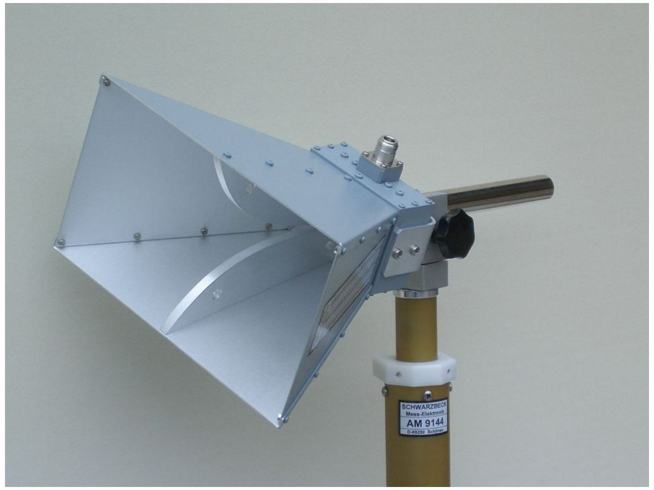
XKCD: What If 131

Qualitative Studies with Microwaves

Prof. Jeff Filippini

Physics 401

Spring 2020



Wikipedia

ILLINOIS

Key Goals of this Lab

Study the optical propagation of microwaves in free space.

- **Electromagnetic waves:** Brief refresher on properties and propagation
- **Introducing microwaves**: Properties and applications
- **Microwave components**: Klystrons, detector diodes, and more
- **Microwave optics**: Demonstrate six classical optical phenomena
- ***Bonuses****: X-ray crystallography, the THz gap, and bolometers*

This is a two-week lab

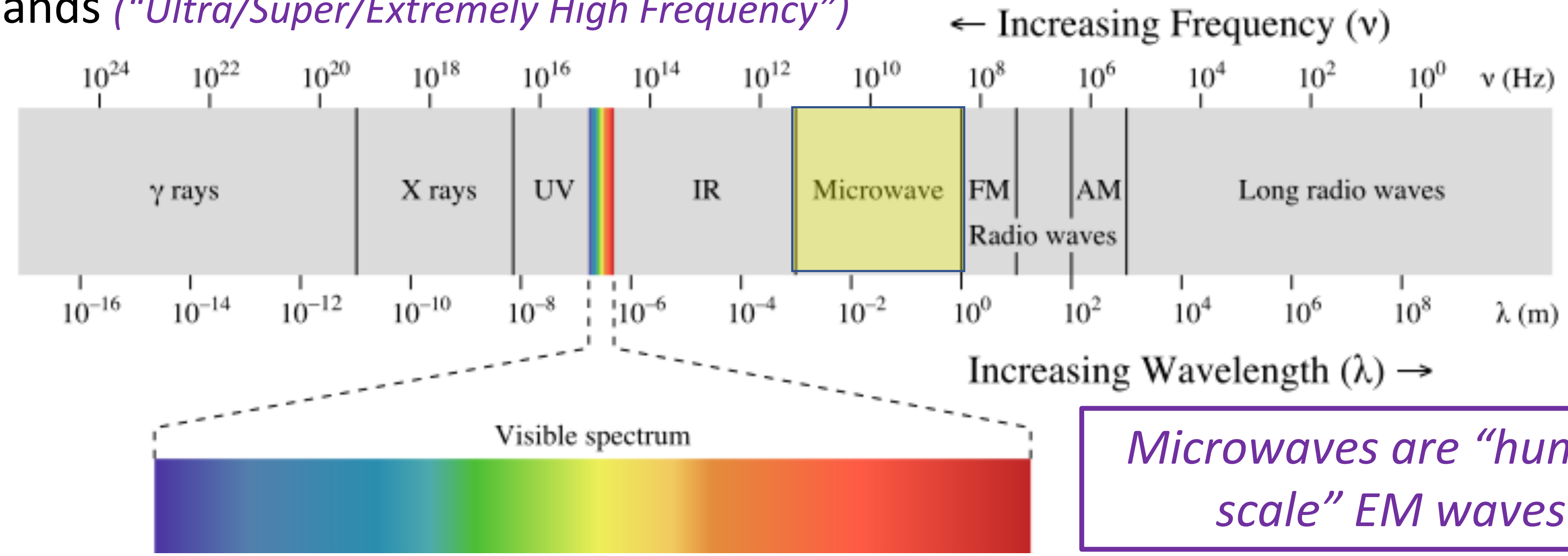
Next week, microwave plumbing: waveguides and cavities

Microwaves on the Electromagnetic Spectrum

- Often defined as wavelengths from 1m (300 MHz) to 1mm (300 GHz)

  Fuzzy definition: RF engineers often use 1 – 100 GHz
- Lies between **radio** (multi-meter wavelengths) and **far-infrared** (sub-mm)

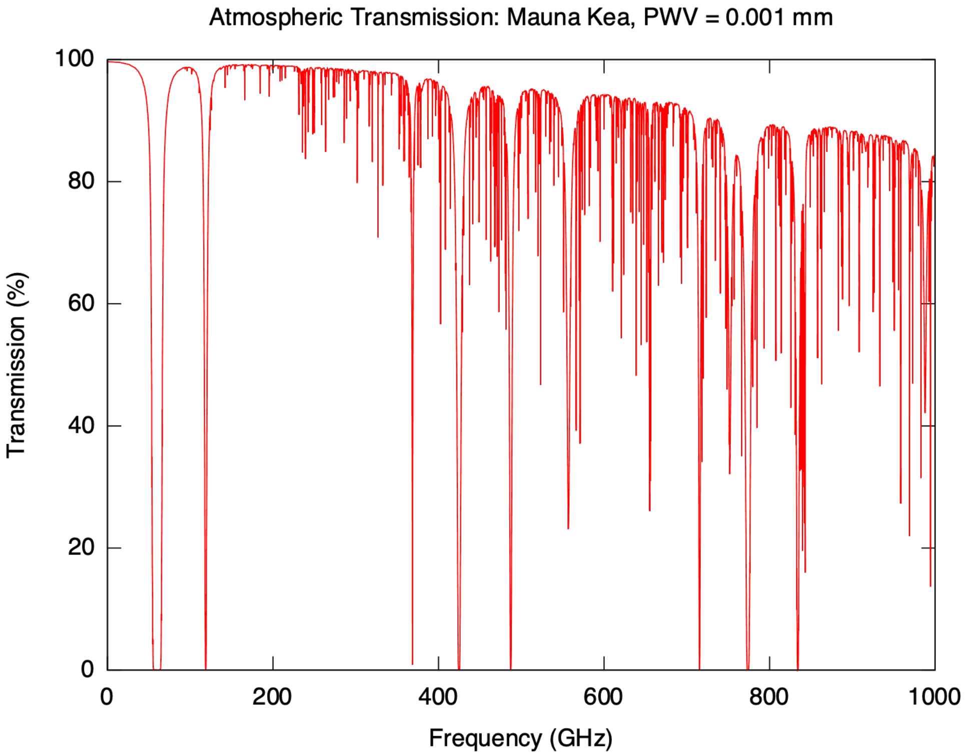
  Today often distinguish a terahertz band (0.3-3 THz) before FIR
- Includes **UHF** (0.3-3 GHz), **SHF** (3-30 GHz), and **EHF**/**mm-wave** (30-300 GHz) bands *("Ultra/Super/Extremely High Frequency")*



Microwaves are "human-scale" EM waves

What's Useful About Microwaves?

- Air is largely transparent to microwaves
  Except e.g. molecular lines of H_2O, O_2
- Line-of-sight propagation: unlike radio, minimal diffraction around obstacles/Earth
- High frequency means higher data transmission rate (bandwidth) than radio
  (…but lower than IR/visible fiber optics)
- Short wavelength means higher resolution for remote sensing (*e.g. radar*) than radio



Transmission of dry air above Mauna Kea
Wikipedia / Caltech Sub-mm Observatory

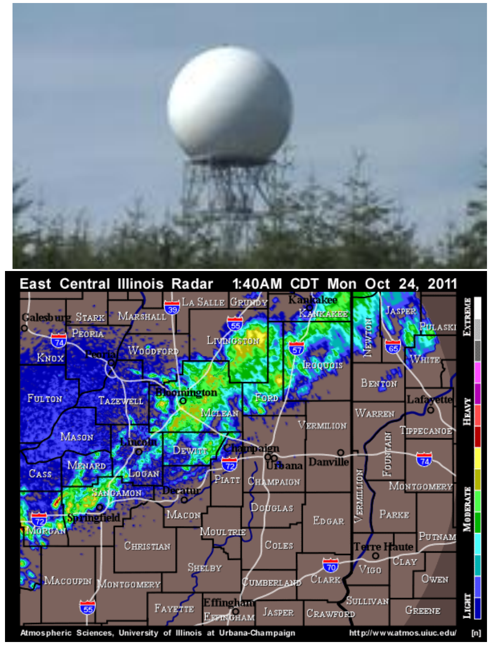
Some Microwave Applications

Microwave oven (2.45GHz)

Communications (0.8-2.69 GHz)

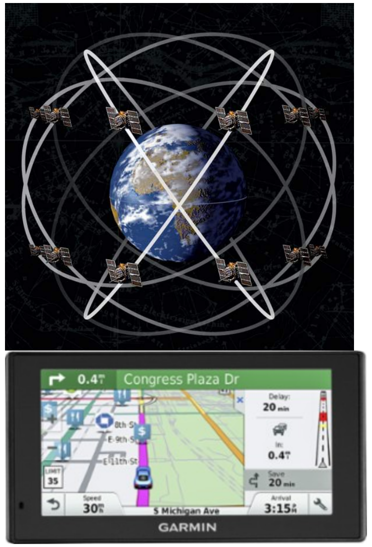
WiFi (2.4/5 GHz)

Satellite TV (4-18GHz)

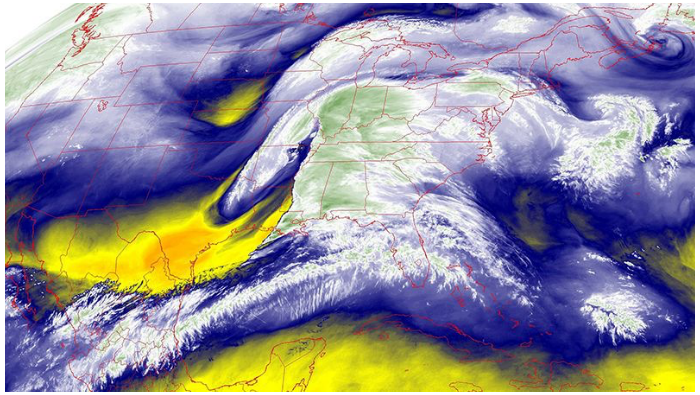


Weather radar (8-12 GHz)

Radar (up to 110GHz)



GPS (1.17-1.575 GHz)



Weather satellite (18-90 GHz)

Images via E.V. Colla, Wikipedia, NOAA/GOES

Maxwell's Equations

In a macroscopic formulation in an isotropic medium, with $\vec{D} = \varepsilon\vec{E}$ and $\vec{B} = \mu\vec{H}$

(1) $$\nabla \cdot \vec{D} = \rho$$

(3) $$\nabla \cdot \vec{B} = 0$$

(2) $$\nabla \times \vec{E} = -\frac{\partial \vec{B}}{\partial t}$$

(4) $$\nabla \times \vec{H} = \vec{J} + \frac{\partial \vec{D}}{\partial t}$$

James Clerk Maxwell (1831–1879)

In media with no free charge or currents (no source terms), (1) and (4) can be rewritten as:

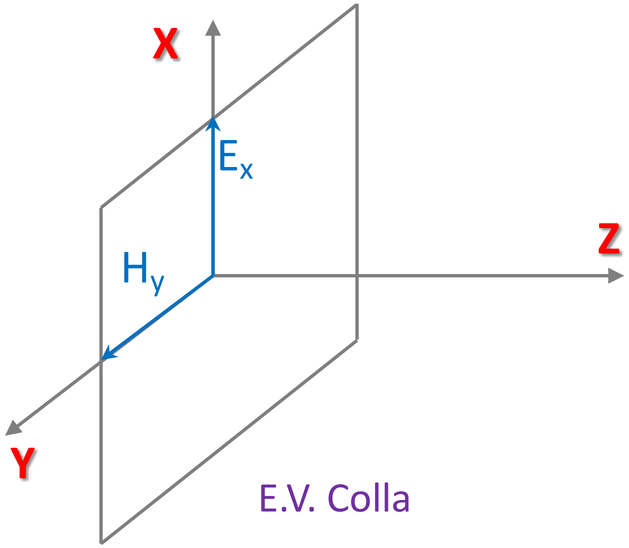
(1') $$\nabla \cdot \vec{D} = \varepsilon\left[\frac{\partial E_x}{\partial x} + \frac{\partial E_y}{\partial y} + \frac{\partial E_z}{\partial z}\right] = 0$$

(4') $$\nabla \times \vec{H} = \frac{\partial \vec{D}}{\partial t}$$

Plane Electromagnetic Waves

Assume a **TEM** (**T**ransverse **E**lectric & **M**agnetic) plane wave propagating in the z-direction

This is correct for propagation in isotropic lossless media, but we won't derive that here.
Recall that waveguide propagation modes can be different!



