Pulses in Transmission Lines

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Physics 401

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ILLINOIS

Key Concepts for this Lab

1. Networks with distributed parameters
   What if the R/L/C are spread out?
   Thevenin-equivalent networks

2. Pulse propagation in transmission lines
   When signals move like waves
   Reflections from resistive and reactive loads

3. Impedance matching
   Getting power where you want it to go

When do Wires Carry Waves?

Thus far we've implicitly assumed that V(t) and I(t) are synchronized throughout our circuits – but signals travel at **finite speed** ($v = c/n$)!

- Speed of light: $c = 3\times10^8$ m/s = 30 cm/ns = **1 ft/ns**
  Real signals are typically slower by a factor (n) of order unity

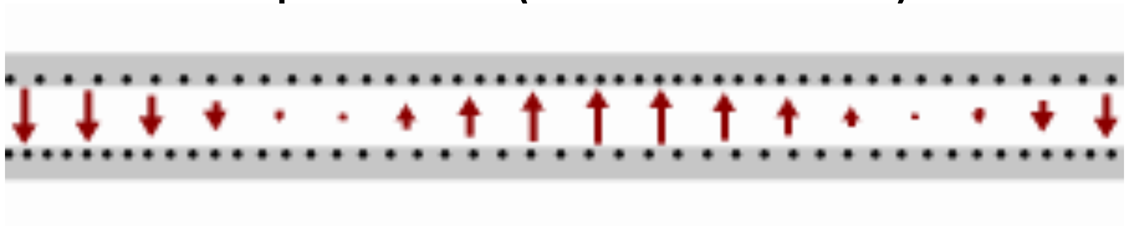
Over distances that are a significant fraction of the **wavelength**, we're better off thinking of **wave propagation**

Frequency	Application	nλ
60 Hz	AC power lines	5000 km
580 kHz	WILL-AM broadcast	500 m (0.3 mile)
2.4 GHz	WiFi	12.5 cm (5 inches)
430 THz	Red light	0.7 μm

Transmission Lines

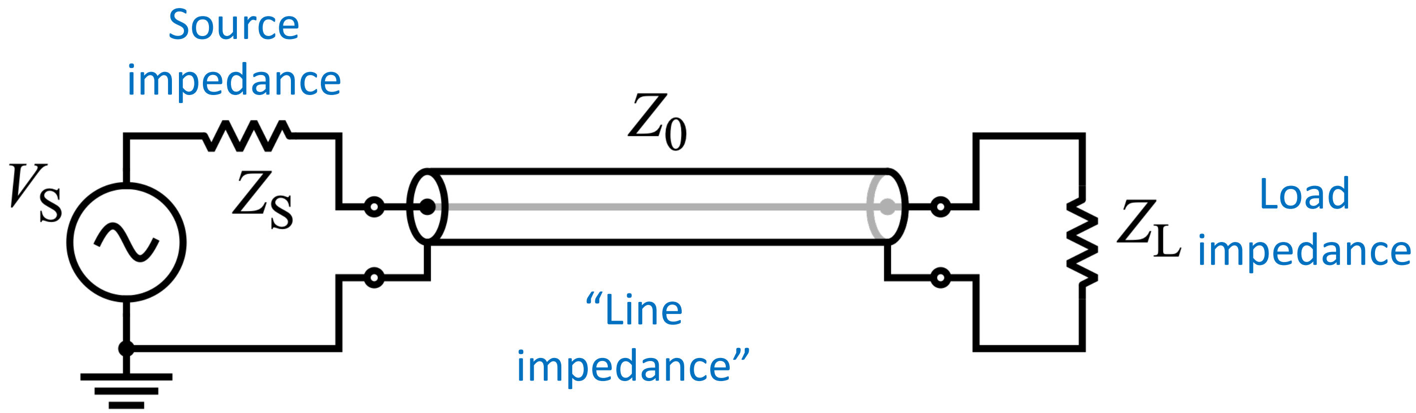
Transmission line: a specialized cable (or other structure) designed to conduct alternating current of radio frequency (RF).

- Limit wave reflection by maintaining uniform impedance (*details below!*)
- Reduce power loss to radiation

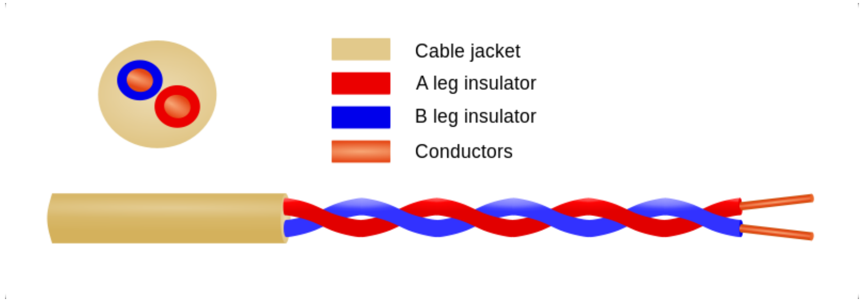


[Wikipedia: Transmission Line]

Design goal:
Maximize power from the source delivered to the load



Common Types of Transmission Lines

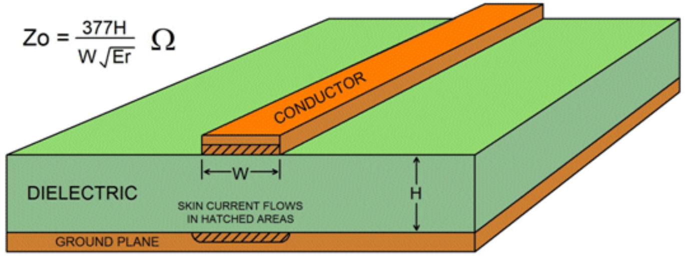


Twisted pair
Wikipedia

Twin lead
Wikipedia

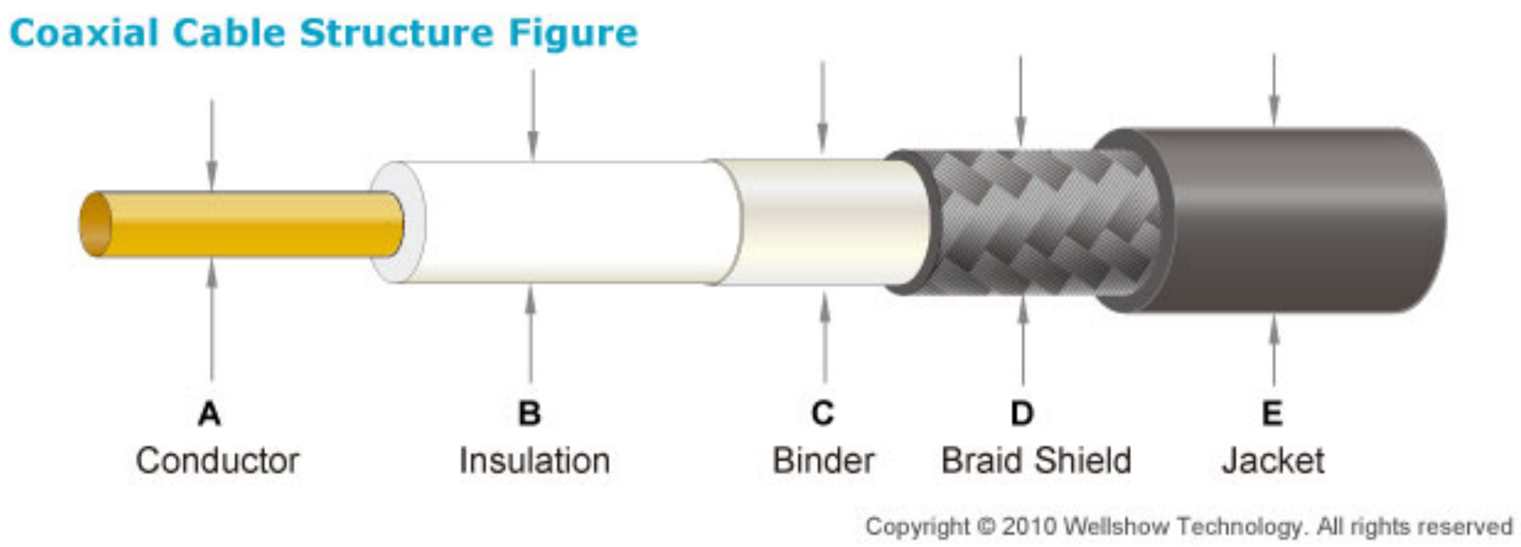
Coaxial cable

MICROSTRIP
TRANSMISSION
LINE



Analog Devices

Coaxial Cable

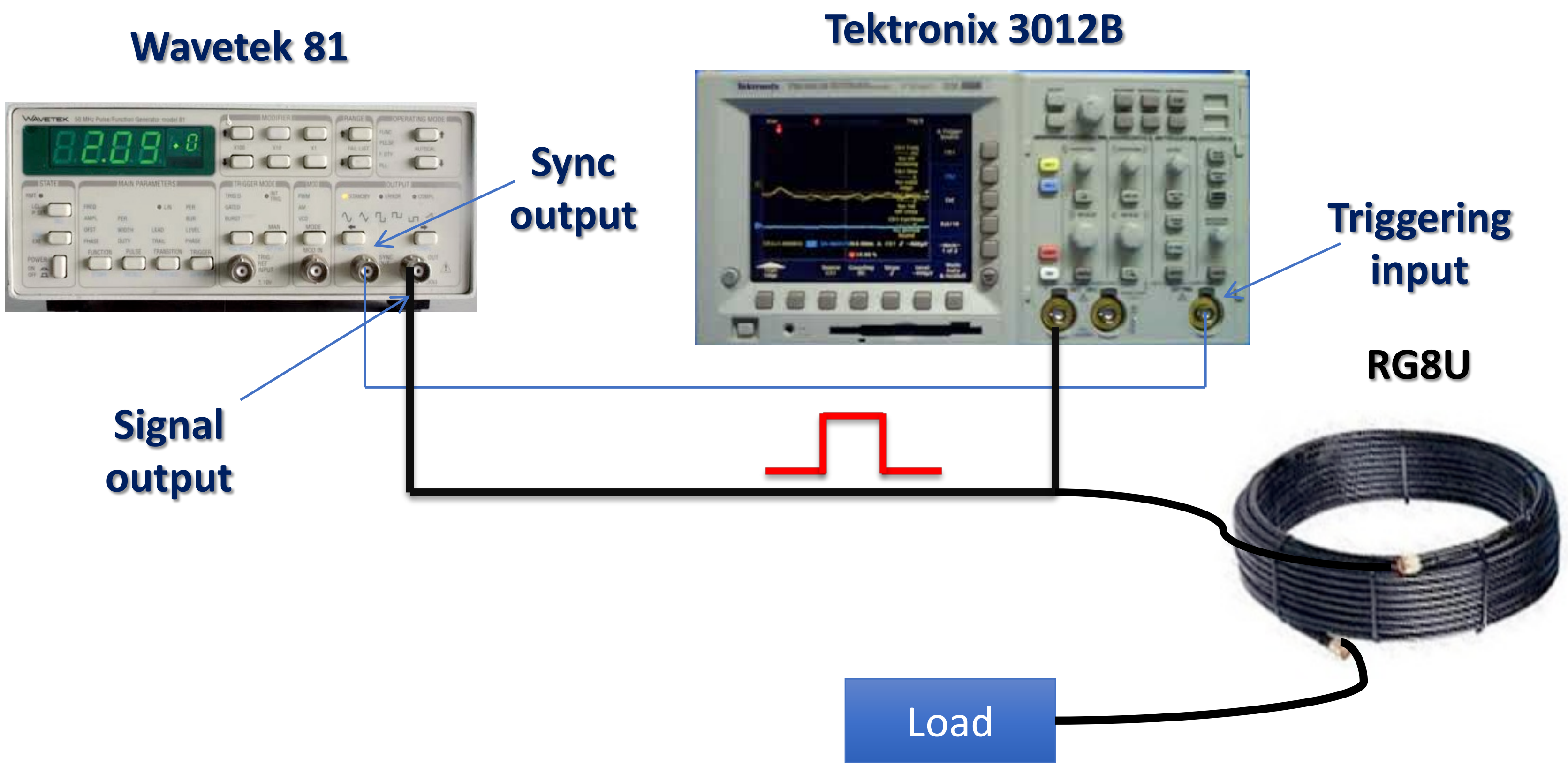


Specification:
Impedance: 53 Ω
Capacitance: 83 pF/m
Conductor: Bare Copper Wire (1/1.02mm)

