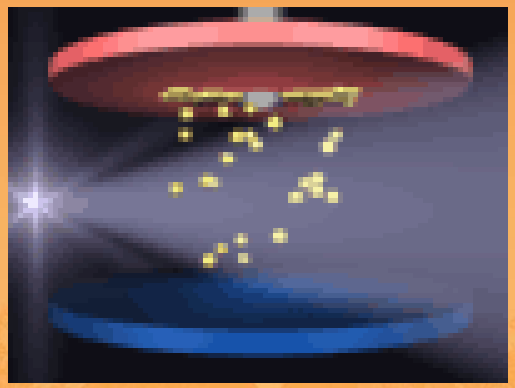
Millikan Oil Drop Experiment

Physics 401, Spring 2015

Eugene V. Colla



Agenda

1. Measuring of the charge of electron.
2. Robert Millikan and his oil drop experiment
3. Theory of the experiment
4. Laboratory setup
5. Data analysis



Measuring of the charge of the electron

1. Oil drop experiment. Robert A. Millikan.. (1909). $e=1.5924(17)\times10^{-19}$ C
2. Shot noise experiment. First proposed by Walter H. Schottky
3. In terms of the Avogadro constant and Faraday constant $e = \frac{F}{N_A}$; F- Faraday constant, N_A - Avagadro constant. Best uncertainty ~1.6 ppm.
4. From Josephson ($K_J = \frac{2e}{h}$) and von Klitzing $\left(R_K = \frac{h}{e^2}\right)$ constants
5. Recommended by NIST value 1.602 176 565(35) 10^{-19} C



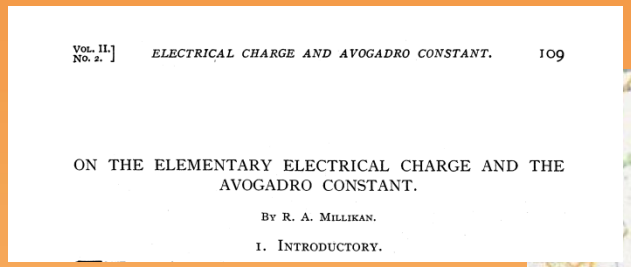
Robert Millikan. Oil drop experiment



ROBERT ANDREWS MILLIKAN
1868-1953



The Nobel Prize in Physics 1923.
Robert A. Millikan "for his work on the elementary charge of electricity and on the photoelectric effect".



Vol. II.] No. 2. ELECTRICAL CHARGE AND AVOGADRO CONSTANT. 109

ON THE ELEMENTARY ELECTRICAL CHARGE AND THE AVOGADRO CONSTANT.

By R. A. Millikan.

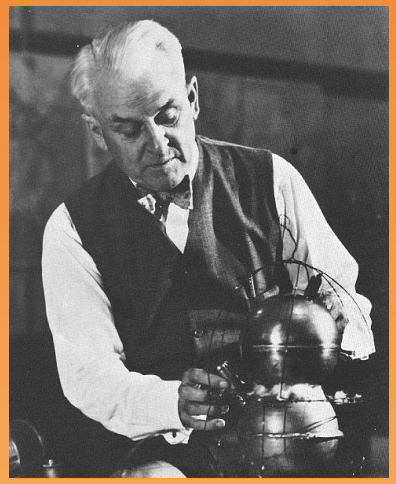
1. Introductory.



University of Chicago



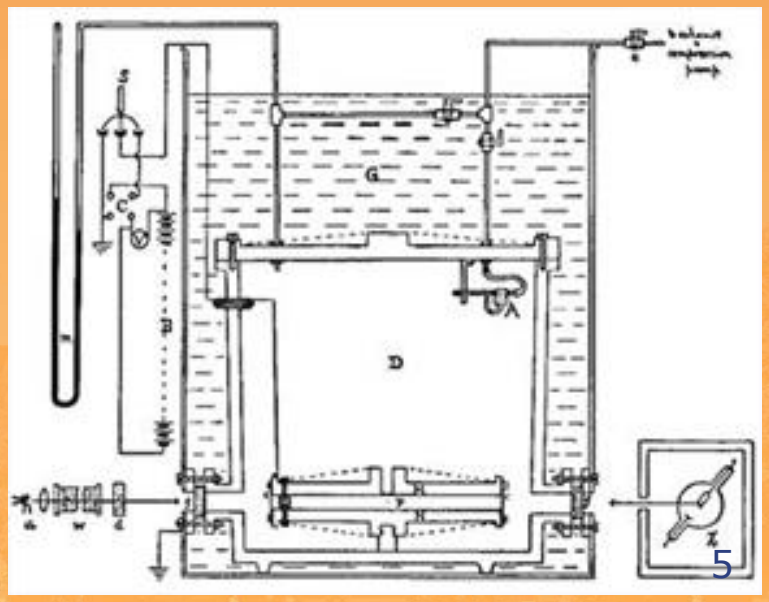
Robert Millikan. Oil drop experiment



ROBERT ANDREWS
MILLIKAN
1868-1953



Diagram and picture of apparatus



Oil drop experiment.

Motivation:

Measurement of the magnitude of the electron charge!

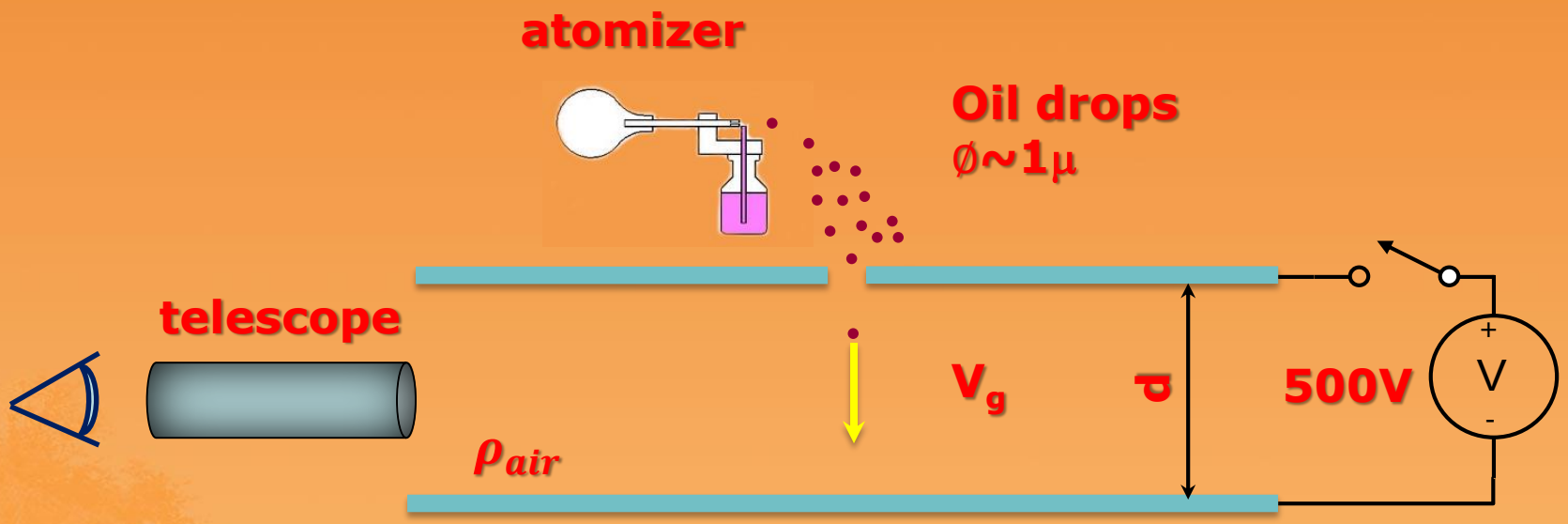
Demonstrate that the electron charge is quantized!



Picture of the PASCO setup

Measure the charge of an electron to ±3%

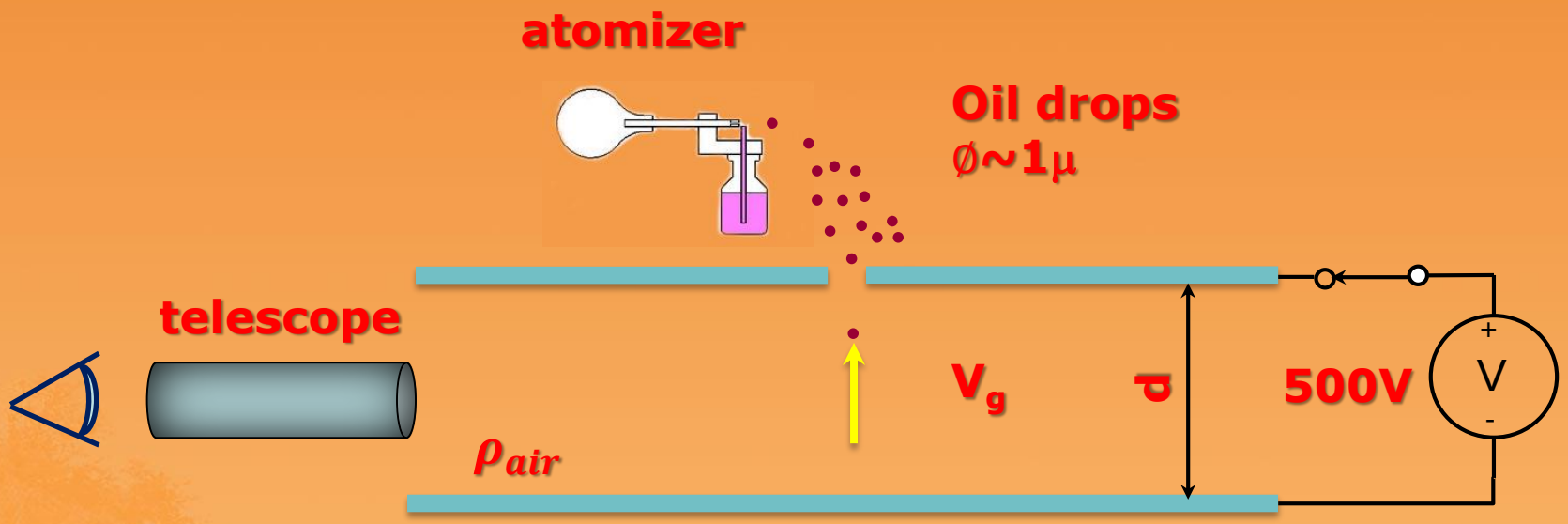
Oil drop experiment.



