# AC Measurement of Magnetic Susceptibility

## Episode 2 – Loss and Thermal Effects

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Physics 401

Spring 2020

ILLINOIS

# Outline

Combine the tools and techniques we've learned to characterize **magnetic properties of materials**

- **Magnetic Materials:** Dia- and paramagnetism (Bonus: superconductors!)
- **Week 1 Refresher:** AC Measurement of Susceptibility
- **Magnetic Losses:** Complex permeability
- **Temperature Dependence:** Curie-Weiss Law
- **Finishing the Final Lab**: What you'll get, what you need to do
- ***Closing out the Semester***

This is the second week of the final lab

Report counts as your final exam

# Reminder: Magnetic Response of Materials

Two things are often called the "magnetic field": **B** and **H**

$$\boxed{\vec{B} = \mu_0\left(\vec{H} + \vec{M}\right)}$$

***B***

*Magnetic induction*
*Magnetic flux density*

Determines forces **on** moving *free* charges via Lorentz force law:

$$\vec{F} = q\left(\vec{E} + \vec{v}\times\vec{B}\right)$$

***M***

*Magnetization*
*Magnetic polarization*

Field created only **by** moving *bound* charges, *i.e.* magnetic response of the medium

***H***

*Magnetic field intensity*
*Magnetizing field*

Field created only **by** moving *free* charges.

In vacuum, $B = \mu_0 H$.

# Reminder: Magnetic Response of Materials

$$\vec{B} = \mu_0\left(\vec{H} + \vec{M}\right) = \mu_0(1+\chi)\vec{H} = \mu_0\mu_r\vec{H}$$

We classify materials into three major categories:

| | | | |
|---|---|---|---|
| **Diamagnetic** | $\chi < 0$ | $\mu_r < 1$ | Weakly repelled |
| **Paramagnetic** | $\chi > 0$ | $\mu_r > 1$ | Weakly attracted |
| **Ferromagnetic** | $\chi \gg 0$ | $\mu_r \gg 1$ | Strongly attracted |

# Magnetic Materials: Diamagnetism

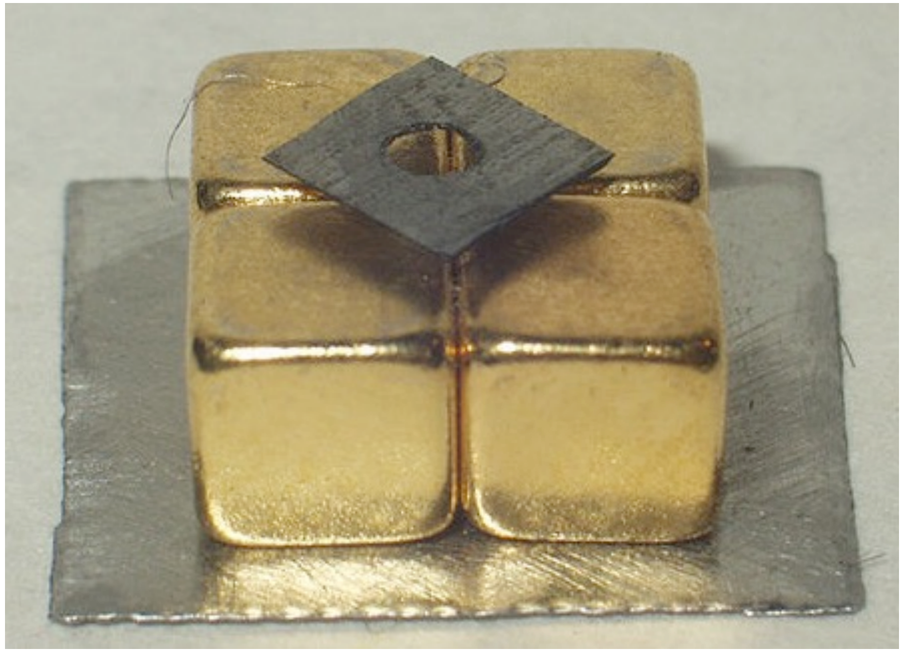
| **Material** | **$\chi_v$ ($10^{-5}$)** |
|---|---|
| Bismuth | −16.6 |
| Carbon (diamond) | −2.1 |
| Carbon (graphite) | −1.6 |
| Copper | −1.0 |
| Lead | −1.8 |
| Mercury | −2.9 |
| **Pyrolytic carbon** | **−40.0** |
| Silver | −2.6 |
| Superconductor | $-10^5$ |
| Water | −0.91 |

Wikipedia

Many materials are **diamagnetic** ($\chi < 0$), i.e. they magnetize in a way that reduces the applied field and are repelled from magnets.

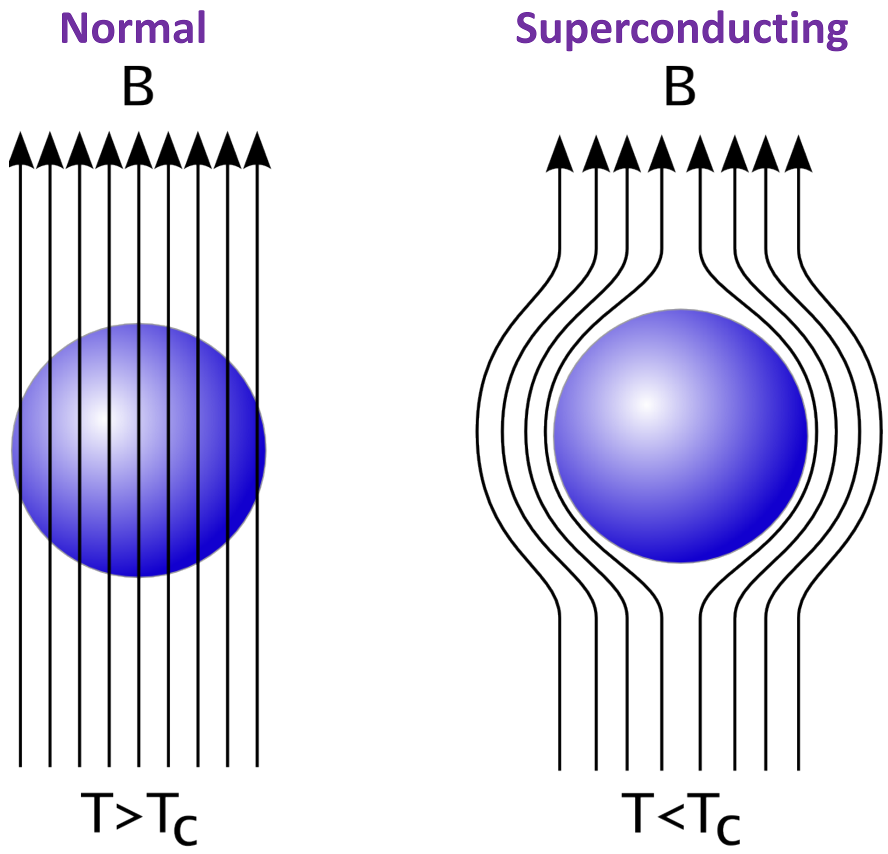
Effect of paired electrons in all materials, generally very weak

Frog levitated in 16T field (Wikipedia)



Levitating sample of pyrolytic carbon (Wikipedia)

# Superconductors: the Meissner Effect



Wikipedia

Superconductors are **perfect diamagnets ($\chi = -1$)**

At T<Tc, electrons condense into a macroscopic quantum state that can flow without loss.

Persistent image currents expel incident fields to maintain B=0. Above some critical field **Hc**, superconductivity is destroyed.

*Important for designing superconducting solenoid magnets*

*Deep analogy with the Higgs mechanism in particle physics!*

# Magnetic Materials: Paramagnetism

| Material | Magnetic susceptibility, χv [$10^{-5}$] |
|---|---|
| Tungsten | 6.8 |
| Caesium | 5.1 |
| Aluminium | 2.2 |
| Lithium | 1.4 |
| Magnesium | 1.2 |
| Sodium | 0.72 |

Wikipedia

In **paramagnetic** ($\chi > 0$) materials, atomic/molecular dipoles from unpaired electron spins align with external fields.

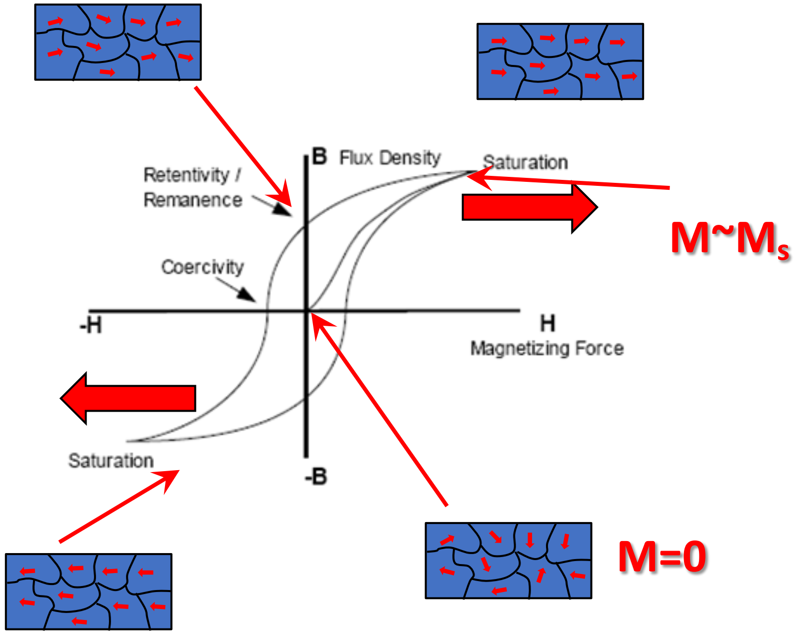
Liquid oxygen trapped between poles of a strong magnet (Wikipedia)

# Magnetic Materials: Ferromagnetism

| Material | $\mu_r$ | $B_{rem}$ (T) |
|---|---|---|
| Fe, 99.8% pure | 5000 | 1.3 |
| Permalloy | 100,000 | 0.7 |
| Superpermalloy | 1,000,000 | 0.7 |
| Co, 99% pure | 250 | 0.5 |
| Ni, 99% pure | 600 | 0.4 |

Wikipedia

In **ferromagnetic** ($\chi \gg 1$) materials, neighboring atomic/molecular dipoles from unpaired electron spins spontaneously align with one another, forming domains.