Signal voltage between central conductor and braid shield

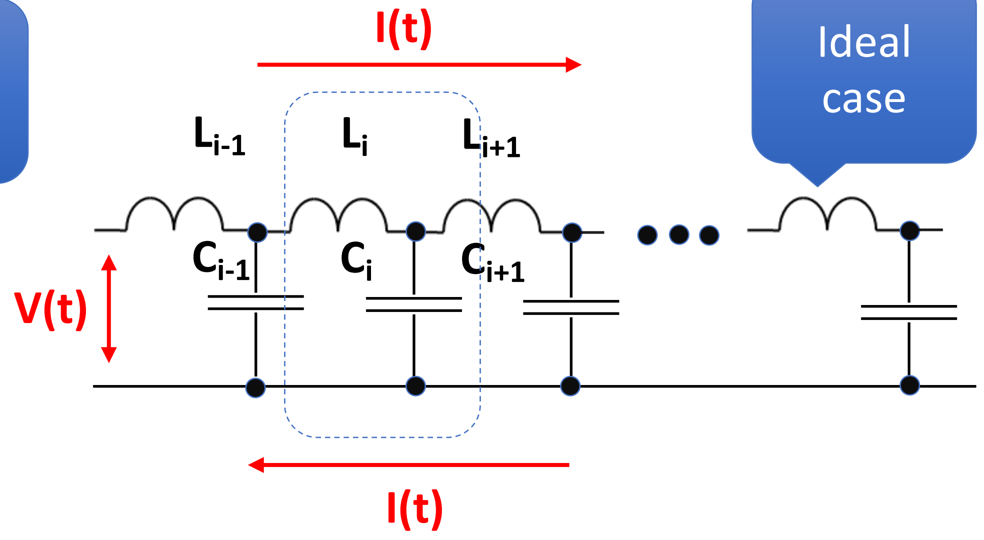
Shield reduces external dipole radiation (and response to RF interference)

Experimental Setup

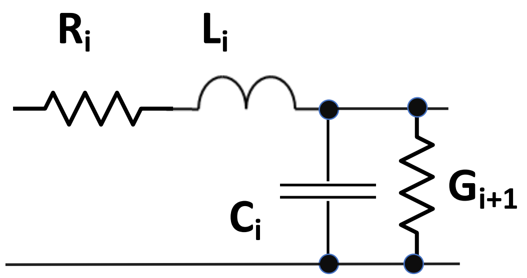


Modeling a Transmission Line

Simplified equivalent circuit



Ideal case



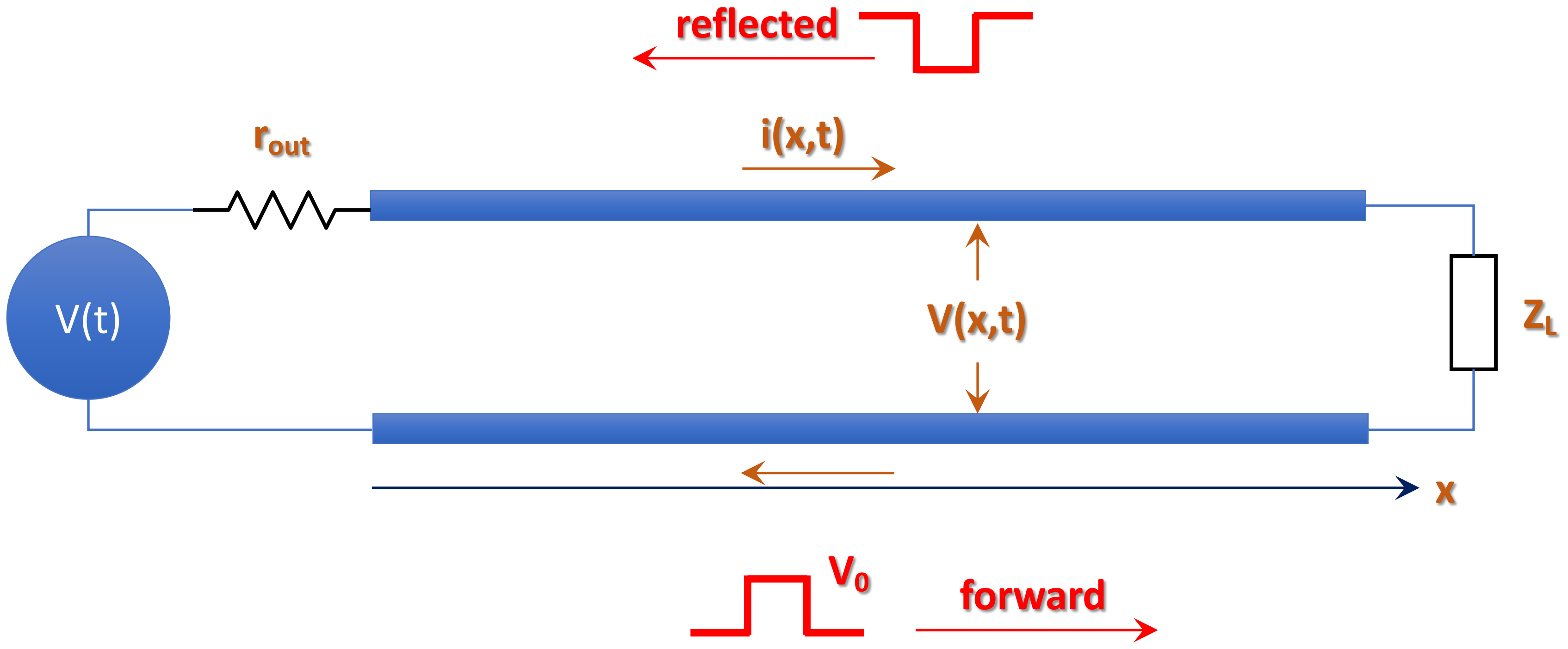
Real situation

Model as **distributed network** rather than lumped components

Ideal line has inductance / capacitance per unit length (lossless)

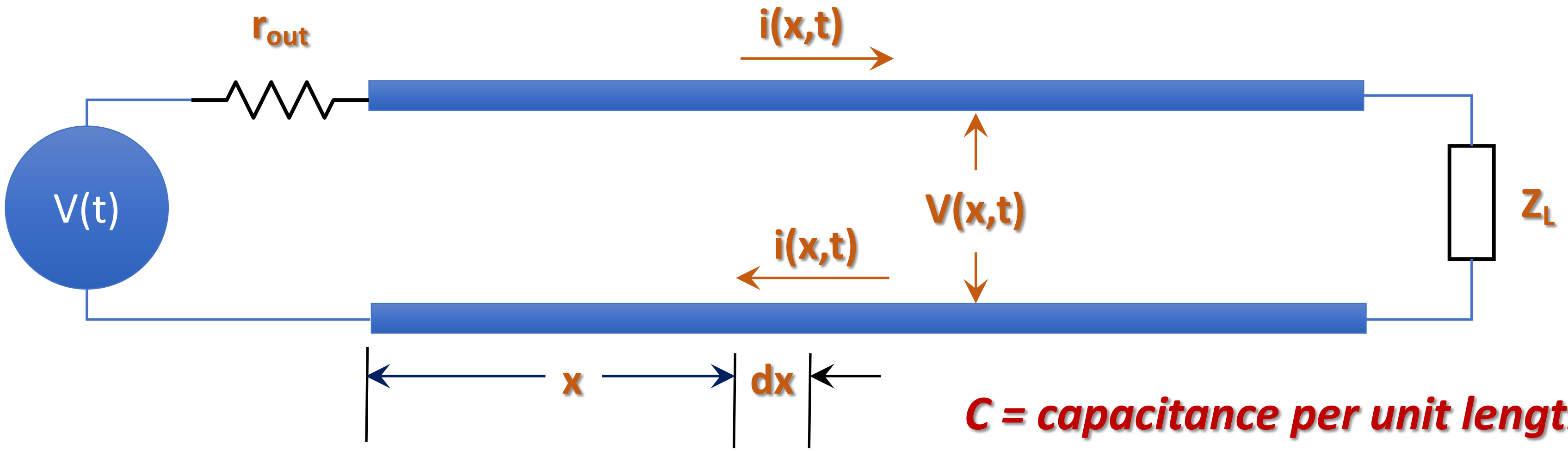
Real lines have finite conductance $G = \frac{1}{R}$ between conductors (*i.e.* loss)

Pulses in Transmission Lines



The Telegrapher's Equations

Lossless line (G=0)



C = capacitance per unit length
L = inductance per unit length

Distributed capacitance

$$(C\,dx)V = -dq$$

$$(C\,dx)\frac{\partial V}{\partial t} = -\frac{\partial q}{\partial t} = -I$$

$$\mathbf{C}\frac{\partial \boldsymbol{V}}{\partial \boldsymbol{t}} = -\frac{\partial \boldsymbol{I}}{\partial \boldsymbol{x}}$$

Distributed inductance

$$dV = -(L\,dx)\frac{\partial I}{\partial t}$$

$$\frac{\partial \boldsymbol{V}}{\partial \boldsymbol{x}} = -\boldsymbol{L}\frac{\partial \boldsymbol{I}}{\partial \boldsymbol{t}}$$

The Wave Equation

Distributed capacitance

$$C\frac{\partial V}{\partial t} = -\frac{\partial I}{\partial x}$$

$$\frac{\partial}{\partial t}$$

$$\frac{\partial^2 I}{\partial x \partial t} = -C\frac{\partial^2 V}{\partial t^2}$$

Distributed inductance

$$\frac{\partial V}{\partial x} = -L\frac{\partial I}{\partial t}$$

$$\frac{\partial}{\partial x}$$

$$\frac{\partial^2 V}{\partial x^2} = -L\frac{\partial^2 I}{\partial x \partial t}$$

Combining

$$\frac{\partial^2 V}{\partial x^2} = \mathrm{LC}\frac{\partial^2 V}{\partial t^2} \qquad \frac{\partial^2 I}{\partial x^2} = \mathrm{LC}\frac{\partial^2 I}{\partial t^2}$$

Voltage and Current Waves

$$\frac{\partial^2 V}{\partial x^2} = \mathrm{LC}\frac{\partial^2 V}{\partial t^2} \qquad \frac{\partial^2 I}{\partial x^2} = \mathrm{LC}\frac{\partial^2 I}{\partial t^2}$$

Seek solutions of the form…

Waves of velocity v

$$V(x,t) = V_0 \sin\omega\left(t - \frac{x}{v}\right)$$

$$\mathrm{I}(x,t) = I_0 \sin\omega\left(t - \frac{x}{v}\right)$$

Substitute $V(x,t)$, $\mathrm{I}(x,t)$ into…

$$\mathrm{C}\frac{\partial V}{\partial t} = -\frac{\partial I}{\partial x} \qquad \frac{\partial V}{\partial x} = -L\frac{\partial I}{\partial t}$$

… to find two key consequences:

1.

$$v = \frac{1}{\sqrt{LC}}$$

Speed of wave propagation

2.

$$Z_k \equiv \frac{V(x,t)}{I(x,t)} = \sqrt{\frac{L}{C}}$$

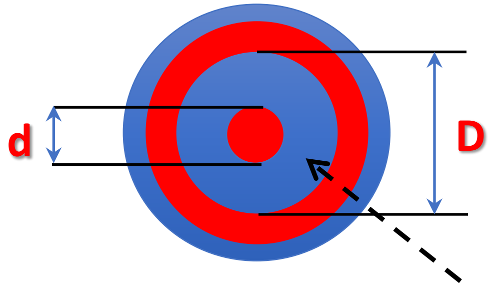
Characteristic impedance of line

Characteristic Impedance

$$Z_k = \sqrt{\frac{L}{C}}$$

C = capacitance per unit length (F/m)
L = inductance per unit length (H/m)

$$\varepsilon_0 = 8.854\times10^{-12}\ F/m$$
$$\mu_0 = 4\pi\times10^{-7}\ H/m$$
$$\varepsilon_0\mu_0 = c^2$$



$$C = \frac{2\pi\varepsilon_0\varepsilon_r}{\ln\left(\frac{D}{d}\right)}$$ **(F/m)**

$$L = \frac{\mu_0\mu_r}{2\pi}\ln\left(\frac{D}{d}\right)$$ **(H/m)**

ε_r – dielectric permittivity
μ_r-magnetic permeability ≈1

Finally for coaxial cable: $Z_k = \frac{138}{\sqrt{\varepsilon_r}}\log_{10}\left(\frac{D}{d}\right)(Ohms)$

Wave Propagation Speed

$$v = \frac{1}{\sqrt{LC}}$$

$$C = \frac{2\pi\varepsilon_0\varepsilon_r}{\ln\left(\frac{D}{d}\right)} \qquad L = \frac{\mu_0\mu_r}{2\pi}\ln\left(\frac{D}{d}\right)$$

$$\varepsilon_0 = 8.854\times10^{-12}\ F/m$$
$$\mu_0 = 4\pi\times10^{-7}\ H/m$$
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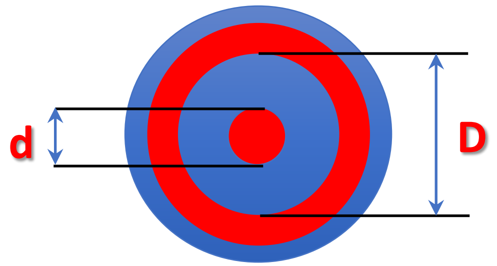


$$v = \frac{1}{\sqrt{LC}} = \frac{1}{\sqrt{\mu_0\mu_r\varepsilon_0\varepsilon_r}} = \frac{c}{\sqrt{\mu_r\varepsilon_r}} \approx \frac{c}{\sqrt{\varepsilon_r}}$$

