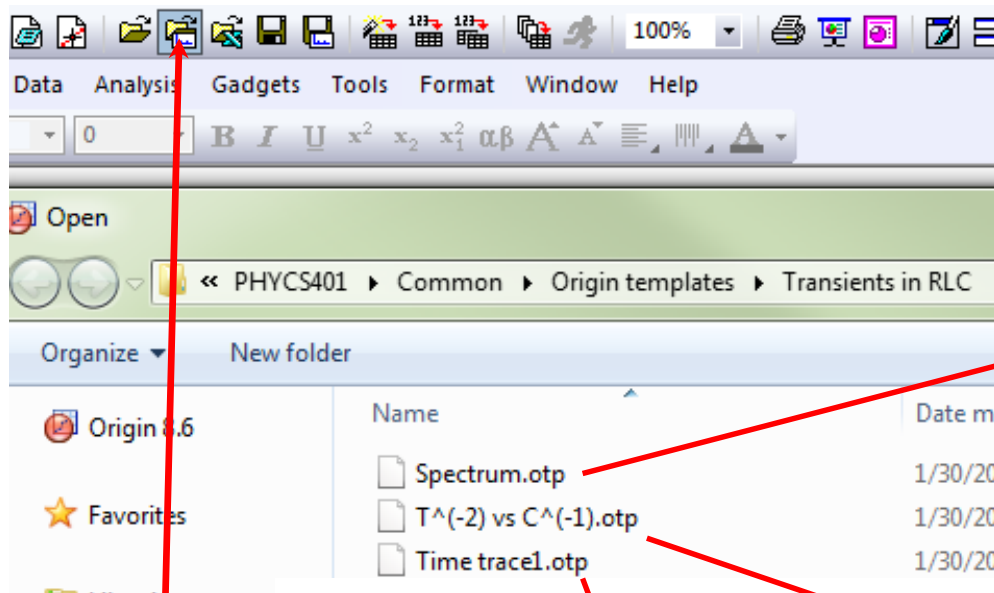
Resonance in RLC circuit.
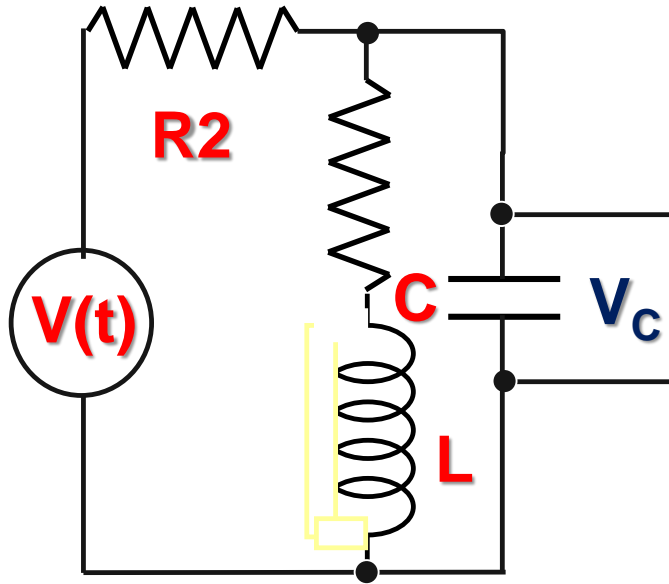
$$Q = \frac{f}{\Delta f} = \frac{1904}{1500} = 1.26$$
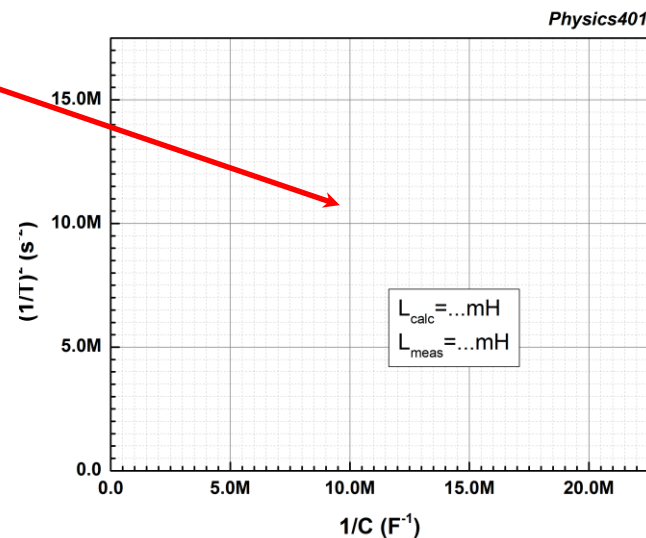
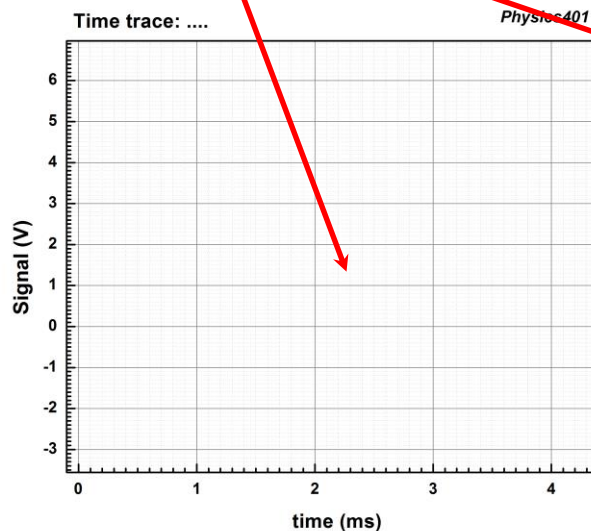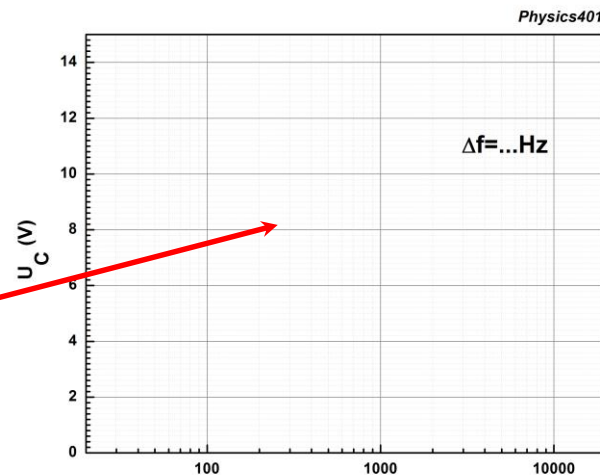# Origin templates for this week Lab.

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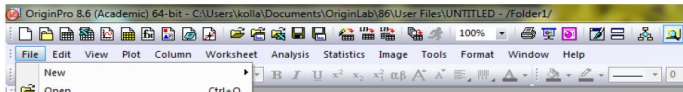

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Origin manuals

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OriginLab®

Working with Origin 8.6.

Step1. Importing data

Very short and simple manual which covers only main general operations with Origin. Document located on server and there is a link from P401 WEB page

There are also manuals from OriginLab.

Do not forget about Origin Help

Video Tutorials at the site of the company

OriginLab® Data Analysis and Graphing Software

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Video Tutorials

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