AC Measurement of Magnetic Susceptibility

Episode 1 – Ferromagnetism and Hysteresis

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Physics 401

Spring 2020

ILLINOIS

Key Goals of this Lab

Combine the tools and techniques we've learned to characterize **magnetic properties of materials**

- Magnetization and (complex) **magnetic susceptibility**
- Ferromagnetism and **hysteresis**
- Thermal effects and the **Curie temperature**

This is the first week of a three-week lab

Counts as your final exam

Outline

Combine the tools and techniques we’ve learned to characterize **magnetic properties of materials**

- **Ferromagnetism**
- **Measuring magnetic properties of materials**
- **Lab setup and measurements**
- **Analysis notes**

This is the first week of a three-week lab

Next week: Temperature dependence of magnetic properties

Reminder: Magnetic Response of Materials

Two things are often called the “magnetic field”: **B** and **H**

_B_

Magnetic induction
Magnetic flux density

Determines forces **on** moving *free* charges via Lorentz force law:

$$\vec{F} = q\left(\vec{E} + \vec{v}\times\vec{B}\right)$$

$$\boxed{\vec{B} = \mu_0\left(\vec{H} + \vec{M}\right)}$$

_M_

Magnetization
Magnetic polarization

Field created only **by** moving *bound* charges, *i.e.* magnetic response of the medium

_H_

Magnetic field intensity
Magnetizing field

Field created only **by** moving *free* charges.

In vacuum, $B = \mu_0 H$.

Reminder: Magnetic Response of Materials

$$\vec{B} = \mu_0\left(\vec{H} + \vec{M}\right)$$

Since many materials have approximately linear response, we define the **magnetic susceptibility**:

$$\boxed{\vec{M} = \boldsymbol{\chi}\vec{H}} \qquad \vec{B} = \mu_0(1+\chi)\vec{H} = \mu_0\mu_r\vec{H} = \mu\vec{H}$$

$$\mu = \mu_0\mu_r = \frac{\partial B}{\partial H} \qquad \mu_r = 1 + \chi = \frac{1}{\mu_0}\frac{\partial B}{\partial H}$$

In general, susceptibility… :

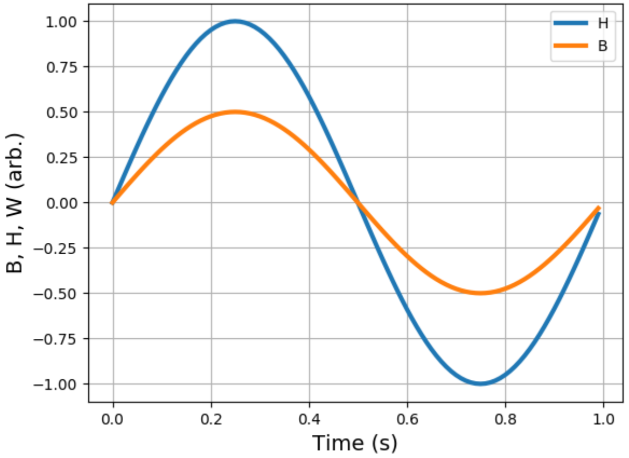
1. … is a function $\chi(H)$ (***nonlinearity***)
2. … is a 2nd-rank **tensor** (*matrix*) (scalar for isotropic materials)
3. … may be **complex** (*phase lag, loss*) $\chi = \chi' - i\chi''$
4. … may have history dependence (***hysteresis***) not captured by this expression *(see below!)*

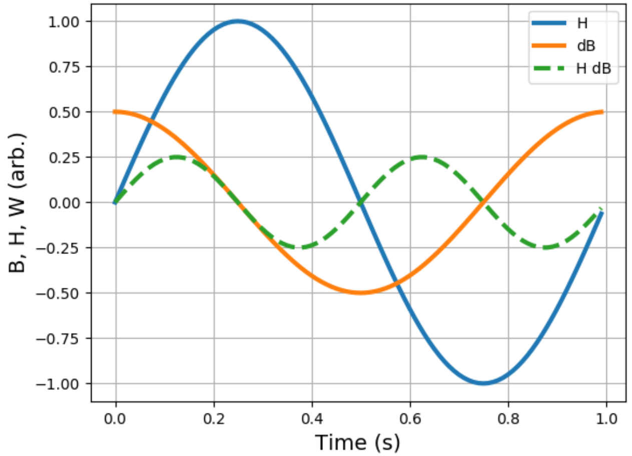
Aside: Loss from Complex Permeability

$$\vec{B} = \mu_0\left(\vec{H} + \vec{M}\right) = (\mu' - i\mu'')\vec{H}$$

Why is a material with complex permeability ($\mu'' \neq 0$) lossy?

Real μ:

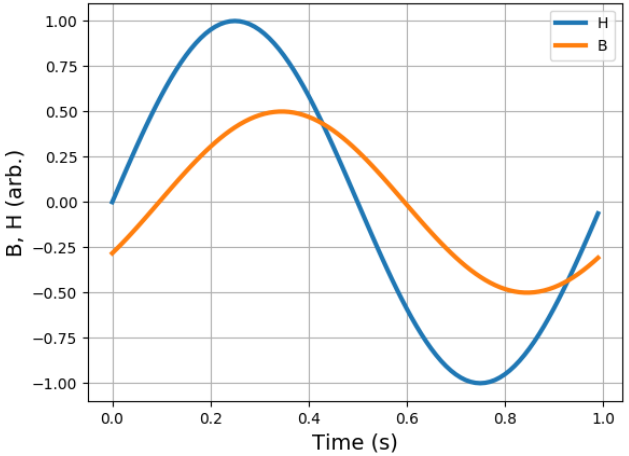


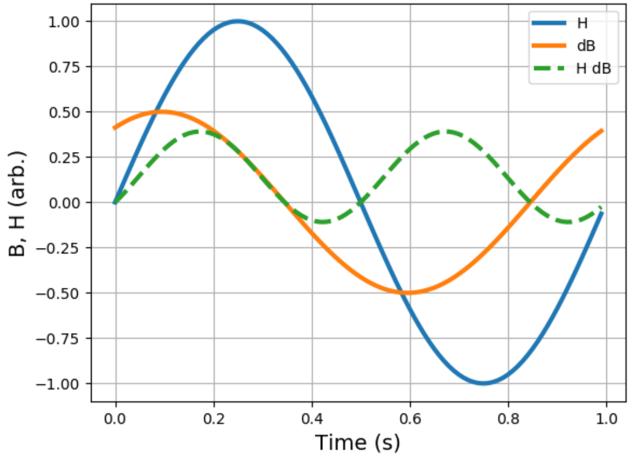


Zero integral

In analogy with $dW = F\,dx$, we have $\boldsymbol{dW = H\,dB}$

Complex μ:





Nonzero integral!

Reminder: Magnetic Response of Materials

$$\vec{B} = \mu_0(\vec{H} + \vec{M}) = \mu_0(1 + \chi)\vec{H} = \mu_0\mu_r\vec{H}$$

We classify materials into three major categories:

Diamagnetic	$\chi < 0$	$\mu_r < 1$	Weakly repelled
Paramagnetic	$\chi > 0$	$\mu_r > 1$	Weakly attracted
Ferromagnetic	$\chi \gg 0$	$\mu_r \gg 1$	Strongly attracted

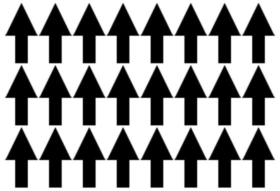
More next time on these… for now, **ferromagnetism**!

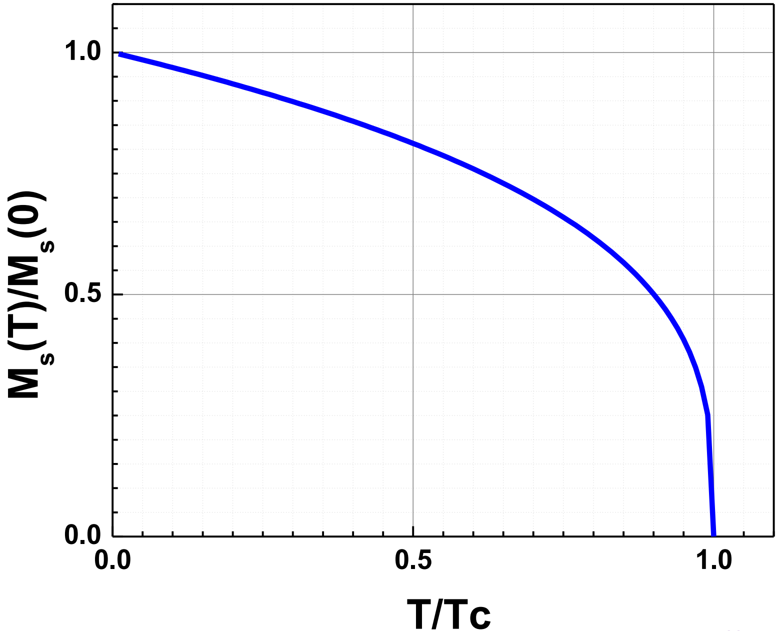
What is Ferromagnetism?

Some materials experience **spontaneous magnetic ordering** in the absence of an applied field.

Happens when aligning interactions among neighboring atomic/molecular dipoles (typically from Pauli exclusion, exchange interactions) exceed magnetic dipole anti-alignment forces and thermal randomization.

This ordering occurs only below some transition temperature, the **Curie temperature (T_c)**.





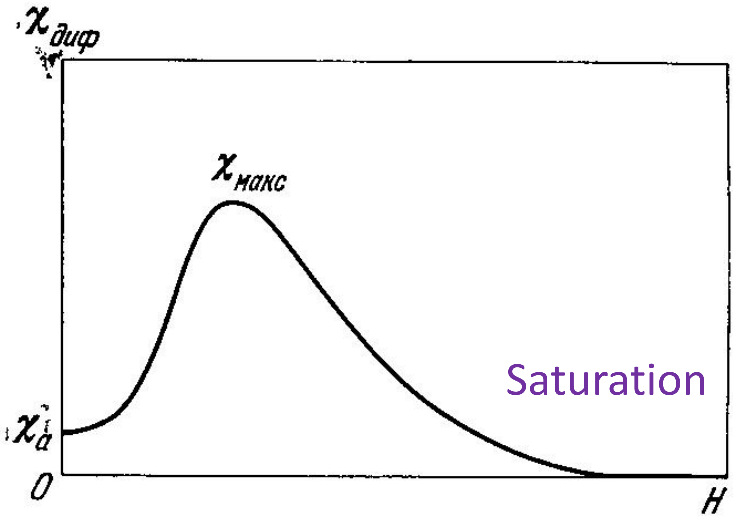
Some materials exhibit spontaneous *<u>anti</u>-alignment of neighbors: antiferromagnetism, ferrimagnetism.*

Typical spontaneous magnetization versus temperature

E.V. Colla

Ferromagnetic Materials

Material	Curie temp. (K)
Co	1388
Fe	1043
$Fe_2O_3{}^*$	948
$FeOFe_2O_3{}^*$	858
$NiOFe_2O_3{}^*$	858
$MgOFe_2O_3{}^*$	713
MnBi	630
Ni	627
MnSb	587
$MnOFe_2O_3{}^*$	573
$Y_3Fe_5O_{12}{}^*$	560
CrO_2	386
MnAs	318
Gd	292



"Stoletov" curve

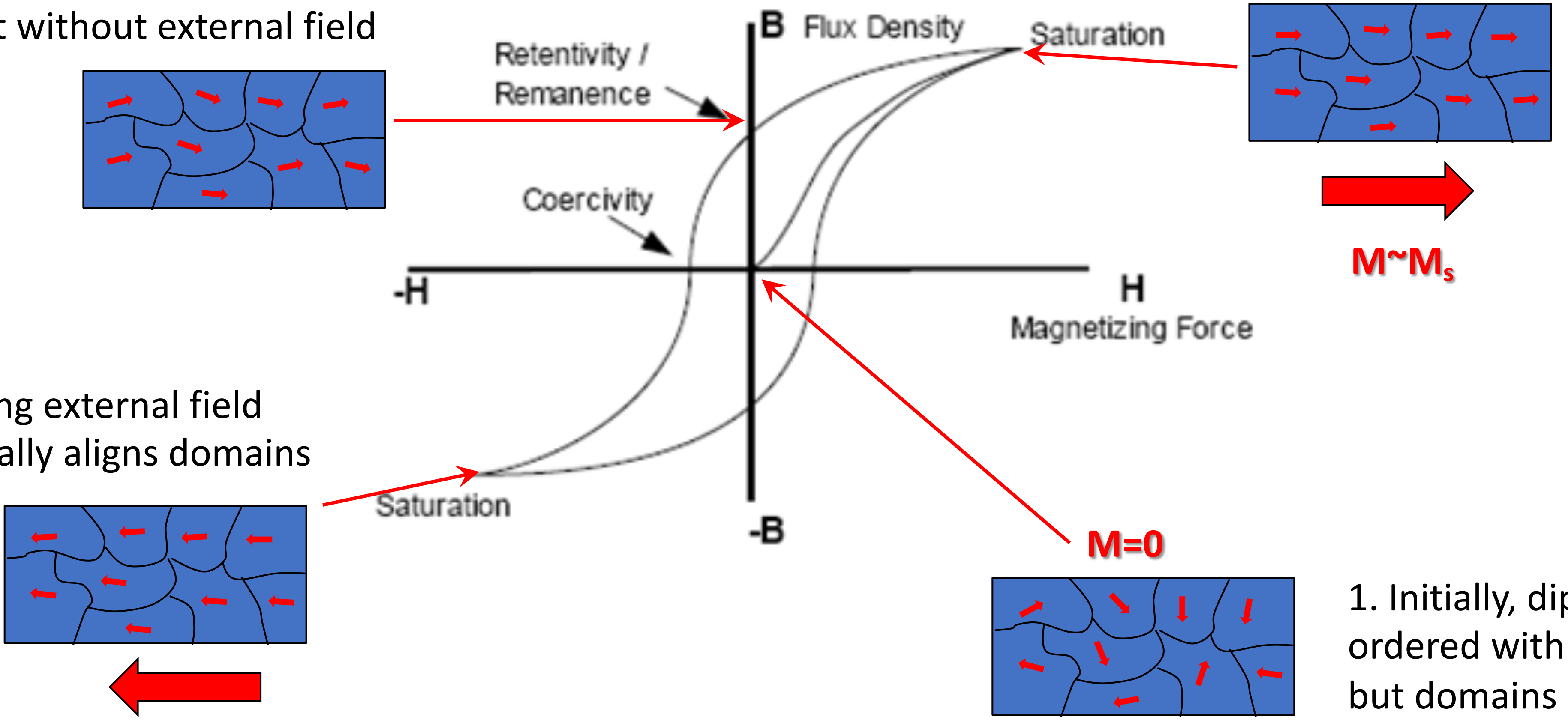
$$\boxed{\chi = \frac{dM}{dH}}$$

Aleksandr Stoletov
(1839 –1896)

** = Ferrimagnetic (local anti-alignment, but unbalanced – acts like a ferromagnet)*

E.V. Colla

Domains and Hysteresis



2. Strong external field maximally aligns domains

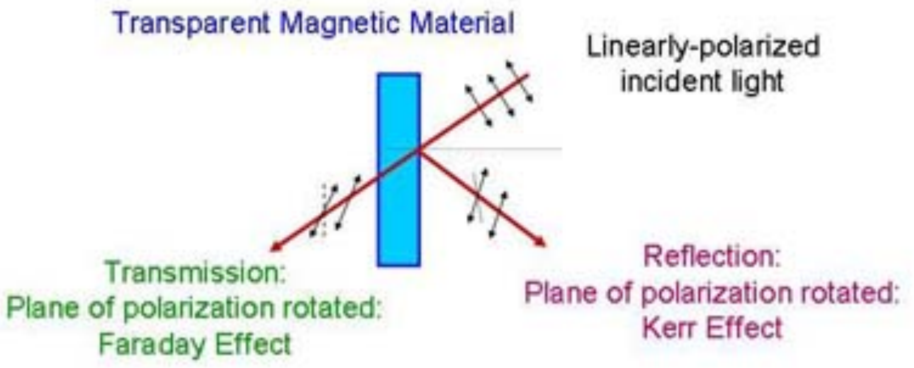
3. Internal fields retain partial alignment without external field

