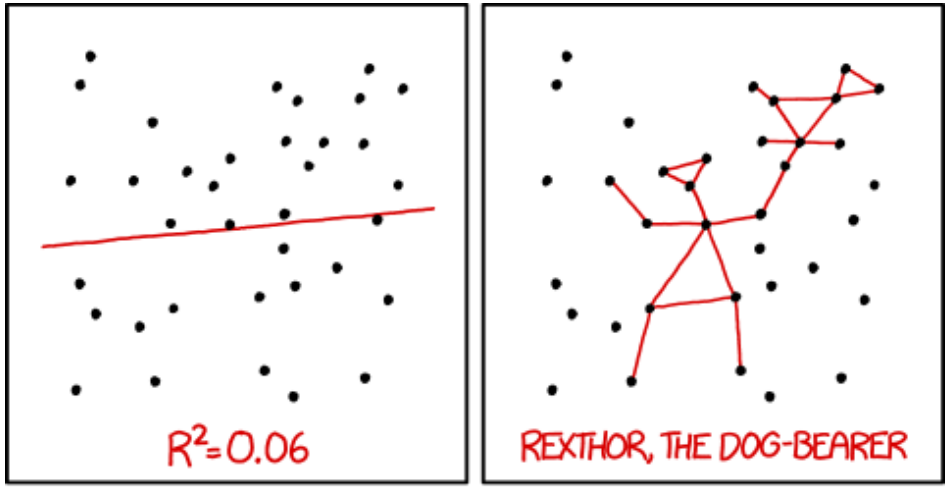
Basic Error Analysis



I DON'T TRUST LINEAR REGRESSIONS WHEN IT'S HARDER TO GUESS THE DIRECTION OF THE CORRELATION FROM THE SCATTER PLOT THAN TO FIND NEW CONSTELLATIONS ON IT.

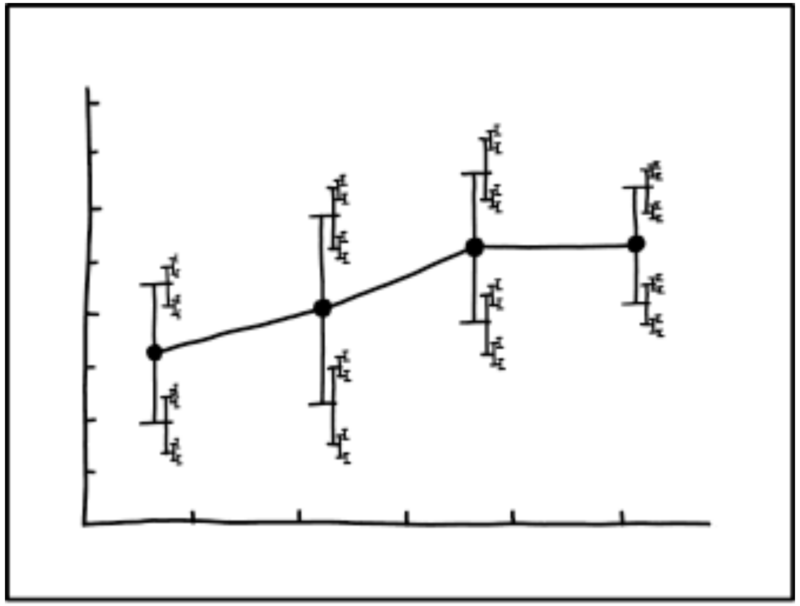
XKCD #1752

Professor Jeff Filippini

Physics 401

Spring 2020

ILLINOIS



I DON'T KNOW HOW TO PROPAGATE ERROR CORRECTLY, SO I JUST PUT ERROR BARS ON ALL MY ERROR BARS.

XKCD #2110

Today's Topics

1. Errors and Uncertainties
2. Instrument Errors
3. Accuracy and Precision
4. Systematic and Statistical Errors
5. Evaluating Fits and Parameter Errors

Appendix: More on oil drop data analysis, fitting

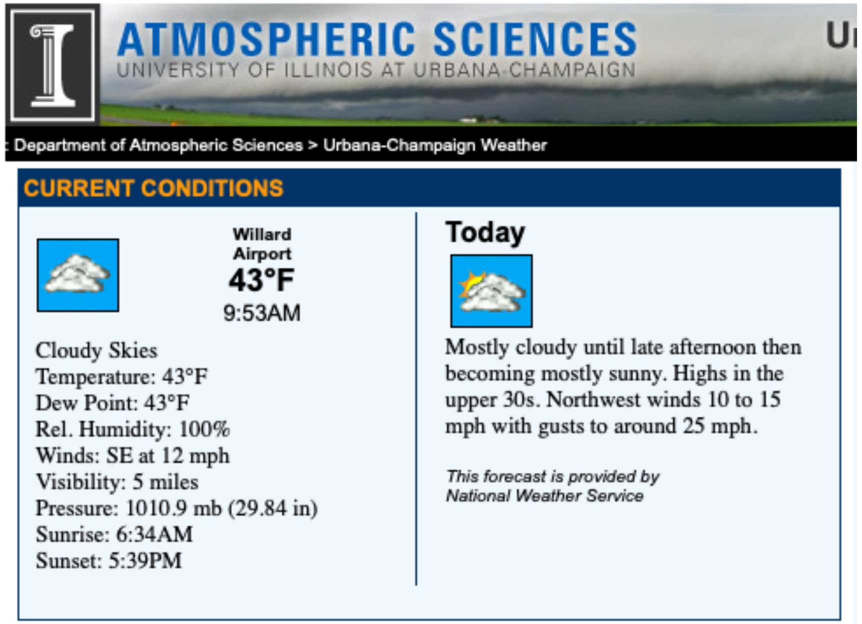
Redeeming some “Bad Words”

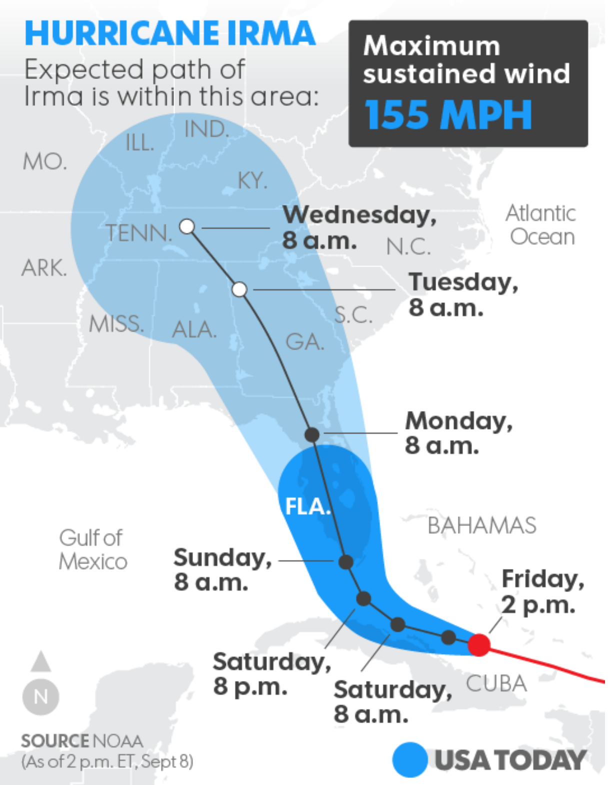
- **Errors** are not mistakes!

- **Statistics** is, in a deep sense, what science is about!
  - “The discipline that concerns the collection, organization, analysis, interpretation and presentation of data” (Wikipedia)
  - What conclusions can you draw about the universe from your observations?

- Careful **error analysis** is critical to learning from data

Errors: Everyday Life





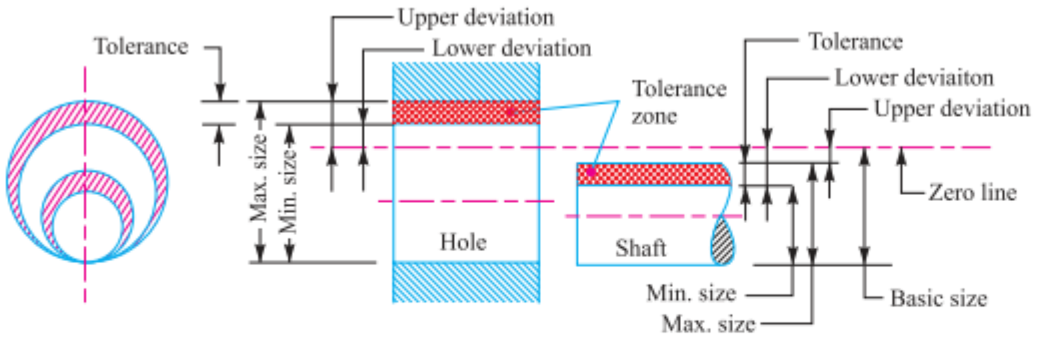
T = 43°F ± ??

Best guess: ΔT ~ 0.5°F

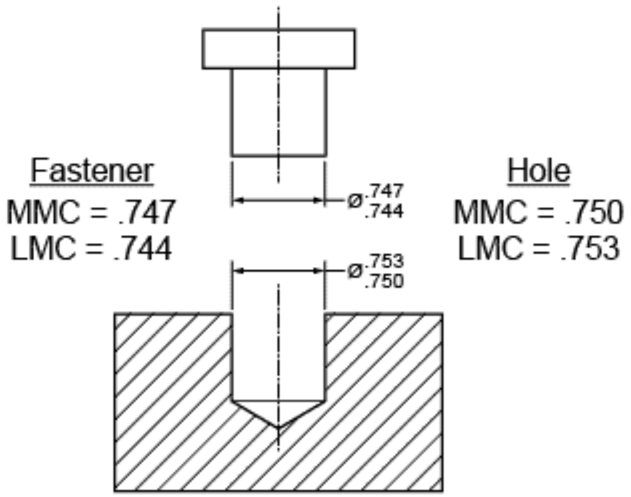
Wind speed 12 mph ± ??

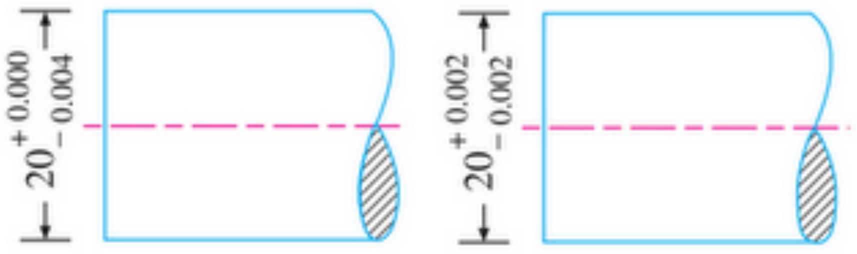
Best guess: Δv ~ 0.5 mph

Errors: Industry



Clearance fit



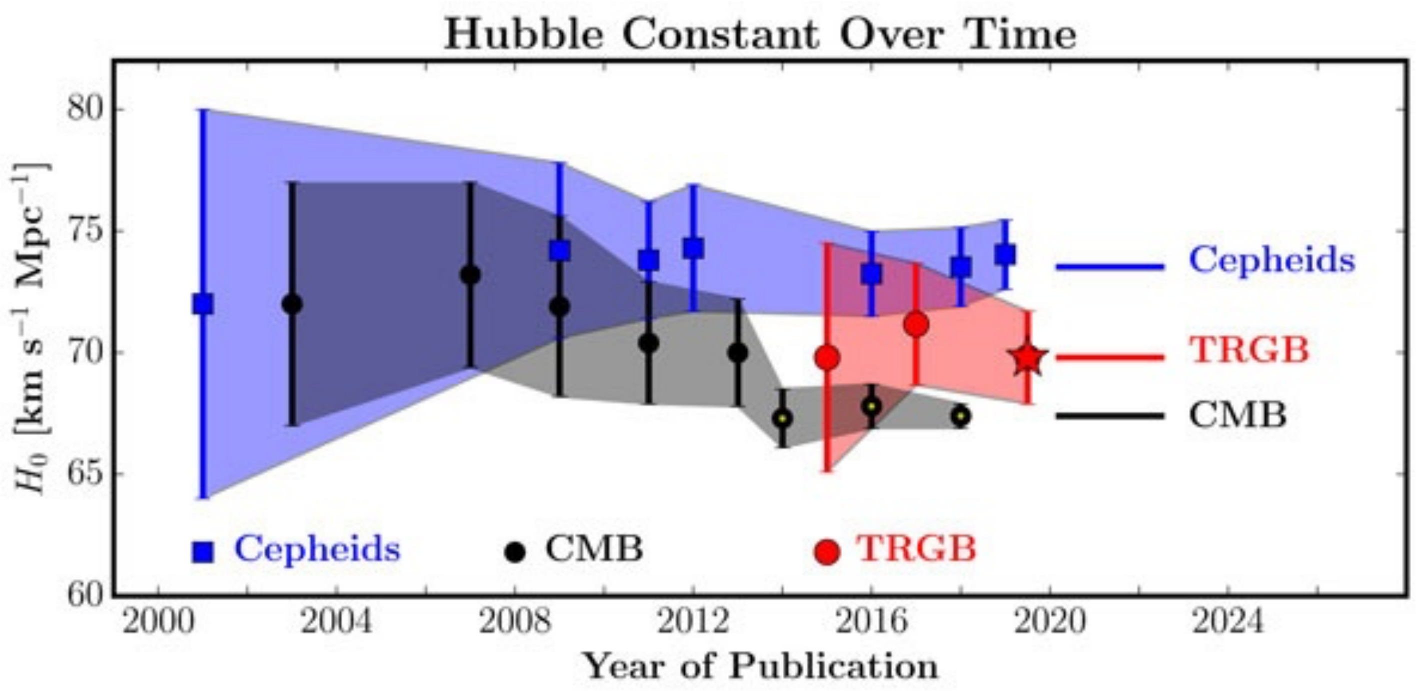


(*a*) Unilateral tolerance (*b*) Bilateral tolerance.

Tight tolerances cost $$$$!!!

Errors: Science

How fast is the universe expanding? The **Hubble Constant**



Multiple methods:

- **Standard candles**: How bright do objects of known properties appear?
  - Peculiar variable stars
  - Largest possible red giant stars
- **Standard rulers:** How big do ripples in the cosmic microwave background look?

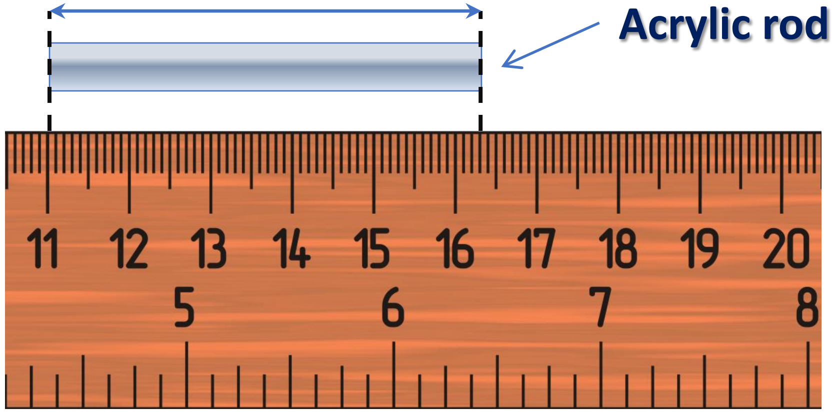
We seek consilience between different techniques
Do we need a new model, or are we missing sources of error??

Measurements: Reading Errors

Typical reading error: ±½ of minimum gradation or count

$\Delta L \cong 0.5mm$

L=53mm±ΔL(?)



$\Delta L \cong 0.03mm$

How far should we go to minimize reading error?