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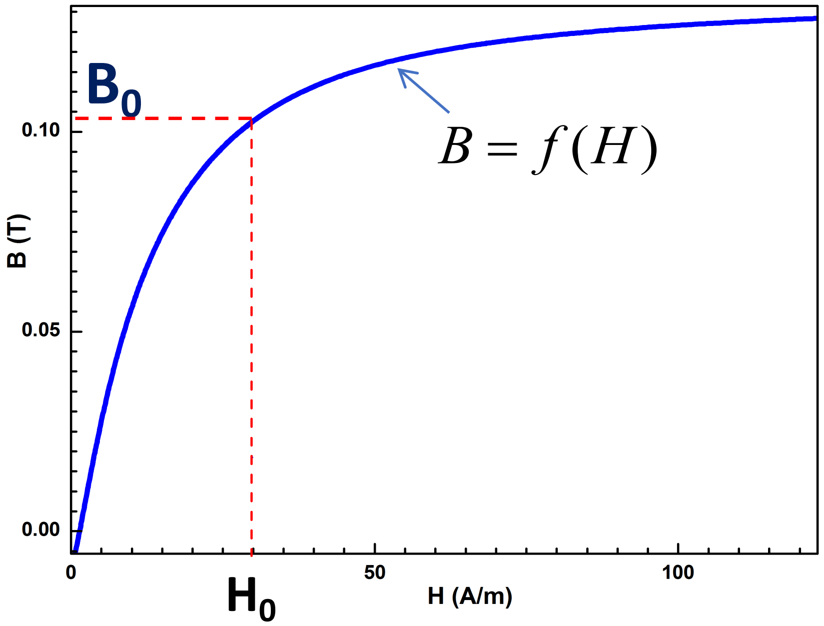
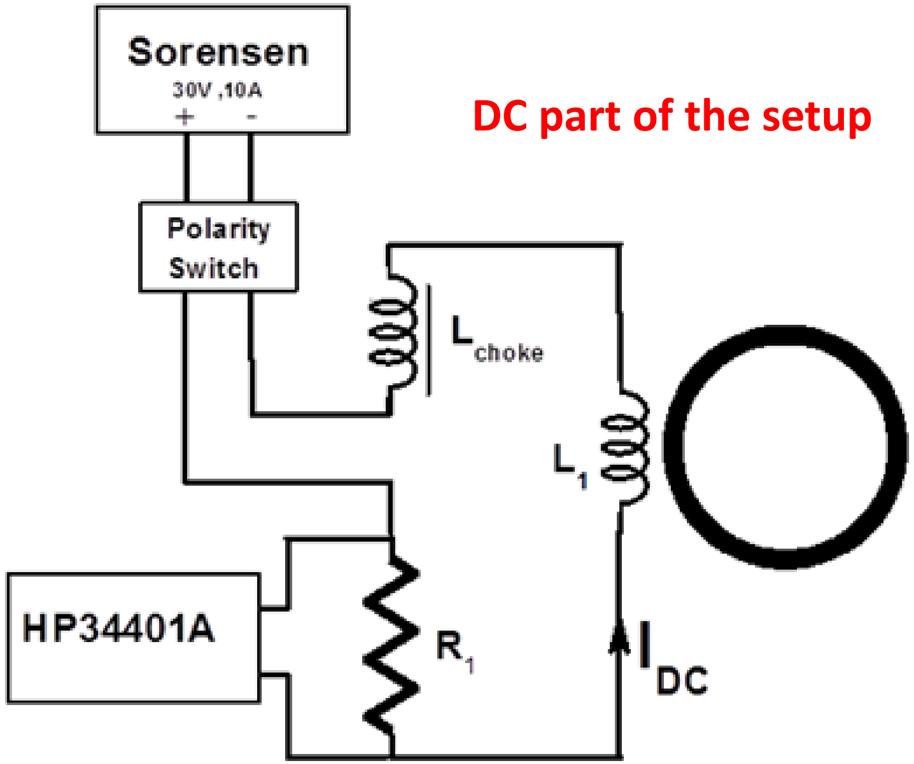
# Measuring Permeability: DC Field



$$H_0 = \frac{N_p I_{DC}}{2\pi r}$$

$$H = H_0 + H_1 \sin\omega t$$



DC part of the setup

Here Np is the number of turns in the DC primary coil

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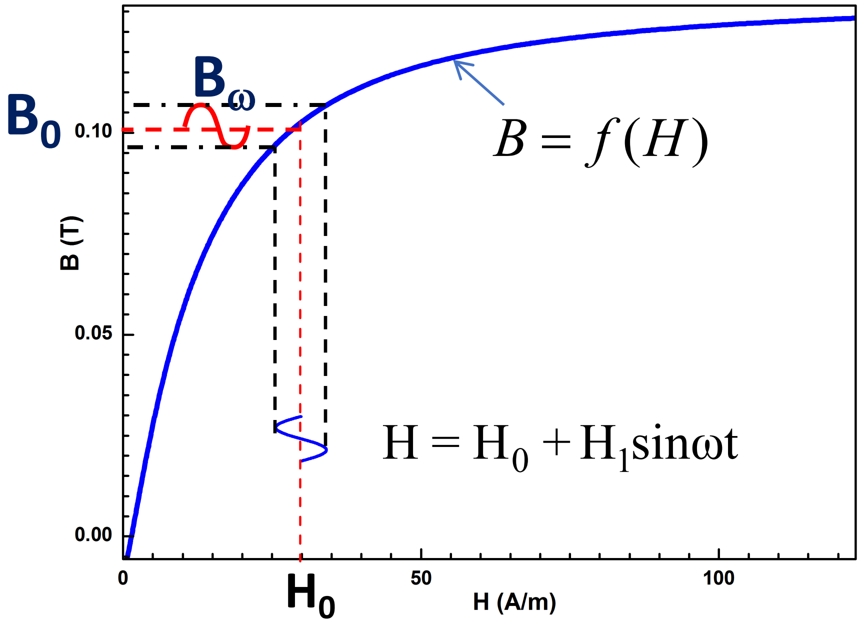
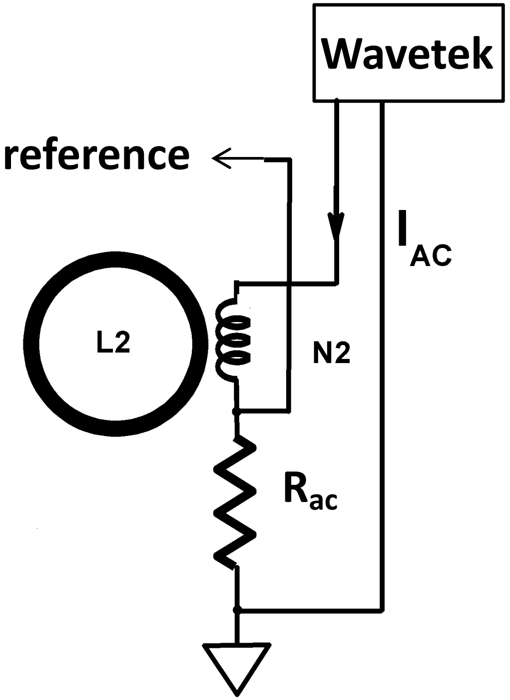
# Measuring Permeability: AC Modulation



$$H = H_0 + H_1\sin\omega t$$



$$H_1 = \frac{N_p I_{AC}}{2\pi r}$$

$$B_\omega \sim \frac{df}{dH} = \frac{dB}{dH} = \mu = \mu_0\mu_r$$

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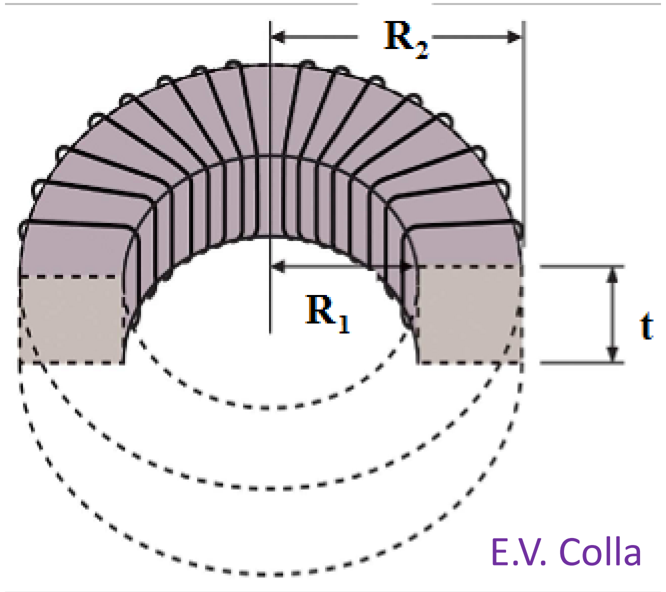
Here Np is the number of turns in the AC primary coil

# Measuring Permeability: AC Toroid Flux

Primary coil is a **toroid** of ***$N_p$*** turns carrying a current ***$I_p$*** creates a magnetic field ***H***:

$$H = \frac{N_p I_p}{2\pi r}$$



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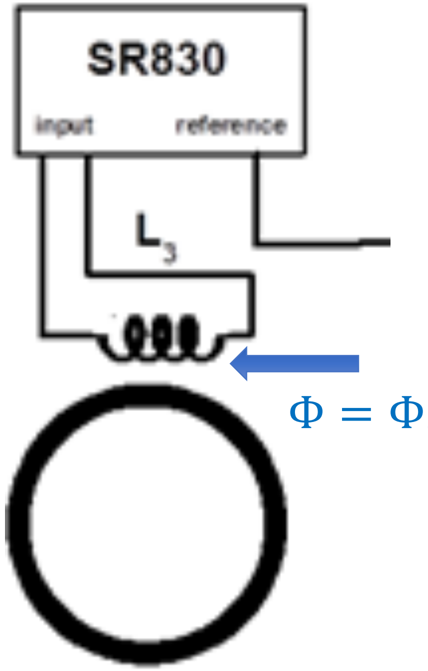
… and this magnetizing field adds a flux ***dΦ*** to *each turn* of the pickup coil

$$d\Phi = \mu \int \vec{H} \cdot d\vec{a} = \frac{\mu\, I\, N\, t}{2\pi} \int_{R_1}^{R_2} \frac{dr}{r} = \frac{\mu\, I\, N\, t}{2\pi} \ln \frac{R_2}{R_1}$$

# Measuring Permeability: Pickup Coil



$\Phi = \Phi_0 + \Phi_1 \sin \omega t$

**Faraday's Law**

$$V_{lock-in} = -N_{pickup} \frac{d\Phi}{dt} \qquad \Phi = \int \vec{B} \cdot d\vec{a} = \mu \int \vec{H} \cdot d\vec{a}$$

Only the AC flux contributes to this time derivative:

$$\Phi_1 = \mu \int \vec{H}_{ac} \cdot d\vec{a} = \frac{\mu\, I_{ac}\, N\, t}{2\pi} \ln \frac{R_2}{R_1}$$

