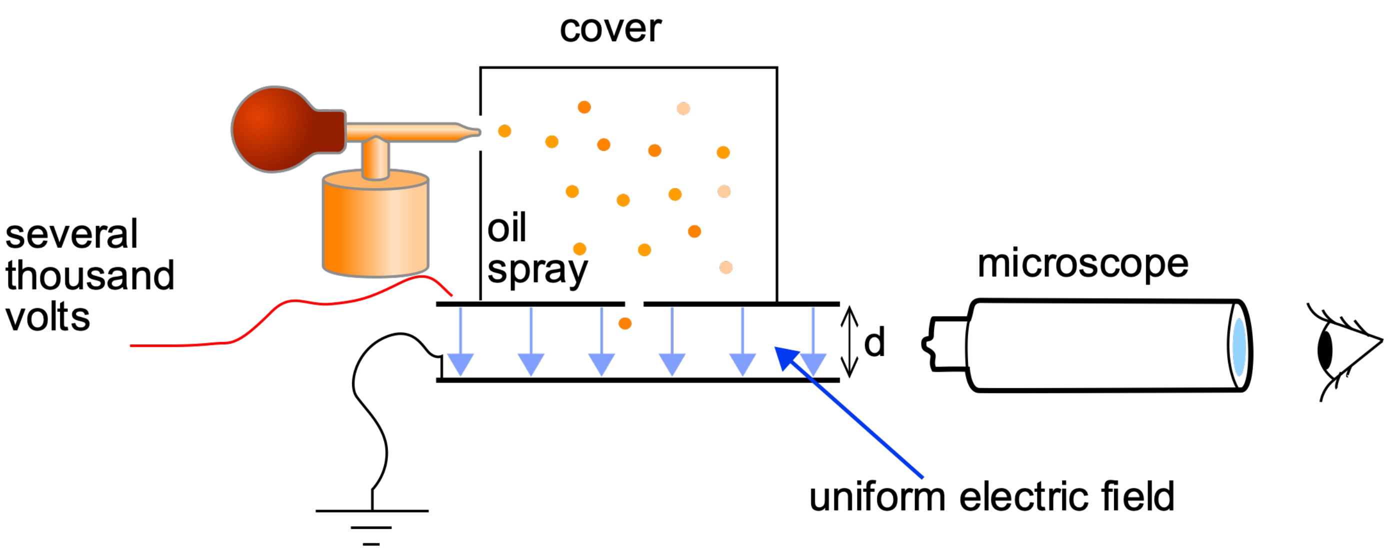


Millikan Oil Drop Experiment

Professor Jeff Filippini

Physics 401

Spring 2020

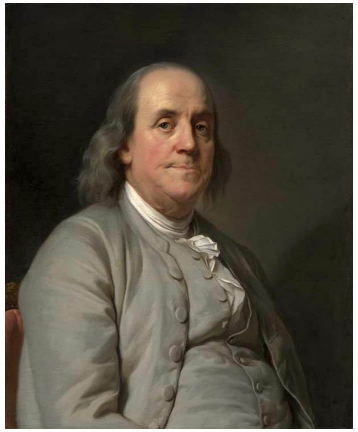
ILLINOIS

Today's Topics

1. History: Measuring the charge of the electron
2. Theory of the Experiment
3. Laboratory Setup
4. Data Analysis

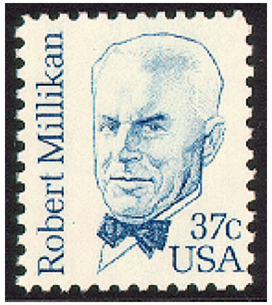
Motivation: The Dawn of Elementary Particles

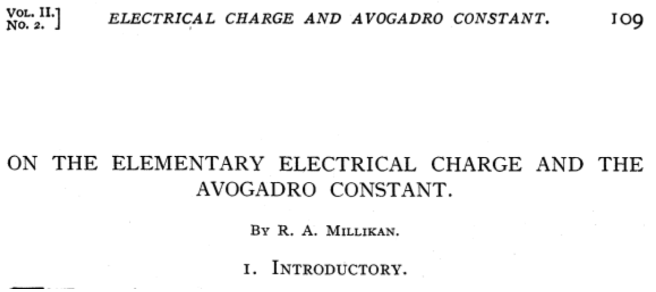
- 1750s: Benjamin Franklin proposes electricity is flow or surplus/deficit of single electrical fluid
- 1881: Hermann von Helmholtz argues for elementary positive and negative electric charges ("atoms of electricity")
- 1897: J.J. Thompson shows particle nature of "cathode rays" emitted in vacuum tubes
  - Universal negatively-charged particles from vacuum tubes, radioactivity, …
  - Measured charge-to-mass ratio (e/m) via magnetic deflection
  - Mass <1/1000 of hydrogen ion (*actually ~1/1836*)



Robert Millikan and the Oil Drop Experiment

ROBERT ANDREWS MILLIKAN
1868-1953



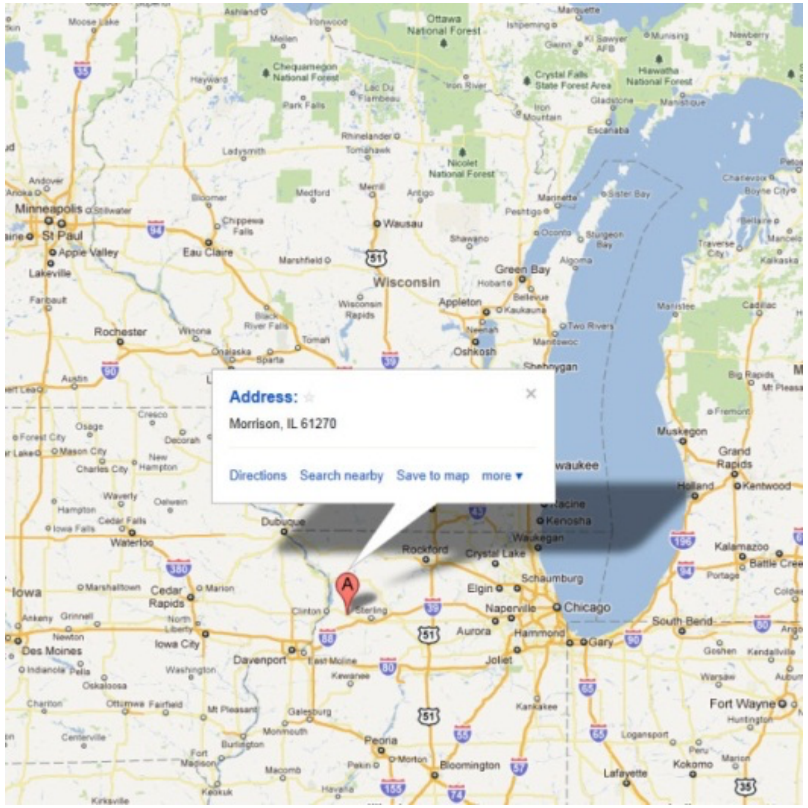


Vol. II.] No. 2. ELECTRICAL CHARGE AND AVOGADRO CONSTANT. 109

ON THE ELEMENTARY ELECTRICAL CHARGE AND THE AVOGADRO CONSTANT.

By R. A. Millikan.

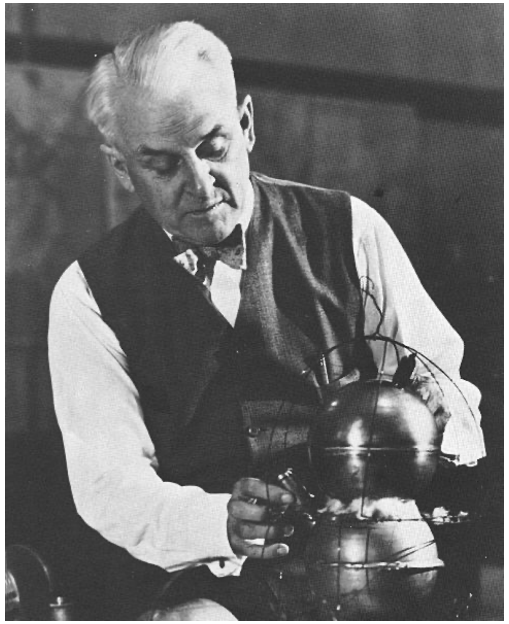
I. Introductory.



The Nobel Prize in Physics 1923.
Robert A. Millikan
"for his work on the elementary charge of electricity and on the photoelectric effect".