Forces on the oil drop:

1) Gravity + buoyant force (air displaced by oil drop)
2) Drag force of the oil drop in the air

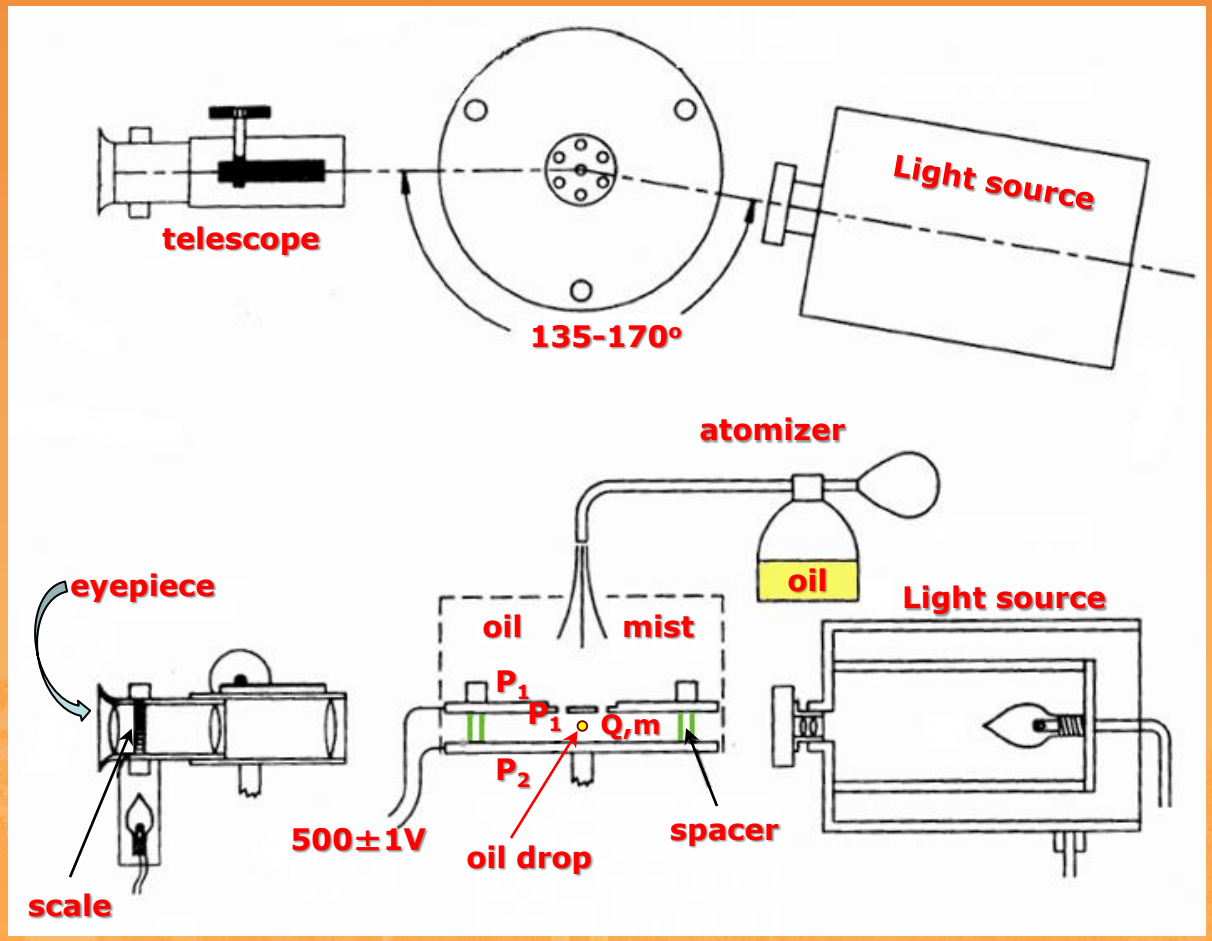
Oil drop experiment.



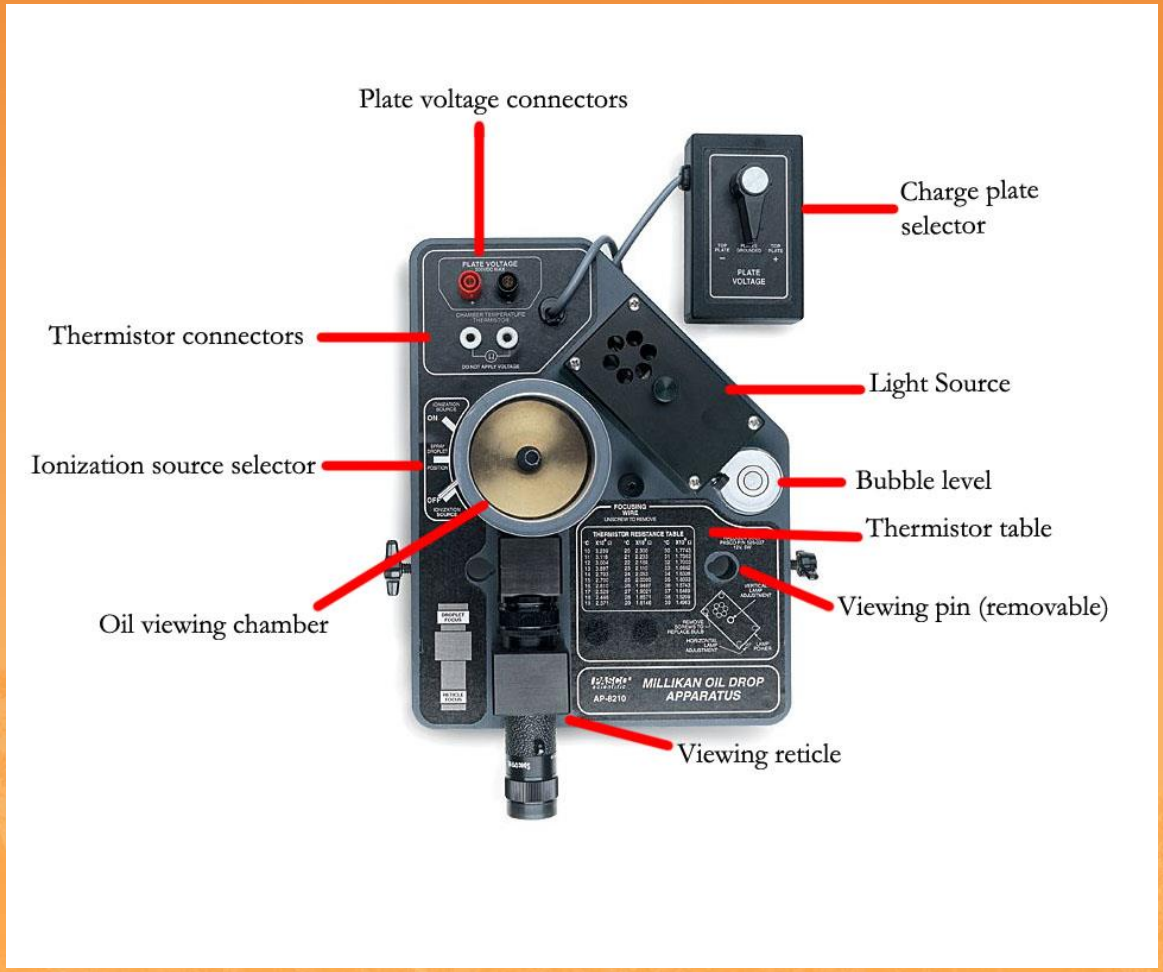
Forces on the oil drop:

1) Gravity + buoyant force (air displaced by oil drop)
2) Drag force of the oil drop in the air
3) Electric force on oil drops which carry charge Q

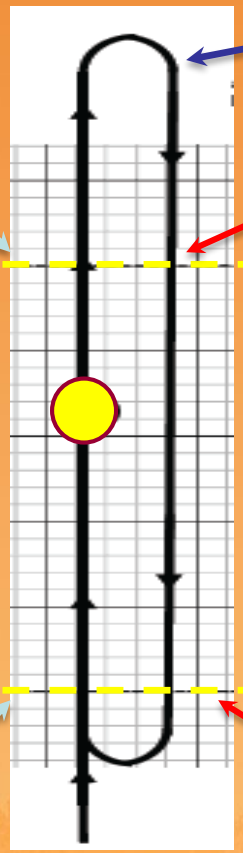
Apparatus. Schematic Layout



Apparatus: actual setup



What is measured



Allow drop to "undershoot" here before starting next rise time experiment

rise time measurement stops here

fall time measurement starts here

fall time t_g

rise time t_{rise}

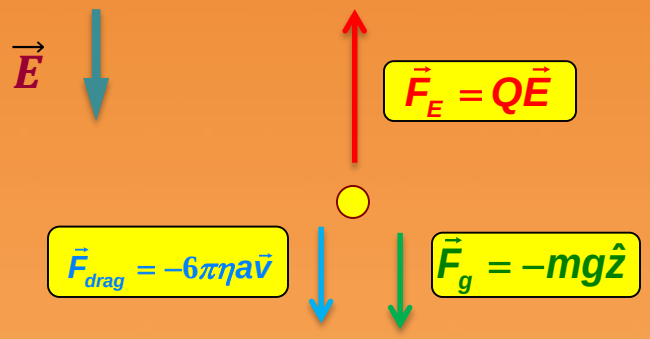
x

x = fall distance = rise distance. *x* must be the same for all drops!

fall time measurement stops here

rise time measurement starts here

Balance of Forces: Newton's Law



$$\vec{F}_g = -mg\hat{z} \qquad (1)$$

$$\vec{F}_{drag} = -6\pi\eta a\vec{v} \qquad (2)$$

$$\vec{F}_E = Q\vec{E} \qquad (3)$$

$$\vec{F} = m\frac{d\vec{v}}{dt} = \vec{F}_g + \vec{F}_{drag} + \vec{F}_E$$

a : radius of drop
ρ : density $\rho = \rho_{oil} - \rho_{air}$
v : velocity of oil drop
Q : charge of oil drop
E : electric field $E=V/d$
V : Voltage across plates
η : viscosity of air
g : gravitational const.