4. Strong external field maximally aligns domains

1. Initially, dipoles are ordered within a domain, but domains are random

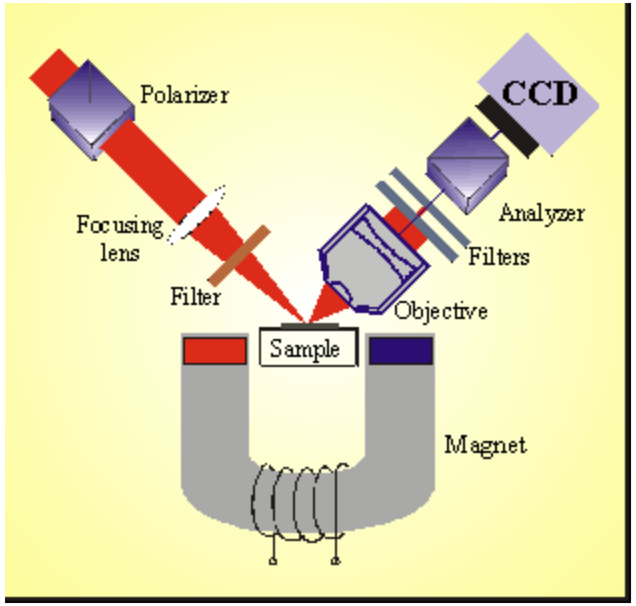
Visualizing Magnetic Domains

Faraday Rotation: Rotation of polarized light passing through a transparent magnetic material



Michael Faraday
1791 – 1867

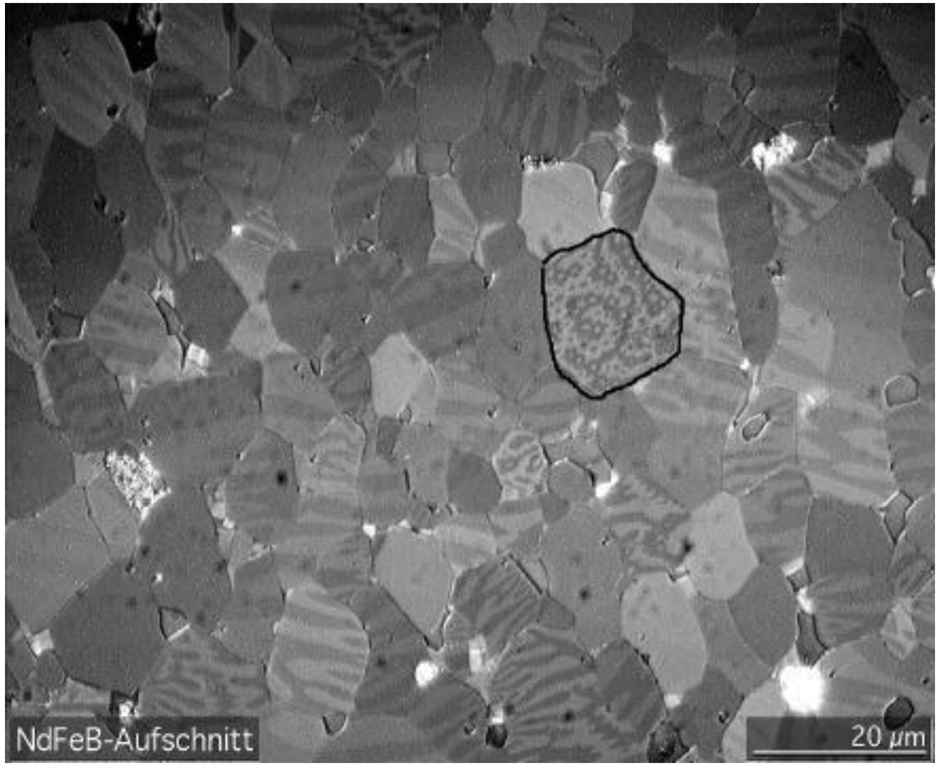
Magneto-Optic Kerr Effect (MOKE): Rotation of polarized light reflected from a magnetic material



Typical Kerr microscope
Radboud Univ., Nijmegen, the Netherlands

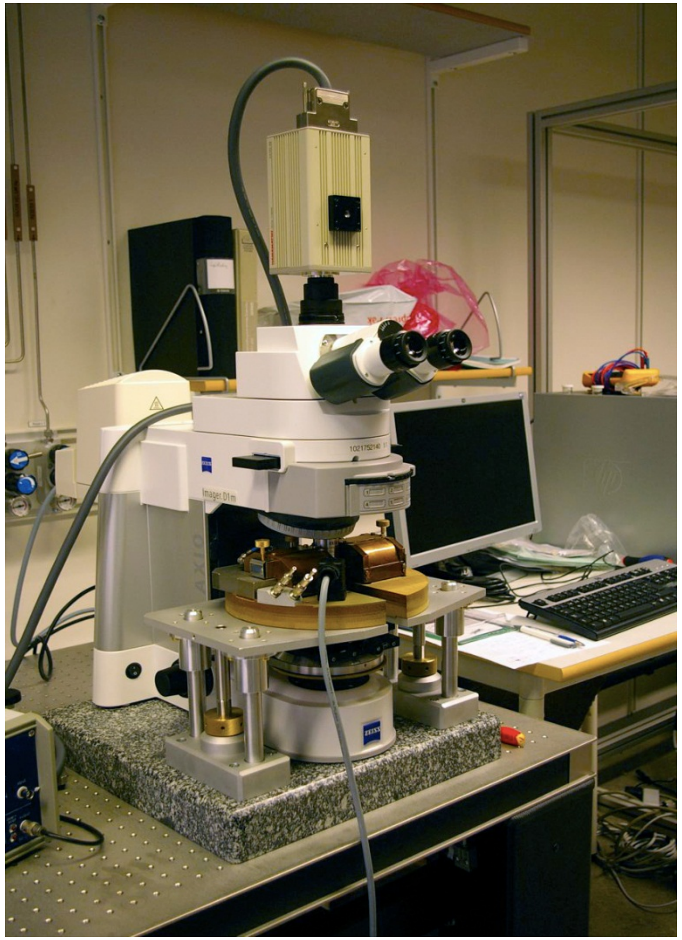
John Kerr
1824 – 1907

Visualizing Magnetic Domains



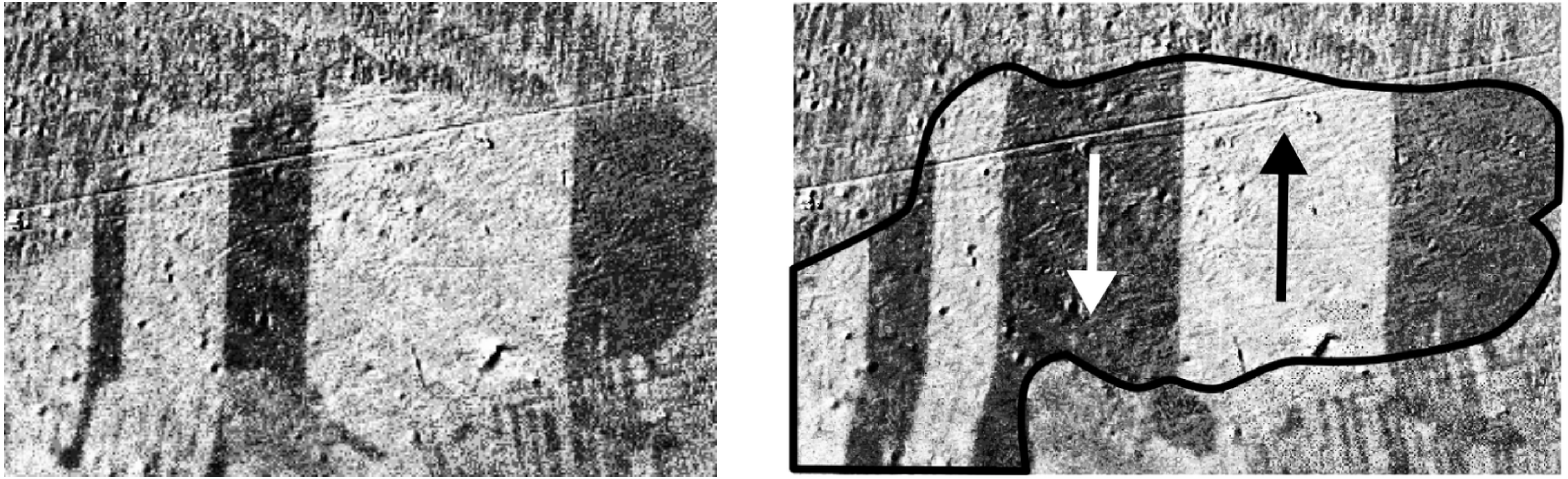
Kerr microscope image of a NdFeB sample, showing domains

Wikipedia



Kerr microscope
University of Uppsala, Sweden

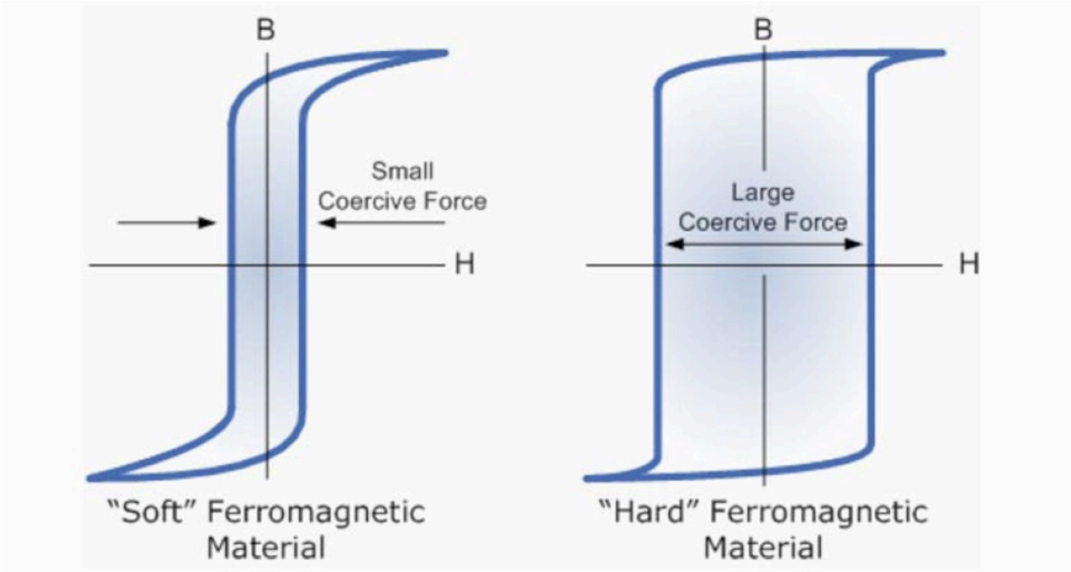
Visualizing Magnetic Domains



Domain walls in a grain of silicon steel, moving as the external magnetic field is increased

Wikipedia

Hysteresis and Coercivity



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Hysteresis necessarily involves energy losses from re-magnetization.
If the domains are "sticky", we need to do work to overcome that.

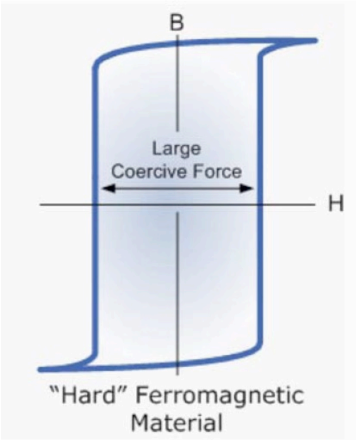
$$W = V \int \vec{H} \cdot d\vec{B}$$

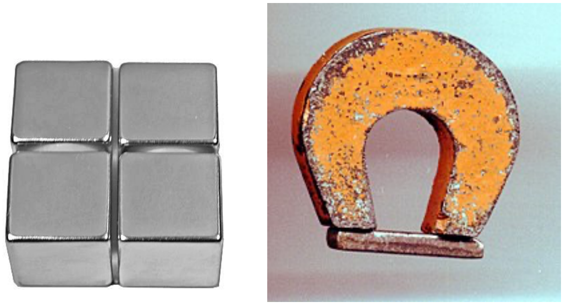
For uniform fields over volume V
(analogous to dW = F dx)

$$\boxed{W_{loop} = V \oint \vec{H} \cdot d\vec{B} = V * A_{loop}}$$

Applications of Magnetic Materials

"Hard" Materials

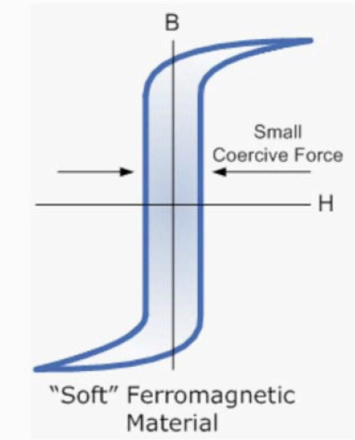




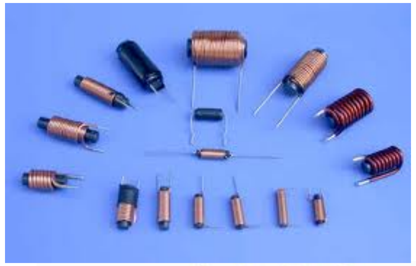
Permanent magnets

Data storage media

"Soft" Materials



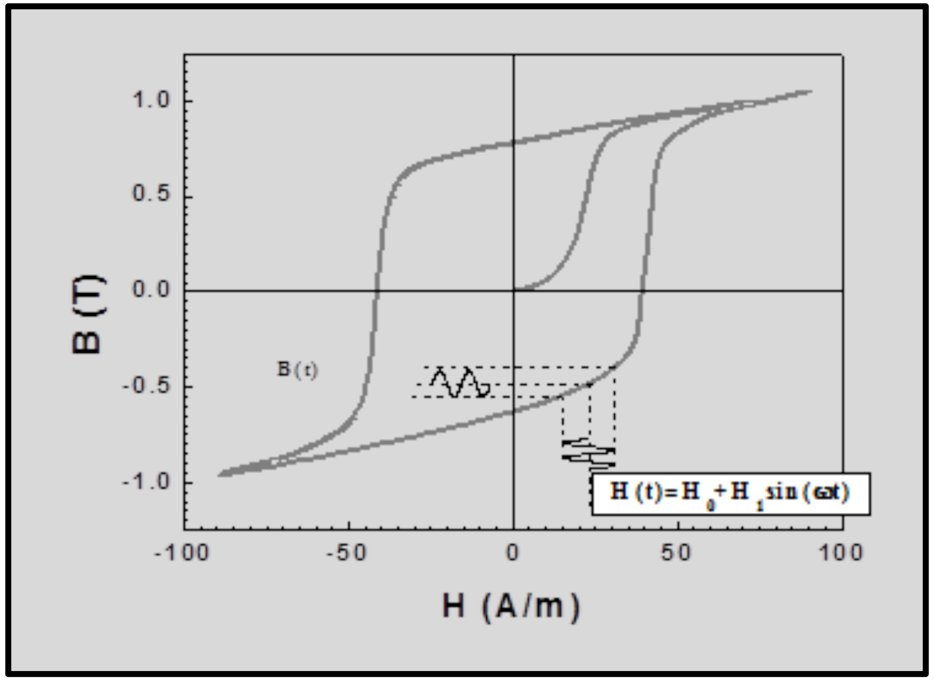
Cores for inductors, electromagnets, power transformers…