The **delay time** of a signal is $\tau = \frac{1}{v}$ (s/m) $\approx 3.336\sqrt{\varepsilon_r}$ ns/m

RG-8/U
RG58U

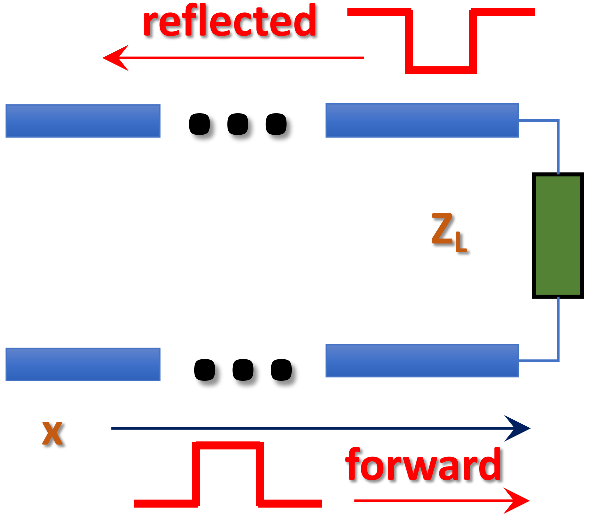
Inner insulation material: Polyethylene

For polyethylene below 1 GHz, $\varepsilon_r \approx 2.25$

Nominal impedance: 52 ohm

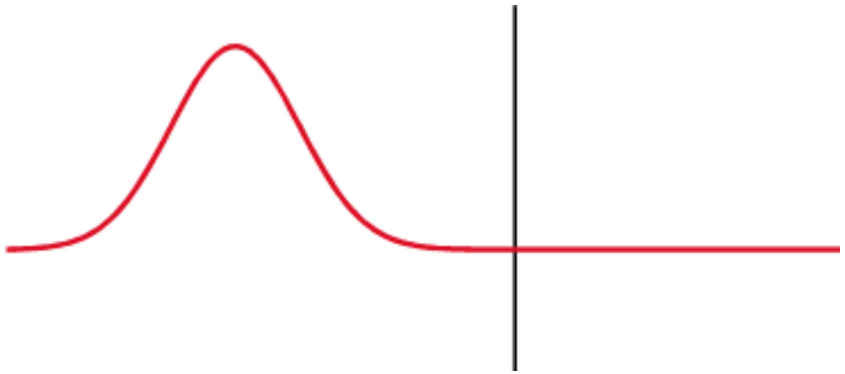
Delay time: ~5 ns/m (speed ~2/3c)

Reflection in Transmission Lines



Any wave will be (partially) **reflected** when it reaches a change in impedance

[Wikipedia: Reflection Coefficient]



Forward wave

$$V(x,t) = V_0 \sin\omega\left(t - \frac{x}{v}\right)$$

$$\mathrm{I}(x,t) = I_0 \sin\omega\left(t - \frac{x}{v}\right)$$

$$V(x,t) = Z_k I(x,t)$$

Reflected wave

$$V_r(x,t) = V_r \sin\omega\left(t + \frac{x}{v}\right)$$

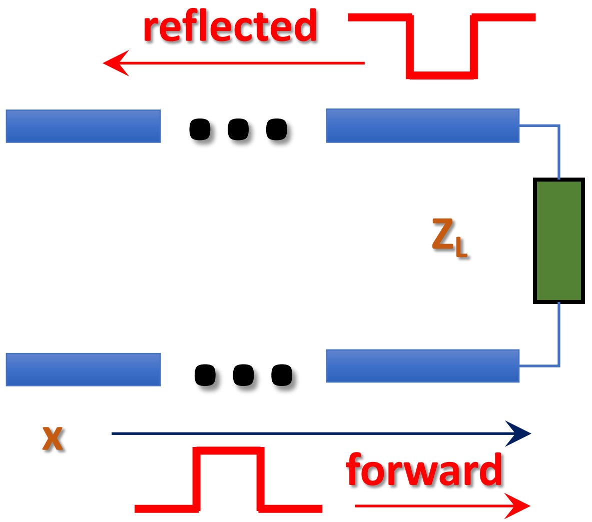
$$\mathrm{I_r}(x,t) = I_r \sin\omega\left(t + \frac{x}{v}\right)$$

$$V_r(x,t) = -Z_k I_r(x,t)$$

At Load

$$V(t) = Z_L I(t)$$

Reflection in Transmission Lines



At Load

$$V = Z_L I$$

Anywhere in Transmission Line

$$V_i = Z_k I_i$$
$$V_r = -Z_k I_r$$

Match at the boundary

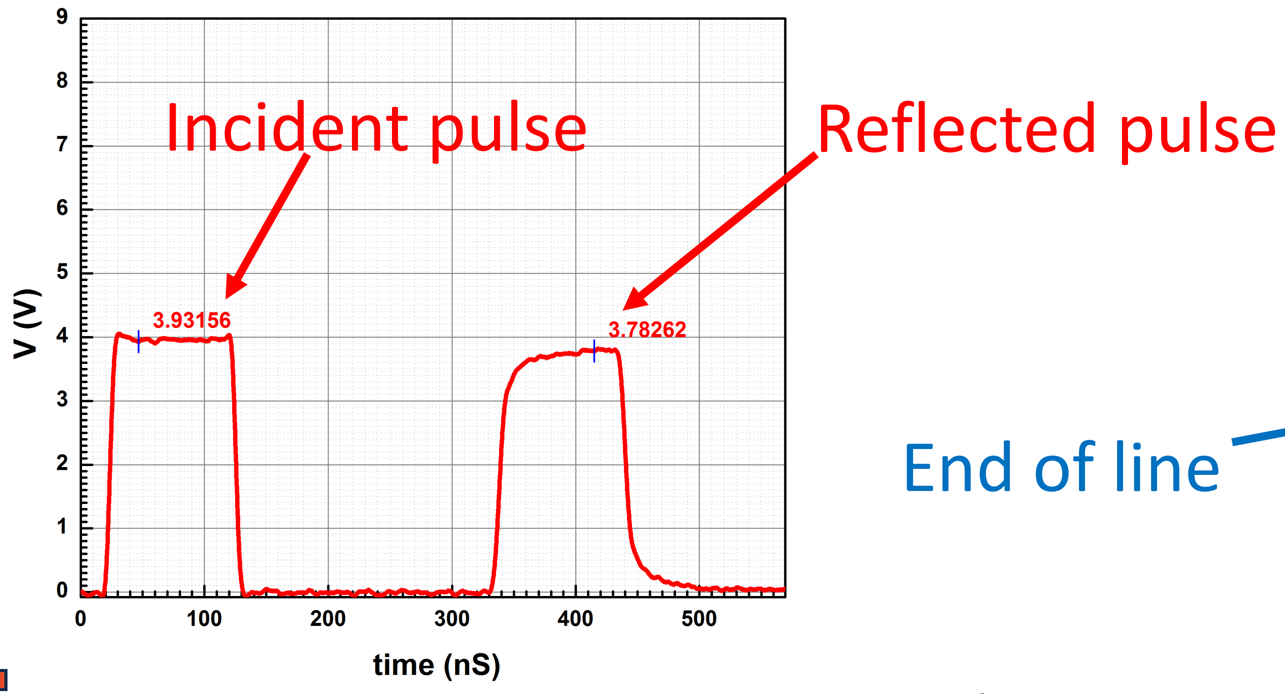
$$V = V_r + V_i$$
$$I = I_r + I_i = \frac{V_i}{Z_k} - \frac{V_r}{Z_k}$$

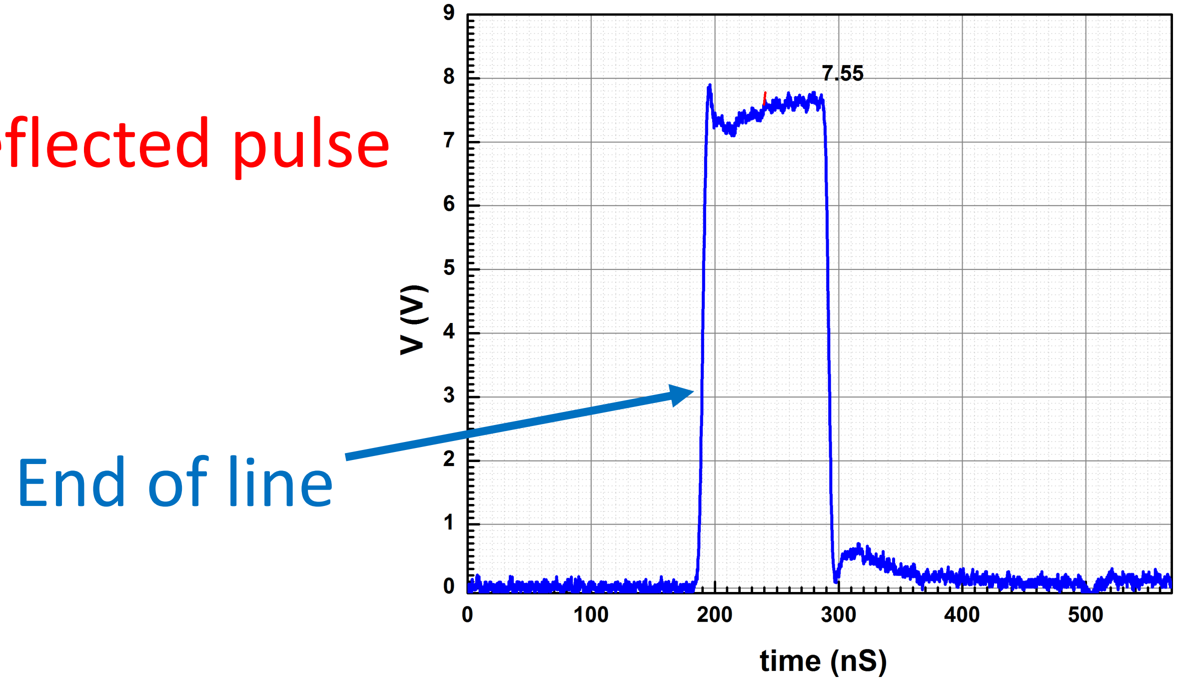
$$\frac{V_i + V_r}{V_i - V_r} = \frac{Z_L}{Z_k} \quad \text{or} \quad V_r = \frac{Z_L - Z_k}{Z_L + Z_k} V_i$$

Reflection from an Open Transmission Line

$$\frac{V_i + V_r}{V_i - V_r} = \frac{Z_L}{Z_k} \quad \text{or} \quad V_r = \frac{Z_L - Z_k}{Z_L + Z_k} V_i$$

Open line: $\boldsymbol{Z_L} = \infty \implies V_r = V_i$, and voltage at load $V_L = V_i + V_r = 2V_i$



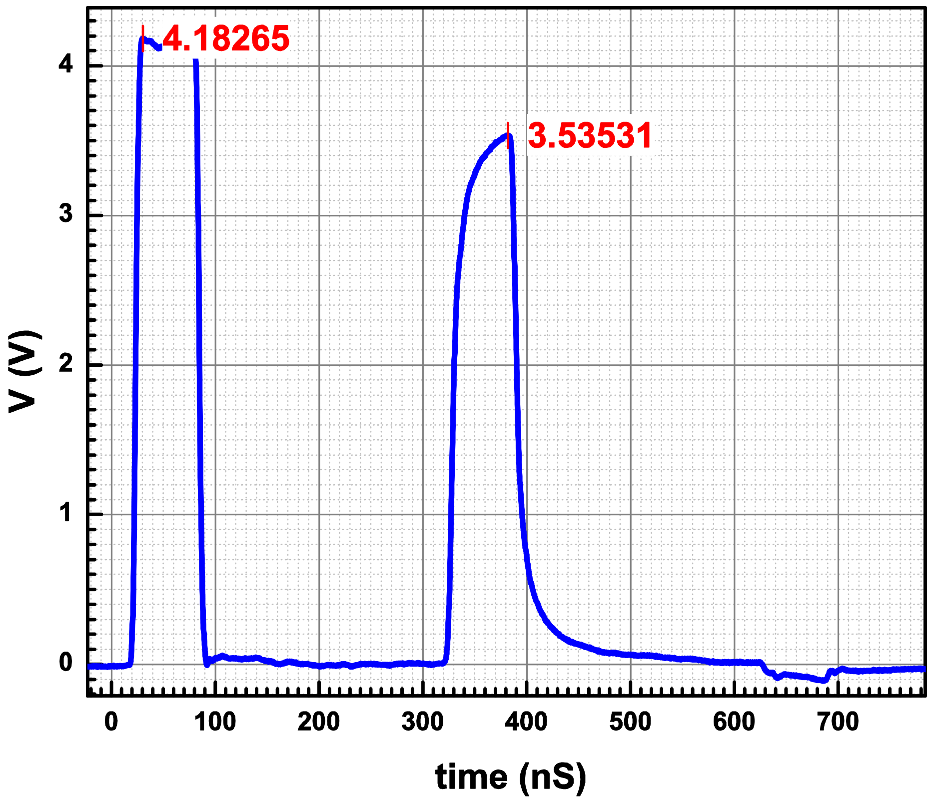


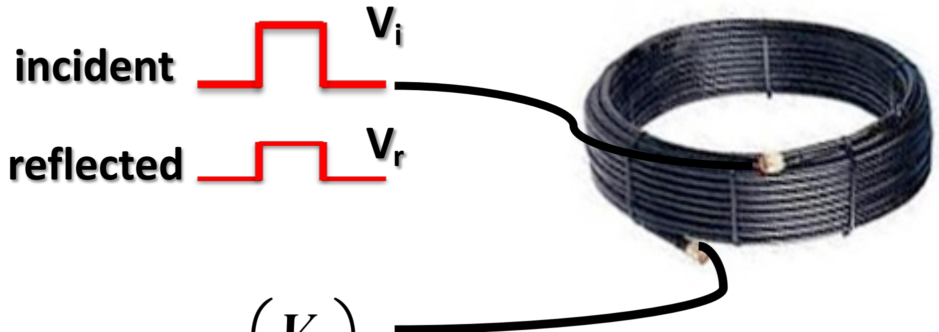
Transmission Line Losses

Why is the reflected pulse ***smaller***?