Forces on the oil drop:
(1) Gravity + buoyant force (air displaced by oil drop)
(2) Drag force of the oil drop in the air
(3) Electric force on oil drops which carry charge Q

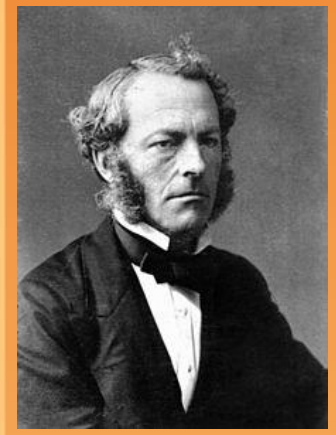
Particle reached terminal velocity $\frac{d\vec{v}}{dt} = 0$

$$\vec{F}_g + \vec{F}_{drag} + \vec{F}_E = 0$$

1μ size particle reaches the terminal velocity in ~10^{-5}s



Modification to Stokes Law



George Gabriel Stokes
(1819-1903)

$$\vec{F}_{drag} = -6\pi\eta a\vec{v}$$

For small particle radius (a<15μ) Stokes law need to be corrected. This correction was derived by E. Cunningham.

$$\vec{F}_{drag} = -6\pi\eta \frac{a}{f_c}\vec{v}$$



Ebenezer Cunningham
(1881-1977)

$$f_c = 1 + A\frac{\lambda}{a} + B\frac{\lambda}{a}e^{-C\frac{a}{\lambda}}, \quad A = 1.246,\ B = 0.42,\ C = 0.78$$

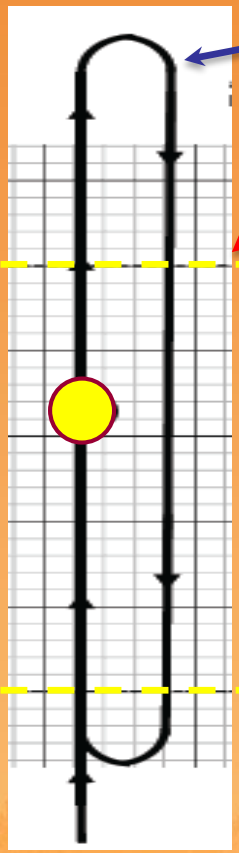
$$f_c \approx 1 + A\frac{\lambda}{a} = 1 + \frac{r_c}{a} \approx 1.1, \quad \text{for } a \approx 10^{-6}\text{m}, \quad r_c = \frac{6.18\times10^{-5}}{p[\text{mmHg}]}$$

Here **a** – particle radius; **λ** – mean free path of the gas molecules

negligible term

$$\lambda\ [\text{m}] = 6.53\times10^{-8}\frac{760}{p[\text{mmHg}]}$$

We measure: t_g and t_{rise}



Allow drop to "undershoot" here before starting next rise time experiment

rise time measurement stops here

fall time measurement starts here

fall time t_g

rise time t_{rise}

x

x = fall distance = rise distance. *x* must be the same for all drops!

rise time measurement starts here

fall time measurement stops here

Solving Newton's law: $Q(t_g, t_{rise})$

f_c can be found from Newton law equation in the case of V=0 (falling drop)

$$\vec{F}_g + \vec{F}_{drag} = \frac{4}{3}a^3\rho g - 6\pi\eta\frac{a}{f_c}\vec{v} = 0$$



(see write-up)

$$\frac{1}{f_c^{\frac{2}{3}}} \approx 1 - \left(\frac{t_g}{\tau_g}\right)^{\frac{1}{2}}; \quad \tau_g = \frac{2\eta x}{\rho g r_c^2}; \quad r_c[m] = \frac{6.18\times10^{-5}}{p[mmHg]}$$



Solving Newton's law: $Q(t_g, t_{rise})$

$$Q = n \times e = \frac{1}{f_c^{3/2}} \frac{9\pi d}{V} \sqrt{\frac{2\eta^3 x^3}{g\rho}} \sqrt{\frac{1}{t_g}} \left[\frac{1}{t_g} + \frac{1}{t_{rise}} \right]$$

- Q : charge of oil drop
- n : number of unpaired electrons in drop
- e : elementary charge
- d : plate separation
- V : Voltage across plates
- ρ : density $\rho = \rho_{oil} - \rho_{air}$
- η : viscosity of air
- g : gravitational constant
- x : drift distance for oil drop
- t_g : fall time
- t_{rise} : rise time



Route of charge calculation $Q(t_g, t_{rise})$.

$$\frac{1}{f_c^{\frac{3}{2}}} \approx 1 - \left(\frac{t_g}{\tau_g}\right)^{\frac{1}{2}} \; ; \quad \tau_g = \frac{2\eta x}{\rho g r_c^2} ; \quad r_c[m] = \frac{6.18\times10^{-5}}{p[mmHg]}$$

$$Q = F \bullet S \bullet T = \left(\frac{1}{f_c^{\,3/2}}\right) \frac{9\pi d}{V} \sqrt{\frac{2\eta^3 x^3}{g\rho}} \frac{1}{\sqrt{t_g}} \left(\frac{1}{t_g} + \frac{1}{t_{rise}}\right)$$

$$F = \frac{1}{f_c^{\,3/2}} \approx 1 - \left(\frac{t_g}{\tau_g}\right)^{\frac{1}{2}}$$

$$S = \frac{9\pi d}{V} \sqrt{\frac{2\eta^3 x^3}{g\rho}}$$

$$T = \frac{1}{\sqrt{t_g}} \left(\frac{1}{t_g} + \frac{1}{t_{rise}}\right)$$



Route of charge calculation. Origin projects. Data collecting.

Projects ***Section L1.opj … Section L4.opg***

Locations:

\\engr-file-03\PHYINST\APL Courses\PHYCS401\Common\Origin templates\Oil drop experiment

\\engr-file-03\PHYINST\APL Courses\PHYCS401\Students\1.Millikan Oil Drop experiment

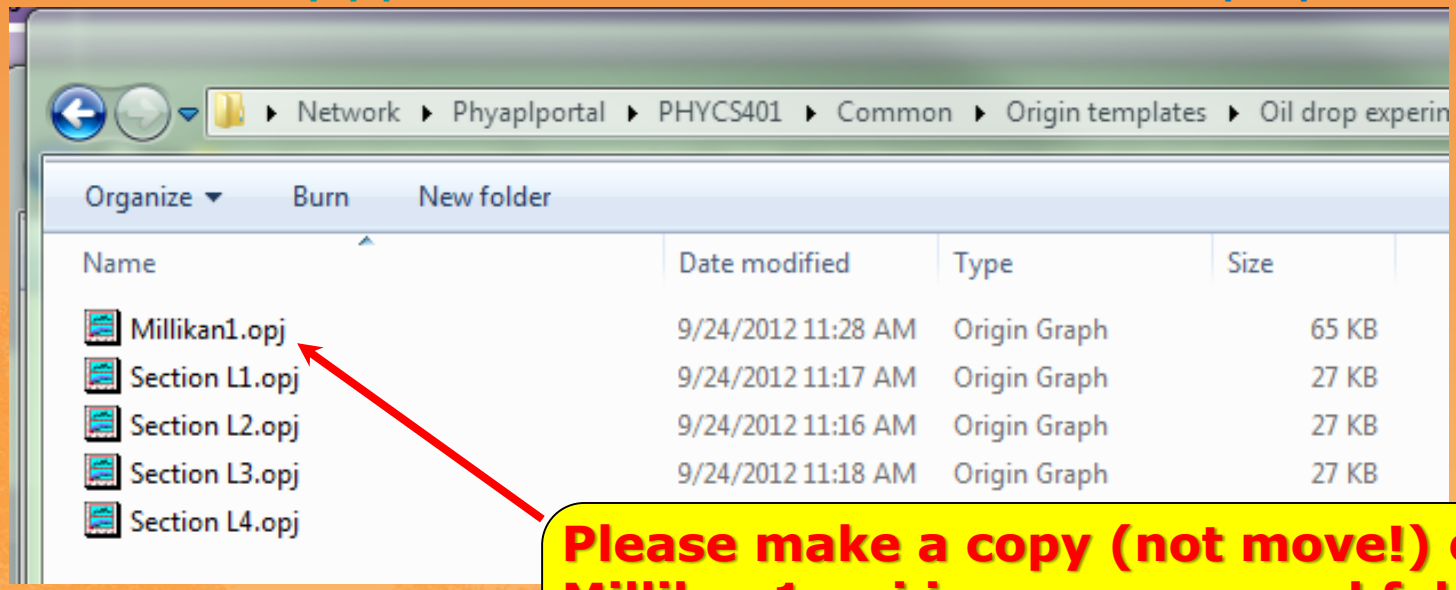
	A(L)	B(Y)	C(Y)	D(Y)	E(Y)	F(Y)	G(Y)	H(Y)	
Long Name	parameter label	Par	tg	tr	parameter label	Par	tg	tr	pa
Units									
Comments	*student1, student2*	*student1, student2*	*student1, student2*	*student1, student2*	*student3, student4*	*student3, student4*	*student3, student4*	*student3, student4*	*s*
1	η				η				η
2	Δη/ΔT				Δη/ΔT				Δη
3	ρ1				ρ1				ρ1
4	ρ2				ρ2				ρ2
5	ρ1−ρ2				ρ1−ρ2				ρ1
6	g				g				g
7	p				p				p
8	x				x				x
9	d				d				d
10	V				V				V
11	Ta				Ta				Ta
12									
13									
14									
15									
16									
17									



Route of charge calculation. Origin projects. Data analysis.