What do we Need?
In some cases we don't care about accuracy better than 1mm

Tools
If a ruler doesn't do the job, use a digital caliper or Vernier caliper

Intrinsic Limits
E.g. thermal expansion
CTE $\Delta L/L = 7.5{\times}10^{-5}K^{-1}$
ΔL ~ 0.004 mm/K

Measurements: Reading Errors

Fluke 8846A multimeter

Example Vdc (reading)=0.85V on 1V range

$$\Delta V = 0.85\times(1.8\times10^{-5})$$
$$+1.0\times(0.6\times10^{-5}) \cong 2.2\times10^{-5} = 22\mu V$$

8846A Accuracy

Accuracy is given as ± (% measurement + % of range)

Range	24 Hour (23 ±1 °C)	90 Days (23 ±5 °C)	1 Year (23 ±5 °C)	Temperature Coefficient/ °C Outside 18 to 28 °C
100 mV	0.0025 + 0.003	0.0025 + 0.0035	0.0037 + 0.0035	0.0005 + 0.0005
1 V	0.0018 + 0.0006	0.0018 + 0.0007	0.0025 + 0.0007	0.0005 + 0.0001
10 V	0.0013 + 0.0004	0.0018 + 0.0005	0.0024 + 0.0005	0.0005 + 0.0001
100 V	0.0018 + 0.0006	0.0027 + 0.0006	0.0038 + 0.0006	0.0005 + 0.0001
1000 V	0.0018 + 0.0006	0.0031 + 0.001	0.0041 + 0.001	0.0005 + 0.0001

Accuracy and Precision: Conceptual

Accuracy

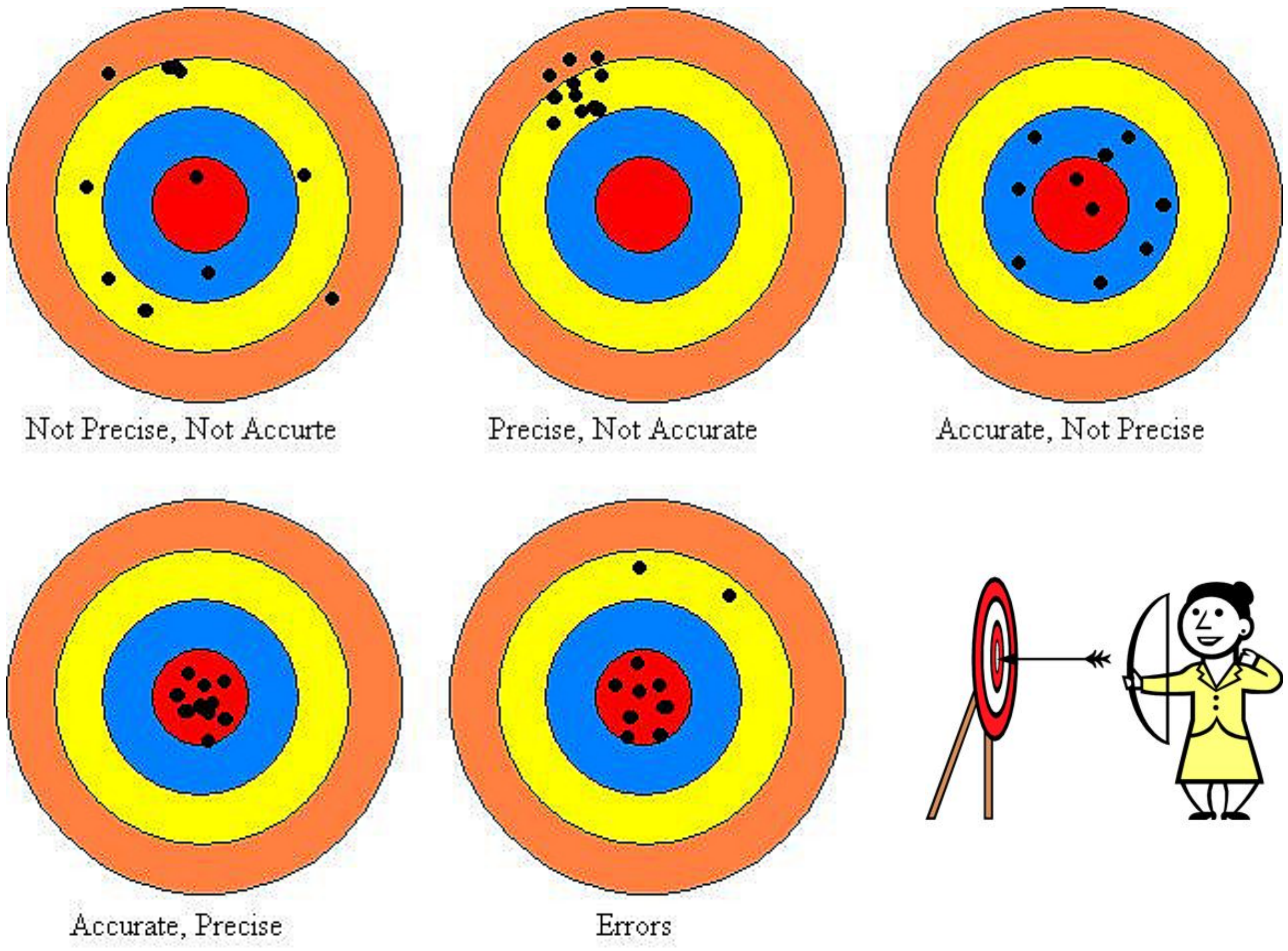
How close does the measurement come to the true value?

Precision

How close do individual measurements come to each other?

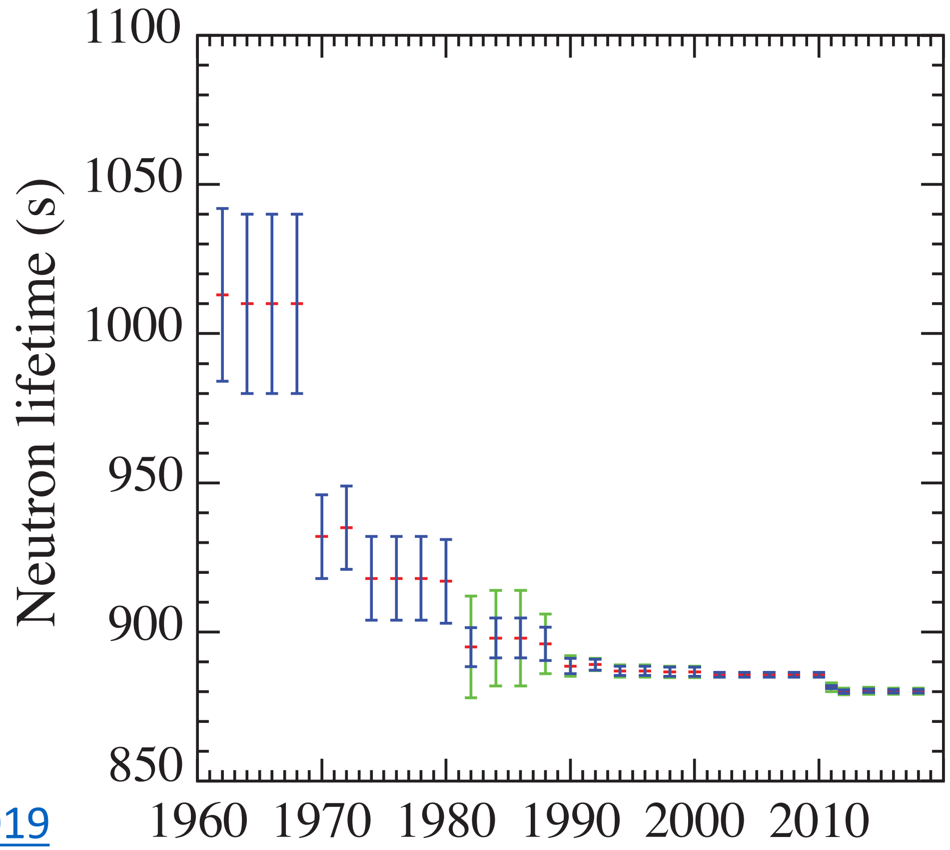
Repeatability, consistency

Accuracy and Precision: Conceptual



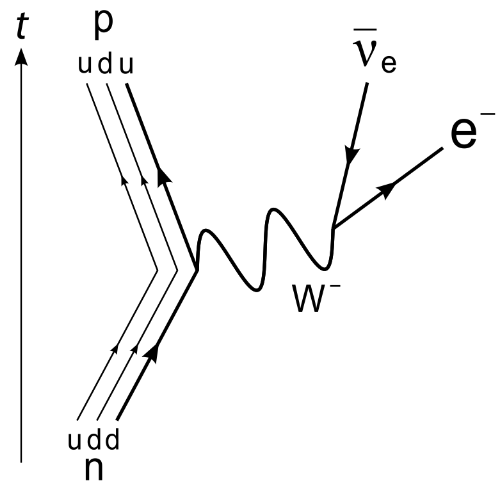
Accuracy and Precision: Reality

… but in experimental science we don't *know* the true value *a priori*!



PDG 2019

How long does a **free neutron** live before decaying to a proton, an electron, and a neutrino?



Wikipedia: Free neutron decay

Two Classes of Error

- **Systematic error**: **Reproducible** inaccuracy introduced by imperfect equipment, calibration or measurement technique.

"Bias"

- **Random error**: Indefiniteness of results due to finite precision of experiment. Measure of **fluctuation** in result after repeatable experimentation.

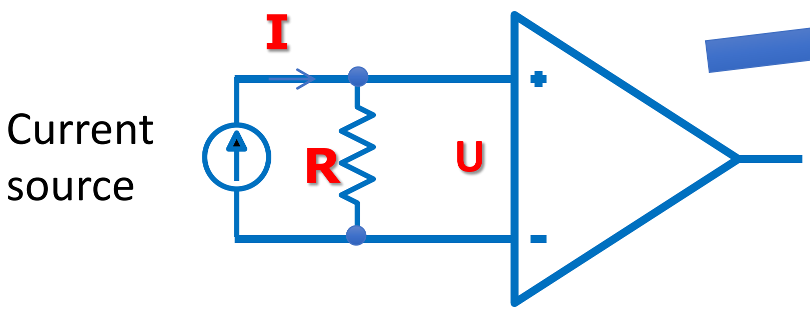
"Noise"

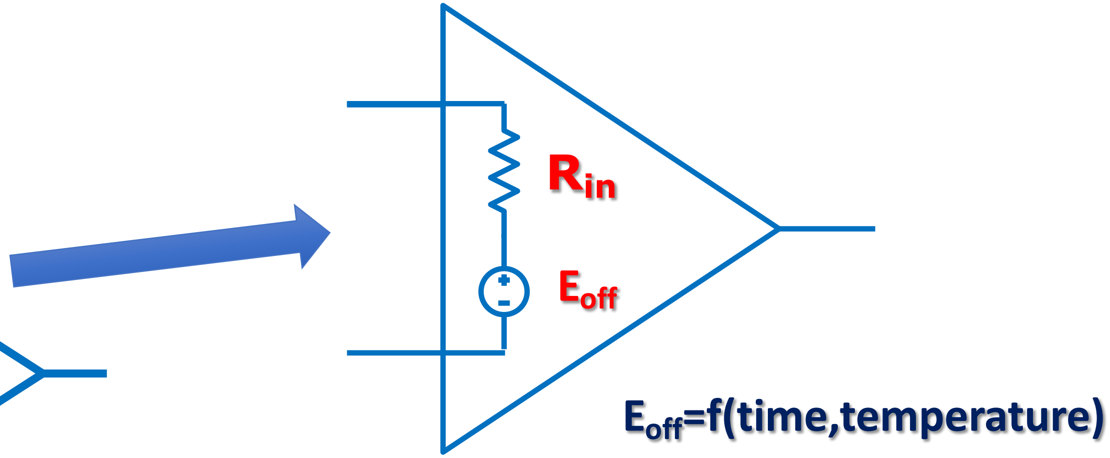
Philip R. Bevington "Data Reduction and Error Analysis for the Physical sciences", McGraw-Hill, 1969

Sources of Systematic Error

- Poor calibration of equipment
- Changes in environmental conditions
- Imperfect methods of measurement
- Offsets and drifts in instruments
- … etc.

Example #1: measuring a DC voltage





expectation

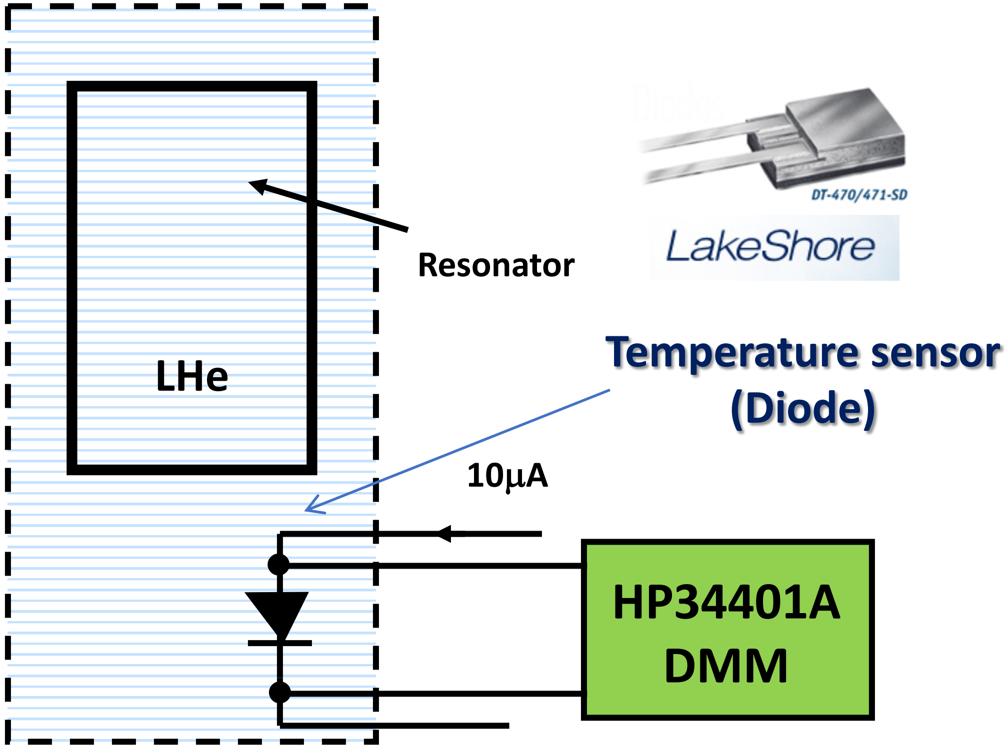
$$U = R * I$$

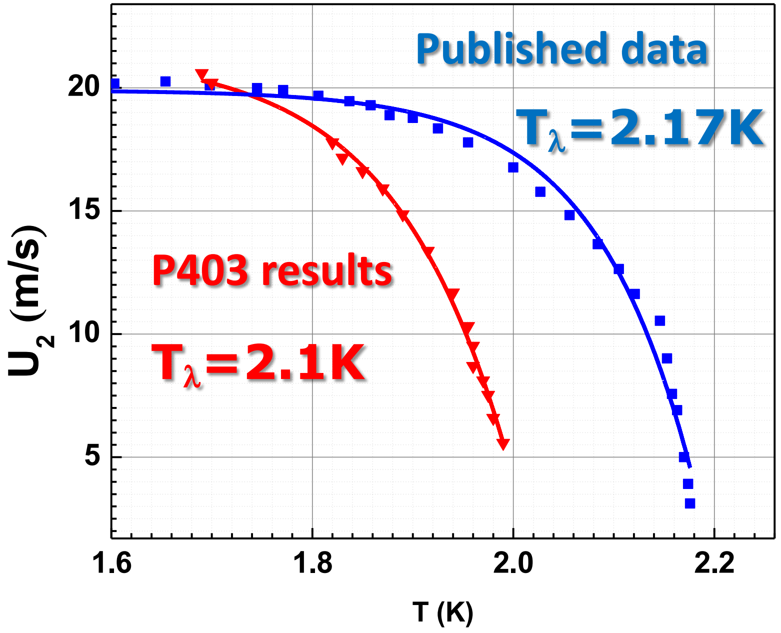
actual result

$$U = \frac{R * I - \left(\frac{R}{R_{in}}\right) E_{off}}{\left(1 + \frac{R}{R_{in}}\right)}$$

Sources of Systematic Error

Example #2: Measuring the speed of "second sound" in superfluid 4He





Explanation is **imperfect calibration**
Common issue with diodes at low T, where dV/dT is small

Random Errors

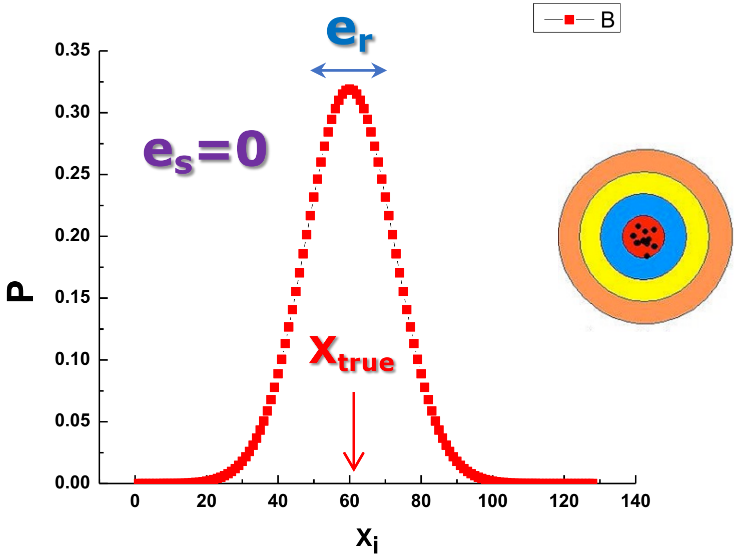
Systematic error

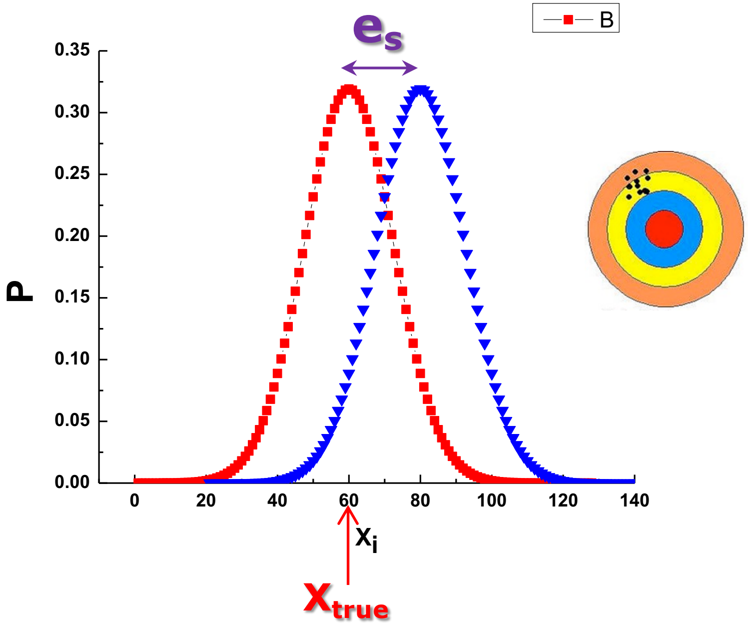
Measured value

$$x_{meas} = x_{true} + \boxed{e_s} + \boxed{e_r}$$

Correct value

Random error



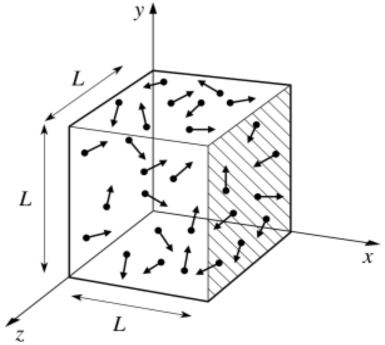


Where Does Randomness Come From?

