Qualitative Studies with Microwaves

Physics 401, Spring 2018
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Qualitative Studies with Microwaves

The main goals of the Lab:

✓ Refreshing the memory about the electromagnetic waves propagation
✓ Microwaves. Generating and detecting of the microwaves
✓ Microwaves optic experiments

This is two weeks Lab
The microwave range includes ultra-high frequency (UHF) (0.3–3 GHz), super high frequency (SHF) (3–30 GHz), and extremely high frequency (EHF) (30–300 GHz) signals.
Application of the microwaves

- Microwave oven (2.45GHz)
- Communication (0.8-2.69GHz)
- Satellite TV (4-18GHz)
- Radar (up to 110GHz)
- Motion detector (10.4GHz)
- Weather radar (8-12GHz)
- GPS 1.17-1.575 GHz

*by courtesy Wikipedia
Maxwell equations

\[ \nabla \vec{D} = \rho \quad (1) \]

\[ \nabla \times \vec{E} = -\frac{\partial \vec{B}}{\partial t} \quad (3) \]

\[ \nabla \vec{B} = 0 \quad (2) \]

\[ \nabla \times \vec{H} = J + \frac{\partial \vec{D}}{\partial t} \quad (4) \]

If \( \rho = 0 \) and \( J = 0 \) and taking in account that \( \vec{D} = \varepsilon \vec{E} \) \( \vec{B} = \mu \vec{H} \) (1) and (4) can be rewritten as

\[ \nabla \vec{D} = \varepsilon \left[ \frac{\partial E_x}{\partial x} + \frac{\partial E_y}{\partial y} + \frac{\partial E_z}{\partial z} \right] = 0 \]

\[ \nabla \times \vec{H} = \frac{\partial \vec{D}}{\partial t} \]
Now assuming that plane wave propagate in z direction and what leads to \(E_y = E_z = 0\) and \(H_x = H_z = 0\)

Now (3) and (4) could be simplified as

\[
\frac{\partial E_x}{\partial z} = -\mu \frac{\partial H_y}{\partial t} \tag{5}
\]

\[
\frac{\partial H_y}{\partial z} = -\varepsilon \frac{\partial E_y}{\partial t} \tag{6}
\]

where \(\mu = \mu_0 \mu_r\) and \(\varepsilon = \varepsilon_0 \varepsilon_r\)

\(\mu_0\) is the free space permeability, \(\varepsilon_0\) is the free space permittivity

\(\mu_r\) is permeability of a specific medium, \(\varepsilon_r\) is permittivity of a specific medium
Combining (5) and (6) (see Lab write-up for more details) we finally can get the equations of propagation of the plane wave:

\[
\frac{\partial^2 E_x}{\partial z^2} = \frac{1}{v^2} \frac{\partial^2 E_x}{\partial t^2} \quad (7) \quad \frac{\partial^2 H_y}{\partial z^2} = \frac{1}{v^2} \frac{\partial^2 H_y}{\partial t^2} \quad (8)
\]

where \( v = \frac{1}{\sqrt{\varepsilon \mu}} \)

\[
E_x = E_{x0} \cos(\omega t - kx)
\]

\[
H_y = H_{y0} \cos(\omega t - kx)
\]

Solution for (7) and (8) can found as

\[
H_y = \sqrt{\frac{\varepsilon}{\mu}} E_x \quad \text{or} \quad E_x = Z H_y
\]

where \( Z = \sqrt{\frac{\mu}{\varepsilon}} \) known as characteristic impedance of medium

k is wave vector and is defined as

\[
k = \frac{2\pi}{\lambda} \quad \text{or} \quad k = \frac{\omega}{v}
\]

For free space (\( \varepsilon_r=1 \) and \( \mu_r=1 \))

\[
Z_{fs} = \sqrt{\frac{\mu_0}{\varepsilon_0}} \approx 377 \text{ ohms}
\]
Plane wave

\[ E_x = E_{x0} \cos(\omega t - kx) \]
\[ H_y = H_{y0} \cos(\omega t - kx) \]

\[ v = \frac{1}{\sqrt{\varepsilon \mu}} \quad H_y = \sqrt{\frac{\varepsilon}{\mu}} E_x \]

\[ Z = \sqrt{\frac{\mu}{\varepsilon}} \quad E_x = ZH_y \quad k = \frac{2\pi}{\lambda} \quad \text{or} \quad k = \frac{\omega}{v} \]

\[ Z_f = \sqrt{\frac{\mu_0}{\varepsilon_0}} \approx 377 \text{ ohms} \]

For free space (\( \varepsilon_r = 1 \) and \( \mu_r = 1 \))

*by courtesy Wikipedia*
Generating of the microwaves

Vacuum tubes: klystron, magnetron, traveling wave tube

Solid state devices: FET, tunneling diodes, Gunn diodes

- Tunable frequency from 9 to 10GHz; maximum output power 20mW
- Microwave oven magnetron; typical power 0.7-1.5kW
Klystron. A piece of history.