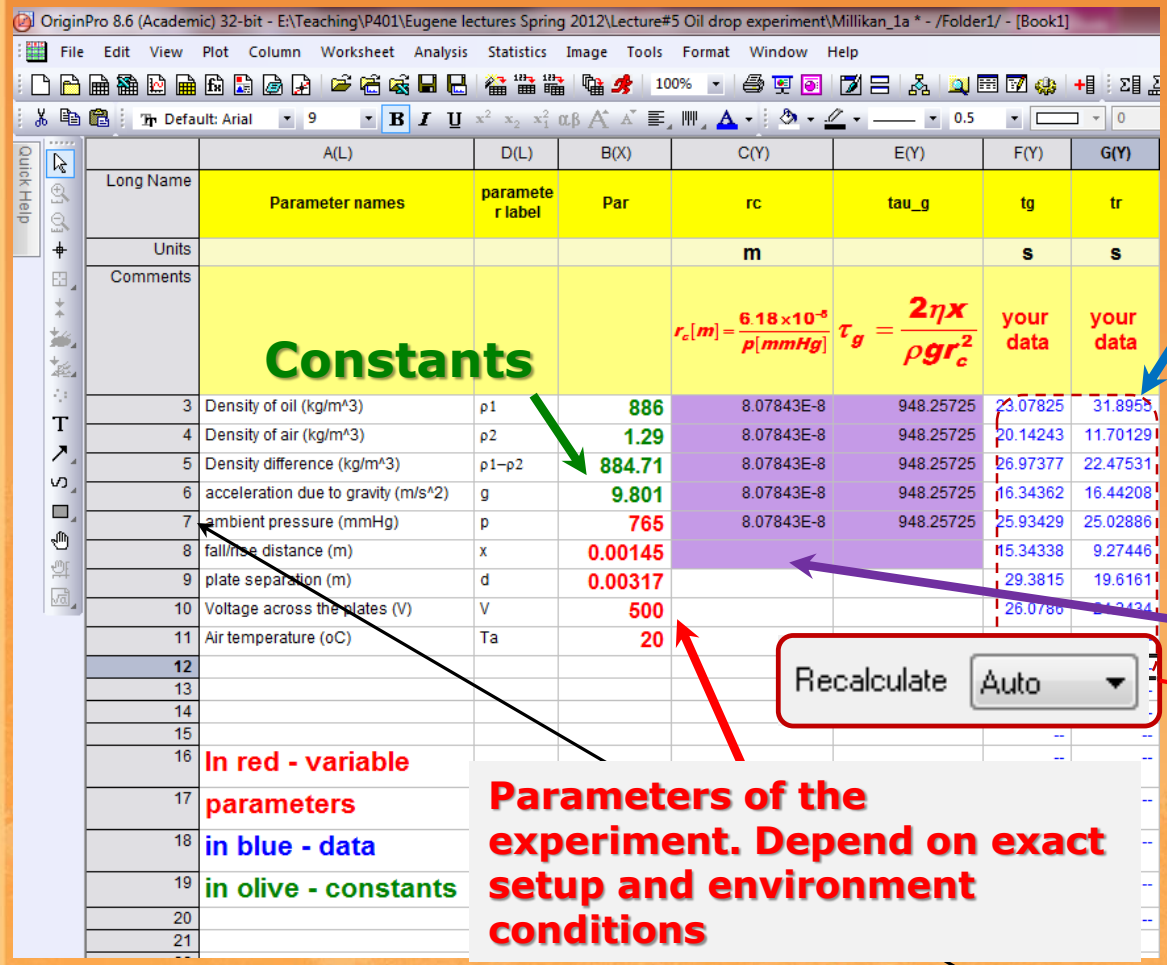
Project: ***Millikan1.opj***

Locations: **\\Phyaplportal\PHYCS401\Common\Origin templates\Oil drop experiment**
\\Phyaplportal\PHYCS401\Students\1. Millikan Oil Drop experiment



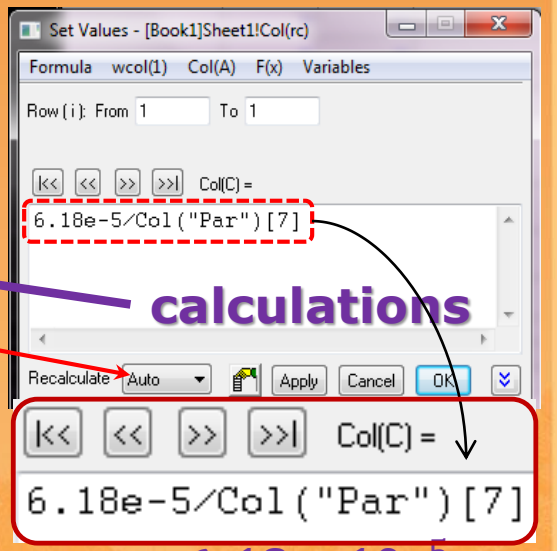
Please make a copy (not move!) of Millikan1.opj in your personal folder and start to work with your personal copy of the project

Route of charge calculation. Origin project. Data analysis.



Project ***Millikan1.opj***

Plugin your data here



calculations

$$r_c[m]=\frac{6.18\times10^{-5}}{p[mmHg]}$$

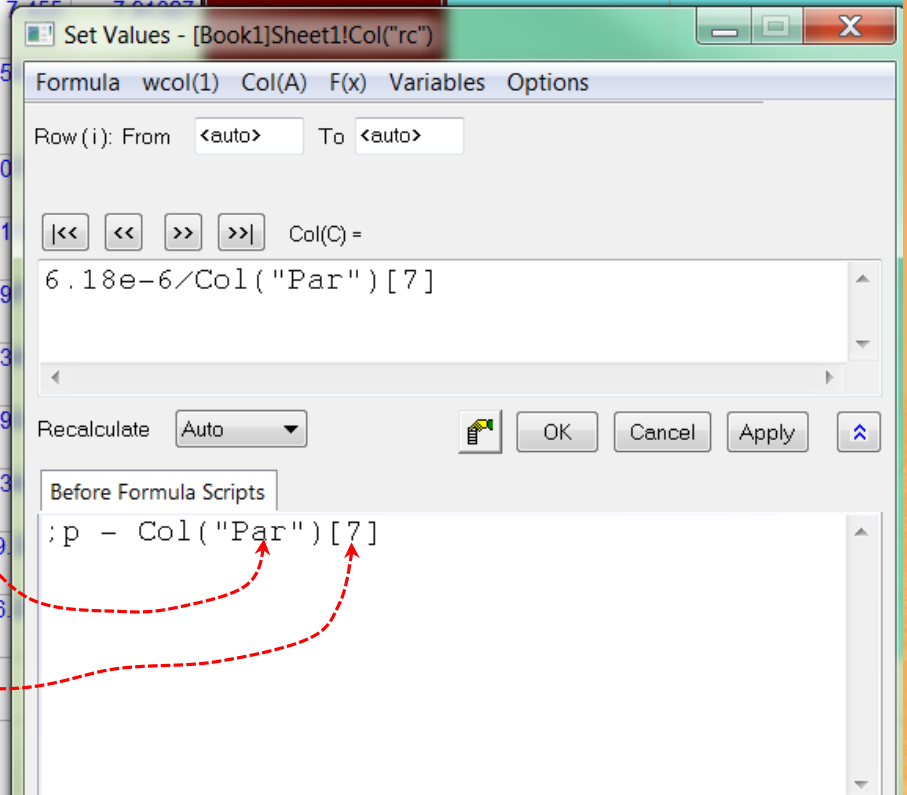
P(mmHg)→Col("Par")[7]

index

Route of charge calculation. Origin project. Data analysis.

Project ***Millikan1.opj***

	D(L)	B(X)	F(Y)	G(Y)	C(Y)	E(Y)	H(Y)
Long Name	parameter label	Par	tg	tr	rc	tau_g	F
Units			s	s	m		
Comments			your data	your data	$r_c[m]=\frac{6.18\times10^{-6}}{p[mmHg]}$	$\tau_g=\frac{2\eta x}{\rho g r_c^2}$	$F=\frac{1}{f_c^{3/2}}\approx 1-$
1	η	1.8478E-5					
2	$\frac{d\eta}{dT}$	4.8E-8	15.5				
3	$\rho 1$	886	23.0				
4	$\rho 2$	1.29	20.1				
5	$\rho 1-\rho 2$	884.71	26.9				
6	g	9.801	16.3				
7	p	765	25.9				
8	x	0.00145	15.3				
9	d	0.00317	29.				
10	V	500	26.				
11	Ta	20					
12	$\eta(T)=\eta(25C^\circ)-\frac{d\eta}{dT}(T^\circ-25)$	1.8478E-5					



Route of charge calculation. Origin project. Data analysis.