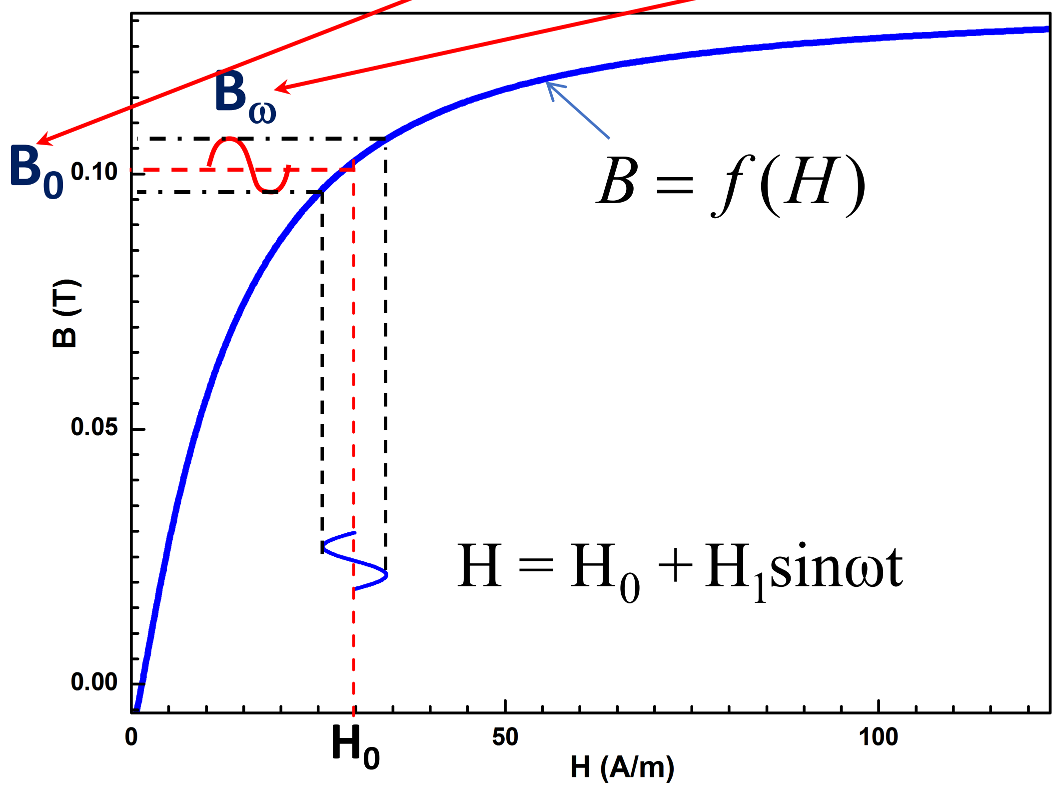
Images: E.V. Colla, Wikipedia

AC Measurement of Magnetic Permeability

Apply a small modulation to H to measure the derivative of the B-H hysteresis loop

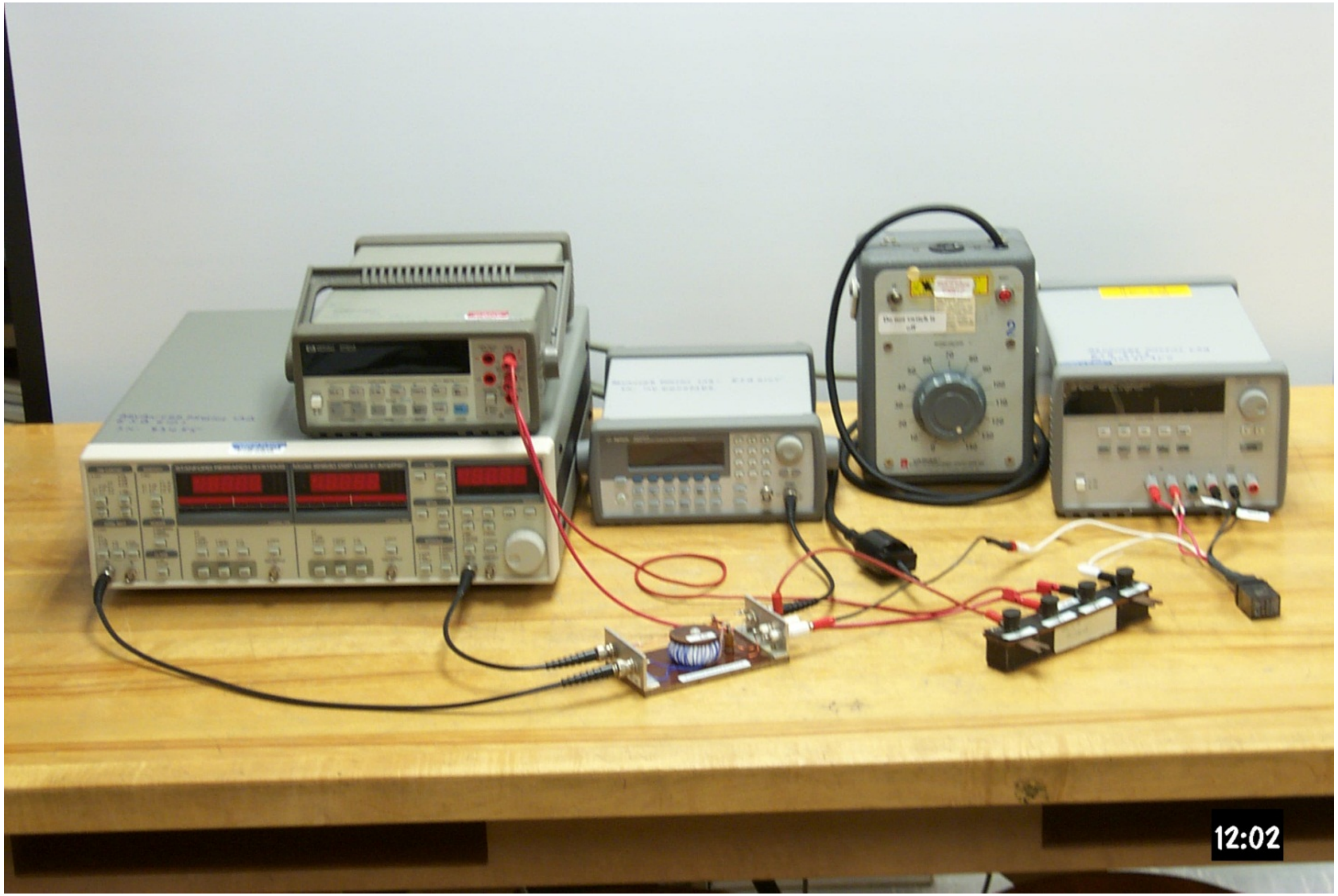


$$B = f(H_0 + H_1 \sin \omega t) = f(H_0) + \frac{df}{dH} H_1 \sin \omega t + \cdots$$



$$\mu(H_0,\omega) = \mu_0(1+\chi(H_0,\omega)) = \left.\frac{dB}{dH}\right|_{H_0,\omega}$$

Setup #1: Mapping the Hysteresis Loops



Setup #1: Mapping the Hysteresis Loops

$H_0 + H_1\, sin\, \omega t$

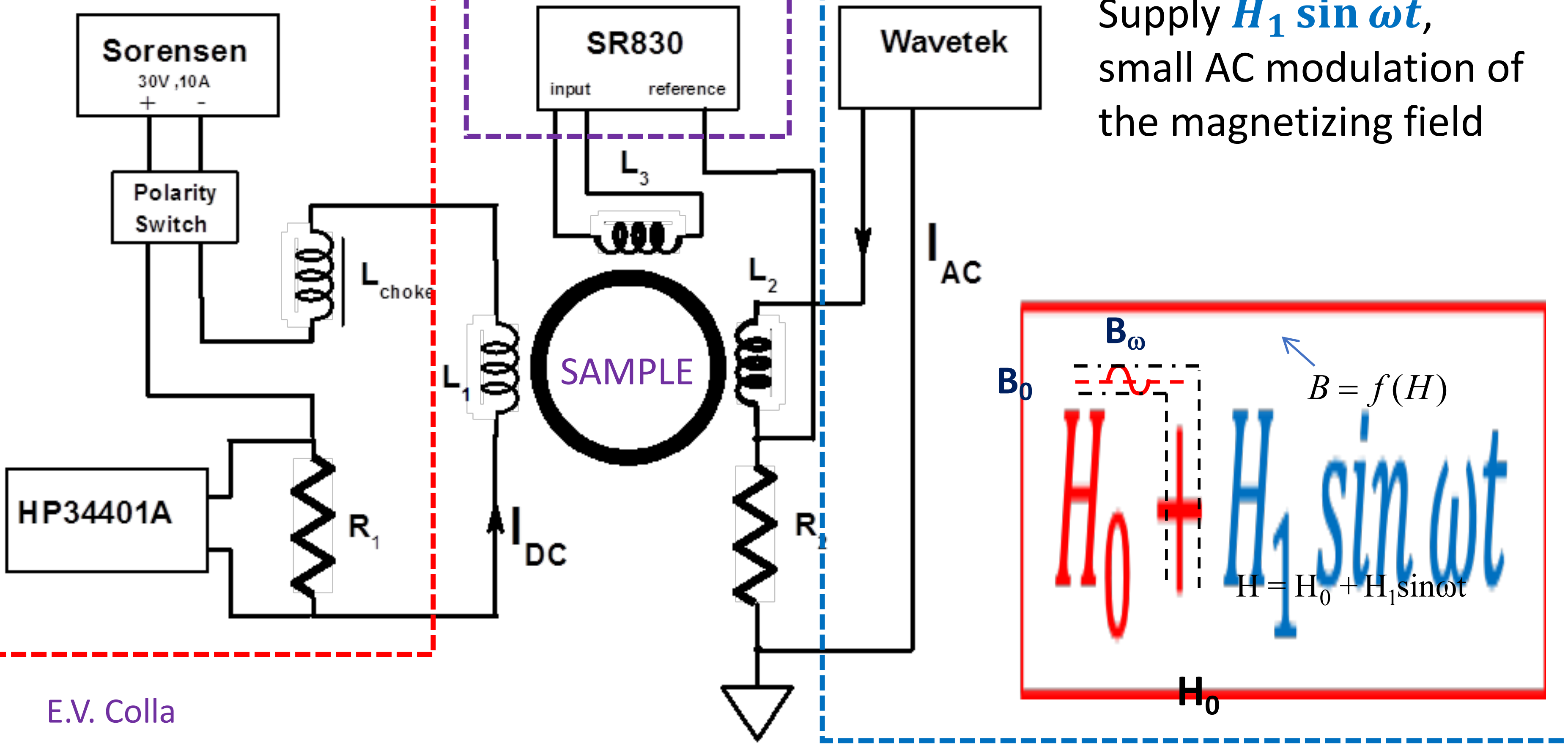
Supply $\boldsymbol{H_0}$, DC magnetizing field

$$H = \frac{N_p I_p}{2\pi r}$$

DMM to monitor $\boldsymbol{H_0}$

Measure $\boldsymbol{B_\omega}$, resulting modulation of the magnetic induction

Supply $\boldsymbol{H_1 \sin \omega t}$, small AC modulation of the magnetizing field



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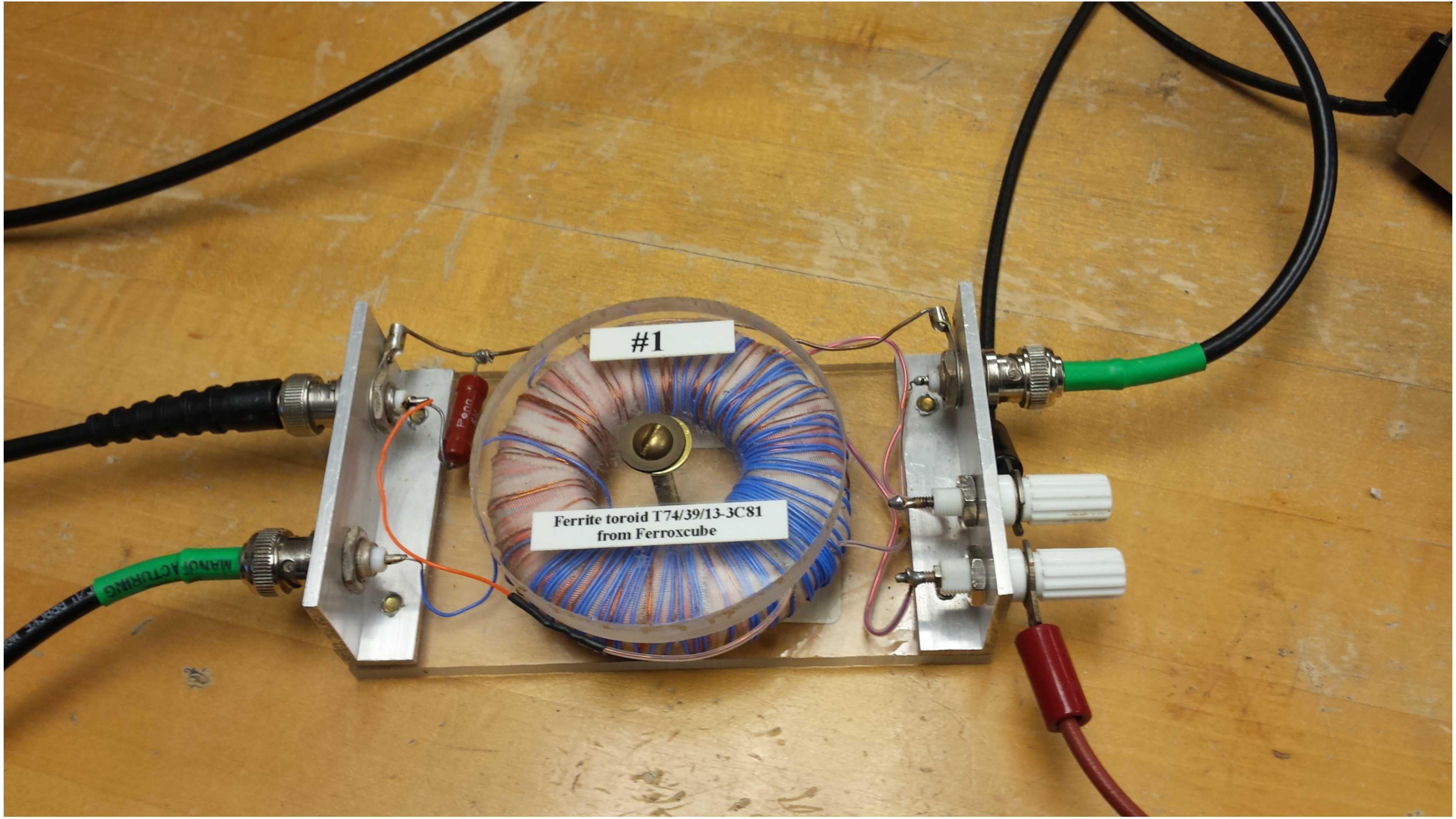
Setup #1: Mapping the Hysteresis Loops



