

UNIVERSITY OF ILLINOIS
AT URBANA-CHAMPAIGN

Transients and Oscillations in RLC Circuits

Physics 401, Spring 2016.

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Transients and Oscillations in RLC Circuits.

Outline

- **Transients. Definition.**
- **Transients in RLC**
- **Resonance in RLC**
- **Data analysis. Origin. Fitting.**



Main goals of this week Lab:

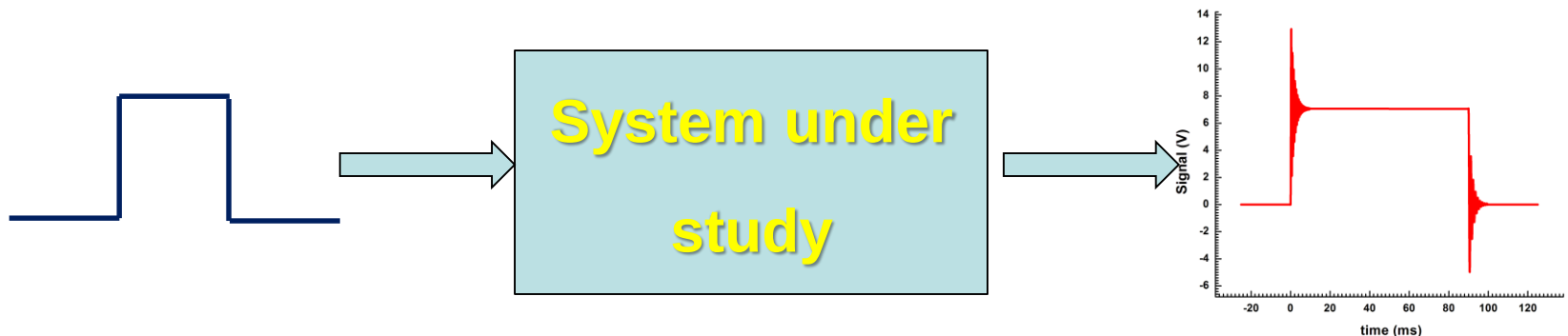
- **To understand what are the transients in general**
- **Transients in RLC circuits. Different regimes of dumping**
- **Data analysis using Origin software**



Transients. Definition.

transient (physics) a short-lived oscillation in a system caused by a sudden change of voltage or current or load

a transient response or natural response is the response of a system to a change from equilibrium.

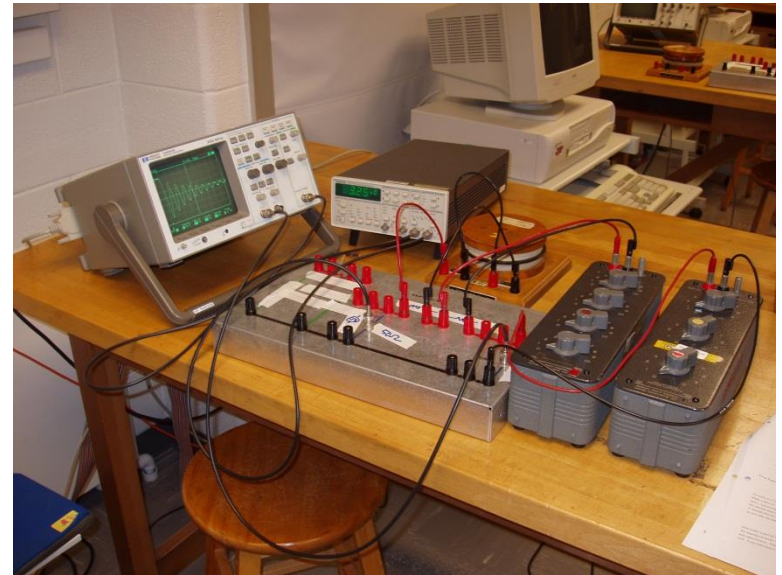
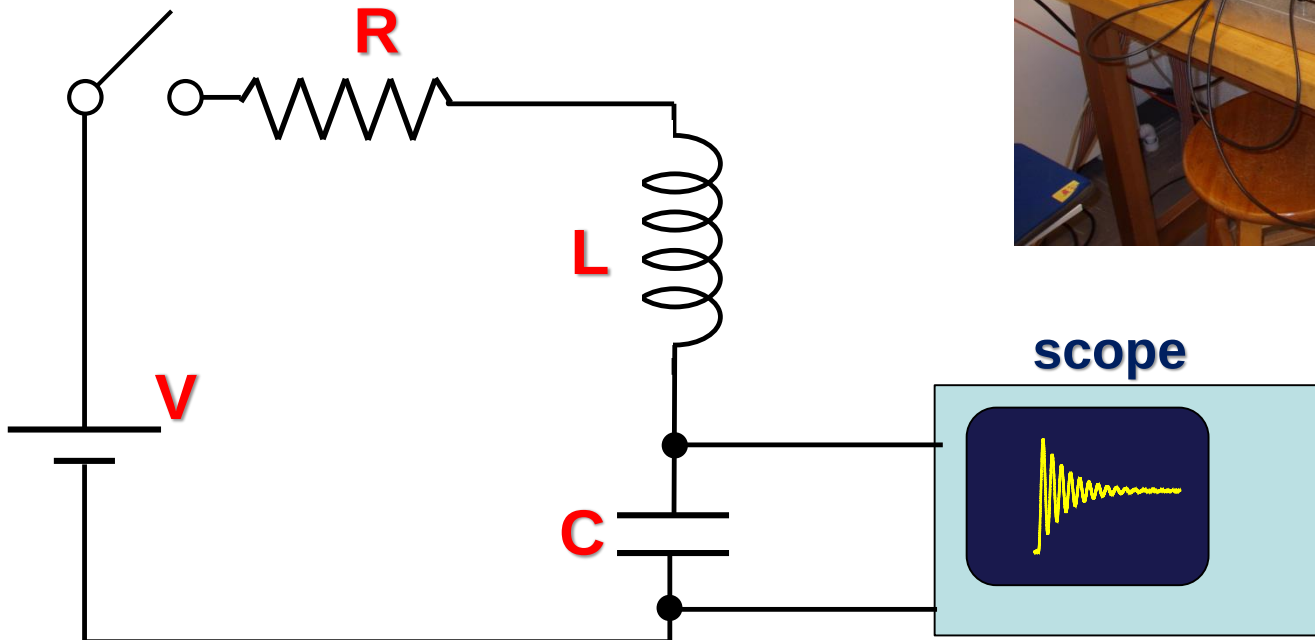


Transients in RLC circuit.

Resistance R [Ohm]

Capacitance C [μF] ($10^{-6}F$)

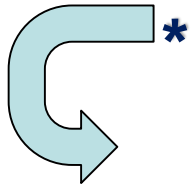
Inductance L [mH] ($10^{-3}H$)



Transients in RLC circuit.