Project ***Millikan1.opj***

H(Y)	I(Y)	J(Y)	K(Y)	L(Y)
F	S	T	Q=FST	n=Q/1.602e-19
$F = \frac{1}{f_c^{3/2}} \approx 1 - \left(\frac{t_g}{\tau_g}\right)^{\frac{1}{2}}$	$S = \frac{9\pi d}{V}\sqrt{\frac{2\eta^3 x^3}{g\rho}}$	$T = \frac{1}{\sqrt{t_g}}\left(\frac{1}{t_g} + \frac{1}{t_{rise}}\right)$	Q=F*S*T	number of elementary charges
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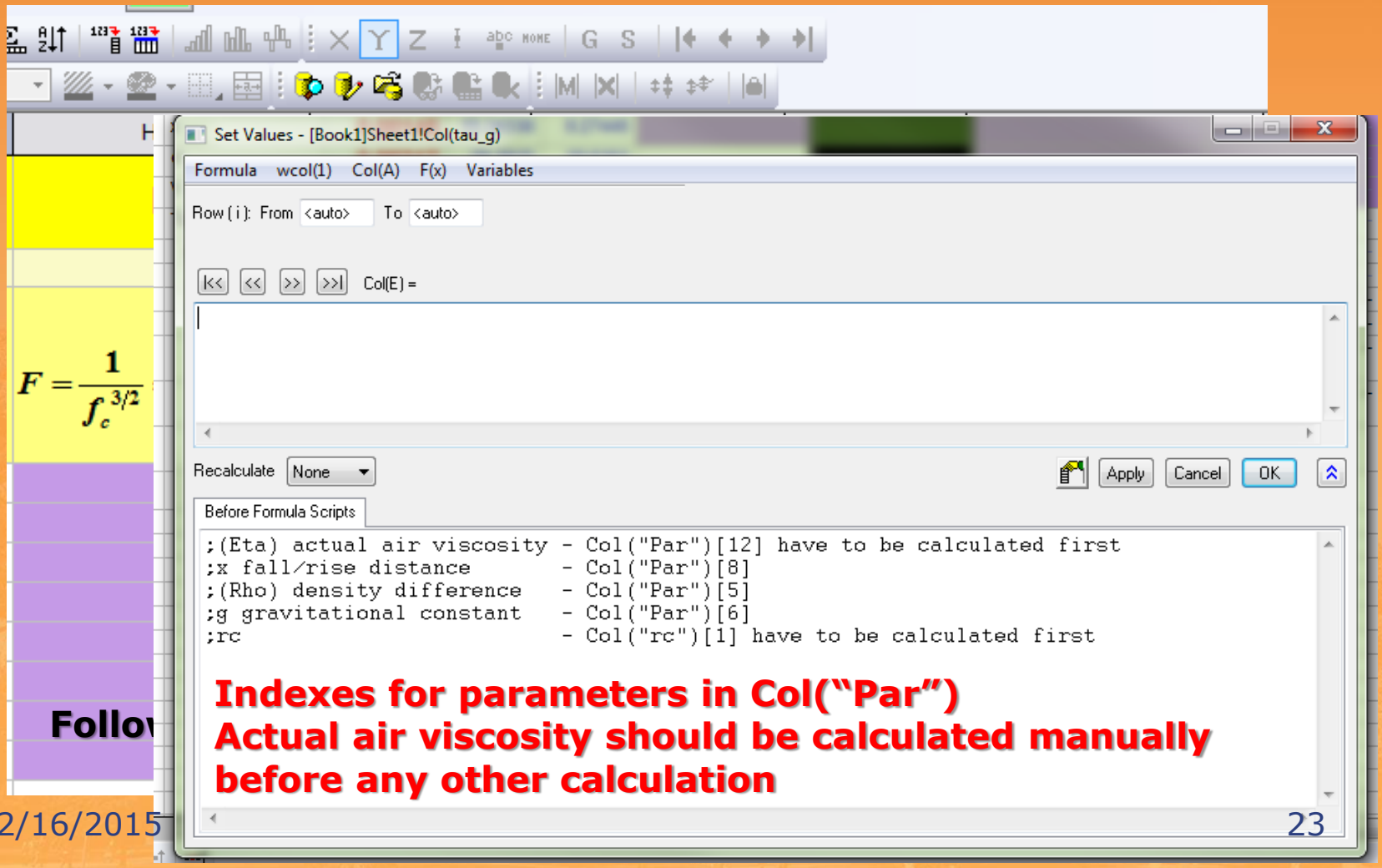
$$Q = F \bullet S \bullet T = \left(\frac{1}{f_c^{3/2}}\right)\frac{9\pi d}{V}\sqrt{\frac{2\eta^3 x^3}{g\rho}}\frac{1}{\sqrt{t_g}}\left(\frac{1}{t_g} + \frac{1}{t_{rise}}\right)$$

Follow correct order of calculations: $r_c \rightarrow \tau_g \rightarrow (F,S,T) \rightarrow Q \rightarrow n$

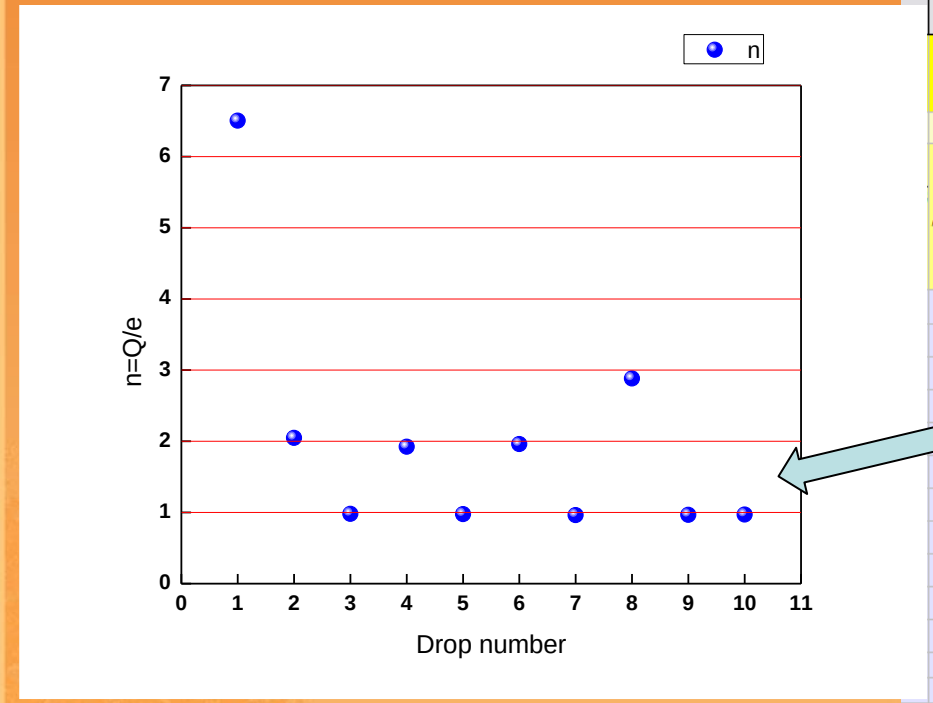


Route of charge calculation. Origin project. Data analysis.

Project *Millikan1.opj*

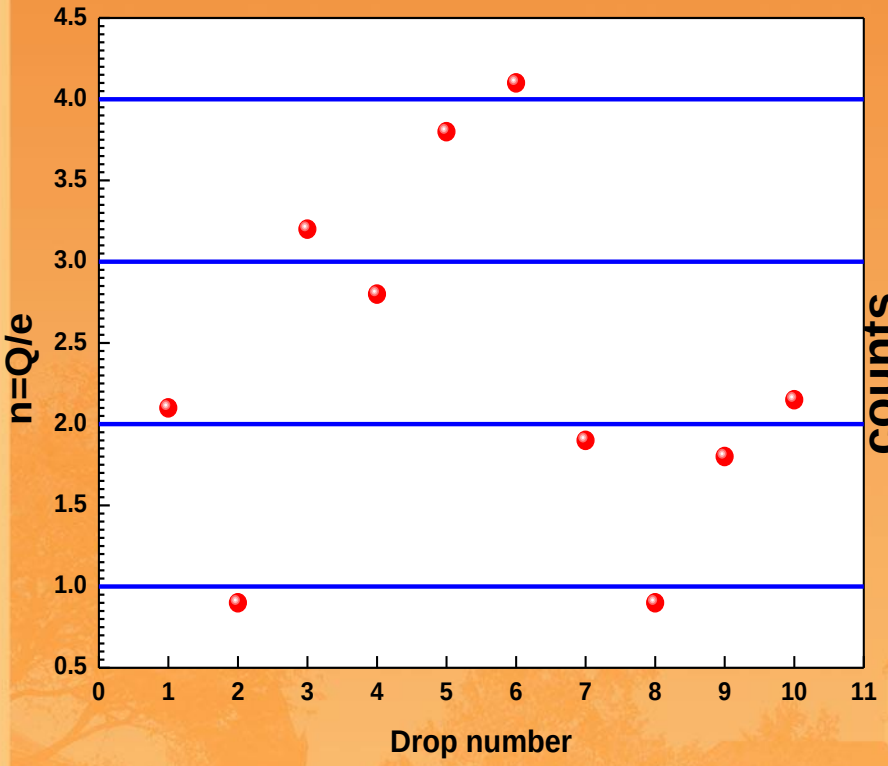


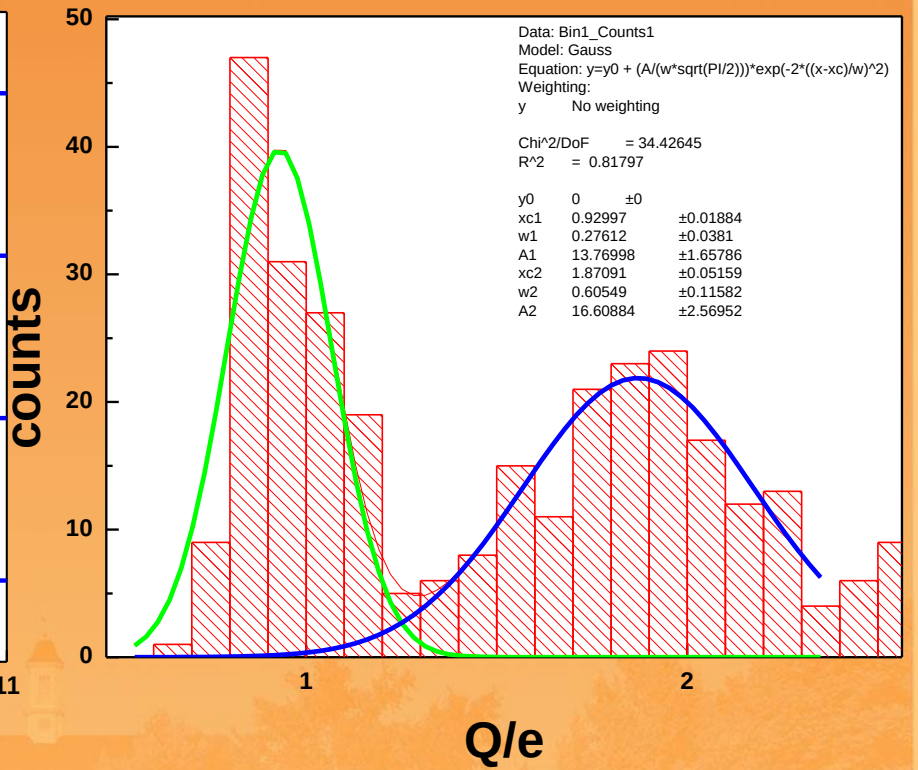
Charge calculation. Origin project.