This AC flux is sourced by the current in coil L2: $I_{ac} = \frac{V_0 \sin \omega t}{R_{ac}}$

$$V_{lock-in} = -N_{pickup} \frac{\mu\, N\, t}{2\pi} \ln \frac{R_2}{R_1} \frac{dI_{ac}}{dt} = -N_{pickup} \frac{\mu\, N\, t}{2\pi} \ln \frac{R_2}{R_1} \boldsymbol{\omega} \cos \omega t$$

$$\boxed{V_{lock-in} = -\mu_r L_0 \frac{V_{ac}}{R_{ac}} \boldsymbol{\omega} \cos \omega t}$$

where $L_0 \equiv N_{pickup} \frac{\mu_0 N t}{2\pi} \ln \frac{R_2}{R_1}$

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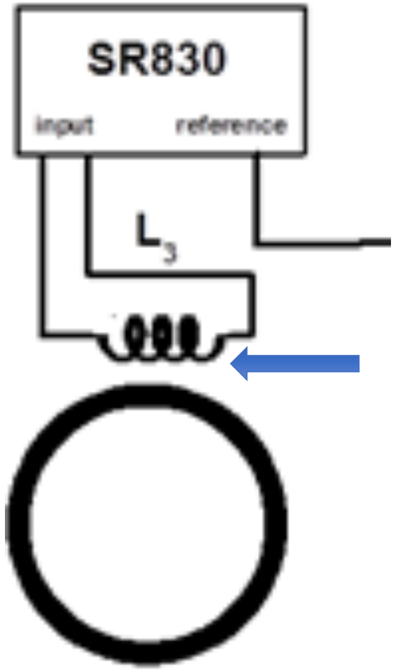
# Measuring Permeability: Pickup Coil



Relative permeability of core

Time derivative of source current

*Geometry of the toroid*

$$V_{lock-in} = -\boldsymbol{\mu_r} L_0 \frac{V_{ac}}{R_{ac}} \omega \cos \omega t$$

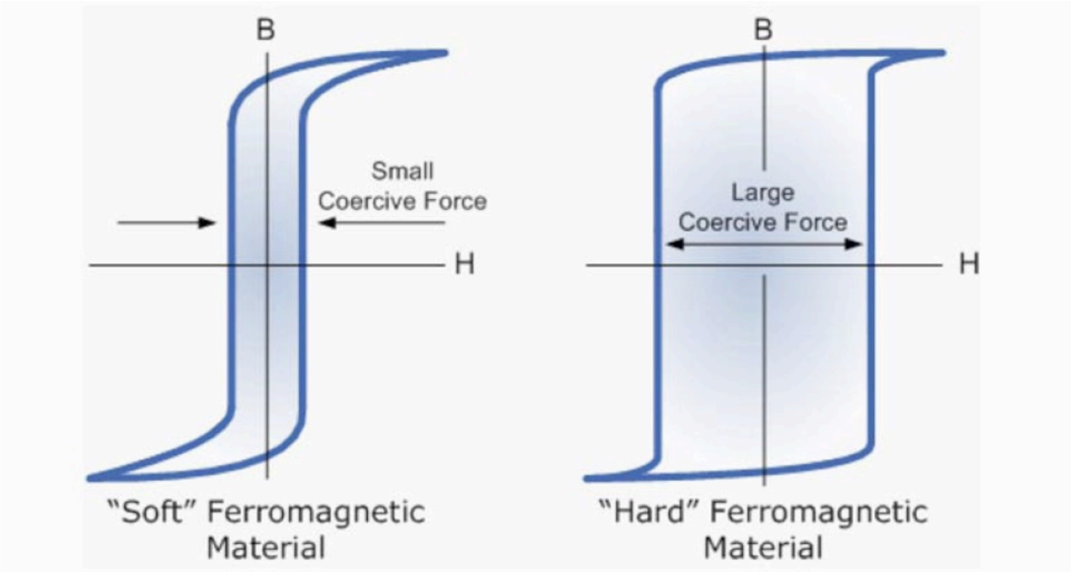
$$L_0 \equiv N_{pickup} \frac{\mu_0 N t}{2\pi} \ln \boxed{\frac{R_2}{R_1}}$$

Geometric mutual inductance in the *absence* of the core

In general: $\boldsymbol{\mu_r = \mu' - i\mu''}$

- The **real** component $\boldsymbol{\mu'}$ appears at the **Y** (quadrature phase) channel of the lock-in, due to its $\pi/2$ phase shift with respect to the driving current (cos vs. sin!)
- The **imaginary** component $\boldsymbol{\mu''}$ characterizes losses, and appears in-phase

# Hysteresis, Coercivity, and Work



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Hysteresis necessarily involves energy losses from re-magnetization.
If the domains are "sticky", we need to do work to overcome that.

$$W = V \int \vec{H} \cdot d\vec{B}$$

For uniform fields over volume V
*(analogous to dW = F dx)*

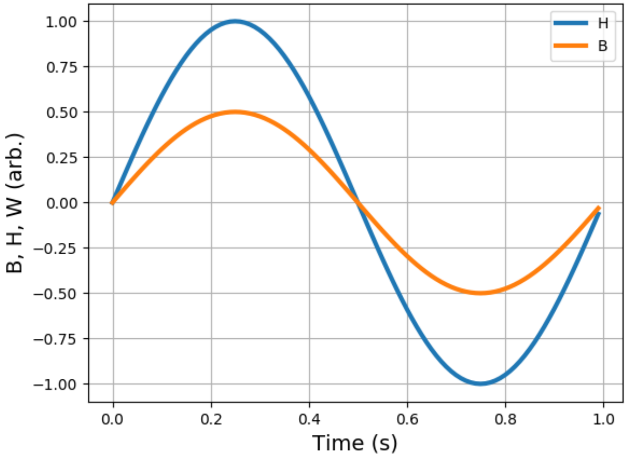
$$\boxed{W_{loop} = V \oint \vec{H} \cdot d\vec{B} = V \, * A_{loop}}$$

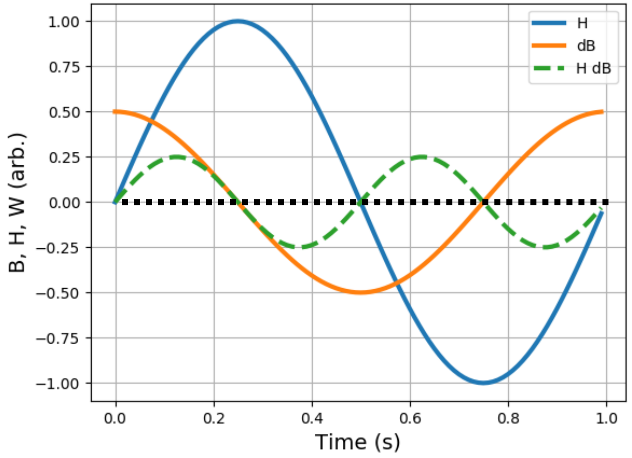
# Aside: Loss from Complex Permeability

$$\vec{B} = \mu_0\left(\vec{H} + \vec{M}\right) = (\mu' - i\mu'')\mu_0\vec{H}$$

Why is a material with complex permeability ($\mu'' \neq 0$) lossy?

Real $\mu_r$:

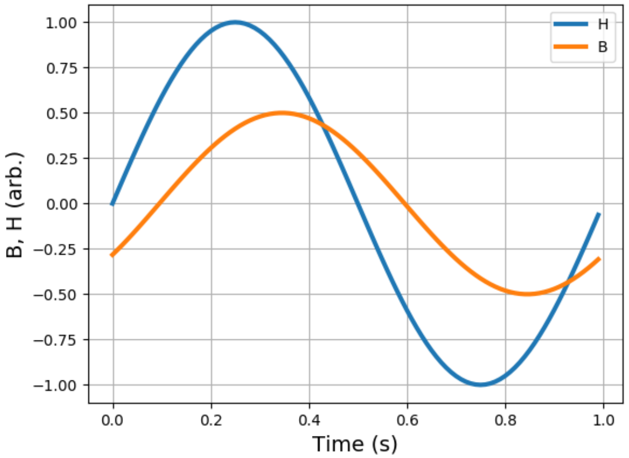


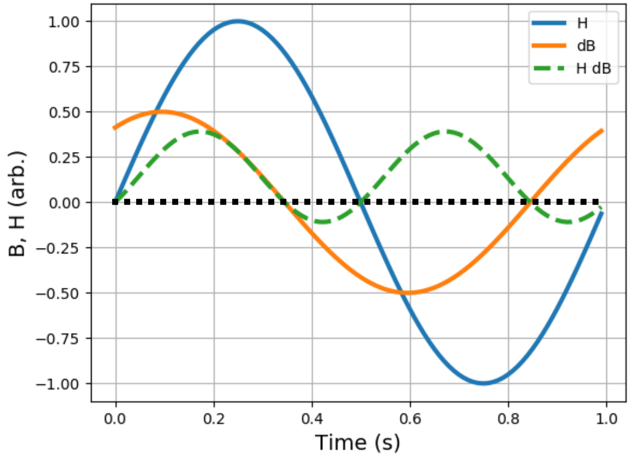


Zero integral

In analogy with
$dW = F\,dx$,
we have
$\boldsymbol{dW = H\,dB}$

Complex $\mu_r$:





Nonzero integral!

# Calculating the Magnetic Induction, B

$$B = \mu_0(1+\chi)H = \mu_0\mu_r H = \mu H$$

$$\mu = \mu_0\mu_r = \frac{dB}{dH} \implies B = \mu_0 \int \mu_r(H)\, dH$$

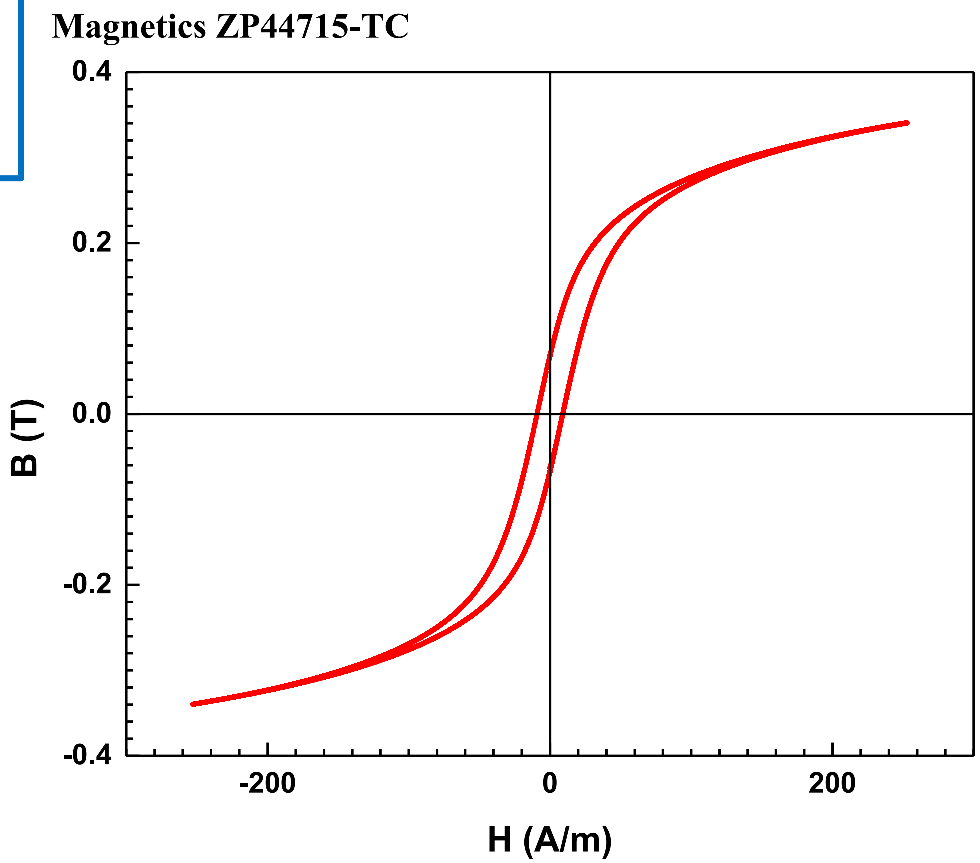
Integrate using OriginPro to find B(H) contour