Russell Harrison Varian (April 24, 1898 – July 28, 1959)

Sigurd Fergus Varian (May 4, 1901 – October 18, 1961)

Patented May 20, 1941

UNITED STATES PATENT OFFICE

2,242,275
ELECTRICAL TRANSLATING SYSTEM AND METHOD

Russell H. Varian, Stanford University, Calif., assignor to The Board of Trustees of The Leland Stanford Junior University, Stanford University, Calif., a corporation of California

Application October 11, 1940 Serial No. 68,492

Varian Brothers...Klystron Tube (1940)
Generating of the microwaves. Klystron.

**Single transit klystron**

**Advantages:** well defined frequencies, high power output

**Reflection klystron**

High power klystron used in Canberra Deep Space Communications Complex (courtesy of Wikipedia)
2K25 Klystron

GENERAL CHARACTERISTICS
Frequency Range ................................................. 8,500 to 9,660 Mc
Cathode Oxide-coated, indirectly heated
  Heater Voltage ................................................. 6.3Volts
  Heater Current ................................................. 0.44 Amperes
Experimental setup. Main components.

- Klystron
- Frequency meter
- Attenuator
- Detector
- Horn
- Microwave Transmitter Arm
- Digital Volt Meter or Oscilloscope
- Microwave Receiver Arm
- Digital Volt Meter or Oscilloscope
- Detector
- Termination
Experimental setup. Main components.
Detecting of the microwaves

Taylor expansion for \( \exp \) function will give

\[
\exp(x) = 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \ldots
\]

\[
I = I_0[\exp\left(\frac{eV}{kT}\right) - 1]
\]

If \( V = V_0 \sin \omega t \)

And finally

\[
b \cdot \frac{V_0^2}{2} (1 - \cos 2\omega t)
\]

Typical I-V dependence for p-n diode

\[
I_{\text{DC}} \propto b \frac{V_0^2}{2} + \ldots
\]
Detecting of the microwaves

$\text{HSCH-9161}$
$\text{HSCH-9162}$
GaAs Detector Diode

$f_c \sim 200 \text{GHz}$

Coaxial Cable

Modulated r-f

Adjusting Screw

Brass-base Terminal

Cat-whisker Type of Contact

Silicon Wafer

Brass Terminal Pin

Ceramic Body

Note: All dimensions in microns (mils)

Beam Lead = 7-9 um

Die Thickness = 50-60 um
Experiments: Michelson interferometer

Mirror A

Mirror B

Transmitter

Beam splitter

Receiver

L_R, L_B optical paths (OP) for “red” and “blue” rays

OP = n*L_G

n – refraction index;
L_G – geometrical length

Condition for constructive interference

2 |L_R - L_B| = k\lambda
Experiments: Michelson interferometer

Physics 403 Lab Michelson interferometer setup
Experiments: Double slit Interference. T. Young 1801

For constructive Interference \( \Delta r = n\lambda \) or \( dsin\theta = n\lambda \)

The measured envelope of the diffraction pattern can be defined as:

\[
|\psi_{ss}|^2 = |\psi_0|^2 \left( \frac{\sin x}{x} \right)^2 \times \cos^2 \left[ (kd \sin(\theta/2)) \right]
\]

where \( x = kb \sin(\theta/2) \) and \( k = \frac{2\pi}{\lambda} \) is wave vector of the plane wave.
Experiments: Double slit interference

Physics 401 Lab setup and example of the data
Experiments: Double slit interference. Fitting

\[ |\psi_s|^2 = |\psi_0|^2 \left( \frac{\sin x}{x} \right)^2 \times \cos^2 \left[ (kd \sin(\theta/2)) \right] \]

\[ x = kb \sin(\theta/2) \]

Model

<table>
<thead>
<tr>
<th>Equation</th>
</tr>
</thead>
<tbody>
<tr>
<td>( y = I0 \times (\sin(K1 \sin(\pi x/360+f))/(K1 \sin(\pi x/360+f))) )</td>
</tr>
<tr>
<td>Reduced Chi-Sqr</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Adj. R-Square</th>
<th>Value</th>
<th>Standard Error</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.96659</td>
<td>190.6014</td>
<td>3.042882</td>
</tr>
<tr>
<td>4.384042</td>
<td>0.074754</td>
<td></td>
</tr>
<tr>
<td>13.51332</td>
<td>0.052244</td>
<td></td>
</tr>
<tr>
<td>-0.01525</td>
<td>7.19E-04</td>
<td></td>
</tr>
<tr>
<td>9.572049</td>
<td>1.440409</td>
<td></td>
</tr>
</tbody>
</table>