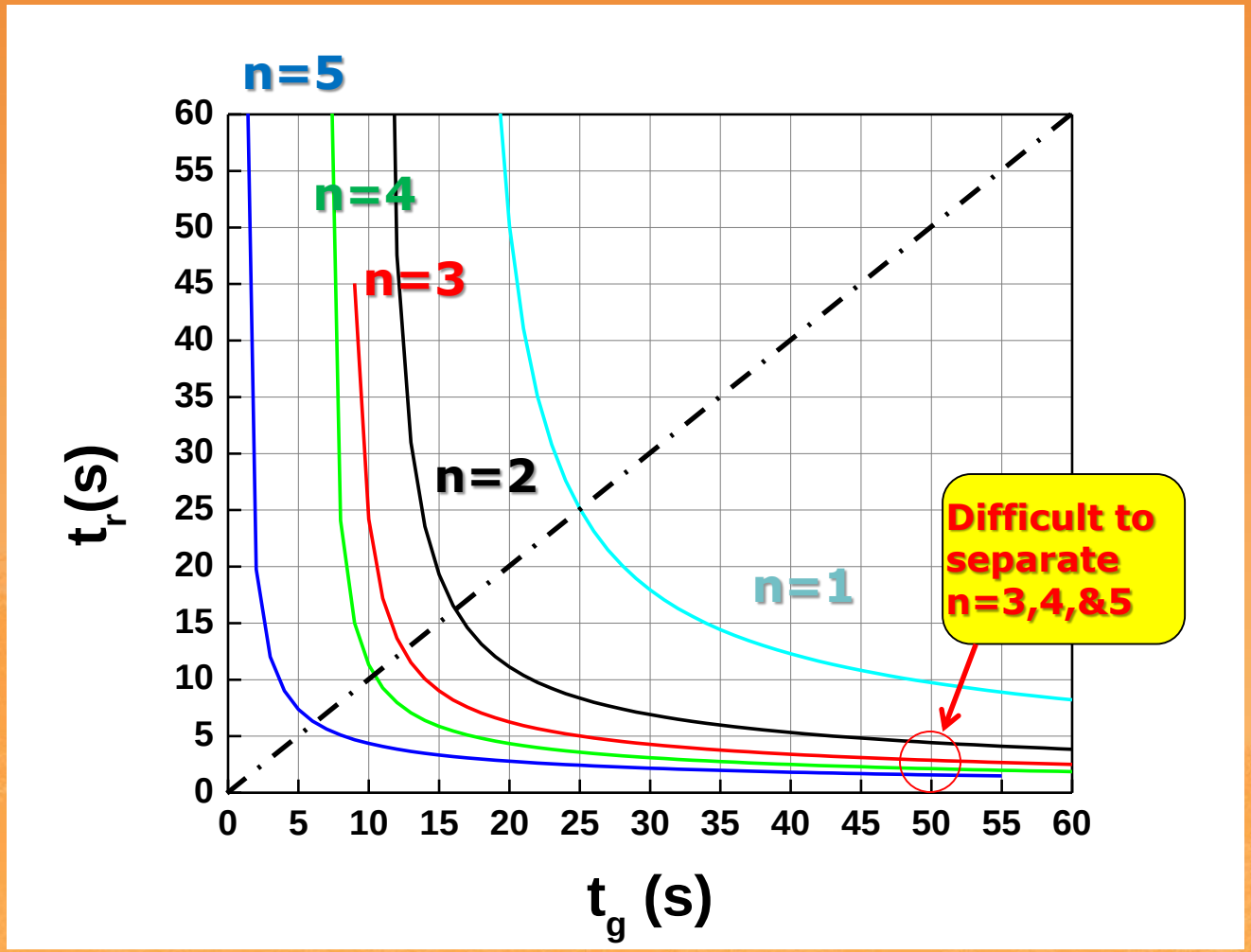
I(Y)	J(Y)	K(Y)	L(Y)
S	T	Q=FST	n=Q/1.602e-19
			c
$S=\frac{9\pi d}{V}\sqrt{\frac{2\eta^3 x^3}{g\rho}}$	$T=\frac{1}{\sqrt{t_g}}\left(\frac{1}{t_g}+\frac{1}{t_{rise}}\right)$	Q=F*S*T	number of elementary charges
--		1.04196E-	6.50414
--		.945E-19	2.0471
--		1.57231E-19	0.98147
		3.08168E-19	1.92365
--		1.56458E-19	0.97664
--		3.14167E-19	1.9611
--		1.54196E-19	0.96252
--		4.61924E-19	2.88342
--		1.54861E-19	0.96668
--		1.55468E-19	0.97046
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Expected results



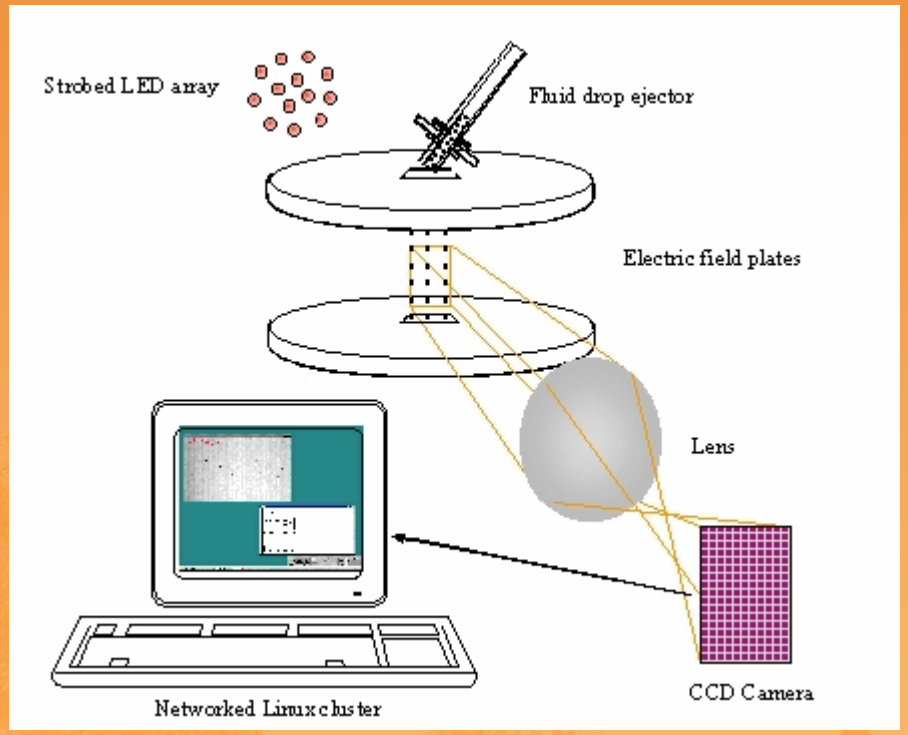


Choice of Oil Drops for the Analysis: rise and fall times



Modern experiments at SLAC

NATIONAL ACCELERATOR LABORATORY



- Drop generation rate 1 Hz
- Fluid - Dow Corning silicon oil
- Number of drops - 17 million
- Mass - 70.1 milligrams
- Duration - 8 months

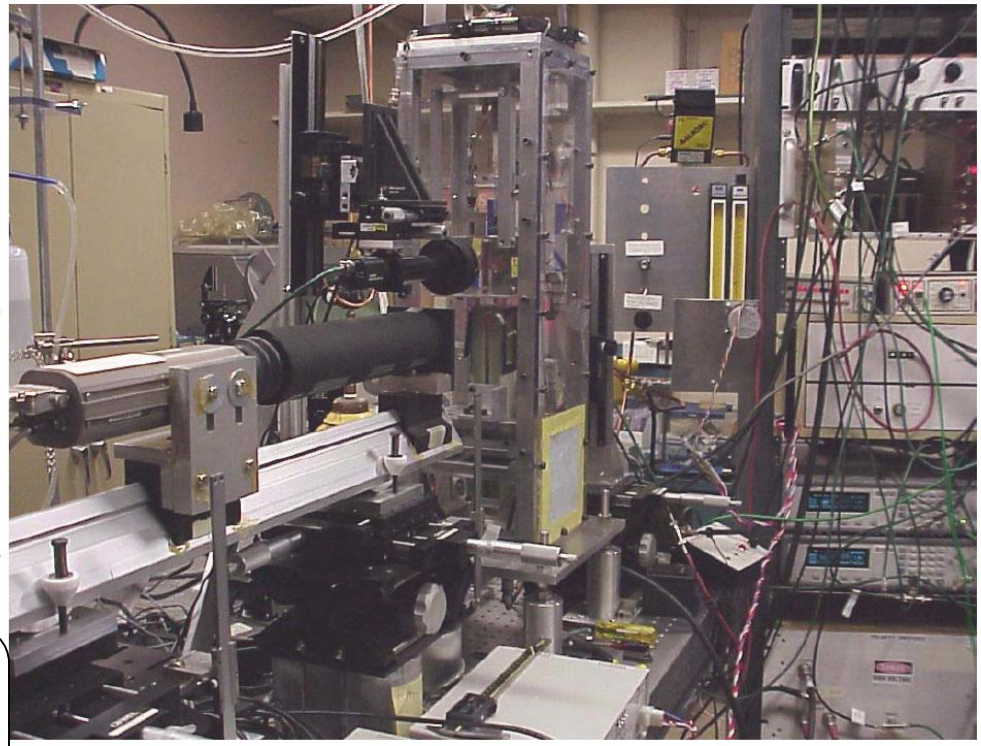
Modern experiments at SLAC

NATIONAL ACCELERATOR LABORATORY

Machine vision mediated auto-control of: the average charges of the drops, the fall path of the drops, the upward laminar air flow, and the electronic drive to the drop ejector are new features of this fluid drop charge measurement system.