Work for any motion through this plane:

$$W = V \int \vec{H} \cdot d\vec{B}$$

For uniform fields over volume V

*(analogous to dW = F dx)*



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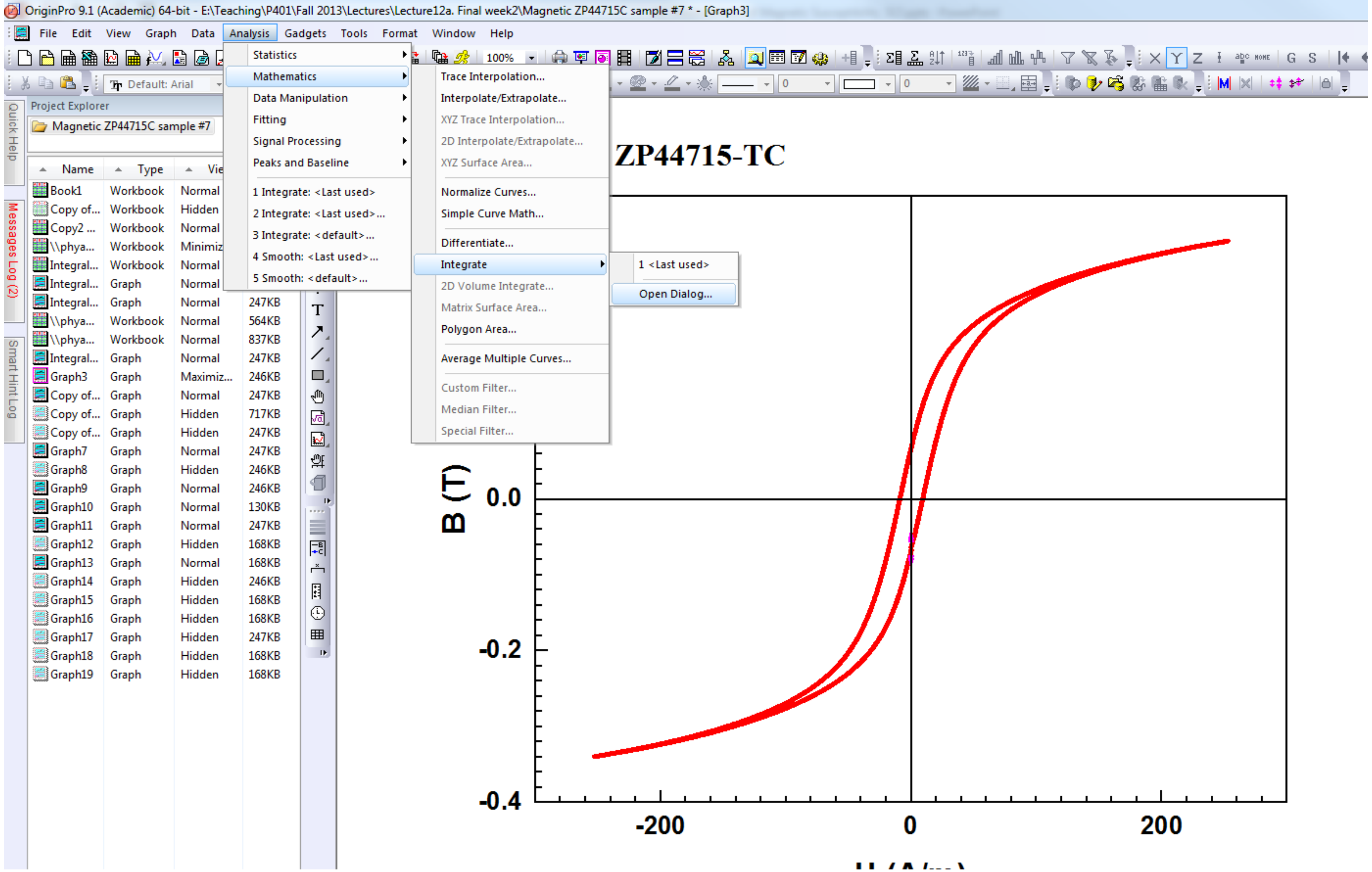
# Hysteresis and Remagnetization Losses

$$W_{loop} = V\oint \vec{H}\cdot d\vec{B} = V\ * A_{loop}$$



OriginPro 9.1 (Academic) 64-bit - E:\Teaching\P401\Fall 2013\Lectures\Lecture12a. Final week2\Magnetic ZP44715C sample #7 * - [Graph3]

Project Explorer

Magnetic ZP44715C sample #7

| Name | Type | Vie | |
|---|---|---|---|
| Book1 | Workbook | Normal | |
| Copy of... | Workbook | Hidden | |
| Copy2 ... | Workbook | Normal | |
| \\phya... | Workbook | Minimiz | |
| Integral... | Workbook | Normal | |
| Integral... | Graph | Normal | |
| Integral... | Graph | Normal | 247KB |
| \\phya... | Workbook | Normal | 564KB |
| \\phya... | Workbook | Normal | 837KB |
| Integral... | Graph | Normal | 247KB |
| Graph3 | Graph | Maximiz... | 246KB |
| Copy of... | Graph | Normal | 247KB |
| Copy of... | Graph | Hidden | 717KB |
| Copy of... | Graph | Hidden | 247KB |
| Graph7 | Graph | Normal | 247KB |
| Graph8 | Graph | Hidden | 246KB |
| Graph9 | Graph | Normal | 246KB |
| Graph10 | Graph | Normal | 130KB |
| Graph11 | Graph | Normal | 247KB |
| Graph12 | Graph | Hidden | 168KB |
| Graph13 | Graph | Normal | 168KB |
| Graph14 | Graph | Hidden | 246KB |
| Graph15 | Graph | Hidden | 168KB |
| Graph16 | Graph | Hidden | 168KB |
| Graph17 | Graph | Hidden | 247KB |
| Graph18 | Graph | Hidden | 168KB |
| Graph19 | Graph | Hidden | 168KB |

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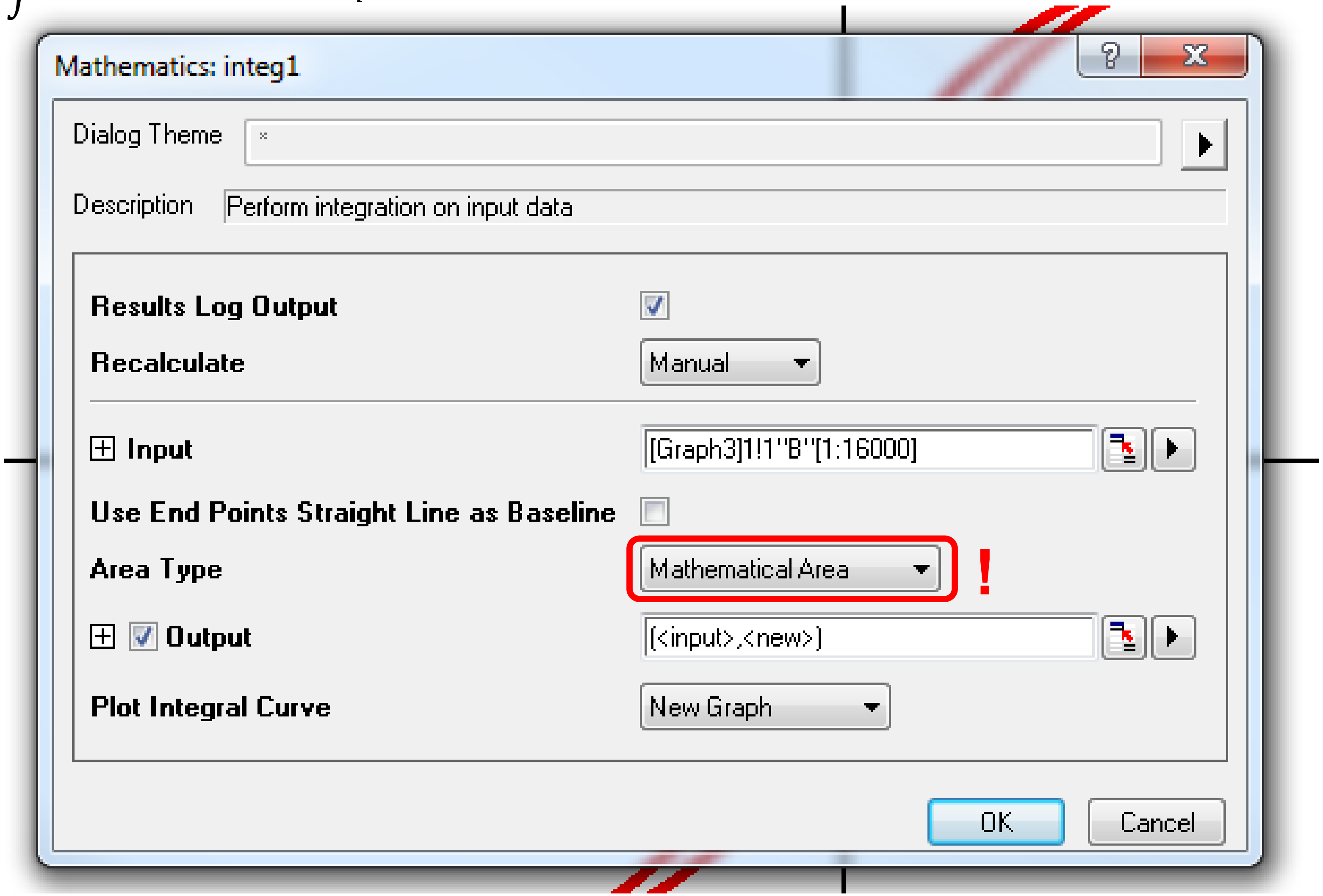
# Hysteresis and Remagnetization Losses

