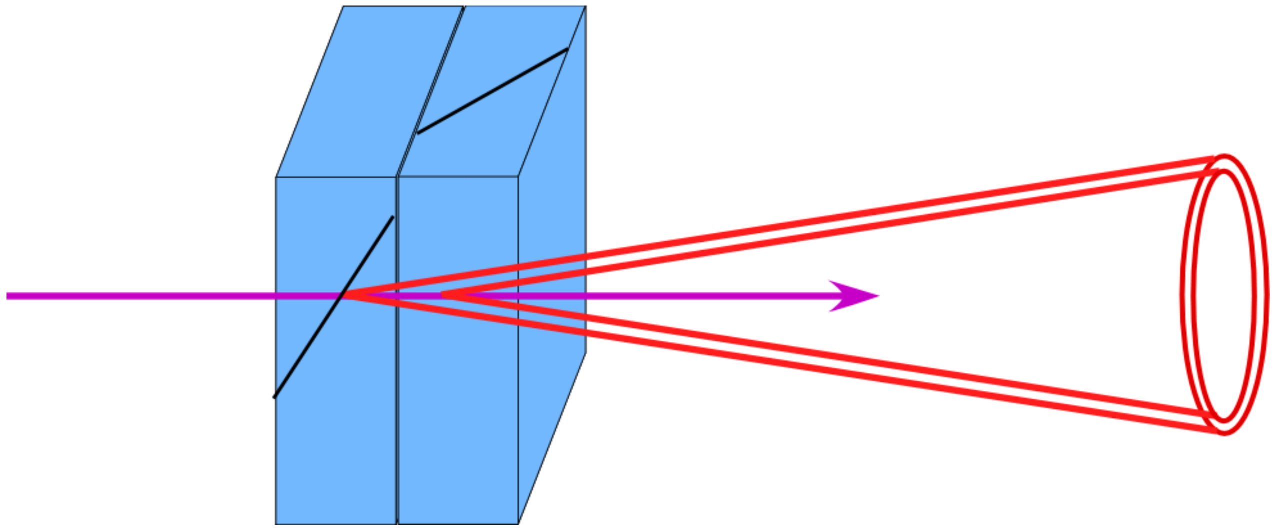
Quantum Entanglement and Applications

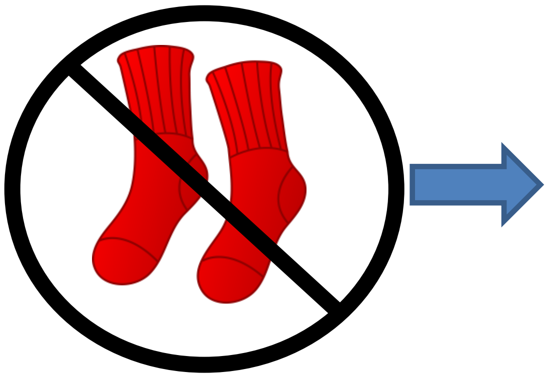
Trent Graham



Advisor: Paul Kwiat
December 3, 2013

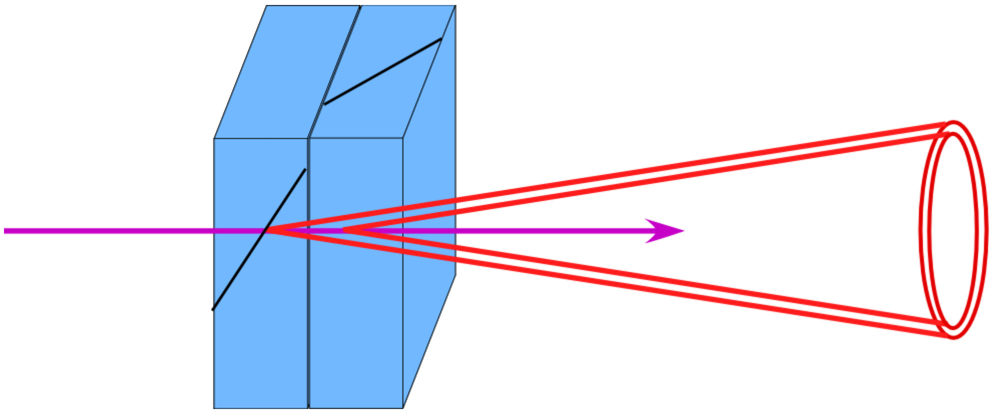
Contents

What is Entanglement

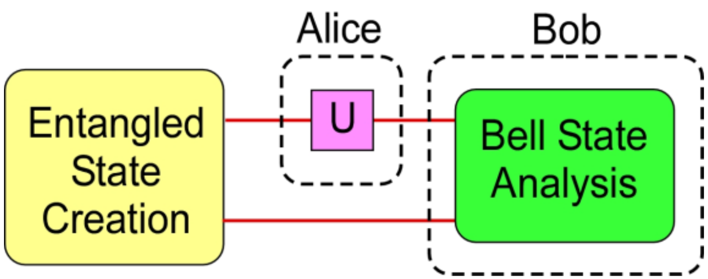


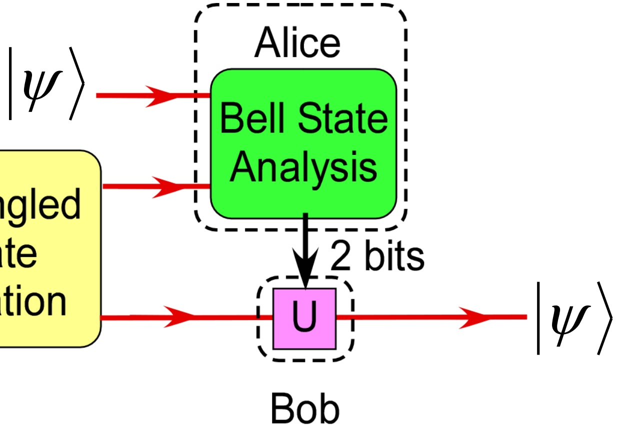
$$\left|\Phi^{\pm}\right\rangle = \frac{1}{\sqrt{2}}\left(|0\rangle\,|0\rangle \pm |1\rangle\,|1\rangle\right)$$