Where have you found randomness in your other physics classes?

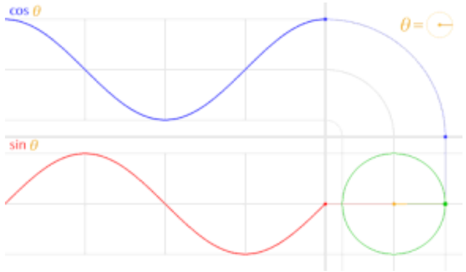
Ignorance

Coupling to large ensembles



Thermal fluctuations

Coupling through complicated functions



Chaos

Psueodo-random #s

Quantum mechanics

Measurement

Random Errors: Poisson Distribution

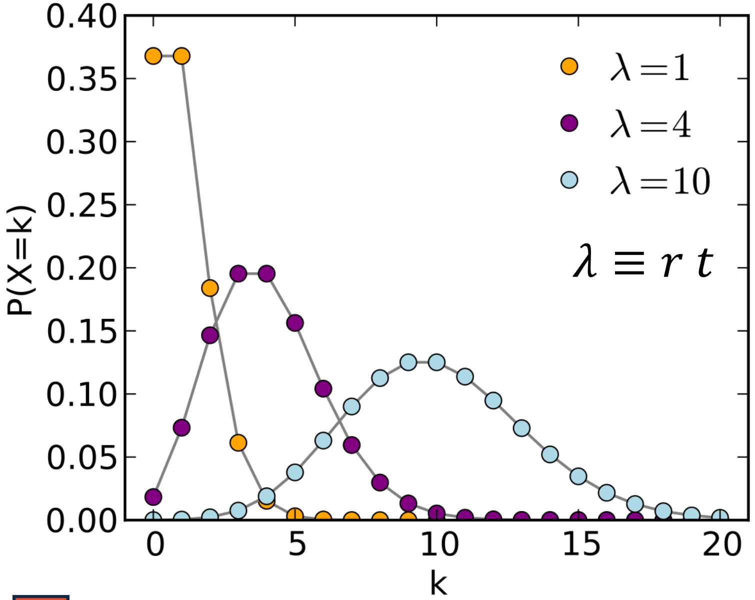
Siméon Denis Poisson (1781-1840)

$$P_n(t) = \frac{(r\ t)^n}{n!} e^{-rt}$$

Probability to observe ***n*** events in a time interval of length ***t***.

Sole parameter is rate ***r*** (average events/second)

[Wikipedia: Poisson distribution]



Number of counts

A statistical process (*e.g.* radioactive decay) is described through a Poisson distribution if it is/has:

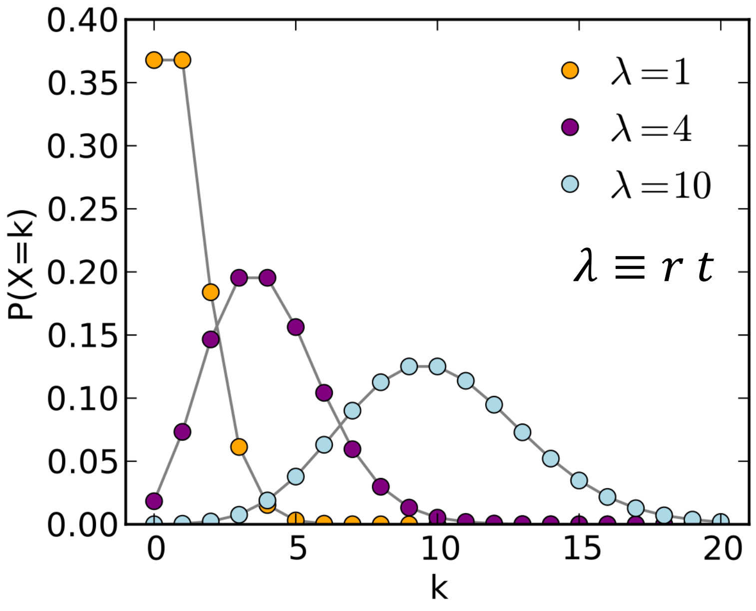
- **Discrete**: something decays or it doesn't (*n=0,1,2,…*)
- **No memory**: Probability for a decay is the same in any time interval, no matter how early or late
- **Universal probability**: the probability to decay in a given time interval is the same for all nuclei
- **Independence**: The decay of one nucleus does not affect the probability of a second nucleus decaying

Random Errors: Poisson Distribution

$$P_n(t) = \frac{(r\,t)^n}{n!}e^{-rt}$$

Probability to observe ***n*** events (n=0, 1, 2, …) in a time interval of length ***t***.

Sole parameter is rate ***r*** (average events/second)



Number of counts

Properties of the Poisson distribution

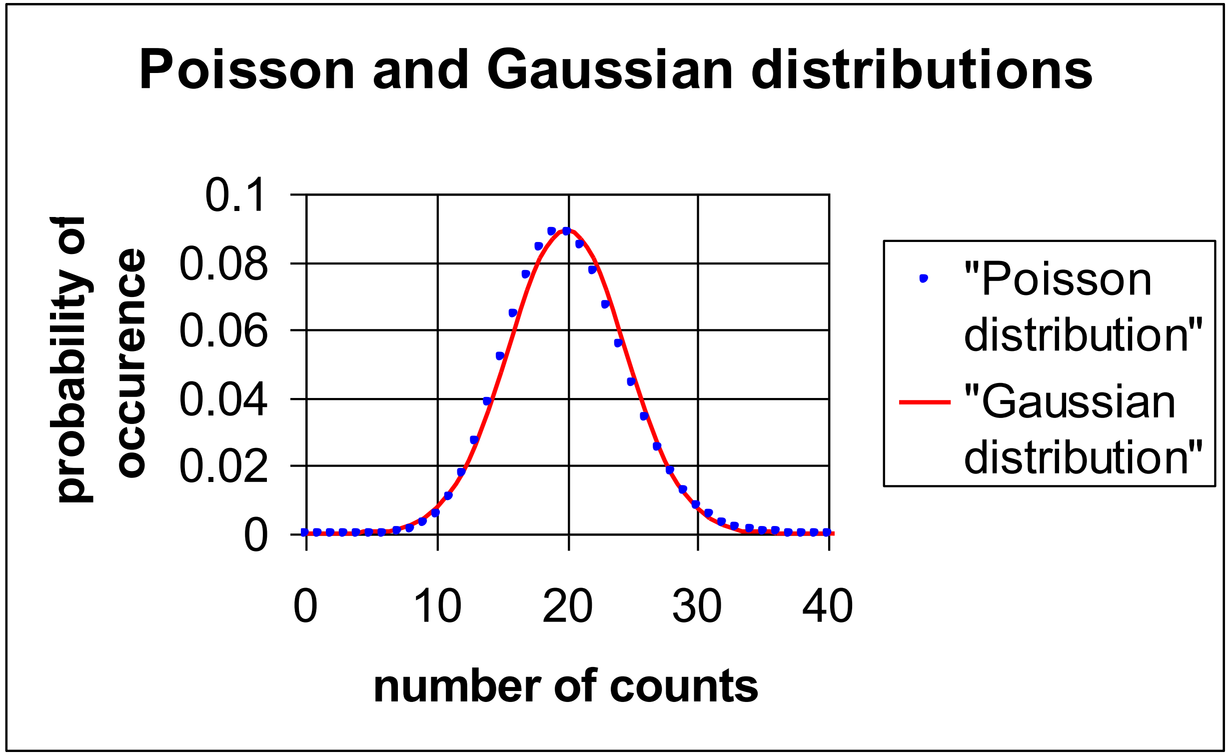
$$\sum_{n=0}^{\infty} P_n(t) = 1$$ Probabilities sum to 1

$$\langle n \rangle = \sum_{n=0}^{\infty} n\, P_n(t) = rt$$ Mean

$$\sigma^2 = \sum_{n=0}^{\infty} (n - \langle n \rangle)^2\, P_n(t) = rt$$ Variance

Poisson Distribution at large *rt*

$$P_n(t) = \frac{(r\,t)^n}{n!} e^{-rt}$$

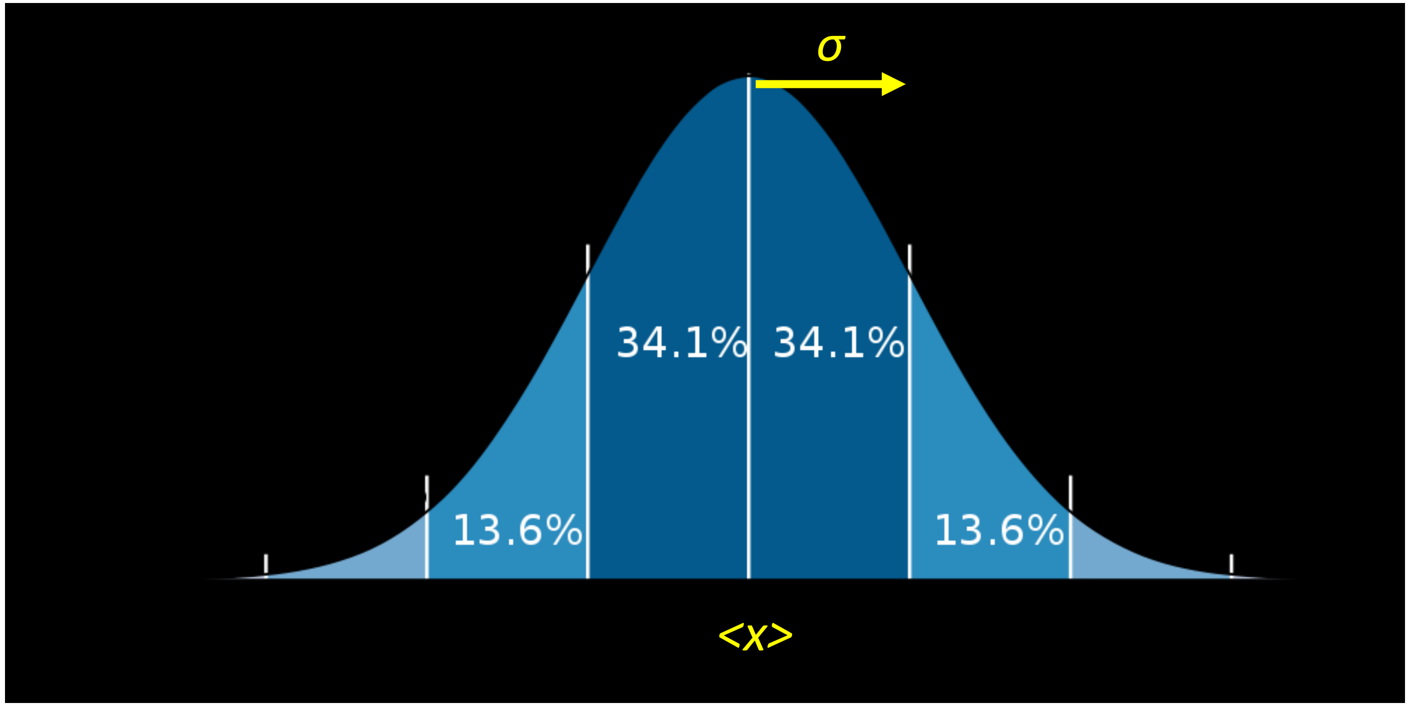


Carl Friedrich Gauss (1777–1855)

$$P_n(x) = \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{(x-\langle x\rangle)^2}{2\sigma^2}}$$

Gaussian (continuous) distribution

Gaussian (Normal) Distribution



Rules of thumb

- ±1σ: 68.3%
- ±2σ: 95.5%
- ±3σ: 99.7%

- ±4σ: 1 in 16k
- ±5σ: 1 in 1.75M
- ±6σ: 1 in 500M

$$P_n(x) = \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{(x-\langle x\rangle)^2}{2\sigma^2}}$$

If I measure *N* events, the "best" estimate of the mean is their arithmetic mean.

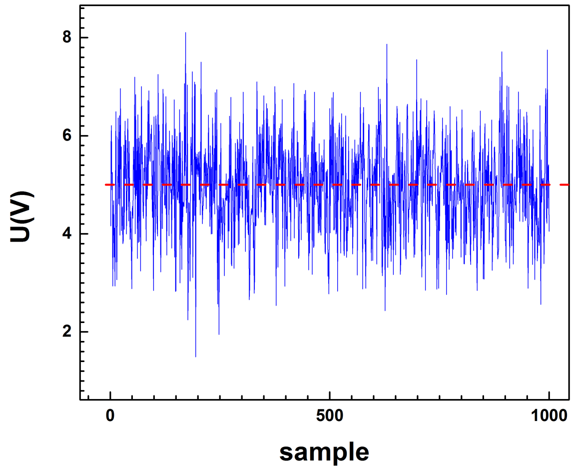
The error on this estimate is $\frac{\sigma}{\sqrt{N}}$

Measurements in the Presence of Noise

Source of noisy signal

Noise is often *(<u>not</u> always!)* coupling between your test setup and some thermal bath

Hence the use of cryogenic detectors!

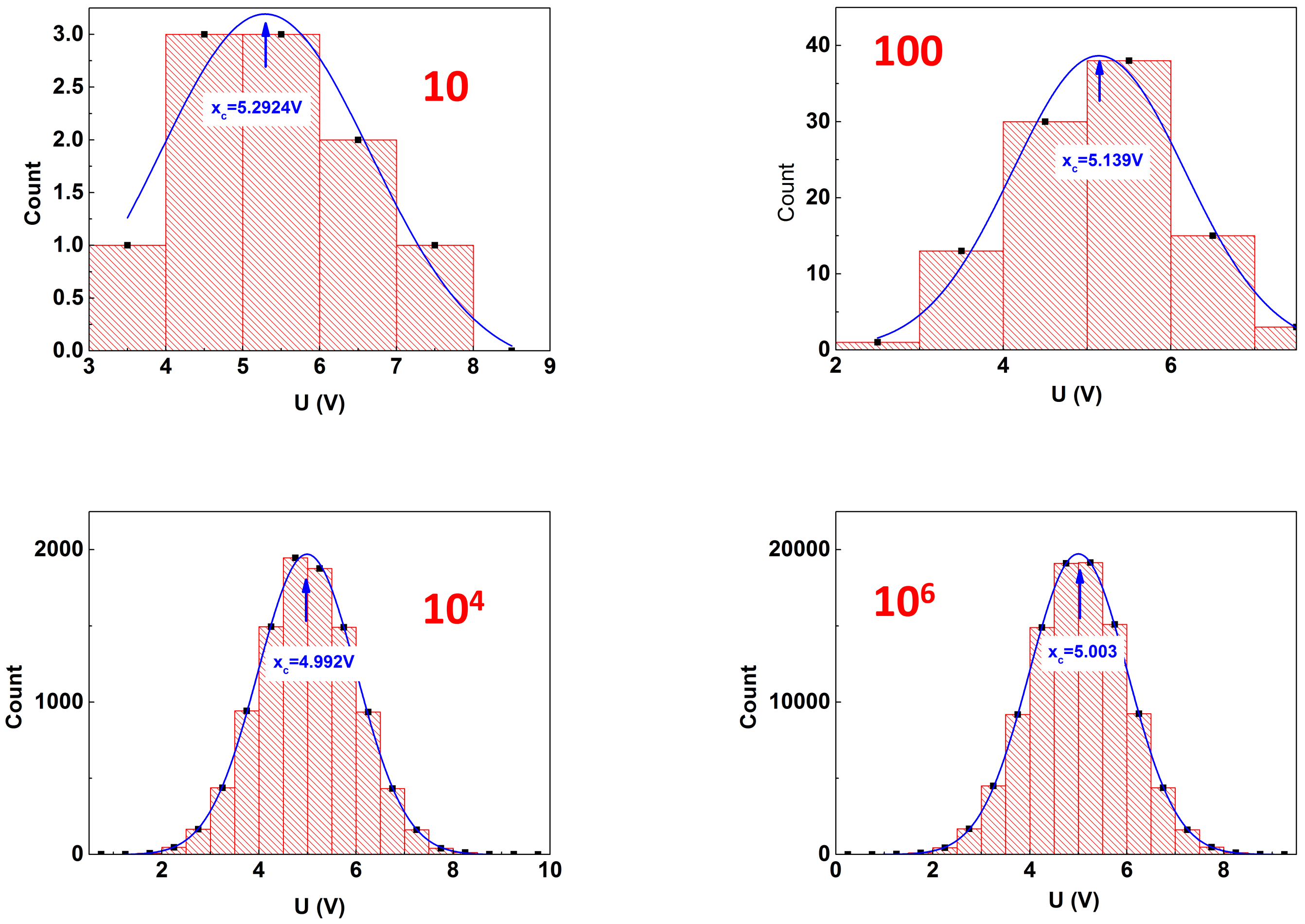


Expected value 5V

4.89855
5.25111
2.93382
4.31753
4.67903
3.52626
4.12001
2.93411

Actual measured values

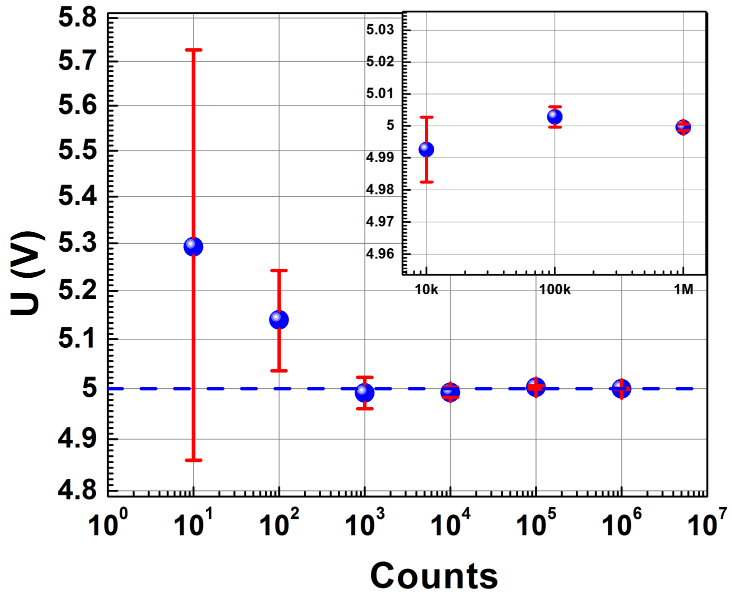
Measurements in the Presence of Noise

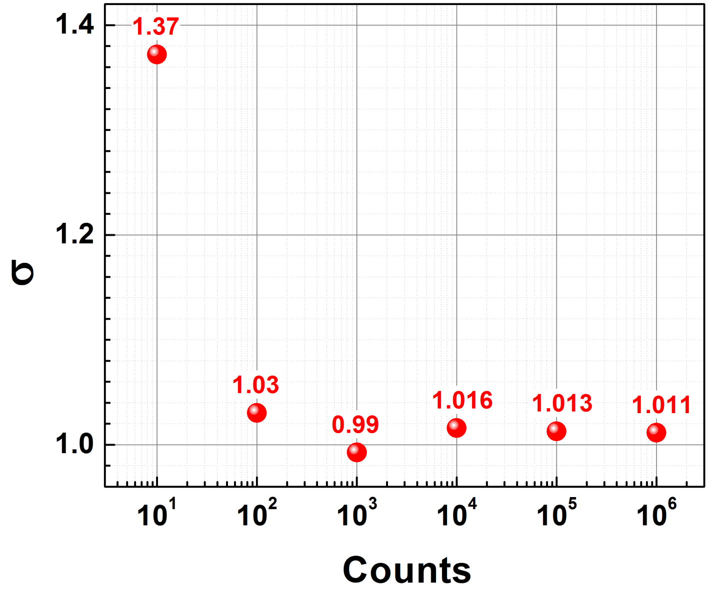


Note that the distribution isn't getting *narrower* at large *N*.