University of Chicago
Moved to Caltech in 1921

Robert Millikan and the Oil Drop Experiment



ROBERT ANDREWS MILLIKAN
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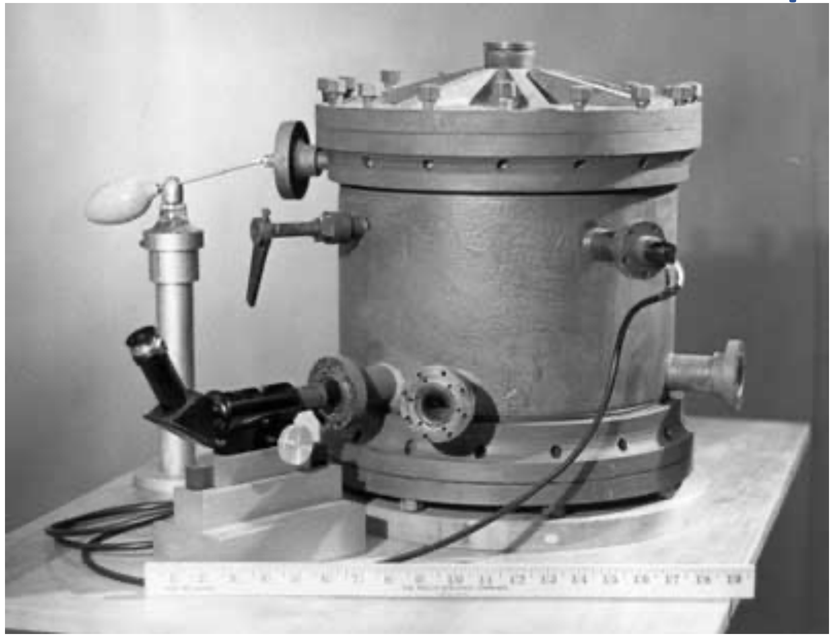
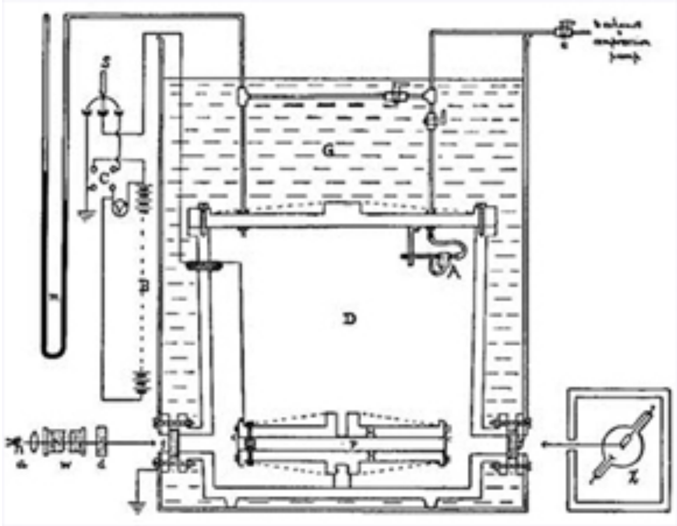
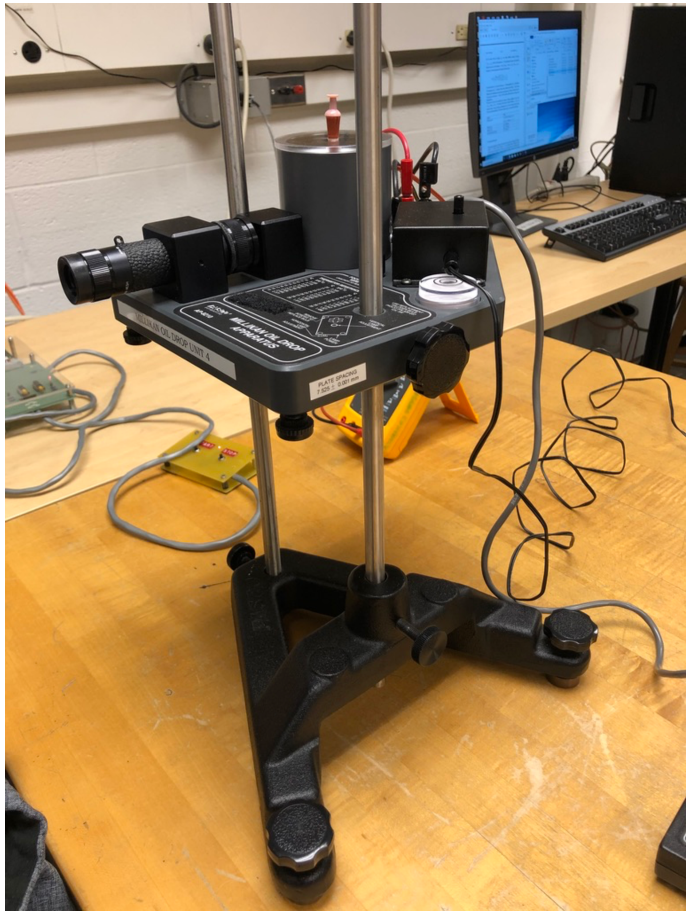




Work with Ph.D. student Harvey Fletcher – not a coauthor on famous paper.
“The father of stereophonic sound”
First functional electronic hearing aid

Oil Drop Apparatus



Motivation:

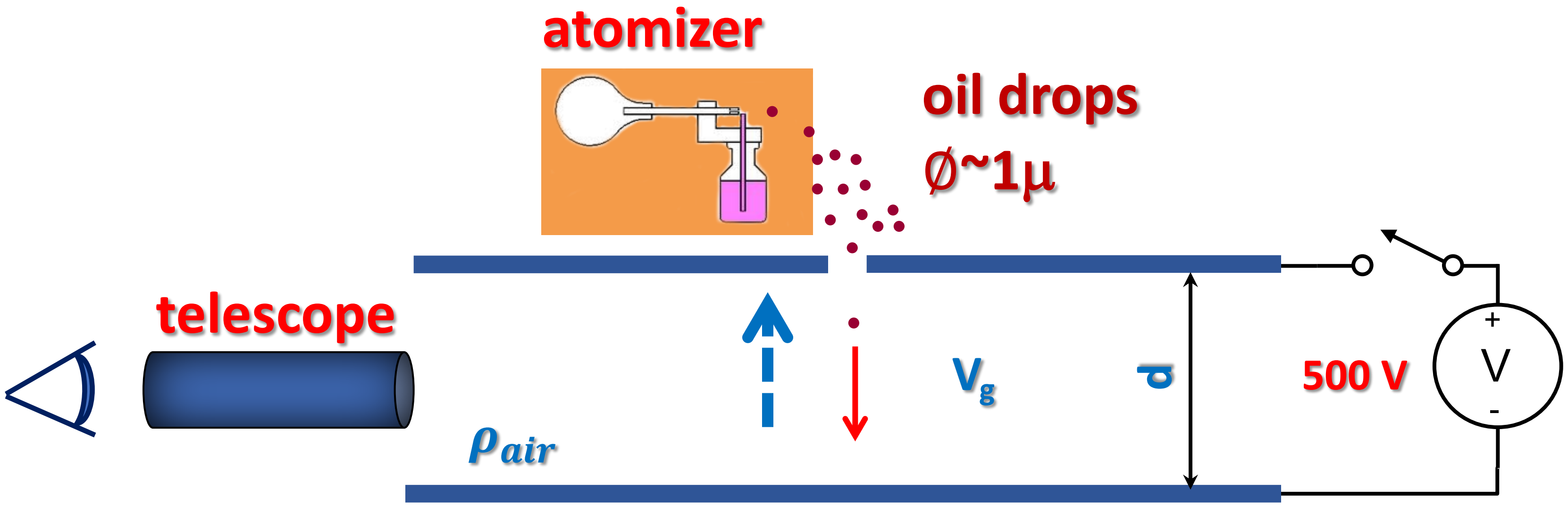
1. Measure the magnitude of the **electron charge**

    Apparatus precision ± 3%

    Millikan original ± 0.1%

2. Demonstrate that electric charge is **quantized**!

Oil Drop Experiment

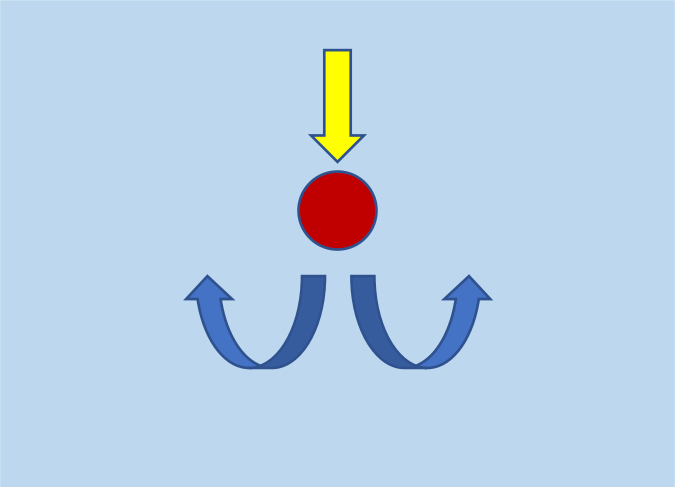


Forces acting on the oil drop:

1. Gravity
2. Electric force on a *charged* oil drop

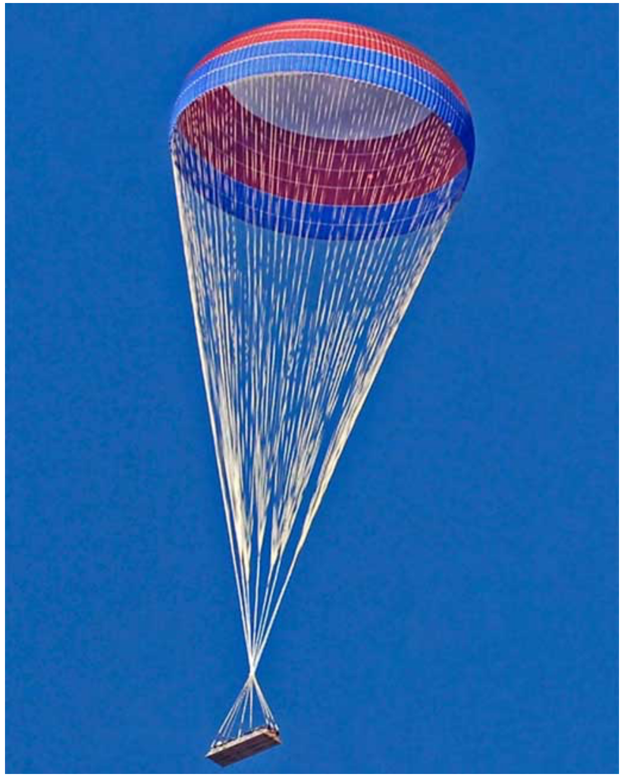
The Role of Air: Buoyancy and Drag

Lowering an object through a fluid means lifting an equal volume of the surrounding fluid

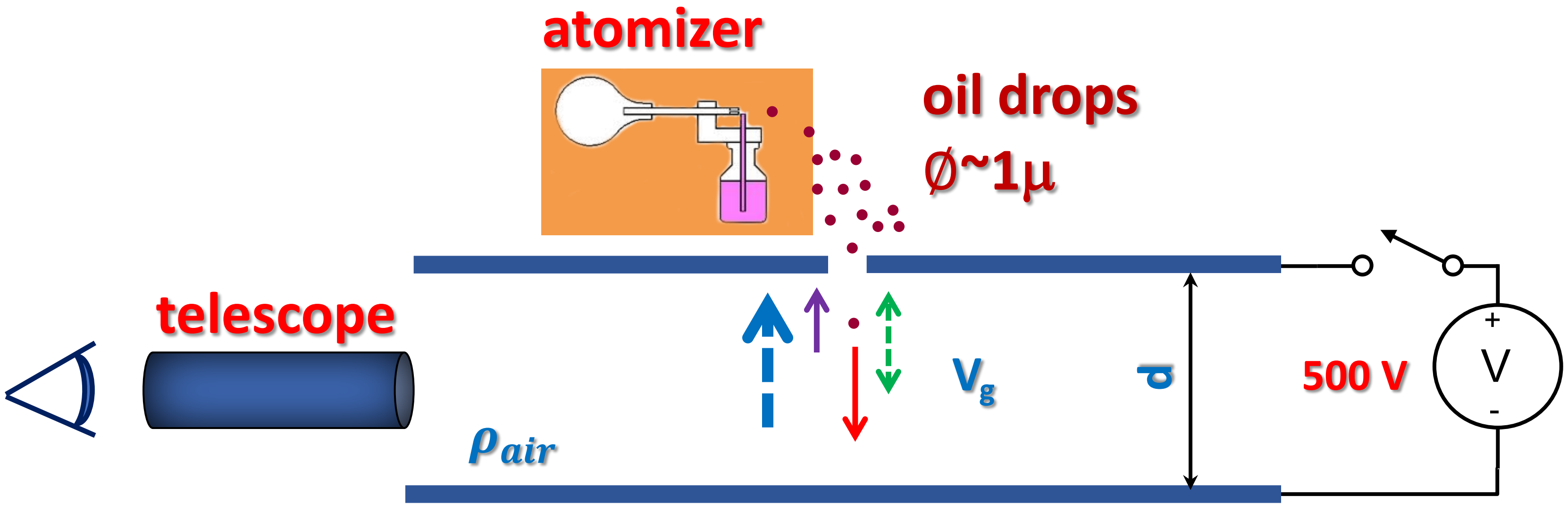


Motion through a fluid opposed by drag: friction with and within the surrounding fluid