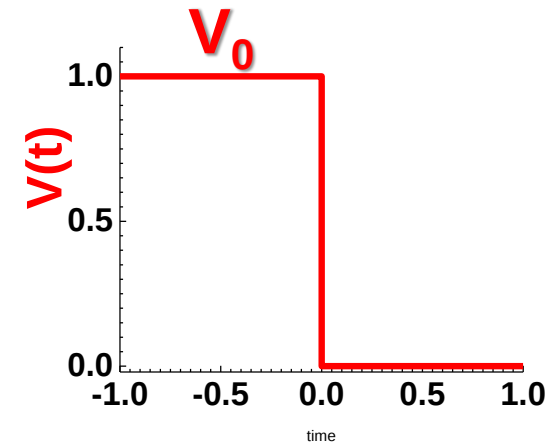
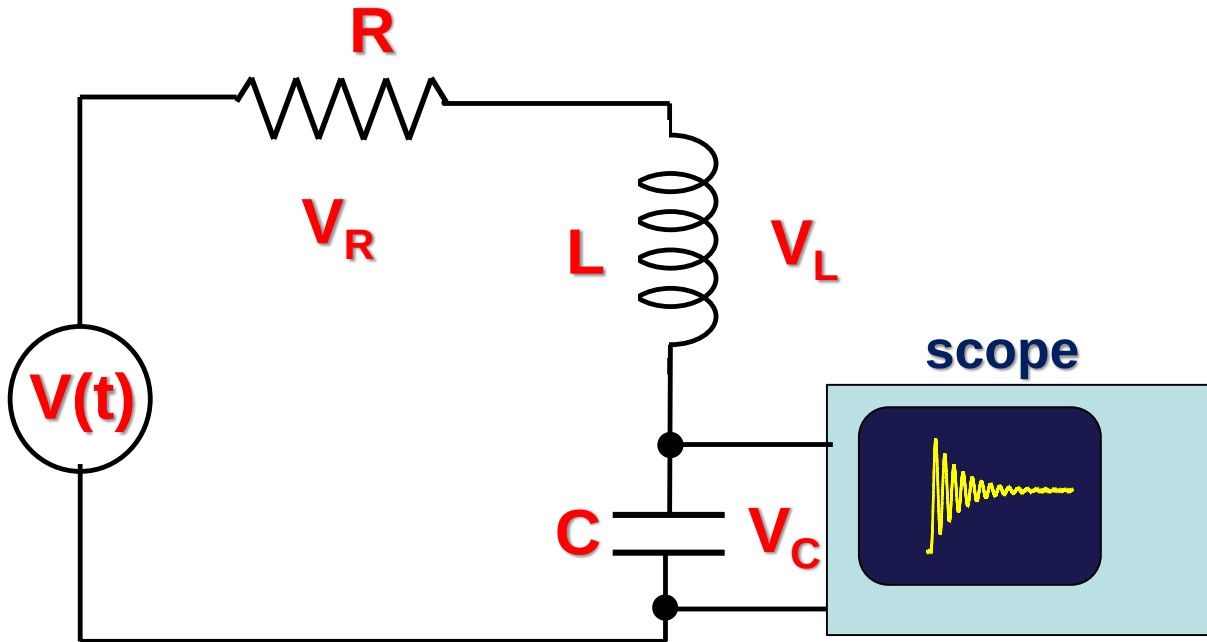
According the Kirchhoff's law

$$V_R + V_L + V_C = V(t)$$



$$L \frac{d^2}{dt^2} q(t) + R \frac{d}{dt} q(t) + \frac{q(t)}{C} = 0 \quad (1)$$

*See Lab write-up for details



Transients in RLC circuit. Three solutions.

The solution of this differential equation can be found in the form

$$q(t) = Ae^{st}$$

This will convert (1) in quadratic equation

$$s^2 + \left(\frac{R}{L}\right)s + \frac{1}{LC} = 0$$

with solutions:

$$s_{1,2} = -\frac{R}{2L} \pm \sqrt{\left(\frac{R}{2L}\right)^2 - \left(\frac{1}{LC}\right)} \equiv -a \pm b$$
$$a = \frac{R}{2L}, \quad b = \sqrt{\left(\frac{R}{2L}\right)^2 - \left(\frac{1}{LC}\right)}$$

$b^2 > 0$ over-damped solution

$b^2 = 0$ critically damped solution

$b^2 < 0$ under-damped solution



Transients in RLC circuit. Over-damped solution: $b^2 > 0$

In this case the solution will be aperiodic exponential decay function with no oscillations:

$$q(t) = e^{-at} (A_1 e^{bt} + B_1 e^{-bt})$$

$$i(t) = \frac{dq}{dt} = -ae^{-at} (A_1 e^{bt} + B_1 e^{-bt}) + be^{-at} (A_1 e^{bt} - B_1 e^{-bt})$$

$$b^2 > 0 \rightarrow R^2 > \frac{4L}{C}$$



Transients in RLC circuit. Over-damped solution: $b^2 > 0$

Taken in account the initial conditions: $q(0)=q_0$ and $i(0)=0$

$$q(t) = q_0 e^{-at} \left(\cosh bt + \frac{a}{b} \sinh bt \right)$$

$$\xrightarrow{(a-b)t \gg 1} \frac{q_0}{2} \left(1 + \frac{a}{b} \right) e^{-(a-b)t}$$

$$i(t) = -\frac{q_0}{2} \left(\frac{a^2 - b^2}{b} \right) e^{-(a-b)t}$$

This is exponential decay function



Transients in RLC circuit. Critically-damped solution: $b^2=0$

For this case the general solution can be found as

$q(t)=(A_2+B_2t)e^{-at}$. Applying the same initial condition

the current can be written as $i=-a^2q_0te^{-at}$

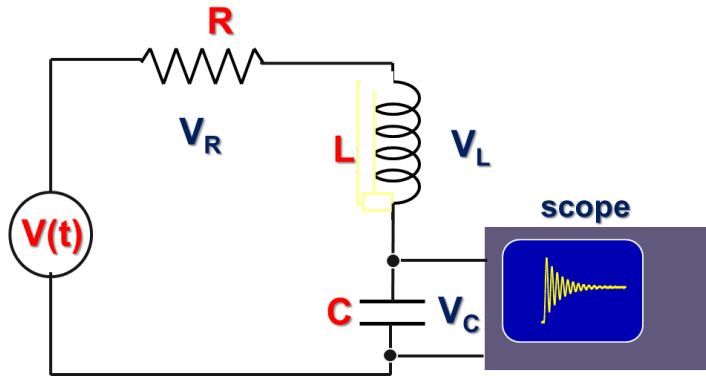
$$b^2 = 0 \rightarrow R^2 = \frac{4L}{C} \quad \text{and} \quad a = \frac{R}{2L}$$

Critically-damped conditions for our network

Critical damped case shows the fastest decay with no oscillations



Transients in RLC circuit. Critically-damped solution: $b^2=0$. Real data analysis.



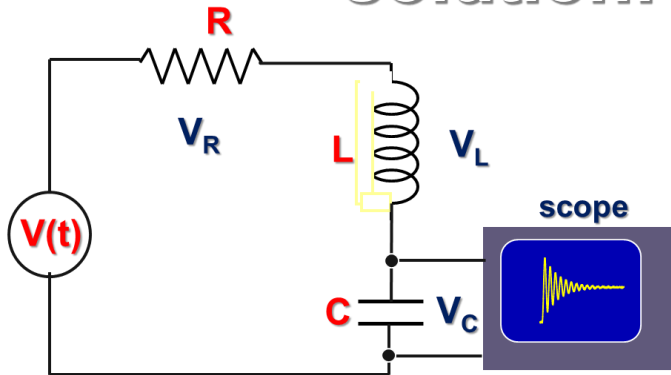
In this experiment $R=300$ ohms, $C=1\mu\text{F}$, $L=33.43\text{mH}$.

The output resistance of Wavetek is 50 ohms and resistance of coil was measured as 8.7 ohms, so actual resistance of the network is $R_a=300+50+8.7=358.7$

Decay coefficient
$$a = \frac{R}{2L} = \frac{358.7}{2 \times 33.43 \times 10^{-3}} \approx 5365$$



Transients in RLC circuit. Critically-damped solution: $b^2=0$. Real data analysis.