	Attenuation (dB per 100 ft)				
MHz	30	50	100	146	150
RG-58U	2.5	4.1	5.3	6.1	6.1

Experiment: RG 58U



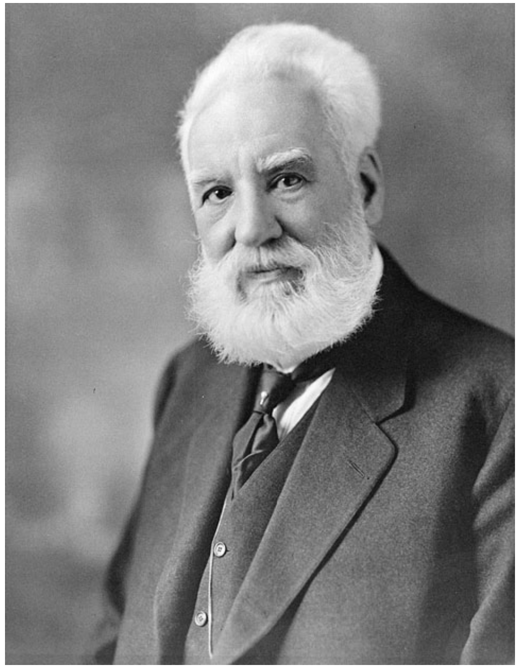


$$ATTN(db) = 20\log\left(\frac{V_i}{V_r}\right)$$

Cable characterized by **attenuation per unit length**

This is slowly frequency dependent!

Reminder: Units of Ratio



Alexander Graham Bell
(1847 – 1922)

We can compare the **relative strength** of two signals by taking the **logarithm** (base-10) of the ratio of their **powers**. This (rarely used) unit was named a **bel**, after A.G. Bell.

We more commonly use the **decibel (dB)**, 1/10th of a bel

$$L[dB] = 10\log_{10}\left(\frac{P_1}{P_2}\right)$$ Power ratio

$$L[dB] = 20\log_{10}\left(\frac{V_1}{V_2}\right)$$ Voltage ratio

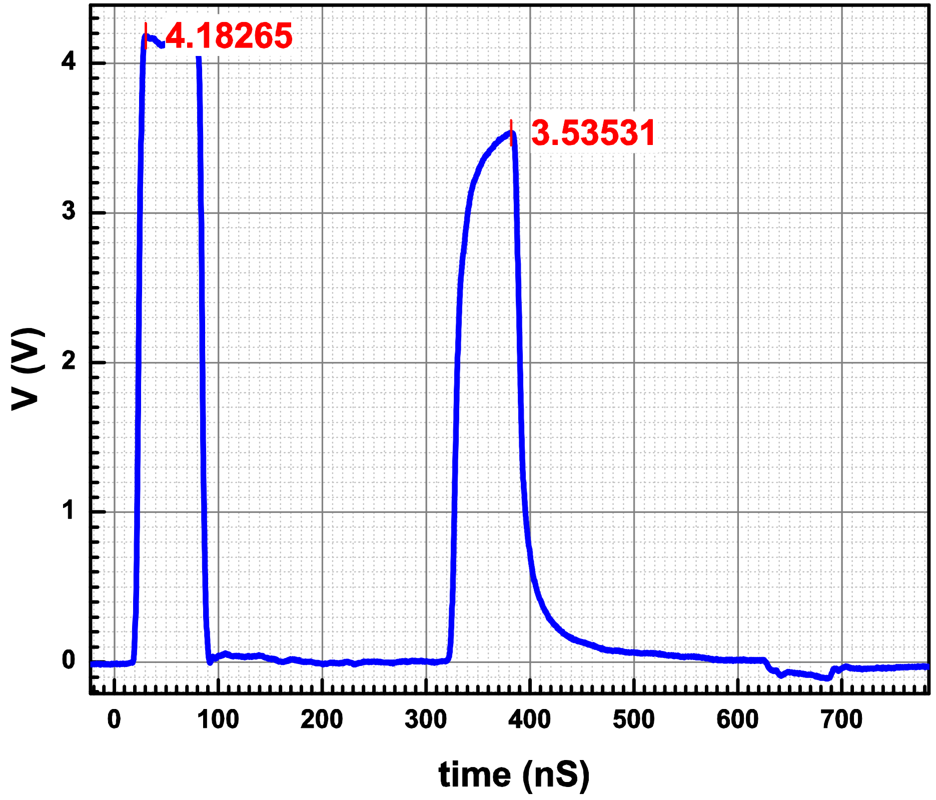
or current, field, …

Ex: -3 dB = ½ power; -20 dB = 1/100 power (1/10 voltage)

Related units: **dBm** = dB relative to 1 mW (absolute unit)
Neper (Np) = like a bel, but natural log (ln)

Transmission Line Losses

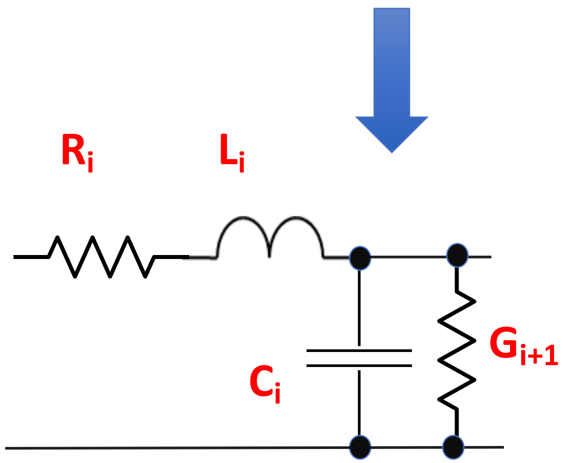
Experiment: RG 58U



In our case:

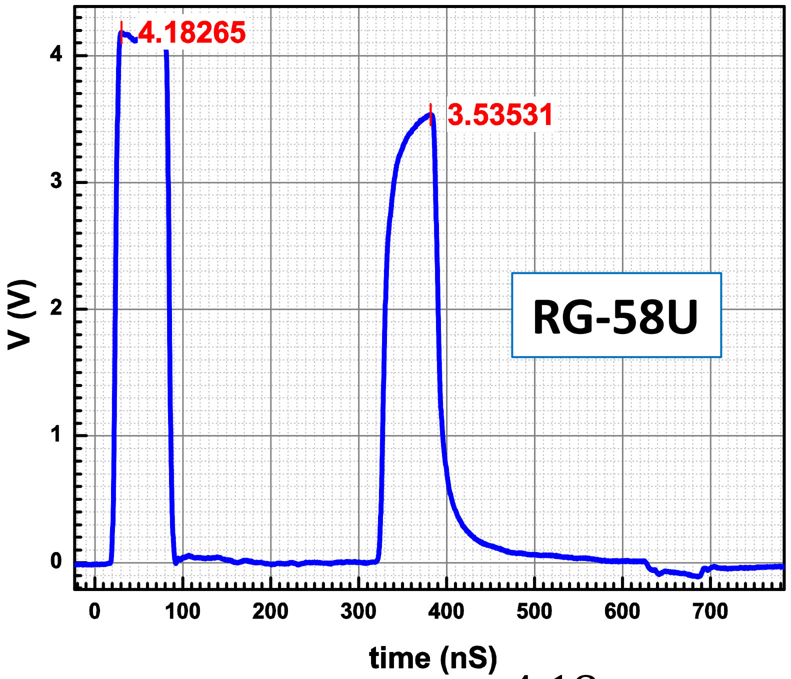
$$Attn(200ft) = 20\log\left(\frac{4.18}{3.54}\right) \approx 1.46\ dB$$

Where does it come from?



Finite conductance

Losses in Different Cables



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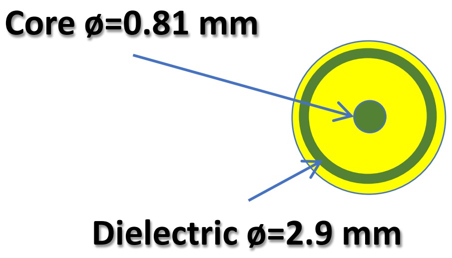
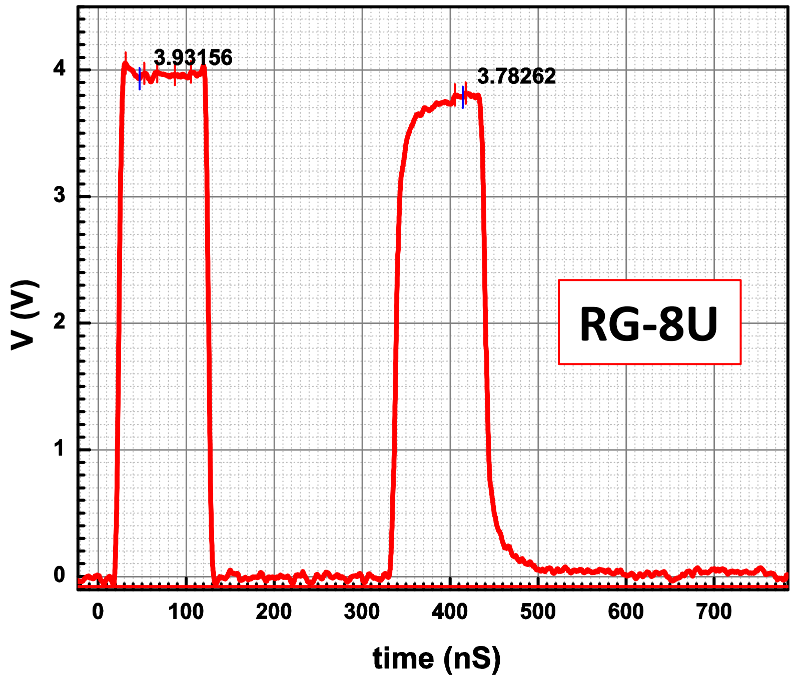
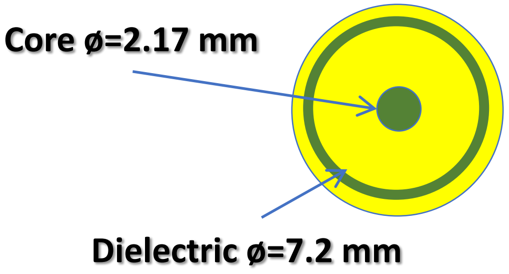
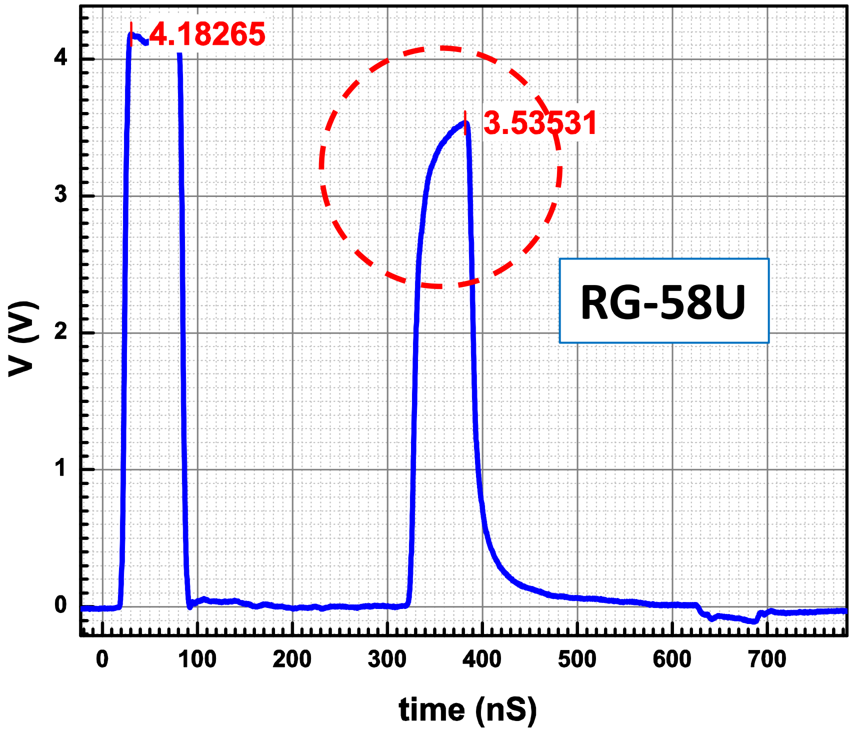




