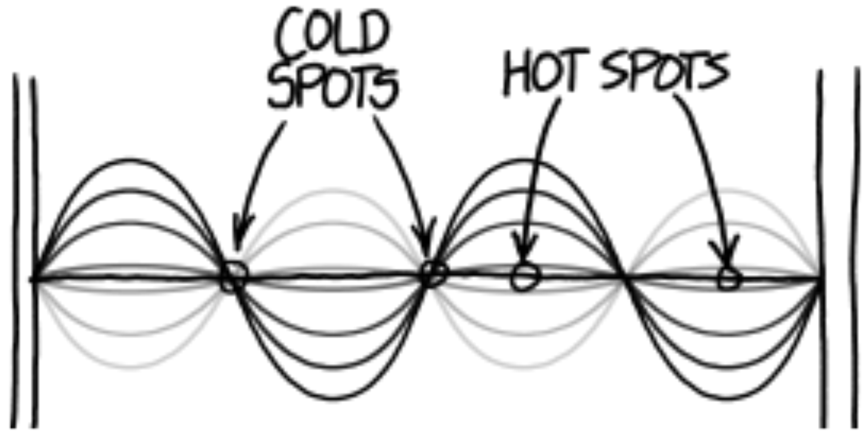
[XKCD: What If 131]

Microwave Cavities

Prof. Jeff Filippini

Physics 401

Spring 2020



[XKCD: What If 131]

ILLINOIS

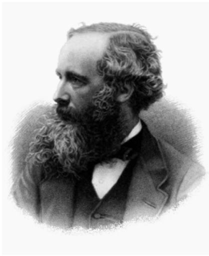
Key Goals of this Lab

Study the behavior of microwaves in resonant cavities.

- **Waves in waveguides**
- **Standing waves and resonance**
- **Lab setup**
- **Microwave cavity experiment**
- ***Bonus****: The Cosmic Microwave Background*

This is the second week of a two-week lab

Reminder: Plane Waves in Free Space



Source-free Maxwell's Eqns

$$\nabla \cdot \vec{D} = 0 \qquad \nabla \cdot \vec{B} = 0$$

$$\nabla \times \vec{E} = -\frac{\partial \vec{B}}{\partial t} \qquad \nabla \times \vec{H} = \frac{\partial \vec{D}}{\partial t}$$

→ Uniform plane wave in z-direction, with $\vec{H} \perp \vec{E}$

→ Wave equation

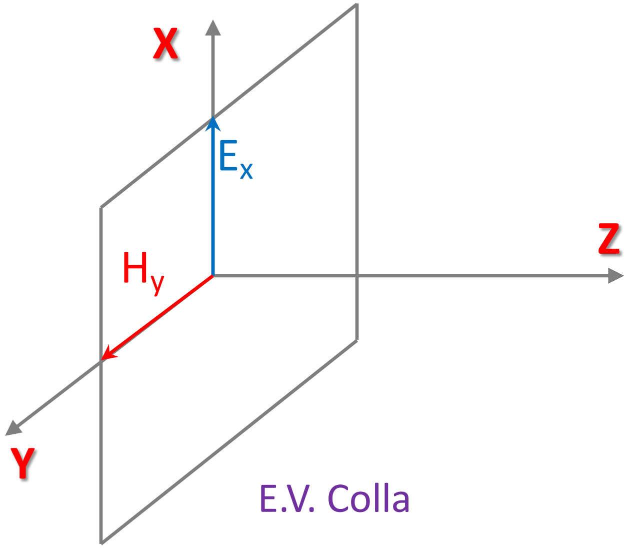
$$\frac{\partial^2 E_x}{\partial z^2} = \frac{1}{v^2}\frac{\partial^2 E_x}{\partial t^2}$$

↓ General solution

$$E_x(z,t) = f\left(t - \frac{z}{v}\right) + \mathrm{g}\left(t + \frac{z}{v}\right)$$

← Fourier components

$$E_x = E_0 e^{i(\omega t - kz)}$$



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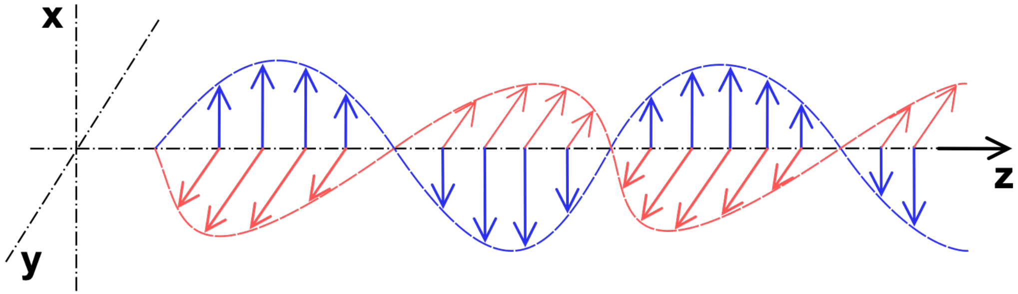

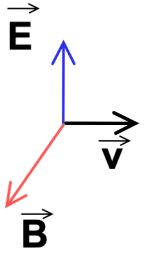

$$v = \frac{1}{\sqrt{\varepsilon\mu}} = \frac{c}{\sqrt{\varepsilon_r \mu_r}}$$

$$E_x = \sqrt{\frac{\mu}{\varepsilon}} H_y \equiv Z H_y$$

$$Z_0 = \sqrt{\frac{\mu_0}{\varepsilon_0}} \approx 377\ \Omega$$

[Wikipedia: Electromagnetic Radiation]

Wave Propagation in Waveguides

Lord Rayleigh
1842-1919
Wikipedia

Waveguides allow **1D routing** of EM waves (vs. $1/r^2$ spreading in 3D)

Impose specific **boundary conditions** on the propagating modes
E.g., zero E-field tangential to conducting wall

Wikipedia

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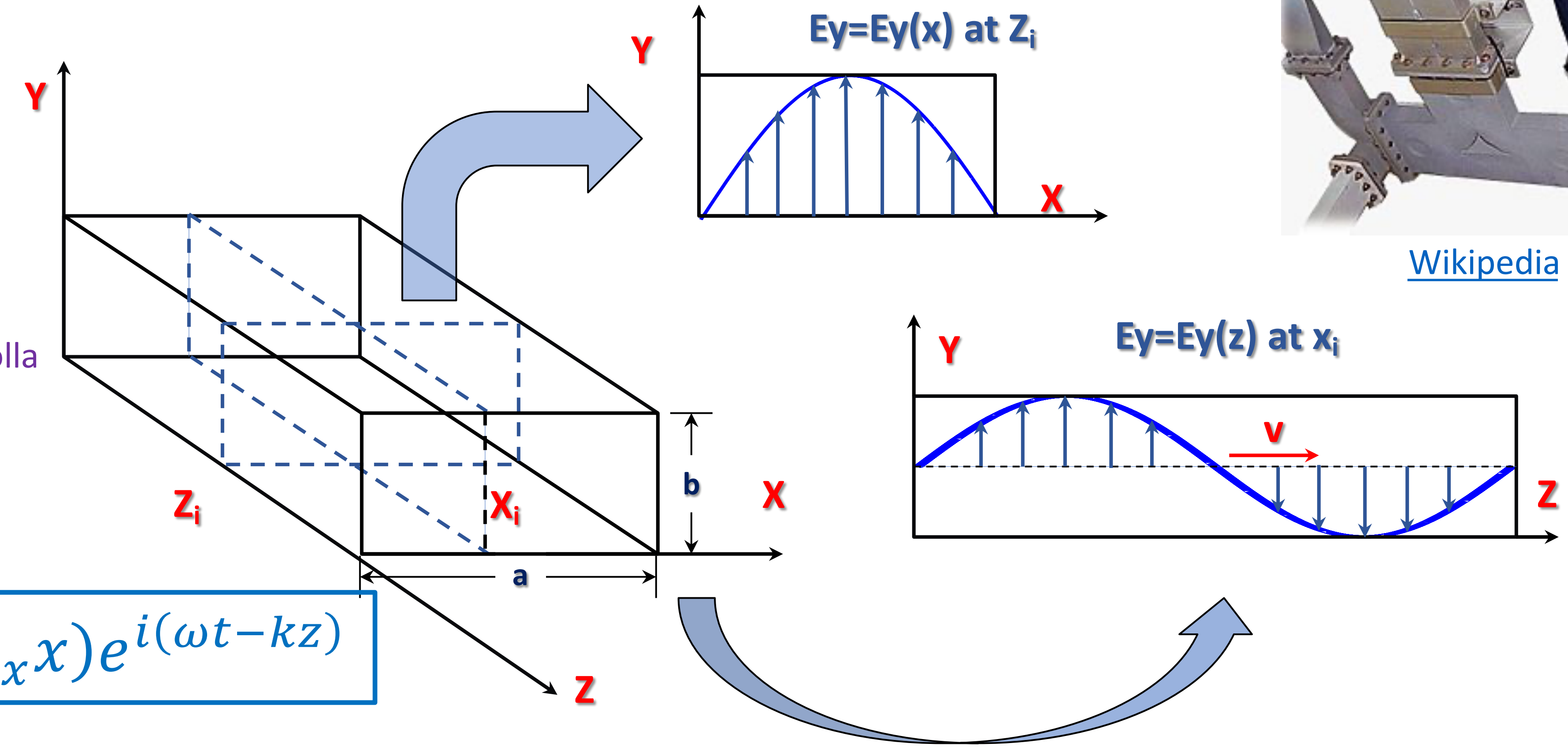


TE_{10} mode

$$E_y = E_0 \sin(k_x x) e^{i(\omega t - kz)}$$

Standing Waves in Cavities

Like particle in a box: Arbitrary wavelengths can't sustain in a finite cavity

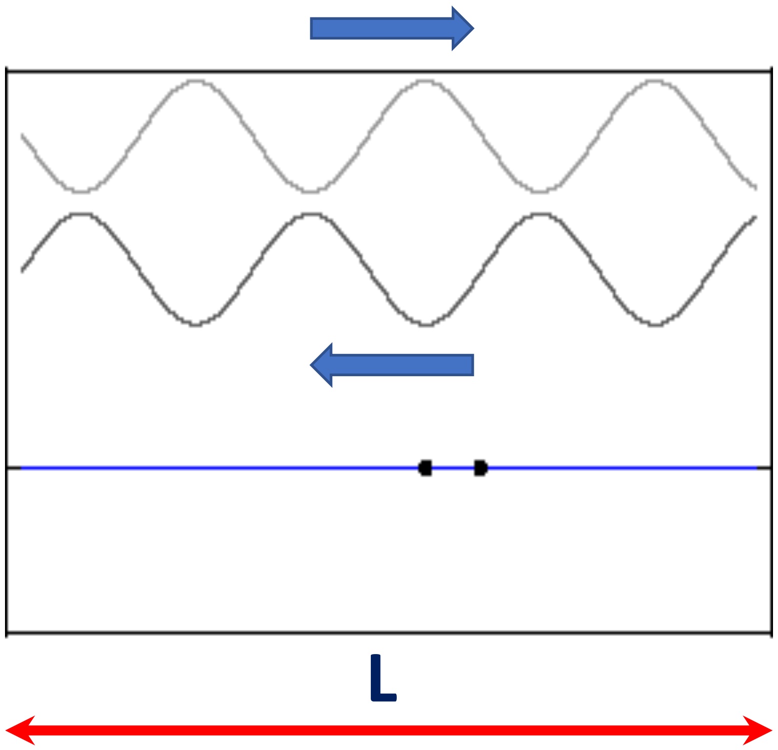
$$E_y = E_0 \sin(k_x x)\, e^{i(\omega t - kx)}$$

+

$$E_y = E_0 \sin(k_x x)\, e^{i(\omega t + kx)}$$

=

L=n*λ/2



Phase shift by π at reflection

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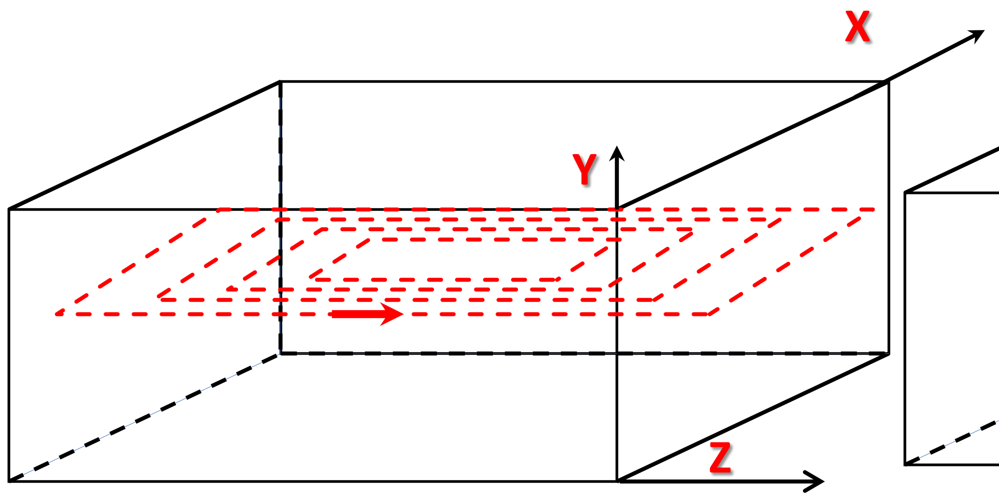
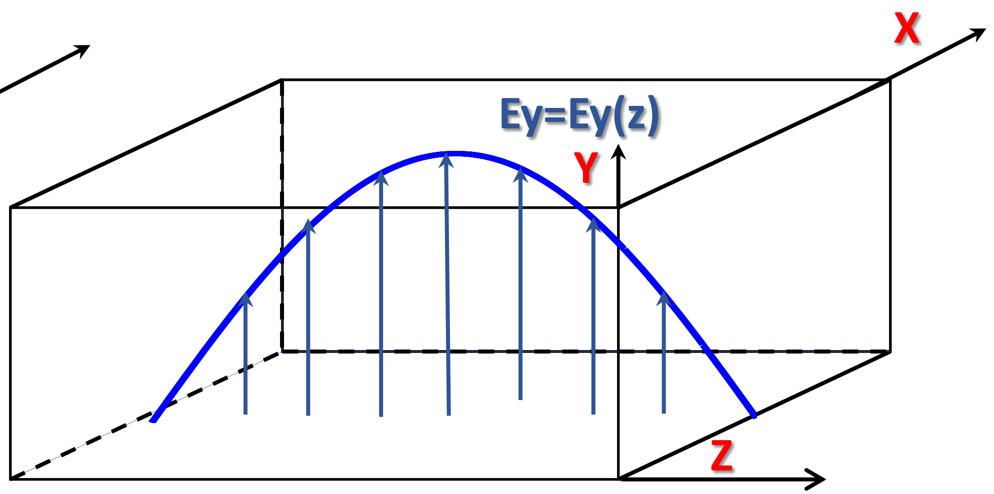
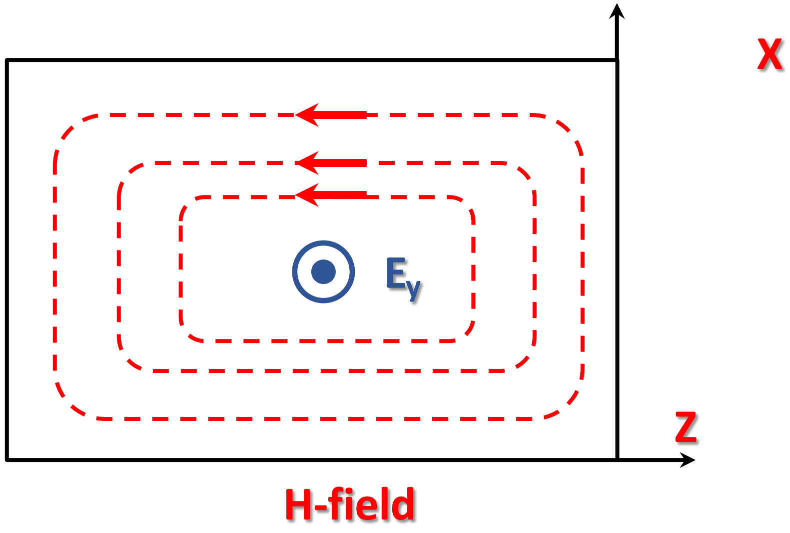
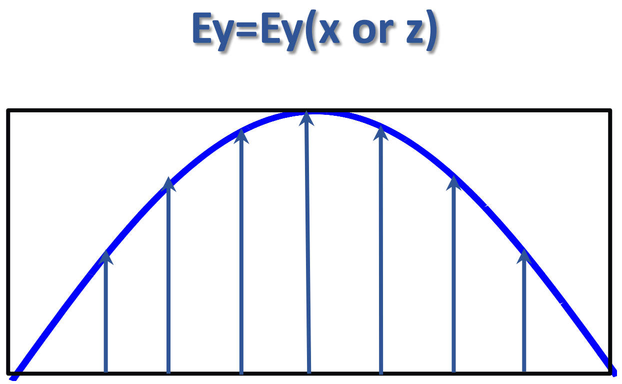
Standing Waves in Cavities