E.V. Colla

We can freely choose our axes such that $E_y = E_z = H_x = H_z = 0$

Note that (1') and (2) just imply that *E* and *H* vary only in z

And (3) and (4') can be simplified to:

$$\frac{\partial E_x}{\partial z} = -\mu \frac{\partial H_y}{\partial t} \quad (5)$$

$$\frac{\partial H_y}{\partial z} = -\varepsilon \frac{\partial E_y}{\partial t} \quad (6)$$

Recall that for an isotropic medium, we define:

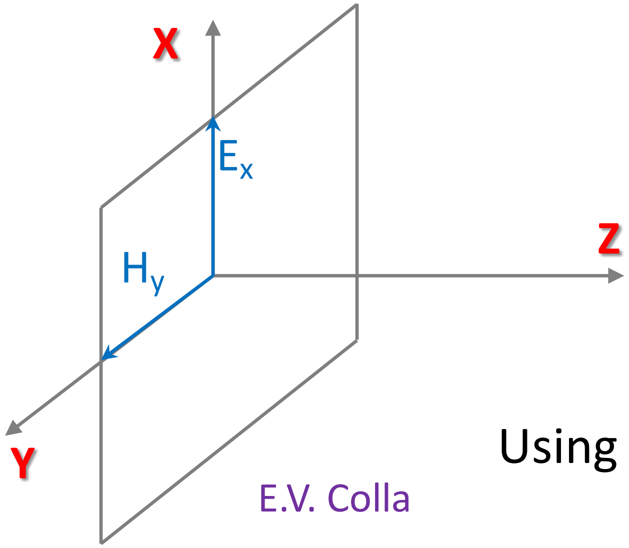
Free space **permittivity ε_0** and **permeability μ_0**

Medium **relative permittivity $\varepsilon_r \equiv \varepsilon/\varepsilon_0$** and **relative permeability $\mu_r \equiv \mu/\mu_0$**

Relative quantities are dimensionless, and for free space (vacuum) $\varepsilon_r = \mu_r = 1$

Plane Electromagnetic Waves

Combining (5) and (6) (*see write-up for details*) we obtain the plane wave propagation equations:



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$$(7)\quad \frac{\partial^2 E_x}{\partial z^2} = \frac{1}{v^2}\frac{\partial^2 E_x}{\partial t^2} \qquad (8)\quad \frac{\partial^2 H_y}{\partial z^2} = \frac{1}{v^2}\frac{\partial^2 H_y}{\partial t^2}$$

$$v = \frac{1}{\sqrt{\varepsilon\mu}} = \frac{c}{\sqrt{\varepsilon_r\mu_r}}$$

… and we thus seek sinusoidal solutions:

$$\boxed{\begin{aligned} E_x &= E_{x0}\cos(\omega t - kx) \\ H_y &= H_{y0}\cos(\omega t - kx) \end{aligned}}$$

$$k = \frac{2\pi}{\lambda} = \frac{\omega}{v}$$

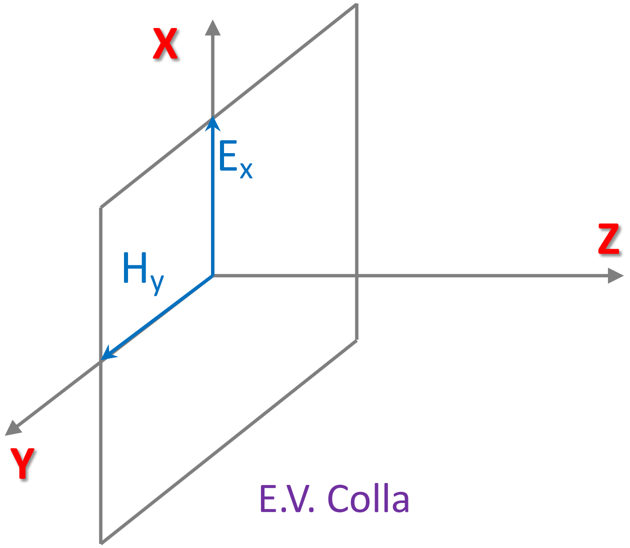
Using (5) or (6), we can relate these two amplitudes:

$$\boxed{E_x = \sqrt{\frac{\mu}{\varepsilon}}H_y \equiv ZH_y}$$

In analogy with our discussion of waveguides, this defines a medium’s **characteristic impedance, Z**

For free space ($\varepsilon_r = \mu_r = 1$), we have $Z_0 = \sqrt{\frac{\mu_0}{\varepsilon_0}} \approx 377\ \Omega$

Plane Electromagnetic Waves



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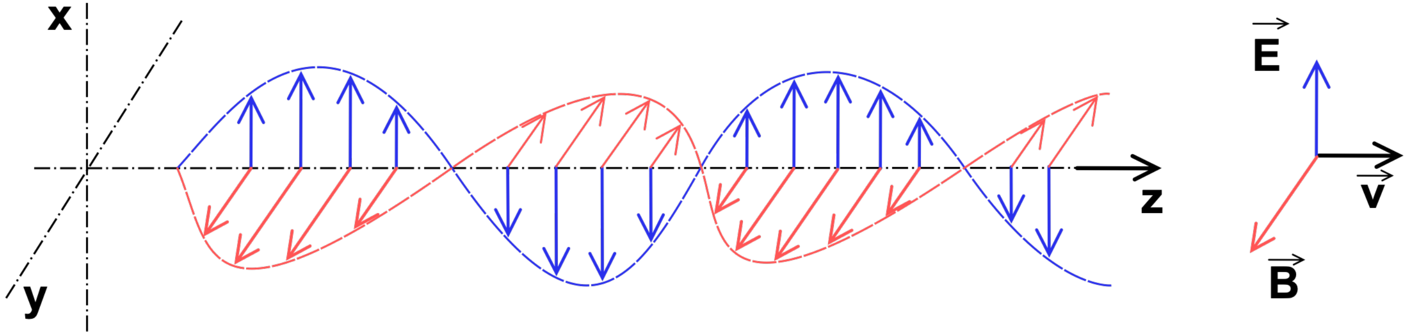


$$v = \frac{1}{\sqrt{\varepsilon\mu}} = \frac{c}{\sqrt{\varepsilon_r\mu_r}}$$

$$E_x = \sqrt{\frac{\mu}{\varepsilon}}H_y \equiv ZH_y$$

$$Z_0 = \sqrt{\frac{\mu_0}{\varepsilon_0}} \approx 377\ \Omega$$

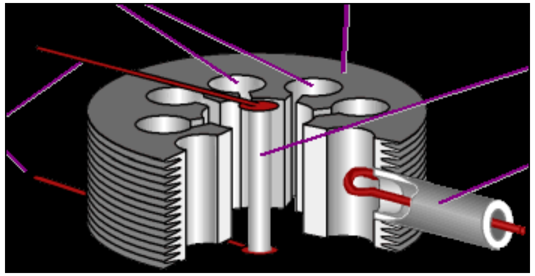
Wikipedia: Electromagnetic Radiation

Generating Microwaves

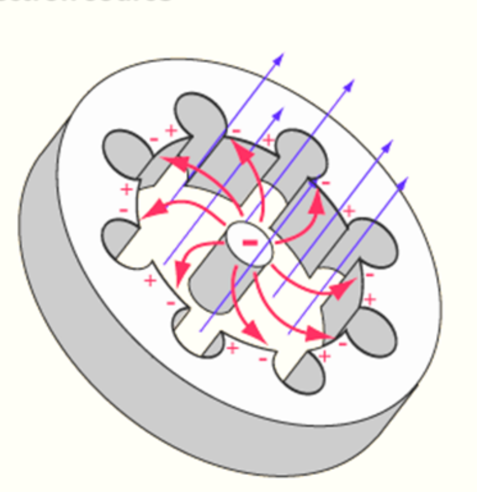
Various ways to make electrons oscillate in the GHz regime:

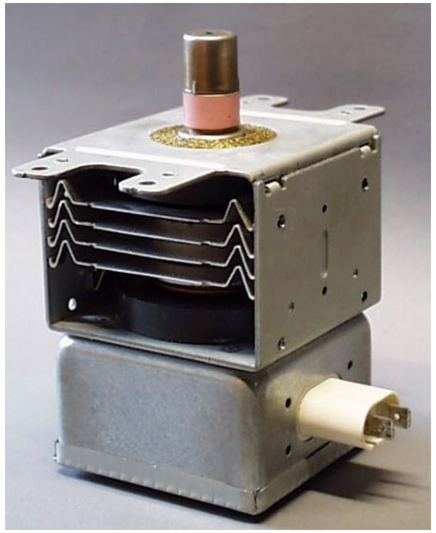
- **Vacuum tubes**: klystron, cavity magnetron, traveling wave tube, …
- **Solid-state devices**: IMPATT diode, tunneling diode, Gunn diode, …

Tunable frequency from 9 to 10GHz maximum output power 20mW



Heated cathode as electron source

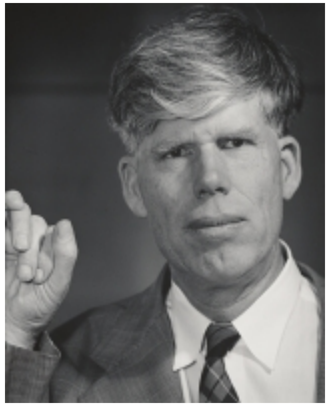




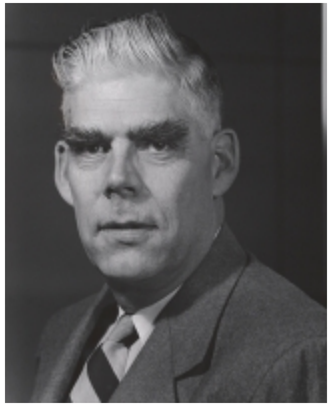
Microwave oven magnetron typical power 0.7-1.5kW

Gunn diode
~10 GHz – 3 THz
max power 10-300 mW

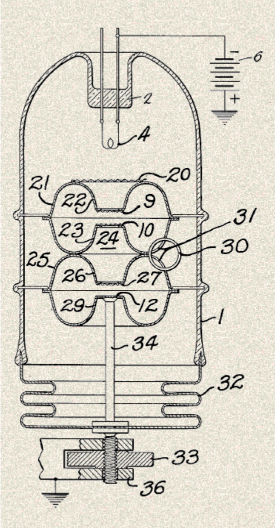
The Klystron: A Piece of History

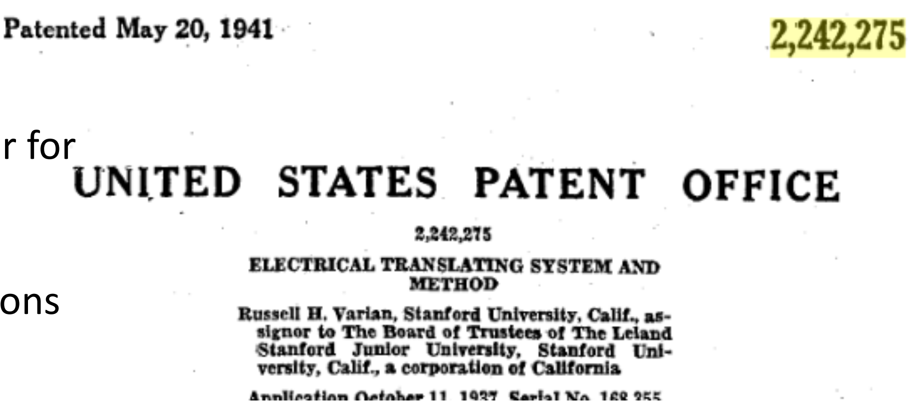


Russell Harrison Varian
(April 24, 1898 –
July 28, 1959)



Sigurd Fergus Varian
(May 4, 1901 –
October 18, 1961)





Patented May 20, 1941 2,242,275

UNITED STATES PATENT OFFICE

2,242,275

ELECTRICAL TRANSLATING SYSTEM AND METHOD

Russell H. Varian, Stanford University, Calif., assignor to The Board of Trustees of The Leland Stanford Junior University, Stanford University, Calif., a corporation of California

Application October 11, 1937, Serial No. 168,255

Groundbreaking microwave amplifier for the microwave range

Influential in radar, telecommunications

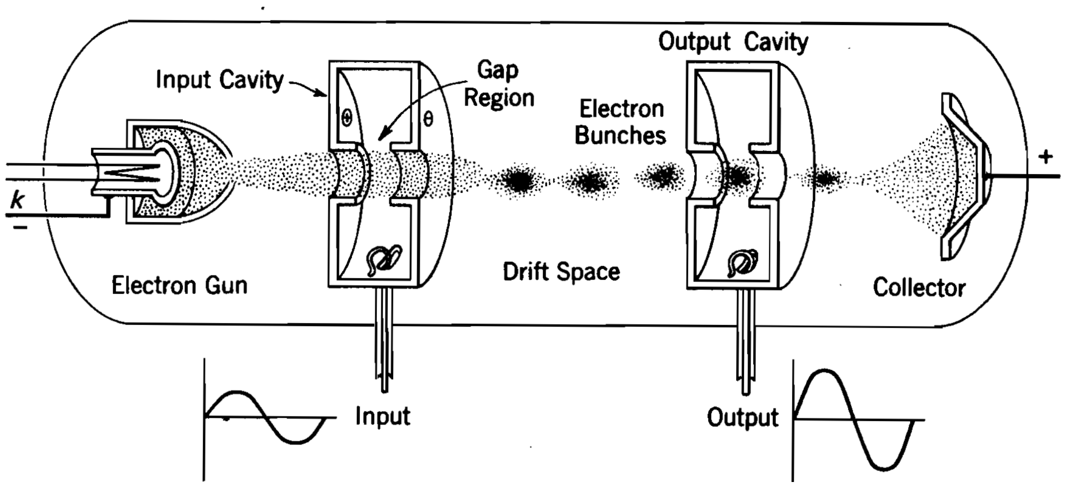
Largely obsolete today as sources

Varian Brothers...Klystron Tube (1940)

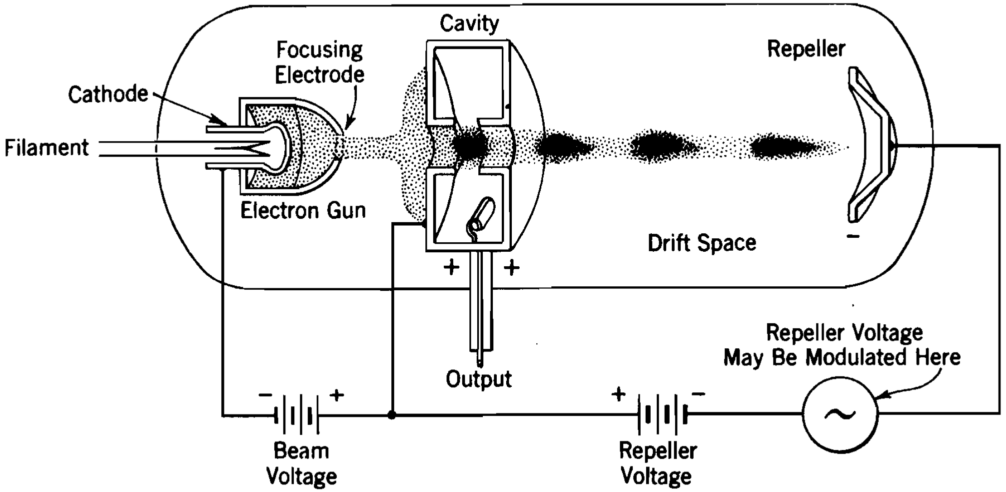
E.V. Colla

The Klystron: How It Works

E.V. Colla



Amplifier: Two-cavity Klystron



Oscillator: Reflex Klystron (Sutton Tube)

1. Strong DC electron beam enters cavity *(input energy)*
2. Electron velocities nudged up/down by RF in a resonant cavity
3. Differential speed causes electrons to bunch while crossing gap
4. Bunched electrons excite RF power in output resonant cavity

Narrow frequency range, high power output (esp. multi-cavity)
With feedback (repeller), can act as an oscillator