Instead, you'd have to be more and more unlucky to have enough fluctuations in one direction to move the arithmetic mean significantly

Measurements in the Presence of Noise





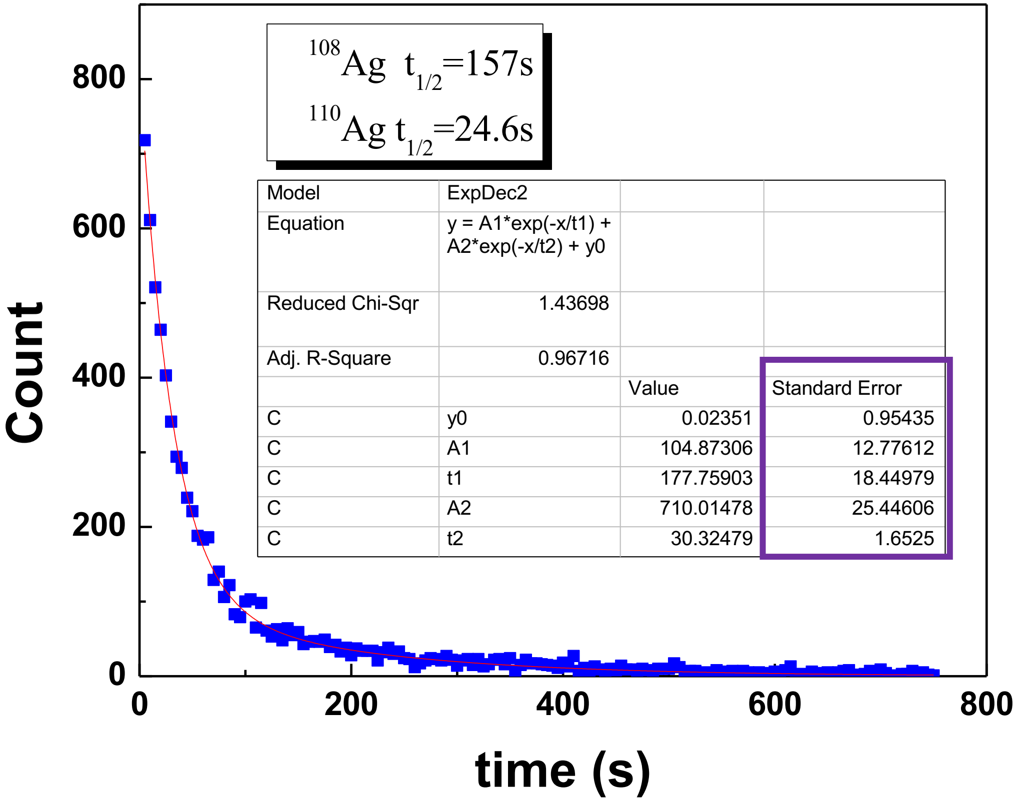
Result → $U = x_c \pm \frac{\sigma}{\sqrt{N}}$

σ - standard deviation
N – number of samples

For N=10^6 U=4.999±0.001 0.02% accuracy

Fitting Errors

Beta decay of neutron-activated silver foil



Model	ExpDec2		
Equation	y = A1*exp(-x/t1) + A2*exp(-x/t2) + y0		
Reduced Chi-Sqr	1.43698		
Adj. R-Square	0.96716		
		Value	Standard Error
C	y0	0.02351	0.95435
C	A1	104.87306	12.77612
C	t1	177.75903	18.44979
C	A2	710.01478	25.44606
C	t2	30.32479	1.6525

$$y = A_1 e^{-t/t_1} + A_2 e^{-t/t_2} + y_0$$

Fitting a parameterized model to data means finding the set of parameter values that minimizes the residuals – distances between data points and model curve.

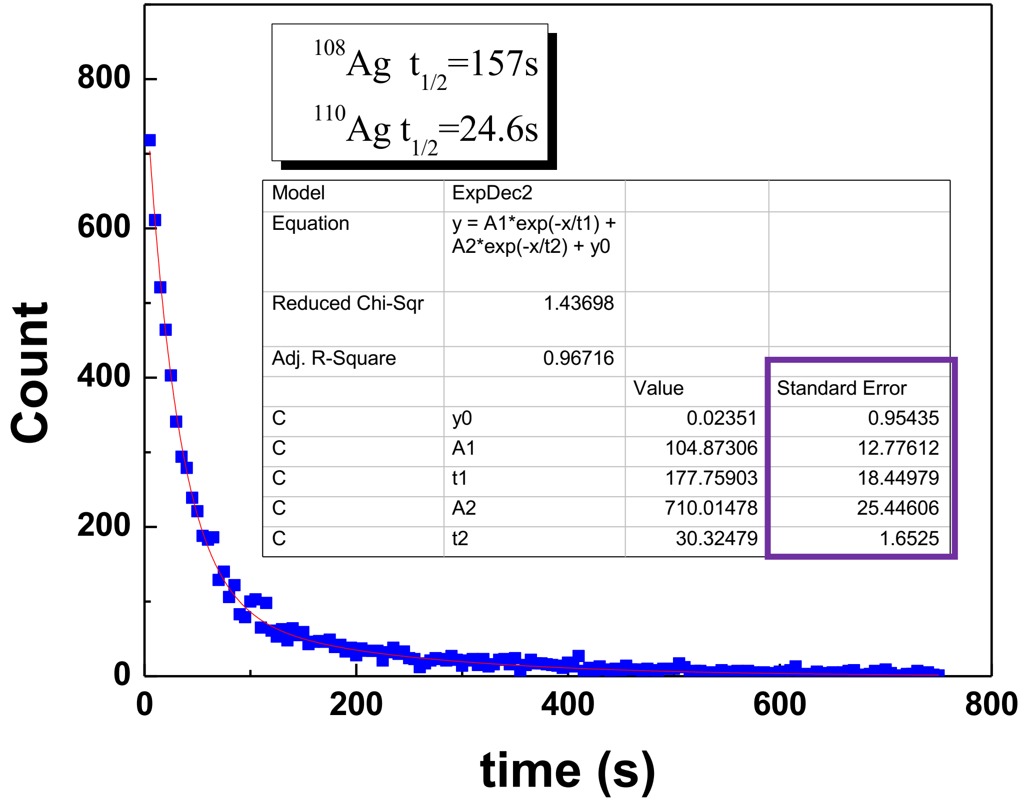
Further details in the appendix slides: minimize what exactly, and how?

Error estimates: how far can we wiggle the parameters before the residuals increase "significantly"?

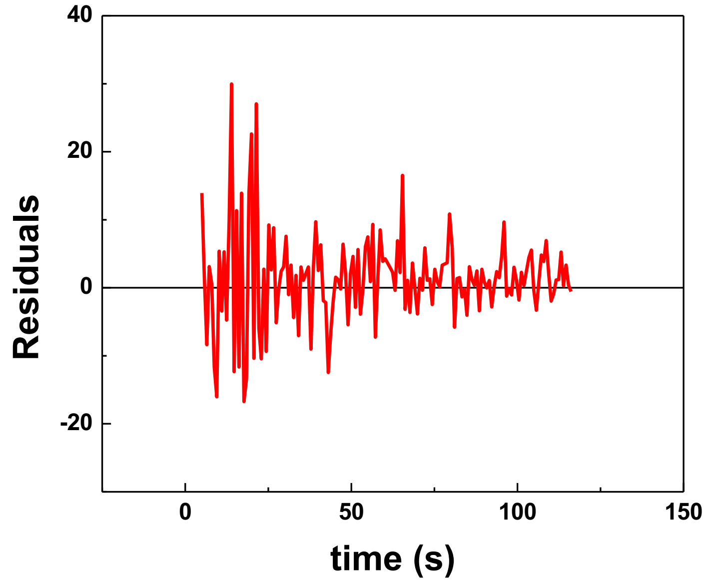
Error estimates assume model is correct and residuals are gaussian noise. Errors not trustworthy if these aren't roughly true!

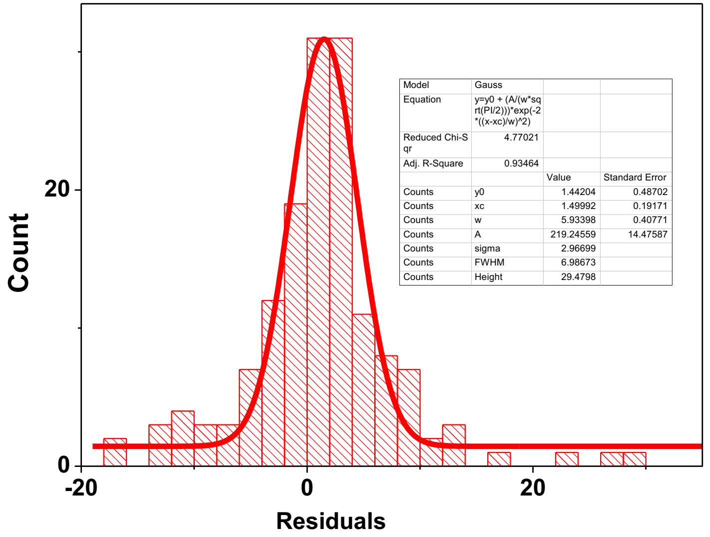
Fitting Errors

Beta decay of neutron-activated silver foil

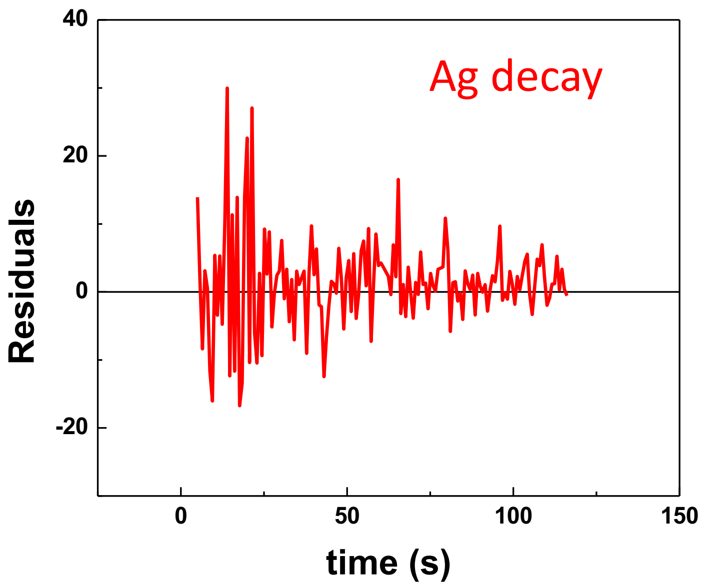


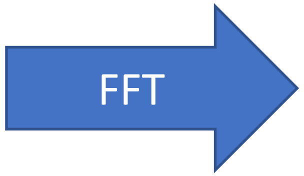
$$y = A_1 e^{-t/t_1} + A_2 e^{-t/t_2} + y_0$$

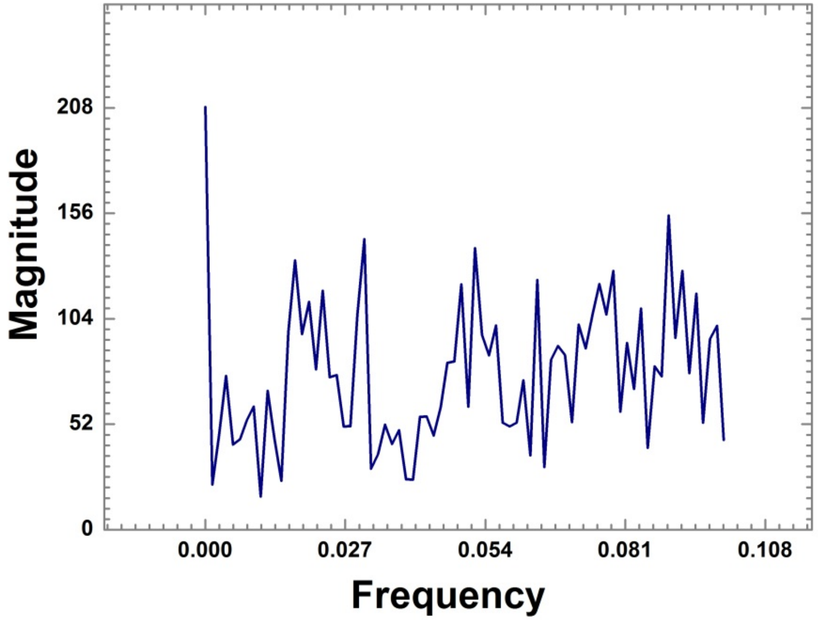




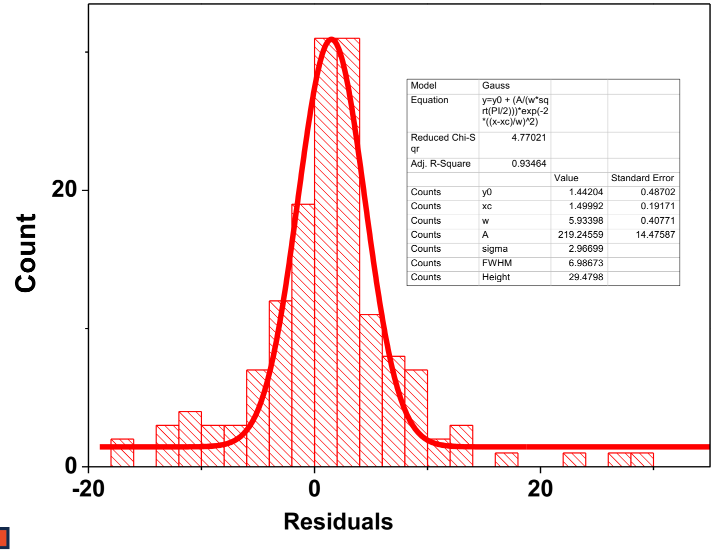
Fitting Errors: Examining the Residuals







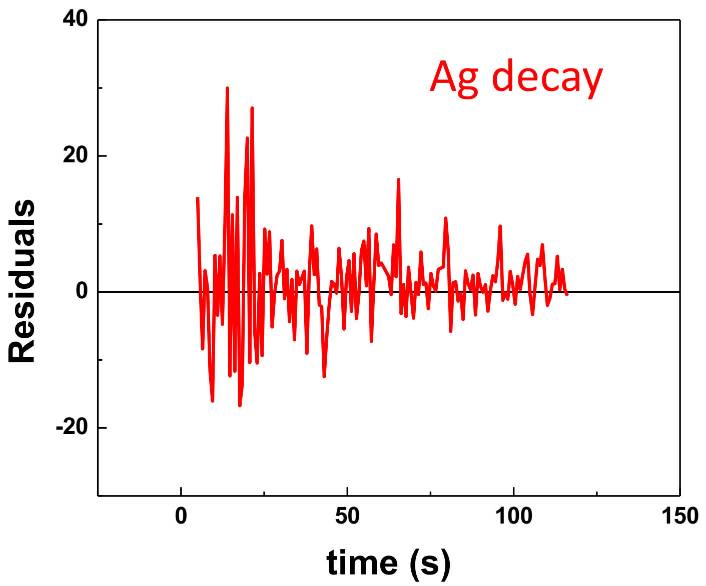
Test #2: No pronounced frequencies visible