$$W_{loop} = V \oint \vec{H} \cdot d\vec{B} = V \; * A_{loop}$$



Mathematics: integ1

Dialog Theme: *

Description: Perform integration on input data

**Results Log Output** ☑

**Recalculate**: Manual

**Input**: [Graph3]1!1"B"[1:16000]

**Use End Points Straight Line as Baseline** ☐

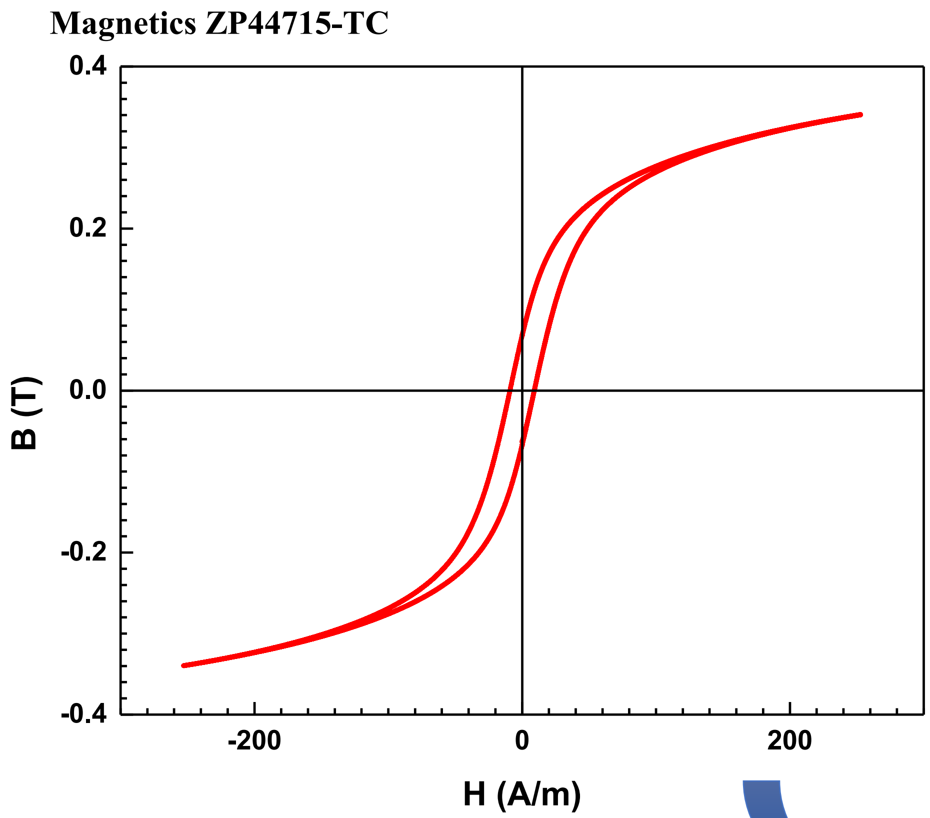
**Area Type**: Mathematical Area !

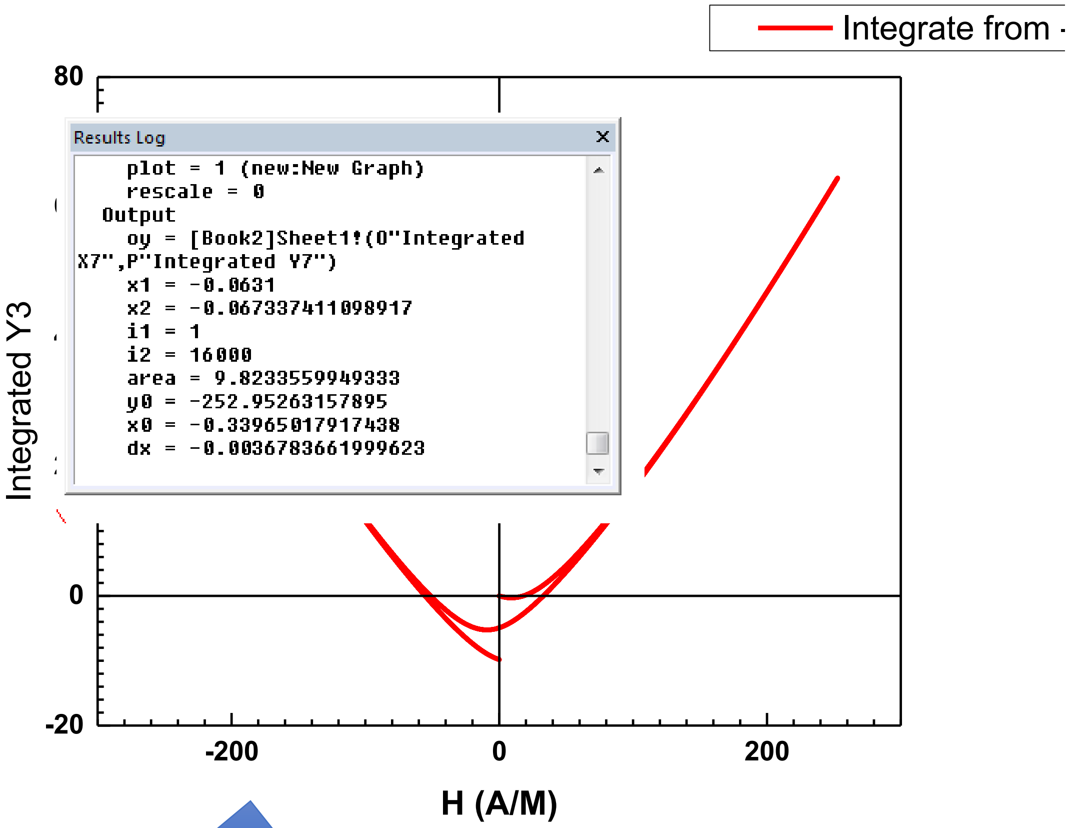
☑ **Output**: [<input>,<new>]

**Plot Integral Curve**: New Graph

# Hysteresis and Remagnetization Losses

$$W_{loop} = V \oint \vec{H} \cdot d\vec{B} = V \ * A_{loop}$$





```
    plot = 1 (new:New Graph)
    rescale = 0
  Output
    oy = [Book2]Sheet1!(O"Integrated
X7",P"Integrated Y7")
    x1 = -0.0631
    x2 = -0.067337411098917
    i1 = 1
    i2 = 16000
    area = 9.8233559949333
    y0 = -252.95263157895
    x0 = -0.33965017917438
    dx = -0.0036783661999623
```

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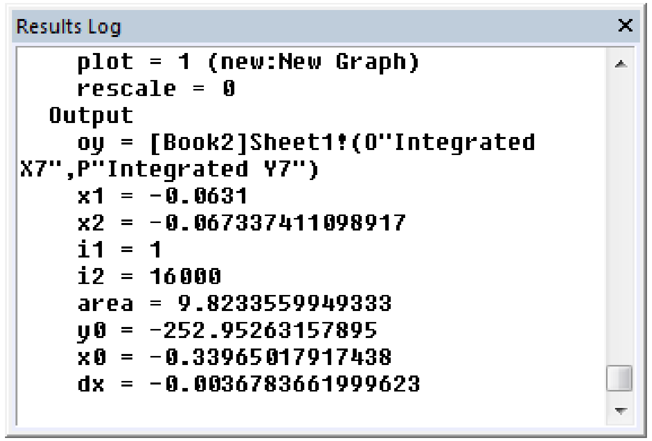
# Hysteresis and Remagnetization Losses

$$W_{loop} = V \oint \vec{H} \cdot d\vec{B} = V \ * A_{loop}$$



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```

***Units :***

$$\boldsymbol{V(volume) \rightarrow m^3}$$

$$\boldsymbol{H(field) \rightarrow A \bullet m^{-1}}$$

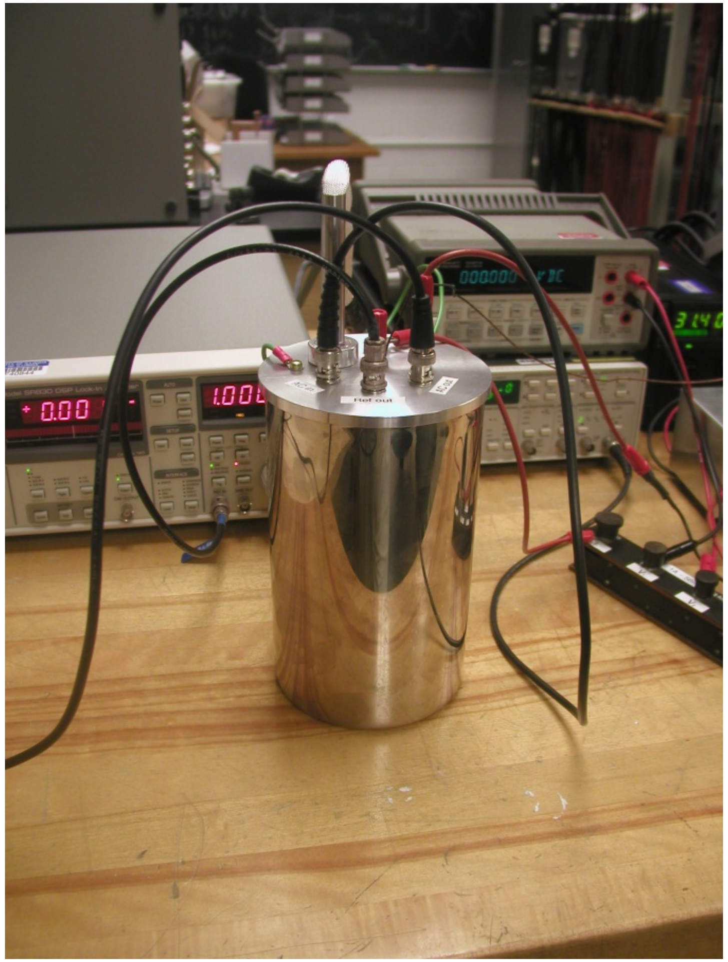
$$\boldsymbol{B(magn.induction) \rightarrow kg \bullet s^{-2} \bullet A^{-1}}$$

$$\boldsymbol{[V \bullet B \bullet H] \rightarrow m^2 \bullet kg \bullet s^{-2} \equiv J(\text{joule})}$$

This is the **energy** lost around a circuit of the loop.
To make this a **loss power**, we need to account for the frequency:

$P_{loss} = \frac{W}{T} = Wf$, where ***T*** is the period and ***f*** is the frequency

# Temperature Dependence of Permeability



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In this experiment we measure $\chi(T)$: magnetic permeability as a function of temperature

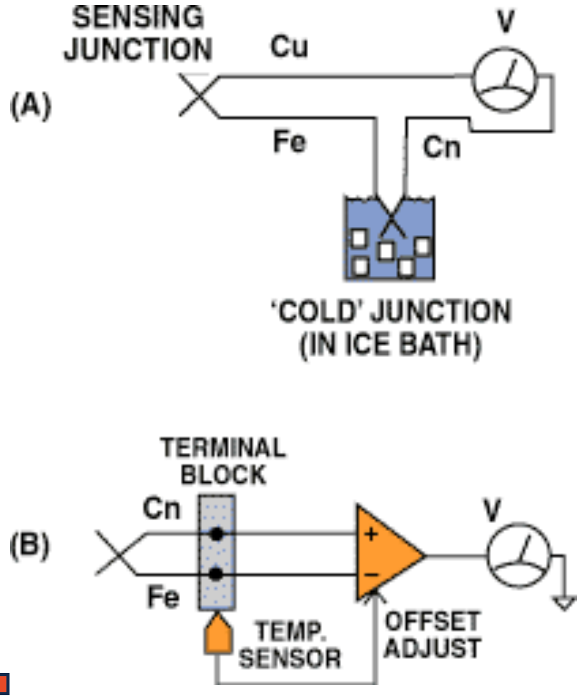
We measure at **fixed $H_0$**, and so fixed **$I_{DC}$**.
The default is to measure at **$I_{DC}=0$** (*zero DC field*).