SLAC 2mi Klystron Gallery

The Classic 2K25 Klystron

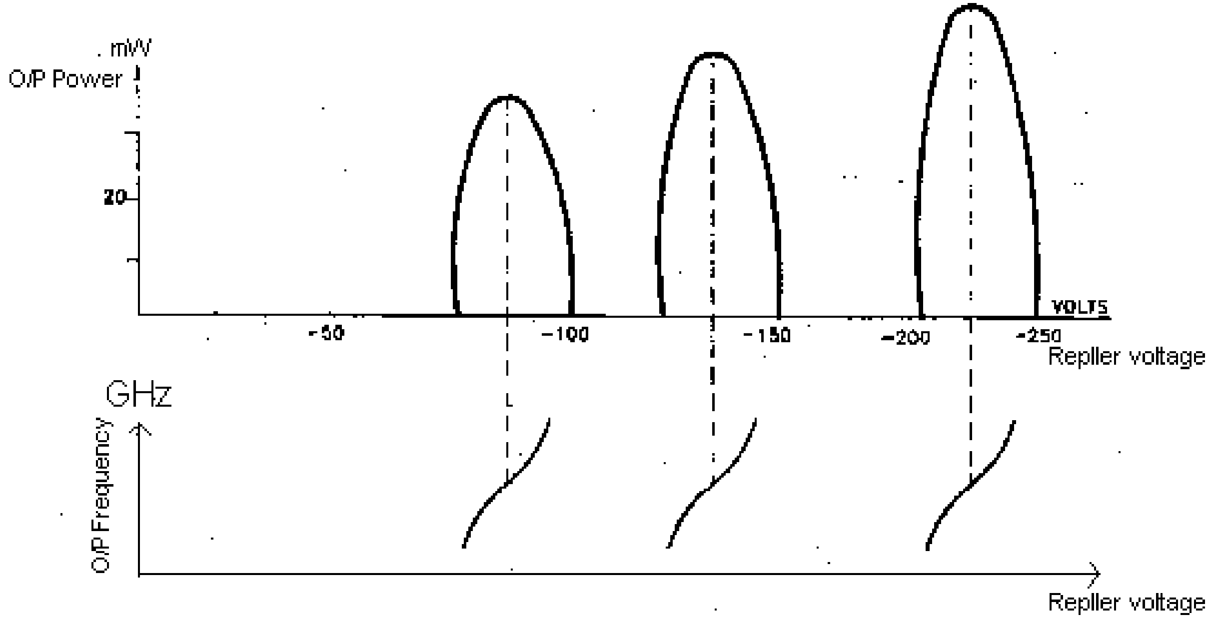
GENERAL CHARACTERISTICS

Frequency Range ······················8,500 to 9,660 Mc

Cathode Oxide-coated, indirectly heated

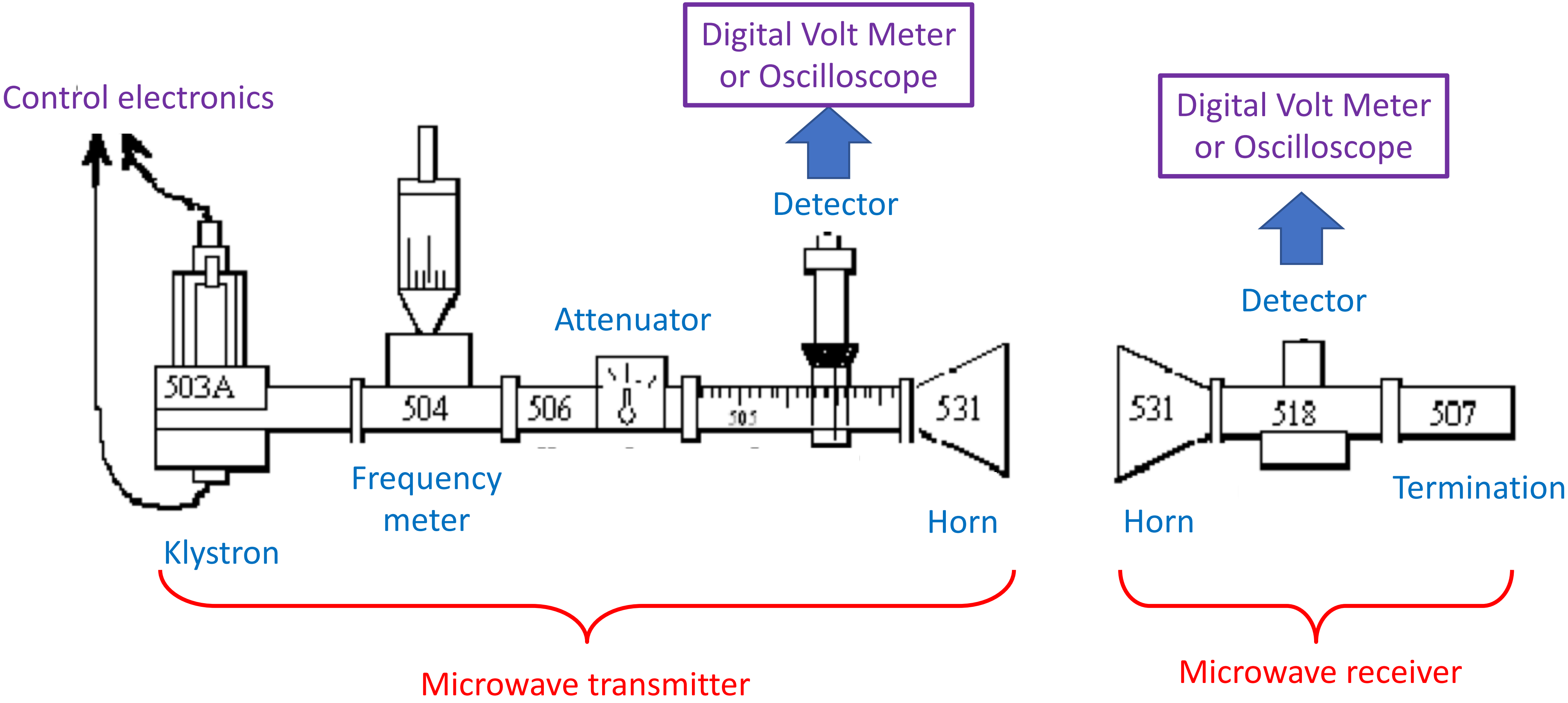
Heater Voltage··································6.3Volts

Heater Current·························0.44 Amperes



Output power and frequency depend on voltage

Experimental Setup: Key Components



Control electronics

Digital Volt Meter or Oscilloscope

Detector

Attenuator

Klystron

Frequency meter

Horn

Microwave transmitter

Digital Volt Meter or Oscilloscope

Detector

Horn

Termination

Microwave receiver

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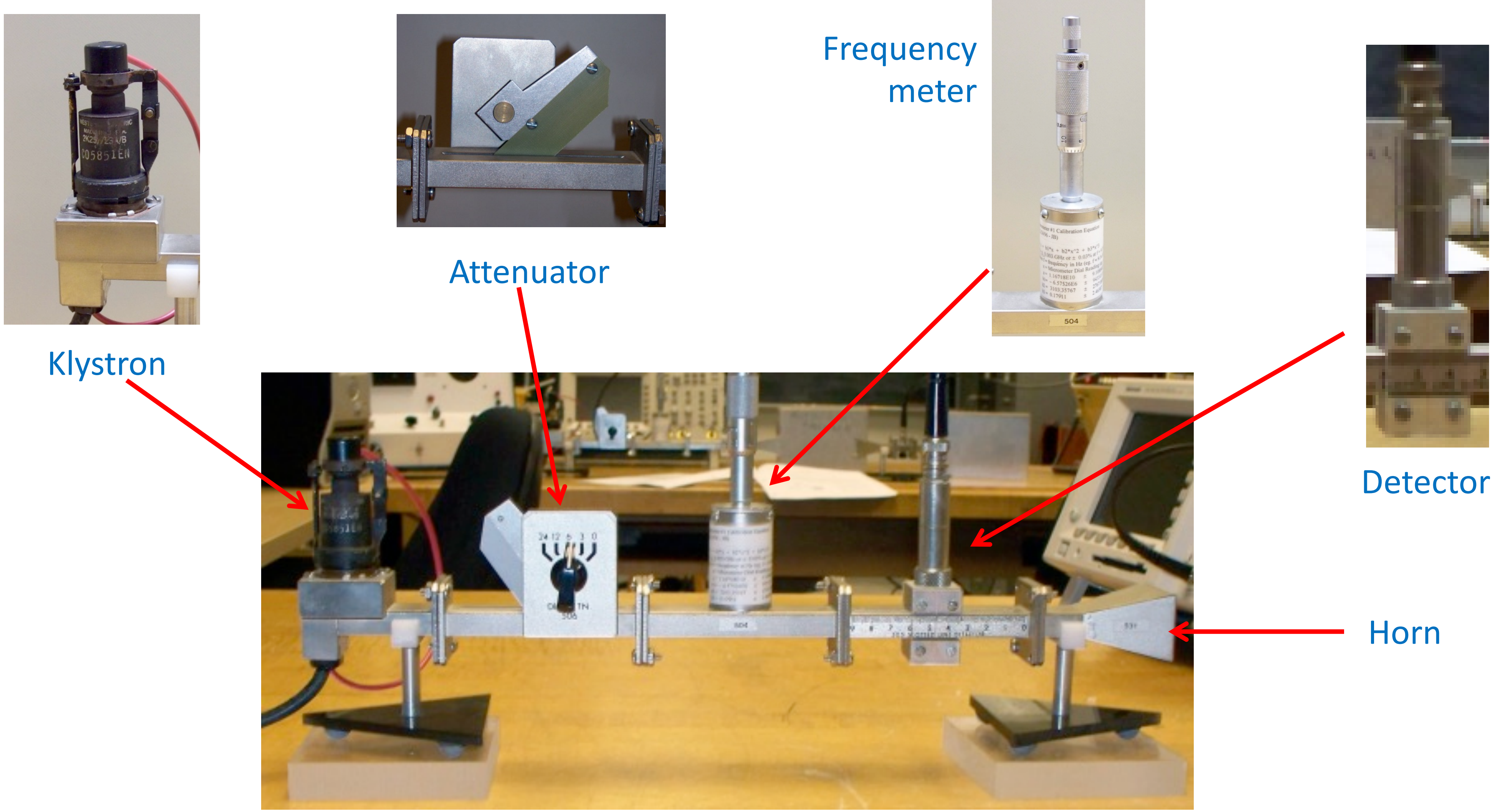
Experimental Setup: Key Components



Klystron

Attenuator

Frequency meter

Detector

Horn

Microwave transmitter

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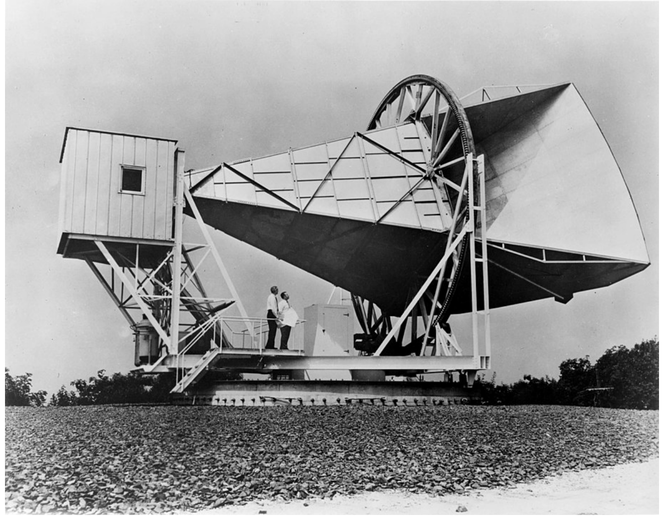
Antennas and Horns

Why do we need **antennas** (including horn antennas)? Two basic functions:

1. Impedance matching: Reduce reflections at transition between transmission line (e.g. *$Z=50\Omega$ coax*) and free space (*$Z_0=377\Omega$*) to maximize power transmission
Antenna efficiency*: radiated power over input power*

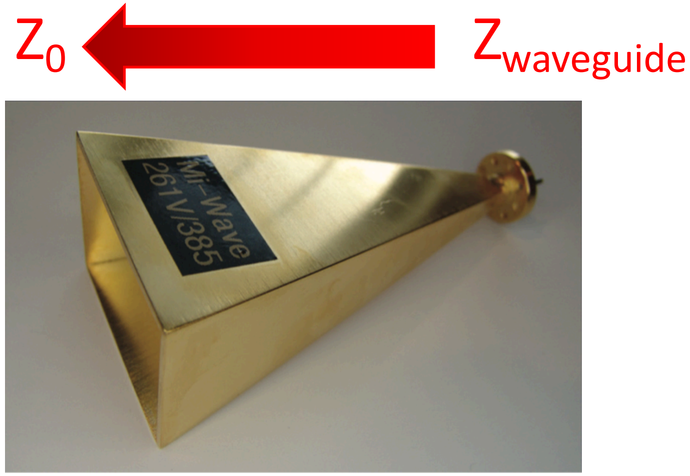
2. Directionality: Focus radiated power into the desired direction
Antenna gain*: Power transmitted in the peak direction relative to an isotropic emitter*



50 ft Holmdel horn, with which the cosmic microwave background was discovered in 1961

Wikipedia

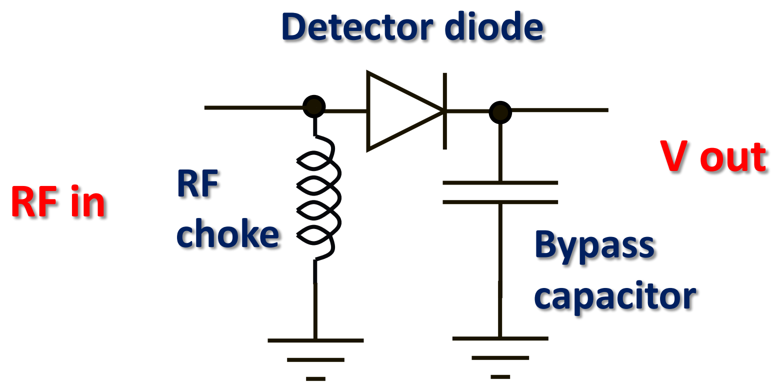
Horn antennas are simple, highly directional, and very broadband (*no resonant structures*)



Mi-Wave V-band (50-75 GHz)

Detecting Microwaves

How to detect a fast AC signal? **Rectify** it!