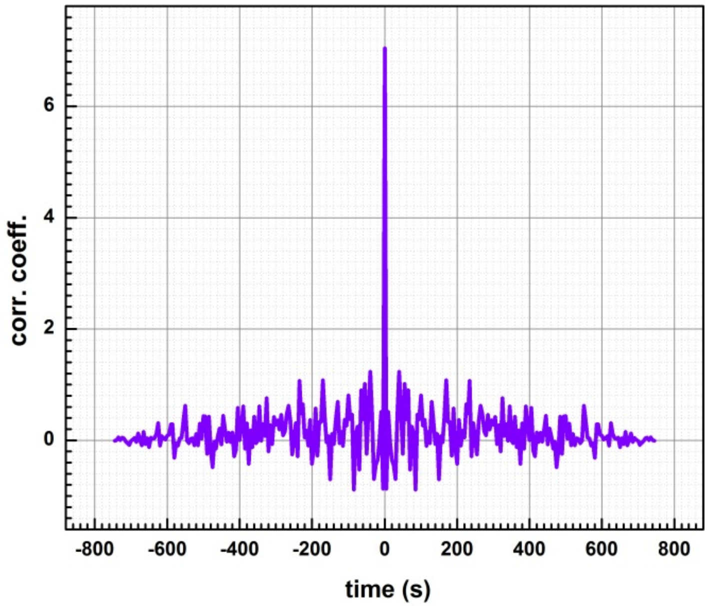


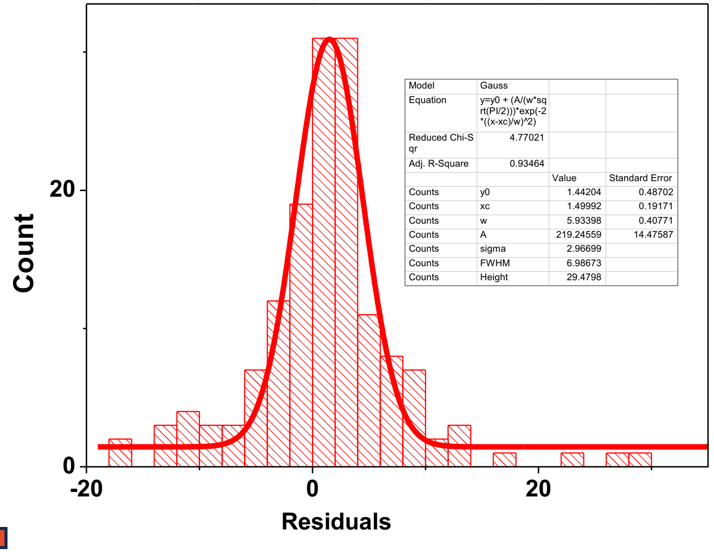
Model	Gauss		
Equation	y=y0 + (A/(w*sqrt(PI/2)))*exp(-2*((x-xc)/w)^2)		
Reduced Chi-Sqr	4.77021		
Adj. R-Square	0.93464		
		Value	Standard Error
Counts	y0	1.44204	0.48702
Counts	xc	1.49992	0.19171
Counts	w	5.93398	0.40771
Counts	A	219.24559	14.47587
Counts	sigma	2.96699	
Counts	FWHM	6.98673	
Counts	Height	29.4798	

Test #1: generally Gaussian/symmetric residuals

Fitting Errors: Examining the Residuals







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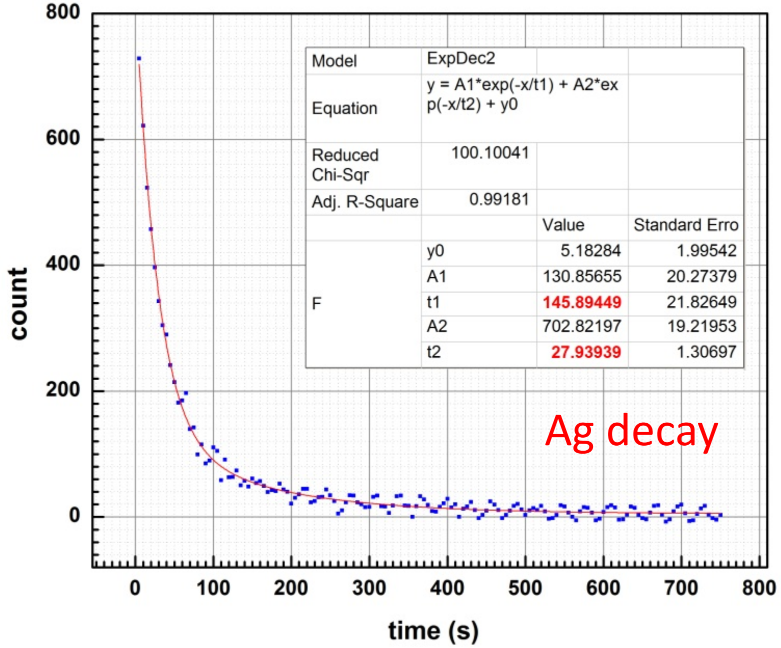
Counts	A	219.24559	14.47587
Counts	sigma	2.96699	
Counts	FWHM	6.98673	
Counts	Height	29.4798	

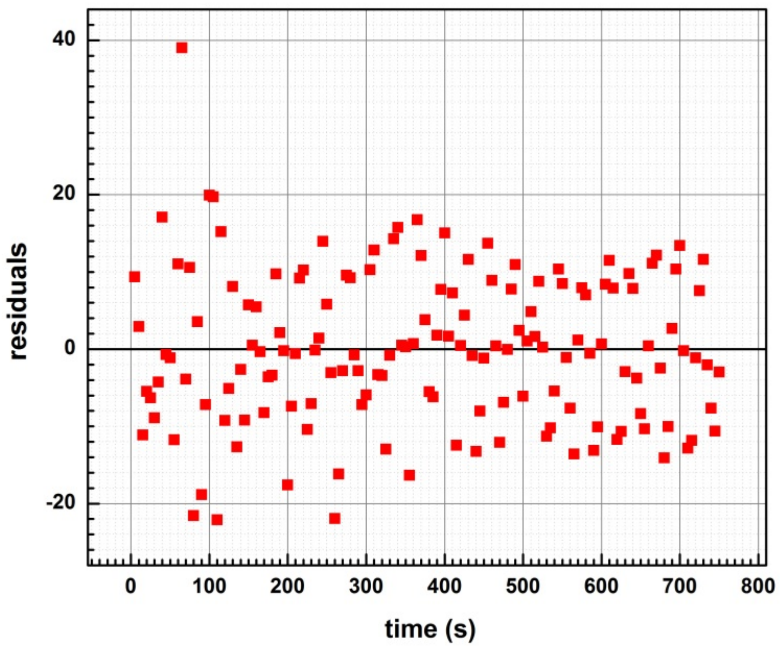
Test #3: Autocorrelation shows no prominent correlation time scales

Correlation
$$y_{fg}(m) = \sum_{n=0}^{N-1} f(n)g(n-m)$$

Auto-correlation
$$y_{ff}(m) = \sum_{n=0}^{N-1} f(n)f(n-m)$$

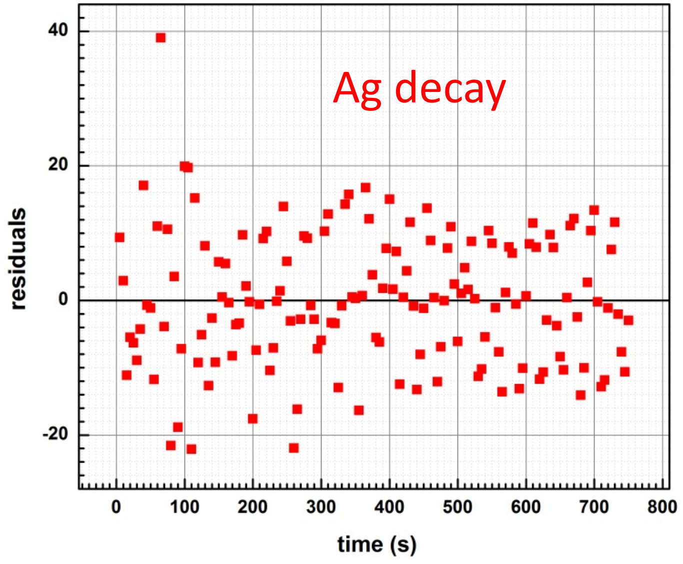
Examining Non-Ideal Fit Residuals

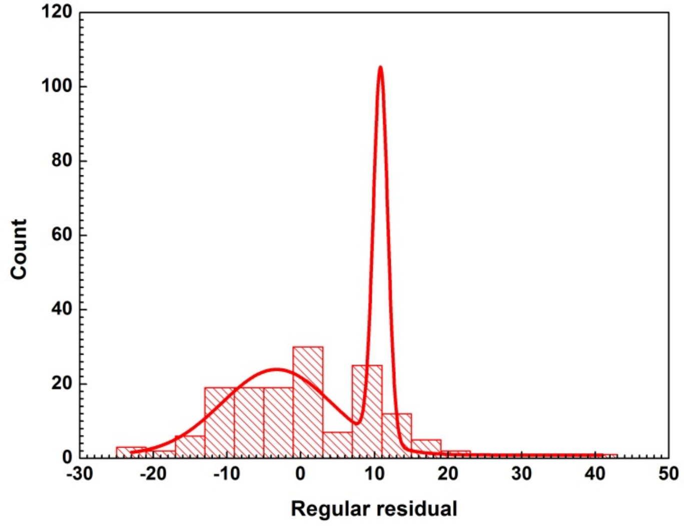


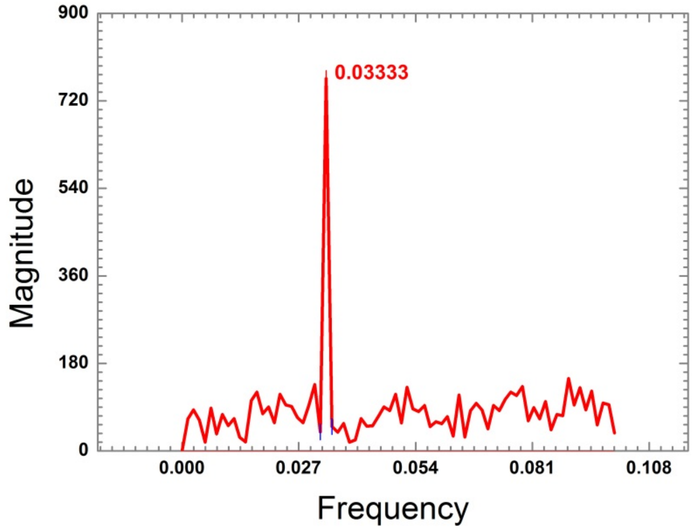


	Clean experiment	Data + "noise"
t_1(s)	177.76	145.89
t_2(s)	30.32	27.94

Examining Non-Ideal Fit Residuals

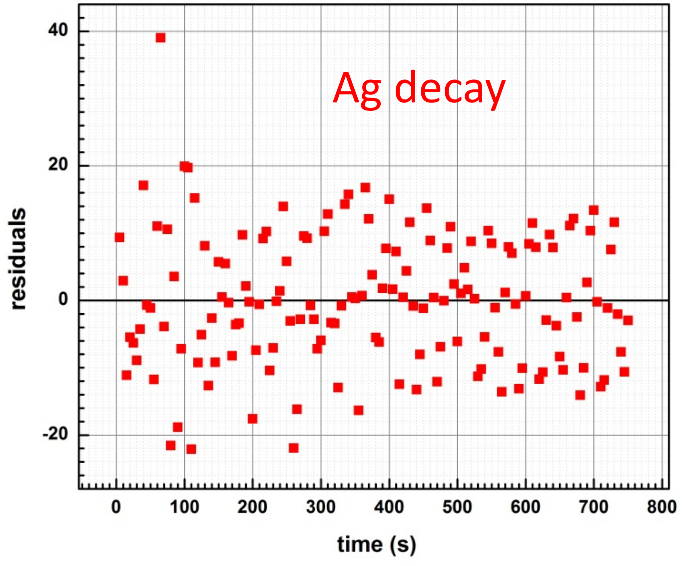


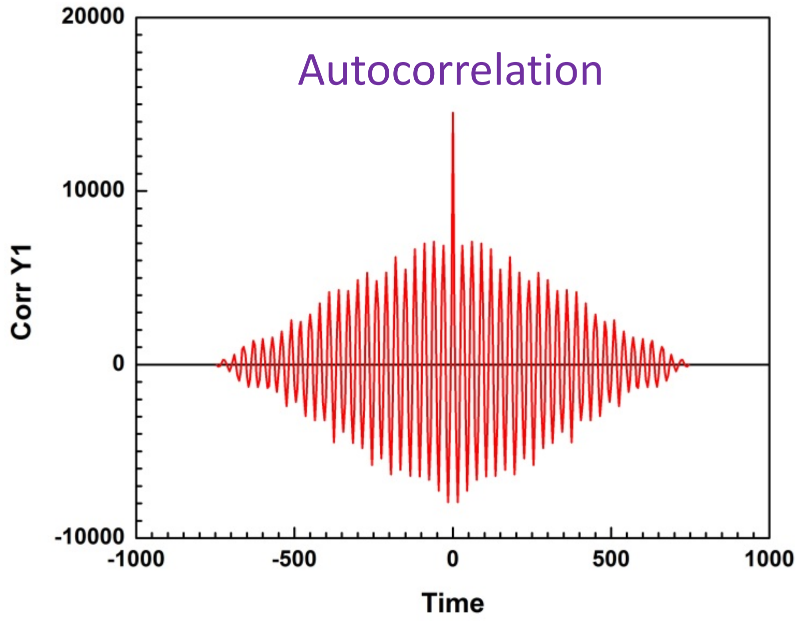




1. Histogram does *not* resemble a normal distribution
2. Prominent component at frequency 0.03333

Examining Non-Ideal Fit Residuals



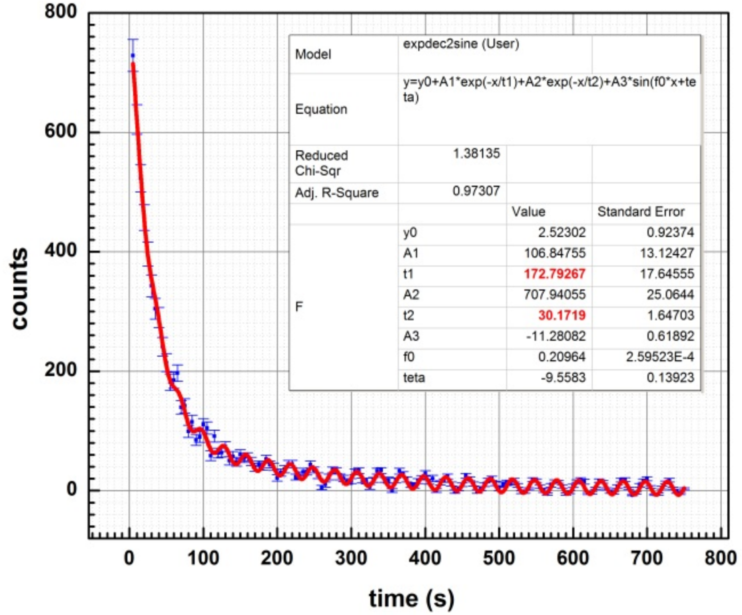


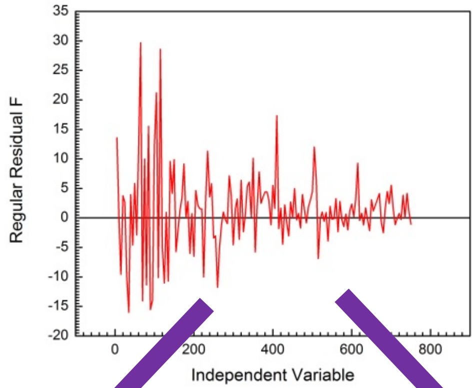
Conclusion: Fitting function needs to be modified to incorporate an additional term

$$y(t) = y_0 + A_0 e^{-t/t_1} + A_1 e^{-t/t_2} + A_3 \sin(\omega t + \theta)$$

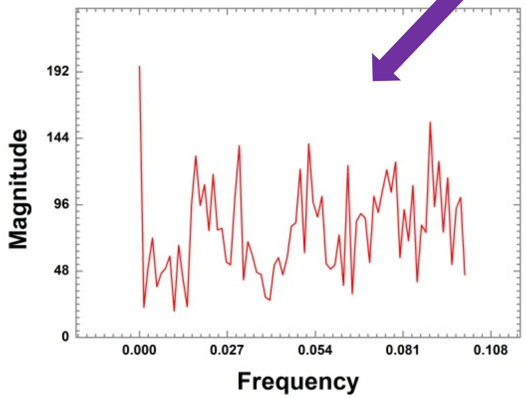
Caution: "With four parameters I can fit an elephant, and with five I can make him wiggle his trunk"
John von Neumann, attributed by Enrico Fermi

Examining Non-Ideal Fit Residuals

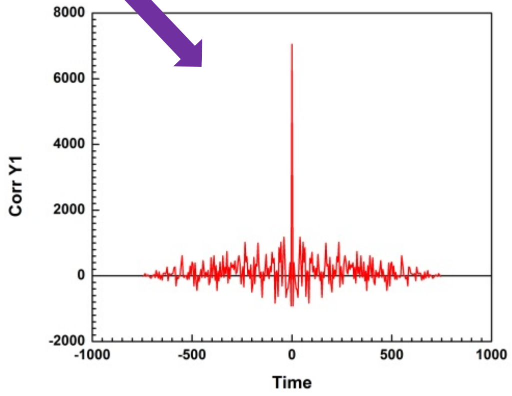




FFT



Autocorrelation



	Clear experiment	Data + noise	Modified fitting
t_1(s)	177.76	145.89	172.79
t_2(s)	30.32	27.94	30.17

Error Analysis: Millikan Oil Drop Experiment

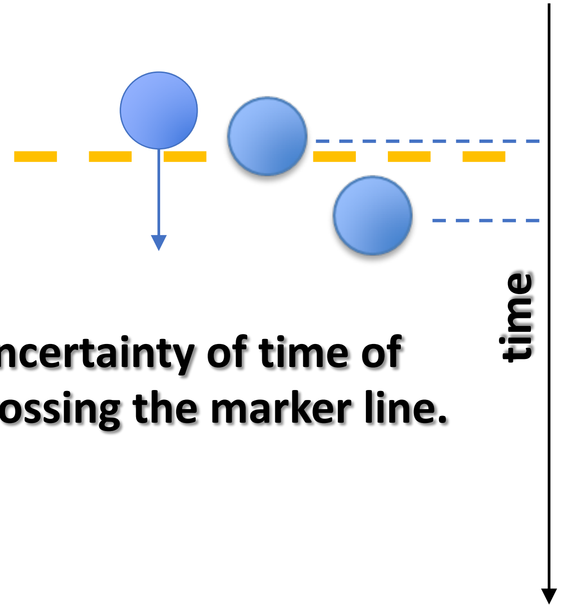
We expect both error components in each measurement

$$Q_{meas} = Q_{true} + e_s + e_r$$

… and there are various sources for each

$\boldsymbol{e_s}$ Systematic error could include uncertainties in plate separation (d), applied DC voltage (V), ambient temperature (T), etc.