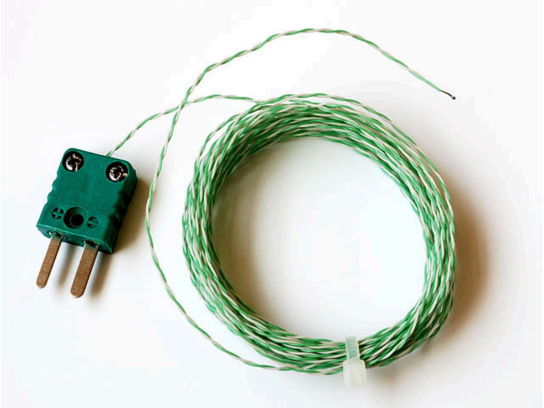
Our thermometer will be a **T-type thermocouple**.
We'll use the DMM to measure the EMF (voltage) across it, and from this infer the temperature.

# Thermocouples

In a thermocouple, a pair of dissimilar metal wires generates a temperature-dependent EMF

**Seebeck effect**: Charge carriers diffuse away from hot areas, setting up a (material-dependent) EMF that is a function of the temperature gradient.





Wikipedia

| Type | Names of Materials | T Range |
|---|---|---|
| B | Platinum30% Rhodium (+)<br>Platinum 6% Rhodium (-) | 2500 -3100F<br>1370-1700C |
| C | W5Re Tungsten 5% Rhenium (+)<br>W26Re Tungsten 26% Rhenium (-) | 3000-4200F<br>1650-2315C |
| E | Chromel (+)<br>Constantan (-) | 200-1650F<br>95-900C |
| J | Iron (+)<br>Constantan (-) | 200-1400F<br>95-760C |
| K | Chromel (+)<br>Alumel (-) | 200-2300F<br>95-1260C |
| N | Nicrosil (+)<br>Nisil (-) | 1200-2300F<br>650-1260C |
| R | Platinum 13% Rhodium (+)<br>Platinum (-) | 1600-2640F<br>870-1450C |
| S | Platinum 10% Rhodium (+)<br>Platinum (-) | 1800-2640F<br>980-1450C |
| **T** | **Copper (+)<br>Constantan (-)** | **-330-660F<br>-200-350C** |

Type T at 0°C: $dV/dT=41.5\ \mu V/°C$

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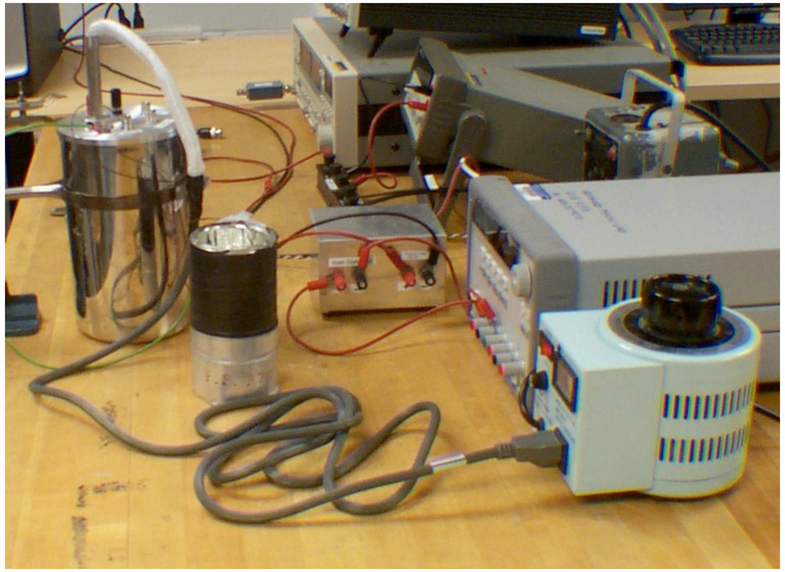
# Temperature Ramp



**Option 1**: Manually change the voltage applied to the heater

**Option 2**: Automate using the Omega **PID*** temperature controller

** **P**roportional, **I**ntegral, **D**ifferential: Three standard linear operations to perform on error signal to determine control (feedback) signal to stabilize input at target*

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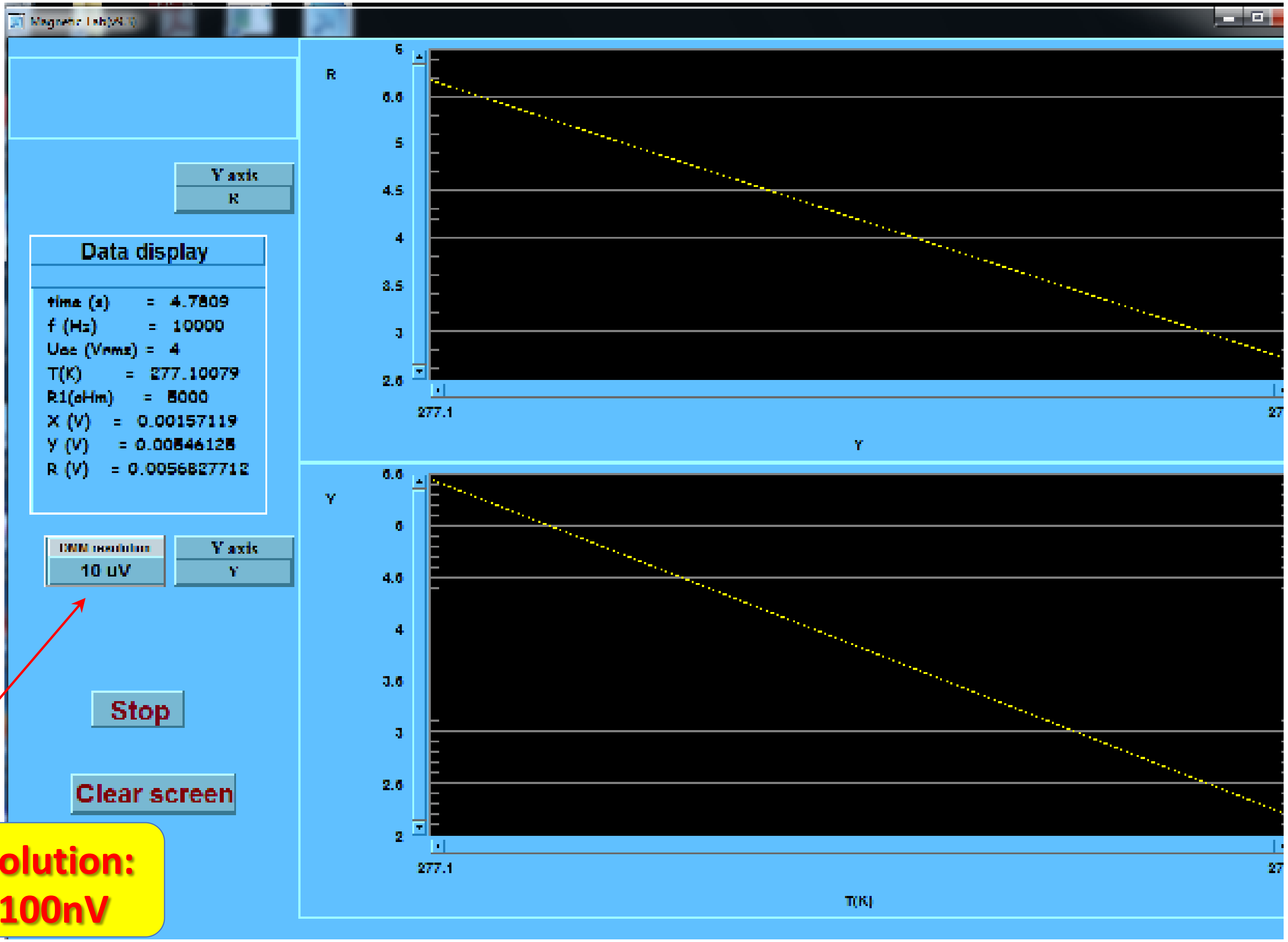
# Temperature Measurement



time (s) = 4.7809
f (Hz) = 10000
Uac (Vrms) = 4
T(K) = 277.10079
R1(oHm) = 5000
X (V) = 0.00157119
Y (V) = 0.00546128
R (V) = 0.0056827712