$$Attn(200ft) = 20\log\left(\frac{3.932}{3.78}\right) \approx 0.335\ dB$$

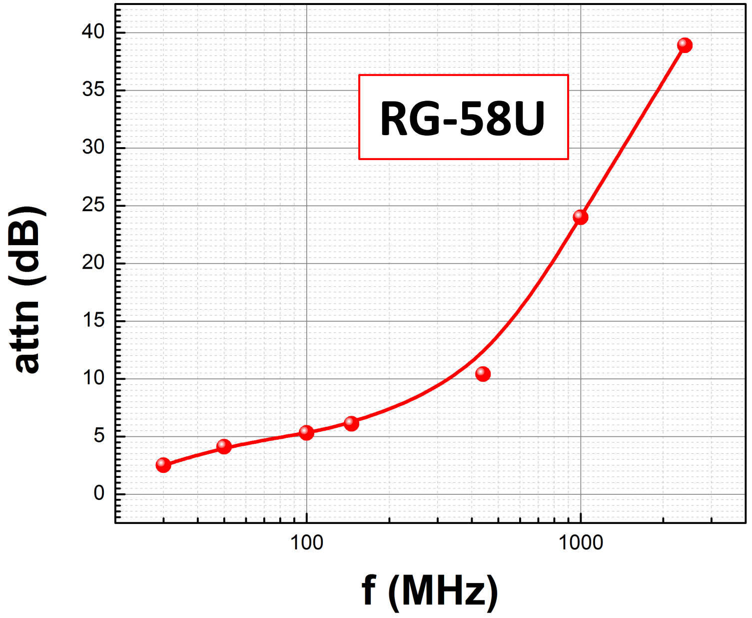


Frequency-Dependent Transmission

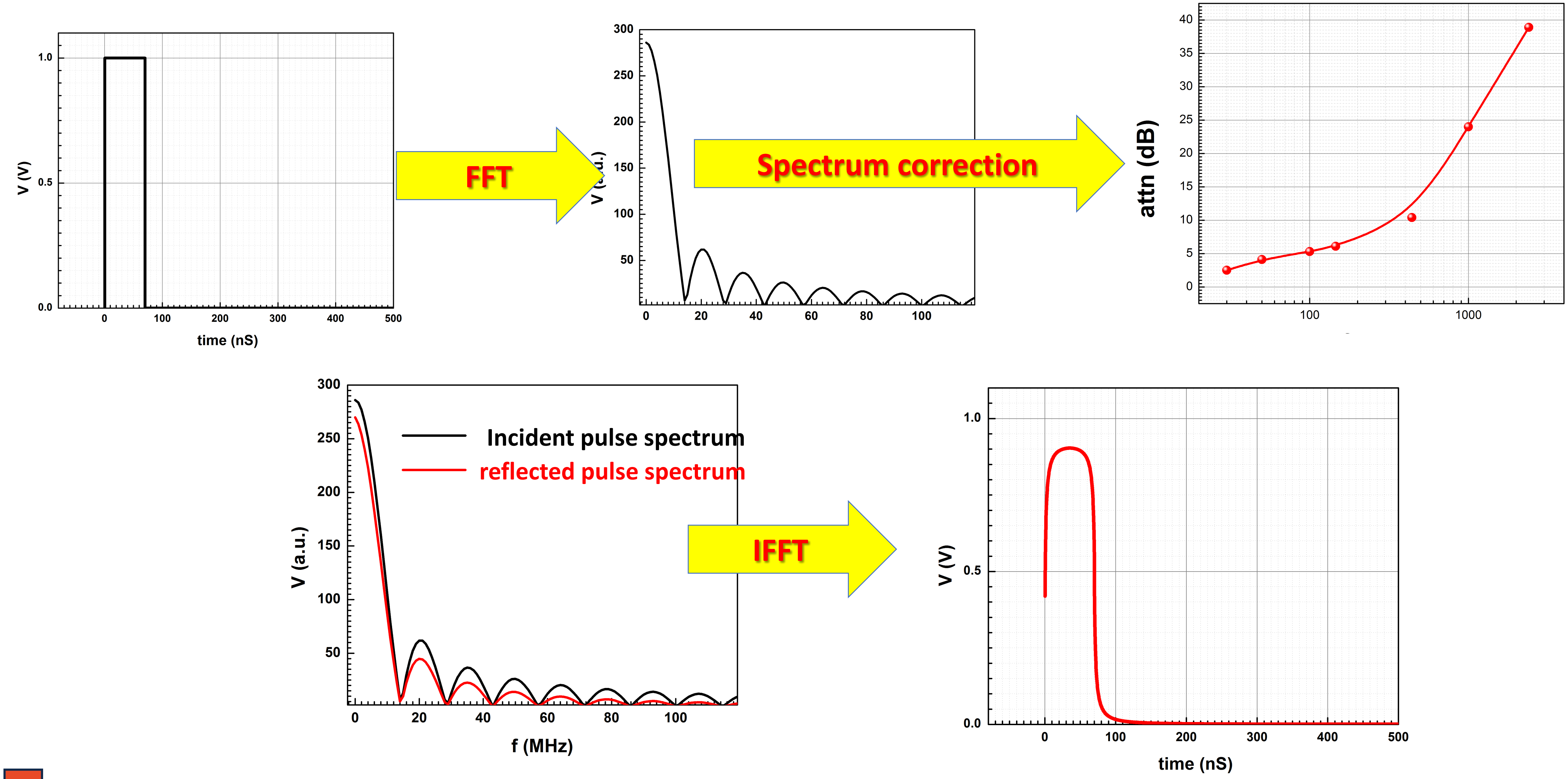


Why is the reflected pulse **distorted** relative to the incident pulse?

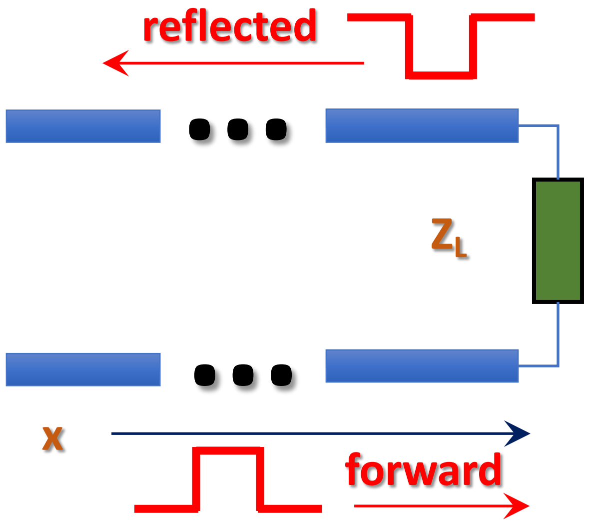
1. **Loss**: Attenuation is frequency-dependent
2. **Dispersion**: Delay (i.e. speed) depends on frequency



Frequency-Dependent Loss Example

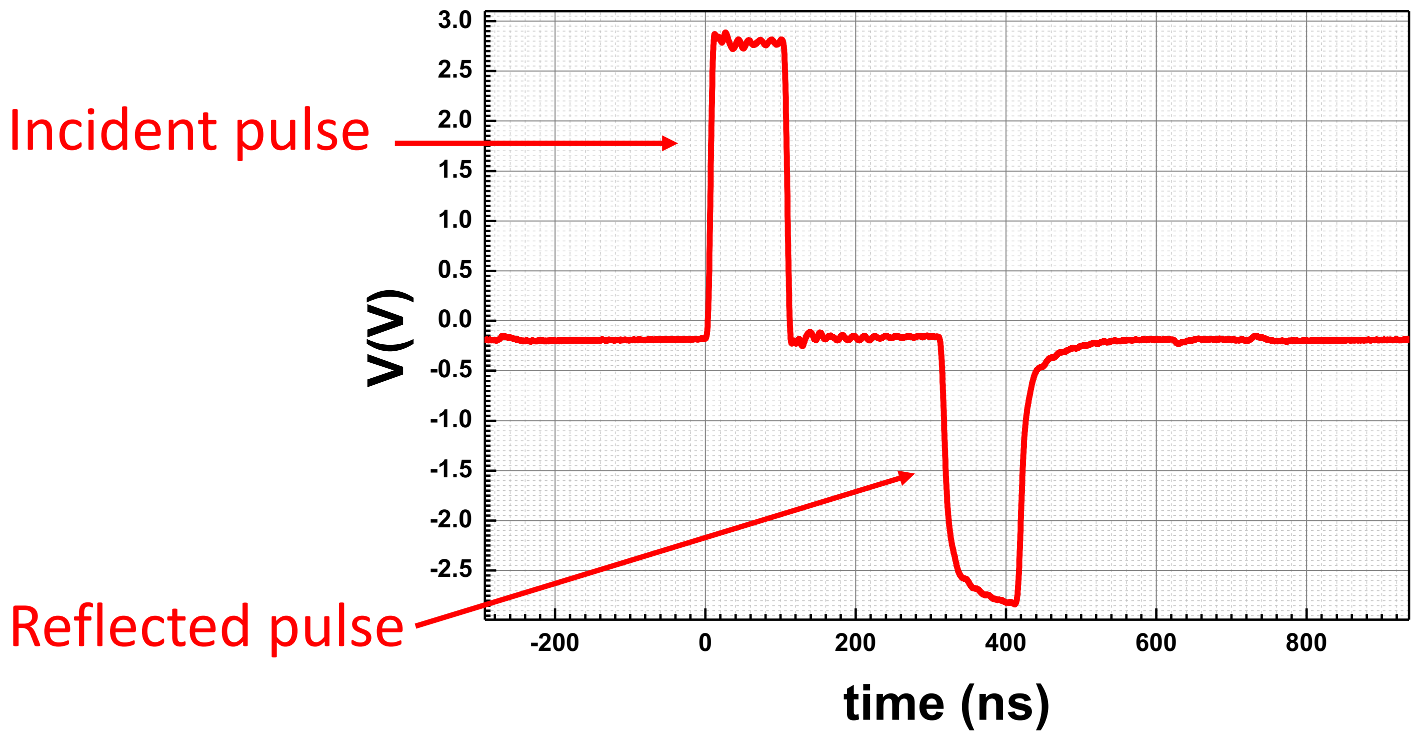


Reflection from a Resistive Load



$$Z_L = R_L$$

$$\frac{V_i + V_r}{V_i - V_r} = \frac{Z_L}{Z_k} \quad \text{or} \quad V_r = \frac{Z_L - Z_k}{Z_L + Z_k} V_i$$



Shorted line

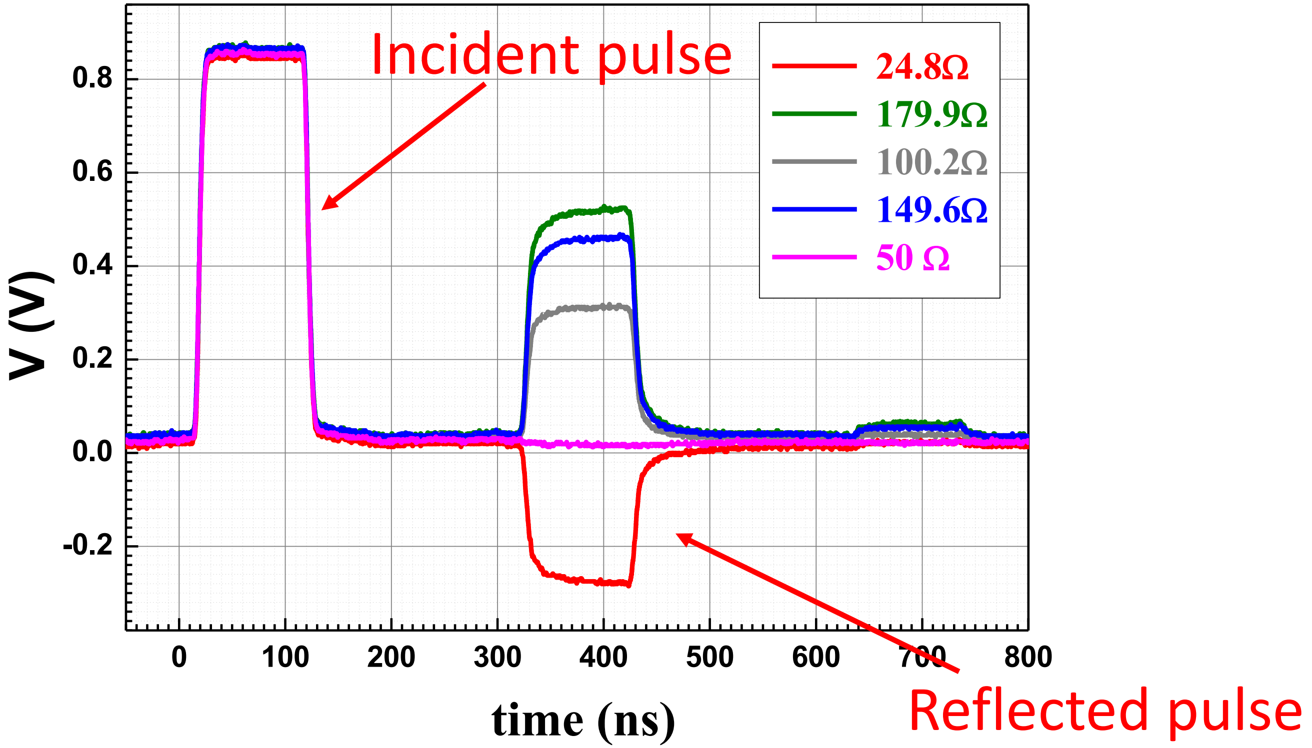
$$R_L = 0 \implies \mathrm{V_r} = -\mathrm{V_i}$$