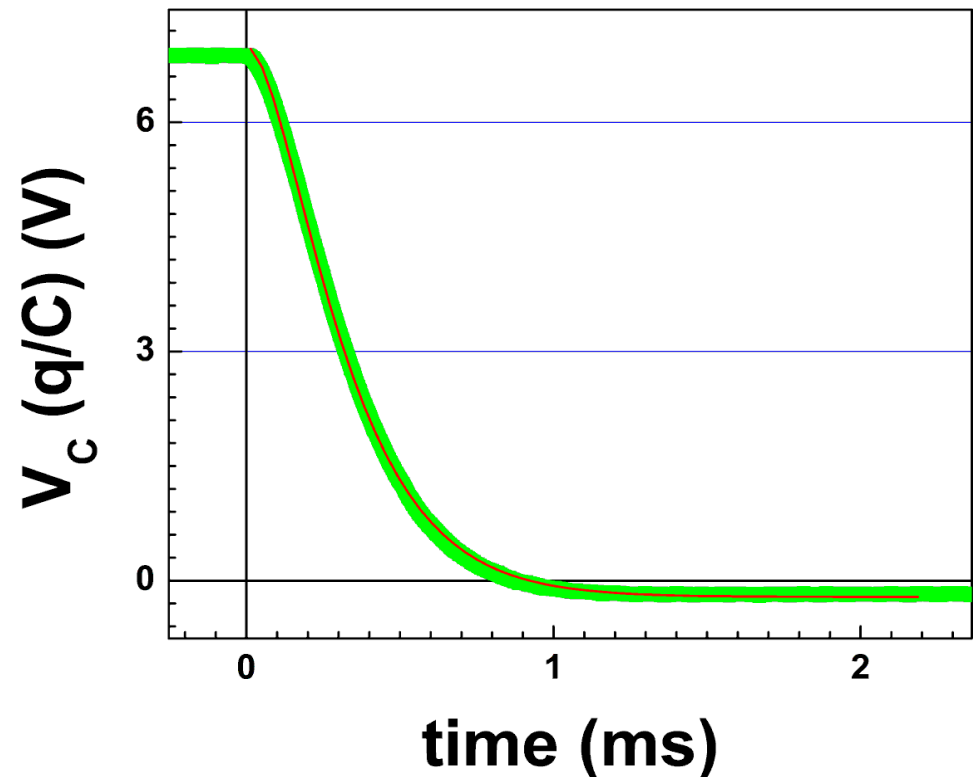
$V_C \sim q$, fitting function: $V_C = V_{co}(1+at)e^{-at}$

Calculated decay coefficient ~ 5385 ,

Obtained from fitting - ~ 5820 .

Possible reason – it is still slightly over damped

Calculated b^2 is $b^2 = 2.99e7 - 2.90e7 > 0$



Transients in RLC circuit. Under-damped solution.

If $b^2 < 0$ we will have oscillating solution. Omitting the details (see Lab write-up) we have the equations for charge and current as:

$$q(t) = q_0 e^{-at} \left(\cos bt + \frac{a}{b} \sin bt \right) = q_0 e^{-at} \sqrt{1 + \frac{a^2}{b^2}} \sin(bt + \varphi)$$

$$i(t) = q_0 e^{-at} \left(\frac{a^2 + b^2}{b} \right) \sin bt$$

$$a = \frac{R}{2L}, \quad b = \sqrt{\left(\frac{R}{2L}\right)^2 - \left(\frac{1}{LC}\right)}; \quad f = \frac{1}{2\pi} \sqrt{\left(\frac{1}{LC}\right) - \left(\frac{R}{2L}\right)^2}$$



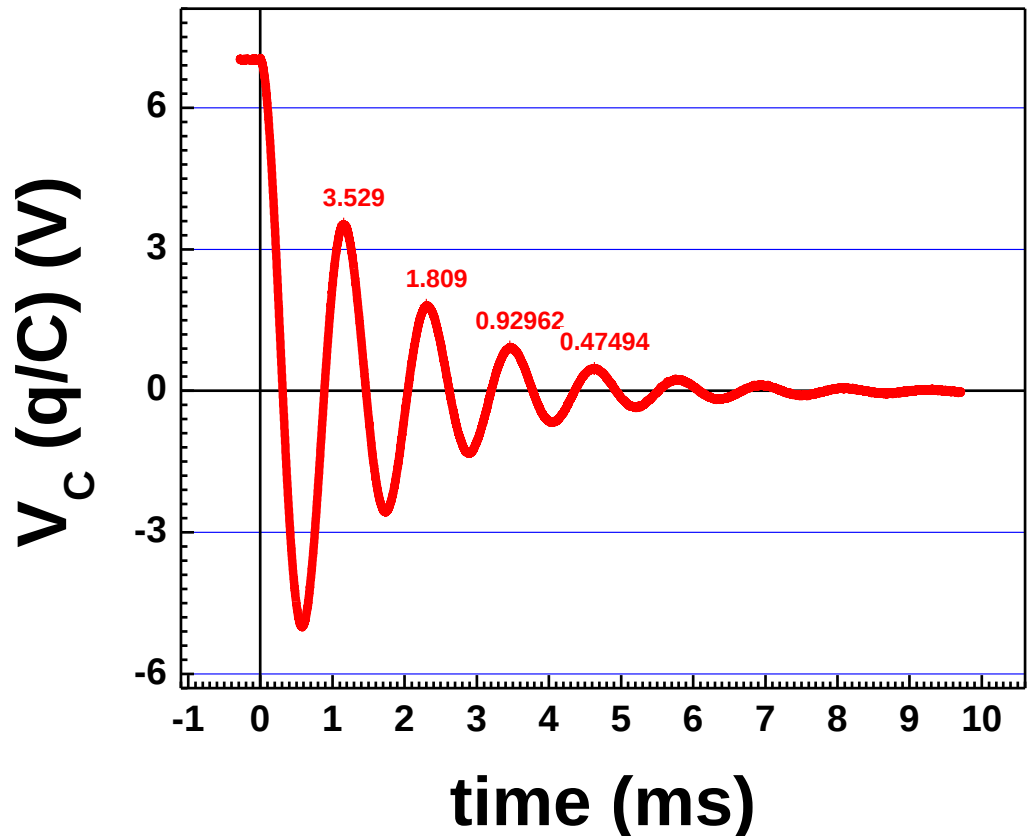
Transients in RLC circuit. Under-damped solution. Log decrement. Quality factor.

Log decrement can be defined as $\delta = \ln \left(\frac{q(t_{\max})}{q(t_{\max}+T_1)} \right) = \ln \left(\frac{e^{-at_{\max}}}{e^{-a(t_{\max}+T_1)}} \right) = aT_1$, where $T_1=1/f_1$

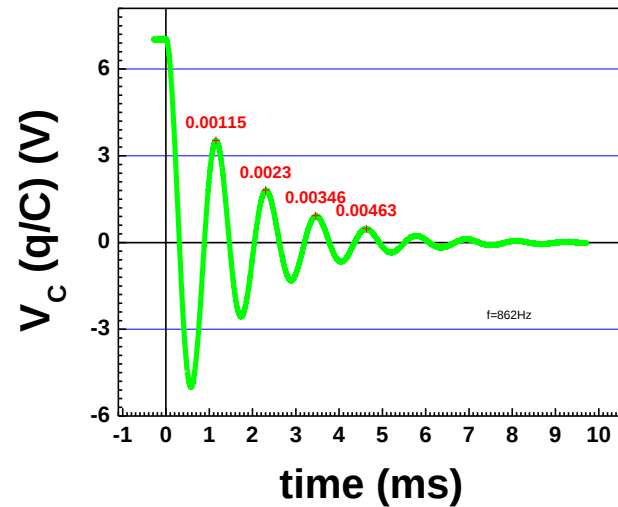
Quality factor can be defined as $Q = 2\pi \frac{E}{\Delta E}$,

For RLC $Q = \frac{\omega_1 L}{R} = \frac{\pi}{\delta}$

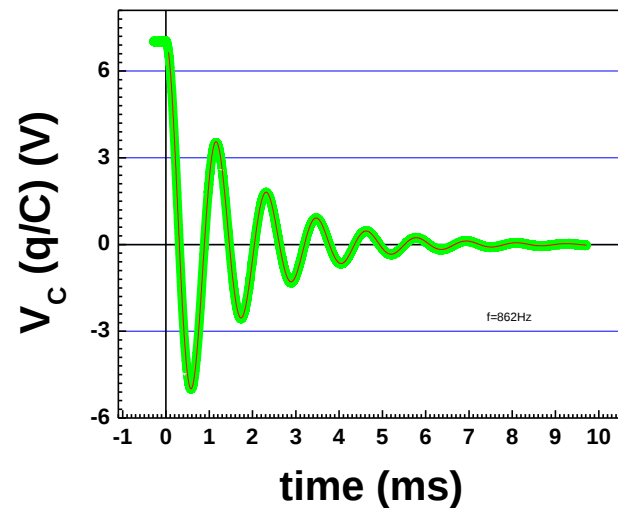
From this plot $\delta \approx 0.67$
 $Q \approx 4.7$



Transients in RLC circuit. Data analysis. Using Origin software.

