Millikan Oil Drop Experiment

Professor Jeff Filippini
Physics 401
Spring 2020
Today’s Topics

1. History: Measuring the charge of the electron
2. Theory of the Experiment
3. Laboratory Setup
4. Data Analysis
Motivation: The Dawn of Elementary Particles

• 1750s: Benjamin Franklin proposes electricity is flow or surplus/deficit of single electrical fluid

• 1881: Hermann von Helmholtz argues for elementary positive and negative electric charges (“atoms of electricity”)

• 1897: J.J. Thompson shows particle nature of “cathode rays” emitted in vacuum tubes
  • Universal negatively-charged particles from vacuum tubes, radioactivity, ...
  • Measured charge-to-mass ratio \((e/m)\) via magnetic deflection
  • Mass <1/1000 of hydrogen ion (actually \(~1/1836\)
Robert Millikan and the Oil Drop Experiment

The Nobel Prize in Physics 1923.
Robert A. Millikan "for his work on the elementary charge of electricity and on the photoelectric effect".

University of Chicago
Moved to Caltech in 1921
Robert Millikan and the Oil Drop Experiment

ROBERT ANDREWS MILLIKAN 1868-1953

Work with Ph.D. student Harvey Fletcher – not a coauthor on famous paper. “The father of stereophonic sound” First functional electronic hearing aid

Physics 401
Oil Drop Apparatus

Motivation:

1. Measure the magnitude of the electron charge
   Apparatus precision $\pm$ 3%
   Millikan original $\pm$ 0.1%

2. Demonstrate that electric charge is quantized!
Forces acting on the oil drop:
1. Gravity
2. Electric force on a charged oil drop
The Role of Air: Buoyancy and Drag

Lowering an object through a fluid means lifting an equal volume of the surrounding fluid.

Motion through a fluid opposed by drag: friction with and within the surrounding fluid.

Objects eventually reach terminal velocity at which drag balances accelerating forces.
Forces acting on the oil drop:

1. Gravity
2. Electric force on a charged oil drop
3. Buoyant force (weight of air displaced by oil drop)
4. Drag force on a moving oil drop
Oil Drop Apparatus: Schematic Layout
Oil Drop Apparatus: Schematic Layout

Fig 1: Oil Drop Apparatus
How the Measurement Looks

- Rise time $t_{rise}$
  - Rise time measurement starts here...
  - ... and stops here

- Fall time measurement starts here...
  - Fall time $t_f$
  - $x = \text{fall distance} = \text{rise distance}$
  - Must be the same for all drops!

- Allow drop to “undershoot” here before starting next rise time experiment

Rise time $t_{rise}$
Balance of Forces: Newton’s Second Law

Forces on the oil drop:
1. Gravity + buoyancy
   \[ \vec{F}_g = -mg \hat{z} \]
2. Drag force
   \[ \vec{F}_{\text{drag}} = -6\pi \eta a \vec{v} \]
3. Electric force
   \[ \vec{F}_E = Q \vec{E} \]

\[ \vec{F} = m \frac{d\vec{v}}{dt} = \vec{F}_g + \vec{F}_{\text{drag}} + \vec{F}_E \]

**Terminal velocity:** speed at which \( \frac{d\vec{v}}{dt} = 0 \)

\[ \vec{F}_g + \vec{F}_{\text{drag}} + \vec{F}_E = 0 \]

Achieved within \( \sim 10^{-5} \) s by a 1 \( \mu \)m particle

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**Drop**
- a: radius of drop
- \( \rho \): net density \( \rho_{\text{oil}} - \rho_{\text{air}} \)
- v: velocity of oil drop
- Q: charge of oil drop

**Apparatus**
- E: electric field \( E = V/d \)
- V: voltage across plates
- \( \eta \): viscosity of air
- g: gravitational constant
Modifications to Stokes’ Law

\[ \vec{F}_{\text{drag}} = -6\pi \eta a \vec{v} \]

For particles of small radii \((a \lesssim 15 \mu m)\), Stokes’ Law must be corrected for the particle nature of air. This correction was derived by E. Cunningham

\[ \vec{F}_{\text{drag}} = -6\pi \eta \frac{a}{f_c} \vec{v} \]

Here:
- \(a = \) particle radius
- \(\lambda = \) mean free path of gas molecules

\[ f_c = 1 + A \frac{\lambda}{a} + B \frac{\lambda}{a} e^{-c \frac{a}{\lambda}}, \quad A = 1.246; \quad B = 0.42; \quad C = 0.78 \]

\[ f_c \approx 1 + A \frac{\lambda}{a} = 1 + \frac{r_c}{a} \approx 1.1 \]

for \(a = 10^{-6} m\), \(r_c = \frac{6.18 \times 10^{-5} m}{p [mmHg]}\)

\[ \lambda = 65.3 \text{ nm} \left( \frac{760 \text{ mmHg}}{p} \right) \]
What We Measure: Rise and Fall Times

Rise time $t_{\text{rise}}$

Rise time measurement starts here...