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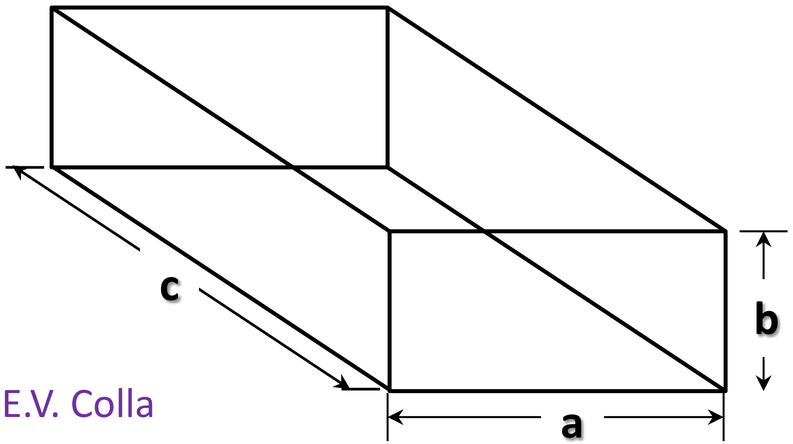
TE Resonant Frequencies

$$\omega_{mnp}^2 = v_0^2 \left[\left(m \frac{\pi}{a} \right)^2 + \left(n \frac{\pi}{b} \right)^2 + \left(p \frac{\pi}{c} \right)^2 \right]$$

Phase velocity
(c in vacuum)

Non-negative integers **m**, **n**, **p**
*At most **one** can be zero*



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Ex: **TE_{101}** mode (m=1, n=0, p=1)

$$\omega_{101}^2 = v_0^2 \pi^2 \left[\left(\frac{1}{a} \right)^2 + \left(\frac{1}{c} \right)^2 \right]$$

Lowest freq. if b smallest dimension

Coupling to the Cavity

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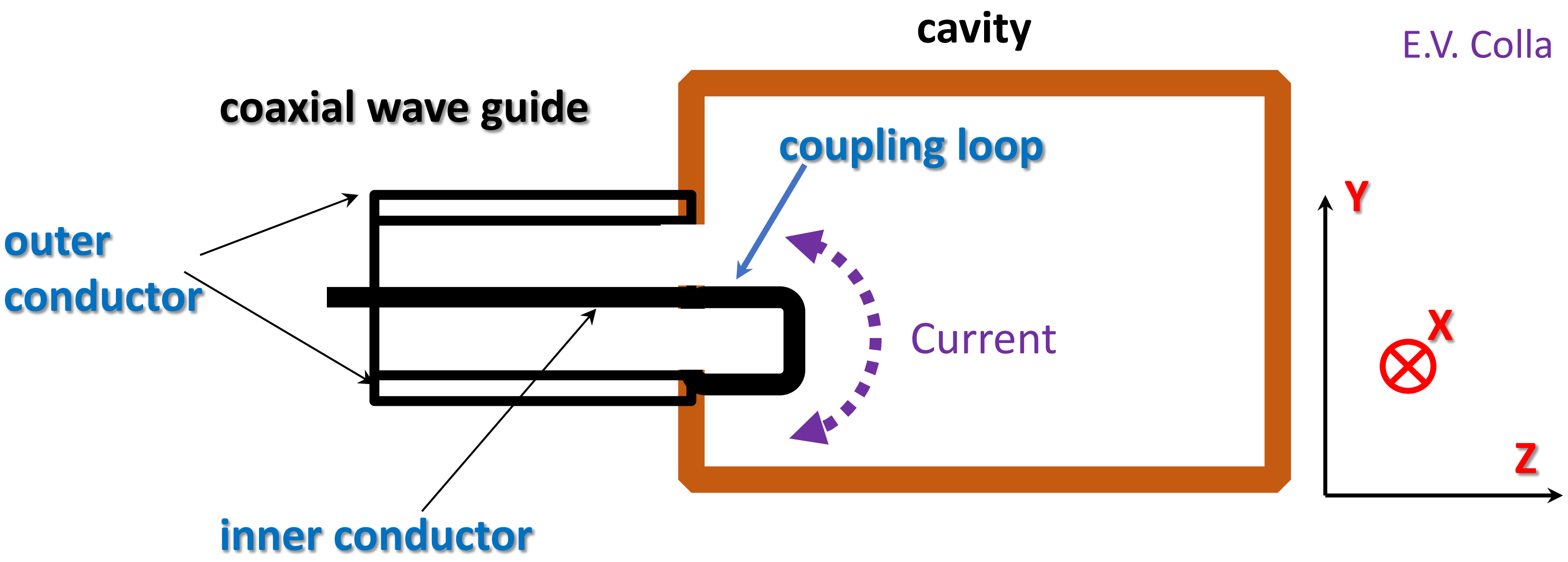
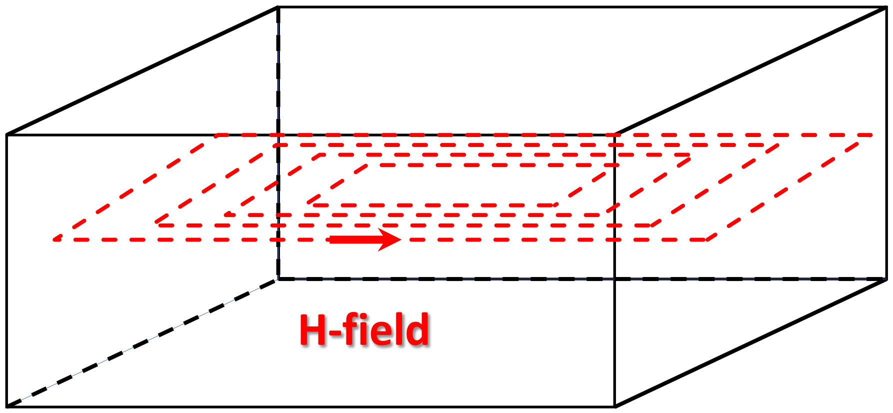
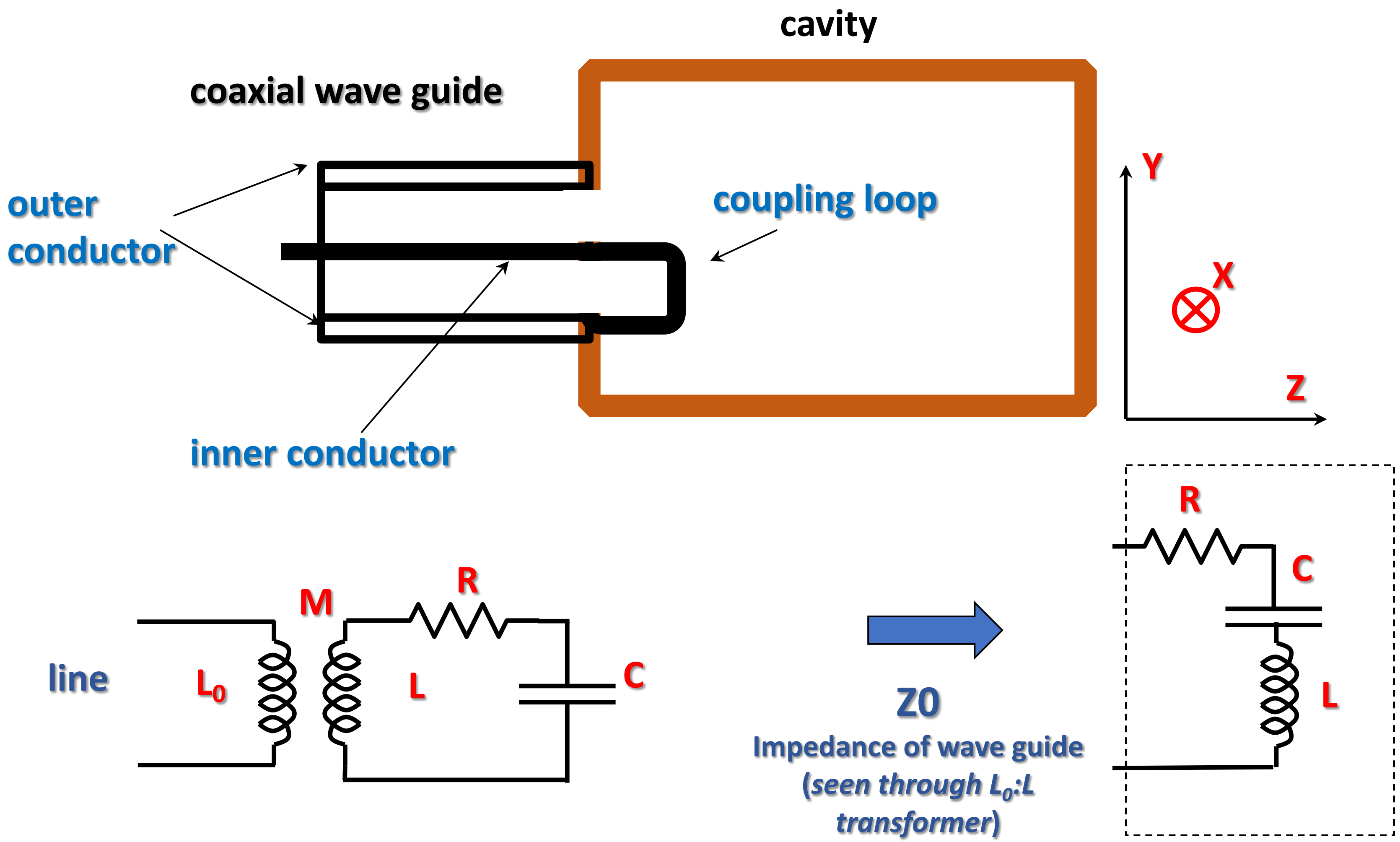


Magnetic dipole coupling antenna

AC current flow around the coupling loop excites an oscillating magnetic field H_x, which couples to the H-field pattern for a cavity TE mode



Equivalent Circuit



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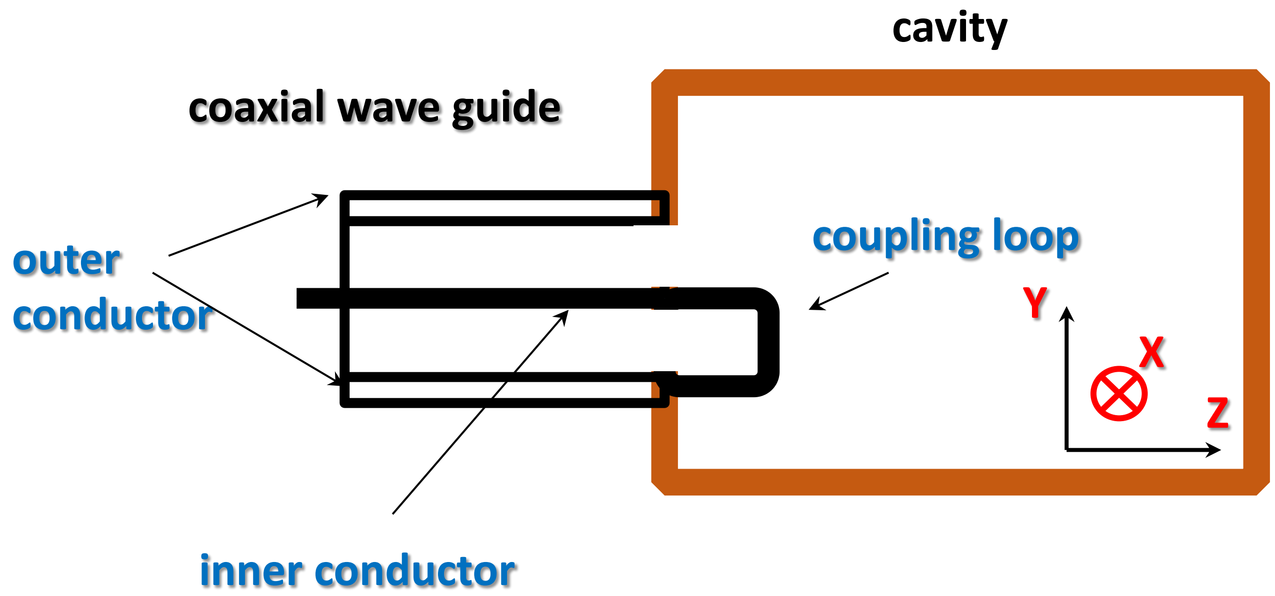
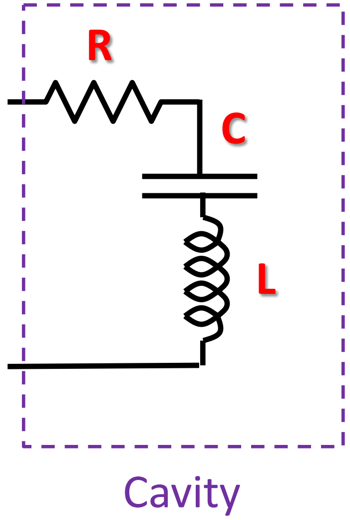
Power Coupling Efficiency



$$Q_L = \frac{\omega L}{R + Z_0}$$

$$Q_L = \frac{\omega L}{Z_0\left(1 + \frac{R}{Z_0}\right)} = \frac{Q_0}{1 + \beta}$$



Z_0
Waveguide impedance

Q_0: Quality factor without external load
β: Coupling coefficient

Maximum power transfer when:

$$Z_0 = R \quad \Longrightarrow \quad \beta = 1$$

$$\Longrightarrow \quad Q_L = \frac{1}{2} Q_0$$

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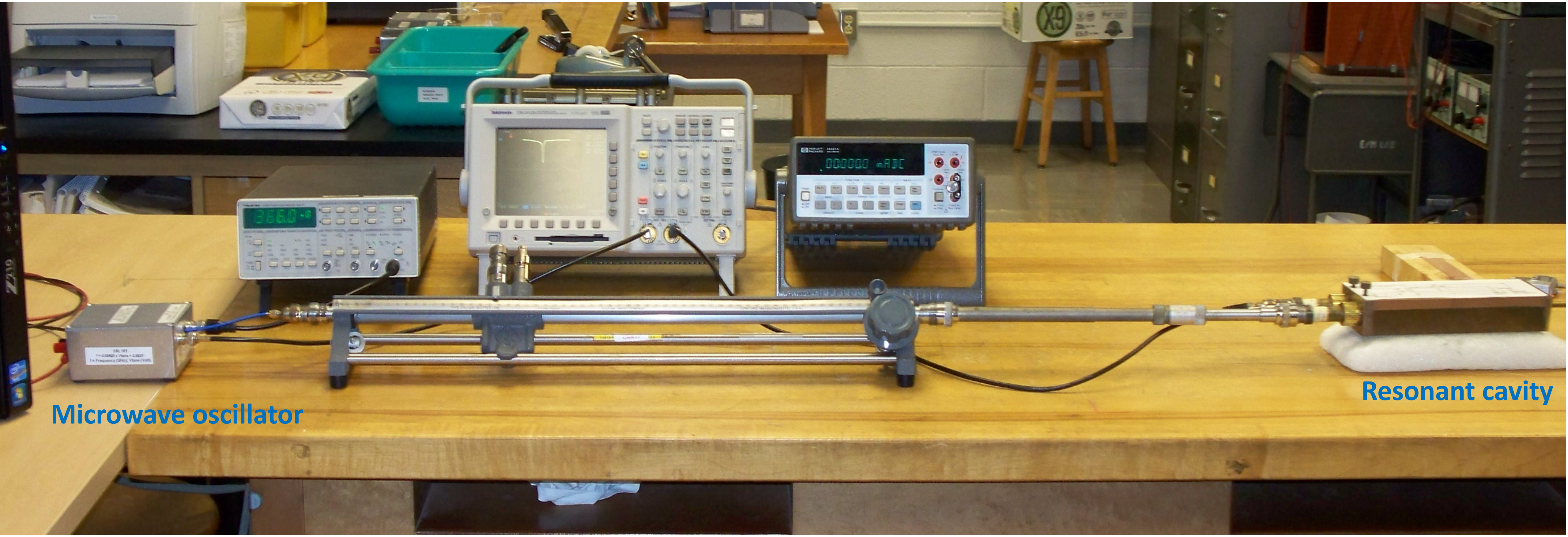
Overview of the Experiment