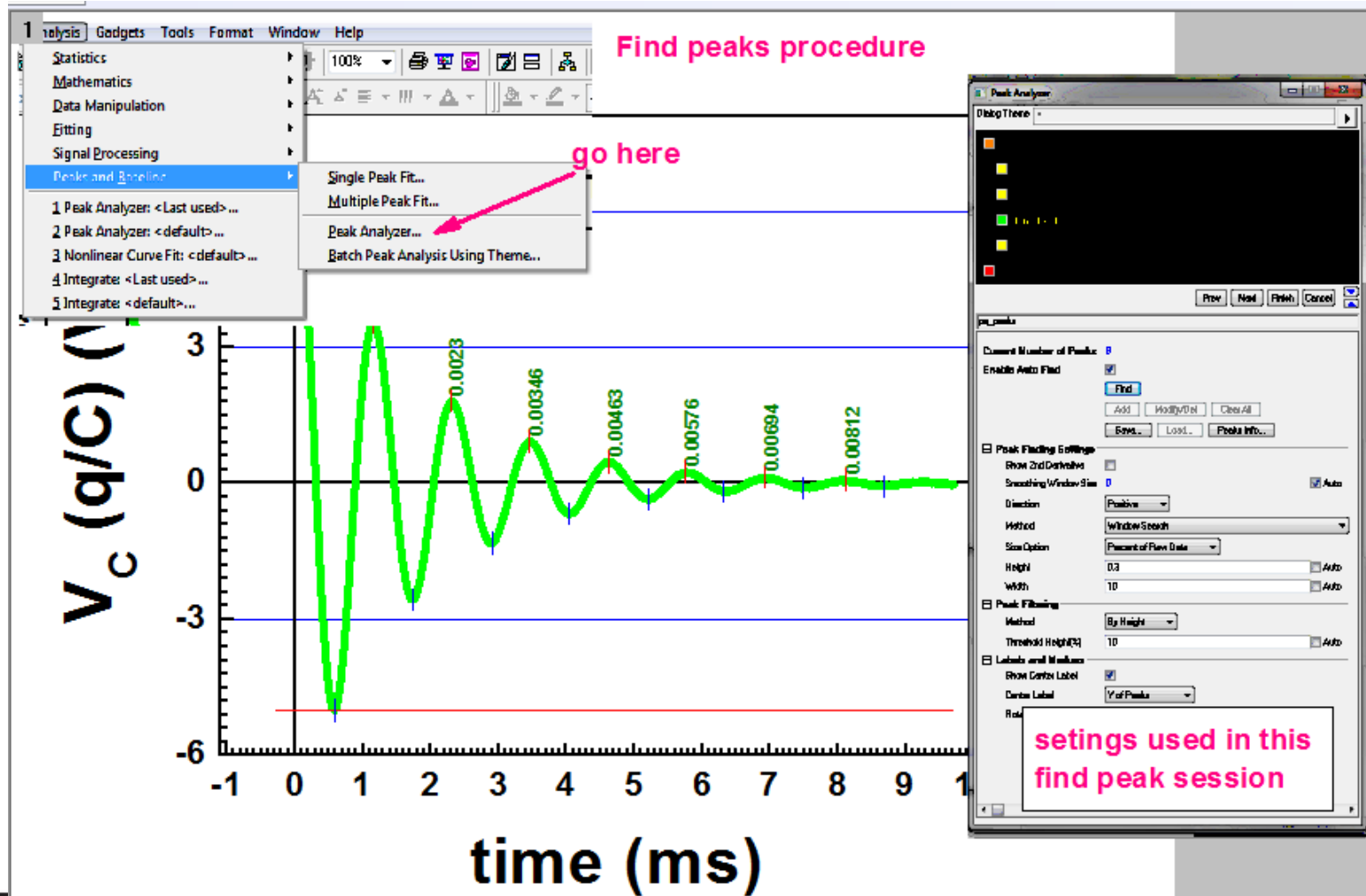


1. Pick peaks
2. Envelope
3. Nonlinear fitting

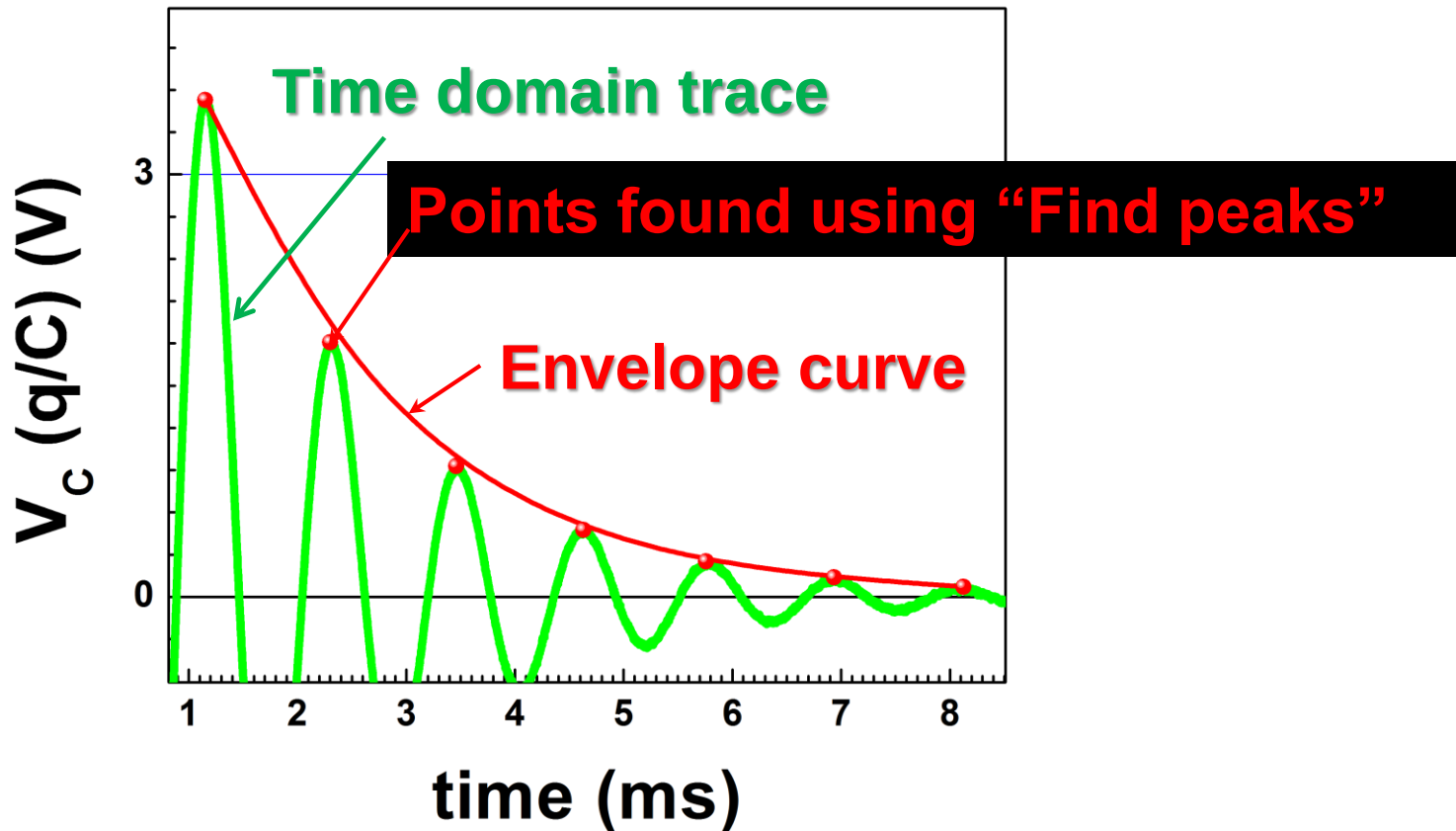


Transients in RLC circuit. Under-damped solution.

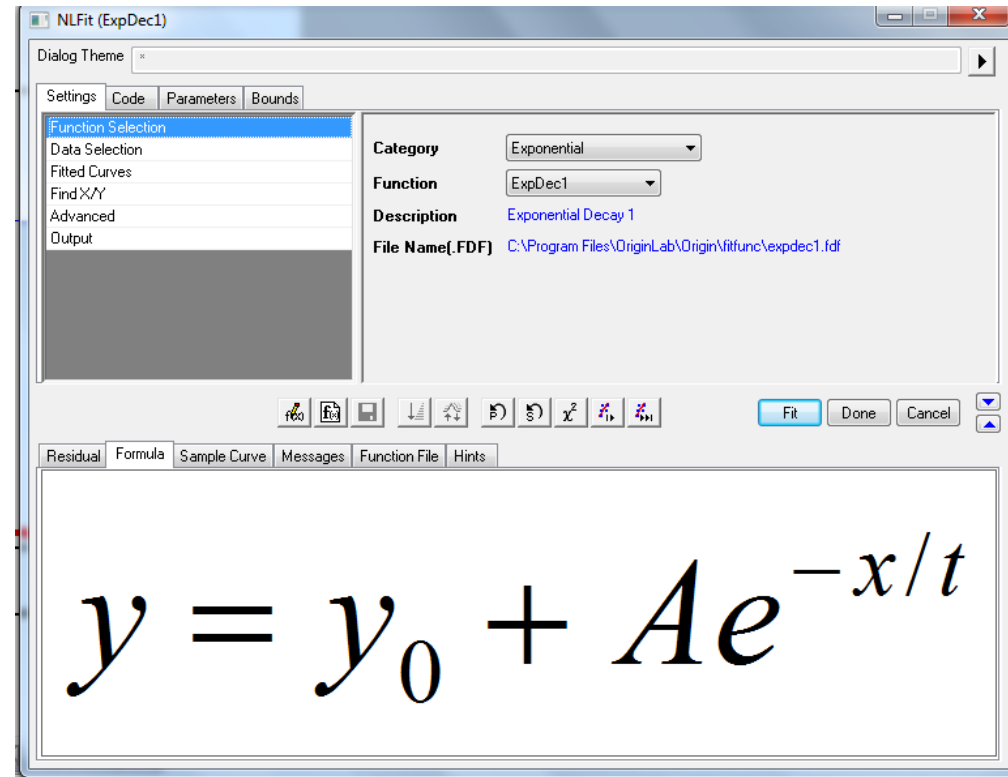
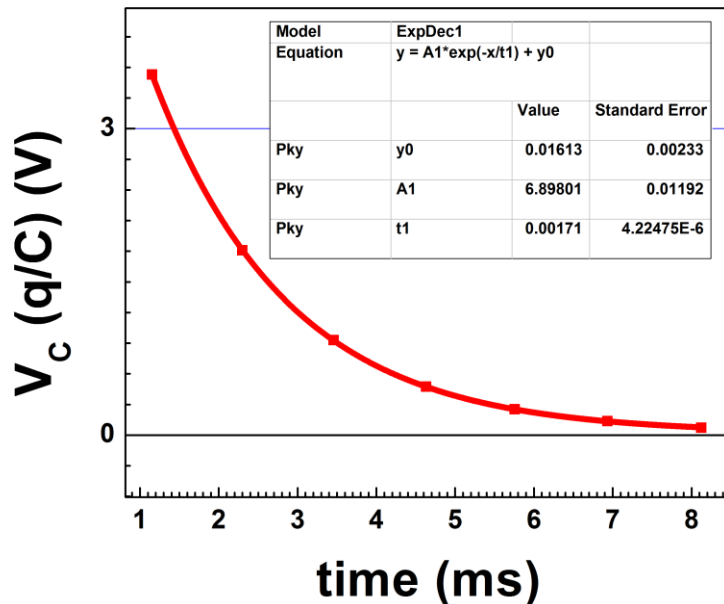
Log decrement. Quality factor.