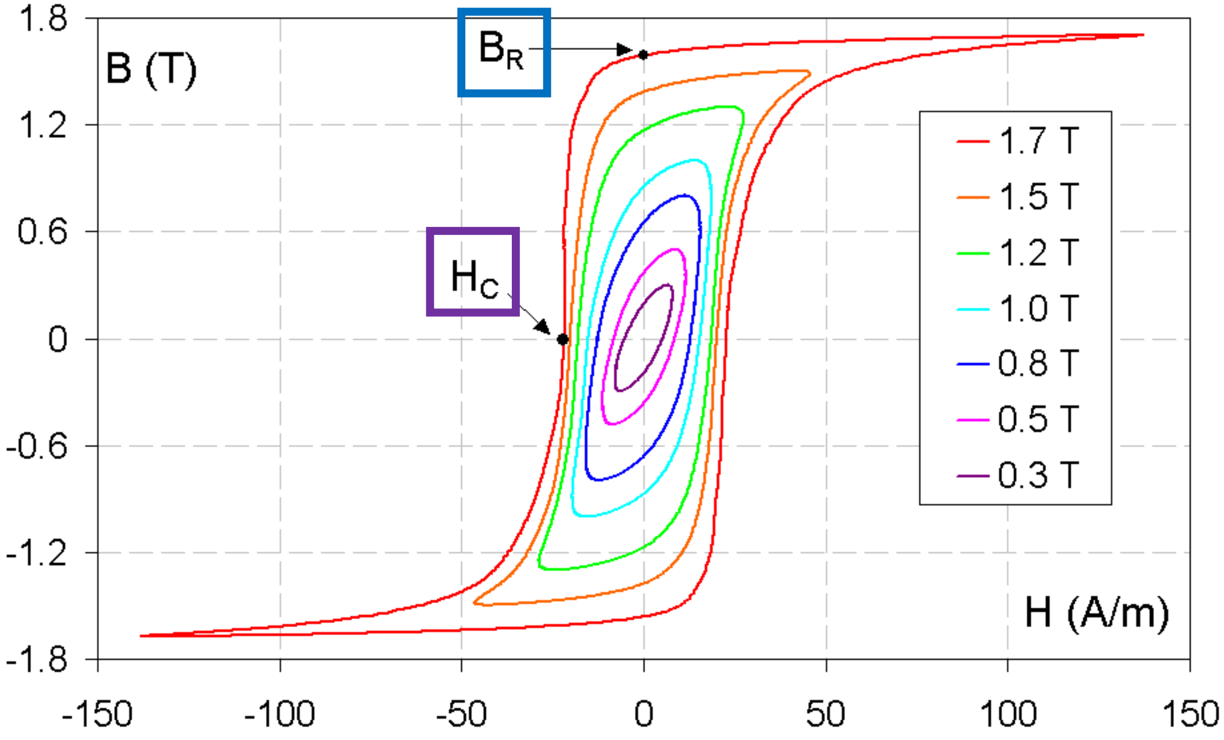
J. Boparai

More on Hysteresis Loops

There isn't just one hysteresis curve!

Key values for saturation curve:

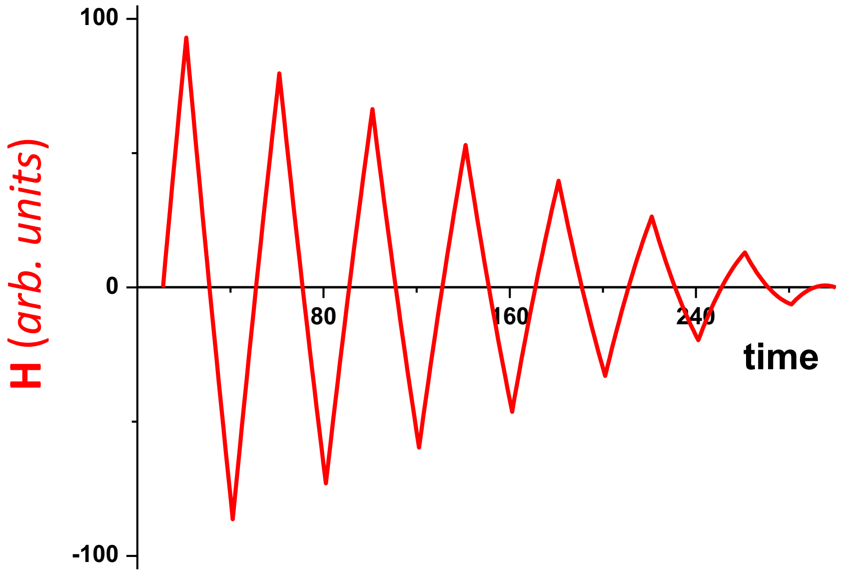
- **B_R** is the **saturation remanence**: the maximum residual magnetism at zero applied field
- **H_C** is the **coercivity**: the applied field required to demagnetize a sample that has been saturated.



A family of AC hysteresis loops for grain-oriented electrical steel (Wikipedia:Remanence)

Demagnetizing the Core

Clear the sample’s unknown residual field (**remanence**) by imposing a slowly-decaying AC H-field



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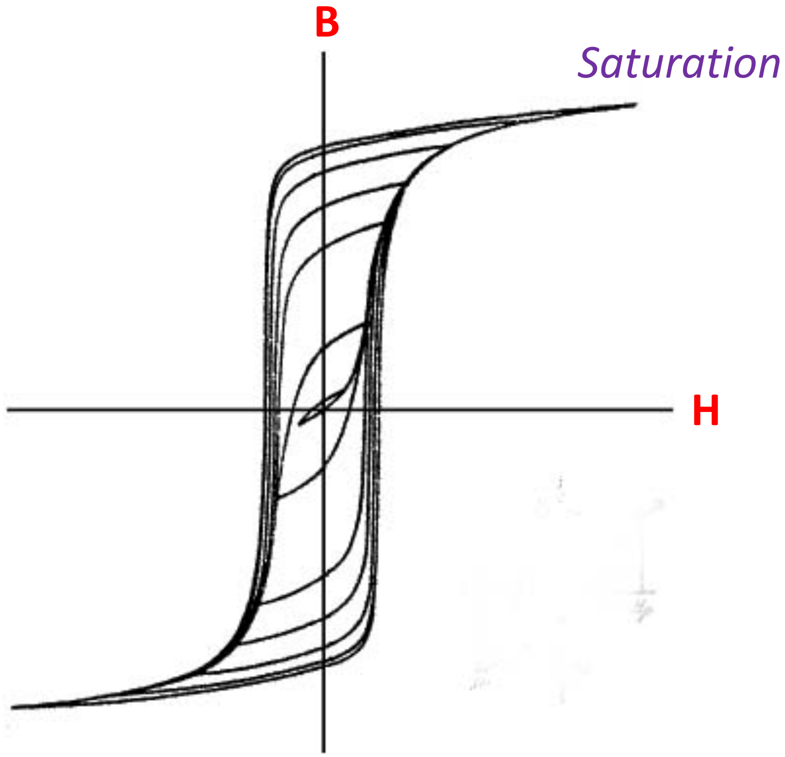
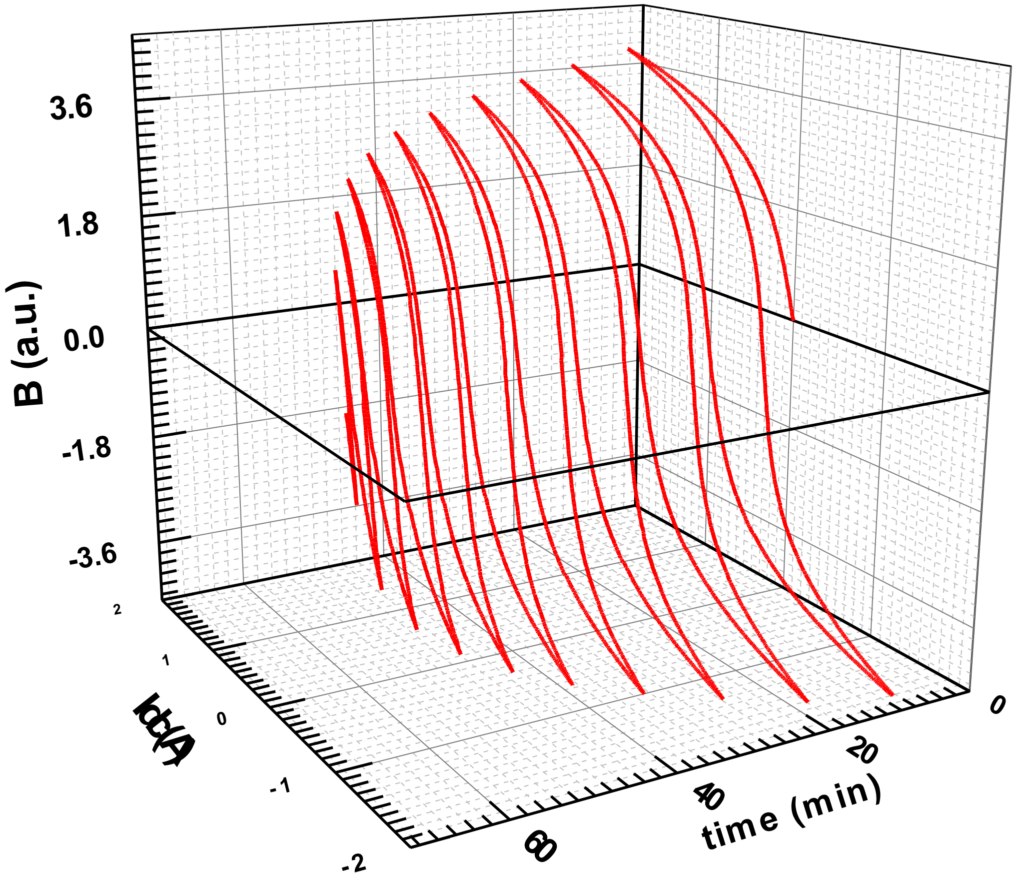
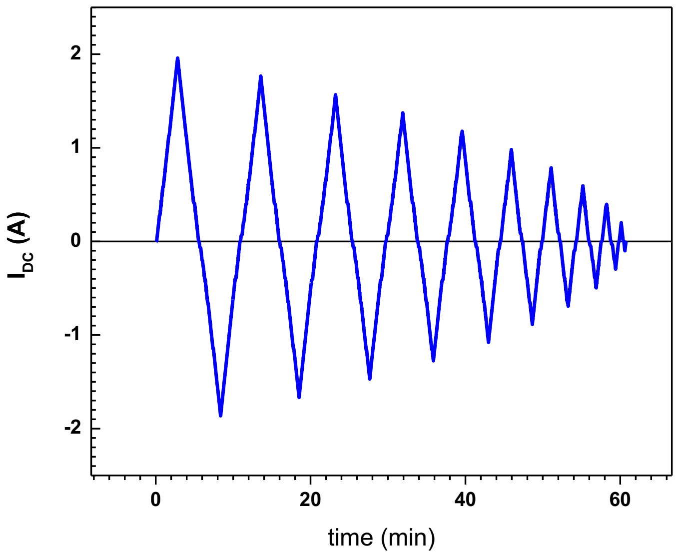


Demagnetizing the Core

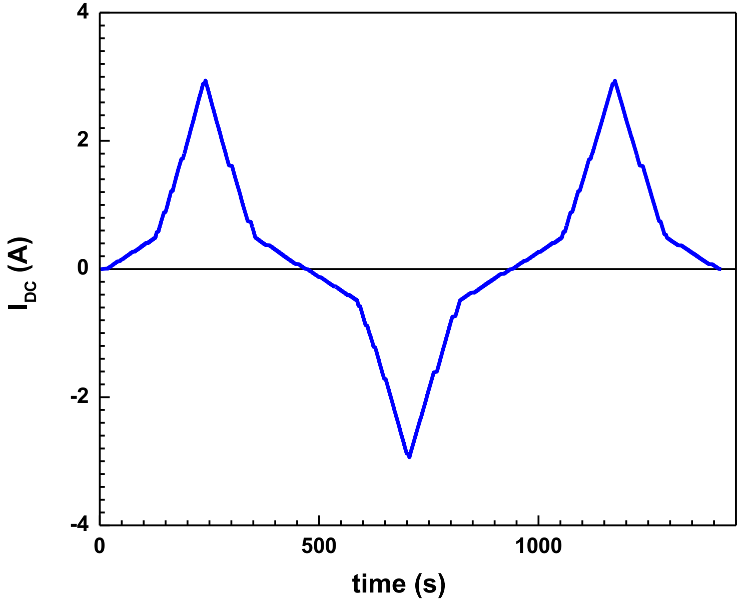


Example: Demagnetization of 4C65 toroid from Ferroxcube

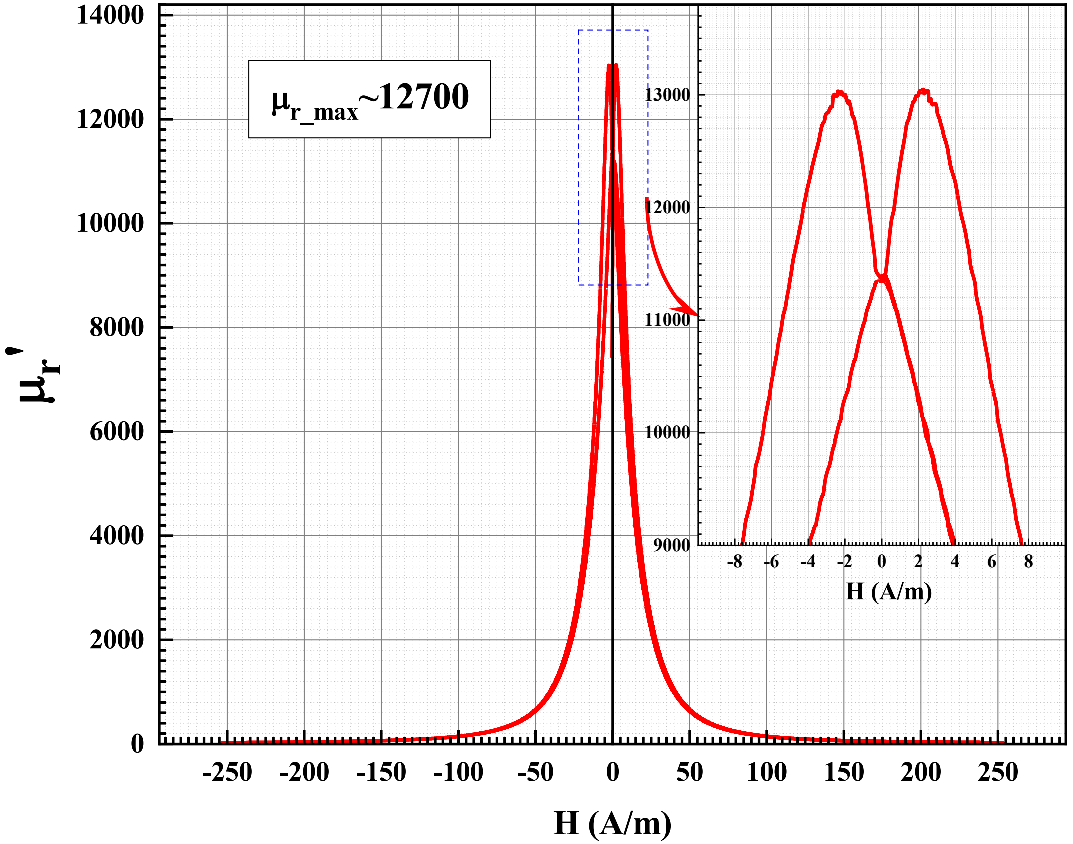


Data and plots by E.V. Colla

Measuring the Magnetic Permeability



Example: DC current profile and magnetic permeability of Magnetics ZW44715TC



Data and plots by E.V. Colla

From Permeability to Hysteresis Loop

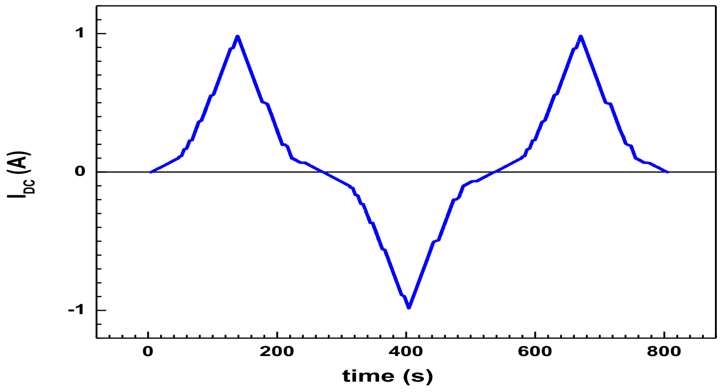
How to get a good data set for an unknown sample?