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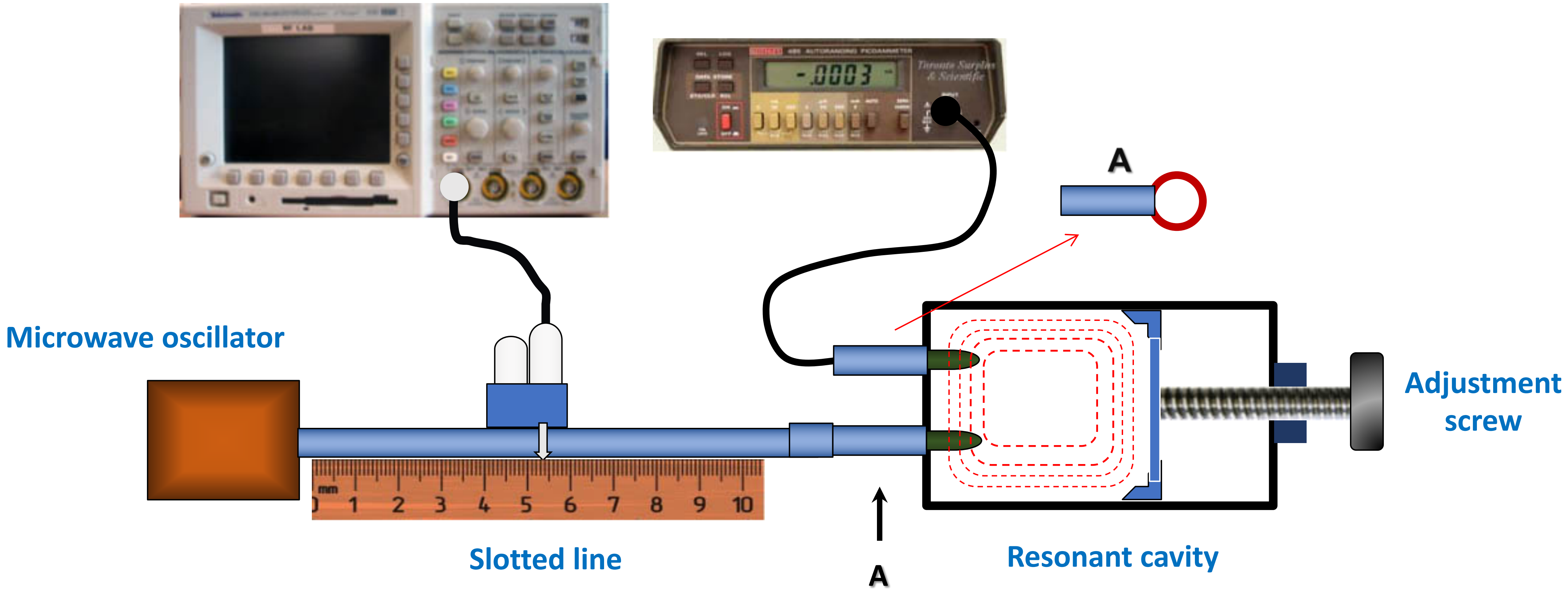
Experimental Setup



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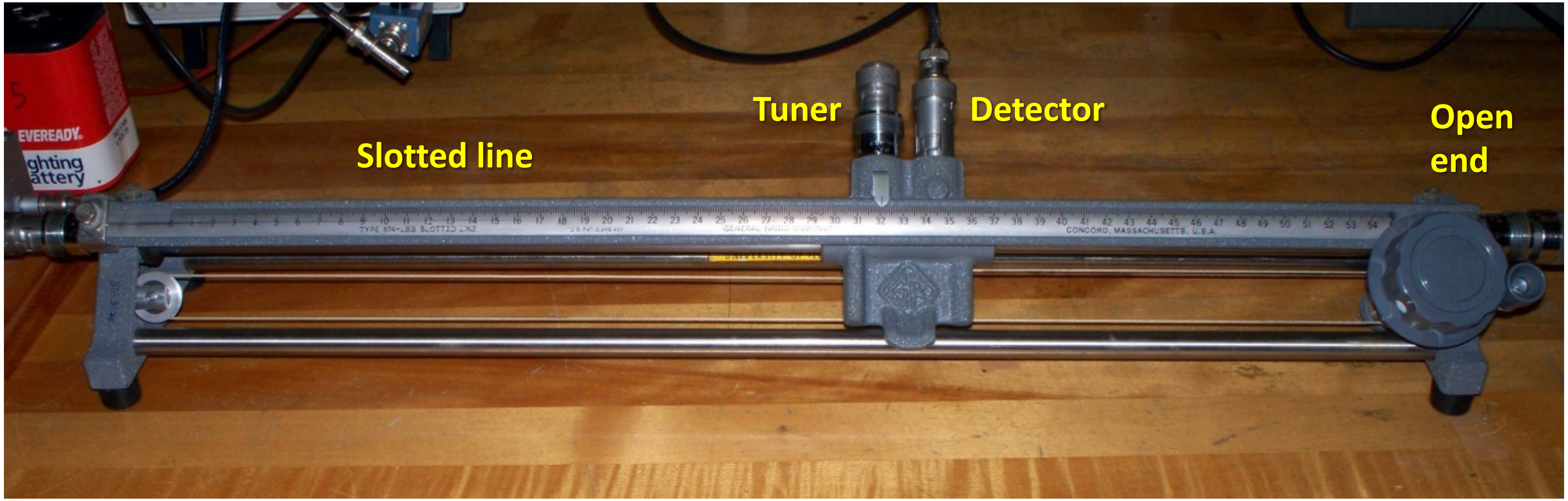
#1: Wavelength Measurement



By sliding the detector along the slotted line (waveguide), find the distance between minima.

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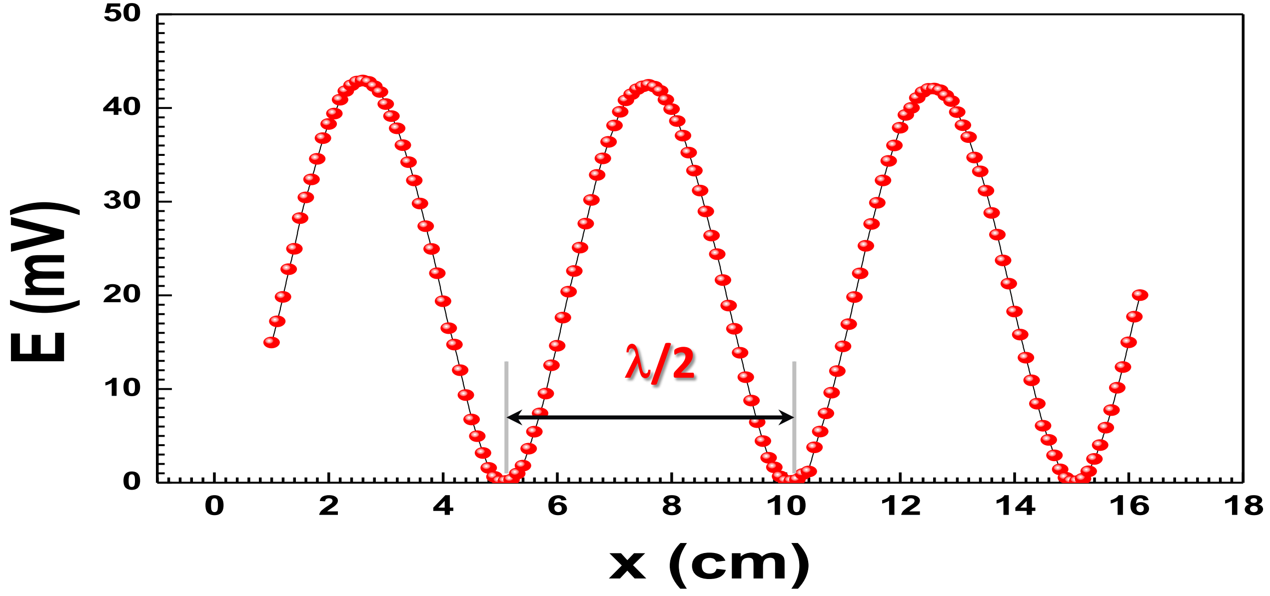
#1: Wavelength Measurement



By sliding the detector along the slotted line (waveguide), find the distance between minima.
Distance between consecutive minima of the standing wave (nodes) is **$\lambda/2$**.

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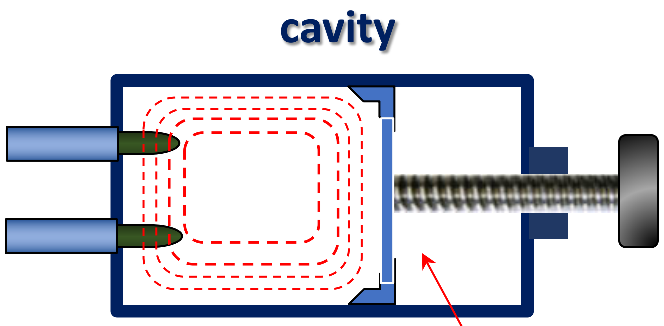
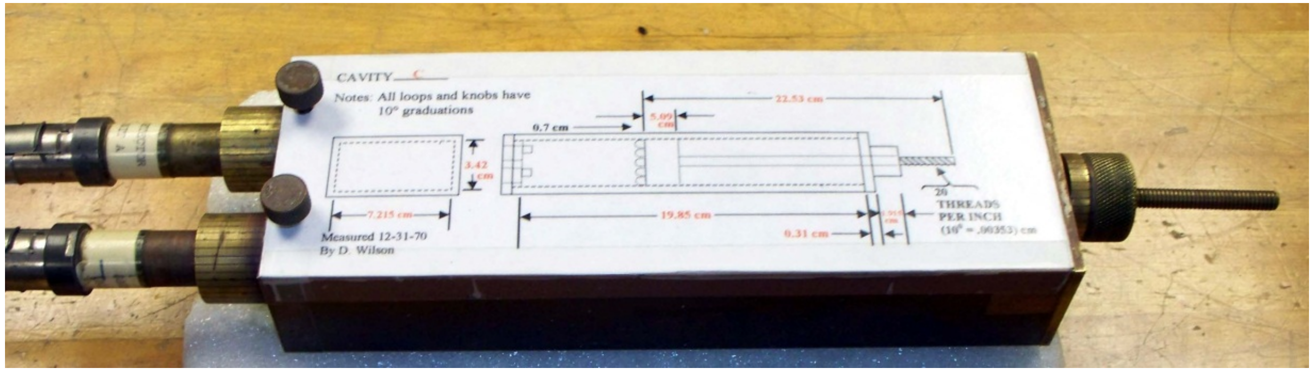
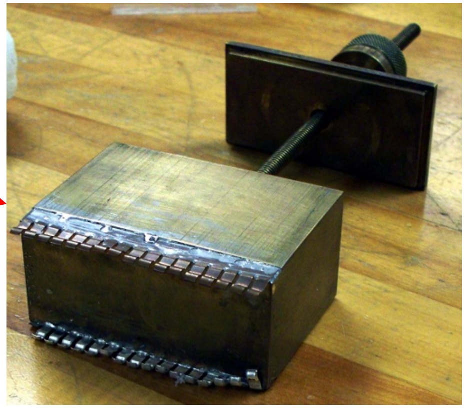
#2: Cavity Resonance





Movable plunger (c direction)



Use the **plunger** to change the length of the cavity in the z-direction.
Use the **cavity detector** to search for maxima in stored power.
Identify plunger positions (dimension *c*) corresponding to **TE_{101}** and **TE_{102}**.

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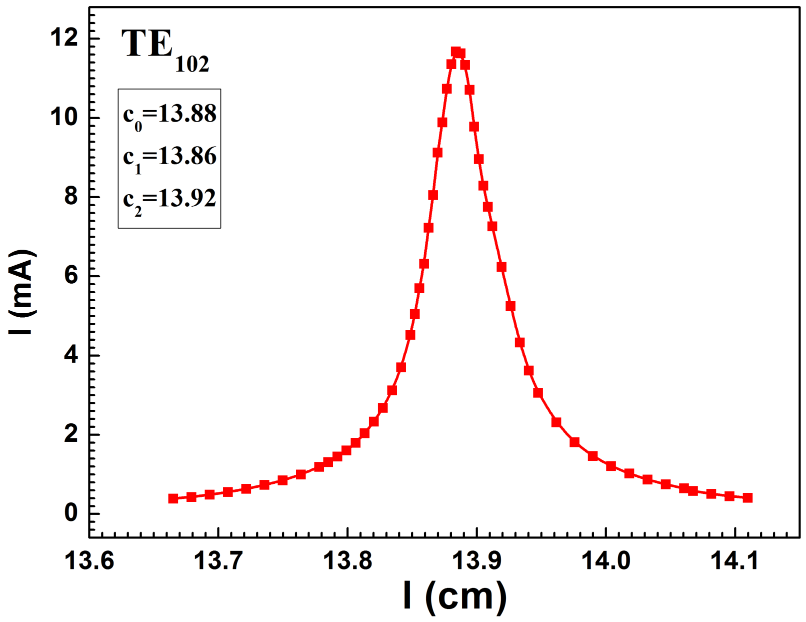
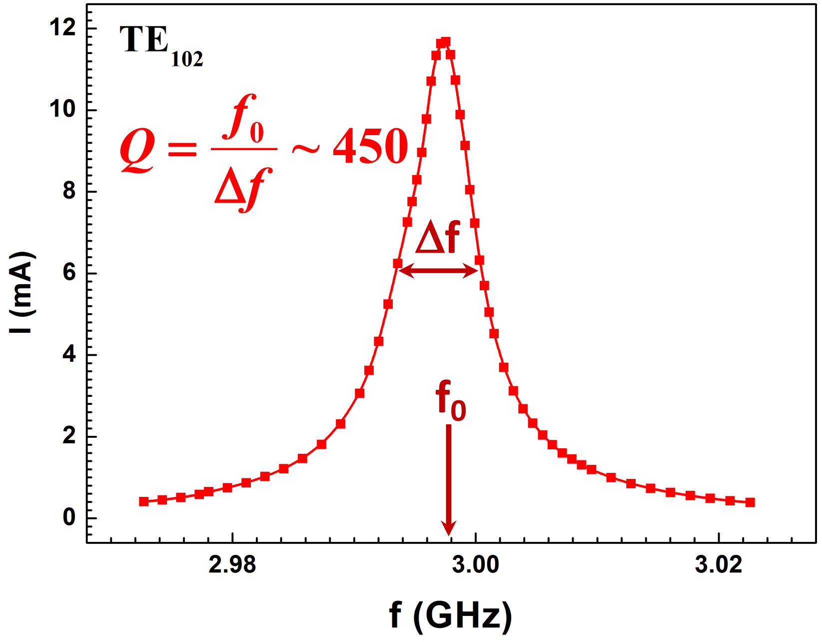
#2: Cavity Resonance

$$\omega_{102}^2 = v_0^2\pi^2\left[\left(\frac{1}{a}\right)^2 + \left(\frac{2}{c}\right)^2\right] \quad \Rightarrow \quad f_{102} = \frac{v_0}{2}\sqrt{\left(\frac{1}{a}\right)^2 + \left(\frac{2}{c}\right)^2}$$