Objects eventually reach **terminal velocity** at which drag balances accelerating forces



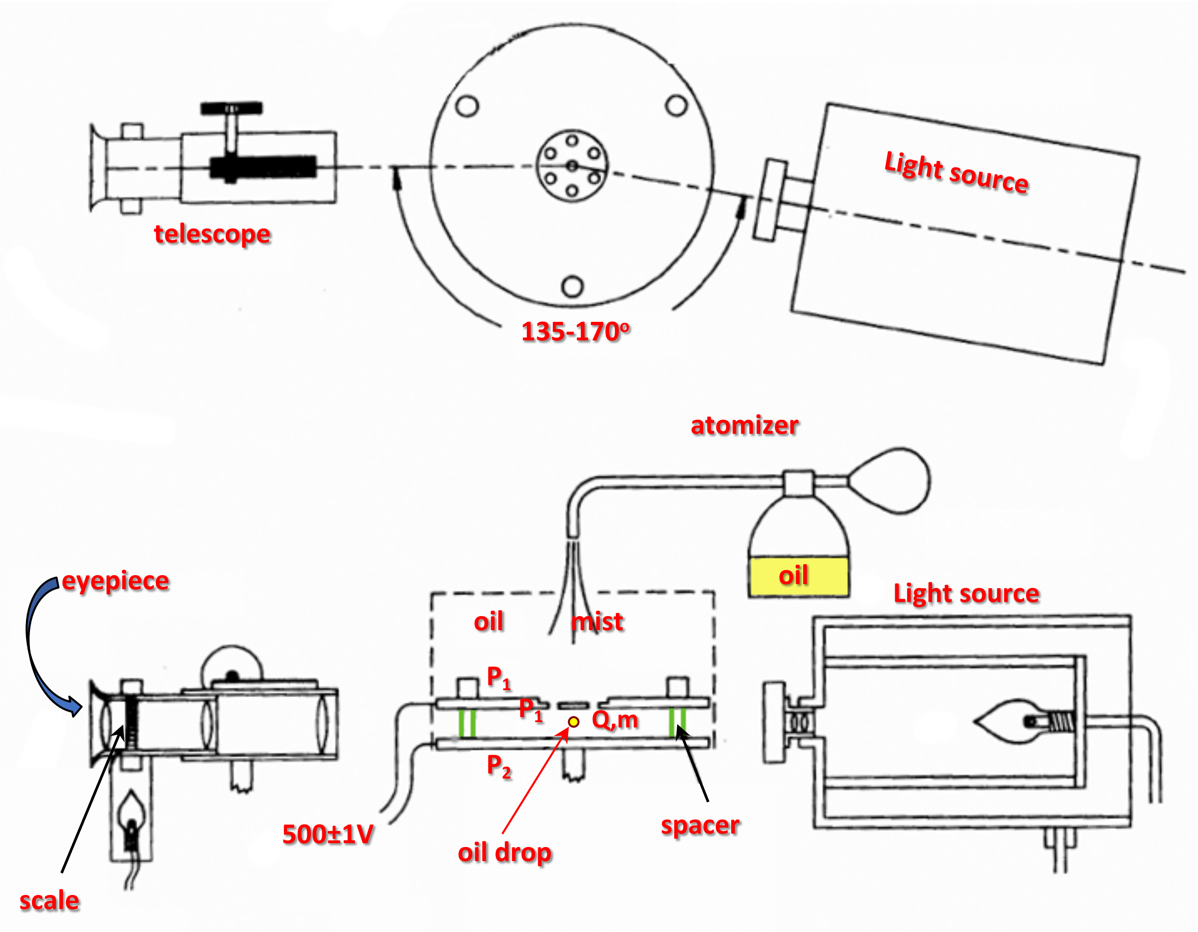
Oil Drop Experiment



Forces acting on the oil drop:

1. Gravity
2. Electric force on a *charged* oil drop
3. Buoyant force *(weight of air displaced by oil drop)*
4. Drag force on a *moving* oil drop

Oil Drop Apparatus: Schematic Layout



Oil Drop Apparatus: Schematic Layout

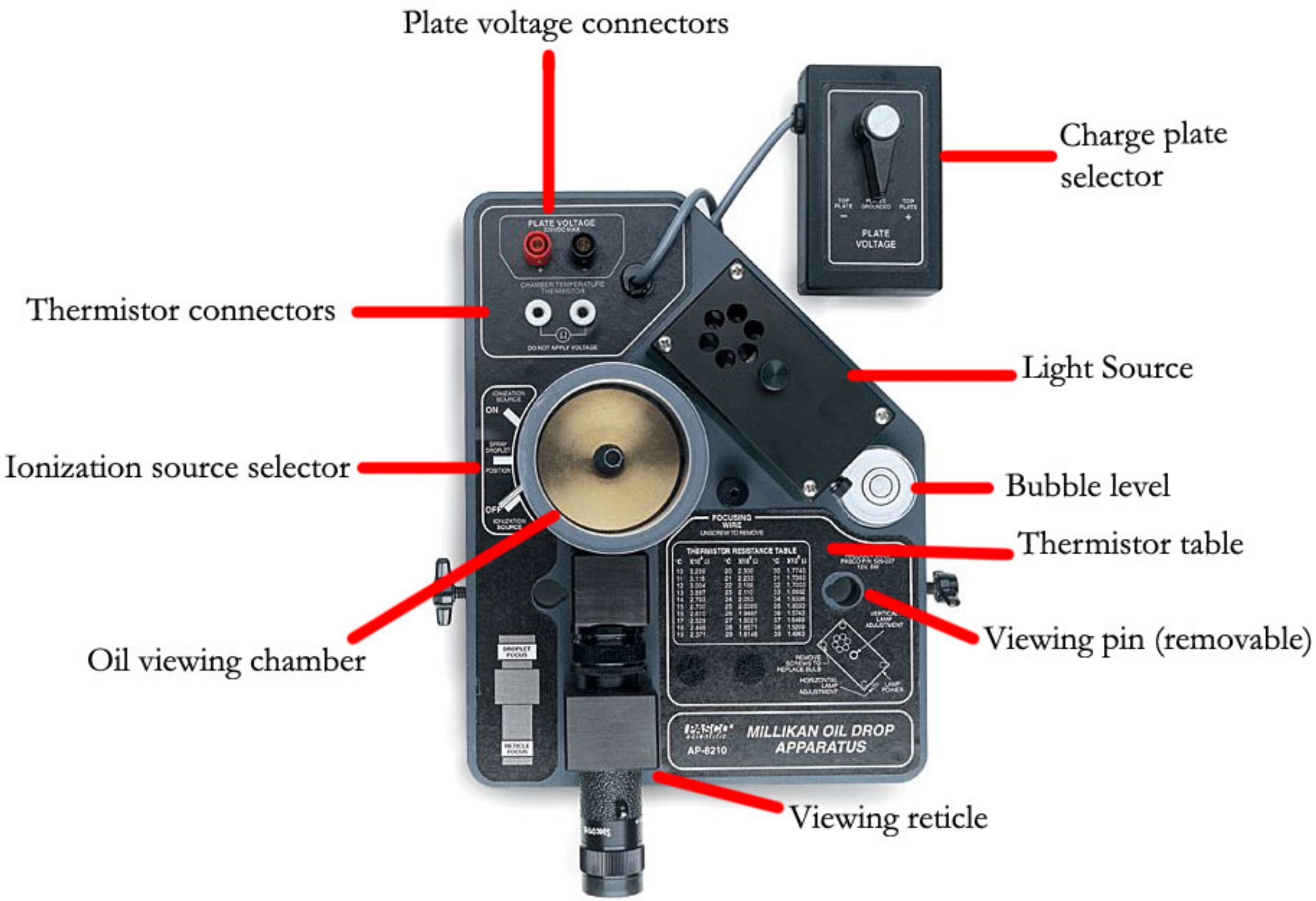


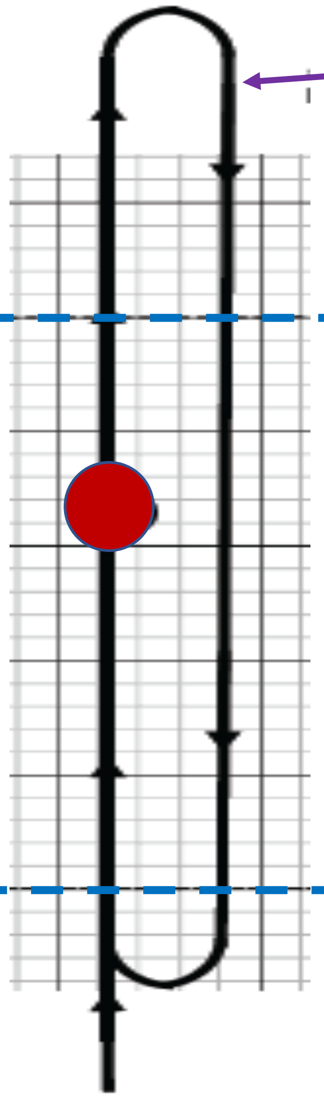
Fig 1: Oil Drop Apparatus

How the Measurement Looks

Allow drop to “undershoot” here before starting next rise time experiment

… and stops here

Fall time measurement starts here…



Fall time t_g

Rise time t_{rise}

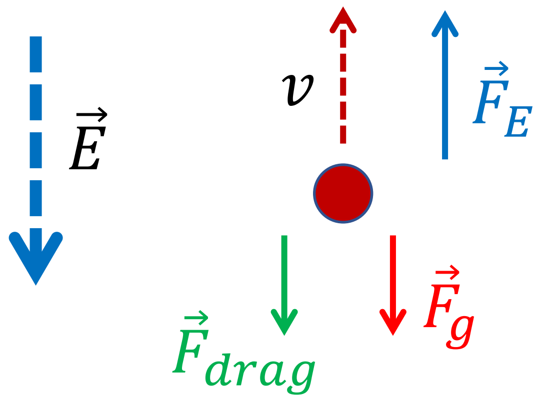
x

x = fall distance = rise distance
Must be the same for all drops!