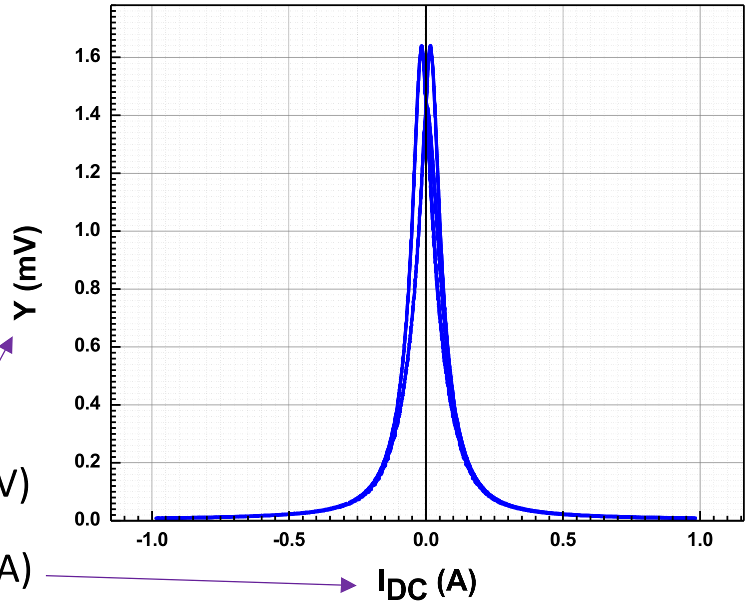
1. Perform one **coarse** (i.e., *fast*) **scan** over I_{DC} (that is, H_0) to find the required dynamic range.
   How wide a range must we cover?
   How small a step size is needed for detail?

   Based upon this data set…

2. Perform a **precision scan** for data analysis

ECE Storeroom unknown material sample #5





Y: amplitude measured by SR830 (in mV)

I_{DC}: current in primary coil (in A)

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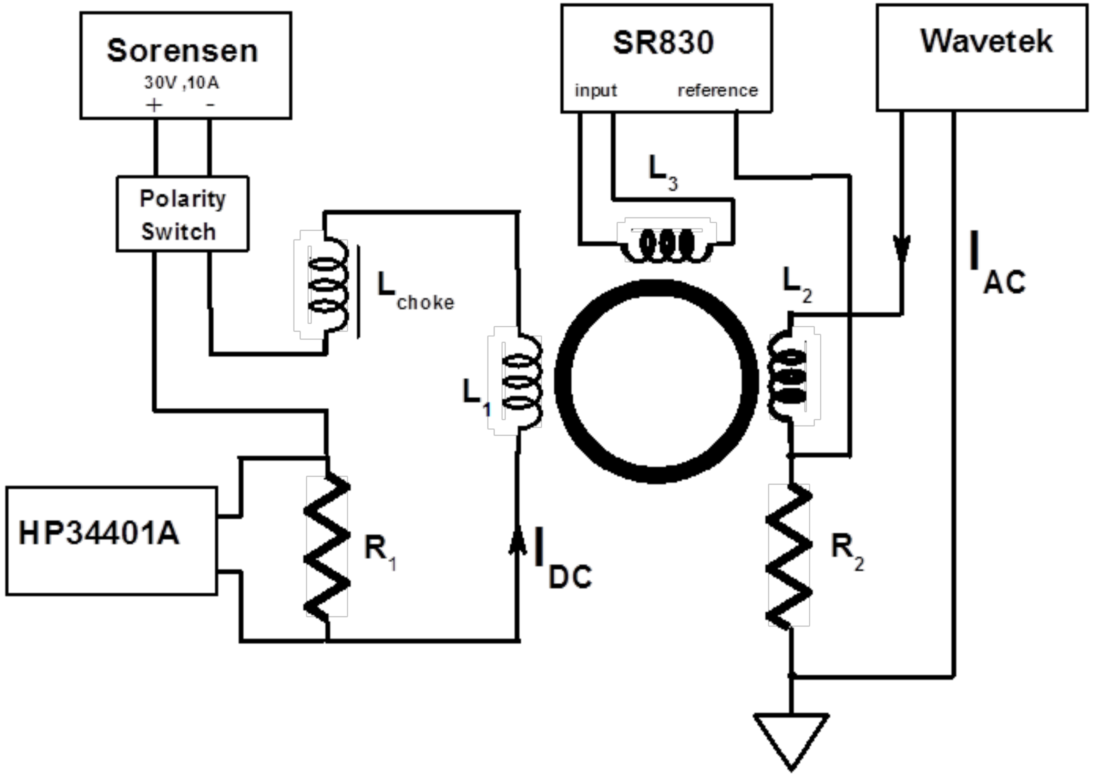
From Permeability to Hysteresis Loop

3. **Calibration**: relating what you measure to what you want to know



What does the lock-in amplifier actually measure?

… the **EMF** imposed on the pickup coil

$$V_{lock-in} = -\frac{d\Phi}{dt} = -\frac{d}{dt}(\vec{B}\cdot\vec{S})$$

The AC current driven in primary coil L_2 is:

$$I_p = \frac{V_0 \sin \omega t}{R_2}$$

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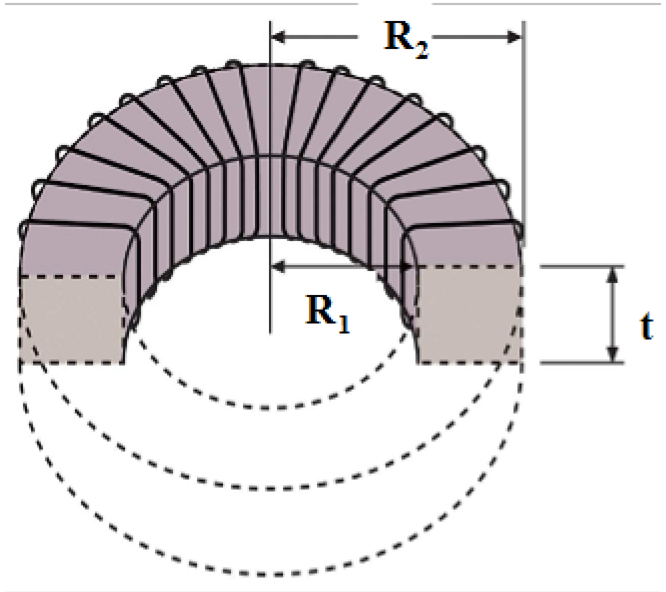
From Permeability to Hysteresis Loop

3. **Calibration**: relating what you measure to what you want to know

$$V_{lock-in} = -\frac{d\Phi}{dt} = -\frac{d}{dt}(\vec{B}\cdot\vec{S})$$

$$I_p = \frac{V_0 \sin \omega t}{R_2}$$



Primary coil is a **toroid** of ***N$_p$*** turns carrying a current ***I$_p$*** creates a magnetic field ***H***, and thus adds a flux ***dΦ***:

$$H = \frac{N_p I_p}{2\pi r}$$

$$d\Phi = \mu \int \vec{H}\cdot d\vec{a} = \frac{\mu\, I\, N\, t}{2\pi}\int_{R_1}^{R_2}\frac{dr}{r} = \frac{\mu\, I\, N\, t}{2\pi}\ln\frac{R_2}{R_1}$$

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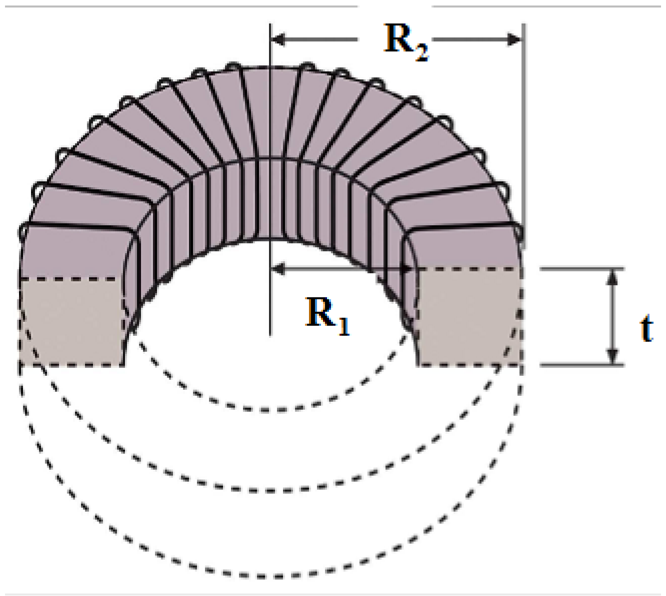
From Permeability to Hysteresis Loop

3. **Calibration**: relating what you measure to what you want to know

$$V_{lock-in} = -\frac{d\Phi}{dt} = -\frac{d}{dt}\left(\vec{B}\cdot\vec{S}\right)$$



Total flux detected by the pickup coil:

$$\Phi = N_{pickup} d\Phi = \frac{\mu\, N_{pickup} I_p\, N_p\, t}{2\pi} \ln\frac{R_2}{R_1}$$

Careful about whether the lock-in is giving you amplitude or r.m.s.!

Toroid inductance:

$$L = \frac{\Phi}{I} = \mu_r L_0 = (\mu' - i\,\mu'')L_0$$

Geometric inductance in vacuum

$$L_0 = \frac{\mu_0\, N_{pickup}\, N_p\, t}{2\pi} \ln\frac{R_2}{R_1}$$

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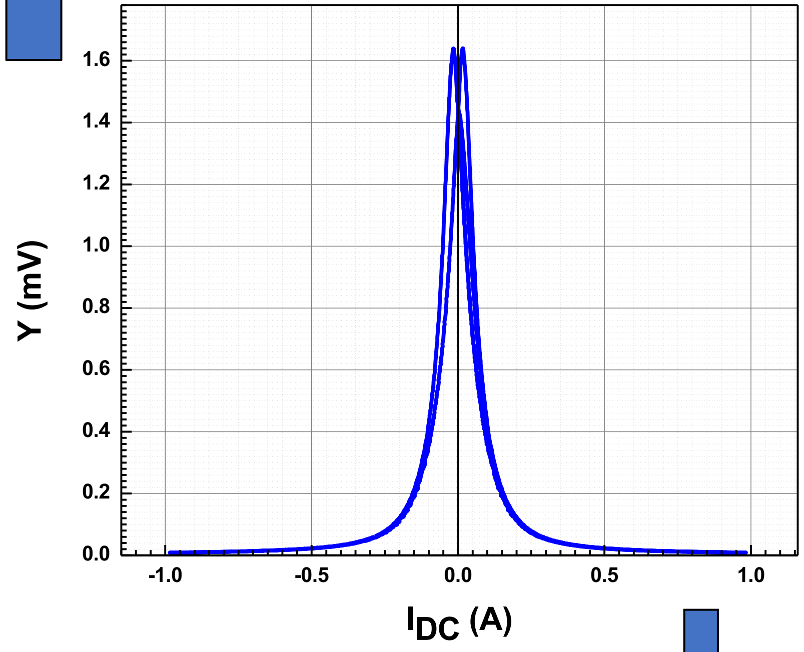
From Permeability to Hysteresis Loop

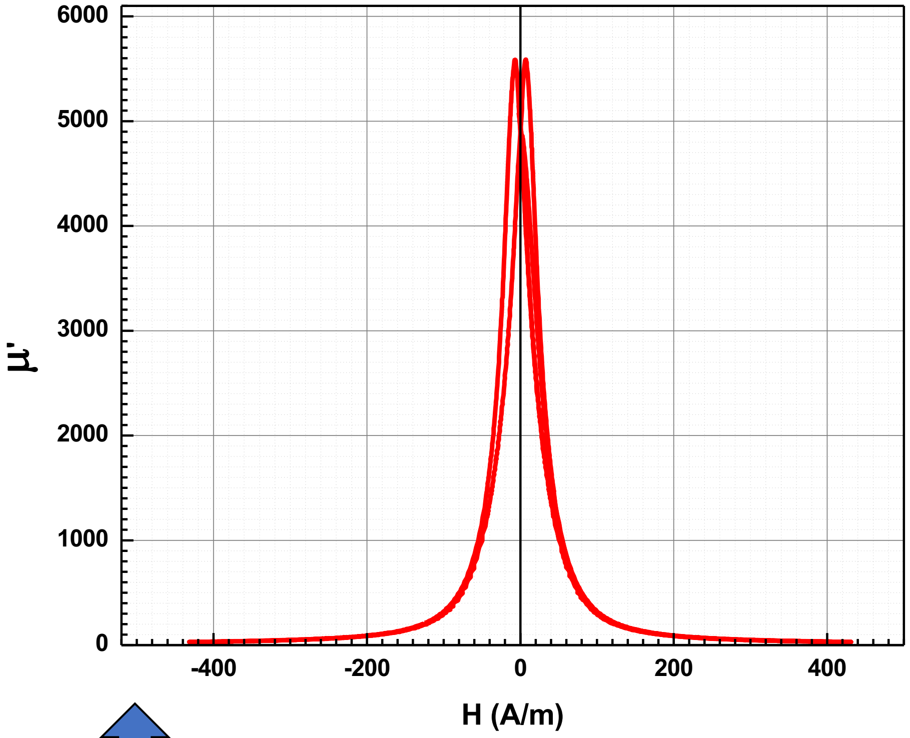
Signal generator: $$\frac{dI_p}{dt} = \omega \frac{V_0}{R_2} \cos \omega t$$

Note: proportional to ω!