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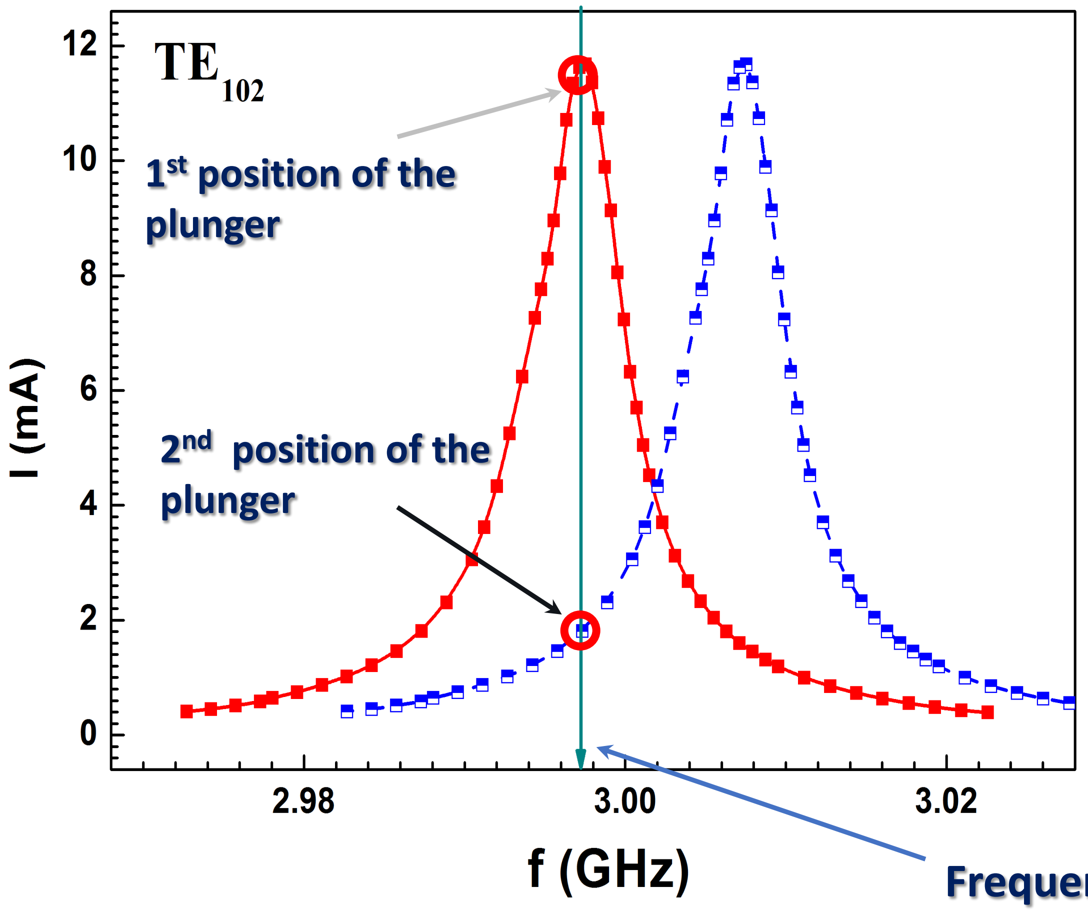
#2: Cavity Resonance



Frequency of the oscillator

By moving the plunger we change the cavity's dimensions, and thus its resonant frequencies

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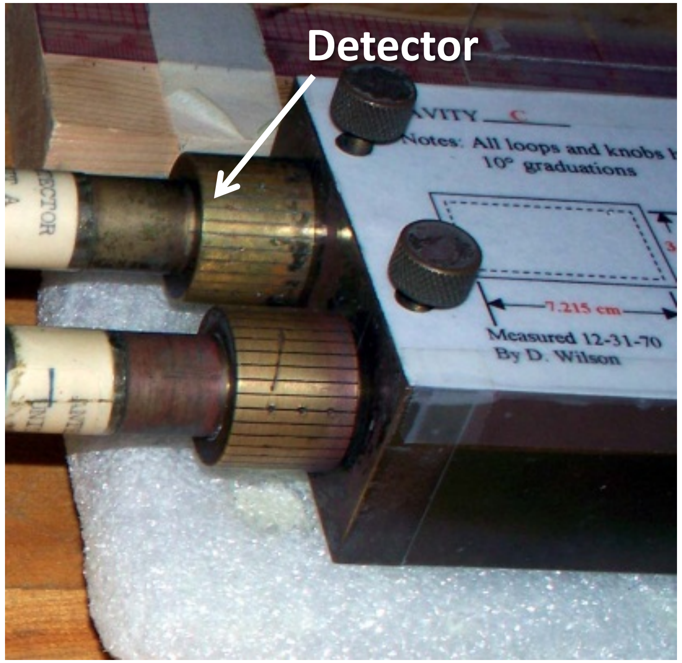
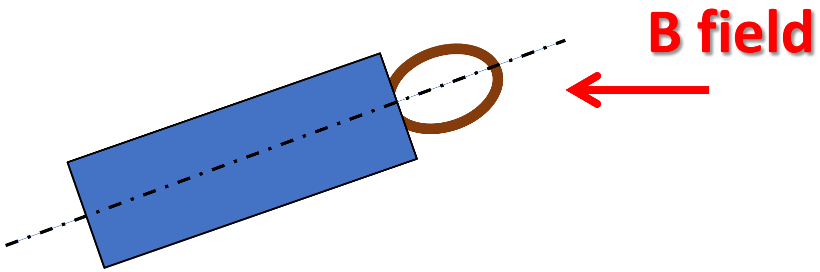
#3: Coupling – Detecting the Magnetic Field





Detector couples to flux threading the pickup loop (magnetic dipole), and thus to the perpendicular H-field

While on resonance, rotate the orientation of the input loop from vertical (in 10° steps to 360°) and read the cavity power detector.

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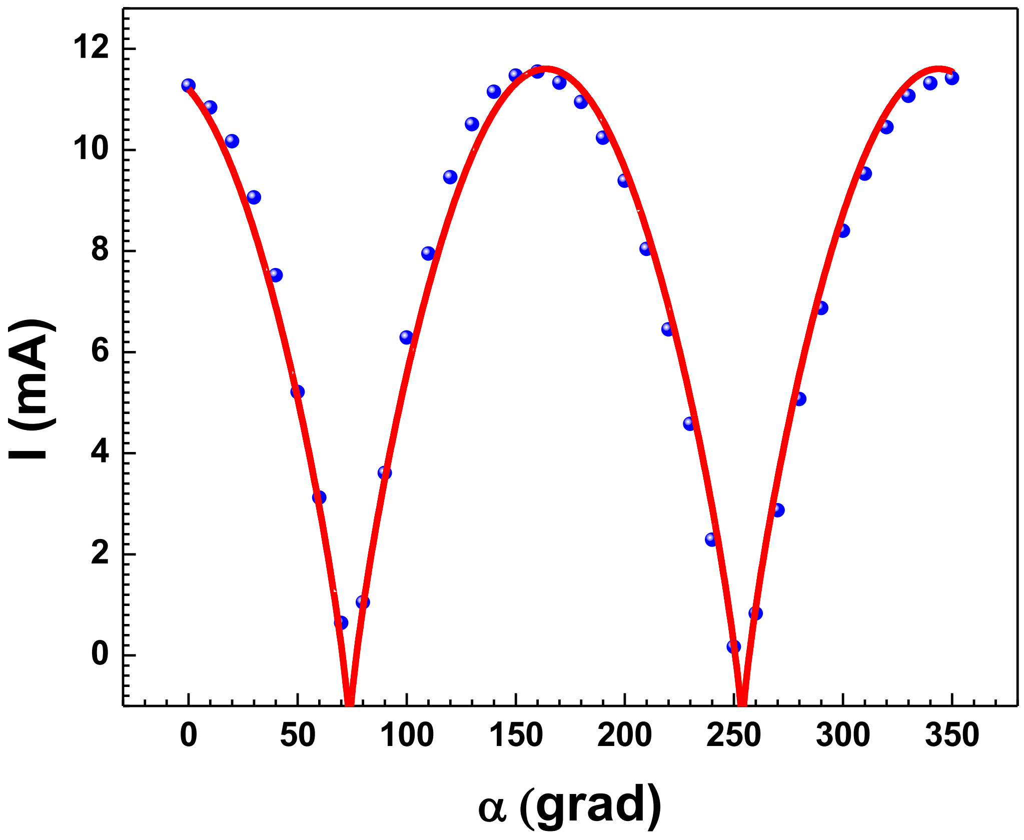
#3: Coupling – Detecting the Magnetic Field



Experimental result

Fitted to $A|(\cos(\alpha+\phi))^{n}|+A_0$

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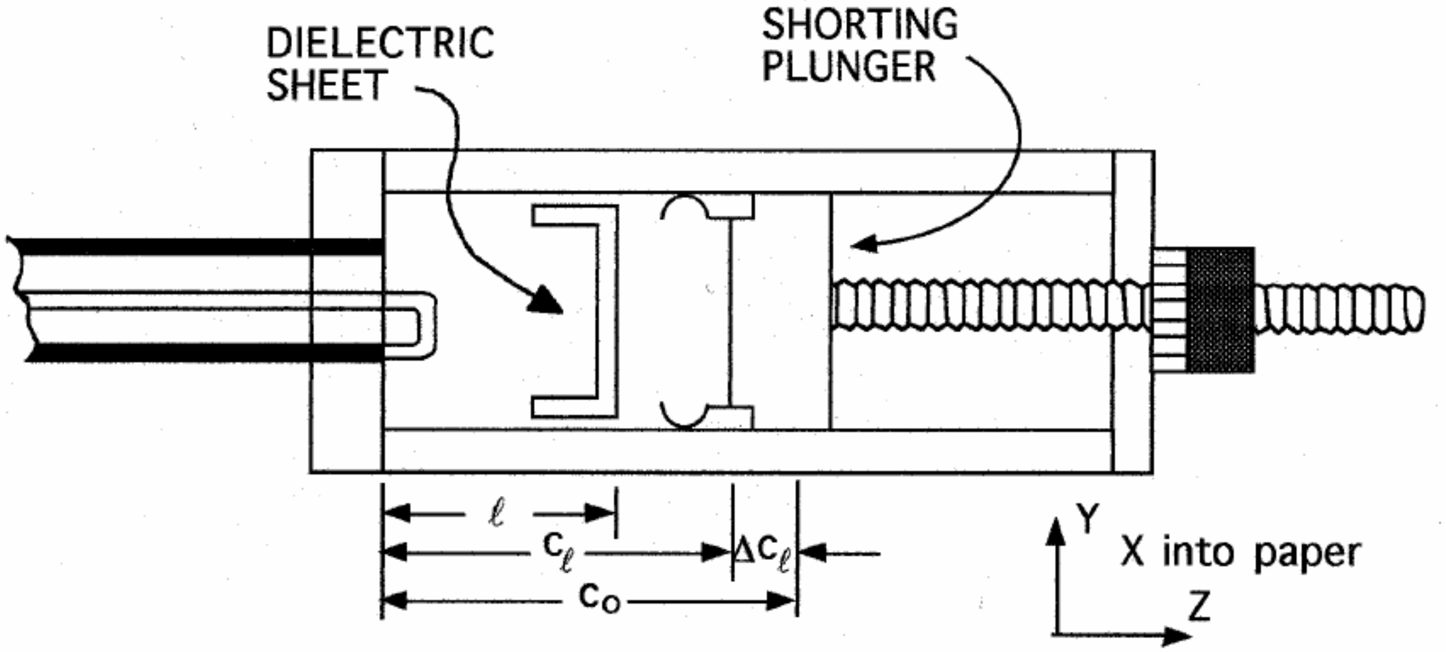
#4: Electric Field Distribution



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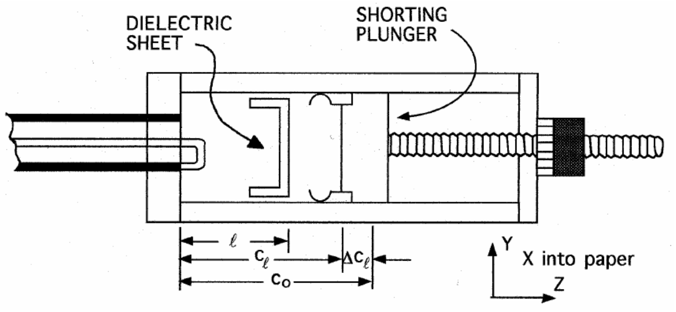
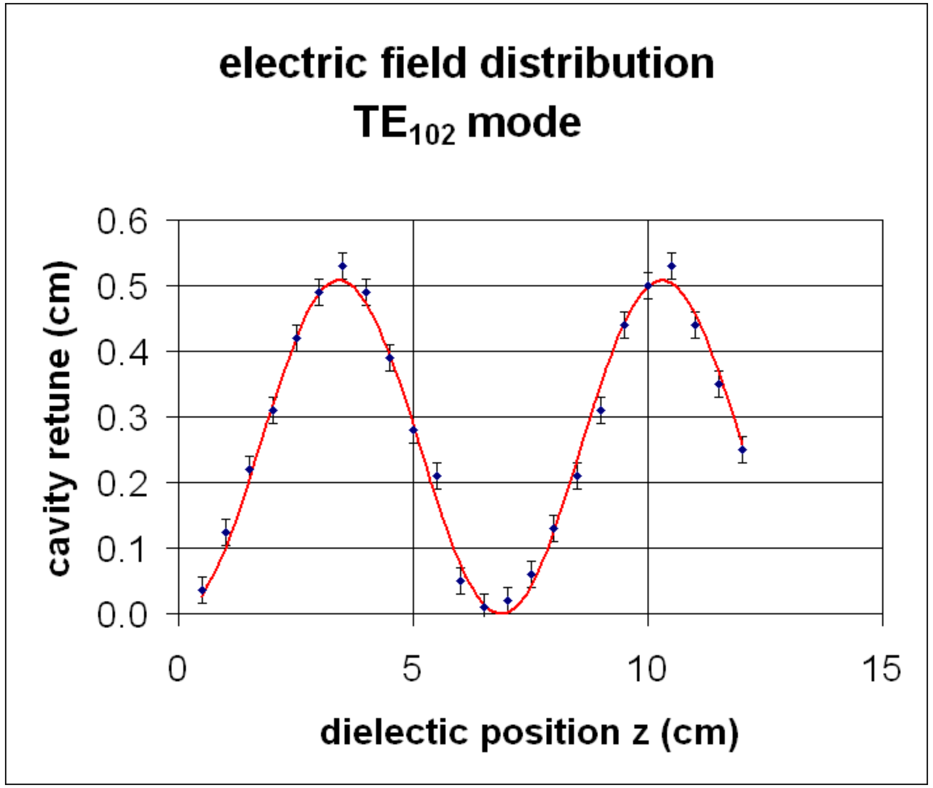
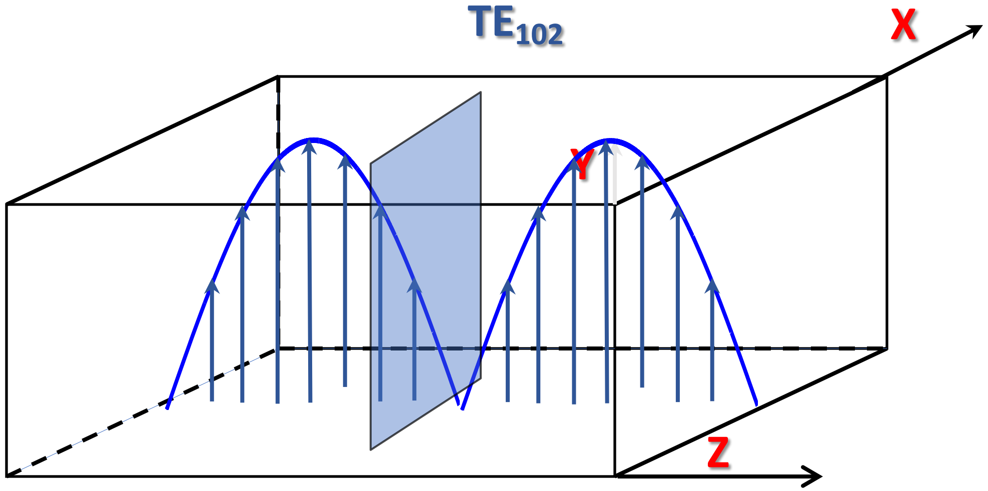
The presence of a dielectric reduces the effective length of the cavity at a given resonance frequency. This effect grows with electric field strength E_y at the dielectric.