$\boldsymbol{e_r}$ Random errors include uncertainties in measurement of t_{rise}, t_g, etc.



Uncertainty of time of crossing the marker line.

Systematic Error Analysis: Millikan Oil Drop Experiment

$$Q_{meas} = Q_{true} + e_s + e_r$$

$$Q = ne = F\,S\,T = \frac{1}{f_c^{2/3}}\;\frac{9\pi d}{V}\sqrt{\frac{2\eta^3 x^3}{g\rho}}\;\sqrt{\frac{1}{t_g}}\left[\frac{1}{t_g} + \frac{1}{t_{rise}}\right]$$

$$\frac{1}{f_c^{2/3}} \approx 1 - \left(\frac{t_g}{\tau_g}\right)^{\frac{1}{2}}$$

$$\Delta Q = \sqrt{\left(\frac{dQ}{dF}\right)^2 (\Delta F)^2 + \left(\frac{dQ}{dS}\right)^2 (\Delta S)^2 + \left(\frac{dQ}{dT}\right)^2 (\Delta T)^2} \approx \sqrt{\left(\frac{dQ}{dS}\right)^2 (\Delta S)^2 + \left(\frac{dQ}{dT}\right)^2 (\Delta T)^2}$$

Generally negligible

$$= \sqrt{(FT)^2 (\Delta S)^2 + (FS)^2 (\Delta T)^2} = Q\sqrt{\left(\frac{\Delta S}{S}\right)^2 + \left(\frac{\Delta T}{T}\right)^2}$$

Systematic Error Analysis: Millikan Oil Drop Experiment

$$Q_{meas} = Q_{true} + \boldsymbol{e_s} + e_r$$

$$\boldsymbol{Q = ne = F\,S\,T = \frac{1}{f_c^{2/3}}\,\frac{9\pi d}{V}\sqrt{\frac{2\eta^3 x^3}{g\rho}}\,\sqrt{\frac{1}{t_g}\left[\frac{1}{t_g}+\frac{1}{t_{rise}}\right]}}$$

$$\Delta Q \approx Q\sqrt{\left(\frac{\Delta S}{S}\right)^2+\left(\frac{\Delta T}{T}\right)^2}$$

$$\frac{\Delta S}{S} = \sqrt{\left(\frac{\Delta d}{d}\right)^2+\left(\frac{\Delta V}{V}\right)^2+\left(\frac{3}{2}\frac{\Delta x}{x}\right)^2+\left(\frac{3}{2}\frac{\Delta\eta}{\eta}\right)^2+\left(\frac{1}{2}\frac{\Delta\rho}{\rho}\right)^2+\left(\frac{1}{2}\frac{\Delta g}{g}\right)^2} \approx \sqrt{\left(\frac{\Delta d}{d}\right)^2+\left(\frac{3}{2}\frac{\Delta x}{x}\right)^2}$$

$$\Delta T = \sqrt{\left(\frac{3/2}{t_g^{5/2}}+\frac{1/2}{t_g^{3/2}}\frac{1}{t_{rise}}\right)^2\Delta t_g^{\,2}+\left(\frac{1}{t_g^{1/2}}\frac{1}{t_{rise}^{\,2}}\right)^2\Delta t_{rise}^{\,2}}$$

Appendix #1: Analyzing the Charge Data

Step 1. Origin Project For Raw Data :

\\engr-file-03\PHYINST\APL Courses\PHYCS401\Students\2. Millikan Raw Data

Data Analysis for Millikan Oil Drop Experi...	2/22/2008 9:36 AM	Adobe Acrobat D...	59 KB
Millikan_raw data.opj	10/5/2017 4:50 PM	OPJ File	15 KB
Millikan_raw data1.opj	9/25/2018 1:38 PM	OPJ File	14 KB
Millikan1_calc.opj	9/26/2018 1:36 PM	OPJ File	95 KB
Millikan1_no_calc.opj	9/25/2017 2:03 PM	OPJ File	66 KB
T measurement.opj	9/26/2018 1:36 PM	OPJ File	316 KB

All project files with raw data should be stored in:
\\engr-file-03\PHYINST\APL Courses\PHYCS401\Students\2. Millikan Raw Data

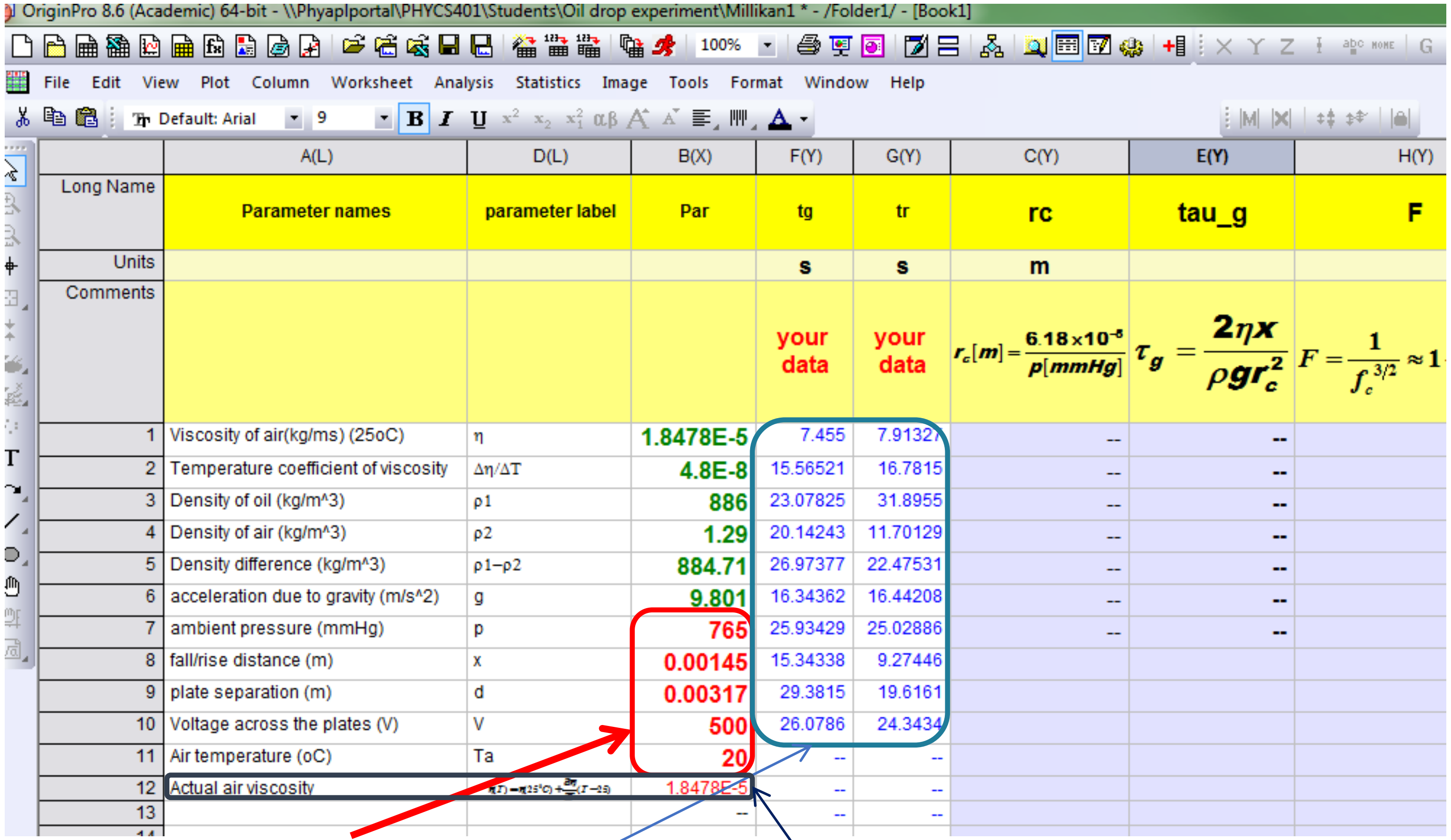
Only files with raw data should be stored in this common folder. Please remove files that are unrelated to the experimental results!

All other files, including those used for calculations, should be saved in your personal folder

Appendix #1: Analyzing the Charge Data

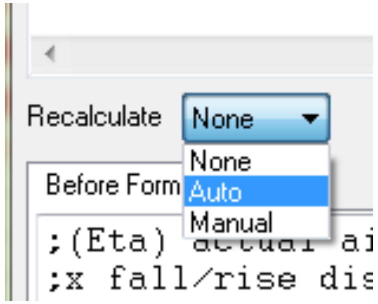
Step 2. Working on your personal Origin project

Make a copy of the Millikan1 project to your personal folder and open it



	A(L)	D(L)	B(X)	F(Y)	G(Y)	C(Y)	E(Y)	H(Y)
Long Name	Parameter names	parameter label	Par	tg	tr	rc	tau_g	F
Units				s	s	m		
Comments				your data	your data	$r_c[m]=\frac{6.18\times10^{-8}}{p[mmHg]}$	$\tau_g=\frac{2\eta x}{\rho g r_c^2}$	$F=\frac{1}{f_c^{3/2}}\approx 1$
1	Viscosity of air(kg/ms) (25oC)	η	1.8478E-5	7.455	7.91327	--	--	
2	Temperature coefficient of viscosity	Δη/ΔT	4.8E-8	15.56521	16.7815	--	--	
3	Density of oil (kg/m^3)	ρ1	886	23.07825	31.8955	--	--	
4	Density of air (kg/m^3)	ρ2	1.29	20.14243	11.70129	--	--	
5	Density difference (kg/m^3)	ρ1−ρ2	884.71	26.97377	22.47531	--	--	
6	acceleration due to gravity (m/s^2)	g	9.801	16.34362	16.44208	--	--	
7	ambient pressure (mmHg)	p	765	25.93429	25.02886	--	--	
8	fall/rise distance (m)	x	0.00145	15.34338	9.27446			
9	plate separation (m)	d	0.00317	29.3815	19.6161			
10	Voltage across the plates (V)	V	500	26.0786	24.3434			
11	Air temperature (oC)	Ta	20	--	--			
12	Actual air viscosity	$\eta(T)=\eta(25^{\circ}C)+\frac{\Delta\eta}{\Delta T}(T-25)$	1.8478E-5	--	--			
13			--	--	--			

Prepare equations calculations of data in next columns (*Set column values…*). Switch *Recalculate* in *Auto* mode



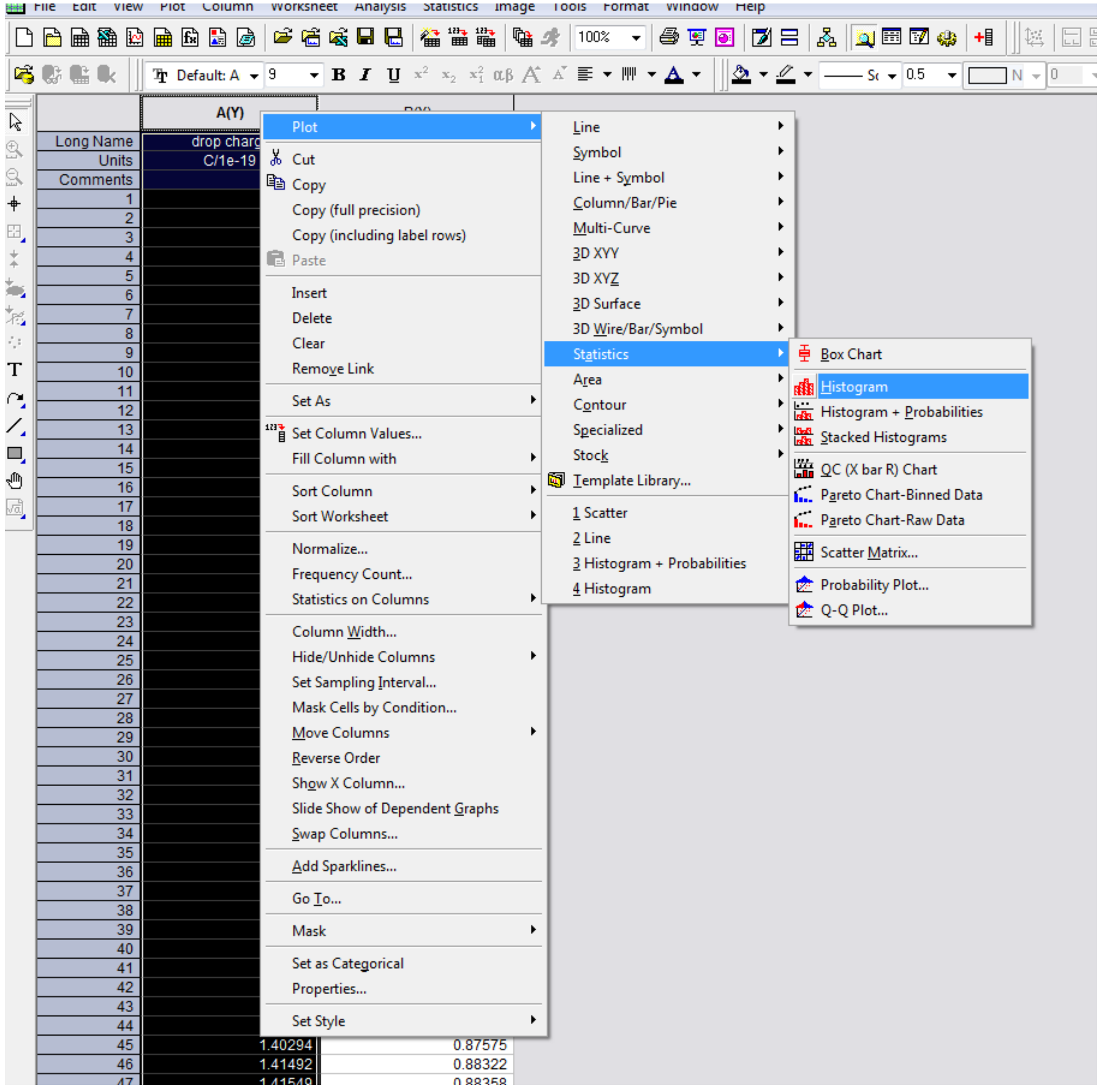
Paste these 5 parameters and raw data from Section L1-L4.opj projects

Manually calculate the actual air viscosity

Appendix #1: Analyzing the Charge Data

Step 3. Make a histogram

Plot a histogram from the data in the column of drop charges



Appendix #1: Analyzing the Charge Data

Step 4. Histogram – set bin size

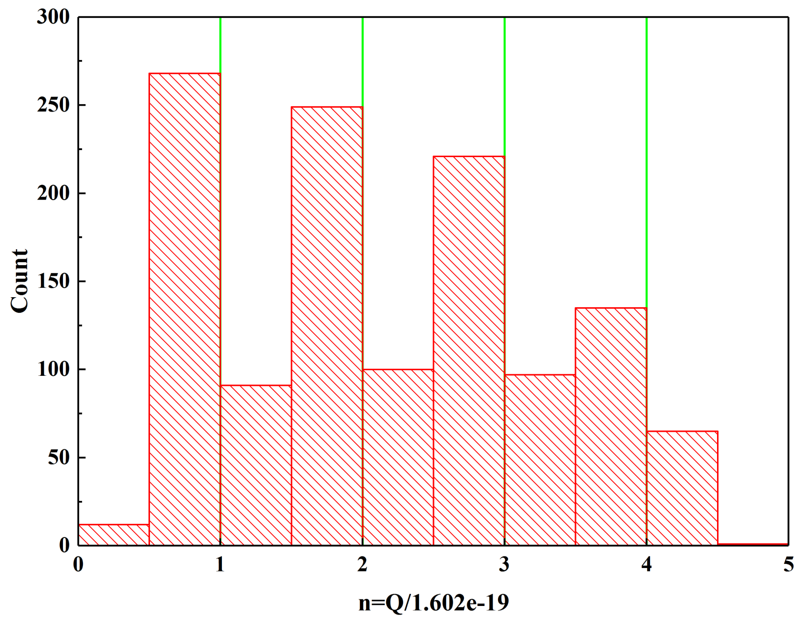
Origin will choose the bin size h *automatically*, but not *optimally*. In the plot at right, *h*=0.5.