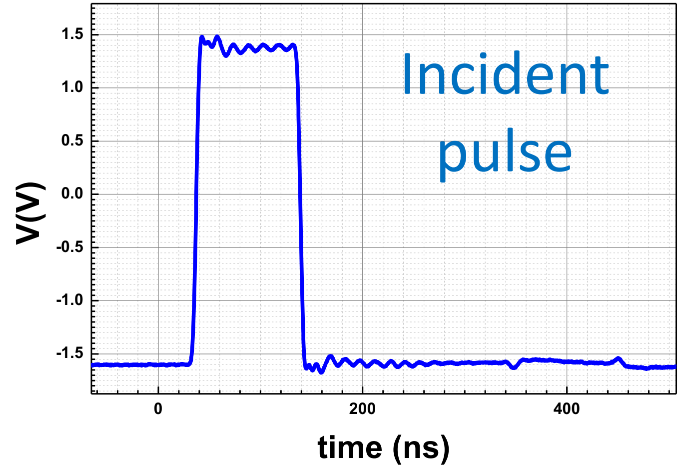
Reflection from a Resistive Load

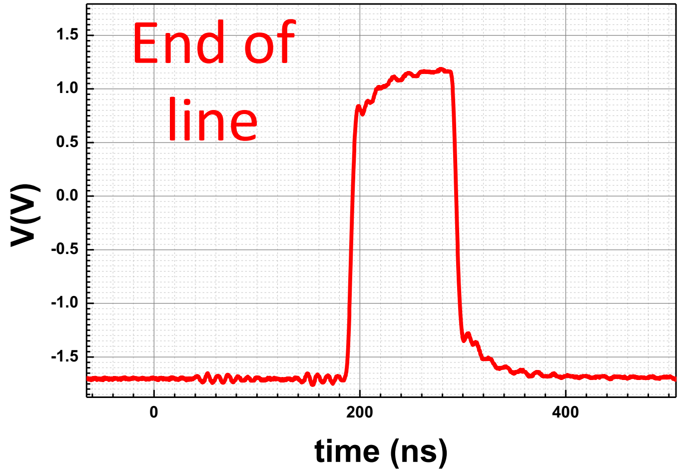
$$\frac{V_i + V_r}{V_i - V_r} = \frac{Z_L}{Z_k} \quad \text{or} \quad V_r = \frac{Z_L - Z_k}{Z_L + Z_k} V_i$$

Matched Impedance

$$R_L = Z_k \implies \mathrm{V_r} = 0$$





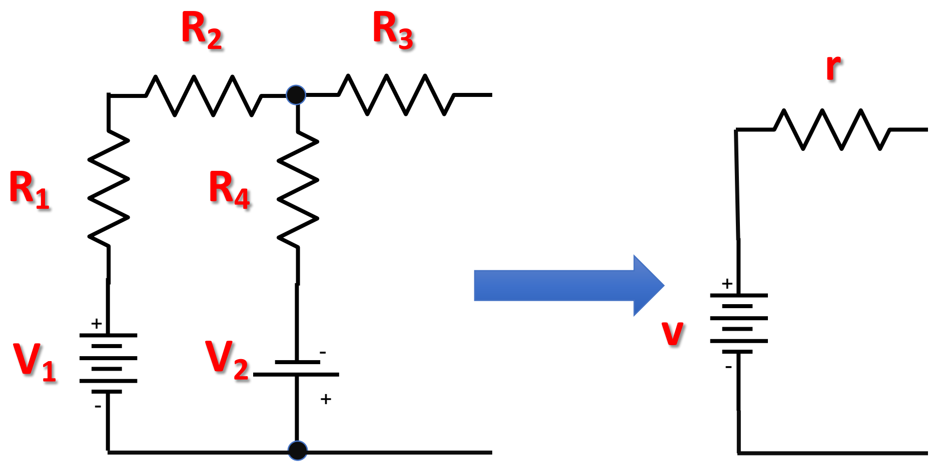


Thevenin's Theorem: Simplifying Networks

Hermann Ludwig Ferdinand von Helmholtz (1821-1894)

Any combination of **voltage sources** and **resistive/reactive impedances** with two terminals ("linear one-port network") can be replaced by an equivalent single **voltage source** and **series impedance**.

Léon Charles Thévenin (1857–1926)



N.B. replacement is exactly equivalent from the **load's** point of view, but e.g. internal power dissipation in the equivalent network may differ

Thevenin's Theorem and Transmission Lines

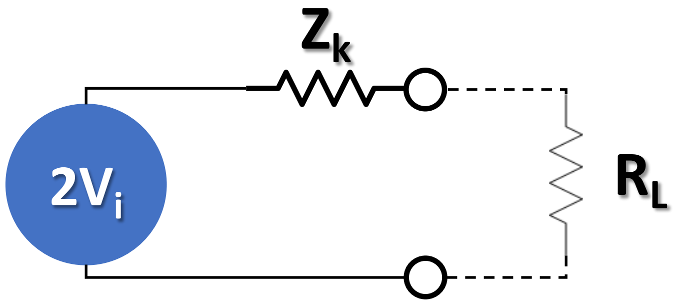
Conditions at the Load

$$V = V_r + V_i = I\,R_L$$

$$I = I_r + I_i = \frac{V_i}{Z_k} - \frac{V_r}{Z_k}$$

Eliminate V_r

$$I = \frac{2\,V_i}{Z_L + Z_k}$$



From this equivalent circuit we can find the maximum possible **power delivered** to the load:

$$P = I^2 R_L = \frac{(2V_i)^2}{(R_L + Z_k)^2} R_L$$

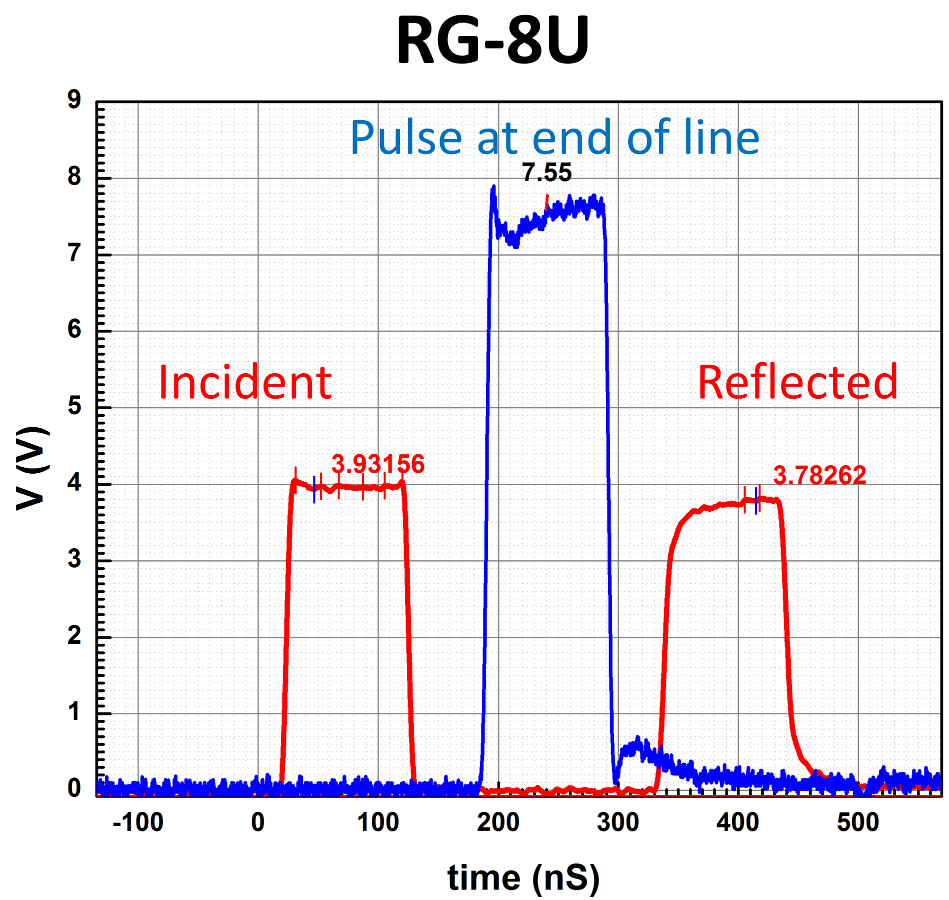
$$P = \frac{(2\,V_i)^2}{R_L\left(1 + \left(\frac{Z_k}{R_L}\right)\right)^2}$$

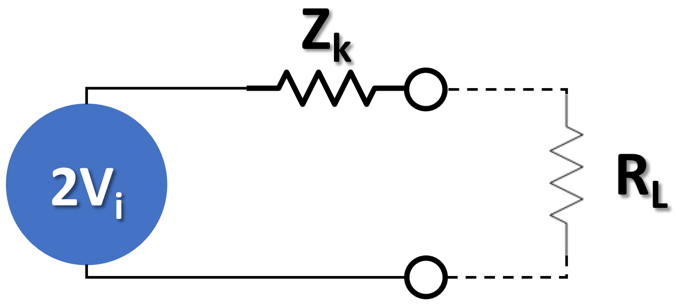
Matched Impedance

$P = P_{max}$ if $R_L = Z_k$

(i.e. no reflection!)

Thevenin's Theorem - Experiment



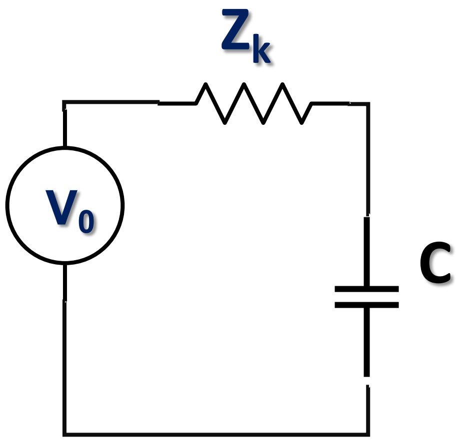


This experiment works best when performed with RG-8U cable due to its lower attenuation

When $R_L=\infty$ (open line) the pulse amplitude at line's end is expected to be $2V_i$, where V_i is the amplitude of the incident pulse