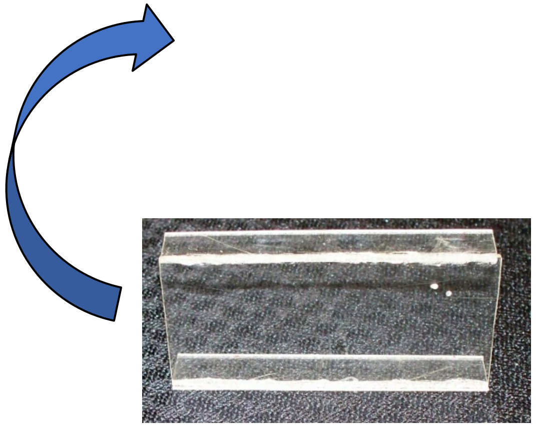
1. Without the dielectric, the cavity's length on resonance is c_0.
2. Place the dielectric in the cavity and change its position l_i in 0.5 cm steps.
3. At each dielectric position, tune the plunger to resonance and record c_i.
4. Plot $\Delta c_i = |c_0 - c_i|$ versus l_i; from this infer E_y vs. l.

#4: Electric Field Distribution





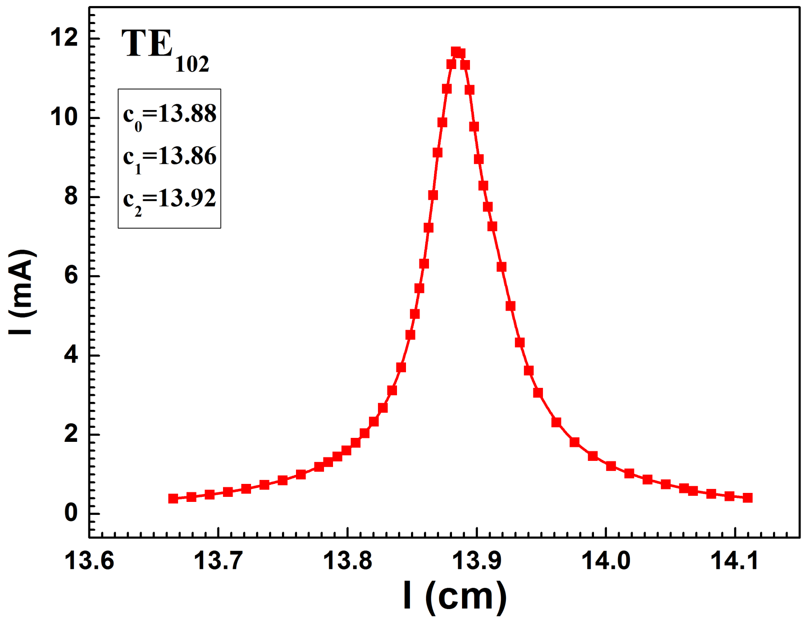


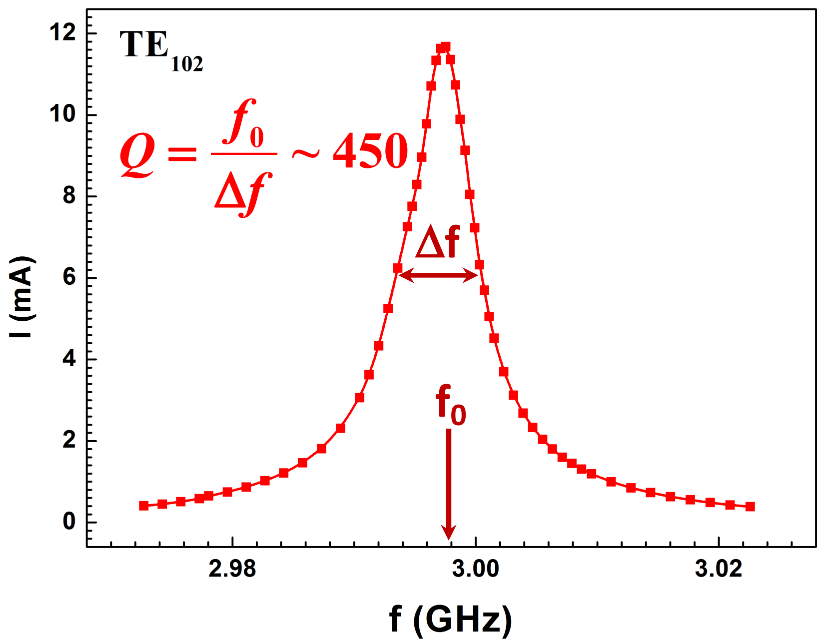


P. Debevec
E.V. Colla

Cavity Resonance in the Frequency Domain

$$\omega_{102}^2 = v_0^2 \pi^2 \left[\left(\frac{1}{a}\right)^2 + \left(\frac{2}{c}\right)^2 \right] \Rightarrow f_{102} = \frac{v_0}{2} \sqrt{\left(\frac{1}{a}\right)^2 + \left(\frac{2}{c}\right)^2}$$





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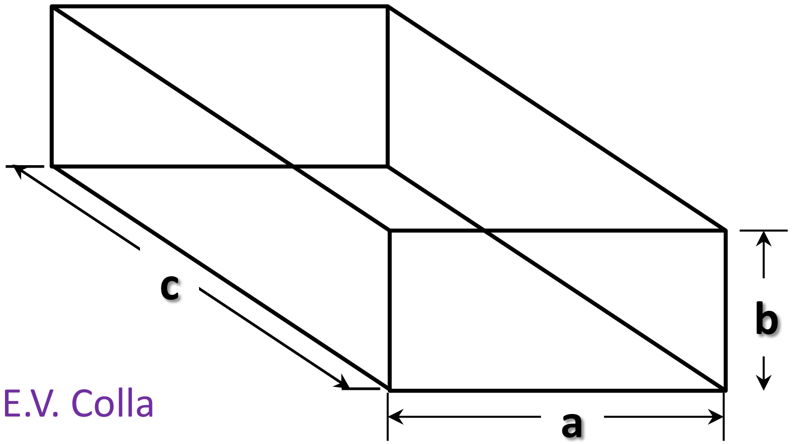
Quality Factor of the Unloaded Cavity

The quality factor of the unloaded cavity's TE_{101} mode is given by:

$$Q_0 = \frac{abc(a^2 + c^2)}{\delta[2b(a^3 + c^3) + ac(a^2 + c^2)]}$$



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In this expression:

- **δ**: skin depth of wall at given frequency
  $$\delta = \sqrt{2\rho/\mu\omega}$$
- **ρ**: resistivity of cavity wall material
- $\boldsymbol{\mu} = \mu_r \mu_0 \approx \mu_0 = 4\pi \times 10^{-7}$

Quality Factor of the Unloaded Cavity

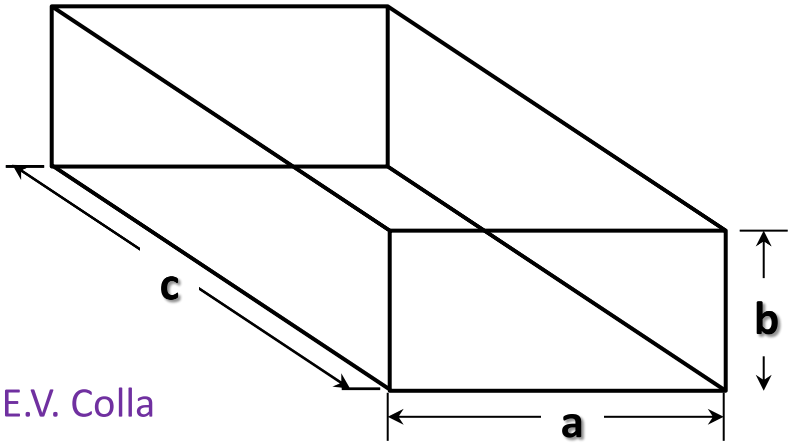
For **red brass**:

- $\rho = 6\times10^{-8}\Omega\cdot m$
- $\mu = 4\pi\times10^{-7}$
- $\delta = \sqrt{2\rho/\mu\omega} = 2.25\times10^{-6}m$

a=7.22 cm, b=3.42 cm, c=6.91 (TE_{101})

$$Q_0 = \frac{abc(a^2+c^2)}{\delta[2b(a^3+c^3)+ac(a^2+c^2)]}$$

$$Q_0 \sim 7700$$



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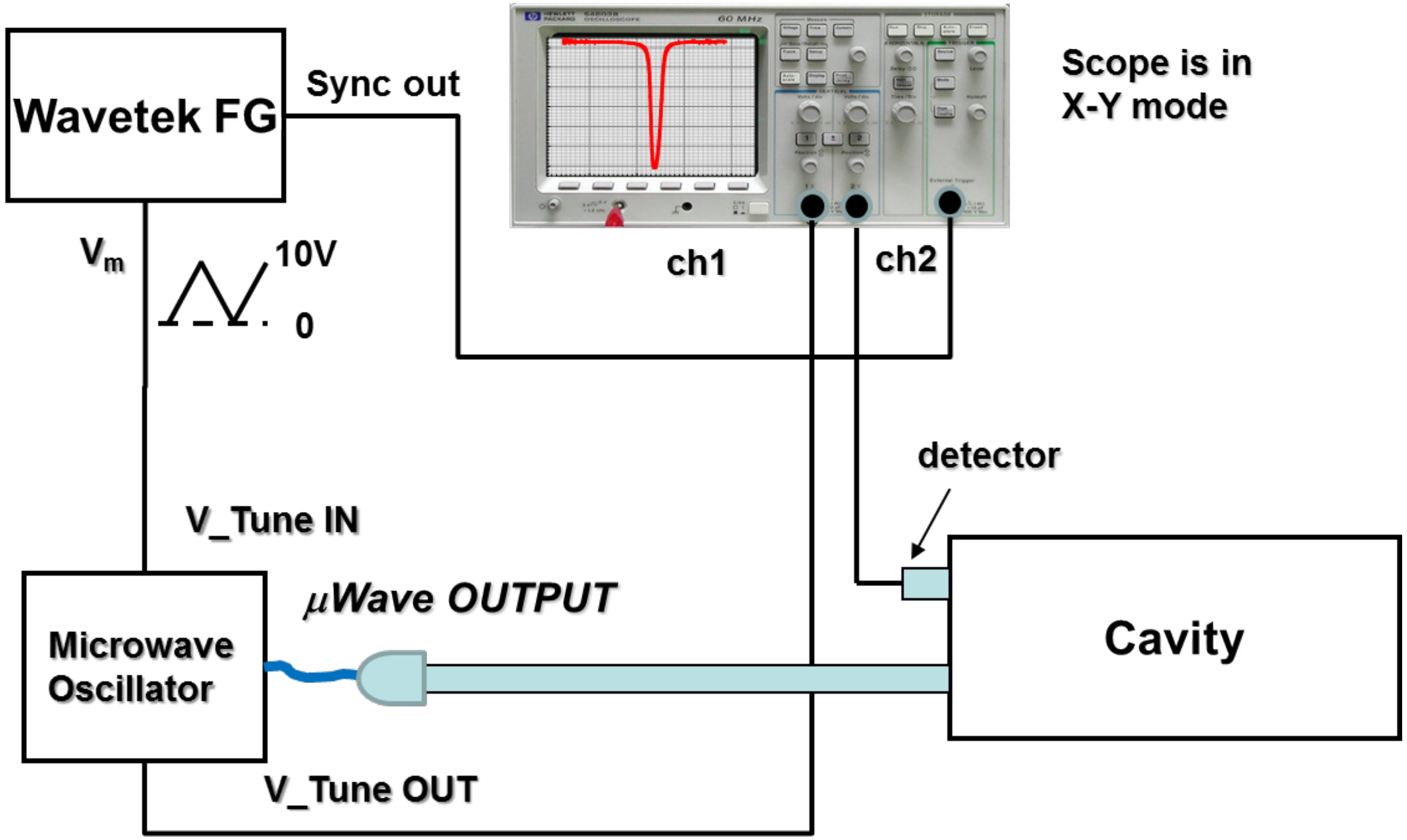
#5: Cavity Resonance: Tuning the Oscillator



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#5: Cavity Resonance: Tuning the Oscillator

1. Acquire data with the oscilloscope in **X-Y mode**
2. To plot **I(f)**, you need to download both Ch1 and Ch2 data
3. Apply a **triangle waveform** to the modulation input of the oscillator
4. Take care when setting the **time scale** setting on the scope – estimate it from the scanning frequency of the triangle wave
5. Apply the **calibration** equation to calculate the frequency of the oscillator from the recorded modulation voltage

	A(X)	B(Y)	C(Y)	D(Y)	E(Y)
Long Name	time	I	time	Vmod	f
Units	s	A	s	V	Hz
1	0	##########	0	3.85055	3.0776
2	1E-6	##########	1E-6	3.84992	3.07758
3	2E-6	##########	2E-6	3.84578	3.07742
4	3E-6	##########	3E-6	3.84297	3.07732

$$f = 0.03706 \cdot V_{mod} + 2.9349$$

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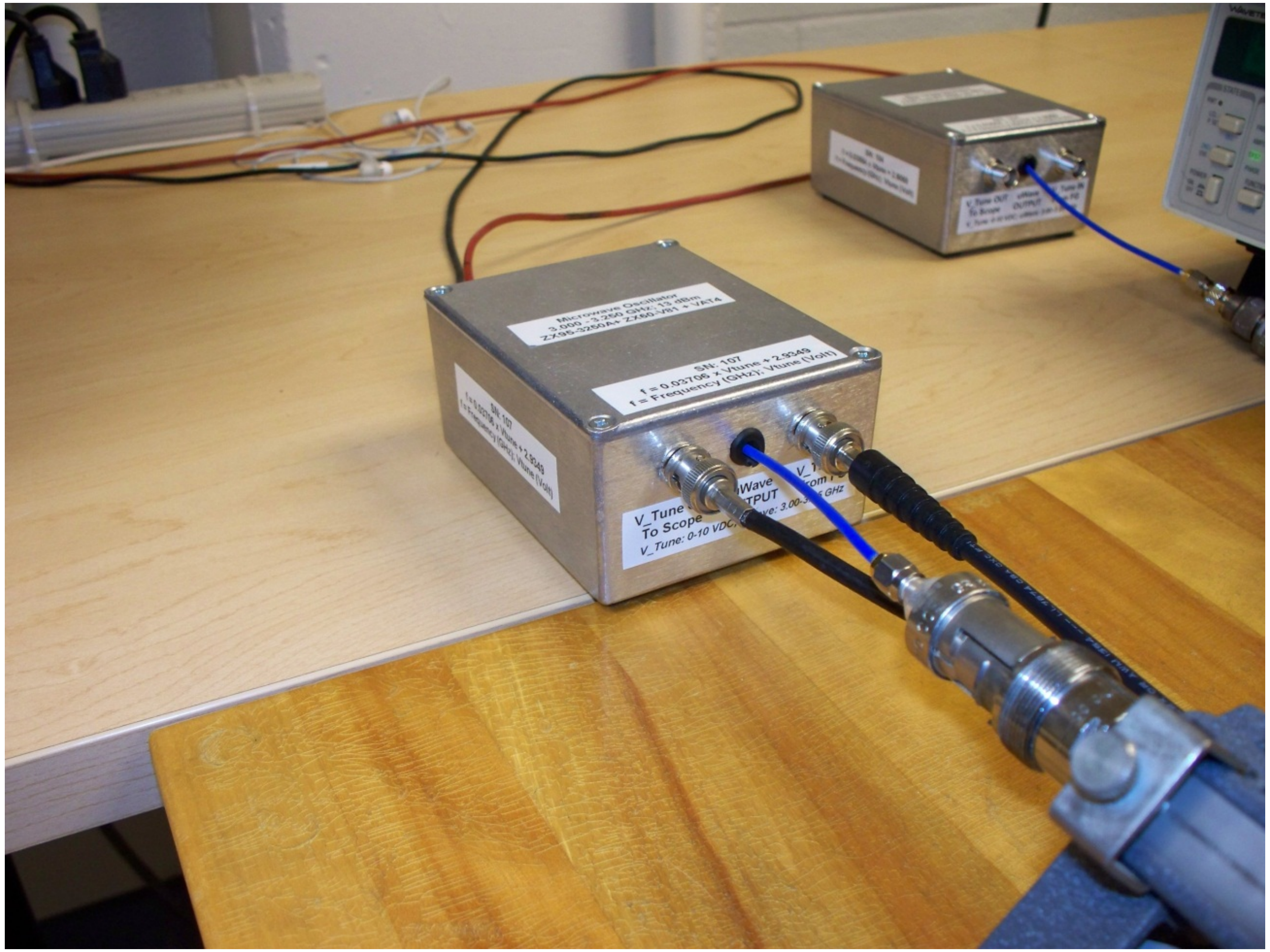
#5: Cavity Resonance: Tuning the Oscillator