Entanglement Creation



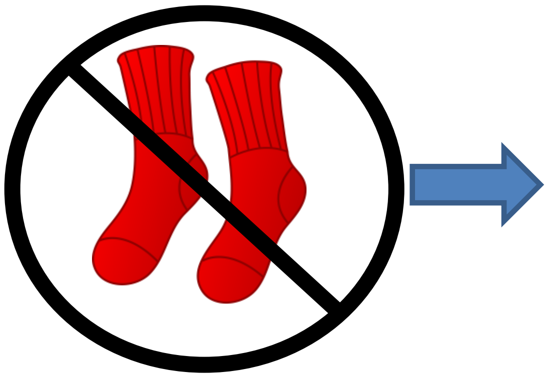
Applications





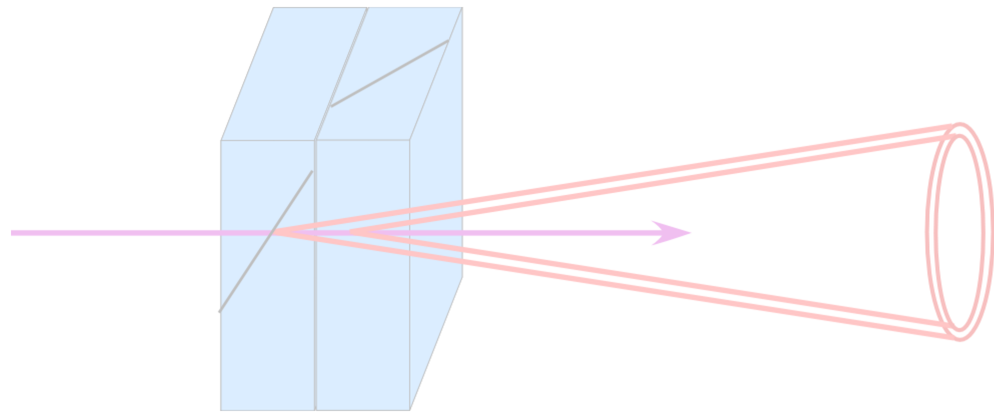
Contents

What is Entanglement

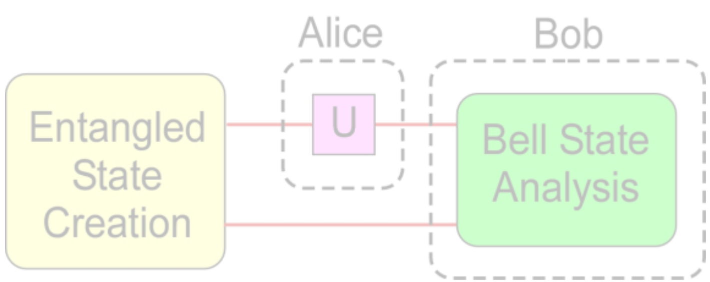


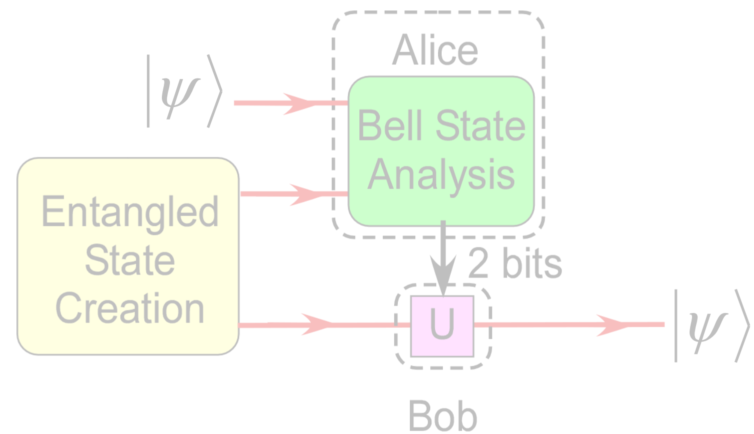
$$|\Phi^{\pm}\rangle = \frac{1}{\sqrt{2}}\left(|0\rangle\,|0\rangle \pm |1\rangle\,|1\rangle\right)$$

Entanglement Creation



Applications





Entanglement is a feature of compound quantum systems

- States that can be written $|\Psi\rangle_{AB} = |\varphi^1\rangle_A|\varphi^2\rangle_B$ are separable