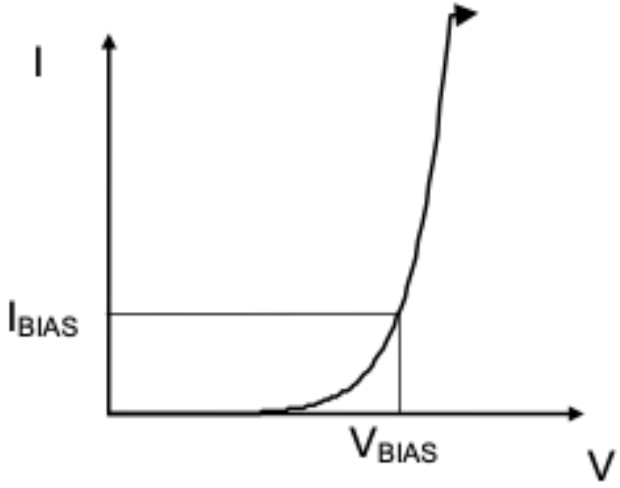


Figure: Macom

Taylor expansion

An ideal diode detector has: $I = I_0\left[\exp\left(\frac{e\,V}{k\,T}\right) - 1\right] \approx a\,V + b\,V^2 + \ \dots$

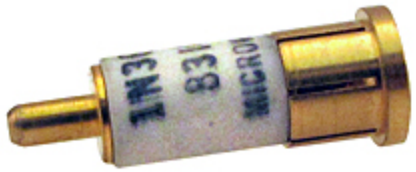
Driven with input $V(t) = V_0 \sin \omega t$ and low-pass filtered…

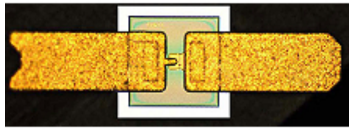
$$I(t) = aV_0 \sin \omega t + \frac{b\,V_0^2}{2}(1 - \cos 2\omega t) + \cdots \Rightarrow \boxed{I_{DC} = b\,\frac{V_0^2}{2} + \ \dots}$$

"Square-law detector"

Typically use **Schottky** (metal-semiconductor) diodes for microwaves, due to their fast carrier response

Detecting Microwaves

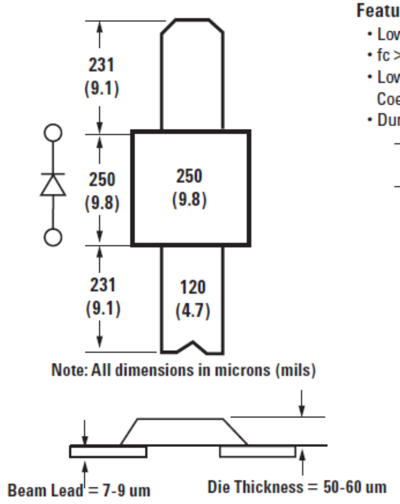




HSCH-9161
HSCH-9162
GaAs Detector Diode

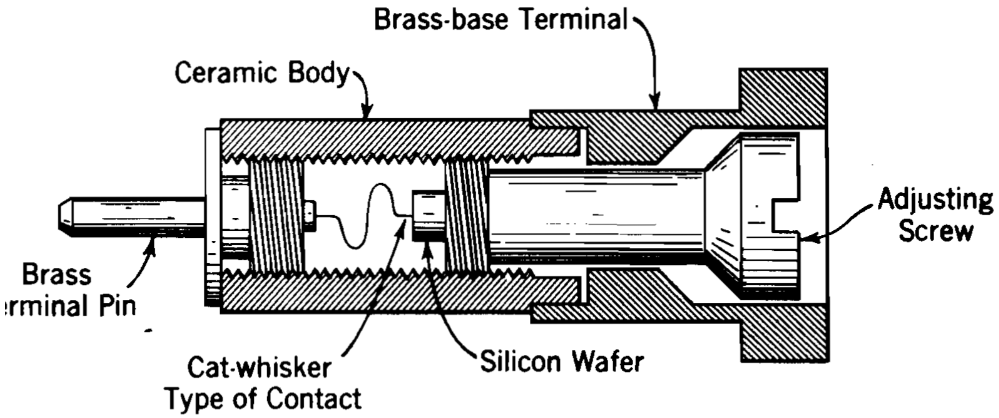
Data Sheet

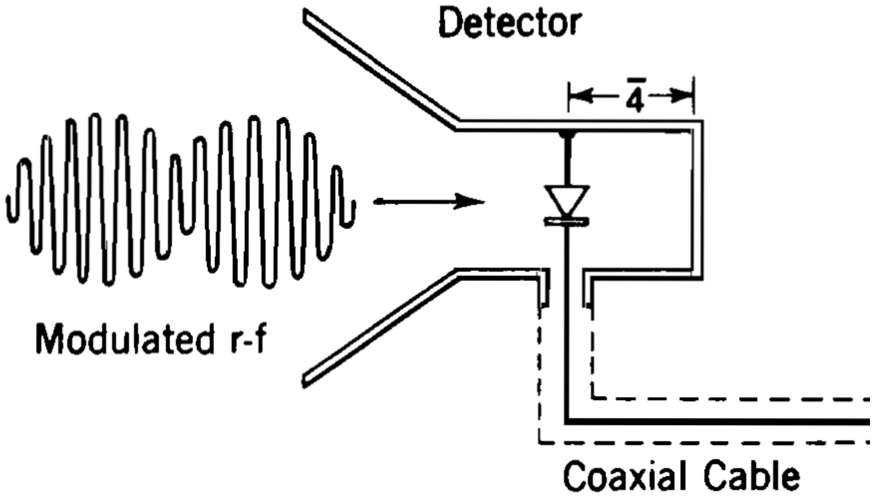
f_c ~ 200GHz



Note: All dimensions in microns (mils)

Features
- Low Junction Capacitance
- fc >200 GHz
- Lower Temperature Coefficient than Silicon
- Durable Construction
  - –Typical 6 gram beam lead strength
  - –High power handling capability





Lab Activities

Demonstrate classical optical phenomena using microwaves

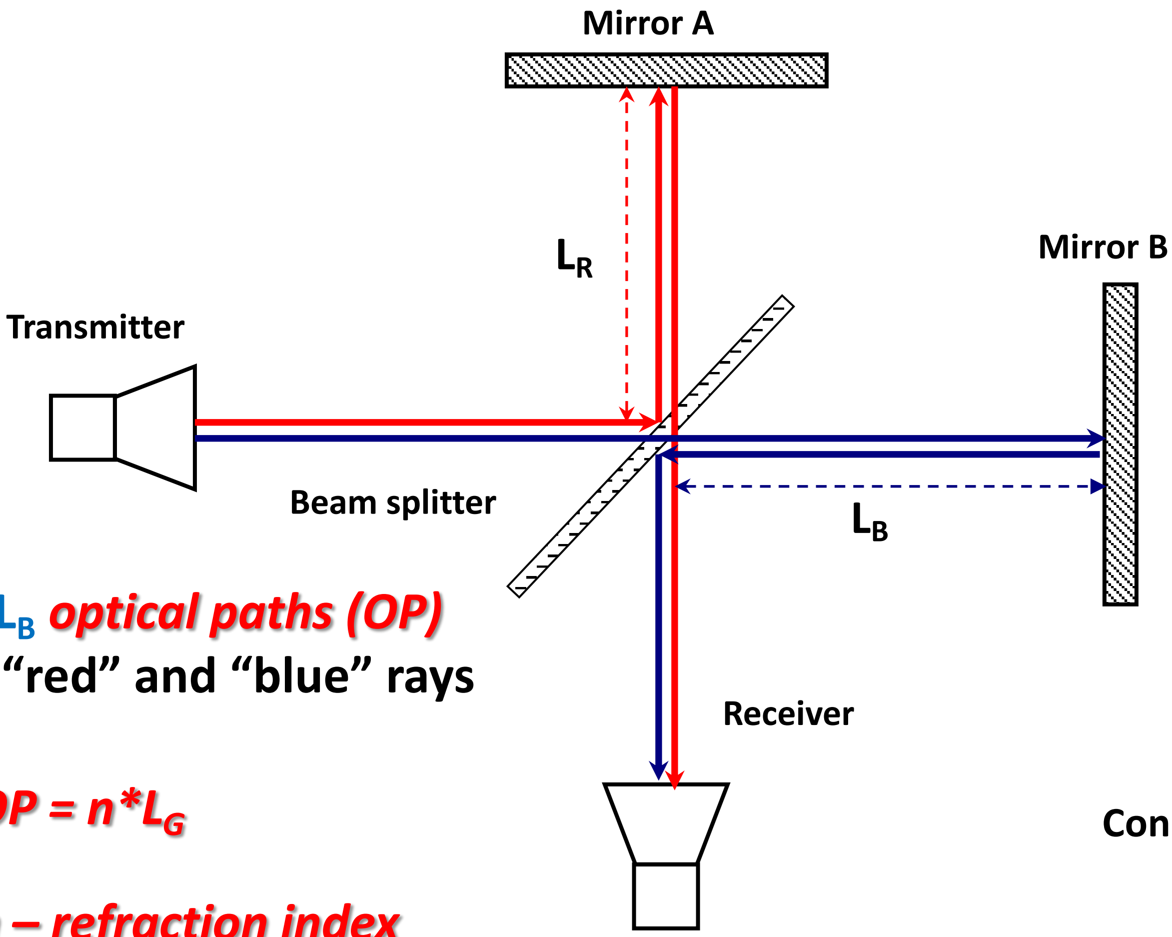
Microwaves are great for this, because of their cm-scale wavelengths
Human-scale “arts and crafts” produces quality optical performance!

Each of the six benches is set up as a “station” for a different optical demonstration. Students rotate through all six

We unfortunately can’t do the hands-on part this year due to COVID-19

Instead we will treat these as demonstrations and focus on the underlying physics, data analysis, and writing

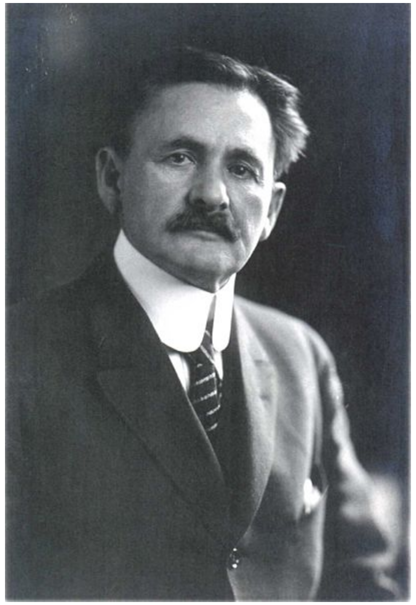
Experiment #1: Michelson Interferometer



L_R, L_B ***optical paths (OP)***
for "red" and "blue" rays

OP = n*L_G

n – refraction index
L_G – geometric length



Albert Michelson
(1852-1931)
1907 Nobel Prize in Physics

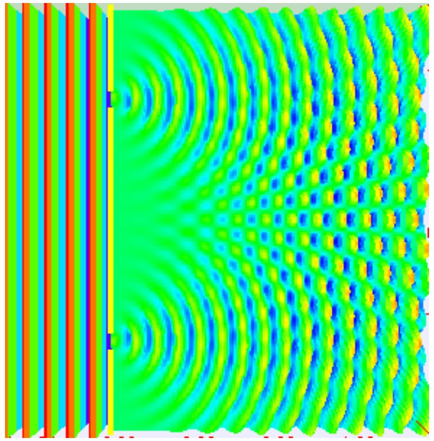
Condition for constructive interference

$$2\left|L_R - L_B\right| = k\lambda$$

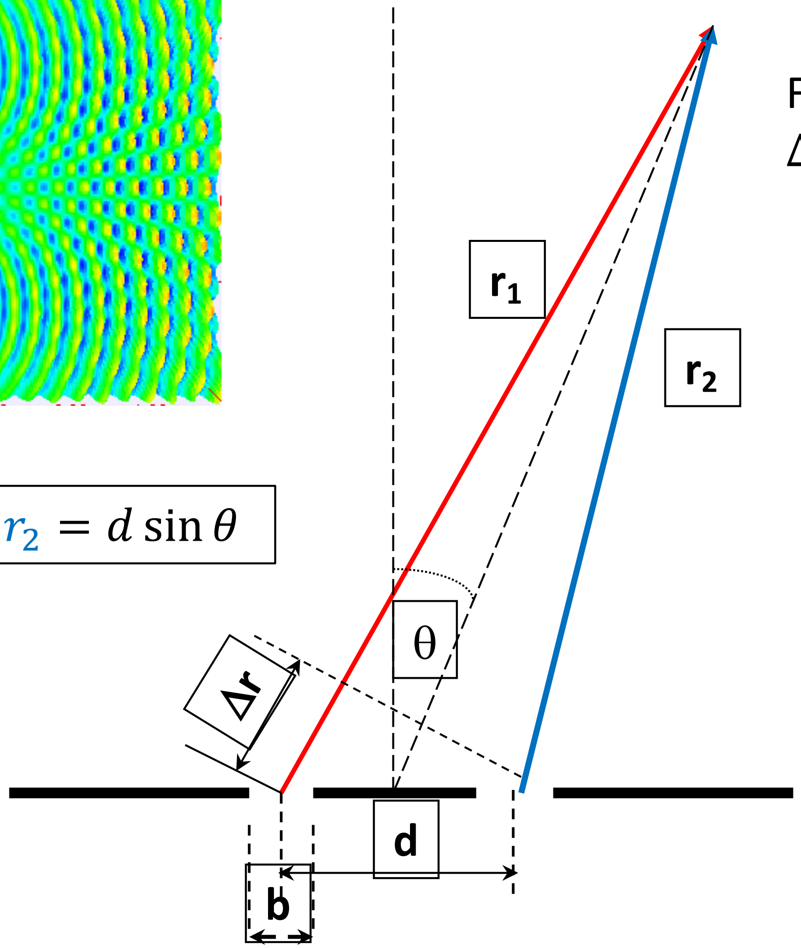
E.V. Colla

Experiment #1: Michelson Interferometer

Experiment #2: Double-Slit Interference



$$\boxed{\Delta r = r_1 - r_2 = d\sin\theta}$$



For constructive interference,
$\Delta r = n\lambda$ or $d\sin\theta = n\lambda$

Thomas Young
(1773 – 1829)