Here in fitting expression:

\[ I_0 = |\psi_0|^2; \]
\[ K1 = kb; \]
\[ K2 = kd \]
Lloyd's Mirror experiment

Difference of the wave paths of “red” and “blue” rays is:

\[ \Delta S = \sqrt{h^2 + d_1^2} + \sqrt{h^2 + d_2^2} - (d_1 + d_2) \]

For constructive interference

\[ \Delta S = n\lambda \]
Total internal reflection experiment. Snell’s law

Snell’s law:

\[ n_1 \sin \theta_1 = n_2 \sin \theta_2 \]

Equation for critical angle:

\[ n_1 \sin \theta_c = n_2 \sin 90^\circ \]

\[ \theta_c = \sin^{-1} \left( \frac{n_2}{n_1} \right) \]
**Total internal reflection experiment**

Experimental setup and the example of the data
Microwave polarization

Transmitter

Polarizer

Receiver

Metallic grid

Etienne-Louis Malus
1775 – 1812

Malus law

\[ E = E_0 \cos \theta \]

\[ I \propto E^2 \]

\[ I = I_0 \cos^2 \theta \]
Microwave polarization

Transmitter

Rotatable receiver

Polarizer

$I = I_0 \cos^2 \theta$

Experimental data

log($I_0 \cos^2 \theta$) vs. log($\cos \theta$)

Equation: $Y = 0.4003 + 2.45233 \times X$

4/2/2018
Bragg diffraction

Interference of the EM waves reflected from the crystalline layers

\[ n\lambda = 2d \sin \theta \]  
Bragg's Law

Sir William Henry Bragg  
1862-1942

William Lawrence Bragg  
1890-1971

The Nobel Prize in Physics 1915  
"for their services in the analysis of crystal structure by means of X-rays"
Bragg diffraction

Different orientations of the crystal

(100)  (110)  (210)
In our experiment $\lambda \sim 3\text{cm}$; For cubic symmetry the angles of Bragg peaks can be calculated from:

$$\left( \frac{\lambda}{2d} \right)^2 = \frac{\sin^2 \theta}{h^2 + k^2 + l^2}$$

where $h, k, l$ are the Miller Indices. For crystal with $d=5\text{cm}$ and $\lambda=3\text{cm}$ the 3 first Bragg peaks for (100) orientation can be found at angles: $\sim 17.5^\circ; 36.9^\circ$ and $64.2^\circ$.

Experimental setup
Bragg diffraction
Bragg diffraction. Results.*

Matthew Stupca
Longxiang Zhang

I (µA)
Θ (degree)

*courtesy of Matthew Stupca
Bragg diffraction. X-rays.

\[ \lambda \ 0.01 \div 10 \text{nm} \]

X-ray tube

*courtesy of Wikipedia*
Bragg diffraction. X-rays.

X-ray K-series spectral line wavelengths (nm) for some common target materials

<table>
<thead>
<tr>
<th>Target</th>
<th>$K\beta_1$</th>
<th>$K\beta_2$</th>
<th>$K\alpha_1$</th>
<th>$K\alpha_2$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Fe</td>
<td>0.17566</td>
<td>0.17442</td>
<td>0.193604</td>
<td>0.193998</td>
</tr>
<tr>
<td>Co</td>
<td>0.162079</td>
<td>0.160891</td>
<td>0.178897</td>
<td>0.179285</td>
</tr>
<tr>
<td>Ni</td>
<td>0.15001</td>
<td>0.14886</td>
<td>0.165791</td>
<td>0.166175</td>
</tr>
<tr>
<td>Cu</td>
<td>0.139222</td>
<td>0.138109</td>
<td>0.154056</td>
<td>0.154439</td>
</tr>
<tr>
<td>Zr</td>
<td>0.70173</td>
<td>0.68993</td>
<td>0.78593</td>
<td>0.79015</td>
</tr>
<tr>
<td>Mo</td>
<td>0.63229</td>
<td>0.62099</td>
<td>0.70930</td>
<td>0.71359</td>
</tr>
</tbody>
</table>


*courtesy of Matthew Stupca*
Bragg diffraction. X-rays.

Study of structural and photoluminescent properties in barium titanate nanocrystals synthesized by hydrothermal process

Ming-Sheng Zhang$^a$,*, Zhen Yin$^a$, Qiang Chen$^a$, Weifeng Zhang$^b$, Wanchun Chen$^c$

*courtesy of Matthew Stupca
Comments and suggestions

1. Klystron is very hot and the high voltage (~300V) is applied to repeller.
2. You have to do 6 (!) experiment in one Lab session – take care about time management. The most time consuming experiment is the “Bragg diffraction”.
3. Do not put on the tables any extra stuff – this will cause extra reflections of microwaves and could result in smearing of the data.
4. This is two weeks experiment but the equipment for the week 2 will be different. Please finish all week 1 measurements until the end of this week

Good luck!