… and stops here

Rise time measurement starts here…

Balance of Forces: Newton's Second Law



Forces on the oil drop:

1. Gravity + buoyancy $\quad \vec{F}_g = -mg\,\hat{z}$
2. Drag force $\quad \vec{F}_{drag} = -6\pi\eta a\vec{v}$
3. Electric force $\quad \vec{F}_E = Q\,\vec{E}$

Drop

- a: radius of drop
- ρ: net density $\rho_{oil} - \rho_{air}$
- v: velocity of oil drop
- Q: charge of oil drop

Apparatus

- E: electric field E=V/d
- V: voltage across plates
- η: viscosity of air
- g: gravitational constant

$$\vec{F} = m\frac{d\vec{v}}{dt} = \vec{F}_g + \vec{F}_{drag} + \vec{F}_E$$

Terminal velocity: speed at which $\frac{d\vec{v}}{dt} = 0$

$$\vec{F}_g + \vec{F}_{drag} + \vec{F}_E = 0$$

Achieved within ~10^{-5} s by a 1 µm particle

Modifications to Stokes' Law

George Gabriel Stokes
(1819-1903)

$$\vec{F}_{drag} = -6\pi\eta a\vec{v}$$

For particles of small radii ($a \lesssim 15\ \mu m$), Stokes' Law must be corrected for the particle nature of air. This correction was derived by E. Cunningham

$$\vec{F}_{drag} = -6\pi\eta \frac{a}{f_c}\vec{v}$$

Ebenezer Cunningham
(1881-1977)

$$f_c = 1 + A\frac{\lambda}{a} + B\frac{\lambda}{a}e^{-c\frac{a}{\lambda}}, \qquad A = 1.246;\ \ B = 0.42;\ \ C = 0.78$$

$$f_c \approx 1 + A\frac{\lambda}{a} = 1 + \frac{r_c}{a} \approx 1.1 \qquad \text{for } a = 10^{-6}\ m,\ r_c = \frac{6.18\times10^{-5}\ m}{p\ [mmHg]}$$

Here:

- a = particle radius
- λ = mean free path of gas molecules

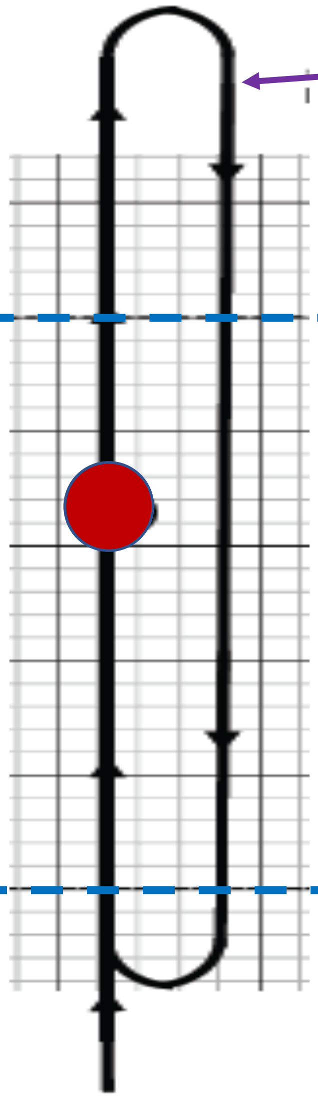
$$\lambda = 65.3\ nm \left(\frac{760\ mmHg}{p}\right)$$

What We Measure: Rise and Fall Times

Allow drop to “undershoot” here before starting next rise time experiment

… and stops here

Fall time measurement starts here…



Fall time t_g

Rise time t_{rise}

x

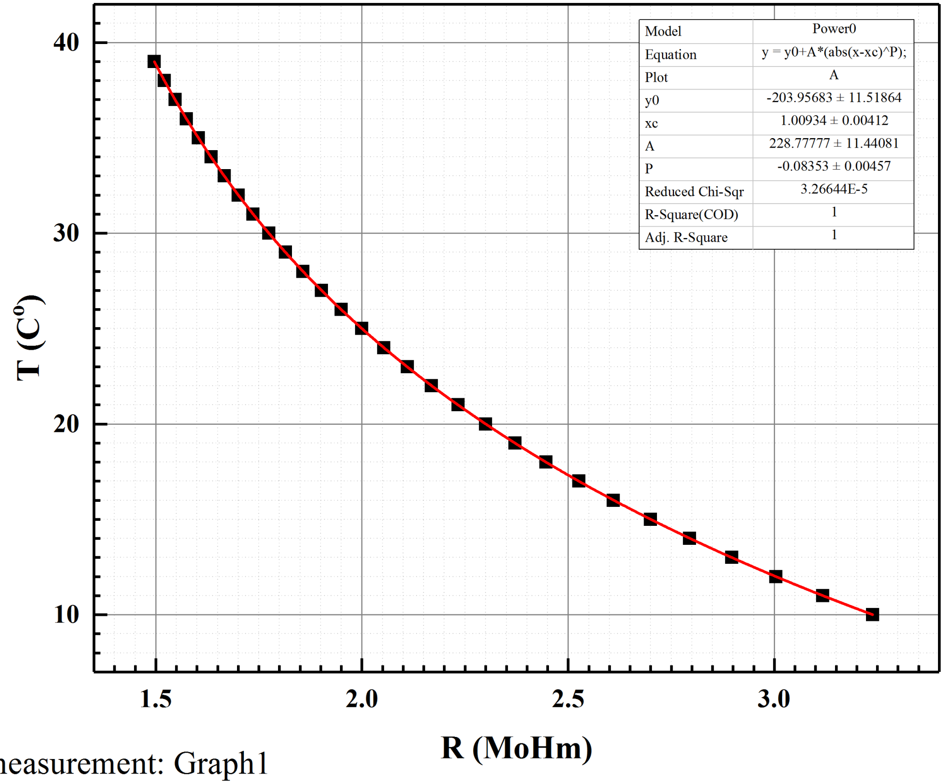
x = fall distance = rise distance
Must be the same for all drops!

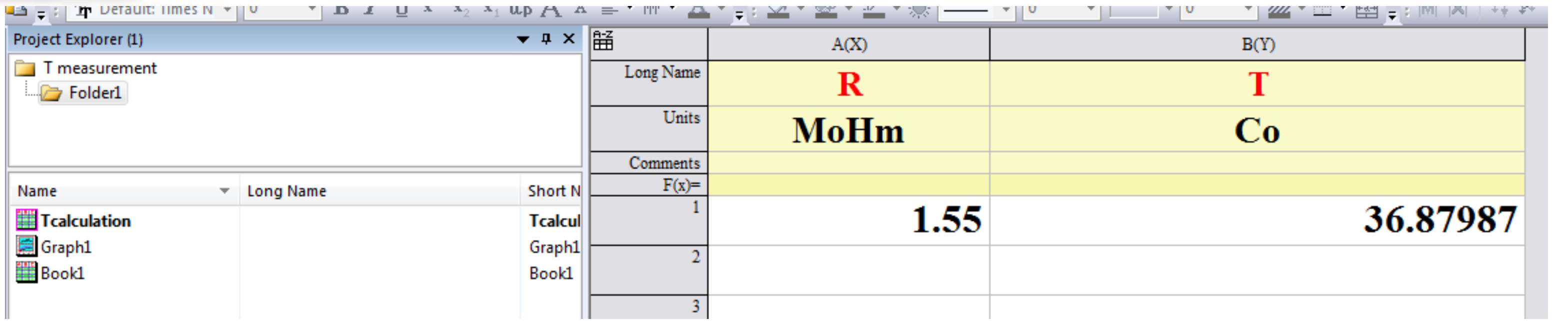
… and stops here

Rise time measurement starts here…

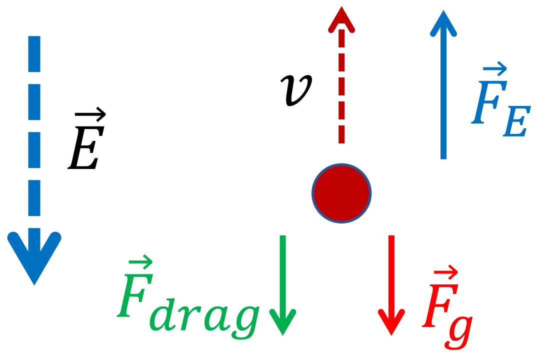
Measuring the Temperature

***Project**: T measurement.opj*





Solving Newton's Law: $Q(t_g, t_{rise})$



Drop

- a: radius of drop
- ρ: net density $\rho_{oil} - \rho_{air}$
- v: velocity of oil drop
- Q: charge of oil drop

Apparatus

- E: electric field E=V/d
- V: voltage across plates
- η: viscosity of air
- g: gravitational constant
- x: drift distance

Measured

- t_g, t_{rise}: fall, rise times

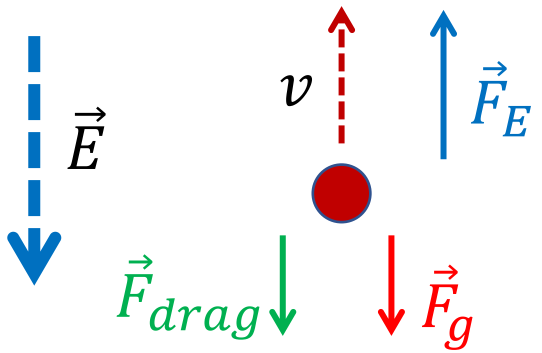
The correction factor f_c can be found from Newton's Law in the case of $v = 0$ (falling drop)

$$\vec{F}_g + \vec{F}_{drag} = \frac{4}{3}a^3\rho g - 6\pi\eta\frac{a}{f_c}\vec{v} = 0$$

See write-up for derivation

$$\frac{1}{f_c^{2/3}} \approx 1 - \left(\frac{t_g}{\tau_g}\right)^{\frac{1}{2}}; \quad \tau_g = \frac{2\eta x}{\rho g r_c^2}; \quad r_c = \frac{6.18\times10^{-5}\, m}{p\, [mmHg]}$$

Solving Newton's Law: $Q(t_g, t_{rise})$



Drop

a: radius of drop
ρ: net density $\rho_{oil} - \rho_{air}$
v: velocity of oil drop
Q: charge of oil drop

Apparatus

E: electric field E=V/d
V: voltage across plates
η: viscosity of air
g: gravitational constant
x: drift distance

Measured

t_g, t_{rise}: fall, rise times

$$\frac{1}{f_c^{2/3}} \approx 1 - \left(\frac{t_g}{\tau_g}\right)^{\frac{1}{2}}; \quad \tau_g = \frac{2\eta x}{\rho g r_c^2}; \quad r_c = \frac{6.18\times10^{-5}\, m}{p\ [mmHg]}$$