We want bins wide enough to control bin-to-bin fluctuations that obscure the pattern, but narrow enough not to wash out the pattern.

There are several theoretical approaches to finding the optimal bin size. One example:

$$h = \frac{3.5\sigma}{n^{1/3}}$$

… where σ is the sample standard deviation, n is the total number of observations.
For the data at right, this yields *h*~0.1

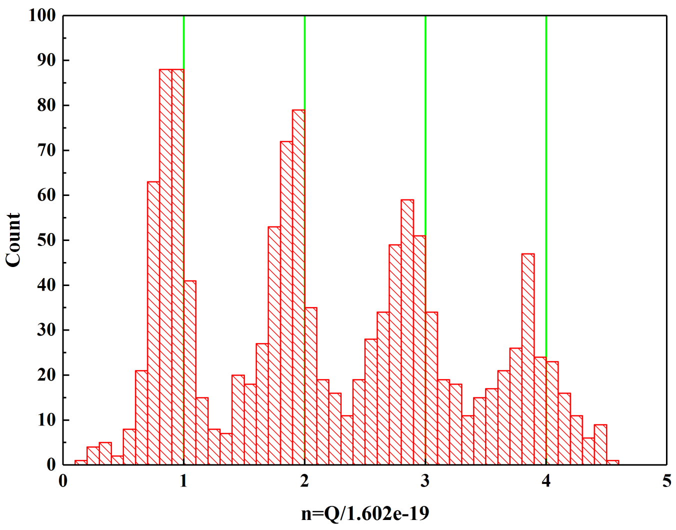


Bin size above is 0.5
Peaks vaguely visible, but can't determine width

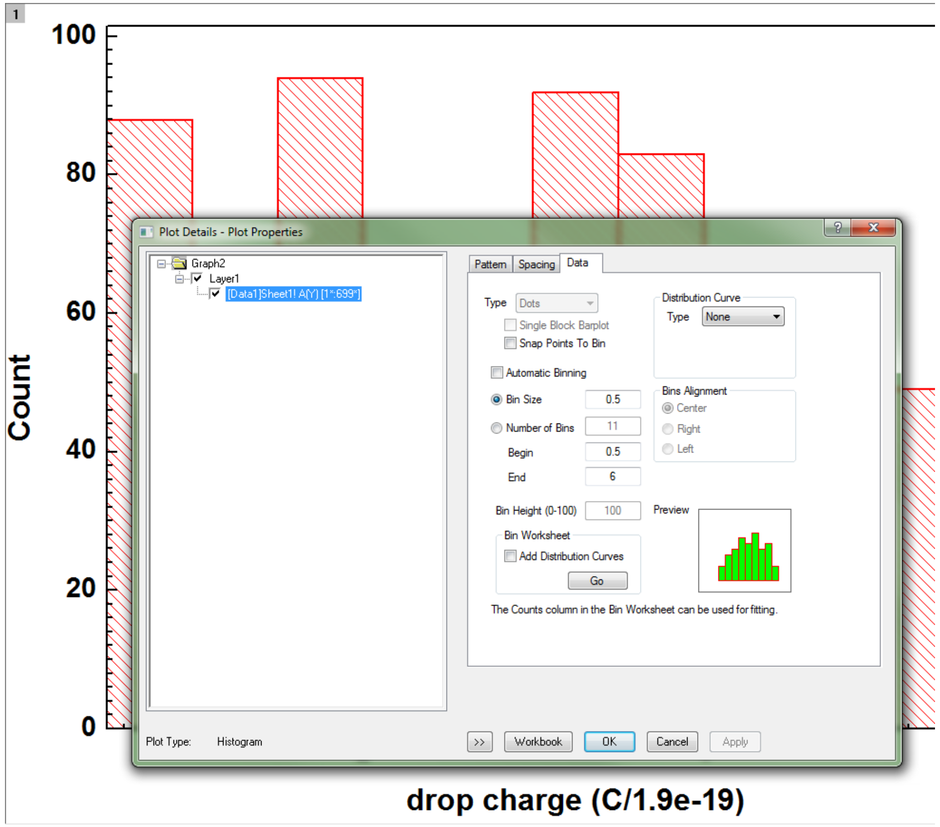
Appendix #1: Analyzing the Charge Data

Step 4. Histogram – set bin size

To change the bin size, click on the graph the uncheck the “*Automatic Binning*” option



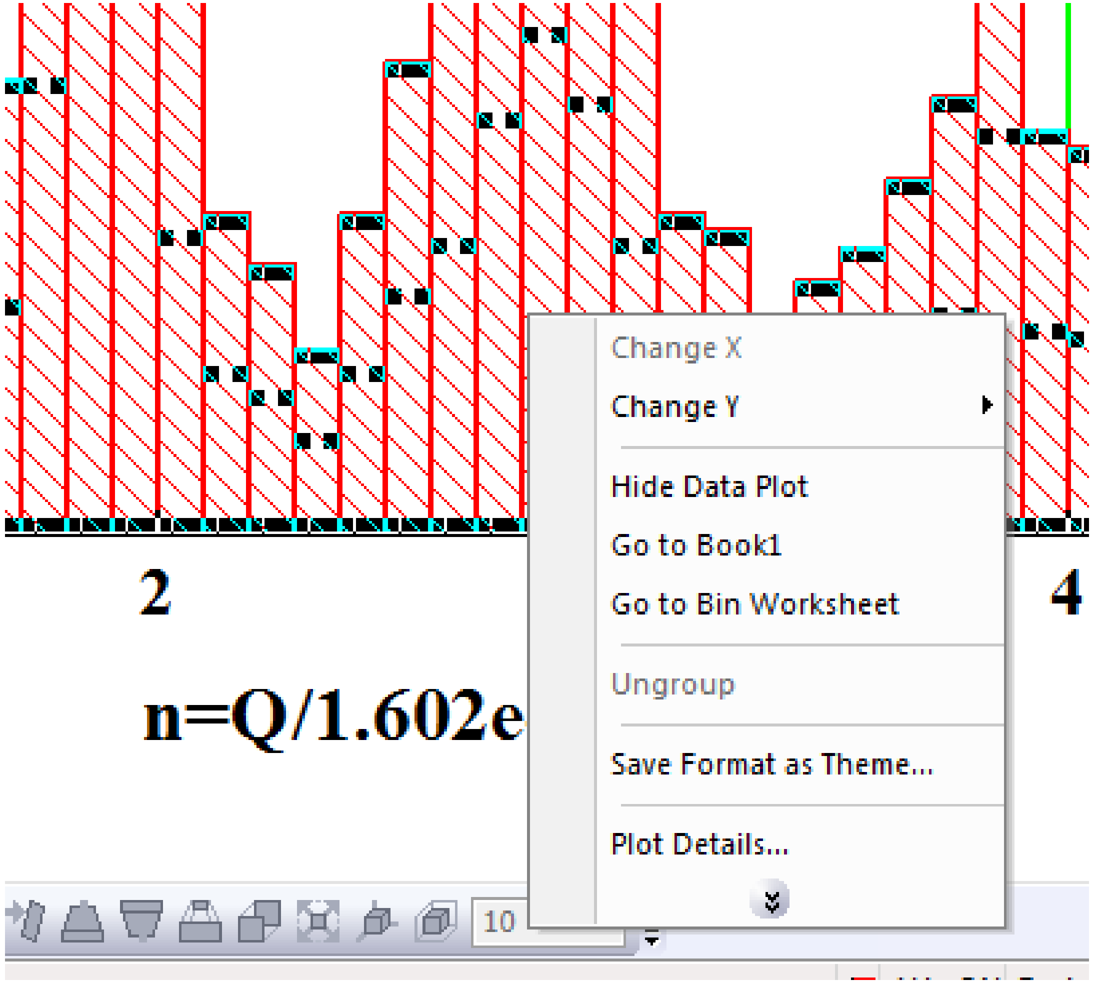
Bin size is now 0.1, pattern is visible!



Appendix #1: Analyzing the Charge Data

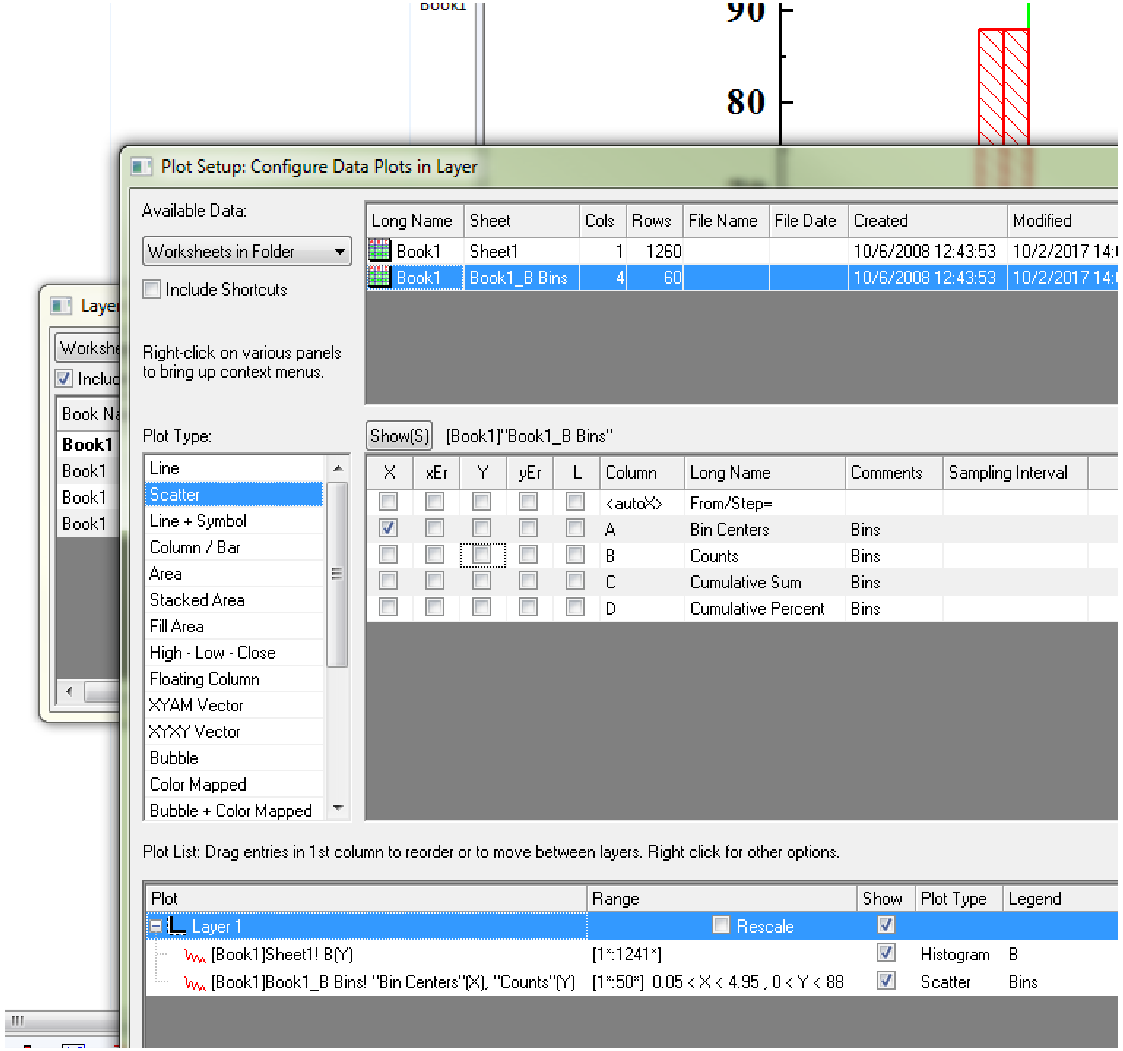
Step 4. Histogram - find the bin worksheet

Right-clock on the histogram and choose "*Go to bin worksheet*"