Fall time $t_{g}$

$x = \text{fall distance} = \text{rise distance}$

Must be the same for all drops!

Allow drop to “undershoot” here before starting next rise time experiment

Fall time measurement starts here...

... and stops here

... and stops here
Measuring the Temperature

Project: T measurement.opj

![Image of apparatus]

![Graph showing temperature vs. resistance]

<table>
<thead>
<tr>
<th>Model</th>
<th>Project</th>
</tr>
</thead>
<tbody>
<tr>
<td>Equation</td>
<td>$T = \frac{AR}{R + A}$</td>
</tr>
<tr>
<td>Plot</td>
<td>A</td>
</tr>
<tr>
<td>y=</td>
<td>-20.359053 + 11.31064</td>
</tr>
<tr>
<td>x=</td>
<td>1.00941 / 0.80042</td>
</tr>
<tr>
<td>A</td>
<td>220.7777 + 1.44083</td>
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<tr>
<td>P</td>
<td>-0.00535 + 0.00537</td>
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<tr>
<td>Rhoard Chi-Sq</td>
<td>1.26446</td>
</tr>
<tr>
<td>R-Squart(OD)</td>
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</tr>
<tr>
<td>Adj. R-Squart</td>
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</table>

<table>
<thead>
<tr>
<th>R (MoHm)</th>
<th>T (°C)</th>
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<tbody>
<tr>
<td>1.55</td>
<td>36.87987</td>
</tr>
</tbody>
</table>

Physics 401
Solving Newton’s Law: \( Q(t_g, t_{rise}) \)

The correction factor \( f_c \) can be found from Newton’s Law in the case of \( v = 0 \) (falling drop)

\[
\vec{F}_g + \vec{F}_{drag} = \frac{4}{3} a^3 \rho g - 6\pi \eta \frac{a}{f_c} \vec{v} = 0
\]

\[ \frac{1}{f_c^{2/3}} \approx 1 - \left(\frac{t_g}{\tau_g}\right)^{\frac{1}{2}}; \quad \tau_g = \frac{2\eta x}{\rho gr_c^2}; \quad r_c = \frac{6.18 \times 10^{-5} \text{ m}}{p [\text{mmHg}]} \]
Solving Newton’s Law: $Q(t_g, t_{rise})$

$$\frac{1}{f_c^{2/3}} \approx 1 - \left(\frac{t_g}{\tau_g}\right)^{1/2}; \quad \tau_g = \frac{2\eta x}{\rho gr_c^2}; \quad r_c = \frac{6.18 \times 10^{-5}}{p \text{ [mmHg]}}$$

Putting it all together:

$$Q = ne = \frac{1}{f_c^{2/3}} \frac{9\pi d}{V} \sqrt{\frac{2\eta^3 x^3}{g \rho}} \sqrt{\frac{1}{t_g} \left[ \frac{1}{t_g} + \frac{1}{t_{rise}} \right]}$$

**F**: Stokes’ law correction

**S**: Stokes’ law force balance

**T**: Your timing measurements
Project: **Millikan_raw data.opj**

Location: `\\engr-file-03\\PHYINST\\APL Courses\\PHYCS401\\Students\\1. Millikan Oil Drop experiment\\Millikan_raw data.opj`

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### Table

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<tr>
<th>Parameter label</th>
<th>Units</th>
<th>Par</th>
<th>tg</th>
<th>tr</th>
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<tbody>
<tr>
<td>$\eta$</td>
<td>kg/ms</td>
<td>1.8478E-5</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\Delta \eta/\Delta T$</td>
<td>kg/msCo</td>
<td>4.8E-8</td>
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<tr>
<td>$\rho_1$</td>
<td>kg/m$^3$</td>
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<td>886</td>
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<td>$\rho_2$</td>
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<td>$g$</td>
<td>m/s$^2$</td>
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<td>9.801</td>
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<tr>
<td>$p$</td>
<td>mmHg</td>
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</tr>
<tr>
<td>$x$</td>
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<tr>
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<tr>
<td>$V$</td>
<td>V</td>
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<tr>
<td>$T_a$</td>
<td>Co</td>
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Parameters, use proper units shown column "Units"  
Raw data, falling time  
Raw data, rising time
Origin Projects: Data Analysis

Location: `\\engr-file-03\PHYINST\APL Courses\PHYCS401\Students\1. Millikan Oil Drop experiment`

Please make a copy *(don’t move!)* of Millikan1.opj and Millikan_raw data.opj in your personal folder and work with your personal copy of the project.
Origin Data Analysis

### Constants

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
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<tbody>
<tr>
<td>$r_c$</td>
<td>$6.18 \times 10^{-5} \text{ m}$</td>
</tr>
<tr>
<td>$P$</td>
<td>$\text{ mmHg}$</td>
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</tbody>
</table>