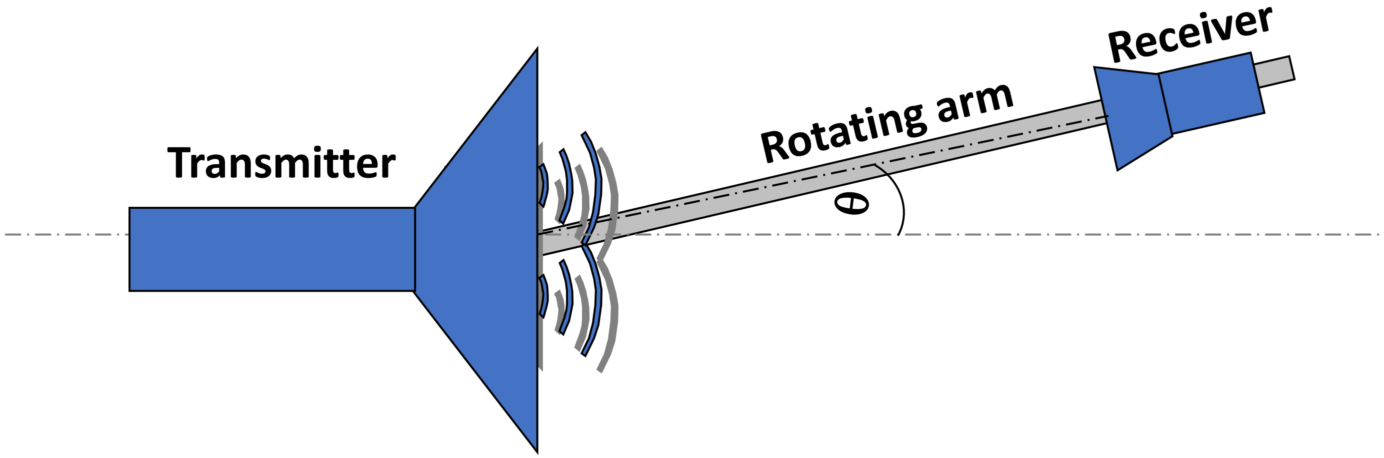
The measured diffraction pattern envelope is:

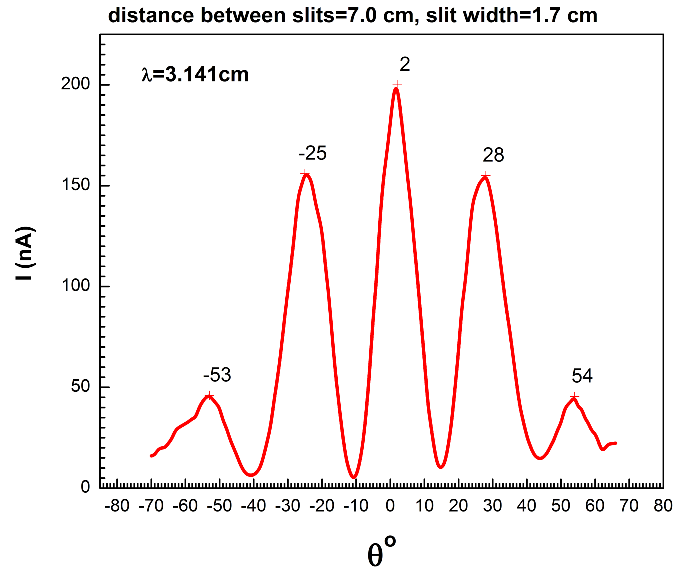
$$|\psi|^2 = |\psi_0|^2 \left(\frac{\sin x}{x}\right)^2 \cos\left[kd\ \sin\frac{\theta}{2}\right]$$

… where $x = kb\ \sin\frac{\theta}{2}$ and $k = \frac{2\pi}{\lambda}$

E.V. Colla

Experiment #2: Double-Slit Interference





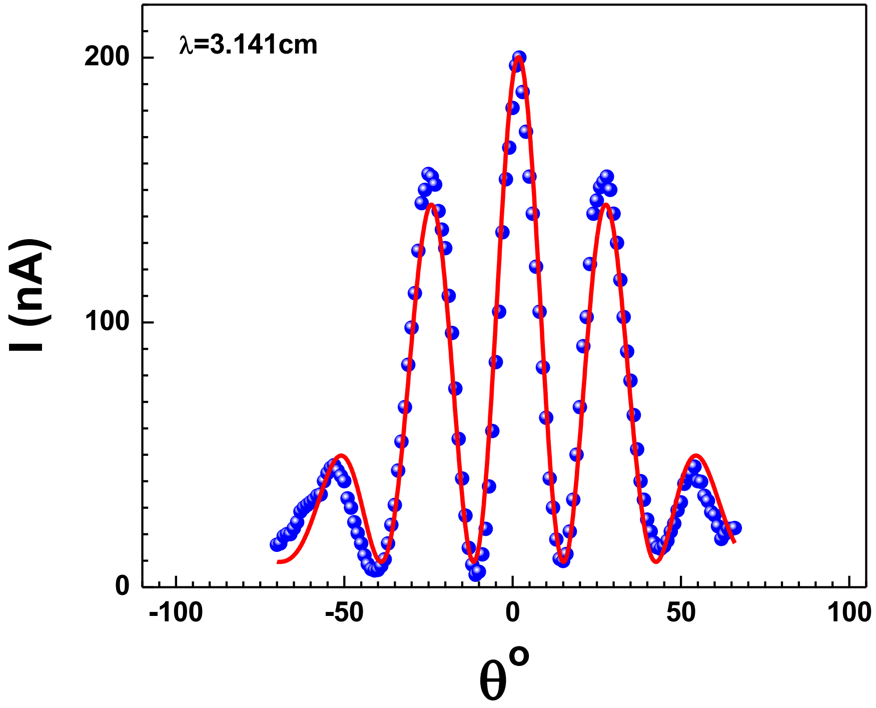
Physics 401 lab setup and example data

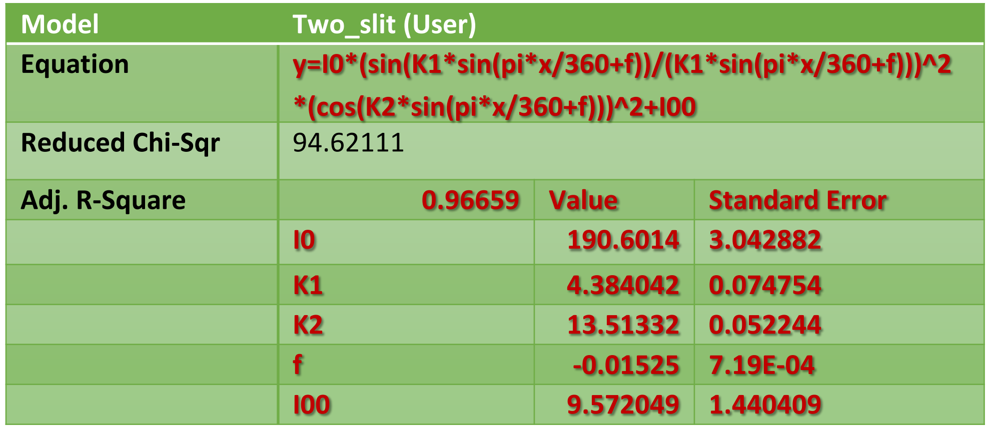
E.V. Colla

Experiment #2: Double-Slit Interference

$$|\psi|^2 = |\psi_0|^2 \left(\frac{\sin x}{x}\right)^2 \cos\left[kd\ \sin\frac{\theta}{2}\right]$$

$$x = kb\ \sin\frac{\theta}{2}$$





Model	Two_slit (User)		
Equation	y=I0*(sin(K1*sin(pi*x/360+f))/(K1*sin(pi*x/360+f)))^2 *(cos(K2*sin(pi*x/360+f)))^2+I00		
Reduced Chi-Sqr	94.62111		
Adj. R-Square	0.96659	Value	Standard Error
	I0	190.6014	3.042882
	K1	4.384042	0.074754
	K2	13.51332	0.052244
	f	-0.01525	7.19E-04
	I00	9.572049	1.440409

Fitting function

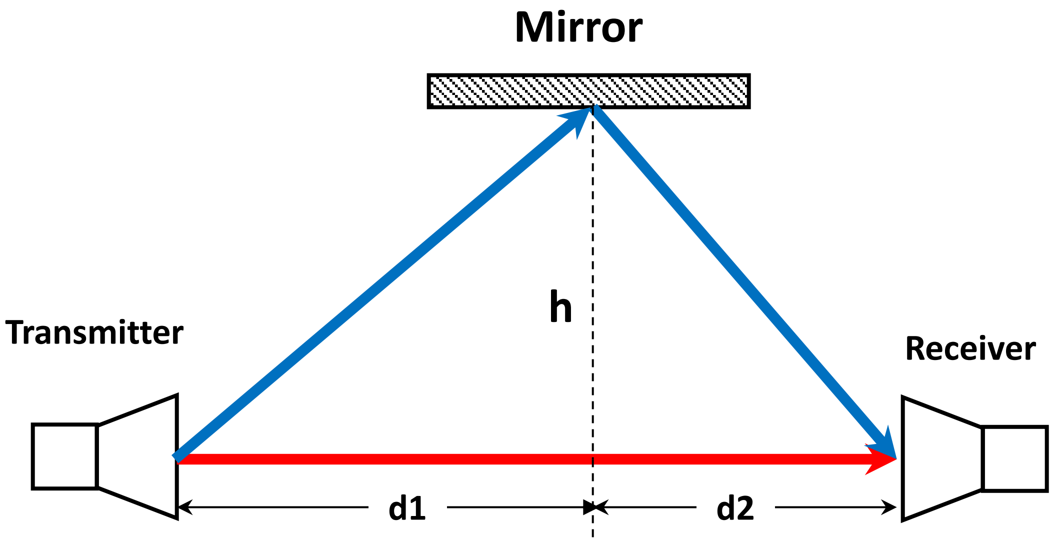
$$y = I_0\left(\frac{\sin\left[K_1 \sin\left(\frac{\pi x}{360} + f\right)\right]}{K_1 \sin\left(\frac{\pi x}{360} + f\right)}\right)^2 \cos^2\left[K_2 \sin\left(\frac{\pi x}{360} + f\right)\right] + I_{00}$$

In this fitting expression:

$I_0 = |\psi_0|^2$
$K_1 = kb$
$K_2 = kd$

E.V. Colla

Experiment #3: Lloyd’s Mirror



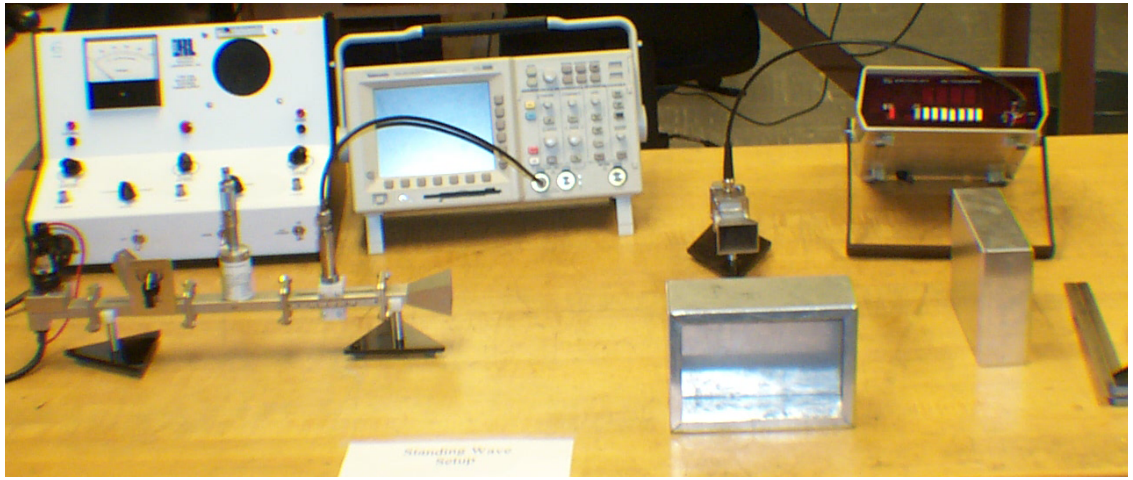
Humphry Lloyd
1802-1881

Difference of the wave path lengths between “red” and “blue” rays is:

$$\Delta S = \sqrt{h^2 + d_1^2} + \sqrt{h^2 + d_2^2} - (d_1 + d_2)$$

For constructive interference:

$$\boldsymbol{\Delta S = n\lambda}$$



E.V. Colla

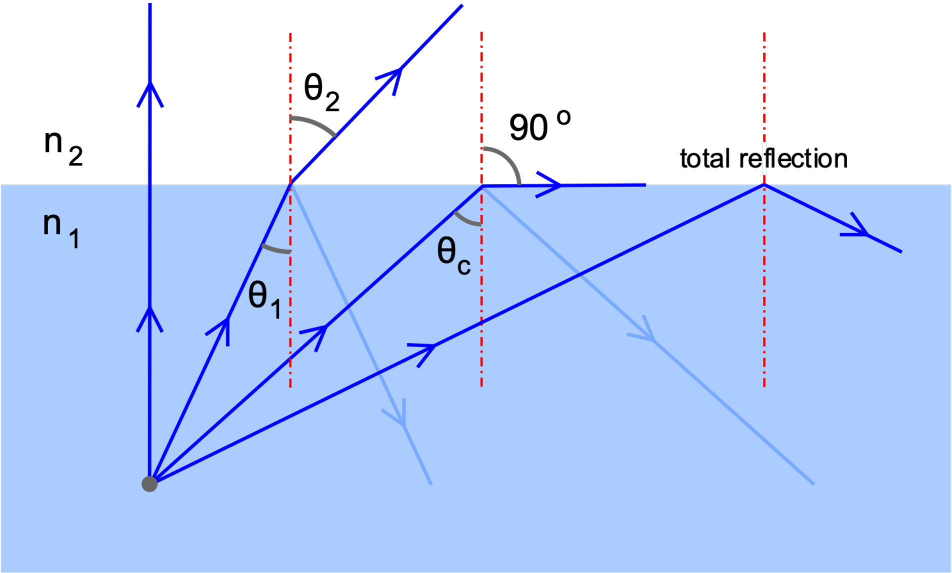
Experiment #4: Total Internal Reflection

Willebrord Snellius
1580-1626

Snell's Law

$$n_1 \sin\theta_1 = n_2 \sin\theta_2$$

Large incidence angles, when $n_1 > n_2$



[Wikipedia](https://en.wikipedia.org)

Claudius Ptolemaeus
after AD 83–*c*.168)