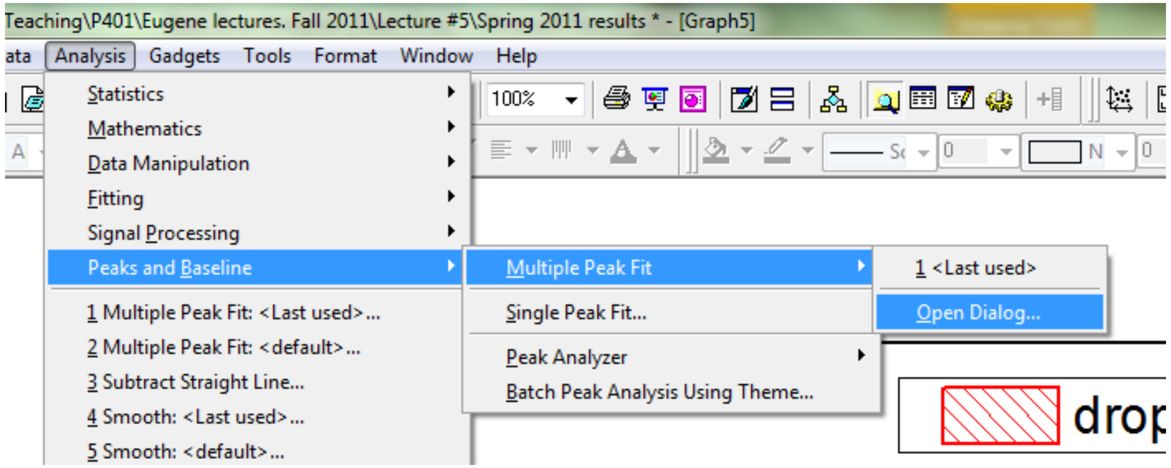
Appendix #1: Analyzing the Charge Data

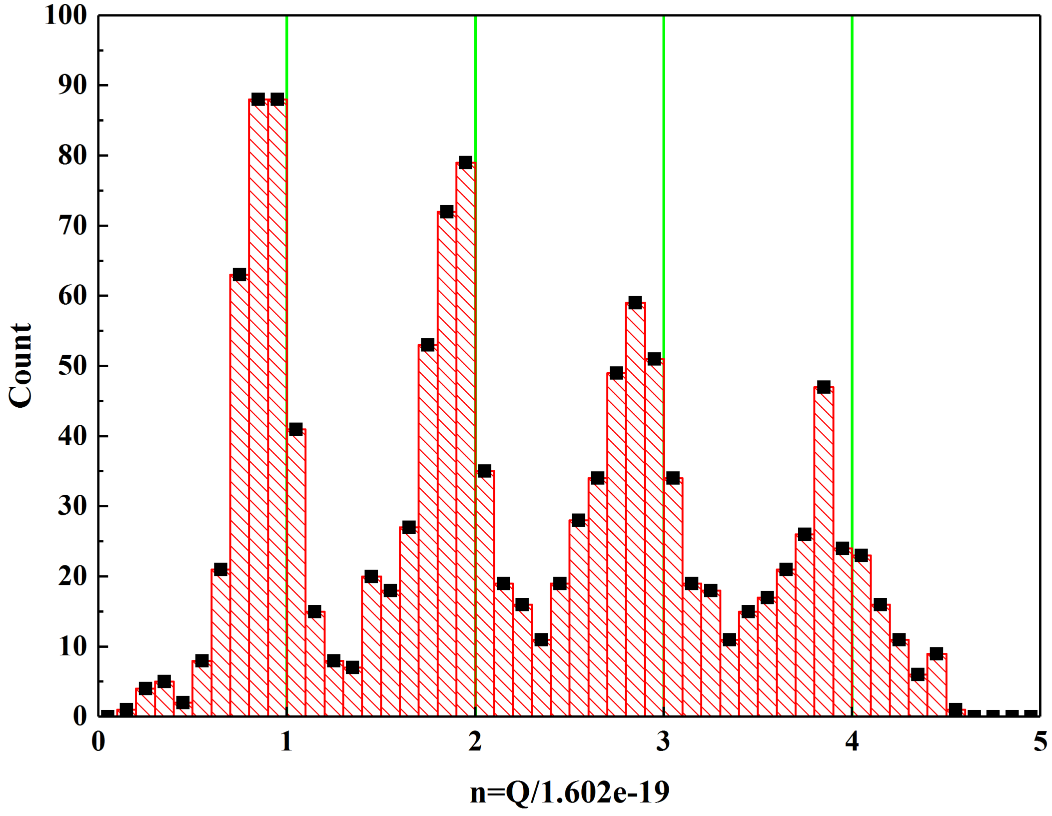
Step 5. Add counts-vs-bin plot



Appendix #1: Analyzing the Charge Data

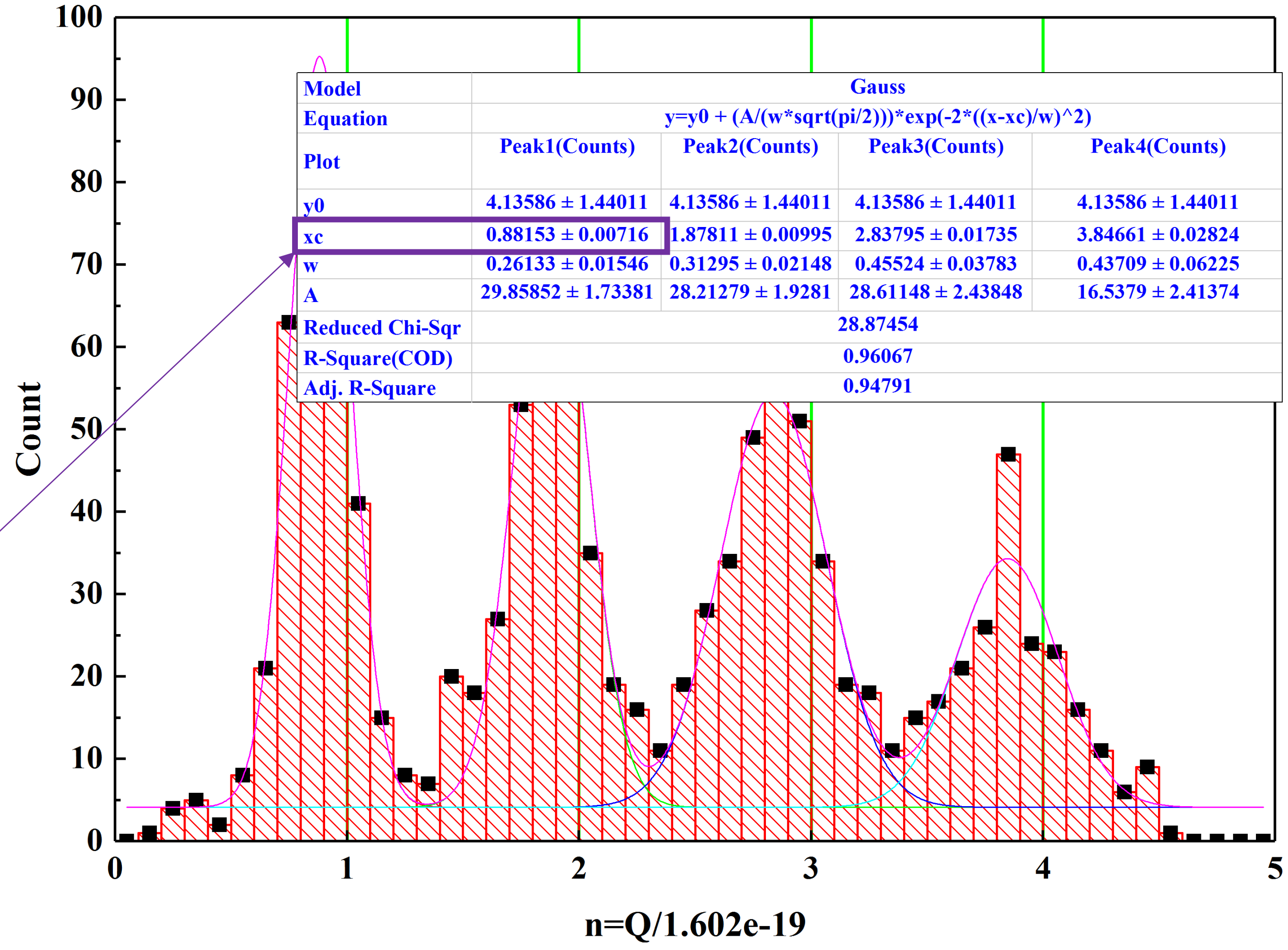
Step 5. Multi-peak gaussian fit





Appendix #1: Analyzing the Charge Data

Step 5. Multi-peak gaussian fit

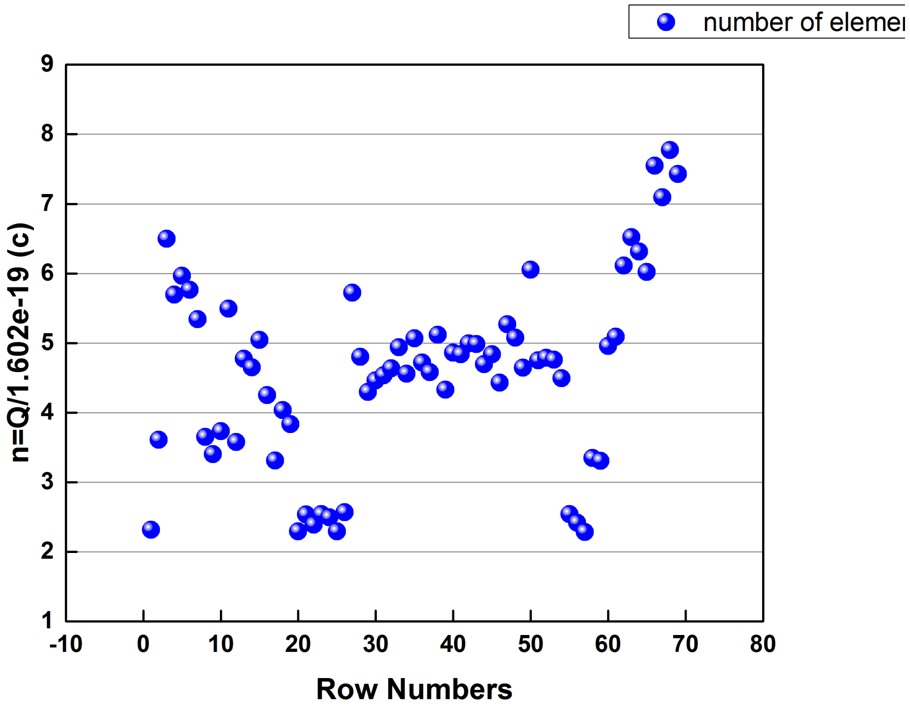


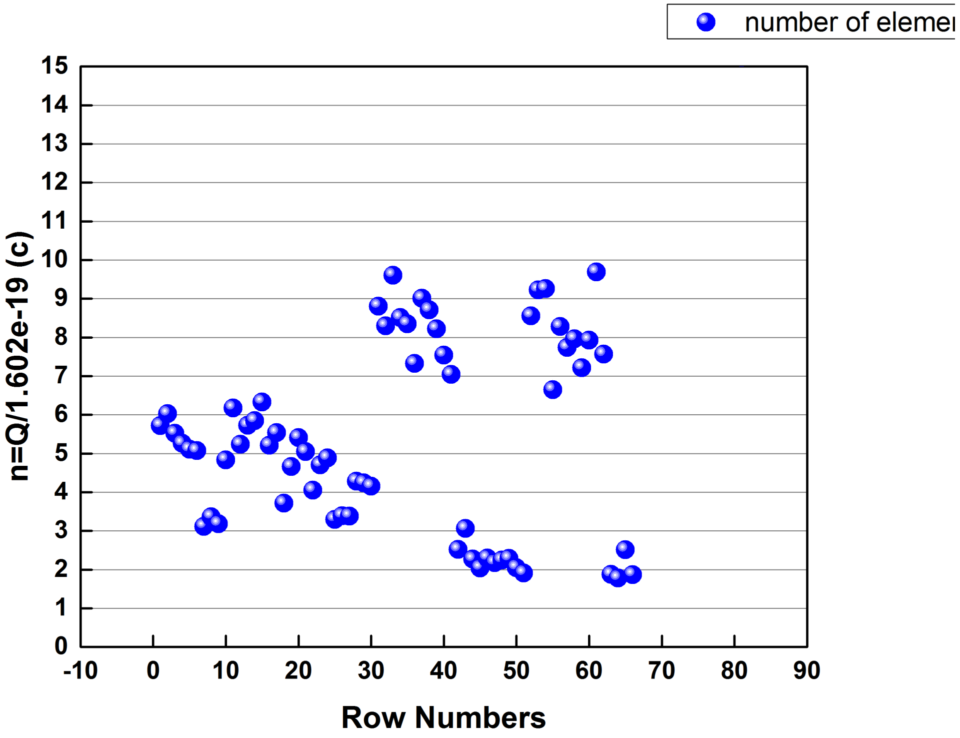
Model	Gauss			
Equation	y=y0 + (A/(w*sqrt(pi/2)))*exp(-2*((x-xc)/w)^2)			
Plot	Peak1(Counts)	Peak2(Counts)	Peak3(Counts)	Peak4(Counts)
y0	4.13586 ± 1.44011	4.13586 ± 1.44011	4.13586 ± 1.44011	4.13586 ± 1.44011
xc	0.88153 ± 0.00716	1.87811 ± 0.00995	2.83795 ± 0.01735	3.84661 ± 0.02824
w	0.26133 ± 0.01546	0.31295 ± 0.02148	0.45524 ± 0.03783	0.43709 ± 0.06225
A	29.85852 ± 1.73381	28.21279 ± 1.9281	28.61148 ± 2.43848	16.5379 ± 2.41374
Reduced Chi-Sqr	28.87454			
R-Square(COD)	0.96067			
Adj. R-Square	0.94791			

First peak: mean 0.882 ± 0.007

Appendix #1: Issues with Oil Drop Data

Be careful with data selections obtained by different teams!





For more details how to create the histogram plot and do the analysis, see "*Working with Histogram Graph. Millikan Oil Drop Experiment*" *(E.V. Colla)*

Appendix #1: Increasing Experimental Accuracy

$$Q_{meas} = Q_{true} + e_s + e_r$$

$\boldsymbol{e_s}$

Systematic error could be improved with more precise knowledge of the experimental parameters: plate separation (*d*), applied DC voltage (*V*), ambient temperature (*T*), etc.
Usually these are limited by existing measurement equipment

$\boldsymbol{e_r}$

Random or statistical errors can be reduced only by increasing the number of data points (*no limit!*)

Appendix #2: Fitting Models to Data

Fitting a model to data means finding the set of *parameter values* that gets the *model function* "closest" to the data

- **Data**: ordered pairs (x_i, y_i), often in the form of an N×2 matrix
  - Independent variable x_i, e.g. frequency, time, etc.
  - Dependent variable y_i, e.g. signal magnitude
- **Model function**: $y = f(x;\ \beta)$, which takes some set of parameters β

What exactly does "closest" mean? Minimizing some *cost function* – most often the sum of squared deviations:

$$S(\beta) = \sum_{i=1}^{N} [f(x_i; \beta) - y_i]^2$$

Appendix #2: Fitting Models to Data

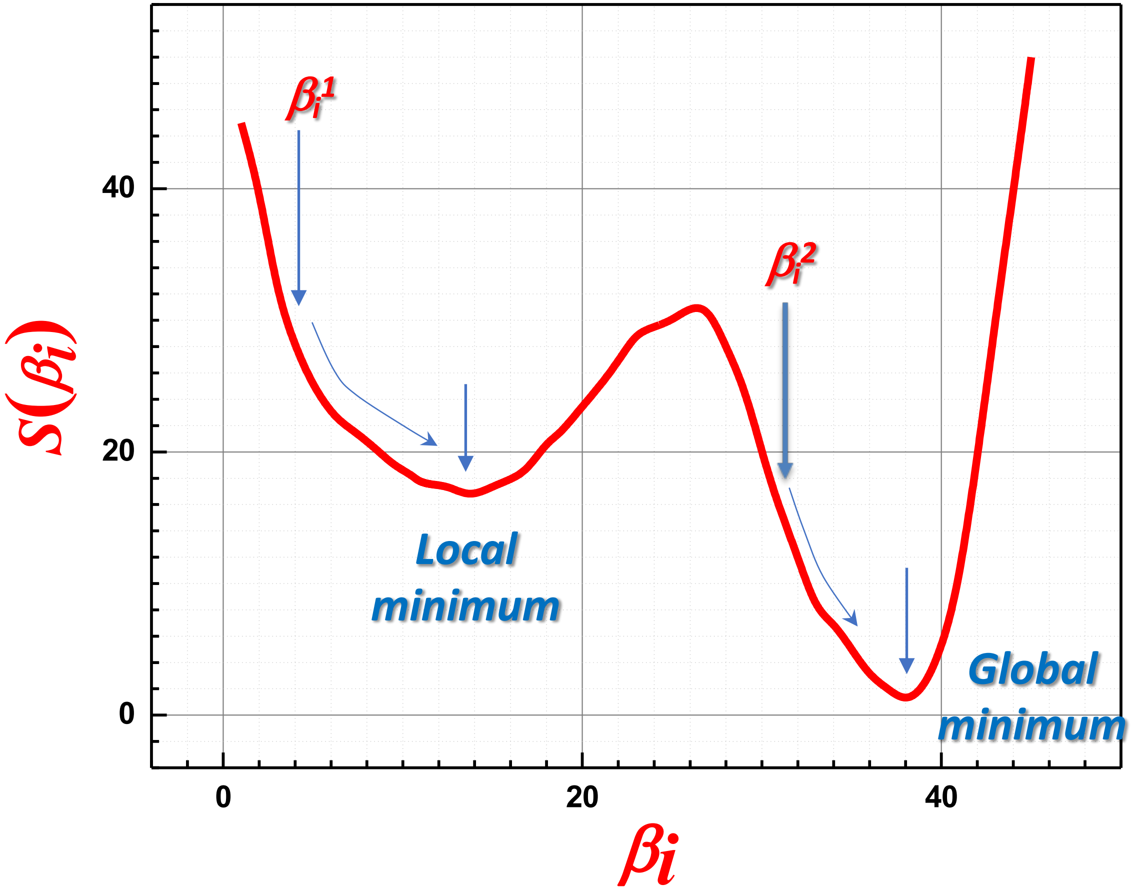
$$S(\beta) = \sum_{i=1}^{N} [f(x_i; \beta) - y_i]^2$$

There are several minimization algorithms for solving such problems. Origins uses the Levenberg-Marquardt algorithm for nonlinear fitting, which is optimized for quadratic cost functions like this one.

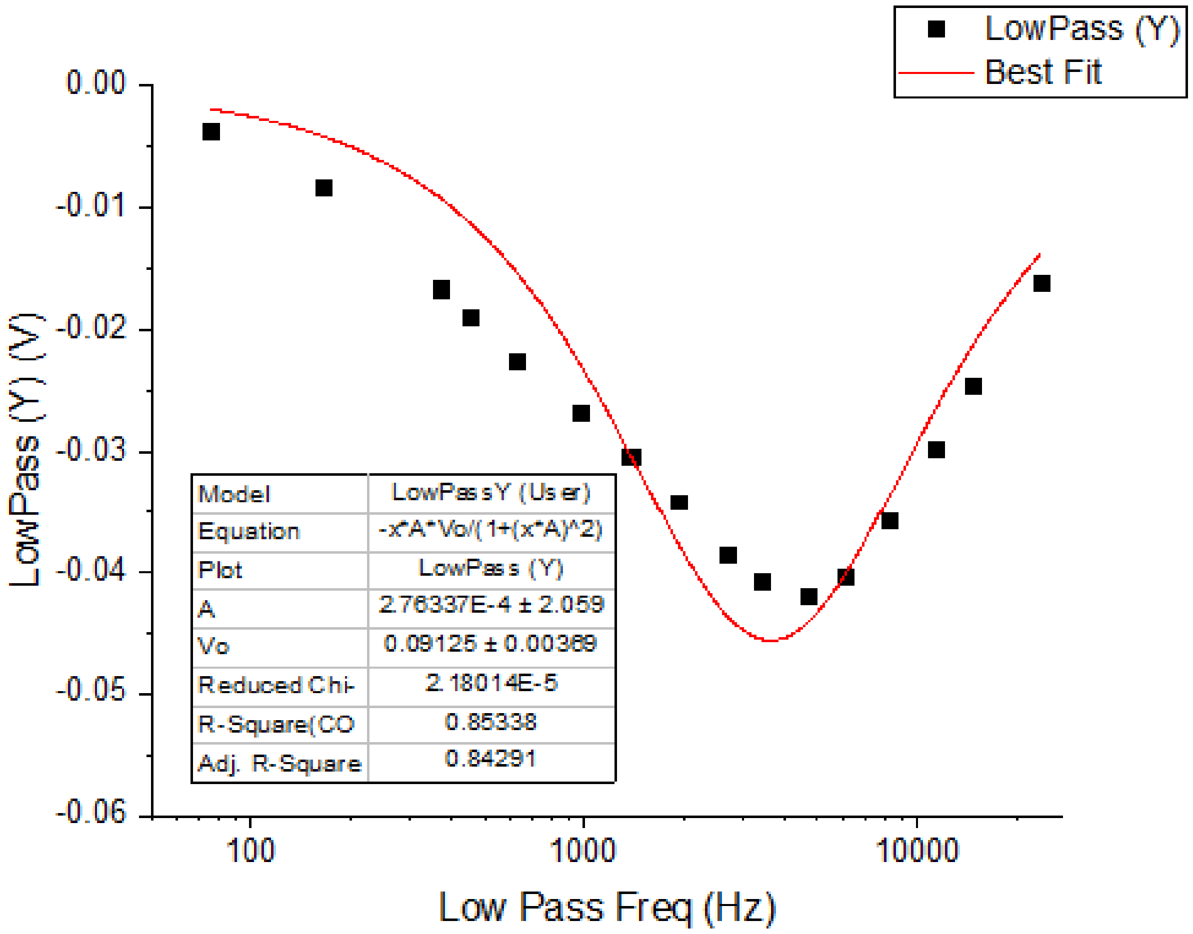
These algorithms proceed from a starting guess and traverse the cost function “landscape” (hopefully efficiently!) to find a global minimum.

How well they do depends on the quality of the starting guess, and it’s easy to get trapped in local minima.

Appendix #2: Fitting Models to Data



Appendix #2: Fitting Models to Data



An example of poor results from the fitting procedure

Appendix #3: Some Reminders

1. Reports should be uploaded *only* to the **proper folder** for your activity and section
   - For example, folder Frequency domain analysis_L1 should only be used by students from section L1
   - Submit only one copy (no need to submit e.g. both Word and PDF)
   - Show your name, and only your name, in the list of authors on the title page of the report
   - I recommend the following file name style:

L1_lab5_LastName