Putting it all together:

$$Q = ne = \frac{1}{f_c^{2/3}} \frac{9\pi d}{V} \sqrt{\frac{2\eta^3 x^3}{g\rho}} \sqrt{\frac{1}{t_g}} \left[\frac{1}{t_g} + \frac{1}{t_{rise}}\right]$$

F: Stokes' law correction

S: Stokes' law force balance

T: Your timing measurements

Origin Projects: Data Collection

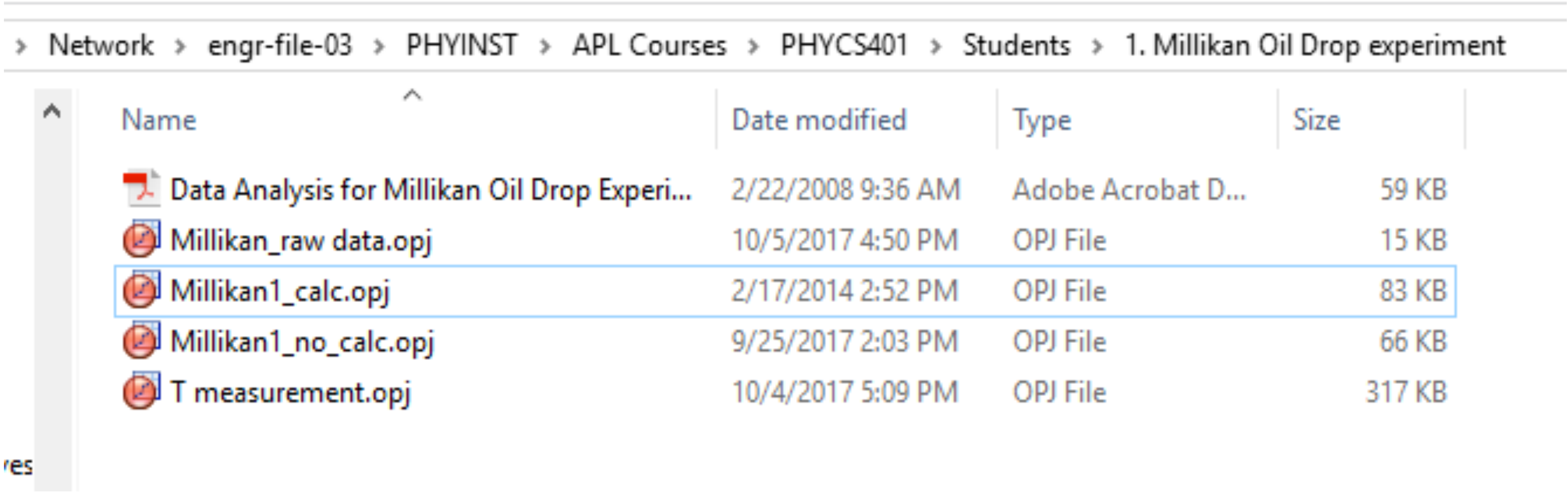
Project: ***Millikan_raw data.opj***

Location: **\\engr-file-03\PHYINST\APL Courses\PHYCS401\Students\1. Millikan Oil Drop experiment\Millikan_raw data.opj**

	A(L)	AE(Y)	B(Y)	C(Y)	D(Y)
Long Name	parameter label	Units	Par	tg	tr
Units				s	s
Comments			*Parameters, use proper units shown column "Units"*	*Raw data, faling time*	*Raw data, rising time*
1	η	kg/ms	1.8478E-5		
2	Δη/ΔT	kg/msCo	4.8E-8		
3	ρ1	kg/m^3	886		
4	ρ2	kg/m^3	1.29		
5	ρ1−ρ2	kg/m^3	884.71		
6	g	m/s^2	9.801		
7	p	mmHg			
8	x	m			
9	d	m			
10	V	V			
11	Ta	Co			
12					

Origin Projects: Data Analysis

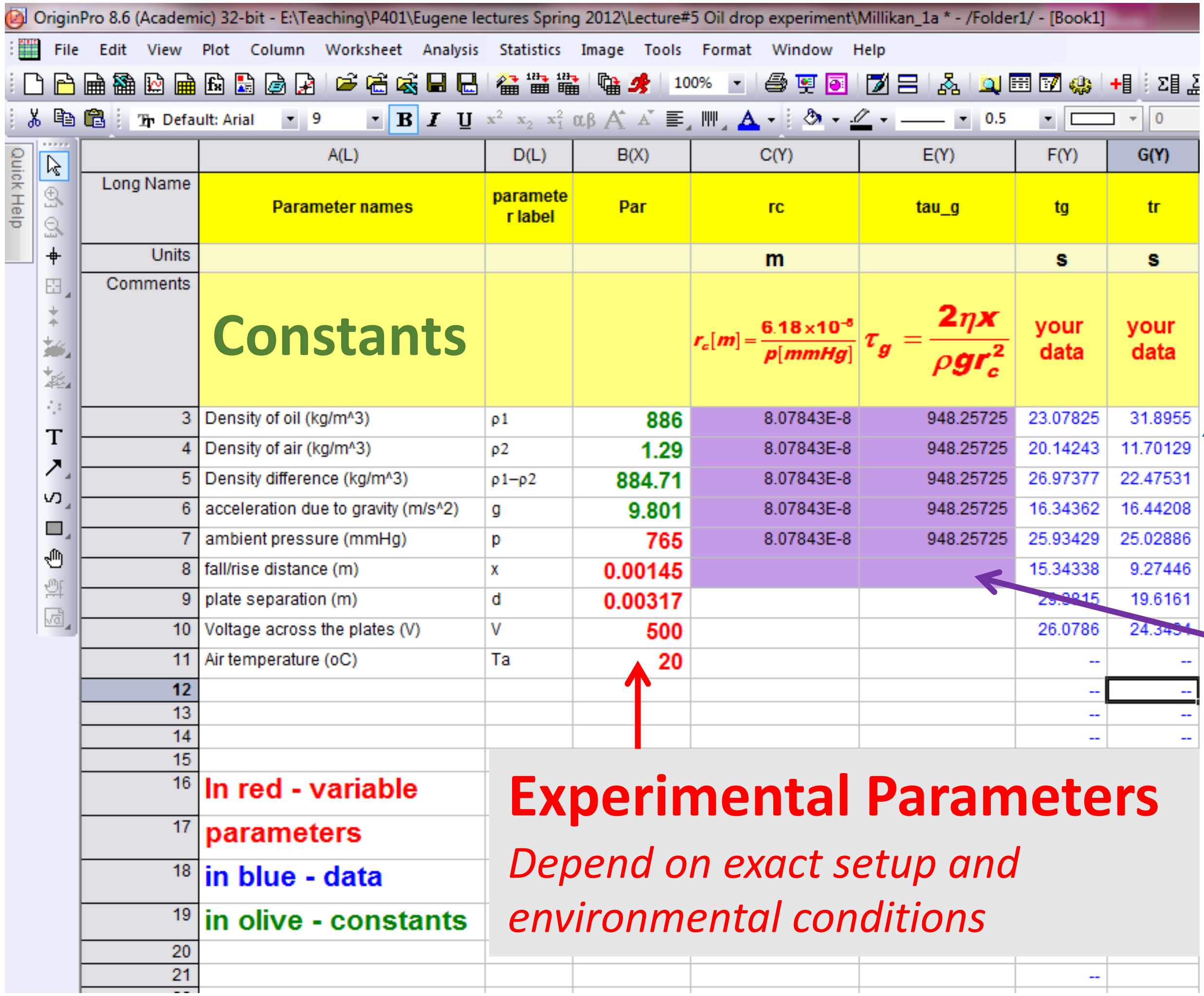
Location: **\\engr-file-03\PHYINST\APL Courses\PHYCS401\Students\1. Millikan Oil Drop experiment**



Please make a **copy** (*don't move!*) of Millikan1.opj and Millikan_raw data.opj in your personal folder and work with your personal copy of the project

Origin Data Analysis

Project: ***Millikan1.opj***



	A(L)	D(L)	B(X)	C(Y)	E(Y)	F(Y)	G(Y)
Long Name	Parameter names	parameter label	Par	rc	tau_g	tg	tr
Units				m		s	s
Comments	Constants			$r_c[m] = \frac{6.18\times10^{-5}}{p[mmHg]}$	$\tau_g = \frac{2\eta x}{\rho g r_c^2}$	your data	your data
3	Density of oil (kg/m^3)	ρ1	886	8.07843E-8	948.25725	23.07825	31.8955
4	Density of air (kg/m^3)	ρ2	1.29	8.07843E-8	948.25725	20.14243	11.70129
5	Density difference (kg/m^3)	ρ1−ρ2	884.71	8.07843E-8	948.25725	26.97377	22.47531
6	acceleration due to gravity (m/s^2)	g	9.801	8.07843E-8	948.25725	16.34362	16.44208
7	ambient pressure (mmHg)	p	765	8.07843E-8	948.25725	25.93429	25.02886
8	fall/rise distance (m)	x	0.00145			15.34338	9.27446
9	plate separation (m)	d	0.00317			[illegible]	19.6161
10	Voltage across the plates (V)	V	500			26.0786	[illegible]
11	Air temperature (oC)	Ta	20			--	--
12						--	--
13						--	--
14						--	--
15							
16	In red - variable						
17	parameters						
18	in blue - data						
19	in olive - constants						
20							
21						--	

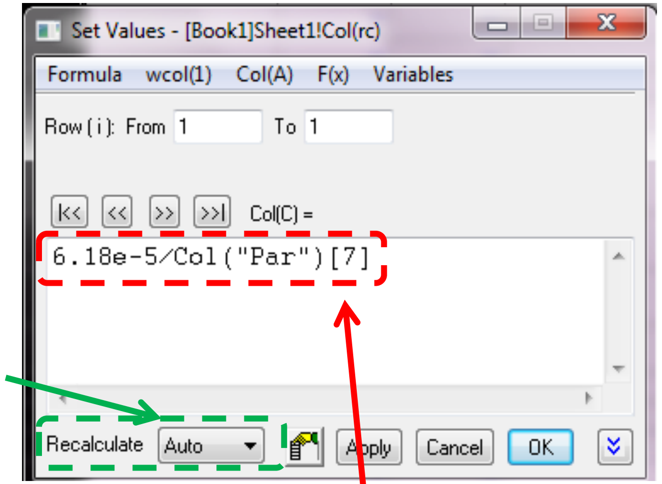
Experimental Parameters

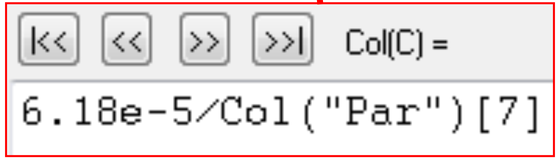
Depend on exact setup and environmental conditions