**DMM resolution: Optimal=100nV**

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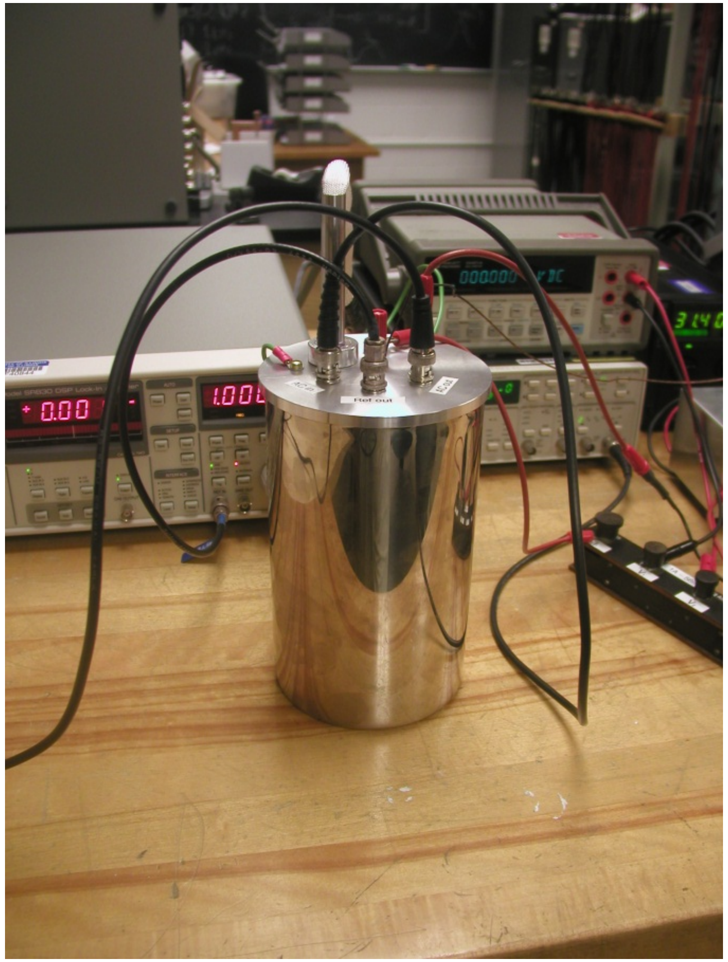
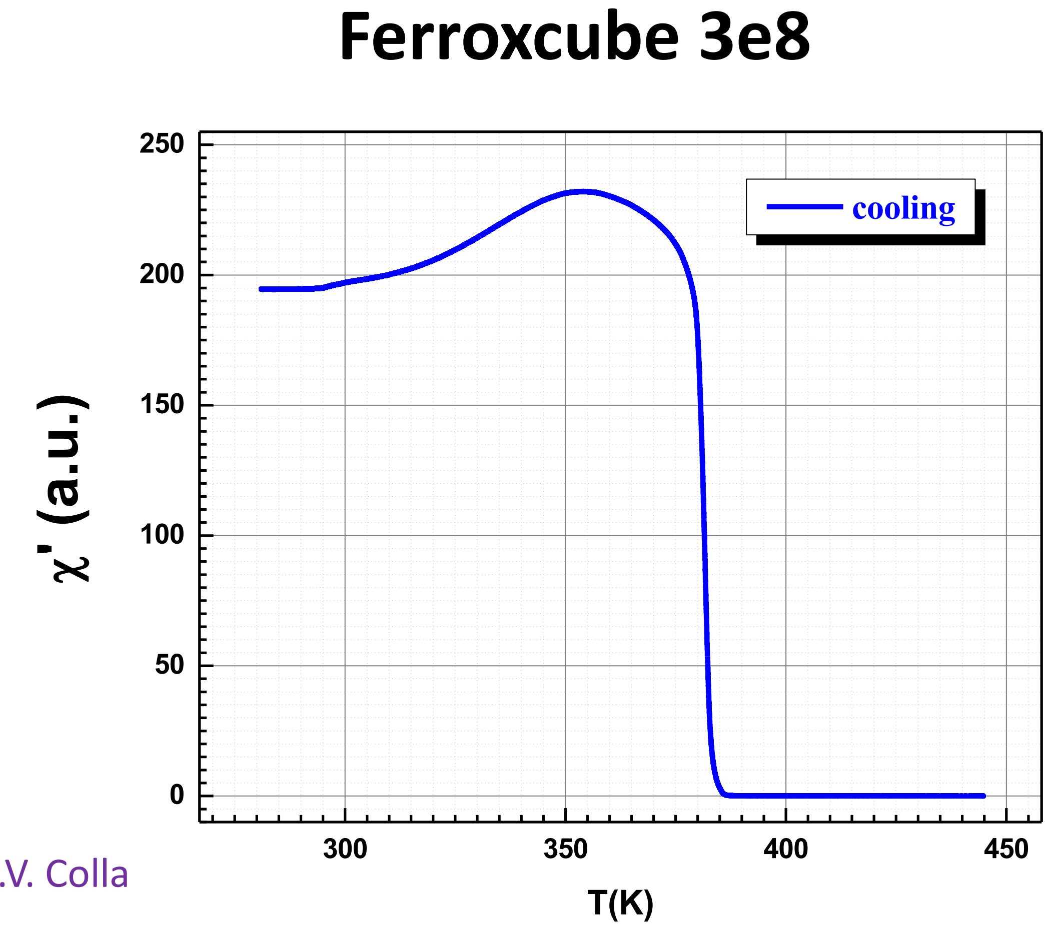
# Temperature Dependence of Permeability





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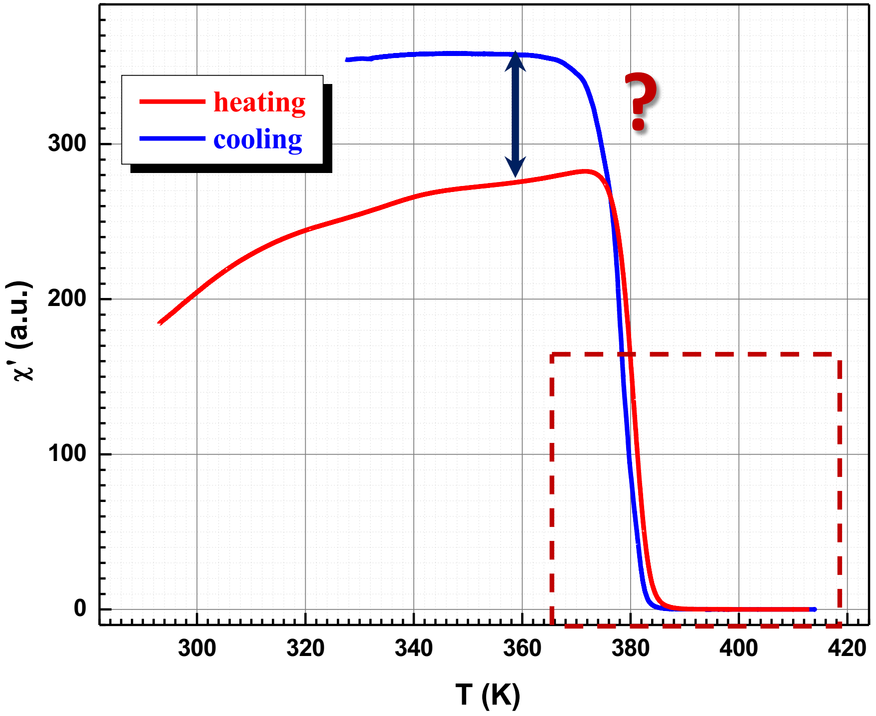
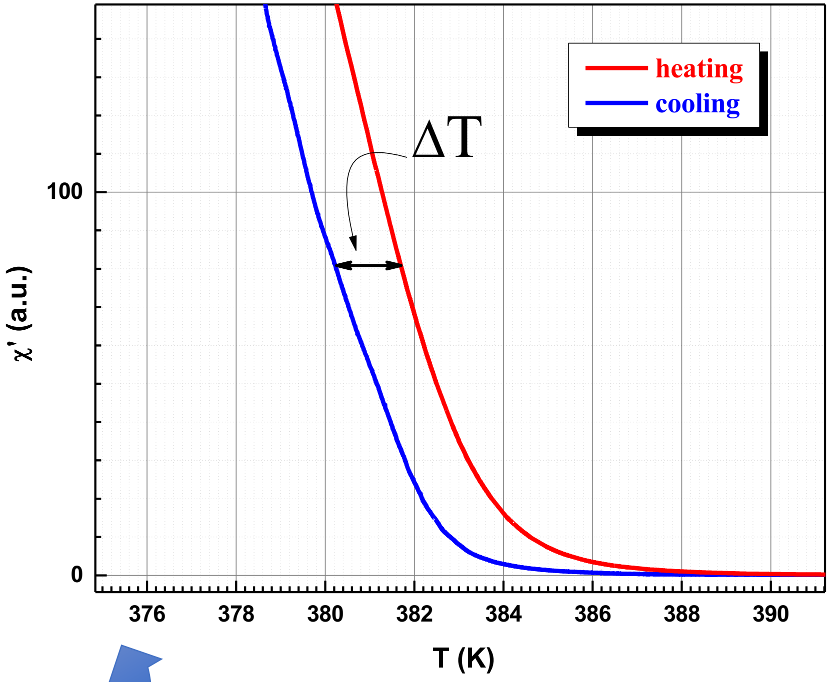
# Temperature Dependence of Permeability

**Ferroxcube 4A20**





**Hysteresis** in Tc and χ

What is the source?

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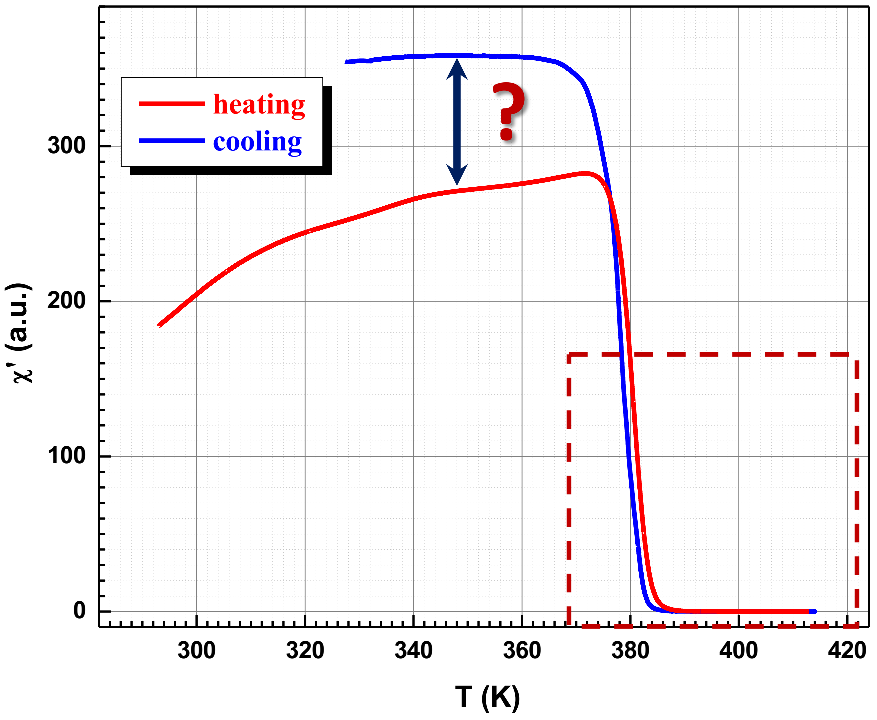
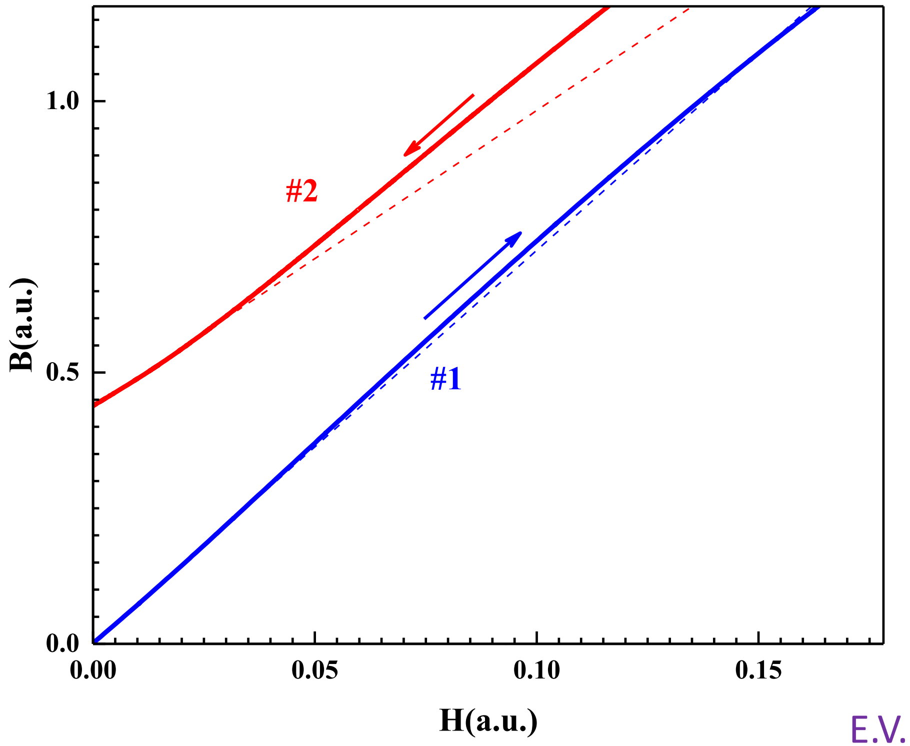
# Temperature Dependence of Permeability