Transients in RLC circuit. Data analysis. Log decrement. Using Origin software. Results.



Transients in RLC circuit. Data analysis. Log decrement. Using Origin software. Results.



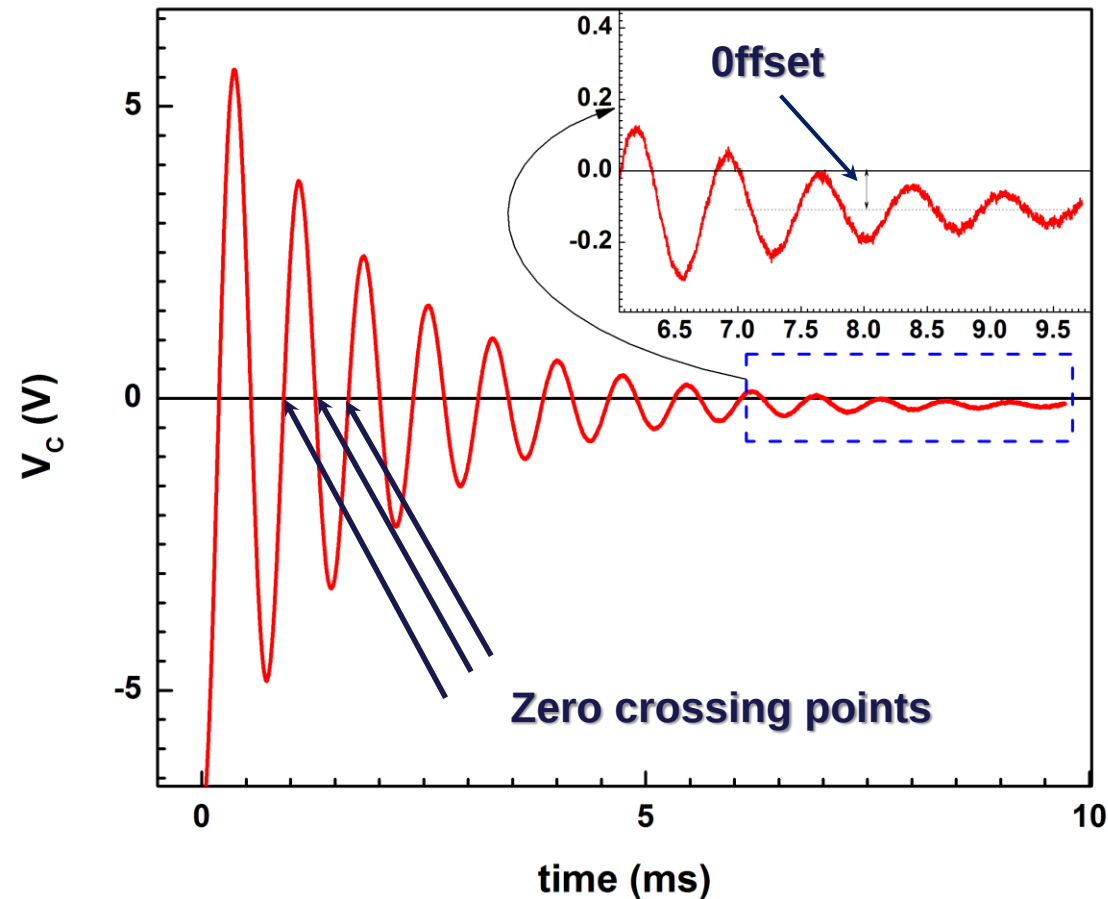
Fitting the “envelope data” to exponential decay function



Transients in RLC circuit. Data analysis.