Critical angle

$$n_1 \sin\theta_c = n_2 \sin 90^\circ$$

$$\theta_c = \sin^{-1}(n_2/n_1)$$

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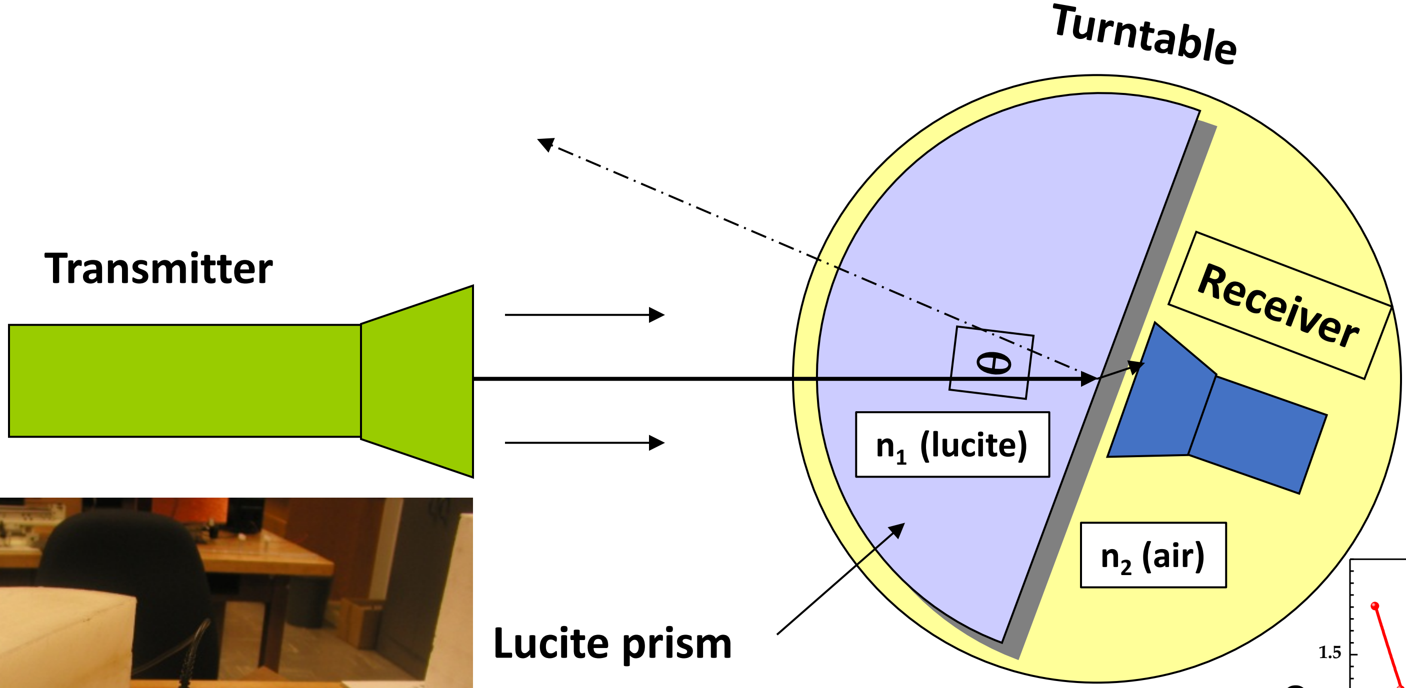
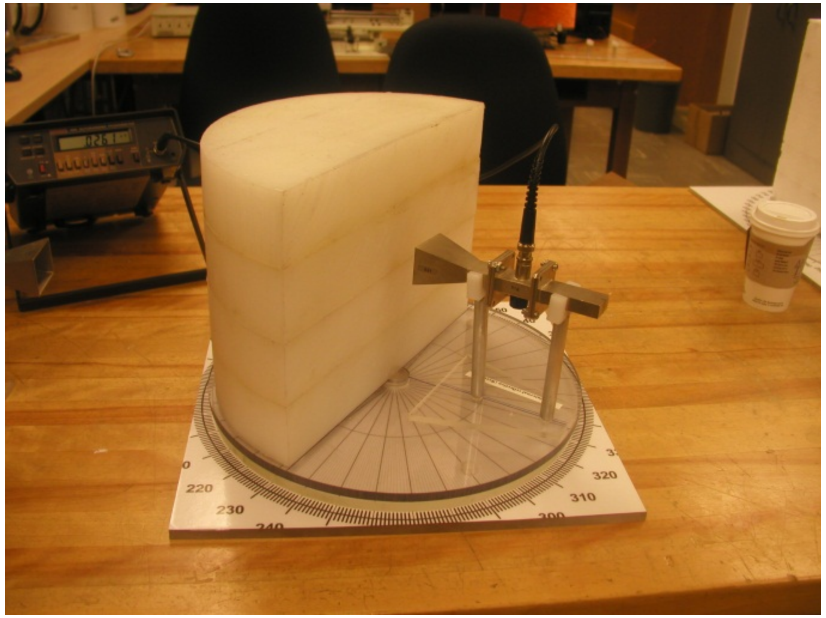
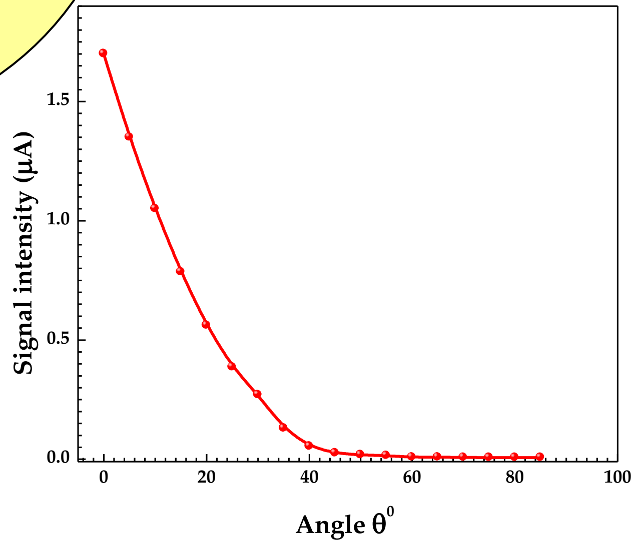
Experiment #4: Total Internal Reflection







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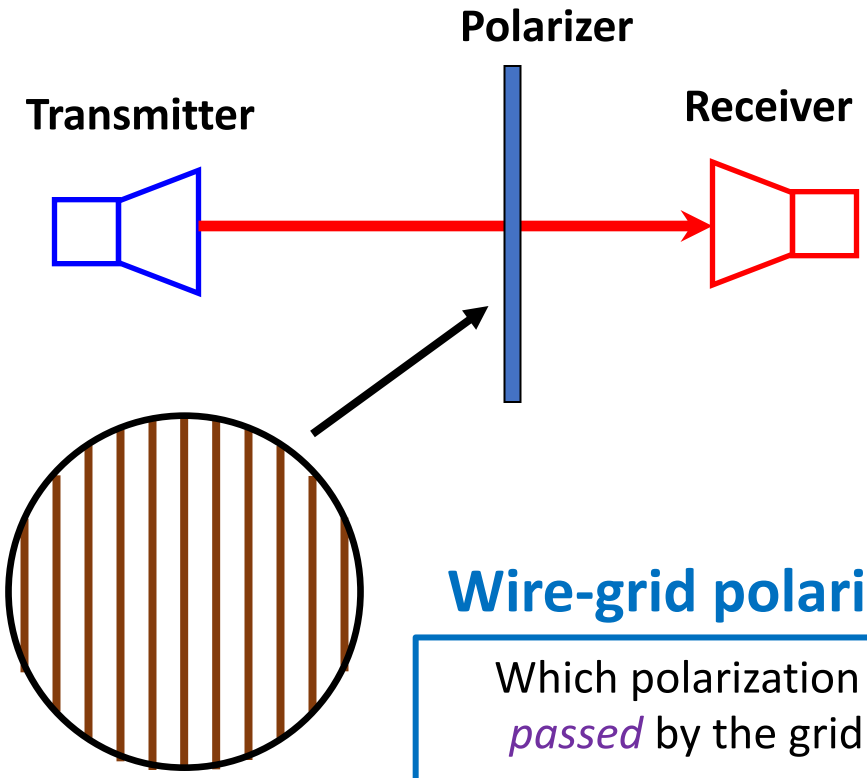
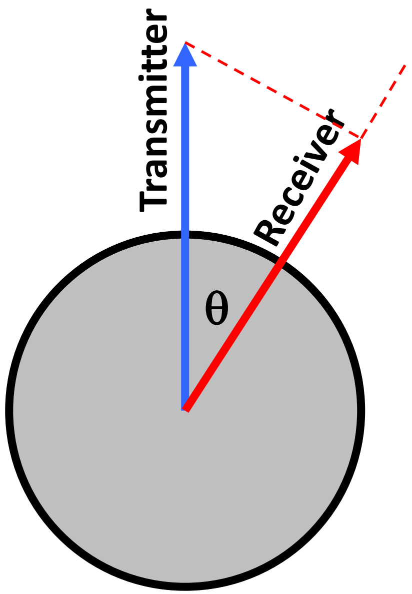
Experiment #5: Microwave Polarization



Wire-grid polarizer

Which polarization is *passed* by the grid?

What happens to the *perpendicular* mode?



Etienne-Louis Malus

1775 – 1812

Malus's Law

$$E = E_0 \cos\theta$$

$$I \propto E^2$$

$$I = I_0 \cos^2\theta$$

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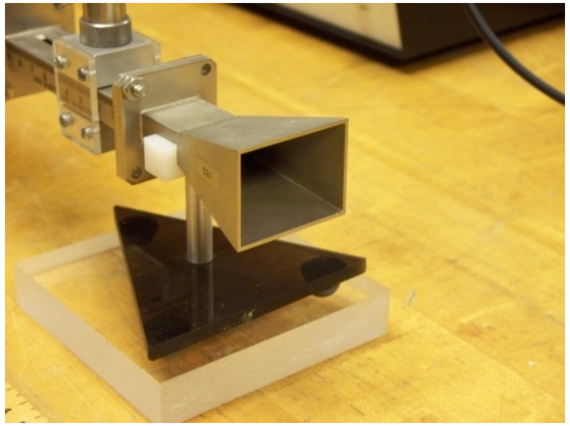
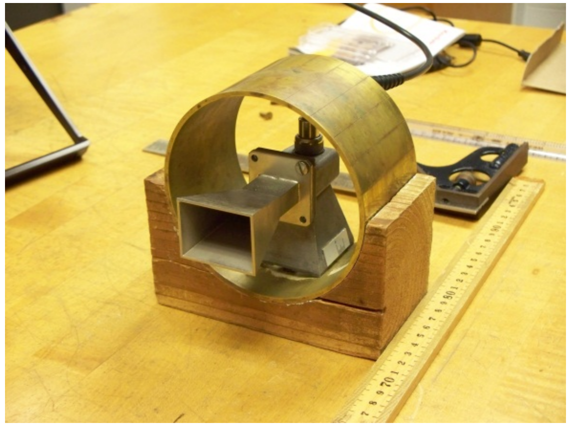
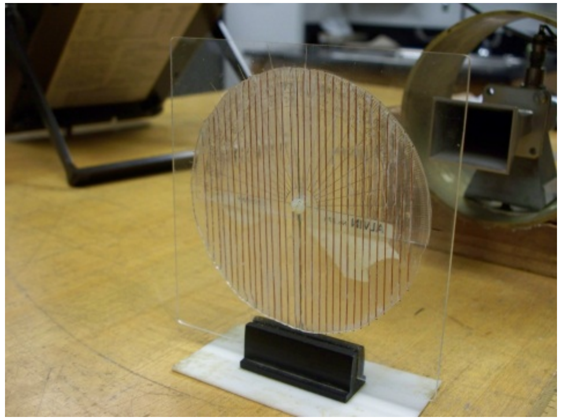
Experiment #5: Microwave Polarization



Transmitter



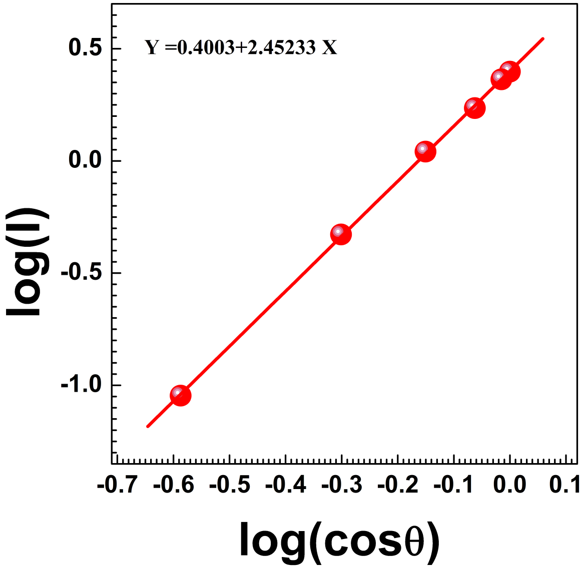
Rotatable Receiver



Polarizer

Experimental data

$I = I_0 \cos^2 \theta$



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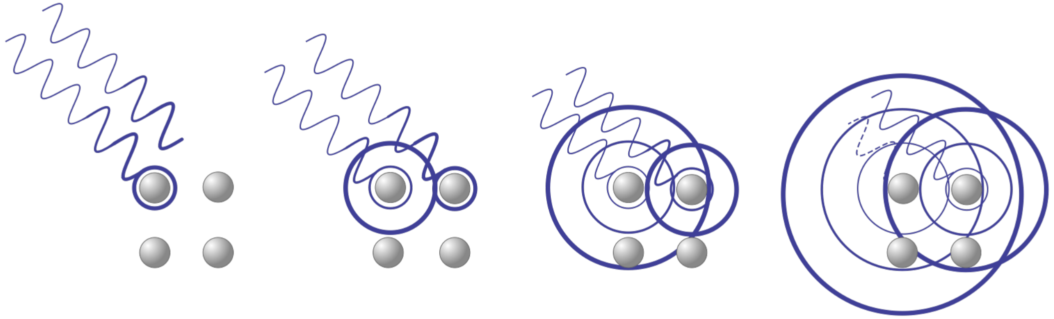
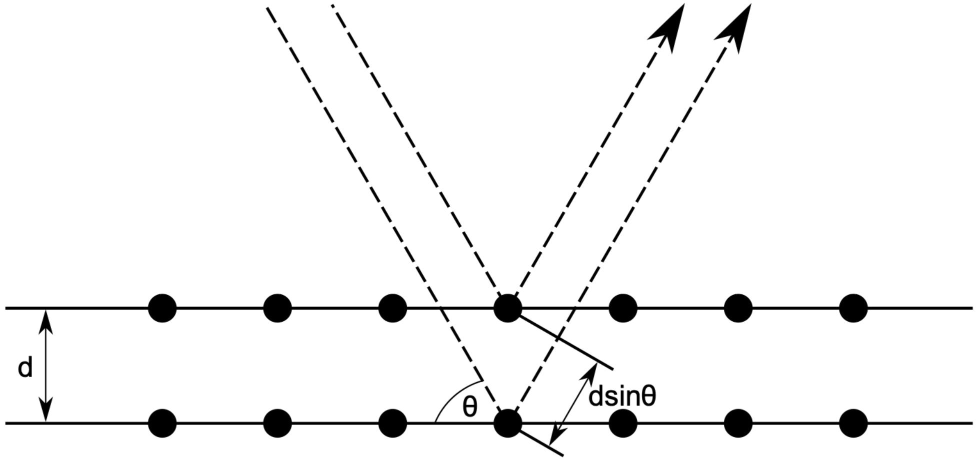
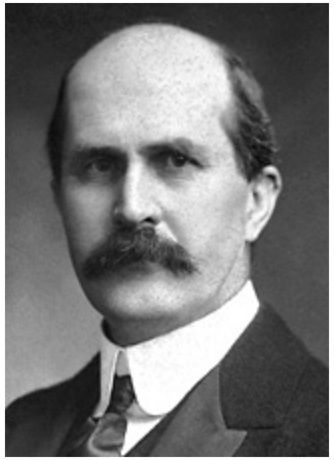
Experiment #6: Bragg Diffraction



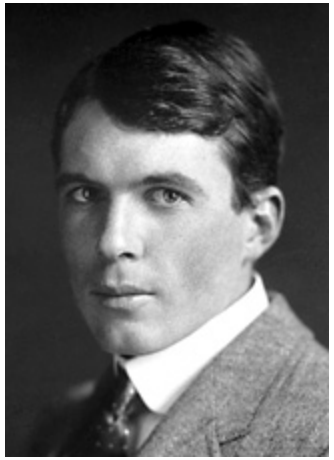
Interference of EM waves scattering from a regular lattice (e.g. X-ray scattering from a crystal).