$$V_{lock-in} = \mu_r L_0 \frac{dI_p}{dt}$$



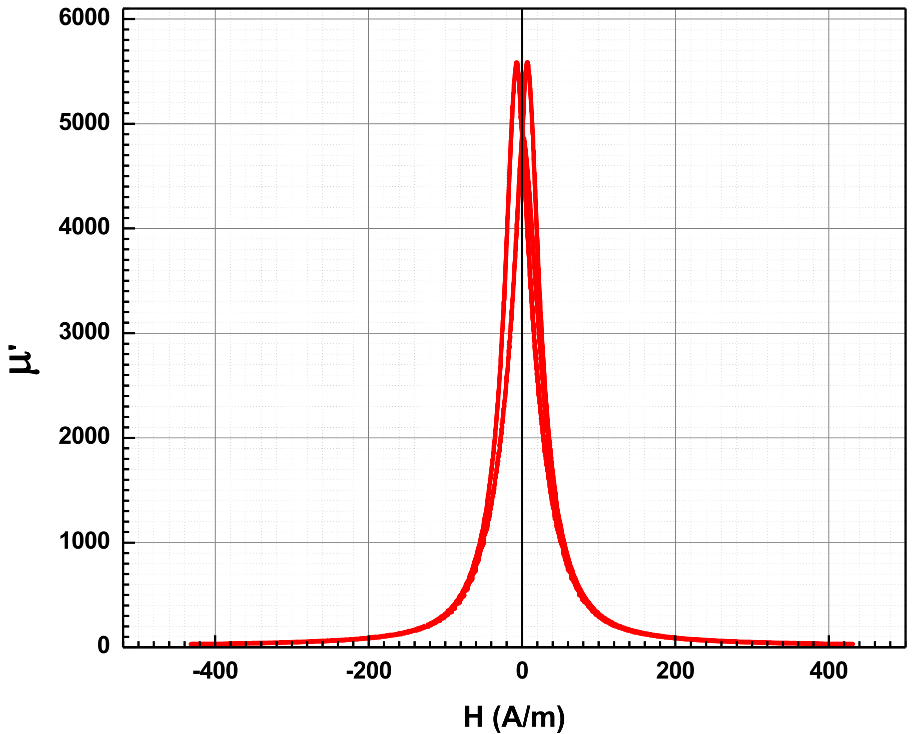
$$\mu_r = \mu' - i\,\mu''$$



$$H_0 = \frac{N_p I_{DC}}{2\pi r}$$

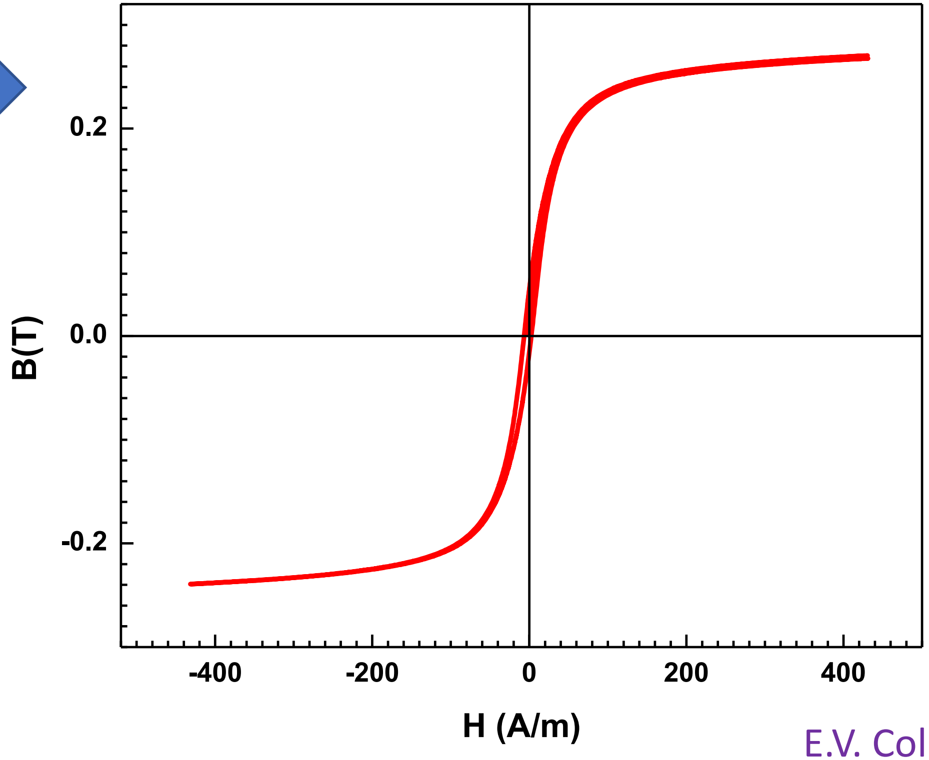
From Permeability to Hysteresis Loop

4. **Integration**: **$\mu_r(H)$** is a local derivative, so we must integrate it to find **B(H)**



$$\mu(H_0) = \mu_0 \mu_r(H_0) = \left.\frac{dB}{dH}\right|_{H_0}$$

$$B(H) = \mu_0 \int \mu_r(H) dH$$



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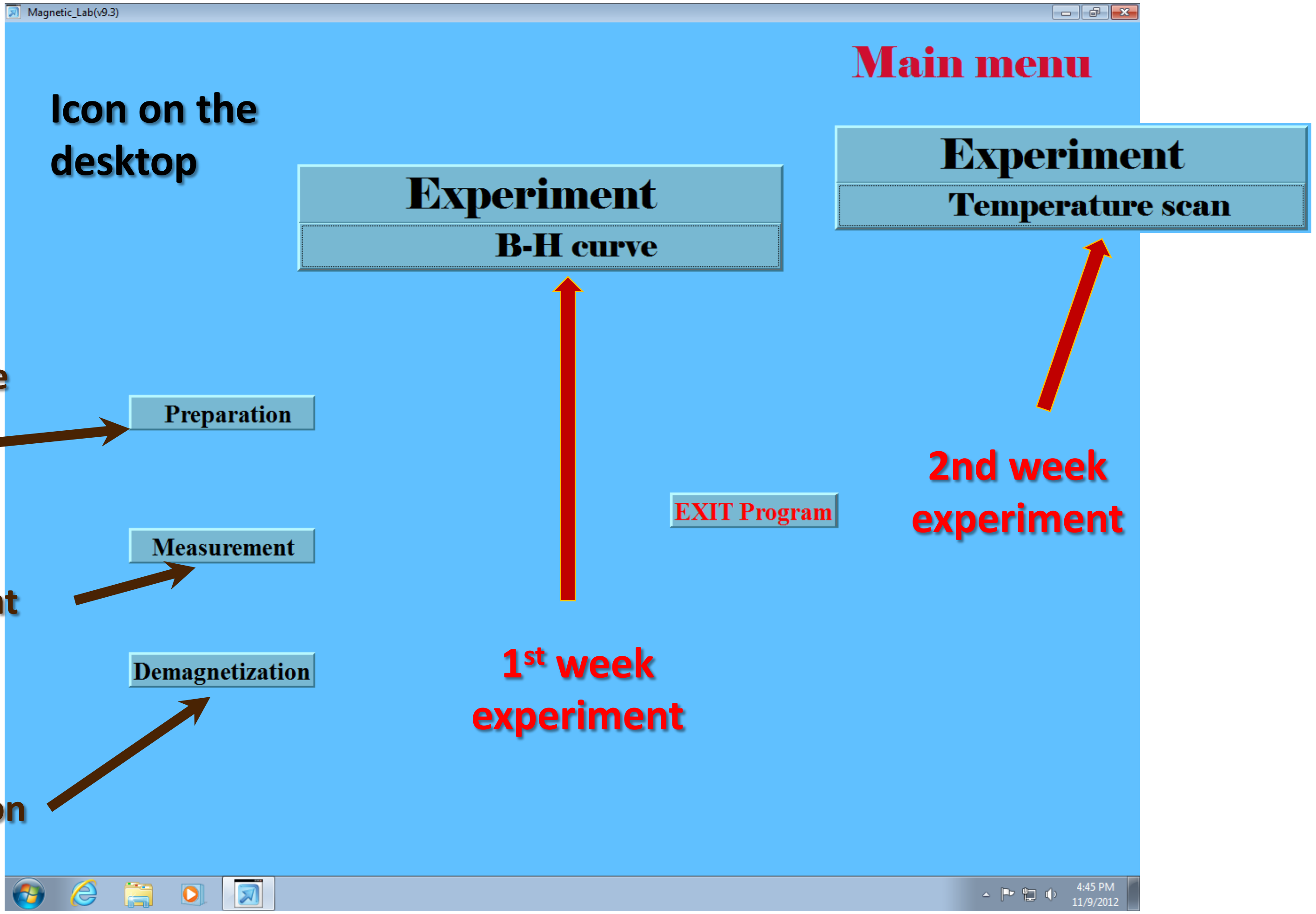
Using the Acquisition Software – E.V. Colla

Magnetic Lab v9.2



Icon on the desktop

Preparation of the profile of the experiment

B-H measurement

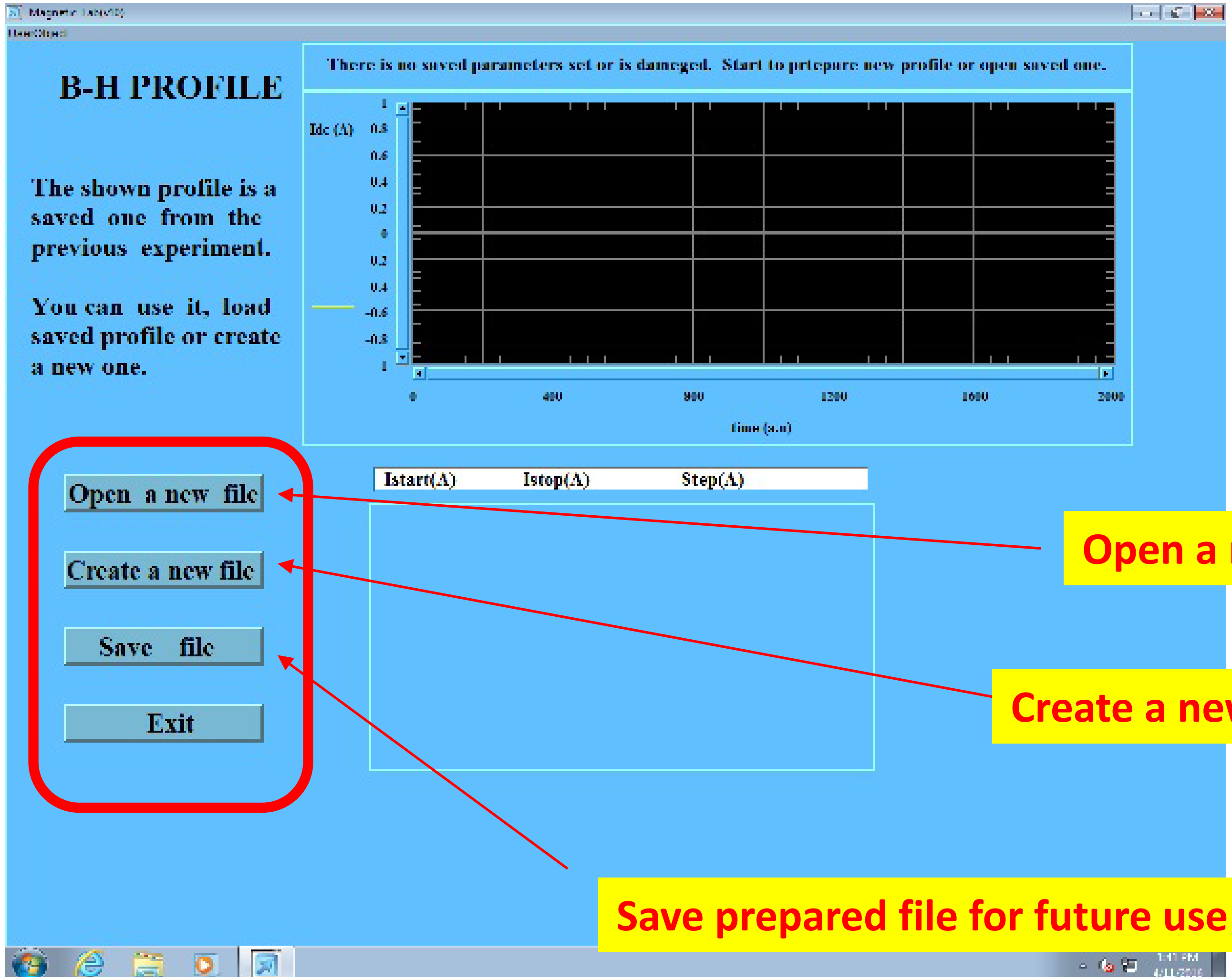
Demagnetization

1st week experiment

2nd week experiment

Using the Acquisition Software – E.V. Colla

Preparing the measurement profile and using the profile template



B-H PROFILE

There is no saved parameters set or is dameged. Start to prtepare new profile or open saved one.

The shown profile is a saved one from the previous experiment.

You can use it, load saved profile or create a new one.