$(1/T)^2$ vs $1/C$ experiment.

$$q(t) = Ae^{-at} \sin(\omega t + \varphi) + \text{offset}$$



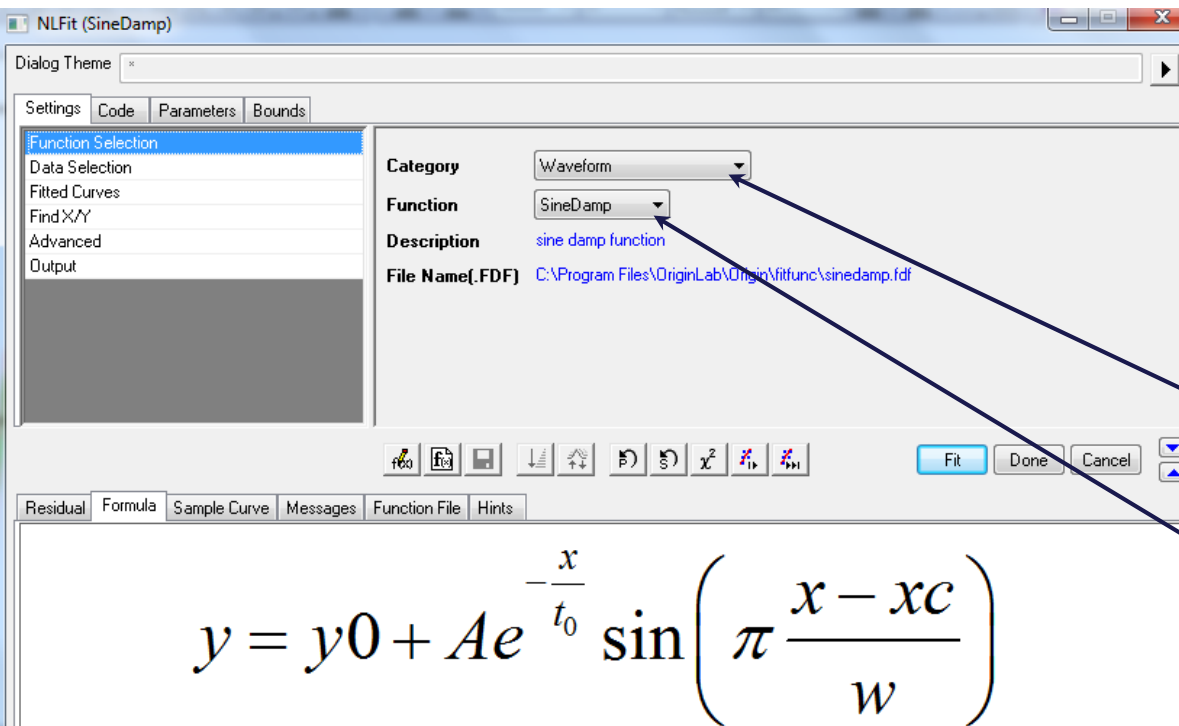
Manual evaluation of the period of the oscillations

Limited accuracy

Results can be effected by DC offset



Transients in RLC circuit. Data analysis. (1/T)² vs 1/C experiment. Using Origin software.



$$q(t) = Ae^{-at} \sin(\omega t + \phi)$$

Use Origin standard
function

Category: Waveform

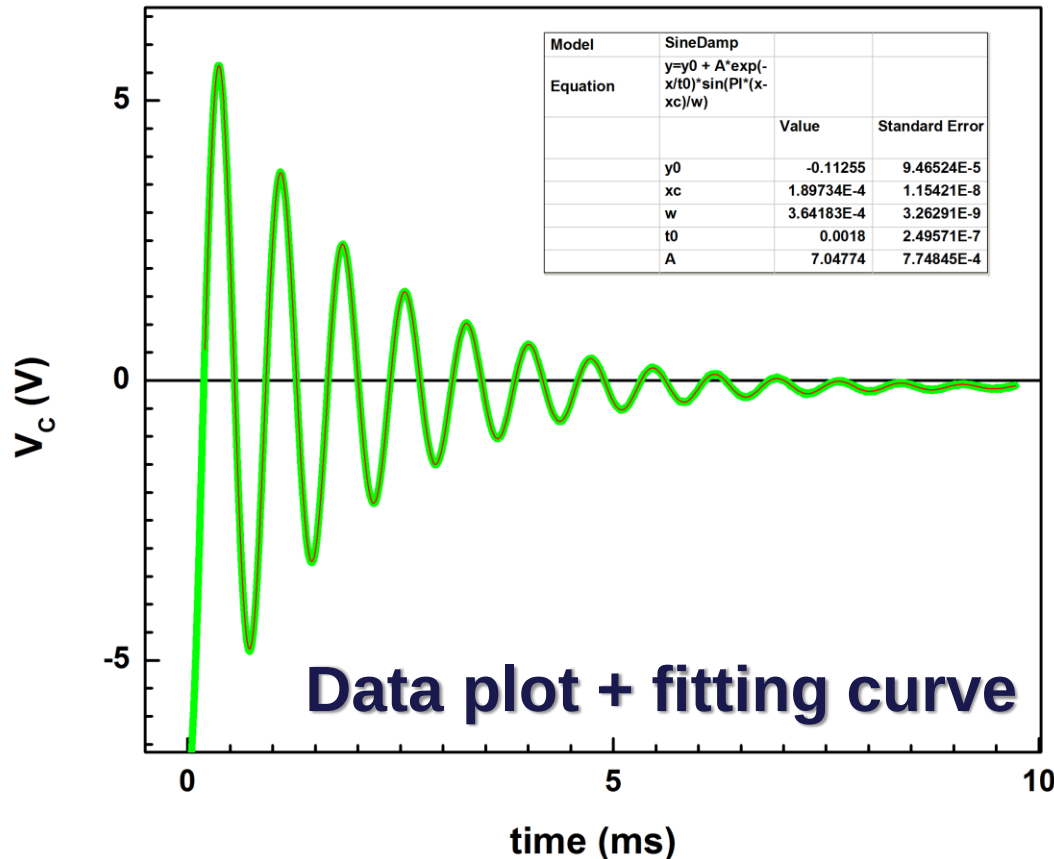
Function: SineDamp

Fitting function ; y_0, A, t_0, x_c, w – fitting parameters

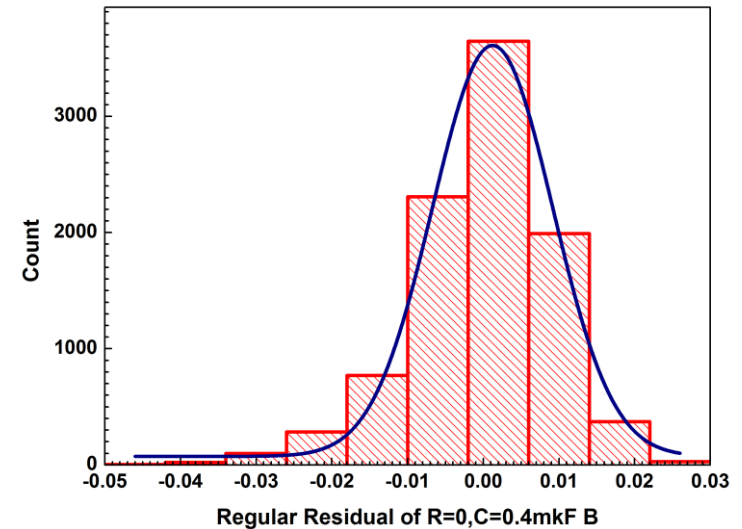
From fitting you can get: $a = \frac{1}{t_0}$ and $T = \frac{1}{f} = 2w$