$$\mu(T) = \mu_0 \mu_r(T) = \frac{dB}{dH}(T)$$

**Slope#1 ~0.728**

**Slope#2 ~0.546**



E.V. Colla

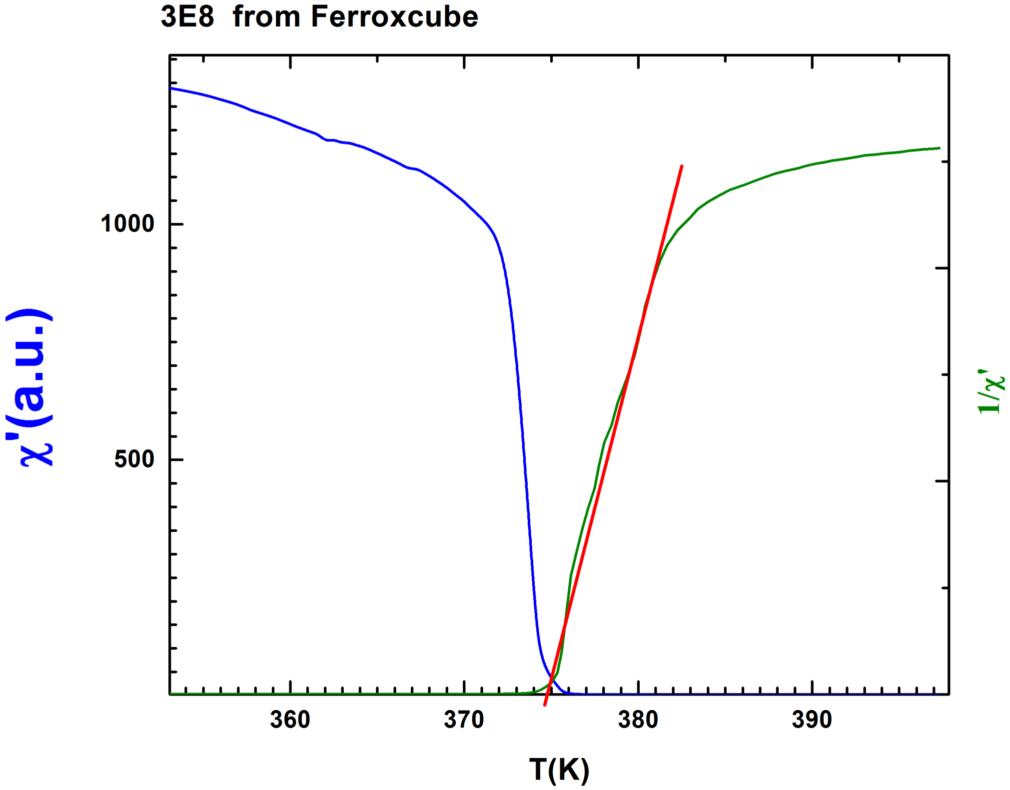
# Curie-Weiss Law

**Pierre Curie**
1859 – 1906

Scanned at the American Institute of Physics

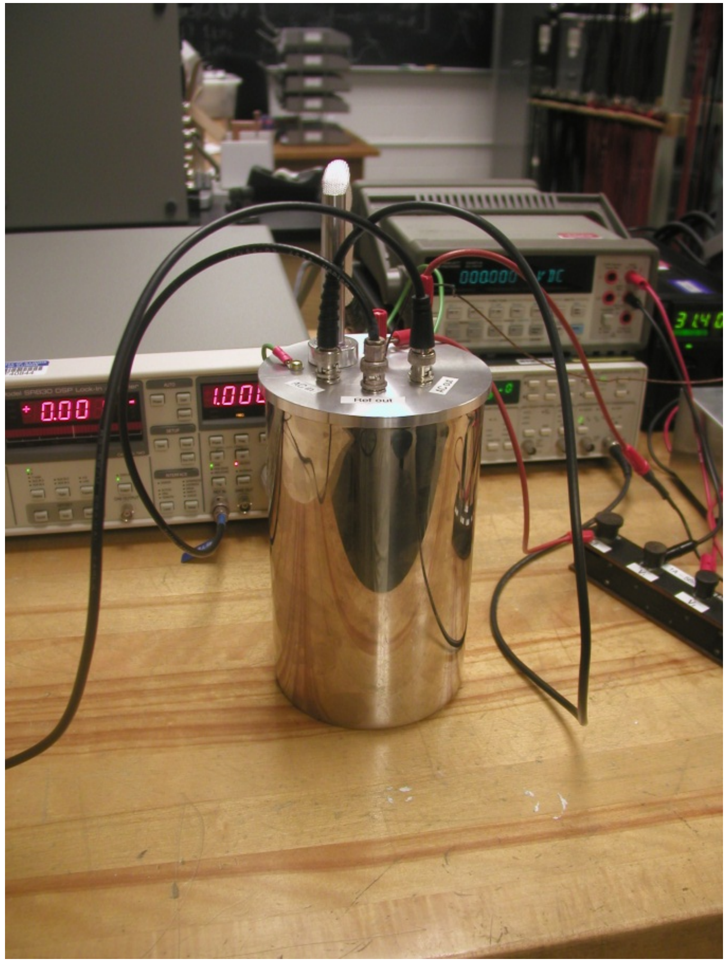
**Pierre Weiss**
1865 – 1940

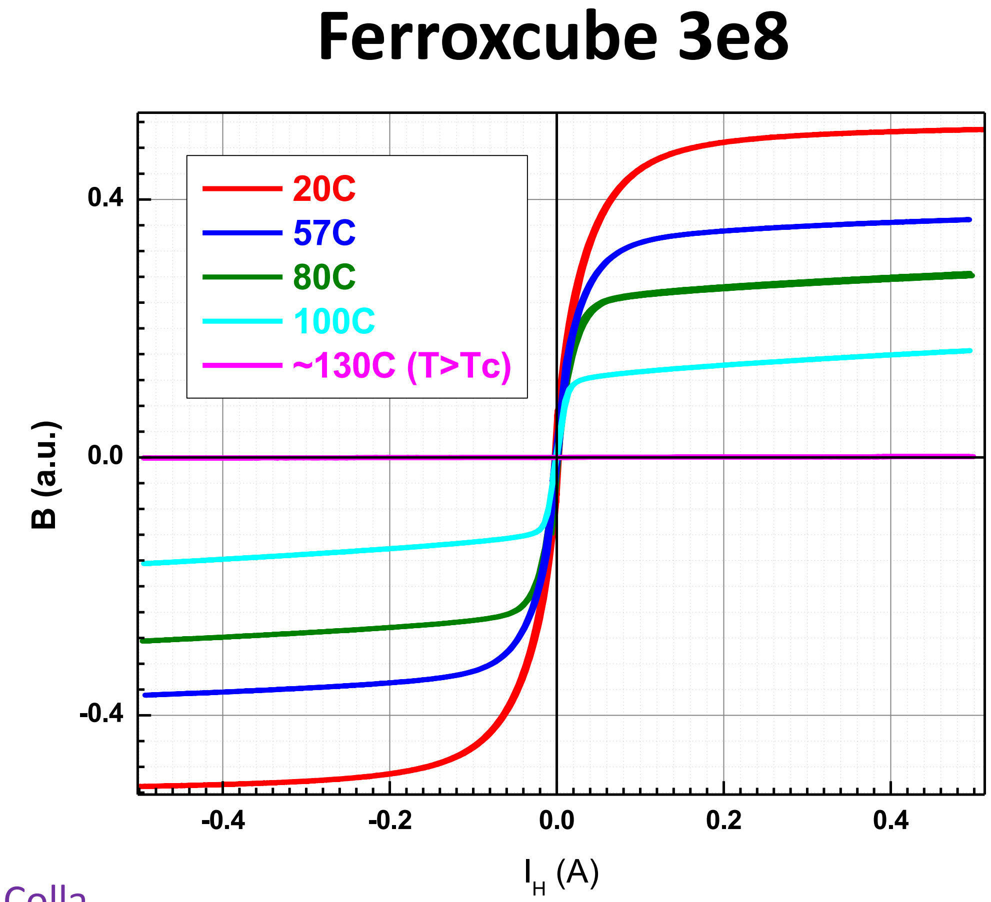
$$\boxed{\chi' = \frac{C}{T - T_c}}$$



E.V. Colla

# Temperature Dependence of Permeability





E.V. Colla

"Week 3": Full curves at varying $H_0$

# Our (Socially Distanced!) Plan for the Final Lab

- Apparatus is largely prepared, data collection is largely automated
- We usually divide lab tasks up into three weeks:
  1. **B(H)** for four samples at room temperature (*$\chi$, $\mu''$, hysteresis*)
  2. **$\chi$(H=0)** for one sample at several temperatures (*Curie-Weiss law ,$T_{Curie}$*)
  3. **B(H,T)** for one sample at several temperatures (*saturation, remanence, …*)
- We will post videos and data on the following schedule:
  - **Week 1**: Wednesday 4/22
  - **Week 2/3**: Sunday 4/26 (*combined, same sample*)
- You will perform the data analysis and write a report as usual, following the lecture and write-up
- *Contact me and your TAs with questions! Don't wait!*

# References

- Information about magnetic materials can be found in:
  \\engr-file-03\phyinst\APL Courses\PHYCS401\Experiments\AC_Magnetization\Magnetic Materials

- SR830 (Lock-in Amplifier) manual
  \\engr-file-03\phyinst\APL Courses\PHYCS401\Common\EquipmentManuals\SR830m.pdf

# End of Semester

- **No lecture** 4/27 or 5/4 – *this is our last meeting!*

- Please turn in any late **lab reports**! You can revise and resubmit one report for a **replacement grade**. *Coordinate with your TA!*

- **Sunday, May 10th, 11:59 pm** is the deadline for the final lab, *and* for all other reports/resubmissions. *No extensions and no vouchers!*

- **Grade thresholds** may be adjusted if needed on a section-by-section basis if substantial disparities arise

# Thank You

for your hard work

and especially for bearing with us during this most unusual and challenging semester

Best of luck in your future endeavors, in and out of the laboratory!

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