Sir William Henry Bragg

1862-1942



Sir William Lawrence Bragg
1890-1971

The Nobel Prize in Physics 1915

"for their services in the analysis of crystal structure by means of X-rays"

Bragg's Law

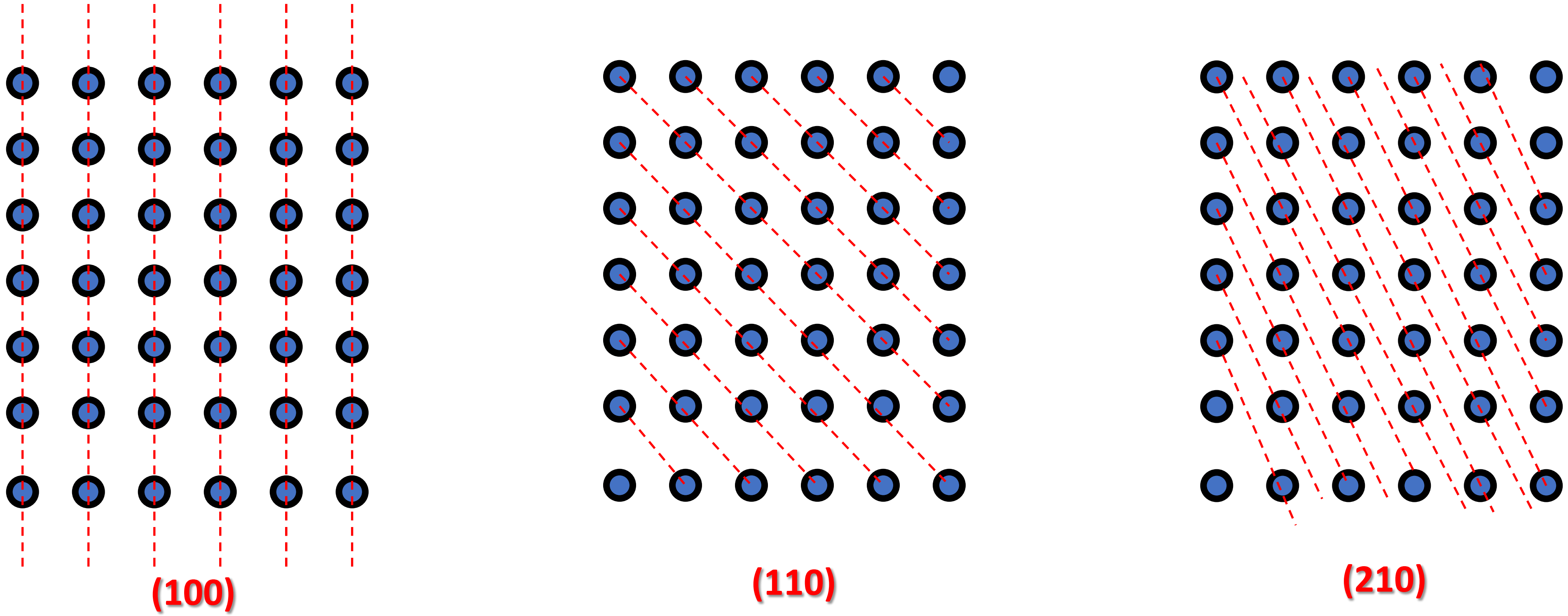
Interference maxima

$$n\lambda = 2d\sin\theta$$

E.V. Colla

[Images: Wikipedia]

Experiment #6: Bragg Diffraction



Different crystal orientations probe interference from different crystal planes, identified by their **Miller indices**, with different spacings (*d*)

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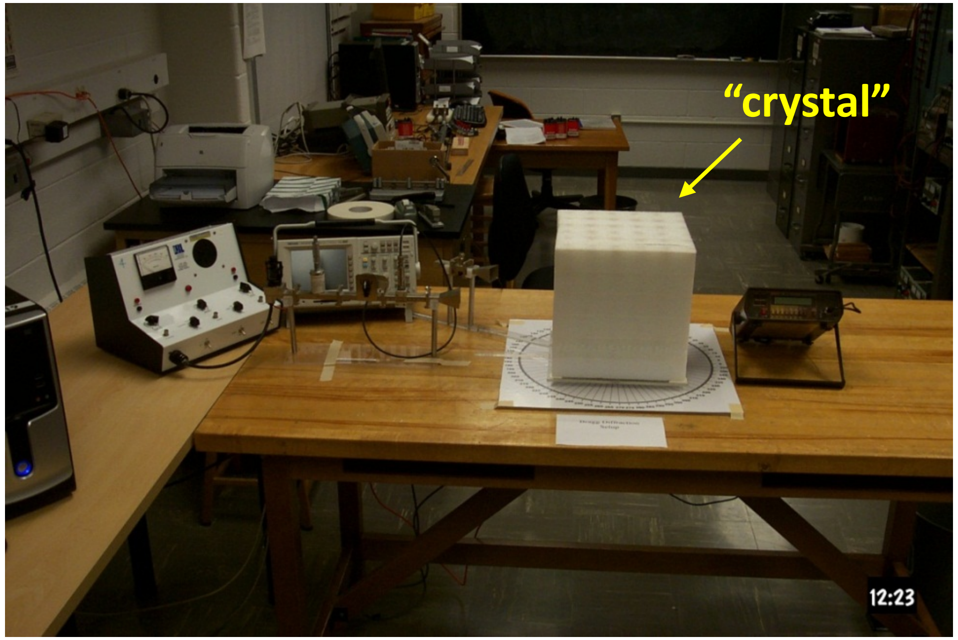
Experiment #6: Bragg Diffraction

Bragg's Law

Interference maxima

$$n\lambda = 2d\sin\theta$$



Experimental setup

In our experiment, ν~10 GHz, so **λ~3 cm**

For cubic symmetry, the angles of the Bragg peaks can be calculated from:

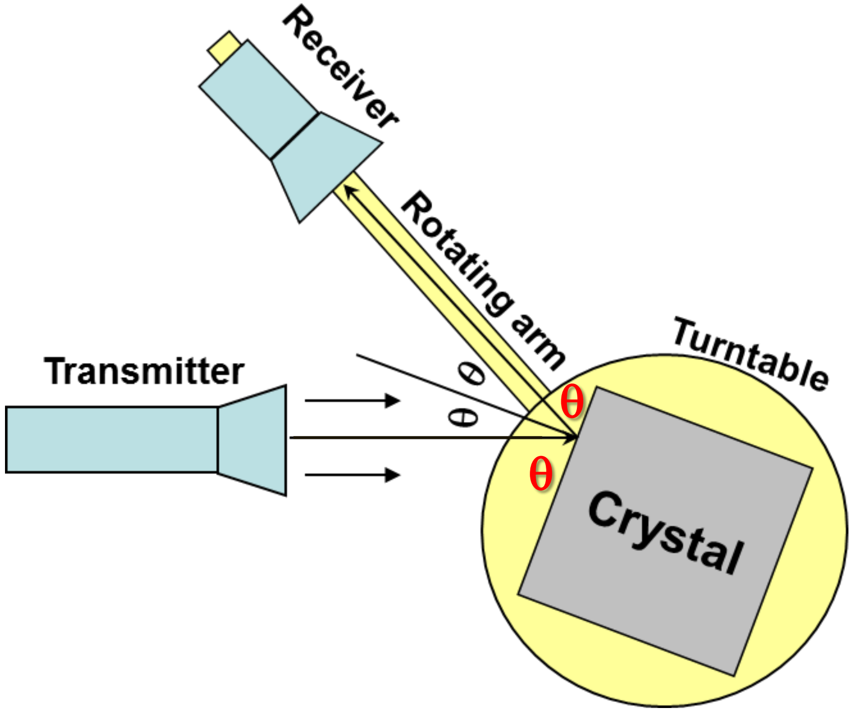
$$\boxed{\left(\frac{\lambda}{2d}\right)^2 = \frac{\sin^2\theta}{h^2+k^2+l^2}}$$

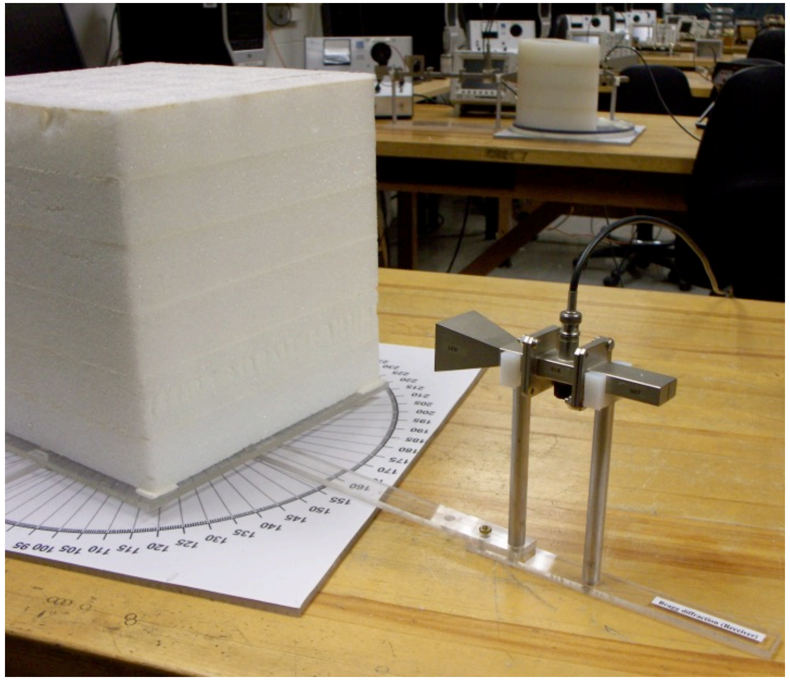
where (*h k l*) are the Miller indices of a crystal plane (*non-neg integer triples*)

The first three Bragg peaks for the (100) orientation are at angles ~17.5°, 36.9°, 64.2°

E.V. Colla

Experiment #6: Bragg Diffraction





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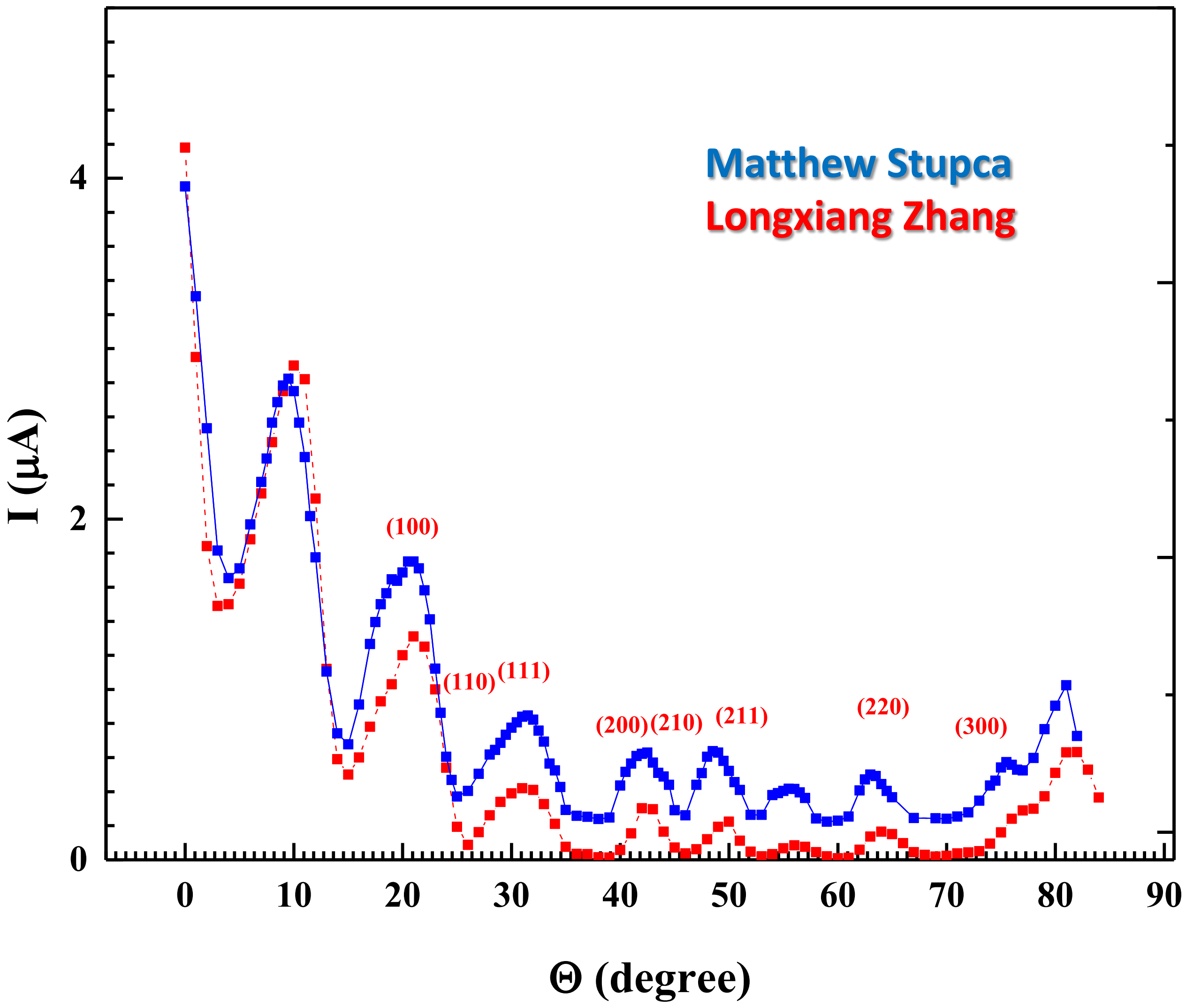
Experiment #6: Bragg Diffraction