- States that <u>cannot</u> be written this way are entangled

Example: the *Bell states* $|\Phi^{\pm}\rangle = \frac{1}{\sqrt{2}}(|0\rangle|0\rangle \pm |1\rangle|1\rangle)$ & $|\Psi^{\pm}\rangle = \frac{1}{\sqrt{2}}(|0\rangle|1\rangle \pm |1\rangle|0\rangle)$ are inseparable

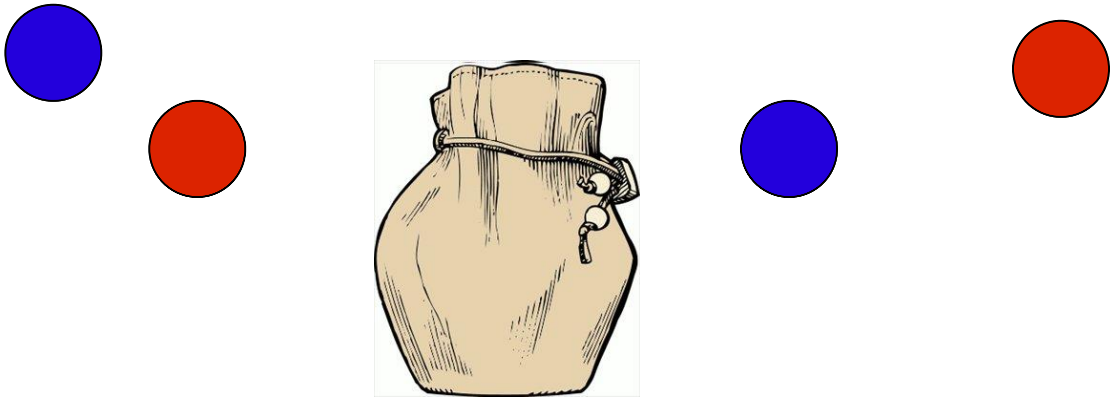
$$\begin{aligned}|\Phi'\rangle &= (\alpha|0\rangle + \beta|1\rangle)(\gamma|0\rangle + \delta|1\rangle) \\ &= \alpha\gamma|0\rangle|0\rangle + \alpha\delta|0\rangle|1\rangle + \beta\gamma|1\rangle|0\rangle + \beta\delta|1\rangle|1\rangle\end{aligned}$$

No solution!