Your data goes here

Recalculate Auto

Calculations



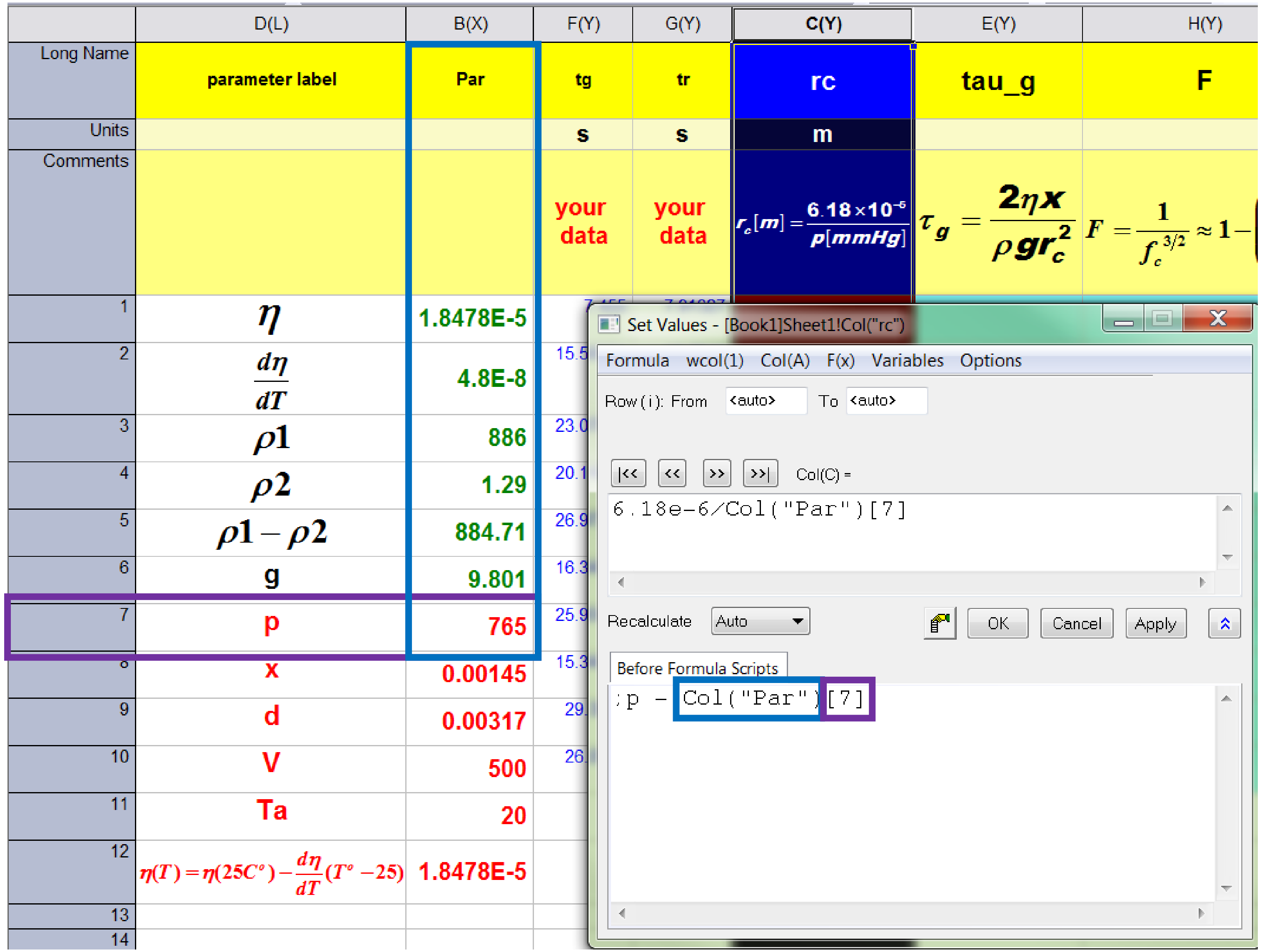


$$r_c = \frac{6.18\times10^{-5}\ m}{p\ [mmHg]}$$

P[mmHg] ⟹ Col(“Par”)[7]

Origin Data Analysis

Project: ***Millikan1.opj***



Origin Data Analysis

Project: ***Millikan1.opj***

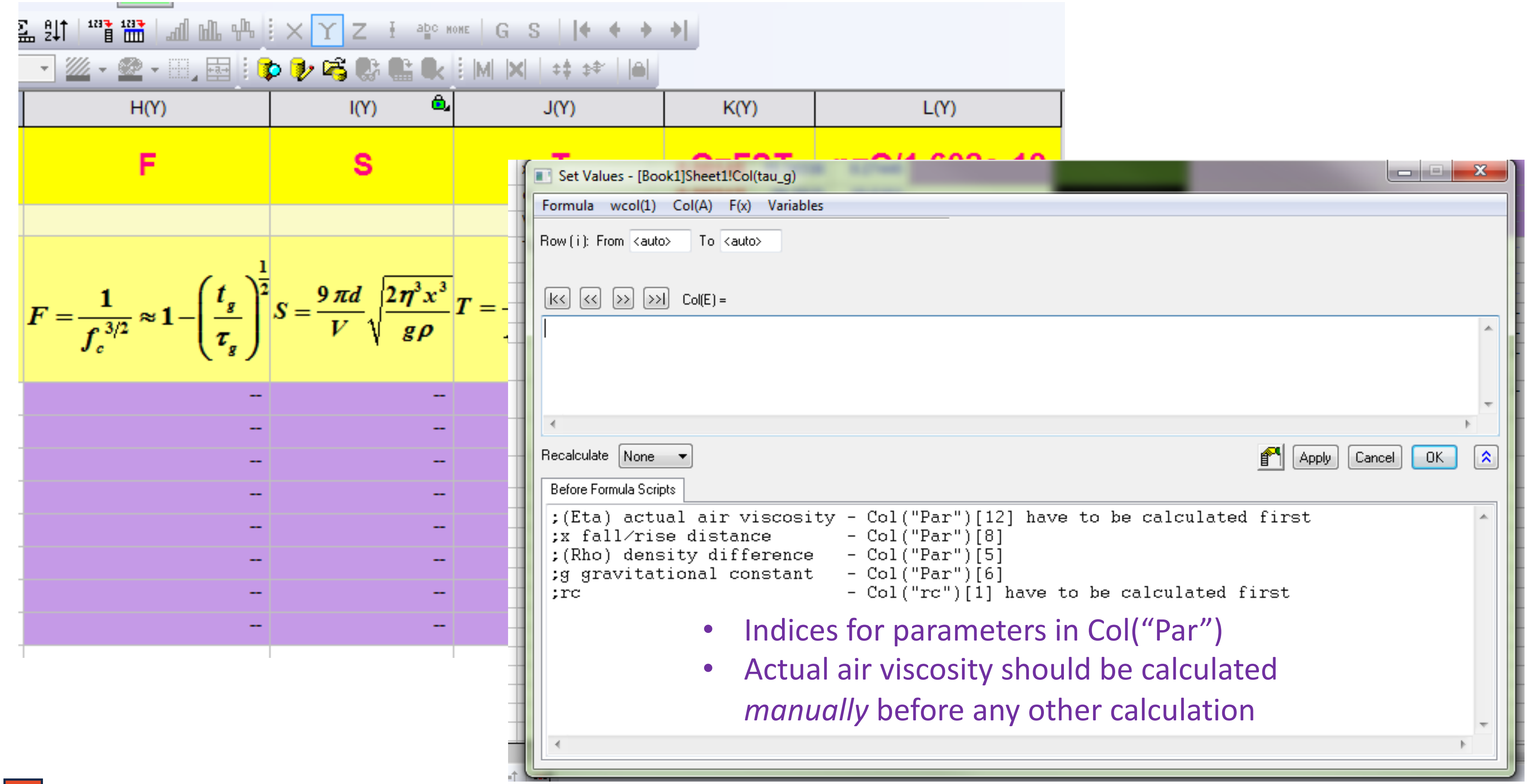
H(Y)	I(Y)	J(Y)	K(Y)	L(Y)
F	S	T	Q=FST	n=Q/1.602e-19
$F = \frac{1}{f_c^{3/2}} \approx 1 - \left(\frac{t_g}{\tau_g}\right)^{\frac{1}{2}}$	$S = \frac{9\pi d}{V}\sqrt{\frac{2\eta^3 x^3}{g\rho}}$	$T = \frac{1}{\sqrt{t_g}}\left(\frac{1}{t_g} + \frac{1}{t_{rise}}\right)$	Q=F*S*T	number of elementary charges
--	--			
--	--			
--	--			
--	--			

$$Q = F \bullet S \bullet T = \left(\frac{1}{f_c^{3/2}}\right)\frac{9\pi d}{V}\sqrt{\frac{2\eta^3 x^3}{g\rho}}\,\frac{1}{\sqrt{t_g}}\left(\frac{1}{t_g} + \frac{1}{t_{rise}}\right)$$

Follow correct dependency **sequence** of calculations: $r_c \rightarrow \tau_g \rightarrow (F, S, T) \rightarrow Q \rightarrow n$

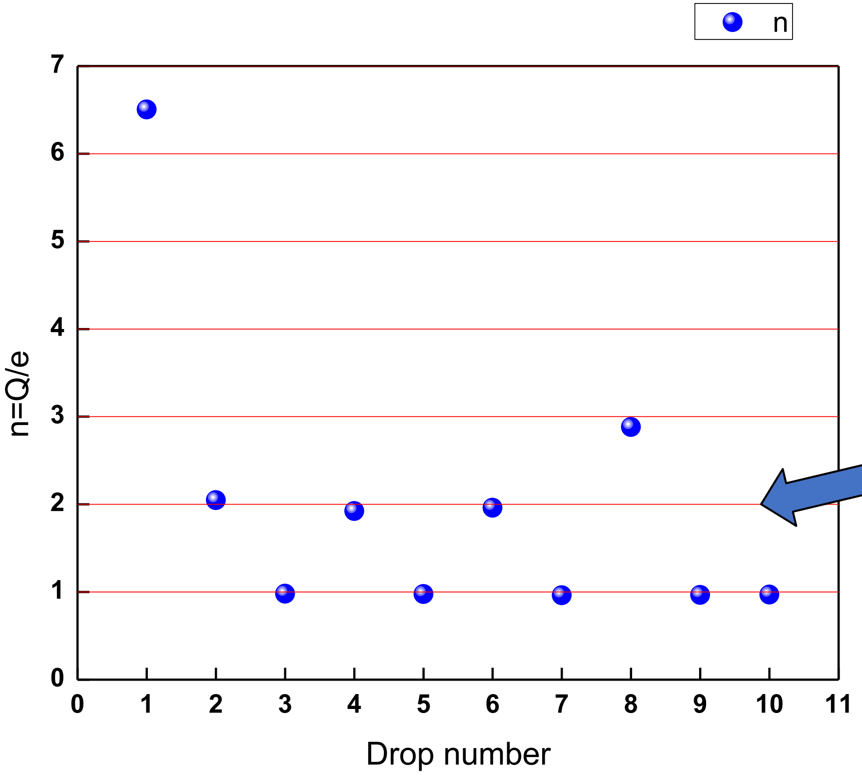
Origin Data Analysis

Project: ***Millikan1.opj***



- Indices for parameters in Col("Par")
- Actual air viscosity should be calculated *manually* before any other calculation