Open a new file

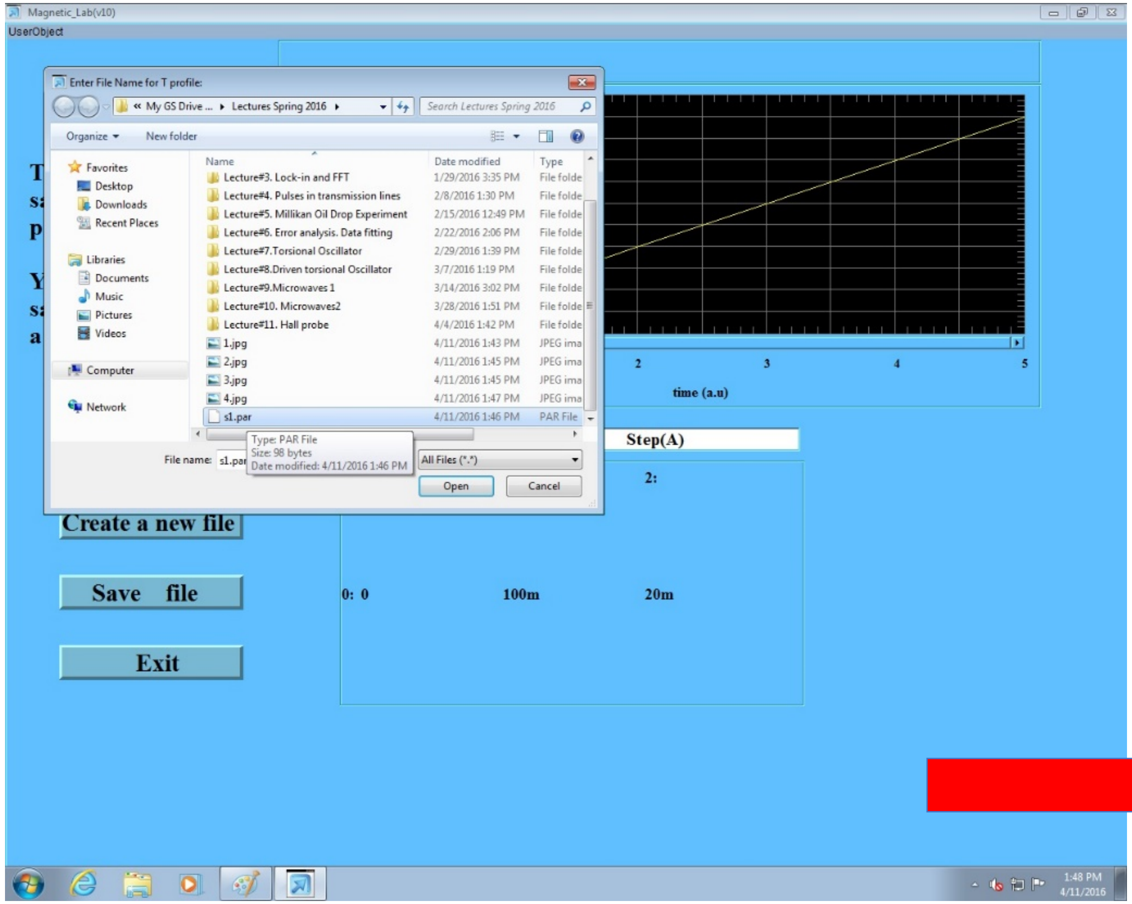
Create a new file

Save prepared file for future use

E.V. Colla

Using the Acquisition Software – E.V. Colla

Preparing the measurement profile and using the profile template



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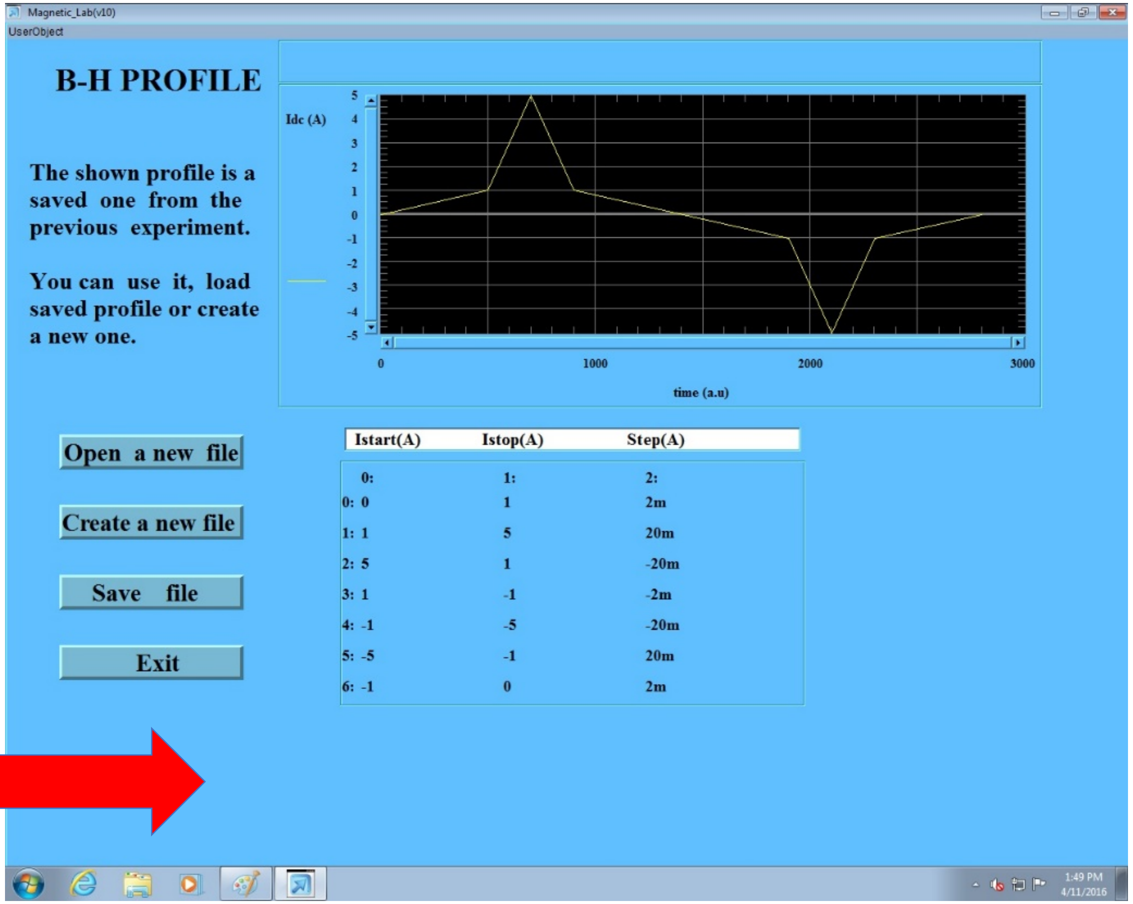
Lecture#4. Pulses in transmission lines	2/8/2016 1:30 PM	File folde
Lecture#5. Millikan Oil Drop Experiment	2/15/2016 12:49 PM	File folde
Lecture#6. Error analysis. Data fitting	2/22/2016 2:06 PM	File folde
Lecture#7.Torsional Oscillator	2/29/2016 1:39 PM	File folde
Lecture#8.Driven torsional Oscillator	3/7/2016 1:19 PM	File folde
Lecture#9.Microwaves 1	3/14/2016 3:02 PM	File folde
Lecture#10. Microwaves2	3/28/2016 1:51 PM	File folde
Lecture#11. Hall probe	4/4/2016 1:42 PM	File folde
1.jpg	4/11/2016 1:43 PM	JPEG ima
2.jpg	4/11/2016 1:45 PM	JPEG ima
3.jpg	4/11/2016 1:45 PM	JPEG ima
4.jpg	4/11/2016 1:47 PM	JPEG ima
s1.par	4/11/2016 1:46 PM	PAR File

File name: s1.par

Type: PAR File
Size: 98 bytes
Date modified: 4/11/2016 1:46 PM

		Step(A)
		2:
0: 0	100m	20m



B-H PROFILE

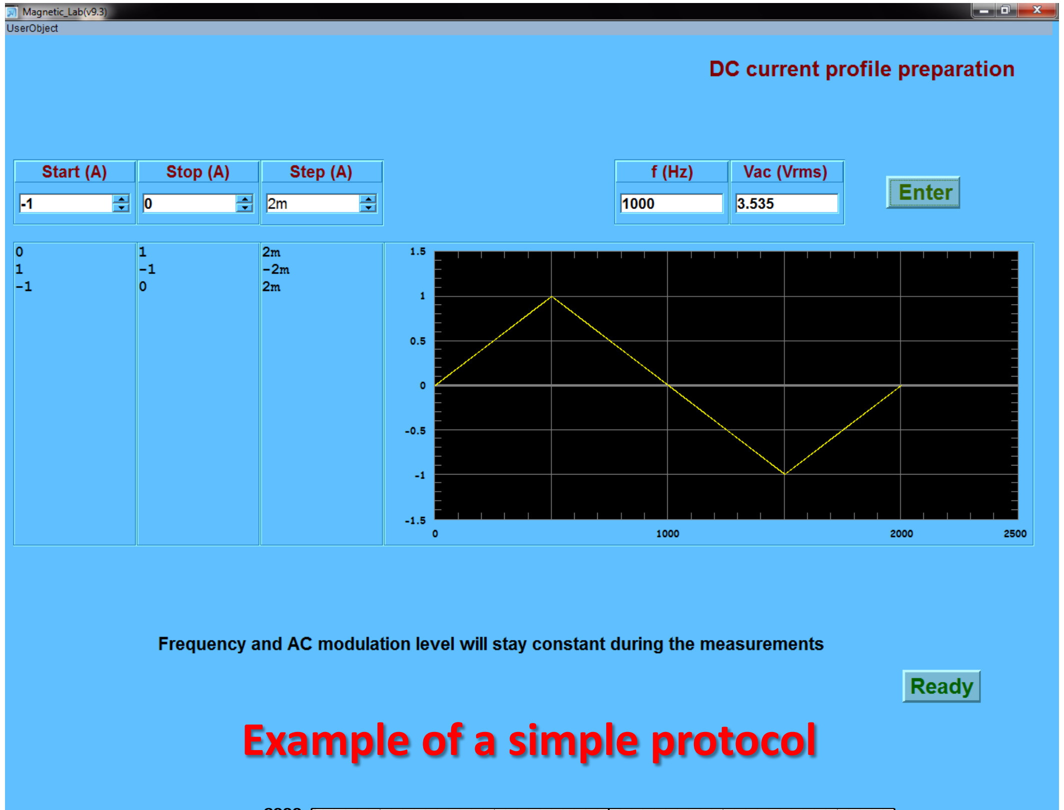
The shown profile is a saved one from the previous experiment.

You can use it, load saved profile or create a new one.

	Istart(A)	Istop(A)	Step(A)
	0:	1:	2:
0:	0	1	2m
1:	1	5	20m
2:	5	1	-20m
3:	1	-1	-2m
4:	-1	-5	-20m
5:	-5	-1	20m
6:	-1	0	2m

E.V. Colla

Using the Acquisition Software – E.V. Colla



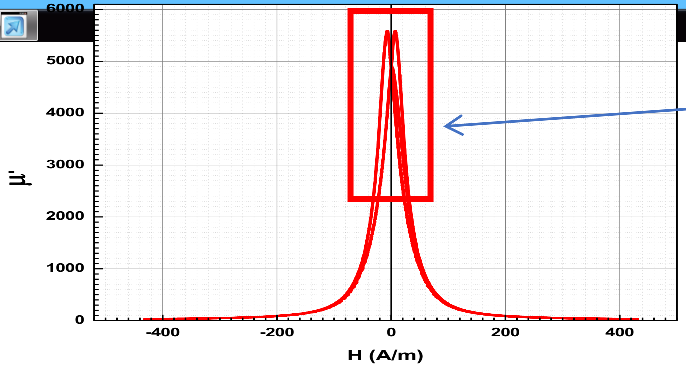
DC current profile preparation

Start (A)	Stop (A)	Step (A)	f (Hz)	Vac (Vrms)
-1	0	2m	1000	3.535

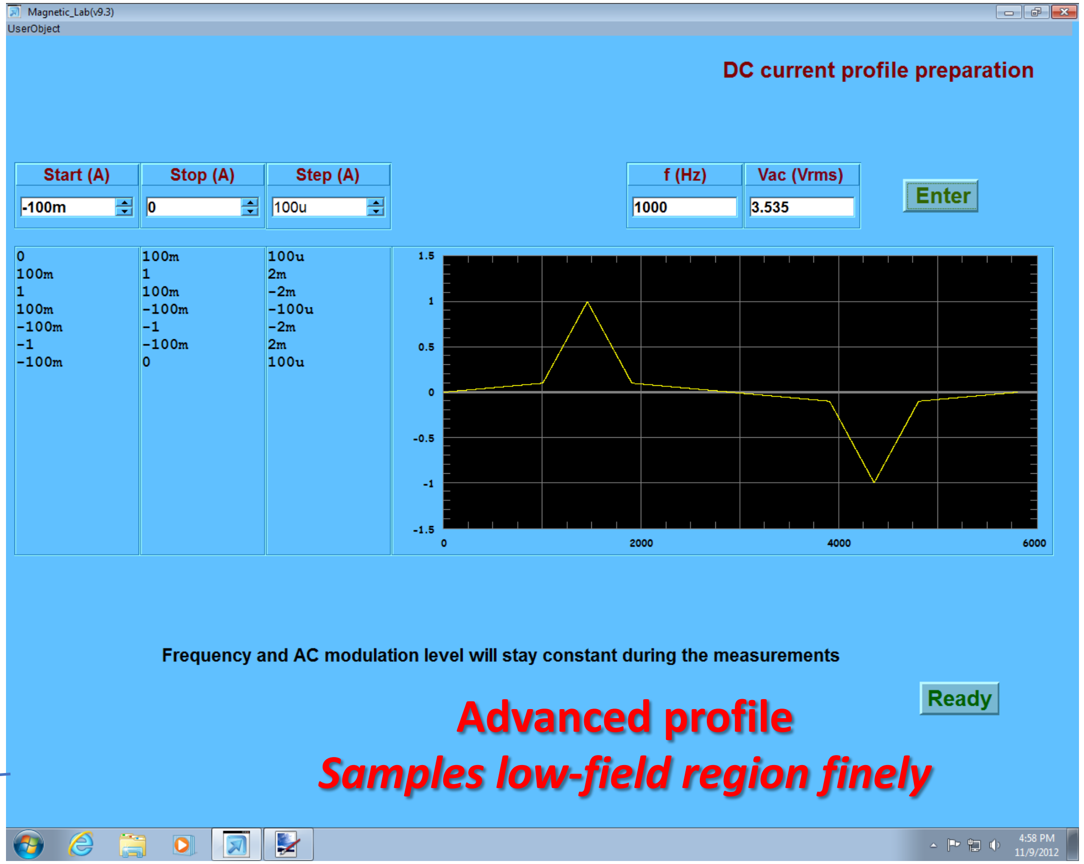
Start (A)	Stop (A)	Step (A)
0	1	2m
1	-1	-2m
-1	0	2m

Frequency and AC modulation level will stay constant during the measurements

Example of a simple protocol



Preparing the measurement profile



DC current profile preparation

Start (A)	Stop (A)	Step (A)	f (Hz)	Vac (Vrms)
-100m	0	100u	1000	3.535

Start (A)	Stop (A)	Step (A)
0	100m	100u
100m	1	2m
1	100m	-2m
100m	-100m	-100u
-100m	-1	-2m
-1	-100m	2m
-100m	0	100u

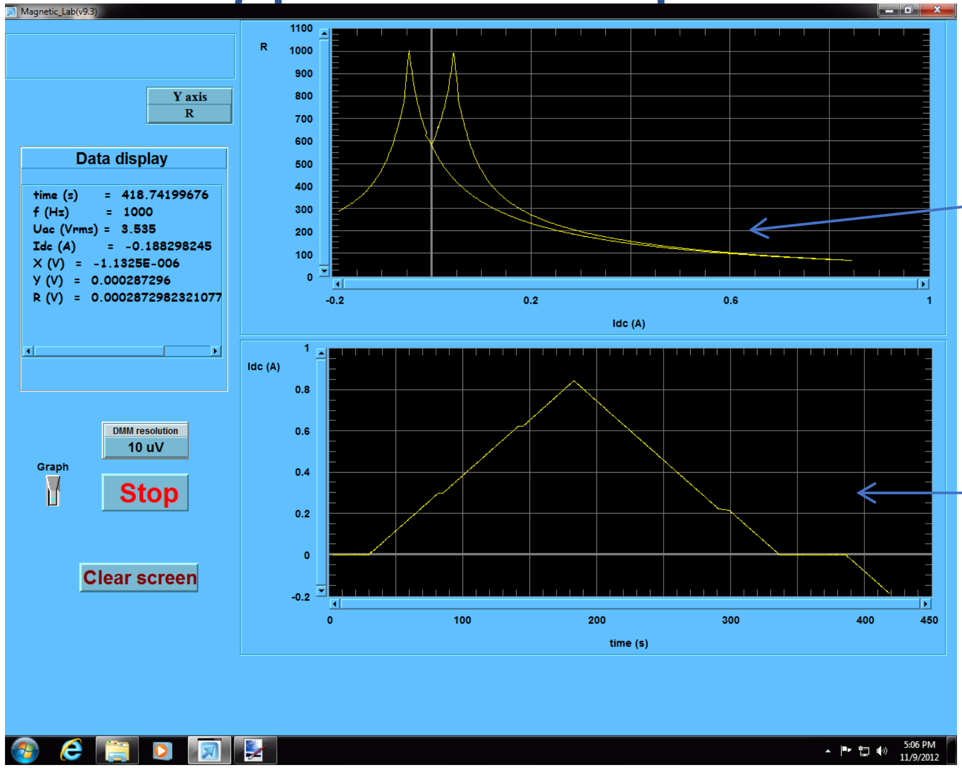
Frequency and AC modulation level will stay constant during the measurements

Advanced profile

Samples low-field region finely

E.V. Colla

Using the Acquisition Software – E.V. Colla



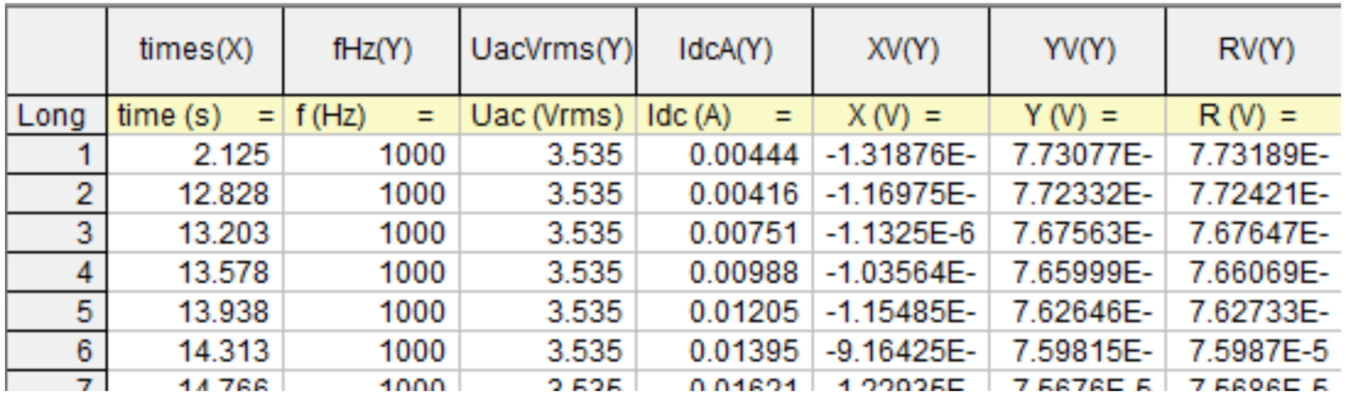
Data display

time (s) = 418.74199676
f (Hz) = 1000
Uac (Vrms) = 3.535
Idc (A) = -0.188298245
X (V) = -1.1325E-006
Y (V) = 0.000287296
R (V) = 0.0002872982321077