Transients in RLC circuit. Data analysis. (1/T)² vs 1/C experiment. Using Origin software.



$$q(t) = A e^{-at} \sin(\omega t + \varphi)$$

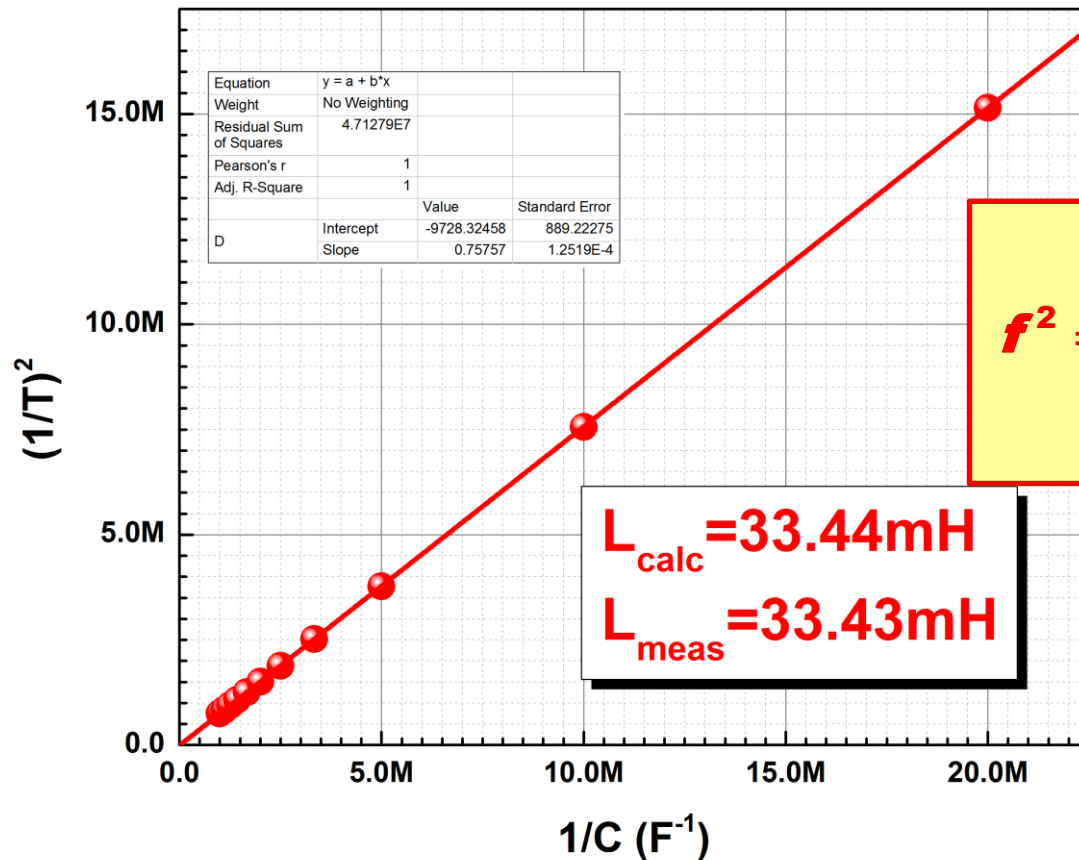


**Residuals - criteria
of quality of fitting**



Transients in RLC circuit. Data analysis.

$(1/T)^2$ vs $1/C$ experiment. Using Origin software.



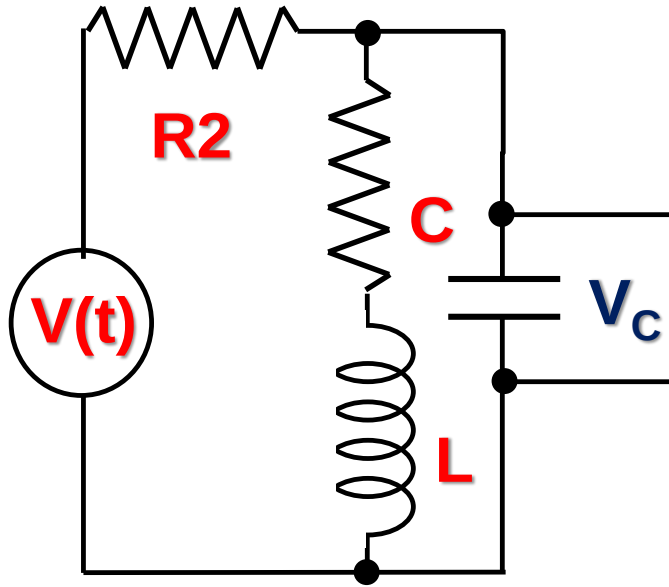
$$q(t) = Ae^{-at} \sin(\omega t + \varphi)$$

$$f^2 = \left(\frac{1}{T}\right)^2 = \frac{1}{2\pi} \left(\left(\frac{1}{LC}\right) - \left(\frac{R}{2L}\right)^2 \right)$$

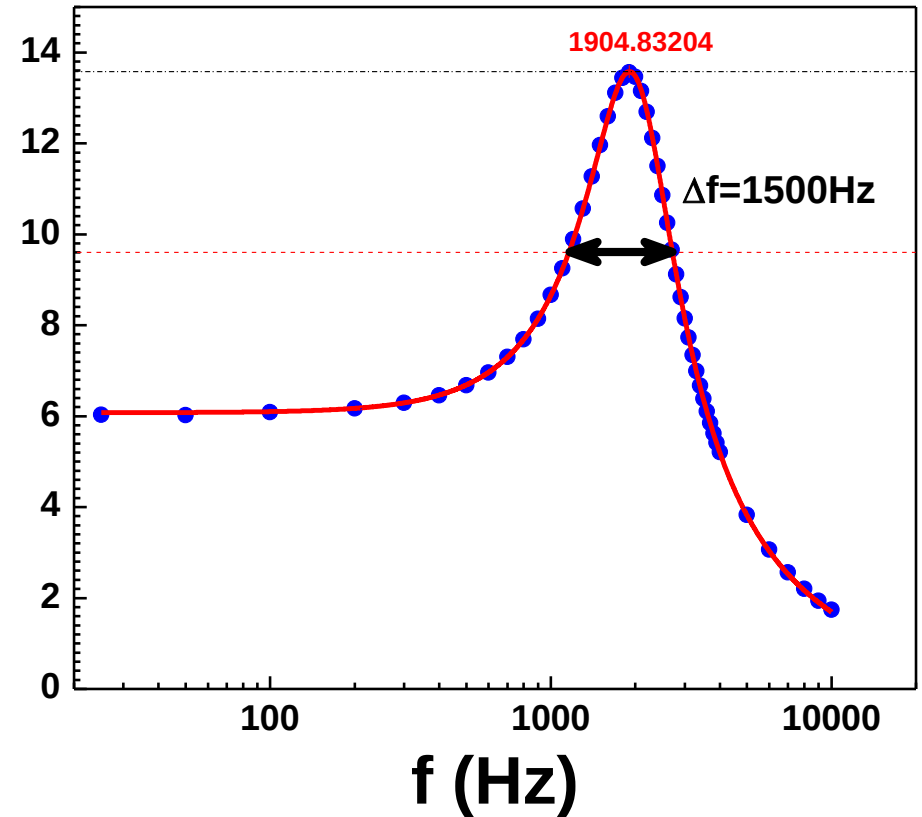
Final results



Resonance in RLC circuit.