Origin Charge Calculation



	I(Y)	J(Y)	K(Y)	L(Y)
	S	T	Q=FST	n=Q/1.602e-19
				c
$)^{\frac{1}{2}}$	$S = \frac{9\pi d}{V}\sqrt{\frac{2\eta^3 x^3}{g\rho}}$	$T = \frac{1}{\sqrt{t_g}}\left(\frac{1}{t_g}+\frac{1}{t_{rise}}\right)$	Q=F*S*T	number of elementary charges
--	--		1.04196E-18	6.50414
--	--		3.27945E-19	2.0471
--	--		1.57231E-19	0.98147
--	--		3.08168E-19	1.92365
--	--		1.56458E-19	0.97664
--	--		3.14167E-19	1.9611
--	--		1.54196E-19	0.96252
--	--		4.61924E-19	2.88342
--	--		1.54861E-19	0.96668
--	--		1.55468E-19	0.97046
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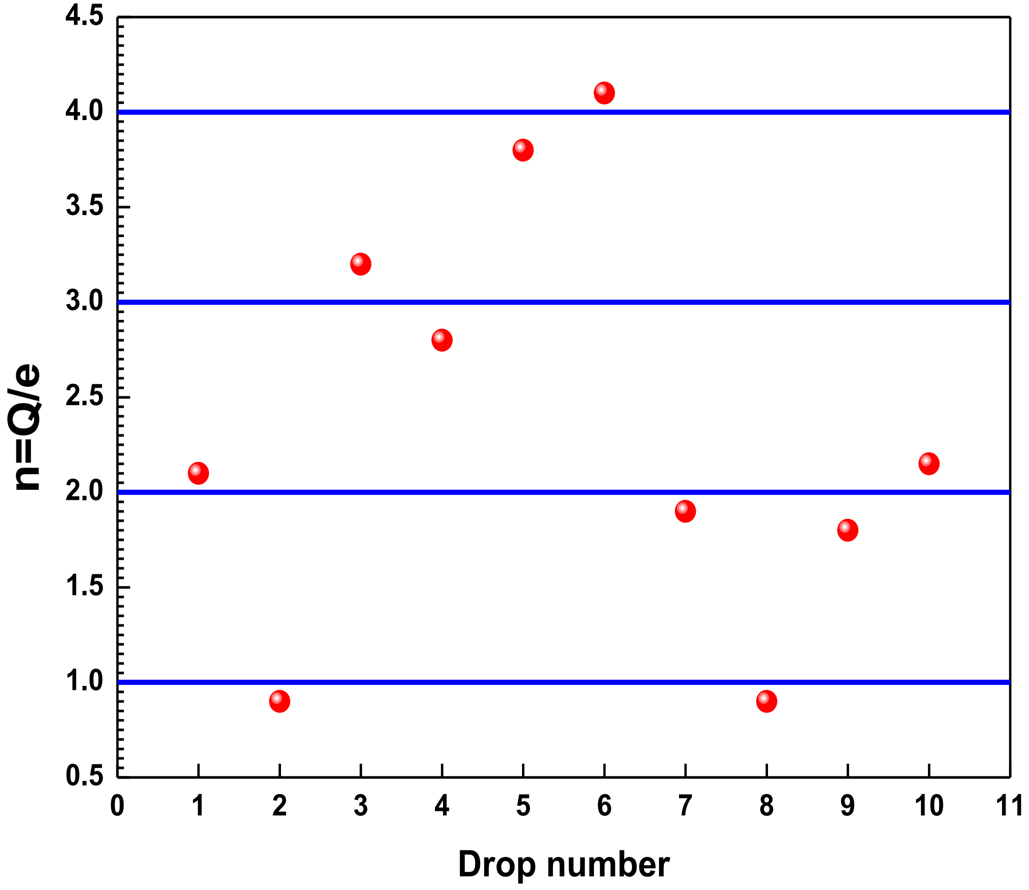
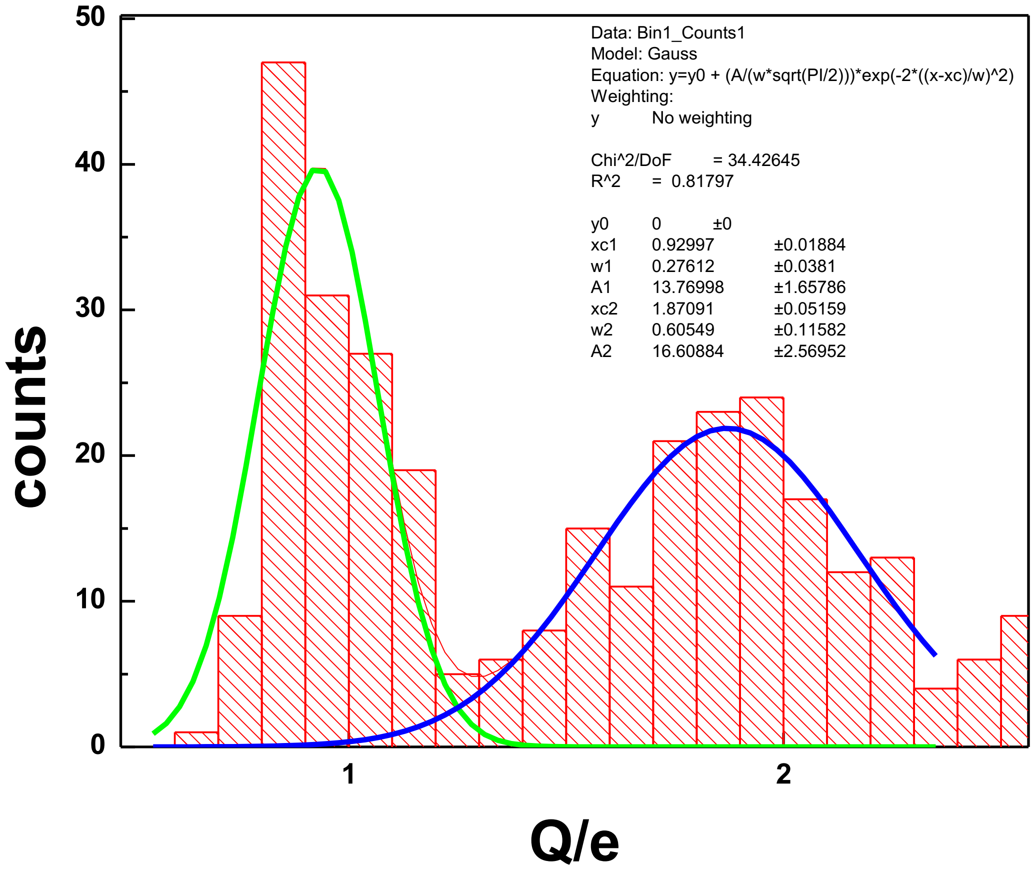
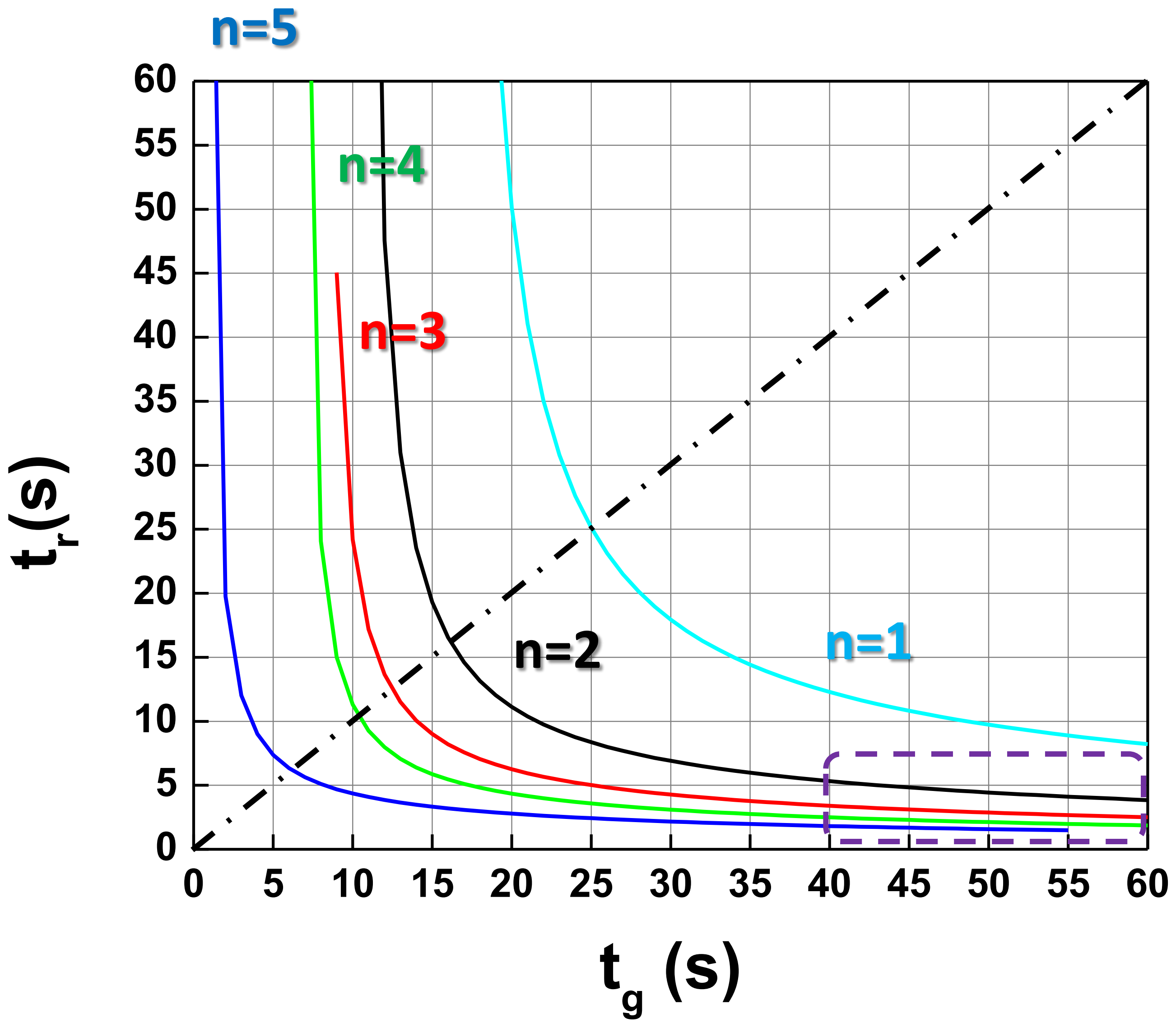
Expected Results