Voltage-controlled oscillator (VCO) ZX95-3250a-S+ from

Mini-Circuits®

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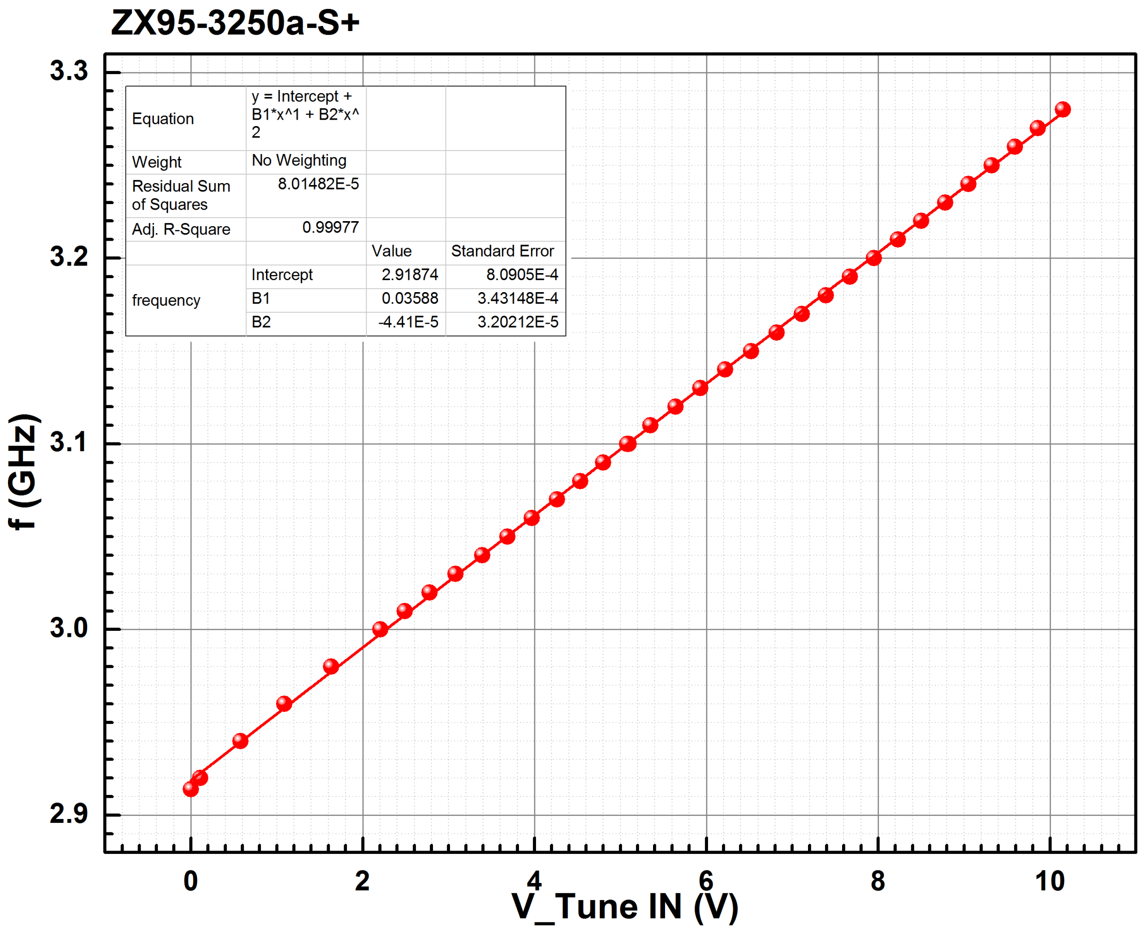
#5: Cavity Resonance: Tuning the Oscillator

FM Calibration for microwave oscillator



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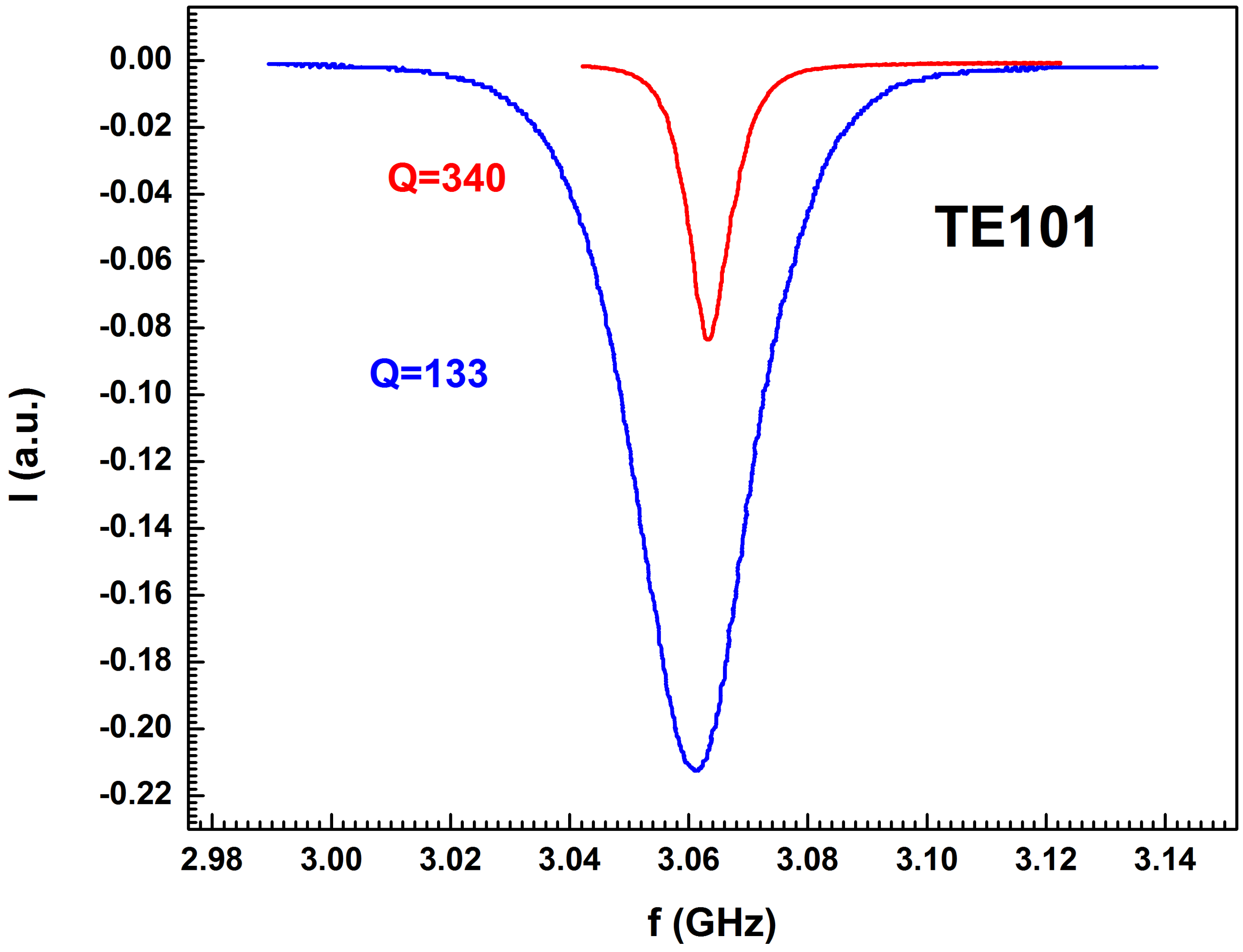
#5: Cavity Resonance



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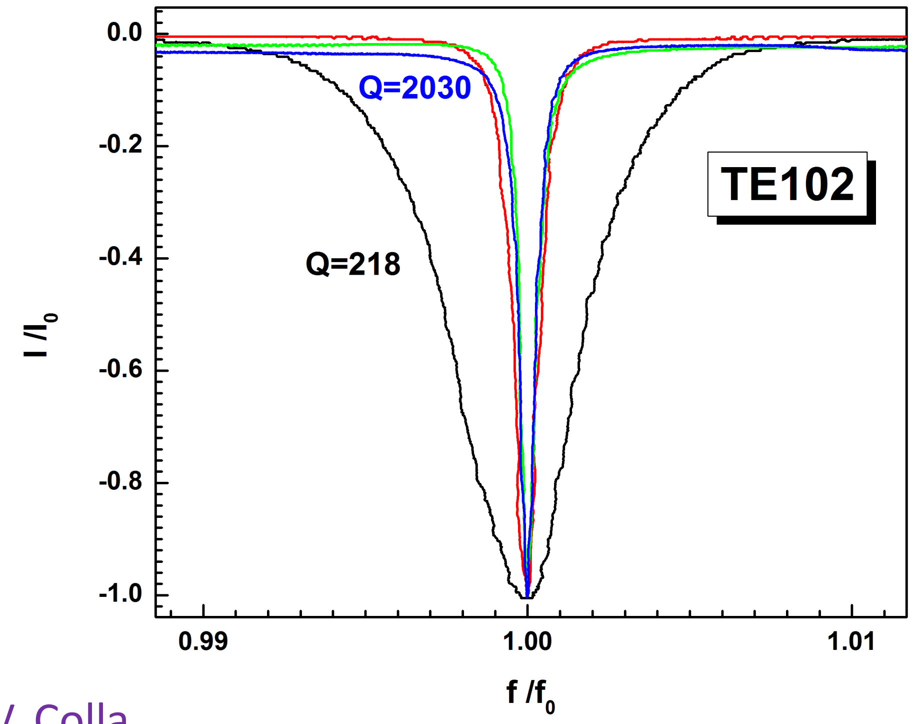
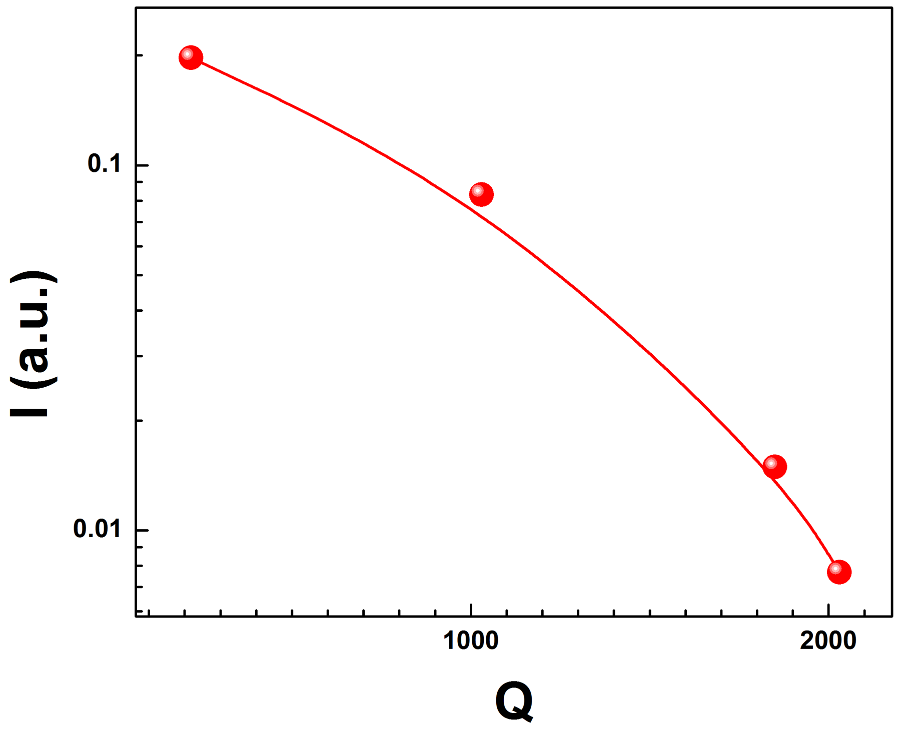
#5: Cavity Resonance

By changing the **coupling** between cavity and transmission line, we simultaneously change both the **quality factor** of the cavity resonance and the **power delivered** to the cavity