Measurement Window

Lock-in amplifier response

The profile of the applied DC current



	times(X)	fHz(Y)	UacVrms(Y)	IdcA(Y)	XV(Y)	YV(Y)	RV(Y)
Long	time (s) =	f (Hz) =	Uac (Vrms)	Idc (A) =	X (V) =	Y (V) =	R (V) =
1	2.125	1000	3.535	0.00444	-1.31876E-	7.73077E-	7.73189E-
2	12.828	1000	3.535	0.00416	-1.16975E-	7.72332E-	7.72421E-
3	13.203	1000	3.535	0.00751	-1.1325E-6	7.67563E-	7.67647E-
4	13.578	1000	3.535	0.00988	-1.03564E-	7.65999E-	7.66069E-
5	13.938	1000	3.535	0.01205	-1.15485E-	7.62646E-	7.62733E-
6	14.313	1000	3.535	0.01395	-9.16425E-	7.59815E-	7.5987E-5
7	14.766	1000	3.535	0.01621	1.22935E	7.5676E-5	7.5686E-5

Structure of the data file (B-H experiment)

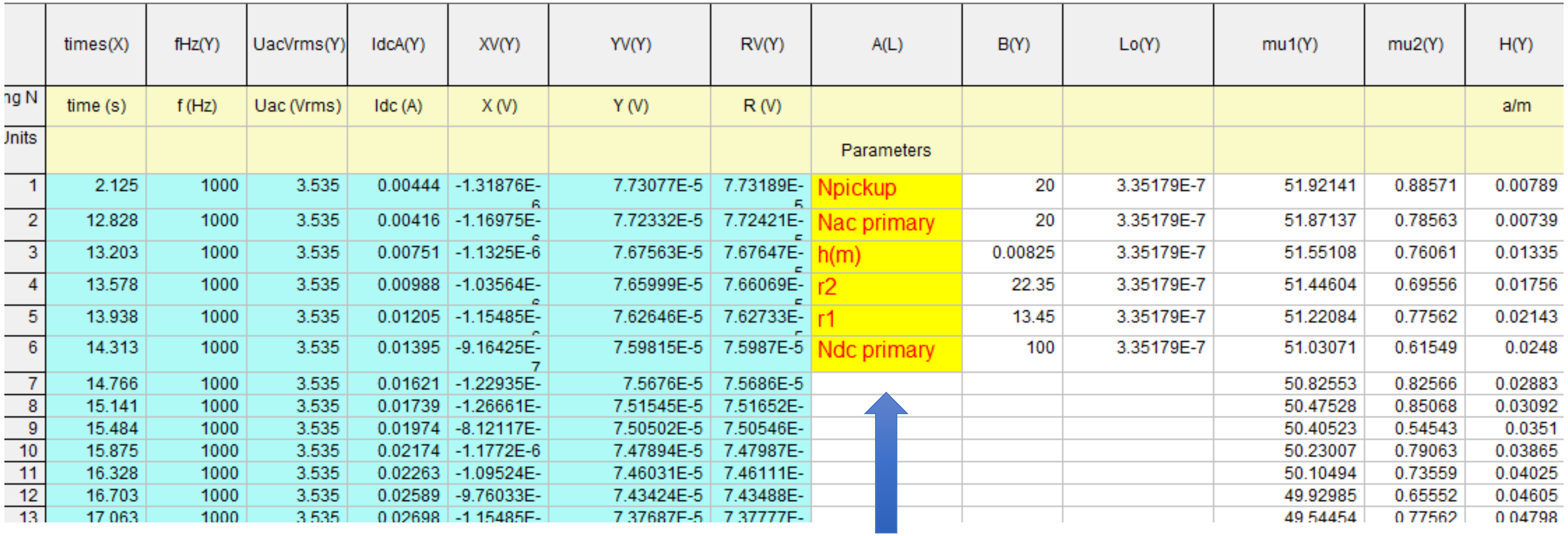
E.V. Colla

Data Analysis Using Origin – E.V. Colla

To calculate the permeability, it’s easiest to use the template :

\\engr-file-03\phyinst\APL Courses\PHYCS401\Common\Origin templates\AC magnetic Lab\MU_CALCULATION.otwu

It does not contain the equations – you have to write them!



	times(X)	fHz(Y)	UacVrms(Y)	IdcA(Y)	XV(Y)	YV(Y)	RV(Y)	A(L)	B(Y)	Lo(Y)	mu1(Y)	mu2(Y)	H(Y)
ng N	time (s)	f (Hz)	Uac (Vrms)	Idc (A)	X (V)	Y (V)	R (V)						a/m
Jnits								Parameters					
1	2.125	1000	3.535	0.00444	-1.31876E-6	7.73077E-5	7.73189E-5	Npickup	20	3.35179E-7	51.92141	0.88571	0.00789
2	12.828	1000	3.535	0.00416	-1.16975E-6	7.72332E-5	7.72421E-5	Nac primary	20	3.35179E-7	51.87137	0.78563	0.00739
3	13.203	1000	3.535	0.00751	-1.1325E-6	7.67563E-5	7.67647E-5	h(m)	0.00825	3.35179E-7	51.55108	0.76061	0.01335
4	13.578	1000	3.535	0.00988	-1.03564E-6	7.65999E-5	7.66069E-5	r2	22.35	3.35179E-7	51.44604	0.69556	0.01756
5	13.938	1000	3.535	0.01205	-1.15485E-6	7.62646E-5	7.62733E-5	r1	13.45	3.35179E-7	51.22084	0.77562	0.02143
6	14.313	1000	3.535	0.01395	-9.16425E-7	7.59815E-5	7.5987E-5	Ndc primary	100	3.35179E-7	51.03071	0.61549	0.0248
7	14.766	1000	3.535	0.01621	-1.22935E-	7.5676E-5	7.5686E-5				50.82553	0.82566	0.02883
8	15.141	1000	3.535	0.01739	-1.26661E-	7.51545E-5	7.51652E-				50.47528	0.85068	0.03092
9	15.484	1000	3.535	0.01974	-8.12117E-	7.50502E-5	7.50546E-				50.40523	0.54543	0.0351
10	15.875	1000	3.535	0.02174	-1.1772E-6	7.47894E-5	7.47987E-				50.23007	0.79063	0.03865
11	16.328	1000	3.535	0.02263	-1.09524E-	7.46031E-5	7.46111E-				50.10494	0.73559	0.04025
12	16.703	1000	3.535	0.02589	-9.76033E-	7.43424E-5	7.43488E-				49.92985	0.65552	0.04605
13	17.063	1000	3.535	0.02698	-1.15485E-	7.37687E-5	7.37777E-				49.54454	0.77562	0.04798

Raw data

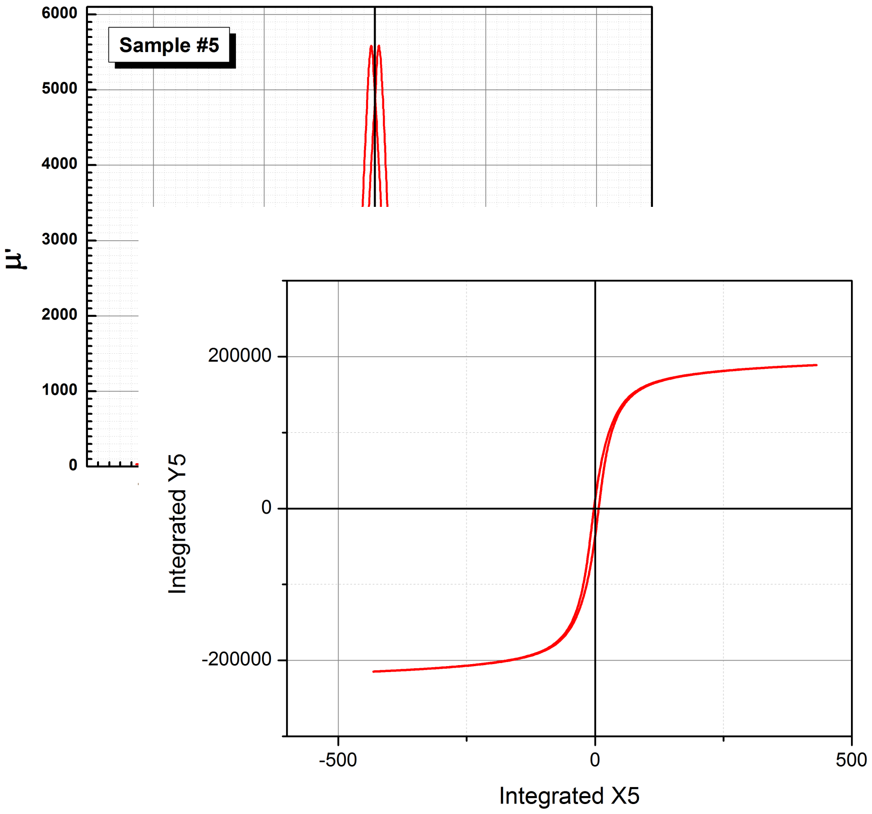
Parameters

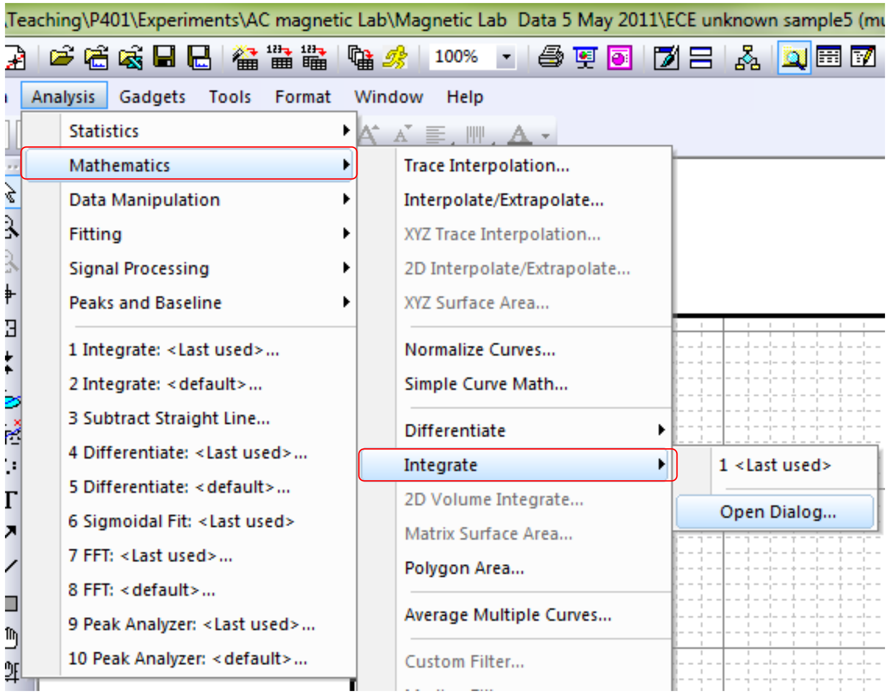
Calculated results

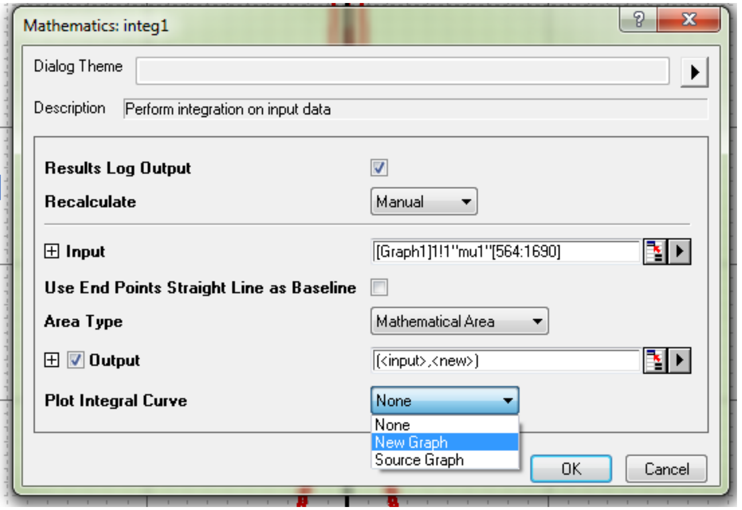
E.V. Colla

Data Analysis Using Origin: Integrating – E.V. Colla

$$B(H) = \mu_0 \int \mu_r(H) dH$$






Mathematics: integ1

Dialog Theme

Description: Perform integration on input data

Results Log Output ☑

Recalculate: Manual

⊞ **Input**: [Graph1]1!1"mu1"[564:1690]

Use End Points Straight Line as Baseline ☐

Area Type: Mathematical Area

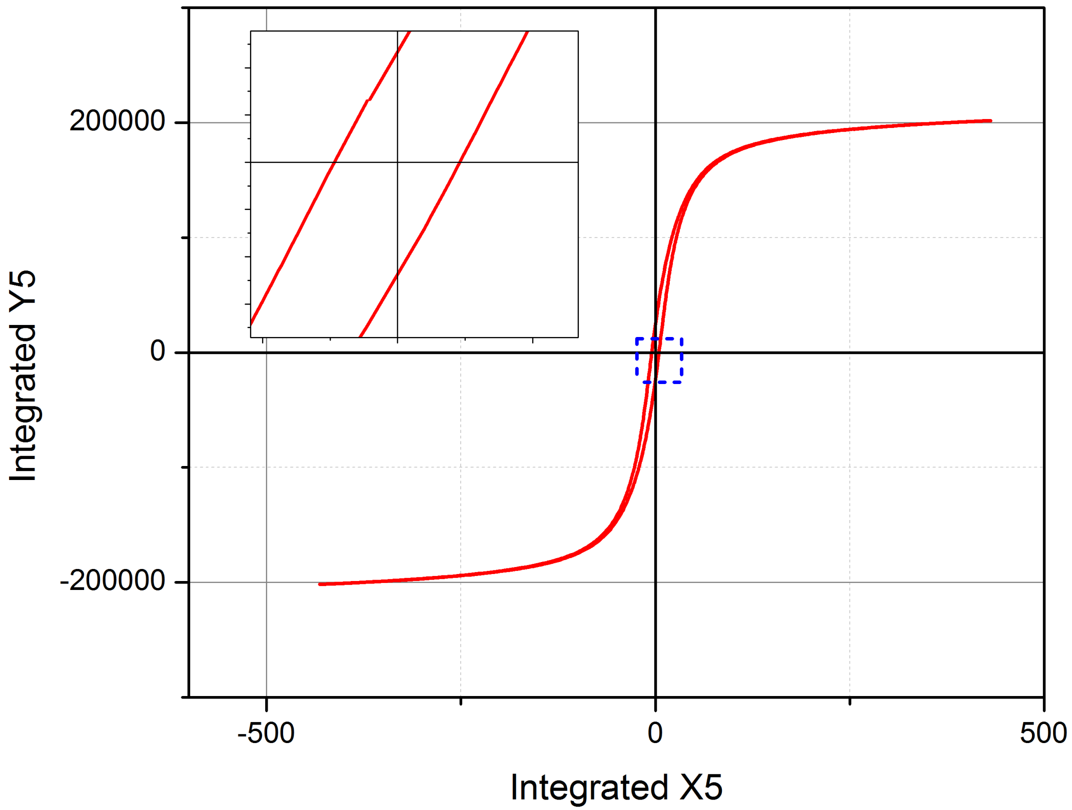
⊞ ☑ **Output**: [<input>,<new>]

Plot Integral Curve: None

E.V. Colla

Data Analysis Using Origin: Integrating – E.V. Colla

$$B(H) = \mu_0 \int \mu_r(H)dH + offset$$



E.V. Colla

References

- Information about magnetic materials can be found in:
  \\engr-file-03\phyinst\APL Courses\PHYCS401\Experiments\AC_Magnetization\Magnetic Materials

- SR830 (Lock-in Amplifier) manual
  \\engr-file-03\phyinst\APL Courses\PHYCS401\Common\EquipmentManuals\SR830m.pdf