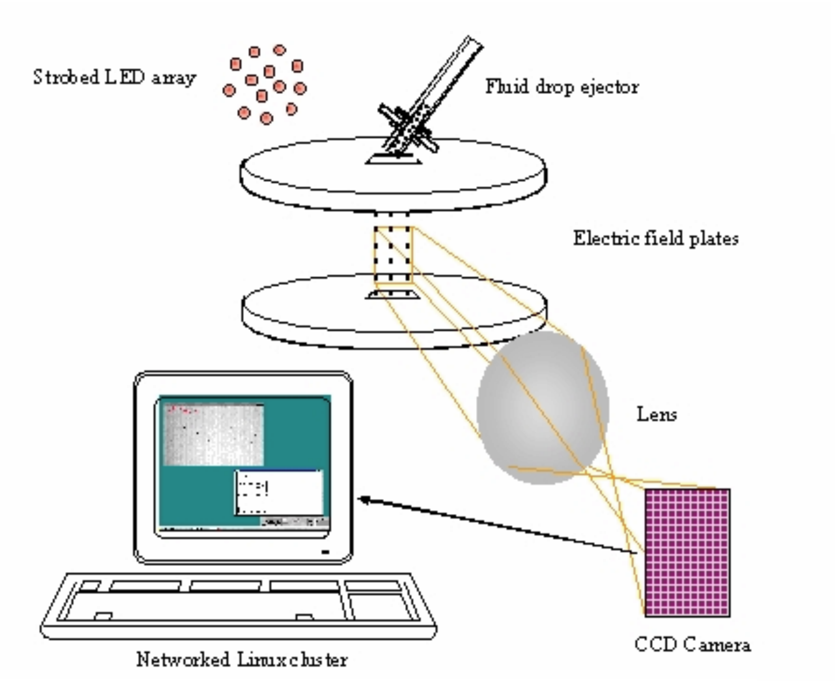


Effect of Choice of Oil Drops



Difficult to separate
n=3, 4, 5, …

Modern Experiments

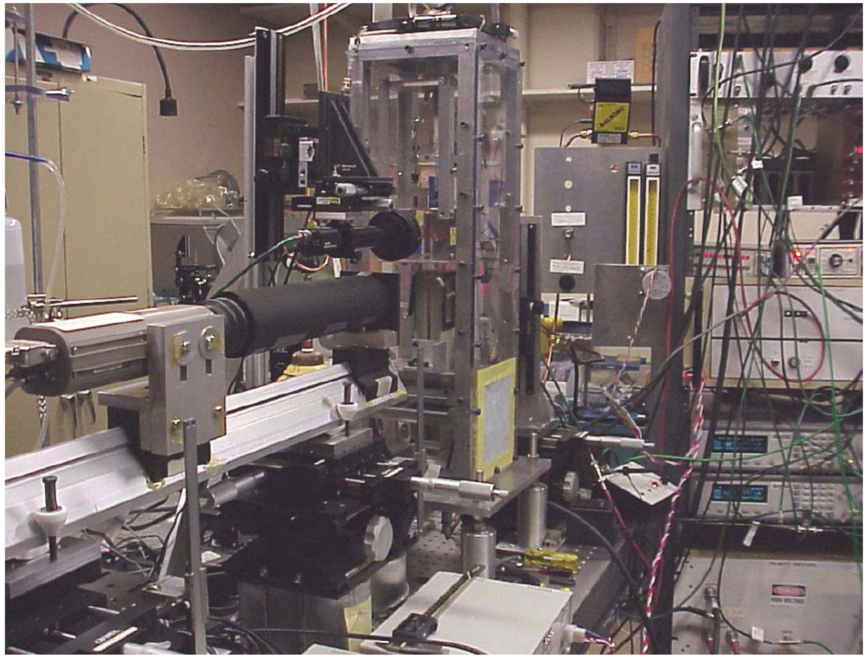


- Drop generation rate: 1 Hz
- Fluid: Dow Corning silicon oil
- Number of drops: 17 million
- Mass: 70.1 milligrams
- Duration: 8 months

Machine vision mediated auto-control of: the average charges of the drops, the fall path of the drops, the upward laminar air flow, and the electronic drive to the drop ejector are new features of this fluid drop charge measurement system.

The experiment is ran from 2004 - 2007.

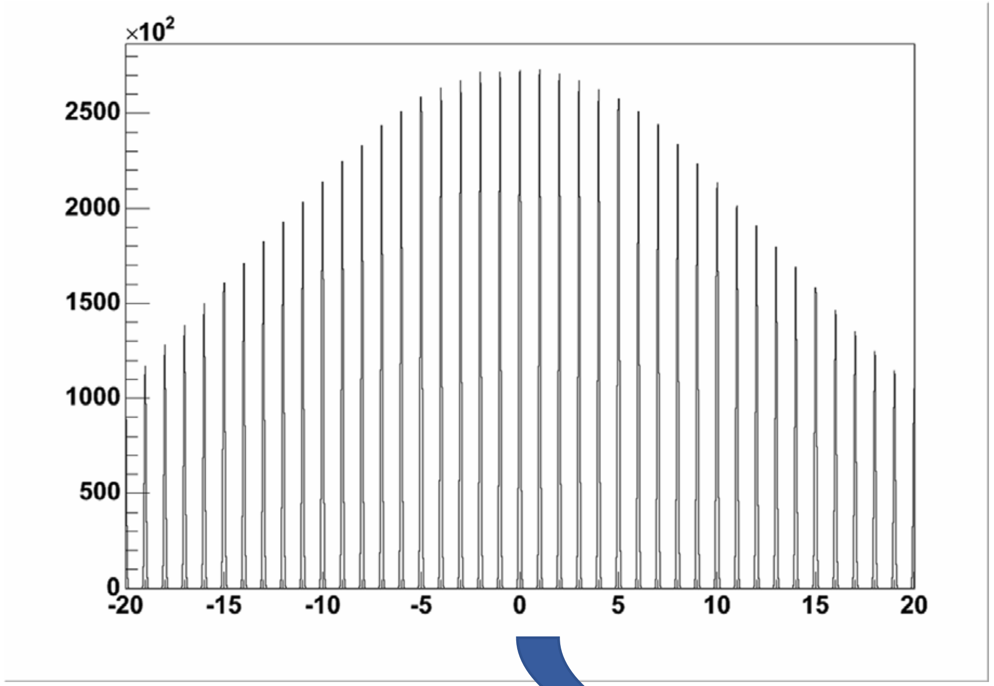
Charge measurement accuracy achieved is better than 1/24 e for drops of up to 26 microns in diameter.

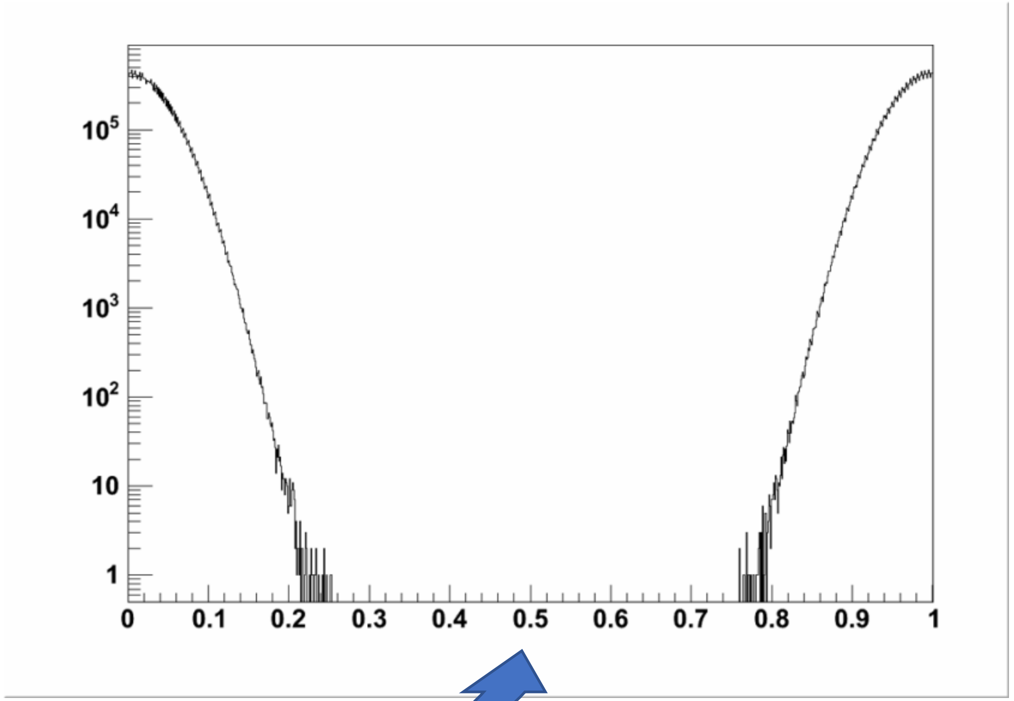


Modern Experiments

Summary as of January 2007

- Total mass throughput for all experiments: 351.4 milligrams of fluid
- Total drops measured in all experiments: 105.6 million
- *No evidence for fractionally charged particles* was found.





Measuring the Electron Charge

- 1909: Oil drop experiment, Robert A. Millikan
  e=1.5924(17)×10⁻¹⁹ C (0.1% error)
- 1918: Shot-noise proposed by Walter H. Schottky
  Poisson fluctuations in current across a barrier appear as noise
  Measure noise variance $\sigma_I^2 = 2eI\,\Delta f$ *over a bandwidth* Δf
- In terms of Avogadro's constant and Faraday constant (Coulombs per mole, from electrolysis)
  $e = F/N_A$*; best uncertainty ~1.6 ppm*
- From superconducting junctions (Josephson constant, $K_J = \frac{2e}{h}$) and quantum Hall effect (von Klitzing constant, $R_K = \frac{h}{e^2}$)

Modern Definition

- SI base units redefined on May 20, 2019
- Move to ground all 7 SI base units in terms of exact defined values of the physical constants *(exactly reproducible in lab!)*
- Response to the "crisis" of infinitesimal variations in the standard kilogram mass

- Modern *exact* definition:

$$e = 1.602176634\times10^{-19}\ Coulomb$$