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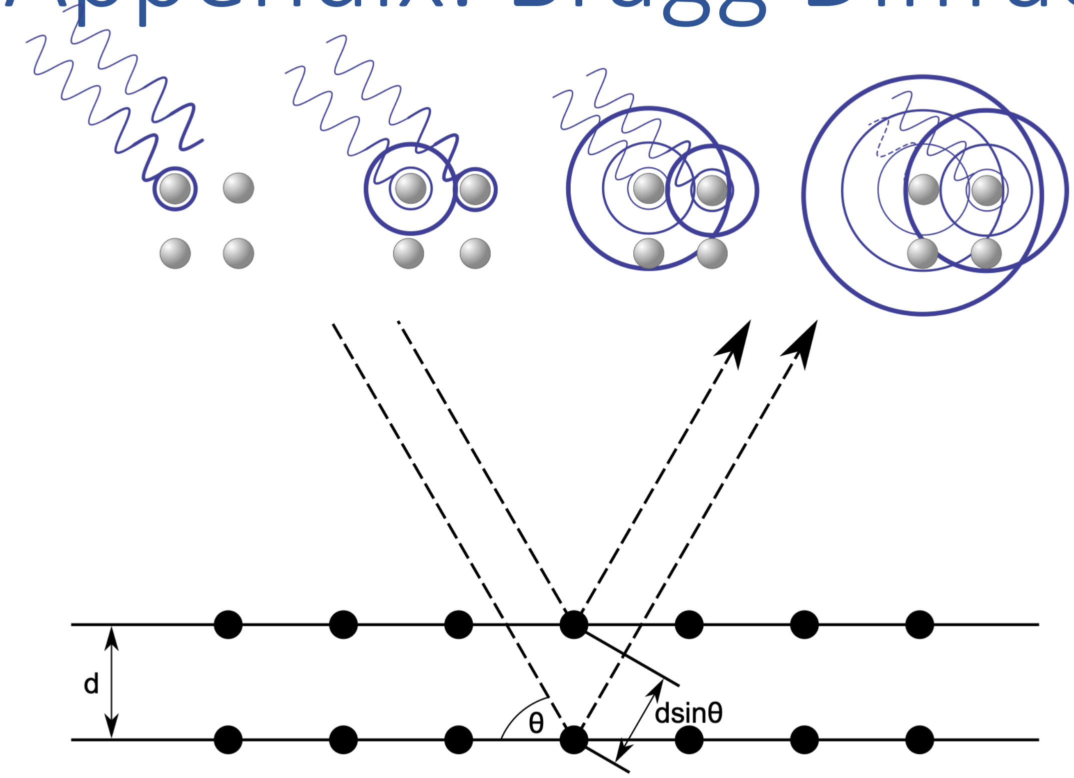
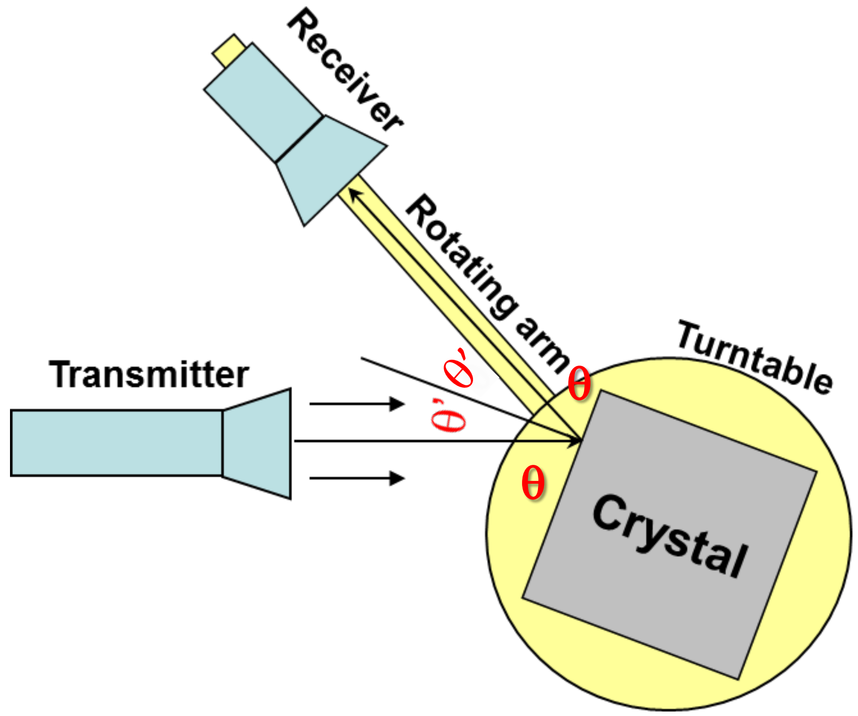
Appendix: Bragg Diffraction





Bragg's Law

Interference maxima

$$n\lambda = 2d\sin\theta$$

$$\theta' = 90° - \theta$$

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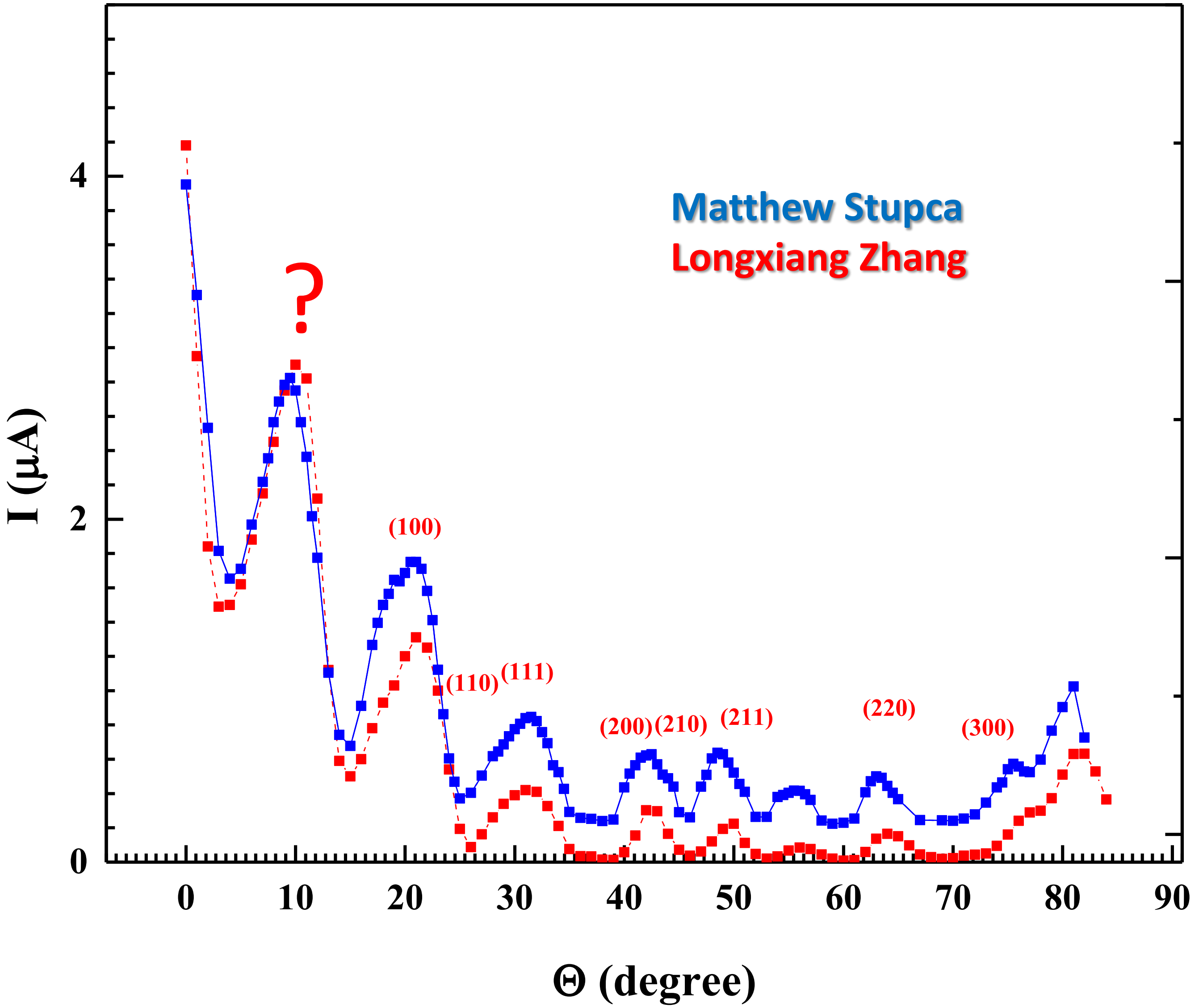
Appendix: Bragg Diffraction



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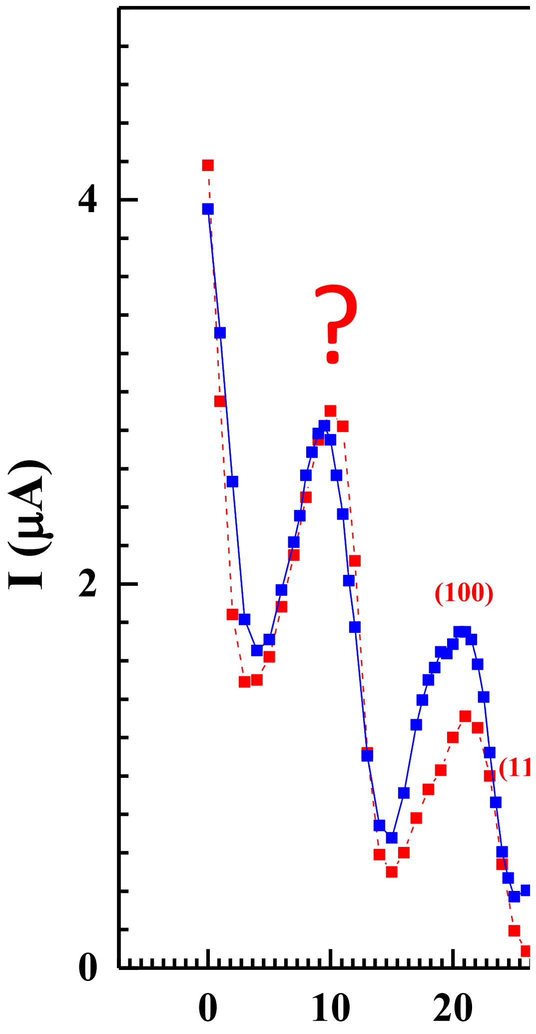
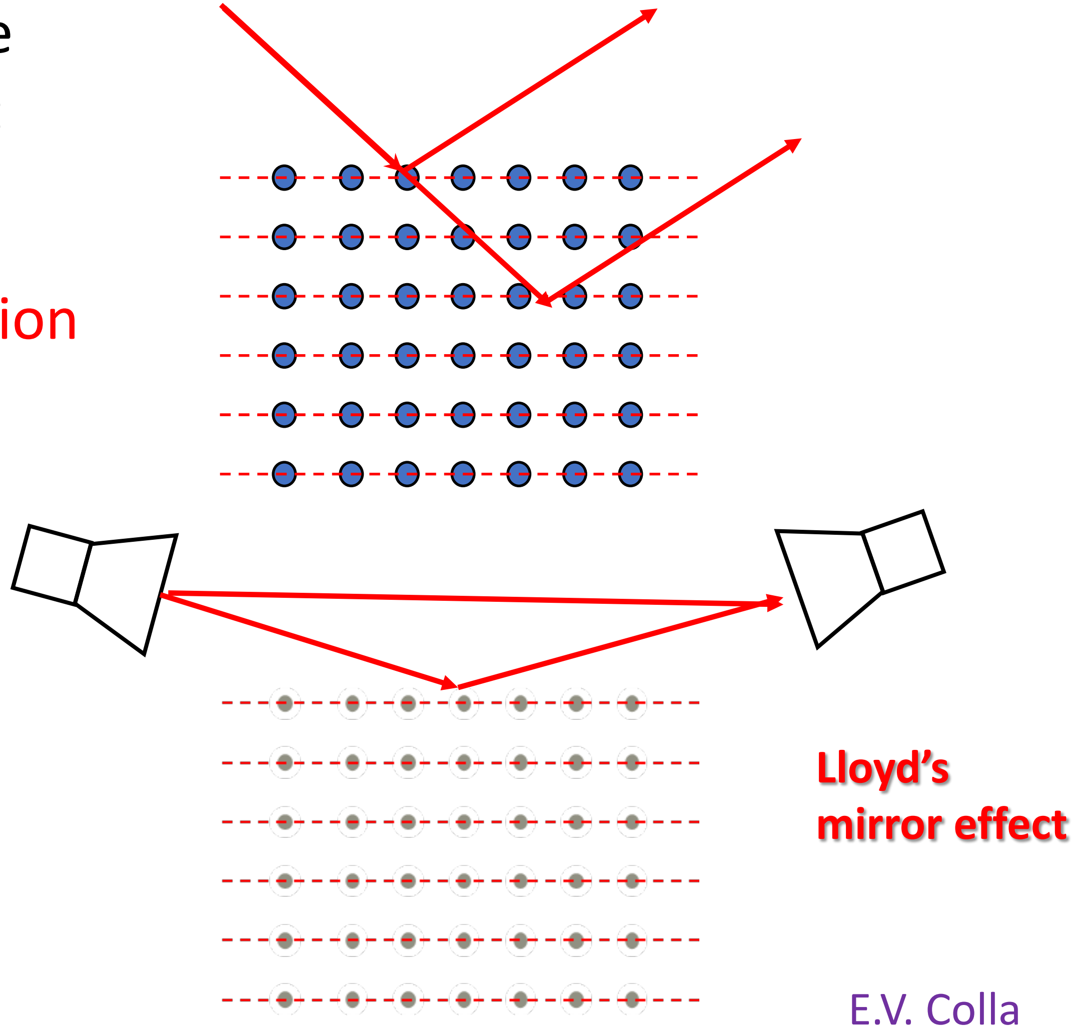
Appendix: Bragg Diffraction



Possible origin of the 10° peak often seen:

Second-order reflection



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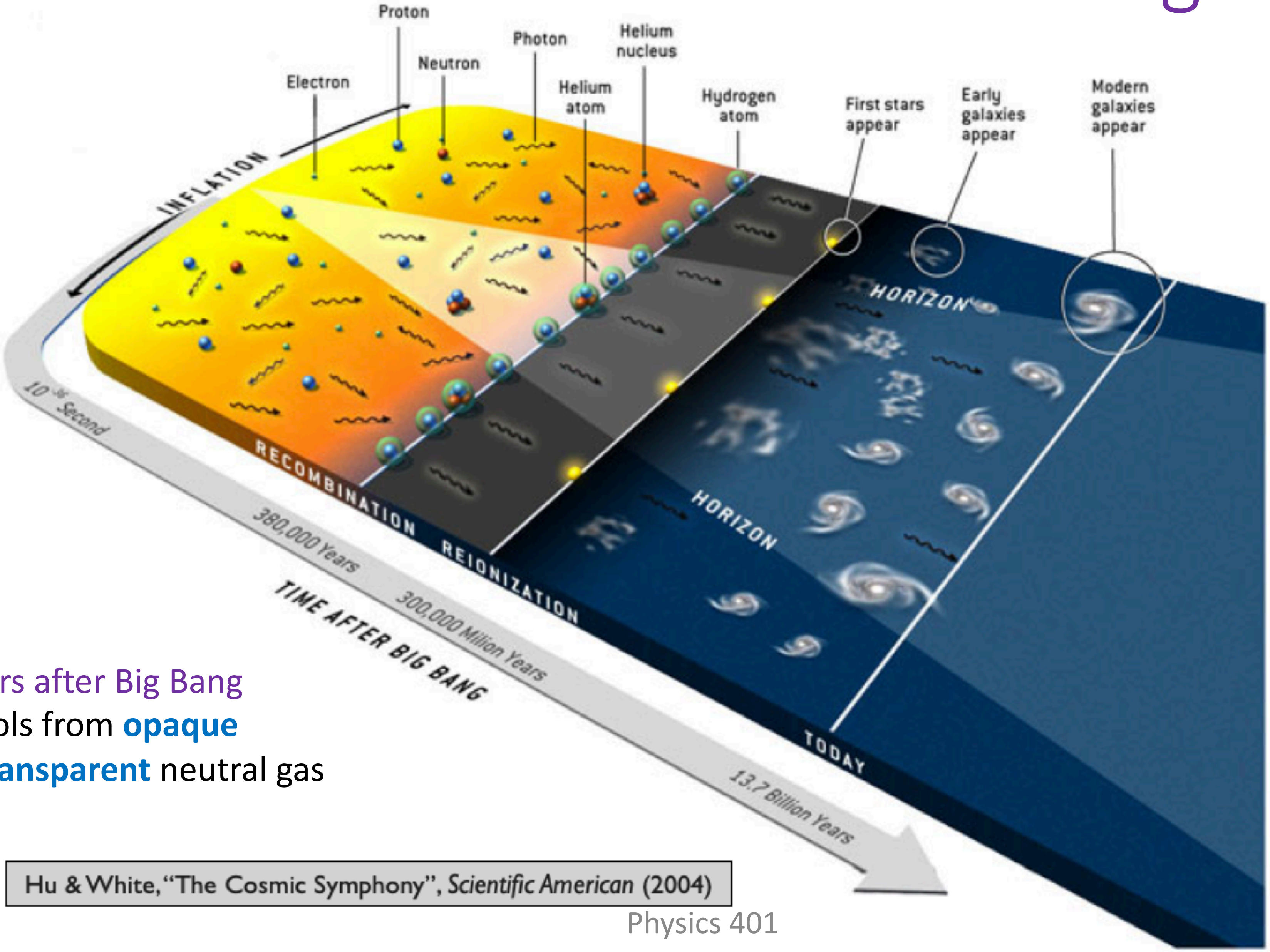
Bonus: The Cosmic Microwave Background