U_C

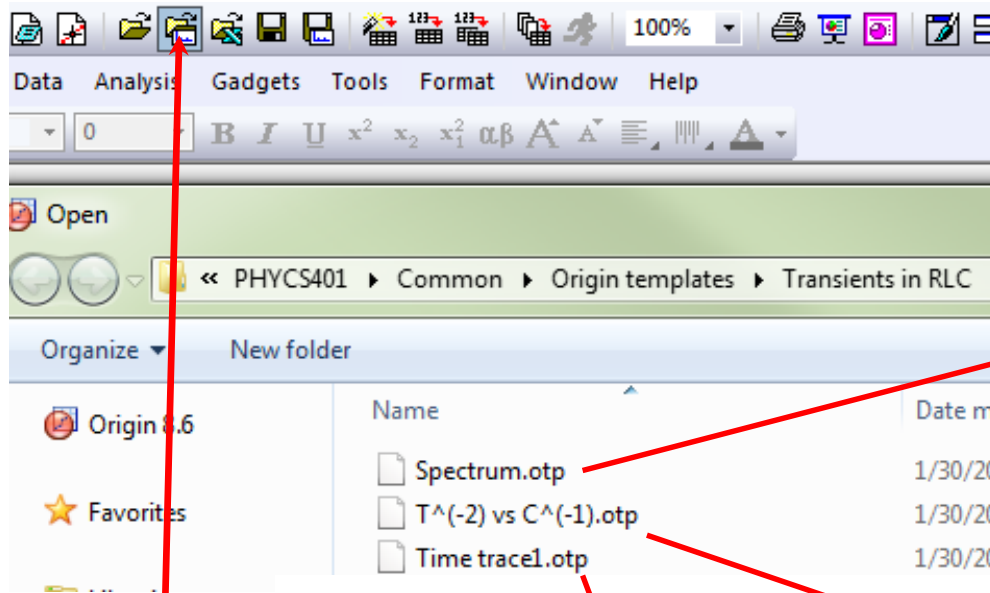


$$Q = \frac{f}{\Delta f} = \frac{1904}{1500} = 1.26$$

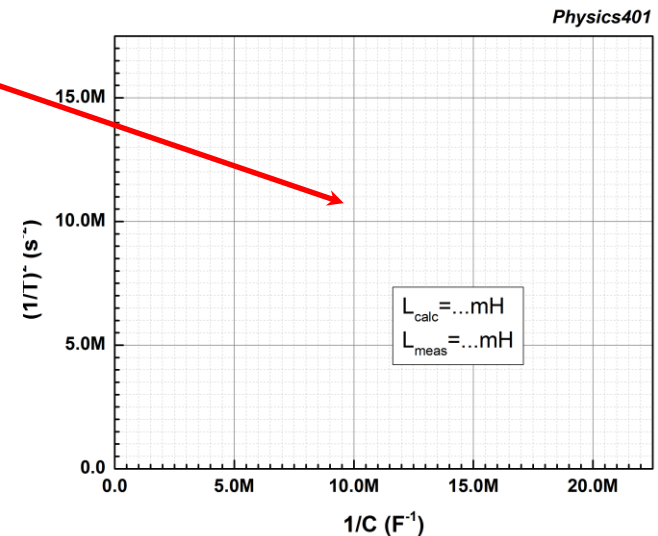
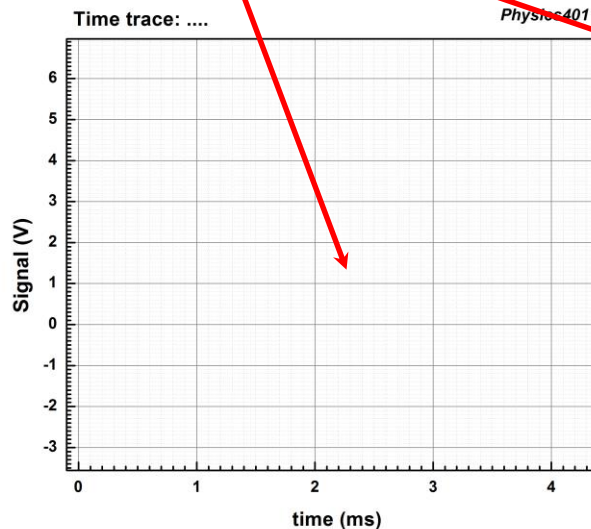
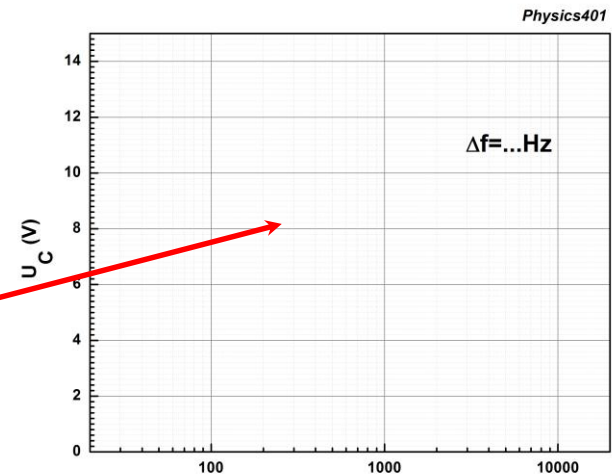


Origin templates for this week Lab.

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**Open template
button**



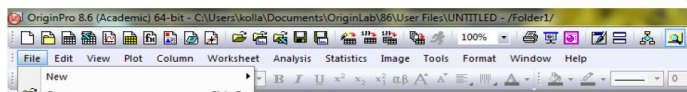
Origin manuals

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Working with Origin 8.6.

Step1. Importing data

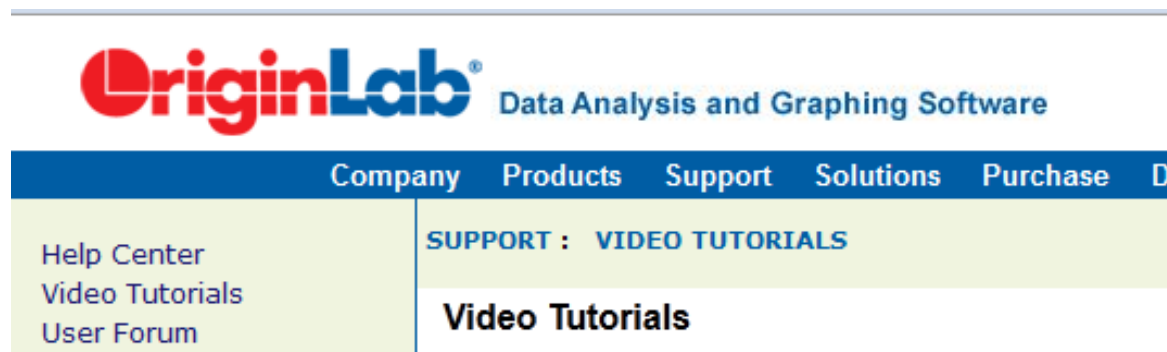


Very short and simple manual which covers only main general operations with Origin. Document located on server and there is a link from P401 WEB page

There are also manuals from OriginLab.

Do not forget about Origin Help

Video Tutorials at the site of the company



<http://www.originlab.com/index.aspx?go=SUPPORT/VideoTutorials>