Reflection from a Capacitive Load

$$I = \frac{2\,V_i}{Z_L + Z_k}$$

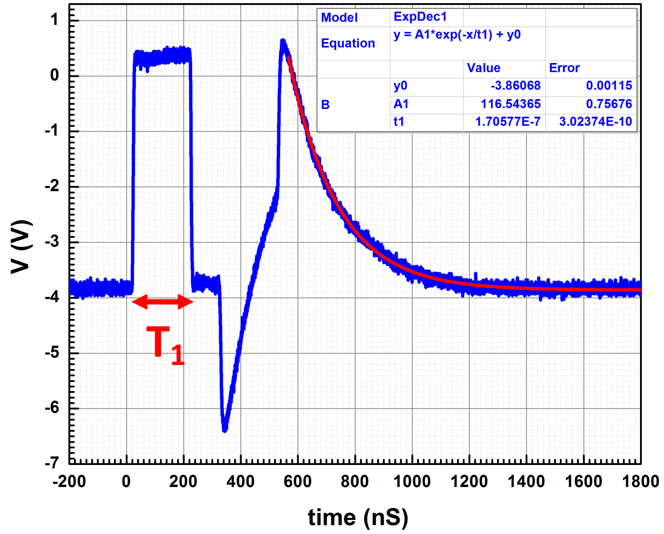


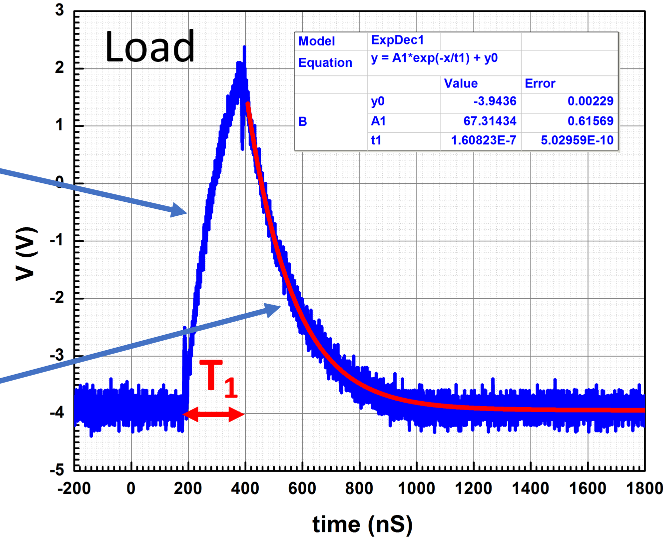
$$\tau = Z_k C$$

$$C = \frac{\tau}{Z_k} \approx 3.2\,nF$$

$$V_L = \left[1 - e^{-t/\tau}\right]$$

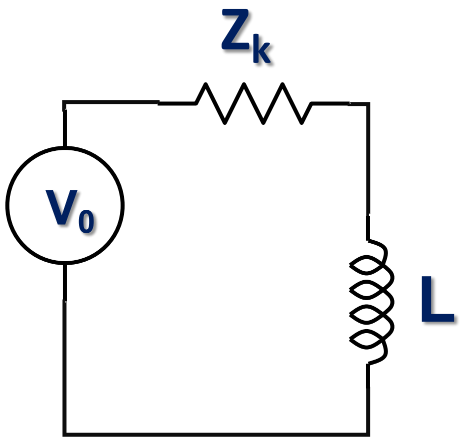
$$V_L = 2\,V_i\left[1 - e^{-T_1/\tau}\right]\left[1 - e^{-(t-T_1)/\tau}\right]$$





Reflection from an Inductive Load

$$I = \frac{2\,V_i}{Z_L + Z_k}$$

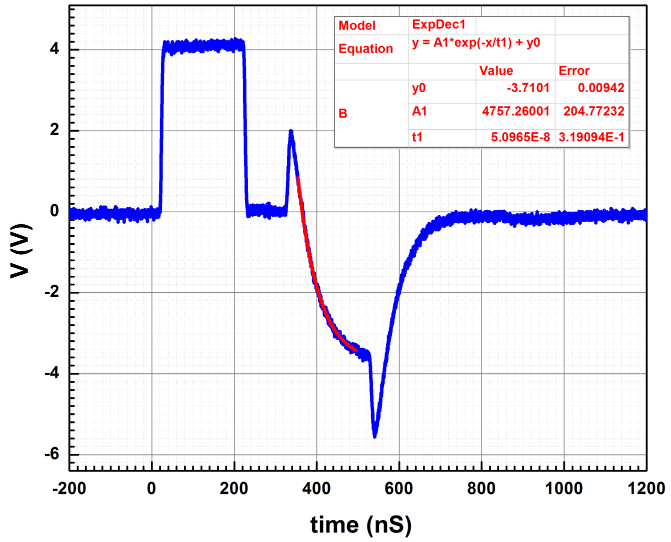


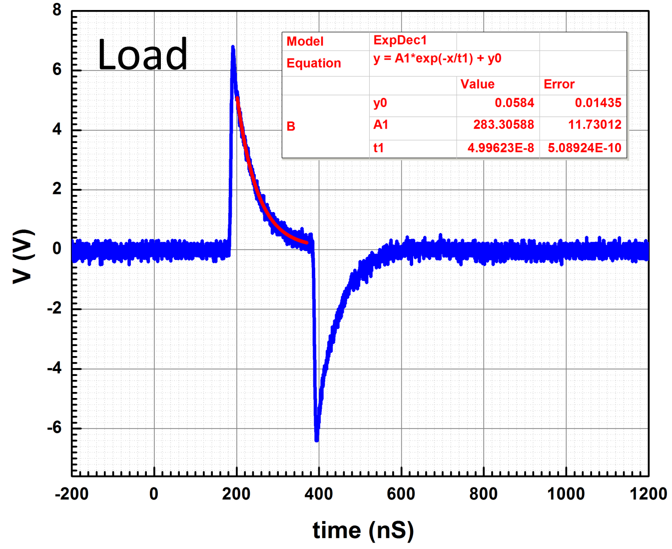
$$2V_i = I\,Z_k - L\frac{dI}{dt}$$

$$I = I_0\left[1 - e^{-t/\tau}\right]$$

$$\tau = \frac{L}{Z_k} \approx 50\,ns$$

$$L = \tau Z_k \approx 2.5\,\mu H$$



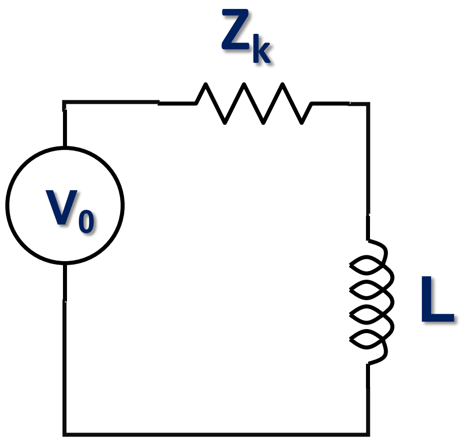


Reflection from an Inductive Load

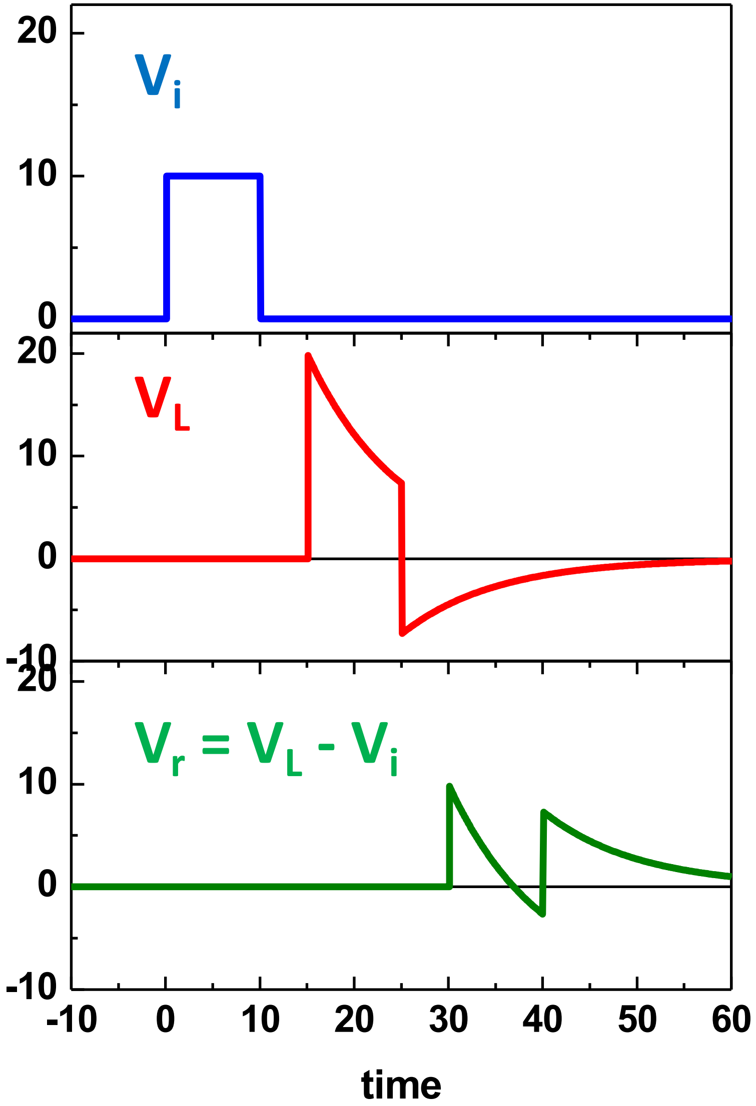
$$I = \frac{2\,V_i}{Z_L + Z_k}$$

$$2V_i = I\,Z_k - L\frac{dI}{dt}$$

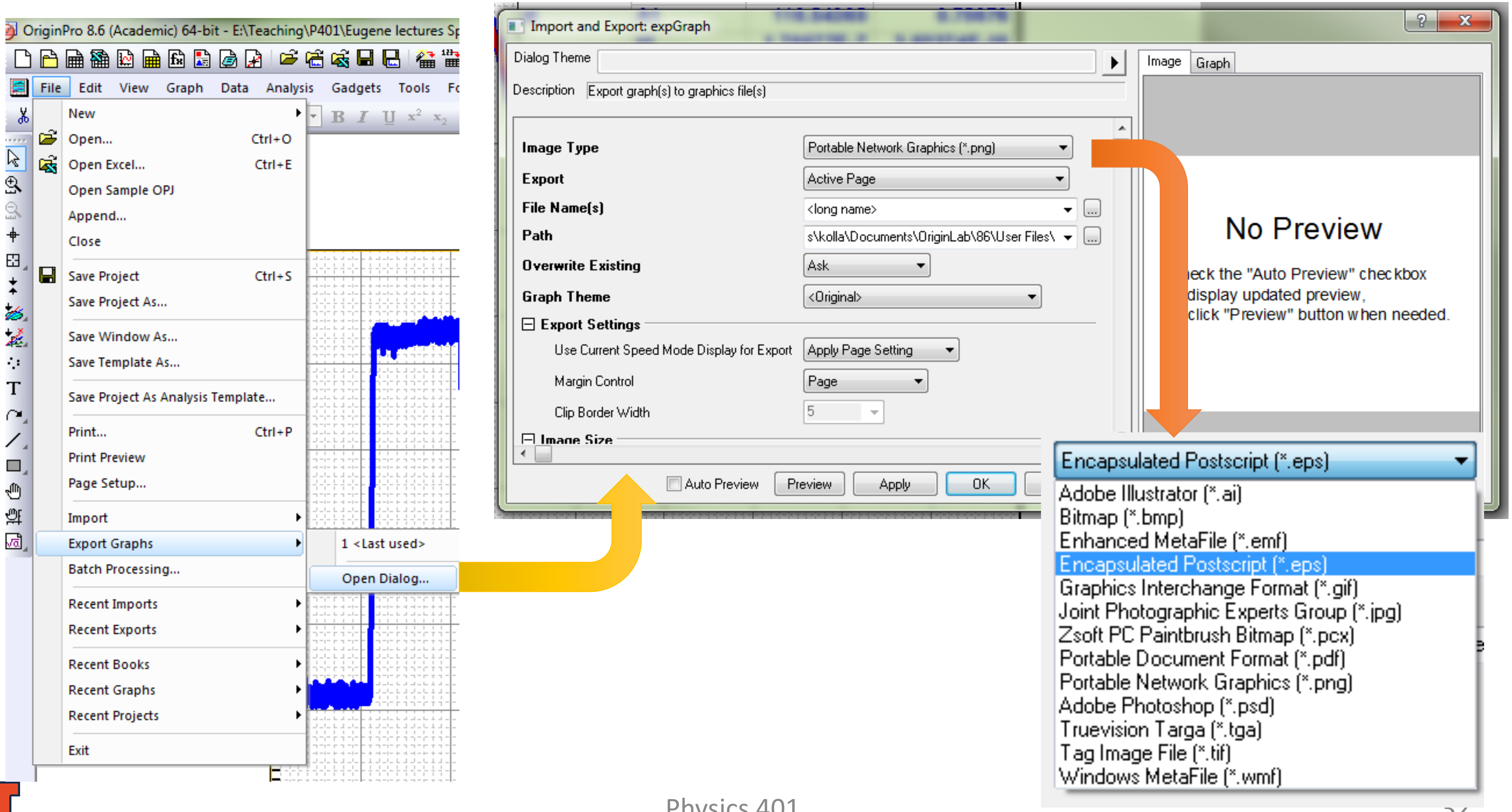
$$I = I_0\left[1 - e^{-t/\tau}\right]$$



$$\tau = \frac{L}{Z_k}$$



Appendix #1: Exporting graphs from Origin



Appendix #2: Some Reminders

1. Reports should be uploaded *only* to the **proper folder** for your activity and section
   - For example, folder Frequency domain analysis_L1 should only be used by students from section L1
   - Submit only one copy (no need to submit e.g. both Word and PDF)
   - I recommend the following file name style:

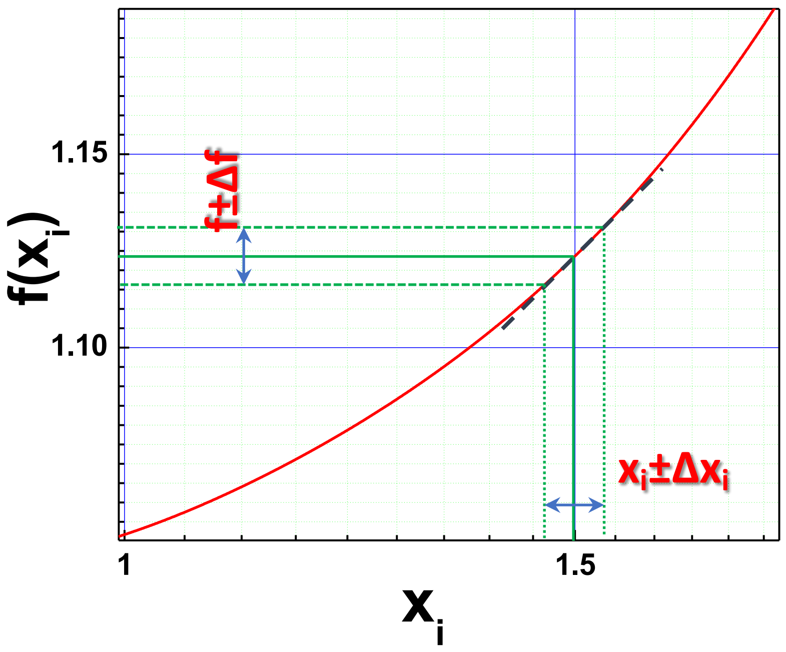
L1_lab3_LastName

2. Origin template for this week's lab:

\\engr-file-03\phyinst\APL Courses\PHYCS401\Common\Origin templates\Transmission line\Time trace.otp

Appendix #3: Error Propagation

Suppose that I'm interested in a **derived quantity** $y = f(x_1, x_2, \dots, x_n)$
I've made lab **measurements** with errors: $x_j \pm \delta x_j$
What is the error on y?



$$(\Delta y)^2 = \left(\Delta f(x_i, \Delta x_i)\right)^2 = \sum_{i=1}^{n} \left[\frac{\partial f}{\partial x_i}(x_i)\right]^2 \cdot \Delta x_i^2$$

Intuition: If *y* depends strongly (weakly) on x_i, then an error in x_i will have a large (small) effect on our estimate of *y*

Technical aside: *This assumes no correlations among errors in the various x's. If this isn't true, we must complicate this formula with a covariance matrix.*

Appendix #3: Error Propagation - Example

Derive the resonance frequency *f* from measured inductance and capacitance

$$f(L,C) = \frac{1}{2\pi\sqrt{LC}}$$

$$L = 10 \pm 1\, mH; \quad C = 10 \pm 2\, \mu F$$

$$(\Delta f)^2 = \left(\Delta f(L, C, \Delta L, \Delta C)\right)^2 = \left[\frac{\partial f}{\partial L}\right]^2 \cdot \Delta L^2 + \left[\frac{\partial f}{\partial C}\right]^2 \cdot \Delta C^2$$

$$\frac{\partial f}{\partial L} = \frac{-1}{4\pi} C^{-\frac{1}{2}} L^{-\frac{3}{2}}$$

$$\frac{\partial f}{\partial C} = \frac{-1}{4\pi} L^{-\frac{1}{2}} C^{-\frac{3}{2}}$$

Results:

f(L,C) = 503.29212104487... Hz

Δf = 56.26977... Hz

f(L,C) = 503 ± 56 Hz

Appendix #3: Error Propagation - Practicalities

$L = 10 \pm 1\, mH; \;\; C = 10 \pm 2\, \mu F$ Where are these numbers coming from?

1. Commercial resistors, capacitors, inductors, … have quoted **tolerances** (use if you haven't measured!)

C=500pF ±5%

F=35mH ±10%

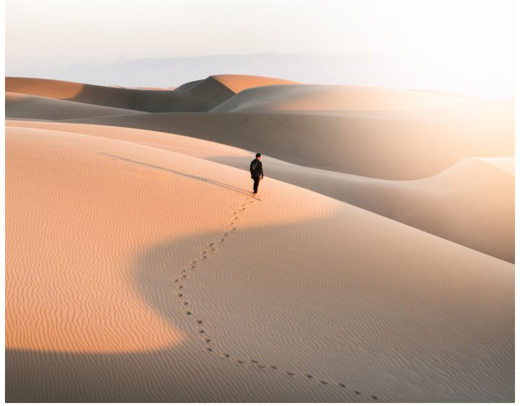
2. Measure components with standard equipment, use equipment accuracy

SENCORE "Z" meter model LC53
Capacitance measurement accuracy ±5%
Inductance measurement accuracy ±2%

Agilent E4980A Precision LCR Meter
Basic accuracy ±0.05%

Appendix #4: Nonlinear Fitting

Fitting is a **minimization** problem: what choice of *parameter values* minimizes some *cost function* that expresses how far the fit function is from the data?



- **Data**: ordered pairs (x_i, y_i), often in the form of an N×2 matrix
  - Independent variable x_i, e.g. frequency, time, etc.
  - Dependent variable y_i, e.g. signal magnitude
- **Parameterized function**: $y = f(x;\ \beta)$, which takes some set of parameters β

Our usual cost function is the sum of squared deviations:

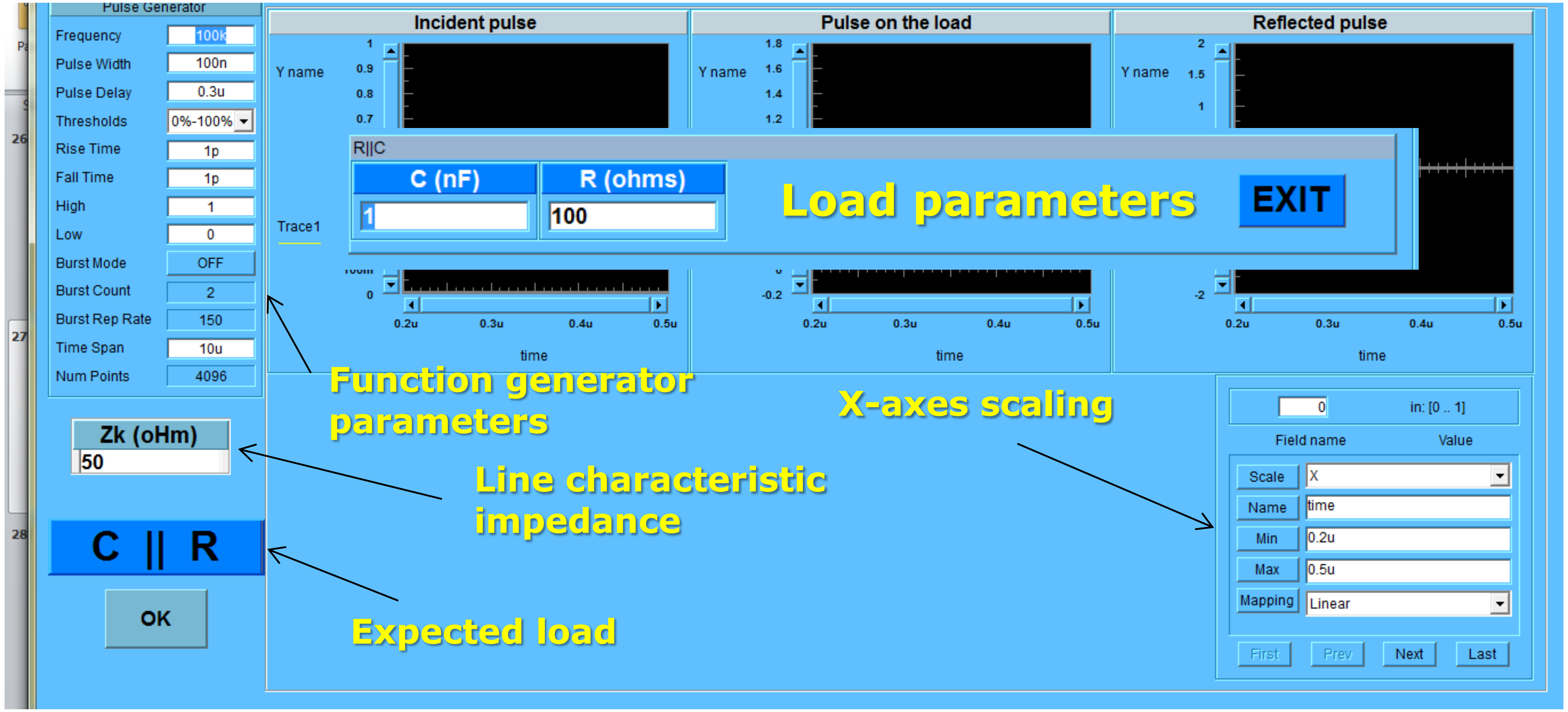
$$S(\beta) = \sum_{i=1}^{N} [f(x_i;\beta) - y_i]^2$$

$\chi^2 = \sum_{i=1}^{N} \frac{[f(x_i;\beta)-y_i]^2}{\sigma_i^2}$ is further normalized by the r.m.s. error, which doesn't matter for fitting

Origins uses the Levenberg-Marquardt algorithm for nonlinear fitting, which is optimized for quadratic cost functions like this one and requires a **starting guess**. In some cases $\beta = (1,1,\dots 1)$ works, but a more reasonable guess is often required

Appendix #5: Unknown Load Fitting

- Transmission line: unknown load simulation



Location:

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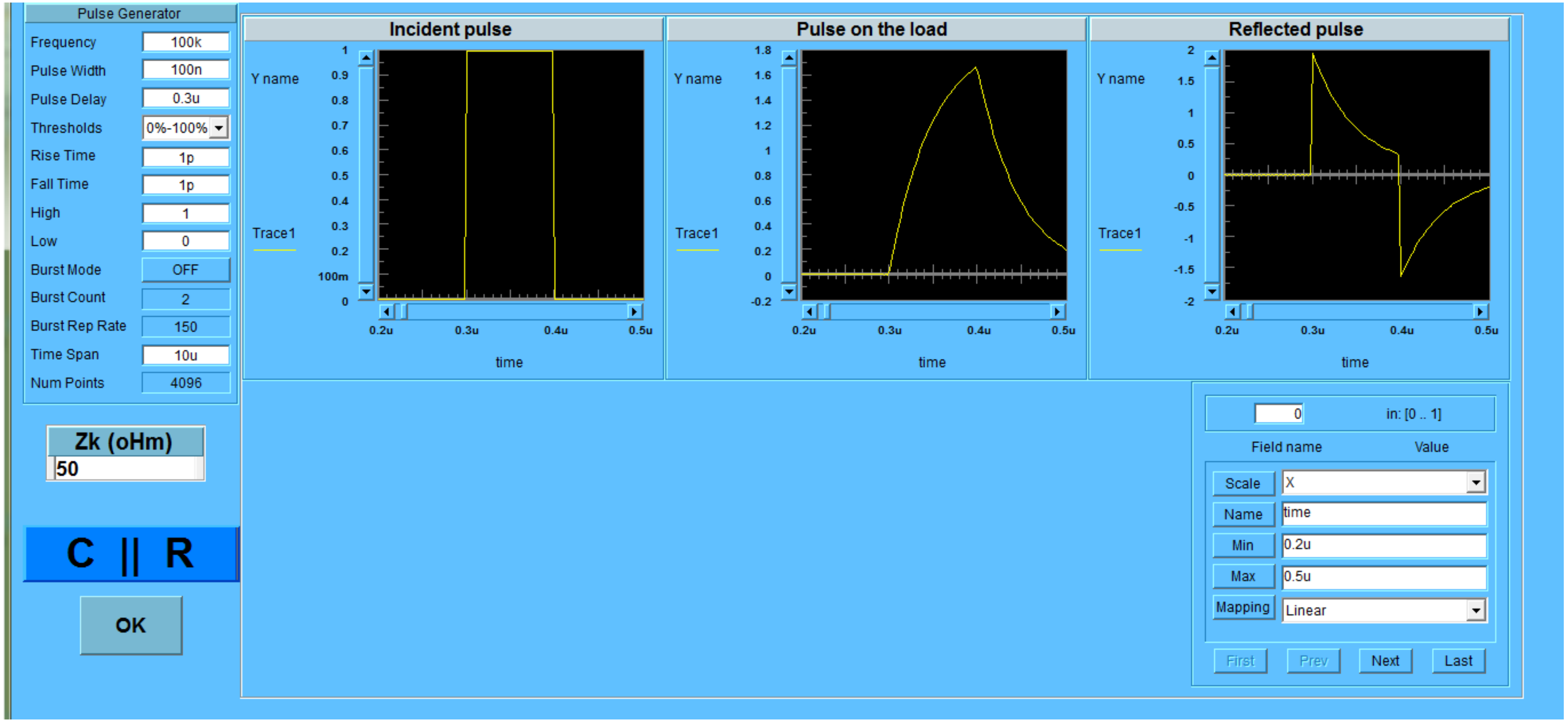
Appendix #5: Unknown Load Fitting

- Transmission line: unknown load simulation



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