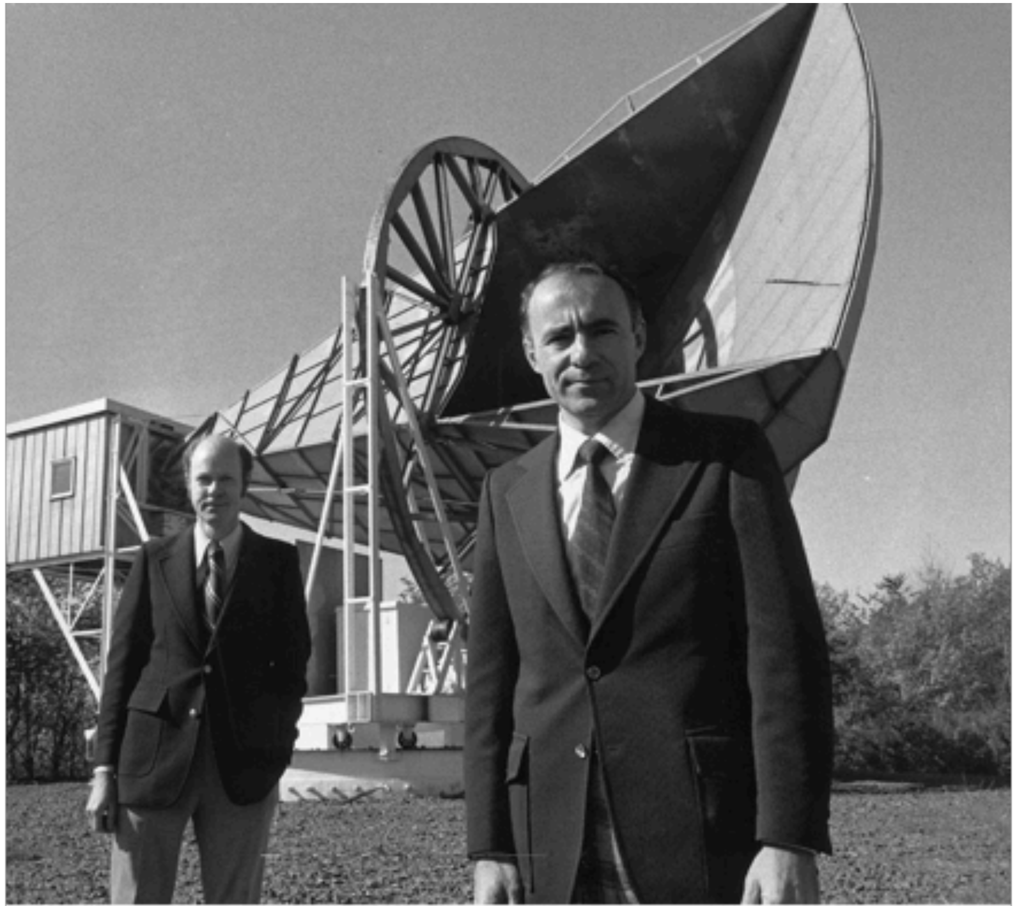


380,000 years after Big Bang

Universe cools from **opaque** plasma to **transparent** neutral gas

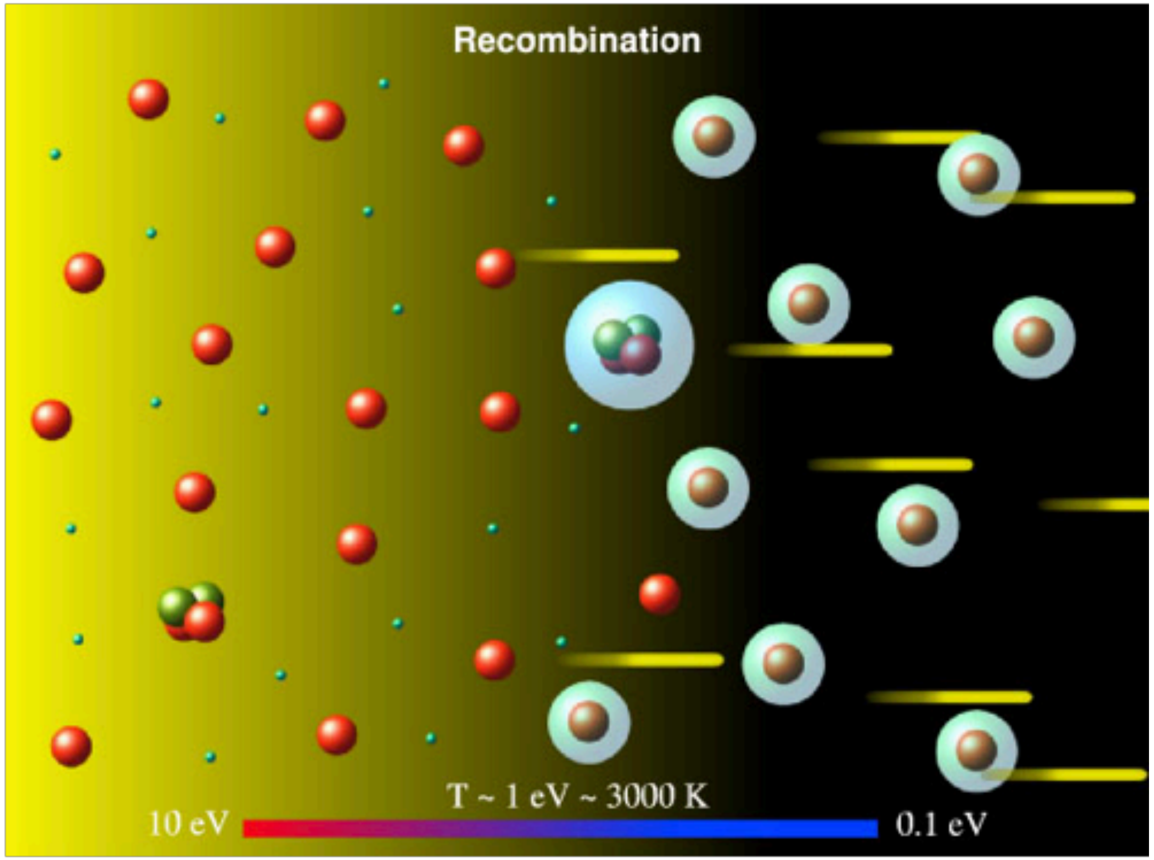
Hu & White, "The Cosmic Symphony", *Scientific American* (2004)

Bonus: The Cosmic Microwave Background



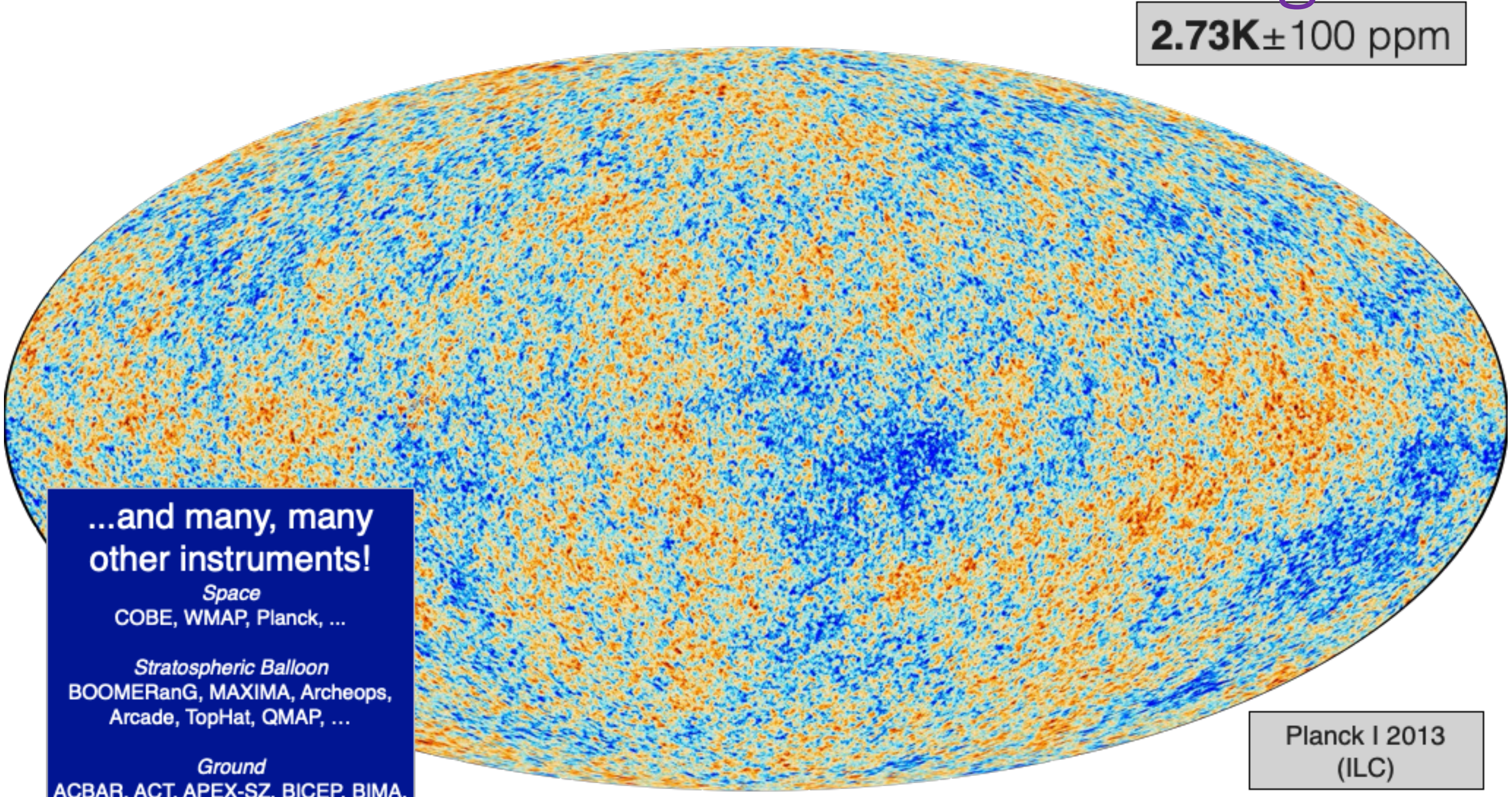
Arno Penzias & Robert Wilson, Bell Labs, NJ

Early, hot, ionized — Late, cool, atoms



Prediction: Alpher & Herman, 1948
Discovery: Penzias & Wilson, 1964 (Nobel 1978)

Bonus: The Cosmic Microwave Background



...and many, many other instruments!

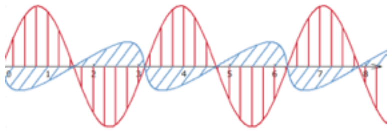
Space
COBE, WMAP, Planck, ...

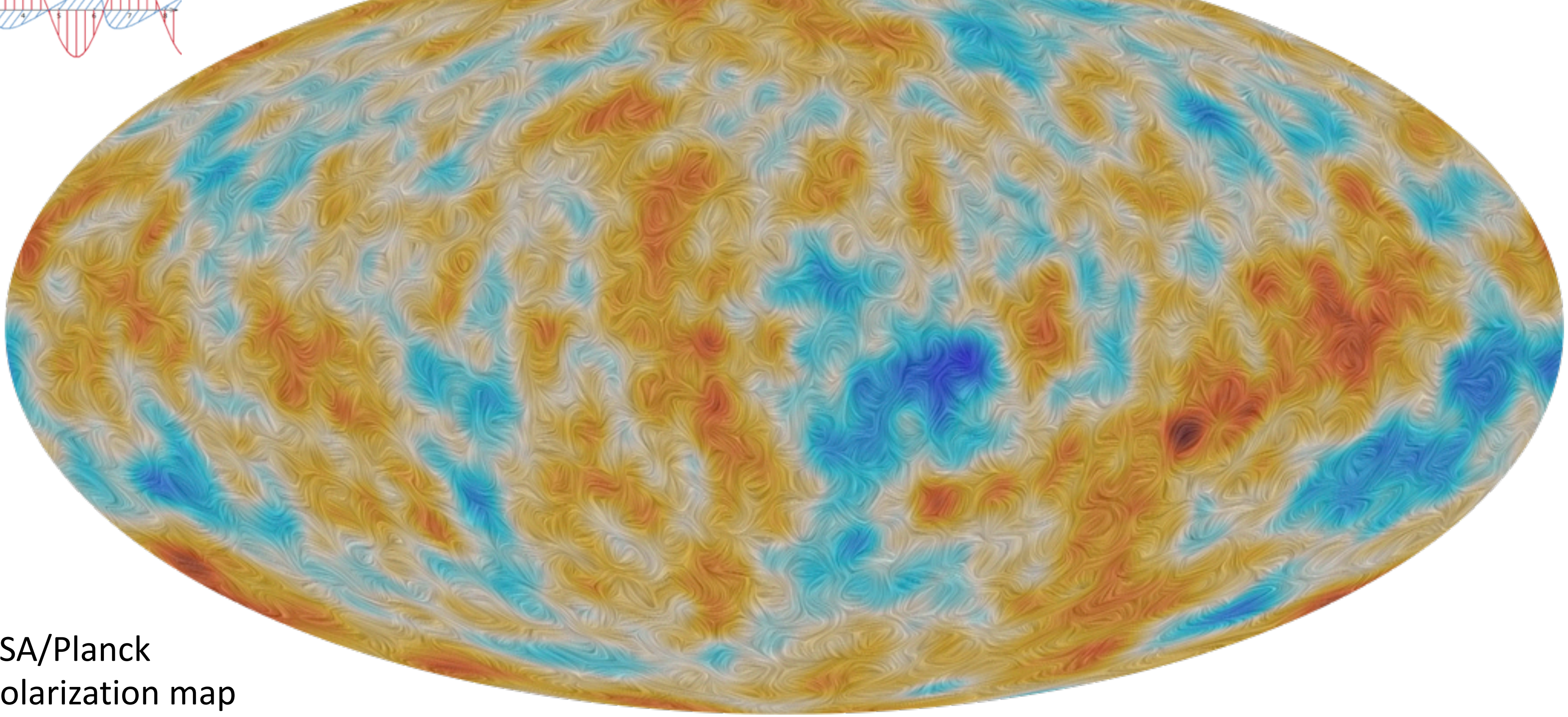
Stratospheric Balloon
BOOMERanG, MAXIMA, Archeops, Arcade, TopHat, QMAP, ...

Ground
ACBAR, ACT, APEX-SZ, BICEP, BIMA, CAPMAP, CBI, DASI, Keck, MAT, POLARBEAR, QUaD, QUIET, Saskatoon, SPT, SuZIE, ...

Anisotropies discovered: COBE, Mather & Smoot (Nobel 2006)
Primordial density fluctuations that grew under gravity into *e.g.* galaxy clusters!

Bonus: The Cosmic Microwave Background

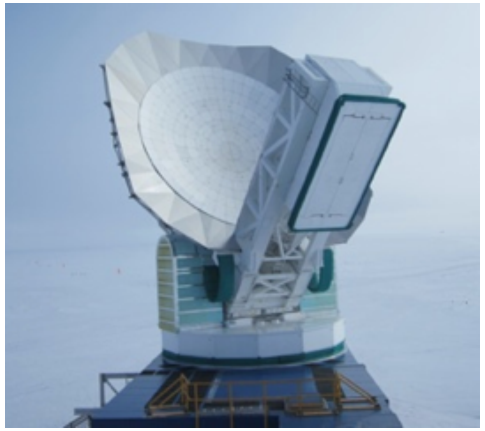


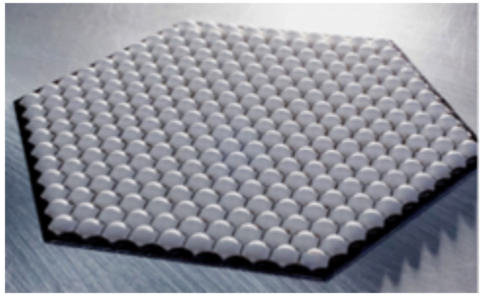


ESA/Planck
Polarization map

Polarization of the CMB could show evidence of primordial gravitational waves from the Big Bang!

Bonus: The Cosmic Microwave Background





Filippini & **Vieira** groups