Lecture 8 - Radiation, Light in Matter

Radiation is the means by which EM waves are created. This allows charged objects to exchange energy and information.

To produce EM waves, charges must accelerate. To see why, consider an electron at rest at the origin.

\[ e \text{ at rest} \quad \rightarrow \quad e \text{ moving at } v \]

The \( e^- \) produces field lines through all space, but nothing is propagating.

Consider now an electron moving uniformly. In this case, the field lines are distorted by the Doppler effect.

\[ e \text{ moving at } v \]

Nothing at a distance \( r > c(t - t') \) can have any information that the acceleration has taken place. So, outside this sphere, the field lines are as they were.

[Note: this is not the same thing as Lorentz contraction, which has no effect on point particles.] The E field is time-dependent now. But still there are no propagating EM waves.

Consider, however, an accelerating charge, that was at rest for \( t < 0 \). But at \( t = 0 \) it suddenly starts moving, arriving at position \( x \) at some later time \( t' \).
Inside the sphere the field lines are distorted. The field lines must match up since \( \nabla \cdot \mathbf{E} = 0 \), so they must exhibit kinks that travel outward at \( c \). Hence EM waves are always tied to accelerating charges.

**Field from an accelerating charge**

**Accelerating point charge at the origin:**

\[
\mathbf{a} = \mathbf{c} \times \mathbf{v}
\]

\[
\mathbf{E} = \frac{q(t)}{4 \pi \varepsilon_0 (\mathbf{r} \cdot \mathbf{a})^2} \left[ (\mathbf{v} \cdot \mathbf{a}) \mathbf{u} + \mathbf{v} \times (\mathbf{a} \times \mathbf{u}) \right]
\]

This term falls off like \( \frac{1}{r^3} \) at large distances, and reduces to the familiar electrostatic form at \( r \to 0 \). This term is usually called the "Coulomb field."

This term falls off like \( \frac{1}{r} \), so is longer ranged. At larger distances, this term dominates. It arises from the acceleration of the charge, and is called the "radiation field."

**Dipole Radiation**

The most straightforward way to accelerate charges is to use an antenna:

\[
\mathbf{I}(t) = I_0 \cos \omega t = \frac{d\mathbf{q}}{dt} = \frac{d\mathbf{q}(t)}{dt}
\]

where

\[
\mathbf{q}(t) = q_0 \sin \omega t \Rightarrow \mathbf{I}_0 = q_0 \omega
\]

If \( \mathbf{d} \) is small we can think of this as an oscillating dipole moment:

\[
\mathbf{p} = \mathbf{q}(t) \mathbf{d} \Rightarrow \mathbf{p} = q_0 \mathbf{d} \sin \omega t \hat{\mathbf{z}}
\]

Let's suppose our antenna is sitting at the origin and oriented along the \( \hat{\mathbf{z}} \) direction:

\[
\mathbf{p} = \mathbf{q}(t) \mathbf{d} \mathbf{\hat{z}} = q_0 \mathbf{d} \sin \omega t \mathbf{\hat{z}}
\]

\[
= p_0 \sin \omega t \mathbf{\hat{z}}
\]
where \( p_0 = q_0 \).

We would like to know what E field results from this source, which requires solving Maxwell's equations. That solution is done in Phys 436 and is beyond the scope of this course. We will just state the result:

\[
E = \frac{M_0 p_0 c^2}{4\pi} \left( \frac{\sin \theta}{r} \right) \sin \left( k_0 r - \omega t \right) \hat{\theta}
\]

\[
|V| = \frac{|E|}{\mu_0 c} \hat{\phi}
\]

\[
S = \frac{1}{\mu_0 M_0 c} \times |V|
\]

\[
I = \langle |S|^2 \rangle = \frac{M_0 p_0^2}{32 \pi^2 c} \frac{\sin^2 \theta}{r^2}
\]

This is usually called "dipole radiation" and is observed from antennae, in atomic transitions, etc.

**Light in Matter**

Light inside matter will experience a different permittivity and permeability than in vacuum, i.e.,

\[
\varepsilon = \varepsilon_0 (1 + \chi_e)
\]

\[
\mu = \mu_0 (1 + \chi_m)
\]

where \( \chi_e \) and \( \chi_m \) are the electric and magnetic susceptibility, respectively.

The phase velocity of light in a medium is therefore different.

\[
v = \frac{1}{\sqrt{\varepsilon \mu}} \neq c
\]

The ratio

\[
n = \frac{c}{v} = \sqrt{\frac{\varepsilon_0 \mu_0}{\varepsilon \mu}} = \sqrt{\frac{\varepsilon E \mu M}{\varepsilon_0 M}}
\]

is the index of refraction. In most cases \( \varepsilon_0 = 1 \), so

\[
n = \sqrt{\varepsilon E} = \sqrt{1 + \chi_e}
\]

we have already seen that \( n \) can cause refraction. Today we are interested in the microscopic origins of \( n \).
Induced Dipoles and Polarizability

\[ \vec{p} = \vec{\beta} \]

Induced dipole

Atom

\[ \alpha = \text{atomic polarizability} \]

How does this happen?

\[ \infty \]