The experiment is ran from 2004 - 2007.

Charge measurement accuracy achieved is better than 1/24 e for drops of up to 26 microns in diameter.



Modern experiments at SLAC

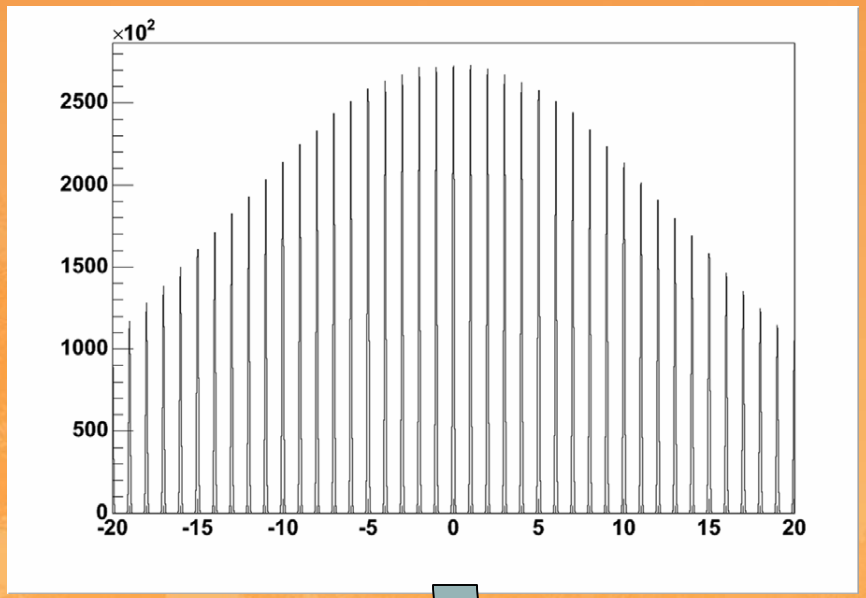
NATIONAL ACCELERATOR LABORATORY

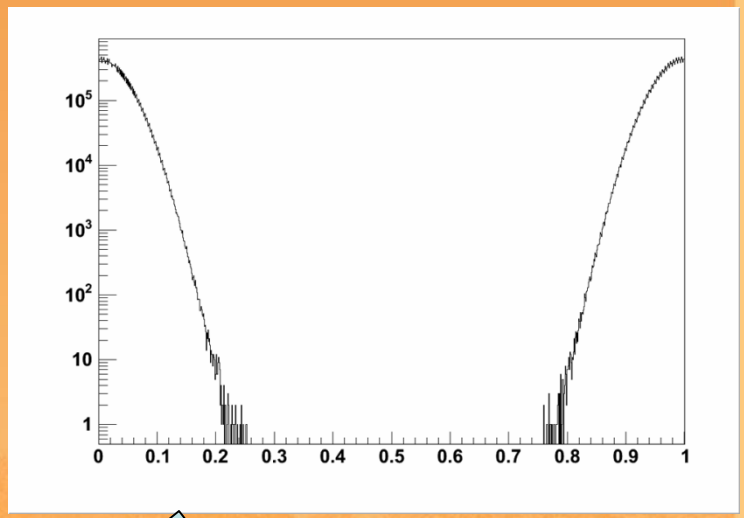
Summary as of January 2007.

Total mass throughput for all experiments- 351.4 milligrams of fluid

Total drops measured in all experiments - 105.6 million

No evidence for fractional charge particles was found.





Appendix #1

- **Traditional reminder:**

L1_Lab3_student name.pdf ~~***Report-exp2.pdf***~~

Please upload the files in proper folder!

This week folders:

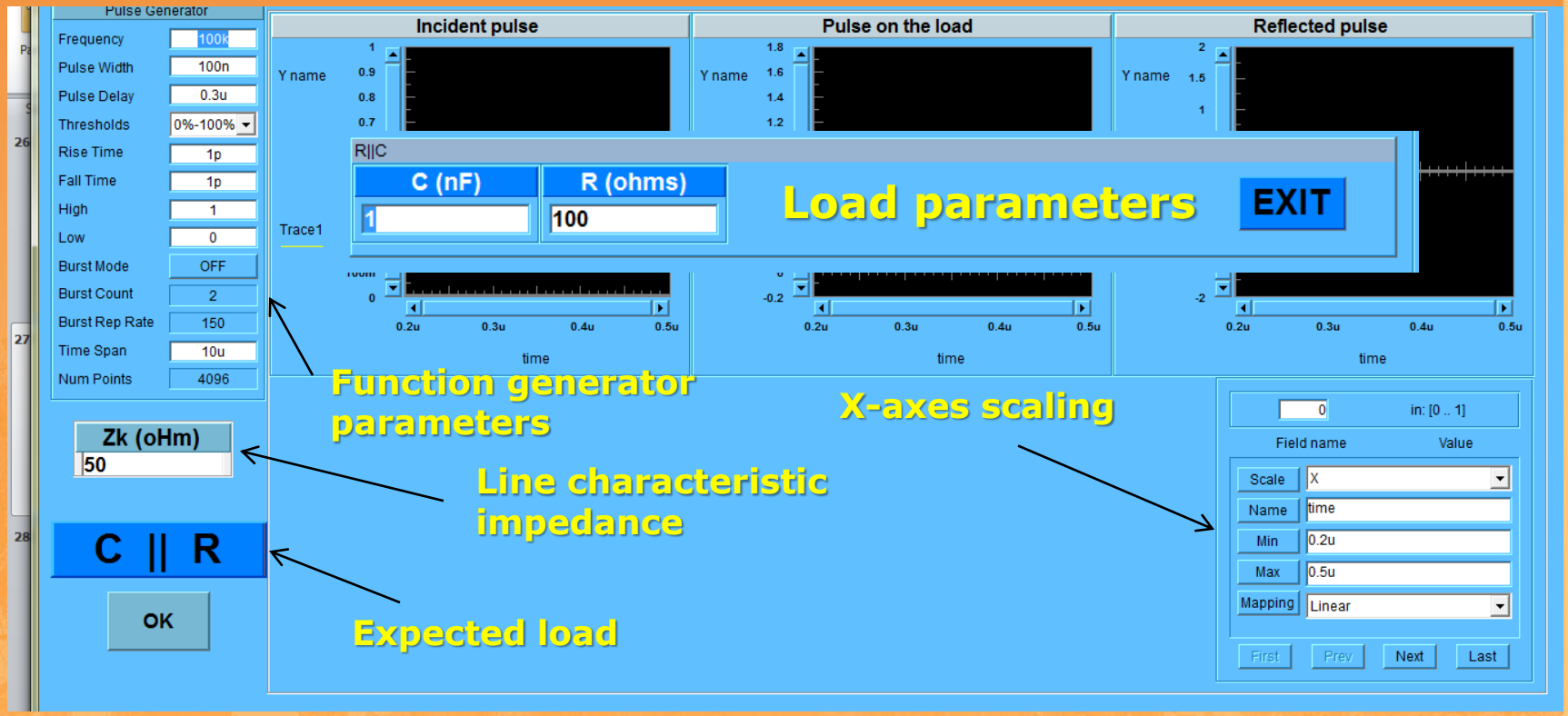
Pulses in transmission lines_L1
Pulses in transmission lines_L3
Pulses in transmission lines_L4
Pulses in transmission lines_L5