*courtesy of Matthew Stupca, E.V. Colla

Lab Suggestions

Less useful this year…

1. Take care in handling the klystron: it gets very hot, and a high voltage (~300 V) is applied to the repeller

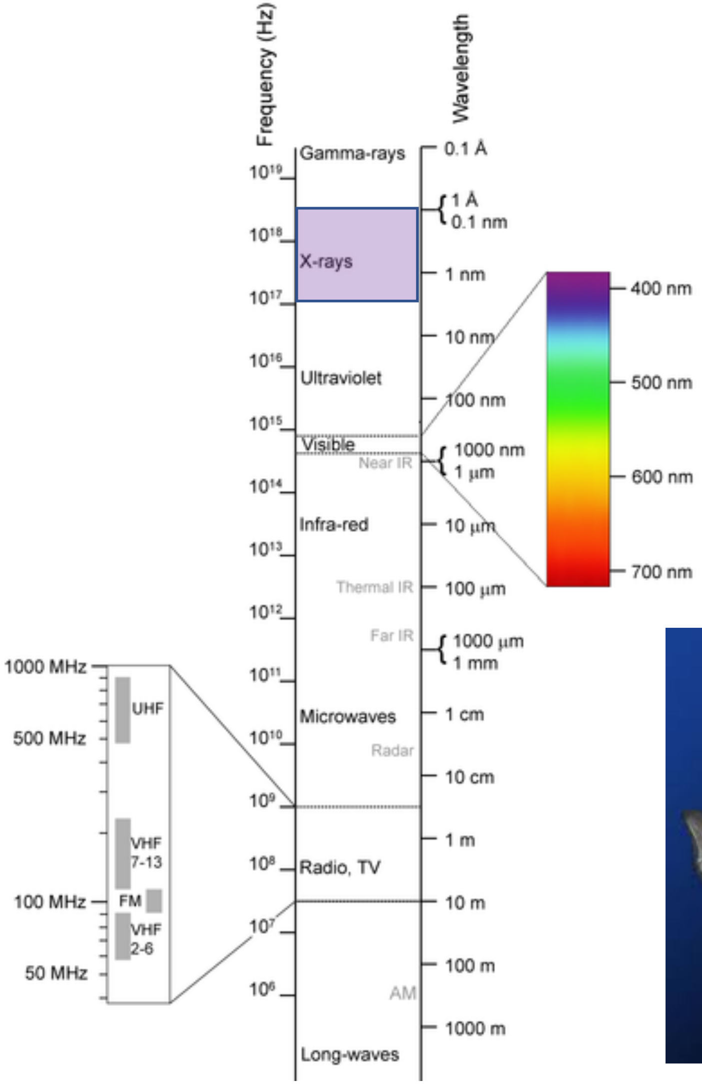
2. Keep the tables clear of extra stuff. Microwaves can reflect from these objects, yielding spurious peaks and smearing.

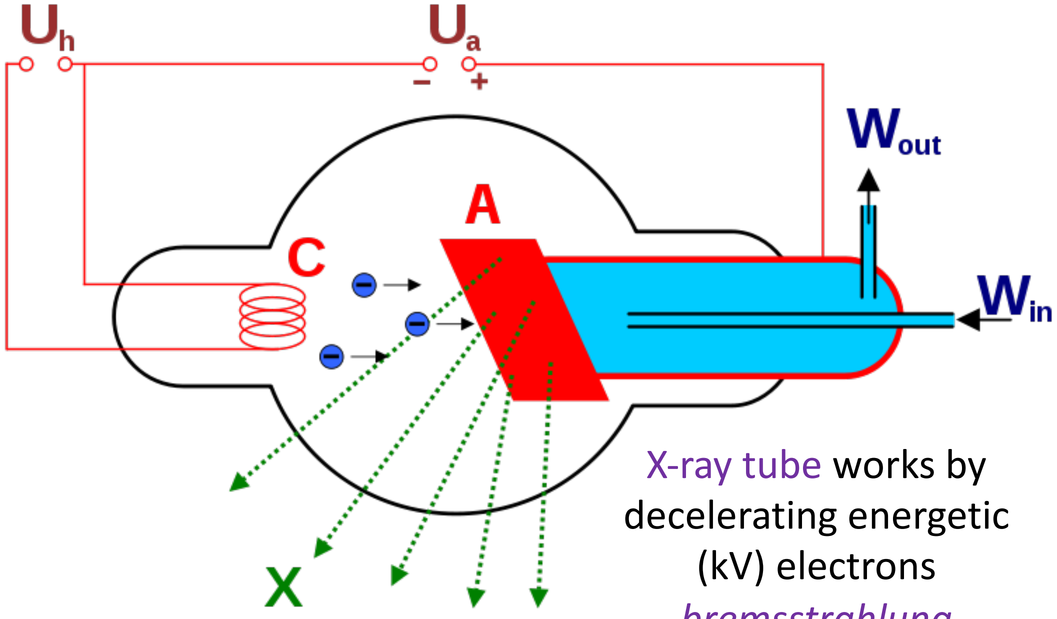
3. You have 6 experiments (!!) to do in one lab session, so take care with time management. The Bragg diffraction experiment is the most time-consuming.

4. The equipment for Week 2 will be different, so please finish all Week 1 measurement in the first week.

via E.V. Colla

Bonus #1: Bragg Diffraction of X-Rays





X-ray tube works by decelerating energetic (kV) electrons
bremsstrahlung

X-rays

- Discovered by Rontgen (1895)
  first Nobel Prize in Physics, 1901
- Used for imaging, cancer treatments, crystallography
- Wavelengths 0.1-10 nm

Bonus #1: Bragg Diffraction of X-Rays

X-ray K-series spectral line wavelengths (nm) for some common target materials

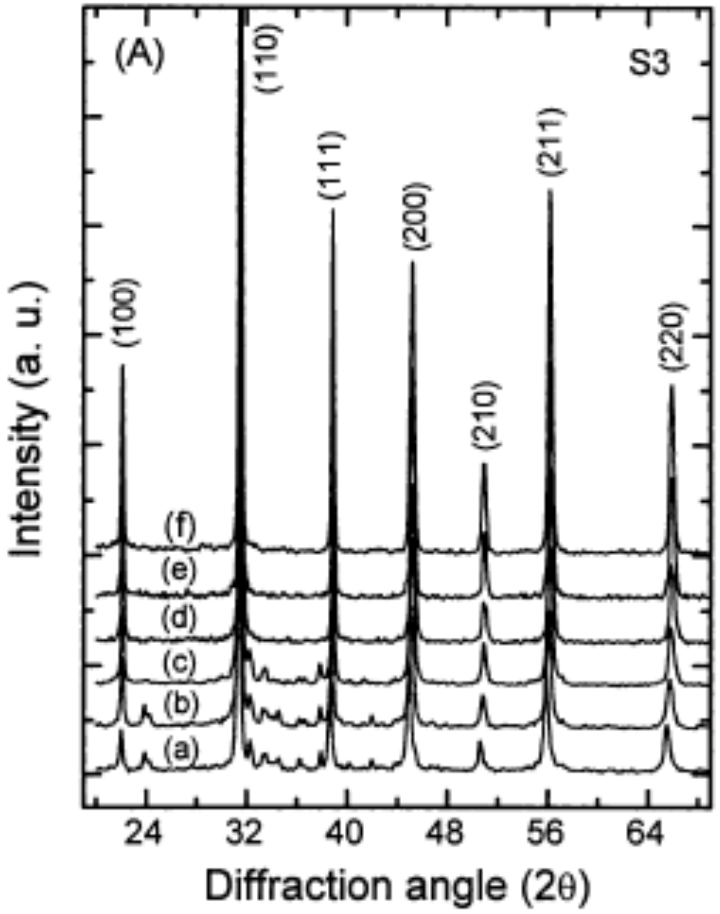
Target	$K\beta_1$	$K\beta_2$	$K\alpha_1$	$K\alpha_2$
Fe	0.17566	0.17442	0.193604	0.193998
Co	0.162079	0.160891	0.178897	0.179285
Ni	0.15001	0.14886	0.165791	0.166175
Cu	0.139222	0.138109	0.154056	0.154439
Zr	0.70173	0.68993	0.78593	0.79015
Mo	0.63229	0.62099	0.70930	0.71359

Characteristic lines from internal electron transition energies

David R. Lide, ed. (1994). *CRC Handbook of Chemistry and Physics 75th edition*. CRC Press. pp. 10–227

***courtesy of Matthew Stupca, E.V. Colla**

Bonus #1: Bragg Diffraction of X-Rays



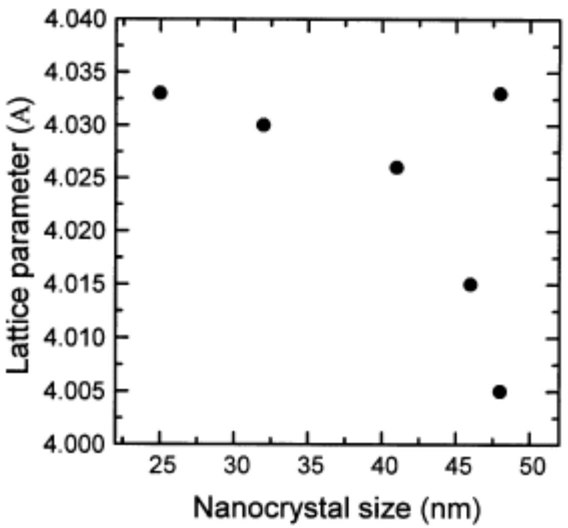


Fig. 4. Lattice parameter c versus the grain size in the $BaTiO_3$ nanocrystal.

Solid State Communications 119 (2001) 659–663

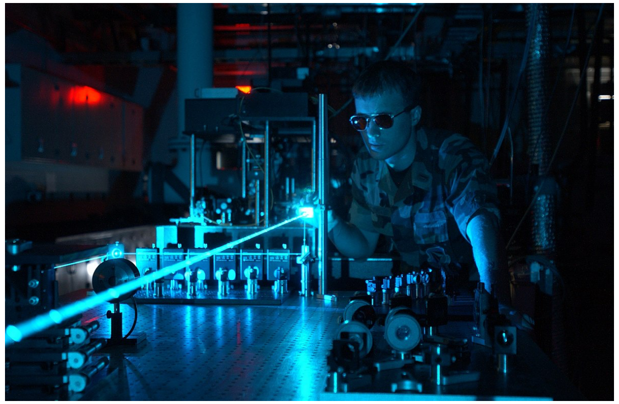
Study of structural and photoluminescent properties in barium titanate nanocrystals synthesized by hydrothermal process

Ming-Sheng Zhang[a,*], Zhen Yin[a], Qiang Chen[a], Weifeng Zhang[b], Wanchun Chen[c]

***courtesy of Matthew Stupca, E.V. Colla**

Bonus #2: The Terahertz Gap

- Below 100 GHz (esp. <30 GHz), we build radios
  - Move electrons around coherently in high-Q electronic systems
  - Electronic amplifiers, oscillators (often crystals), mixers, antennas
  - Heterodyne receivers, diode detectors

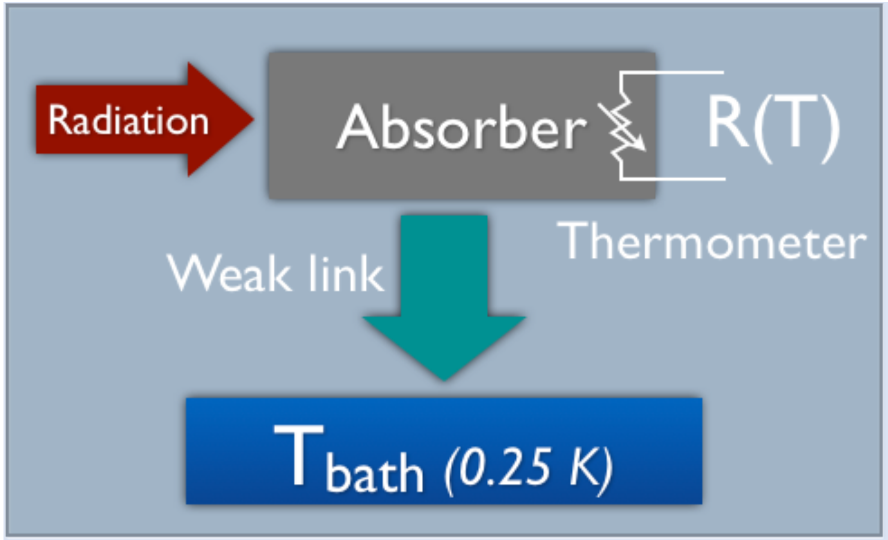


- Above 10 THz (infrared), we build photonics
  - Control discrete energy-level transitions of electrons
  - Lasers, LEDs, nonlinear crystal mixers, lenses, optical cavities
  - CCDs, photomultipliers, photographic plates

Images: Wikipedia

- In between, no mass-produced powerful (*watt-scale*) transmitters are available, nor sensitive detectors that operate at room temperature
  - Switching electrons so rapidly is hampered by stray impedances, heat dissipation
  - Quanta are thermally excited ($h\nu < k_B(300\ K)$), and appropriate transitions are rare

Bonus #3: Bolometric Detectors



You can detect *any* radiation by turning it into **heat**!

Incident radiation heats up an absorber that is thermally isolated from its environment.

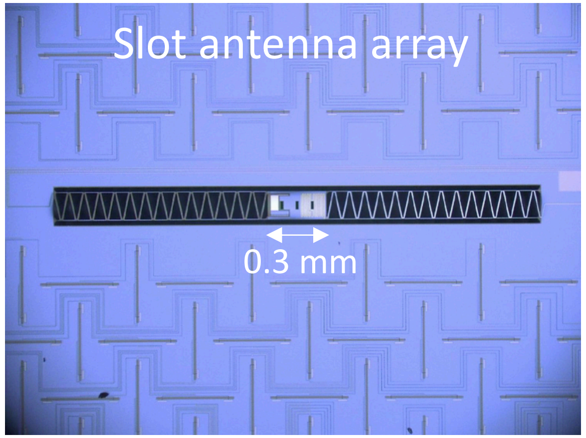
A sensitive thermometer measures the changing absorber temperature

Ultimate limitation is *kT* thermal fluctuations, so we typically must operate at **<1 Kelvin**

Caltech/JPL

"Spiderweb" bolometer for mm-wave radiation
Used on BOOMERANG balloon, Planck satellite

Antenna-coupled transition-edge sensor bolometer (**150 GHz**) used on SPIDER CMB balloon



Caltech/JPL