### Experimental Parameters

Depend on exact setup and environmental conditions

### Calculations

$r_c = \frac{6.18 \times 10^{-5} \text{ m}}{P \text{ [mmHg]}}$

Project: **Millikan1.opj**
### Origin Data Analysis

#### Project: Millikan1.opj

<table>
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<tr>
<th></th>
<th>D(L)</th>
<th>B(X)</th>
<th>F(Y)</th>
<th>G(Y)</th>
<th>C(Y)</th>
<th>E(Y)</th>
<th>H(Y)</th>
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<tr>
<td>Long Name</td>
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<td>Par</td>
<td>t_g</td>
<td>t_r</td>
<td>rc</td>
<td>tau_g</td>
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<td>Comments</td>
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<td>your data</td>
<td>your data</td>
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</tr>
</tbody>
</table>

1. \( \eta \) = 1.8478E-5
2. \( d\eta \) = 4.8E-8
3. \( \rho_1 \) = 886
4. \( \rho_2 \) = 1.29
5. \( \rho_1 - \rho_2 \) = 884.71
6. \( g \) = 9.801
7. \( p \) = 765

\[
\tau_g = \frac{2\eta x}{\rho \rho g' c^2} = \frac{1}{f_c^{3/2}} \approx 1
\]

---

**Formula:** `wcol(1)` Col(A) F(x) Variables Options

Flow: Fion <auto> To <auto>

Before Formula Scripts:

\[ p = \text{Col("Par")[?]} \]
Follow correct dependency sequence of calculations: \( r_c \rightarrow \tau_g \rightarrow (F, S, T) \rightarrow Q \rightarrow n \)
Origin Data Analysis

Project: Millikan1.opj

\[ F = \frac{1}{f_c^{3/2}} \approx 1 - \left( \frac{t_g}{t_s} \right)^{1/2}, \quad S = \frac{9 \pi d}{V} \sqrt{\frac{2m^3}{\rho g}} \quad T = \text{...} \]

- Indices for parameters in Col(“Par”)
- Actual air viscosity should be calculated manually before any other calculation
Origin Charge Calculation

\[ n = \frac{Q}{e} \]

\[ S = \frac{9 \pi d^2}{V} \sqrt{\frac{2 \eta x}{g \rho}} \]

\[ T = \frac{1}{\sqrt{\frac{1}{t_s} + \frac{1}{t_{sw}}}^\frac{1}{2}} \]

\[ Q = F^* S^* T \]

<table>
<thead>
<tr>
<th>S</th>
<th>T</th>
<th>Q=FST</th>
<th>n=Q/1.602e-19</th>
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<td>1.04196E-18</td>
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<td>4.61924E-19</td>
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<td>1.54861E-19</td>
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<tr>
<td>1.55468E-19</td>
<td>0.97046</td>
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</table>

Number of elementary charges.
Expected Results

Data: Bin1_Counts1
Model: Gauss
Equation: y = y0 + A/(w*sqrt(PI/2)) * exp(-2*(x-xc)^2/w^2)

Chi^2/DoF = 34.42645
R^2 = 0.81797

\( y0 = 0.92997 \pm 0.01884 \)
\( w1 = 0.27612 \pm 0.00381 \)
\( A1 = 13.76998 \pm 1.65786 \)
\( xc2 = 1.87091 \pm 0.05159 \)
\( w2 = 0.60549 \pm 0.011582 \)
\( A2 = 16.60884 \pm 2.56952 \)
Effect of Choice of Oil Drops

Difficult to separate n=3, 4, 5, ...
Modern Experiments

- Drop generation rate: 1 Hz
- Fluid: Dow Corning silicon oil
- Number of drops: 17 million
- Mass: 70.1 milligrams
- Duration: 8 months

Machine vision mediated auto-control of the average charges of the drops, the fall path of the drops, the upward laminar air flow, and the electronic drive to the drop ejector are new features of this fluid drop charge measurement system.

The experiment is run from 2004 - 2007.

Charge measurement accuracy achieved is better than 1/24 e for drops of up to 26 microns in diameter.
Modern Experiments

Summary as of January 2007

• Total mass throughput for all experiments: 351.4 milligrams of fluid
• Total drops measured in all experiments: 105.6 million
• *No evidence for fractionally charged particles* was found.
Measuring the Electron Charge

• 1909: Oil drop experiment, Robert A. Millikan
e=1.5924(17)×10^{-19} \text{C (0.1\% error)}

• 1918: Shot-noise proposed by Walter H. Schottky
Poisson fluctuations in current across a barrier appear as noise
Measure noise variance $\sigma_I^2 = 2eI \Delta f$ over a bandwidth $\Delta f$

• In terms of Avogadro’s constant and Faraday constant (Coulombs per mole, from electrolysis)
e = F / N_A; best uncertainty ~1.6 ppm

• From superconducting junctions (Josephson constant, $K_J = \frac{2e}{h}$) and quantum Hall effect (von Klitzing constant, $R_K = \frac{h}{e^2}$)
Modern Definition

• SI base units **redefined** on May 20, 2019
• Move to ground all 7 SI base units in terms of exact defined values of the physical constants (exactly reproducible in lab!)
• Response to the “crisis” of infinitesimal variations in the standard kilogram mass

• Modern *exact* definition:
  \[ e = 1.602176634 \times 10^{-19} \text{ Coulomb} \]