This week you have the last chance to submit "*Transients in RLC*" report



Appendix #2

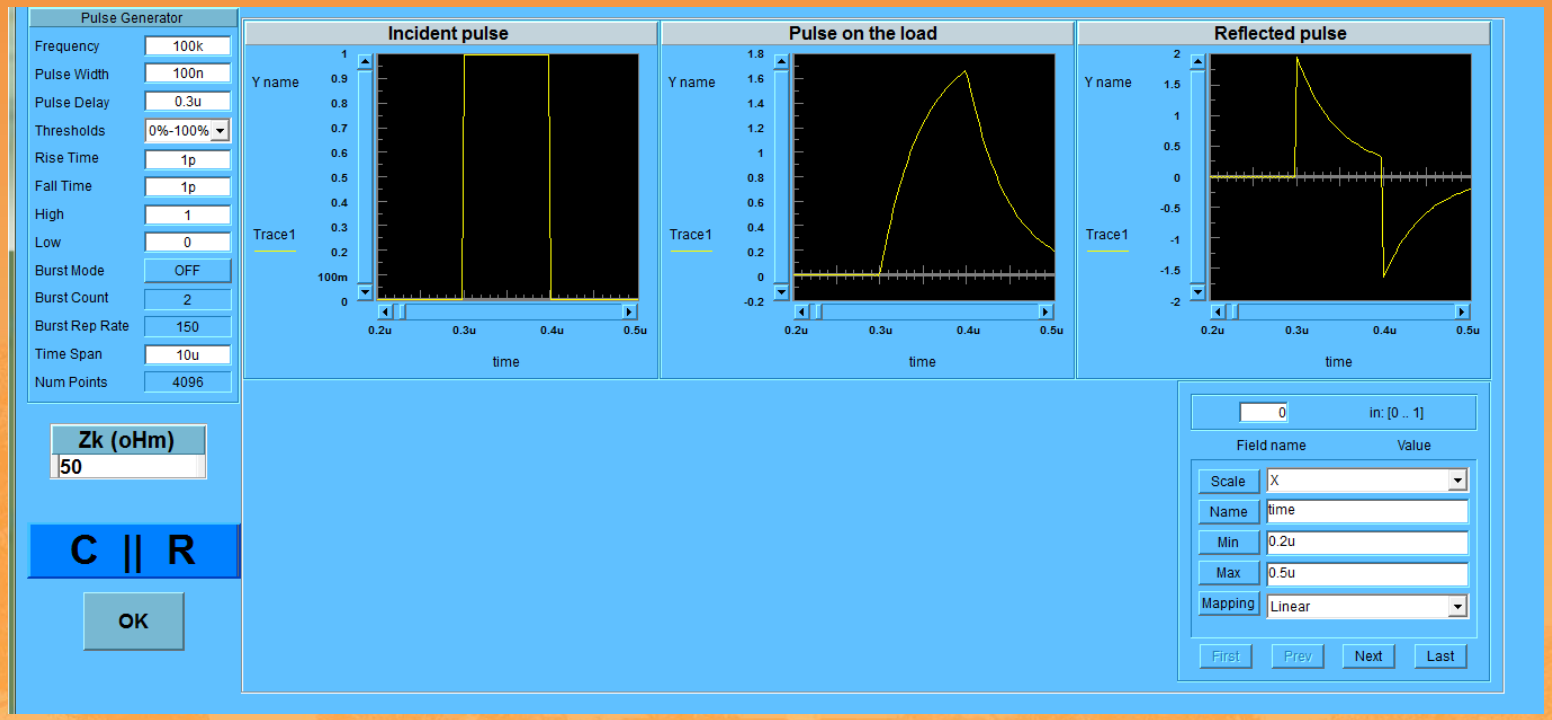
- **Transmission line. Unknown load simulation**



Location: \\Phyaplportal\PHYCS401\Common\Transmission line software

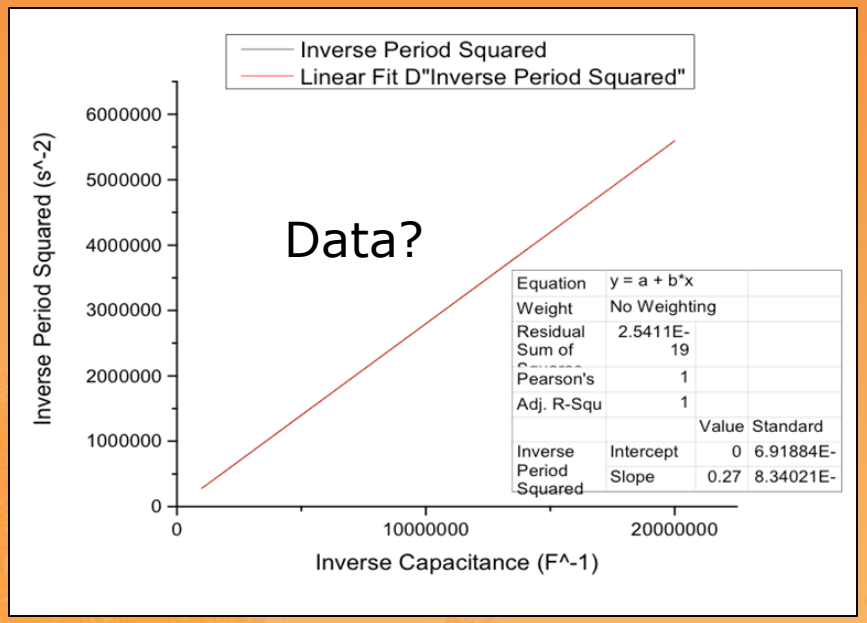
Appendix #2

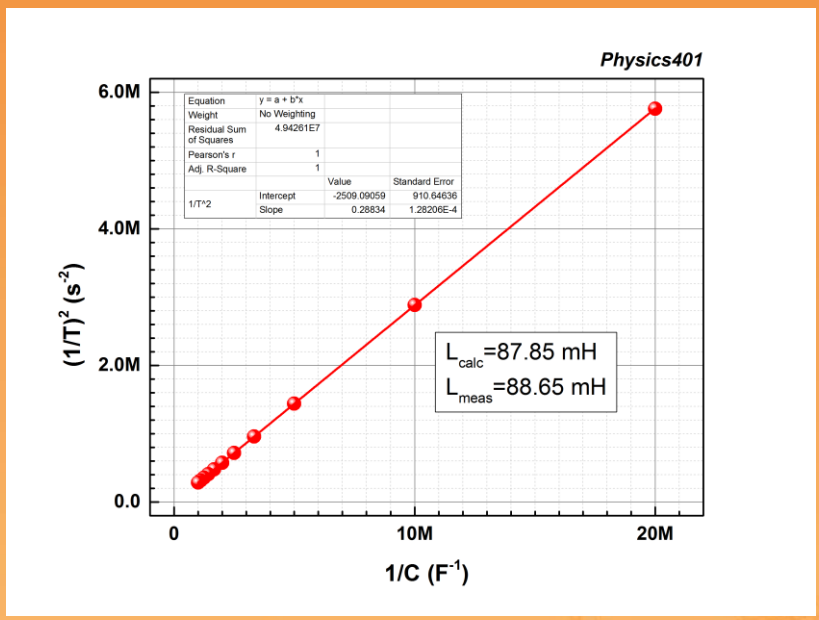
- **Transmission line. Unknown load simulation**



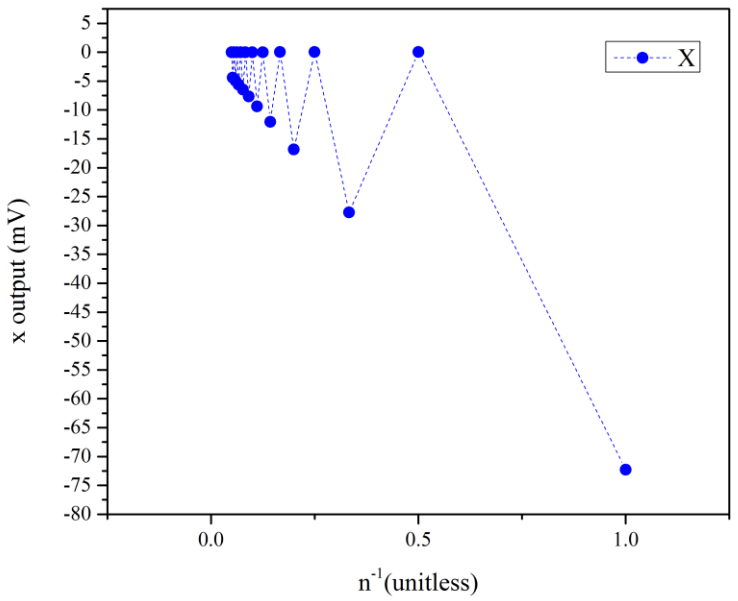
Location: \\Phyaplportal\PHYCS401\Common\Transmission line software

Appendix #3 Graphs





Appendix #3 Graphs



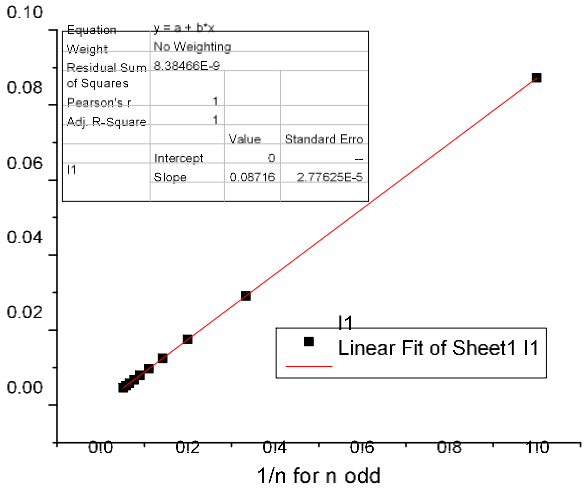


Figure 15: Odd harmonic amplitudes plotted against $1/n$

Appendix #3

Resonance fitting issue

$$K \equiv \frac{R \bullet Q^2}{R_0}; \gamma \equiv \frac{\omega}{\omega_0} = \frac{f}{f_0}; s \equiv (1-\gamma^2)$$

$$\tilde{H} = K \bullet \frac{1 - j\gamma\left(\frac{1}{Q} - Qs\right)}{s^2 + \gamma^2 \frac{1}{Q^2}}$$

Model	res_X (User)		

Equation	g=x/f0;
	s=(1-g^2);
	D=Q^2*s^2+g^2;
	y=A*Q/D		

Reduced Chi-Sqr	0.0016		
Adj. R-Square	0.99983		
		Value	Standard Error
X	A	1.48573	0.0062
	f0	1169.54077	0.24889
	Q	8.24921	0.04749

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in Origin fitting equation **A** is a **K**

Variable parameters for fitting:

1. f0 - resonance frequency (can be raugthly calculated fro L and C)
2. A - scaling coefficient. Initial estimation $\frac{R \bullet Q^2}{R_0}$
3. Q -quality factor. For initialization you can you some number from 2 to 10