Measurement outcomes are <u>random</u> and <u>correlated</u>

What's so special about entanglement?

- Classical things can be random and correlated, too…



- … but not entangled!

How is this different from an entangled state?

- Each marble has a defined color from the beginning (local hidden variable)
- The processes are <u>distinguishable</u> in principle
- There is no conjugate measurement basis

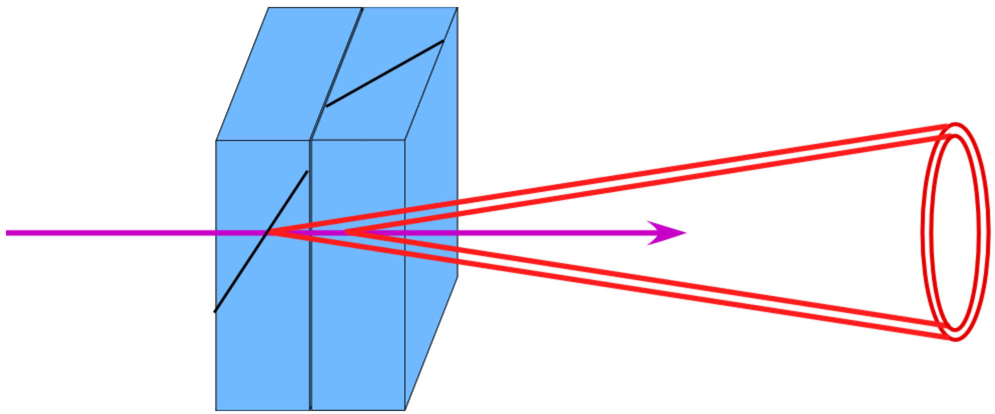
Entangled systems give
random and correlated measurement outcomes
<u>in every measurement basis!</u>

Contents

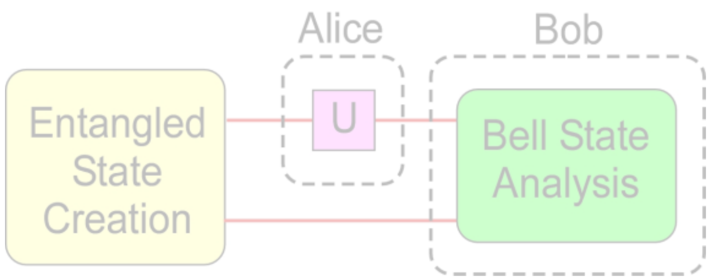
What is Entanglement

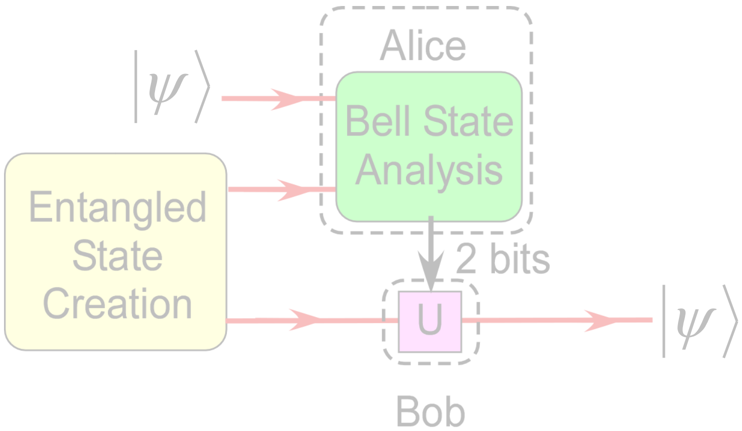
$$|\Phi^{\pm}\rangle = \frac{1}{\sqrt{2}}(|0\rangle|0\rangle \pm |1\rangle|1\rangle)$$

Entanglement Creation

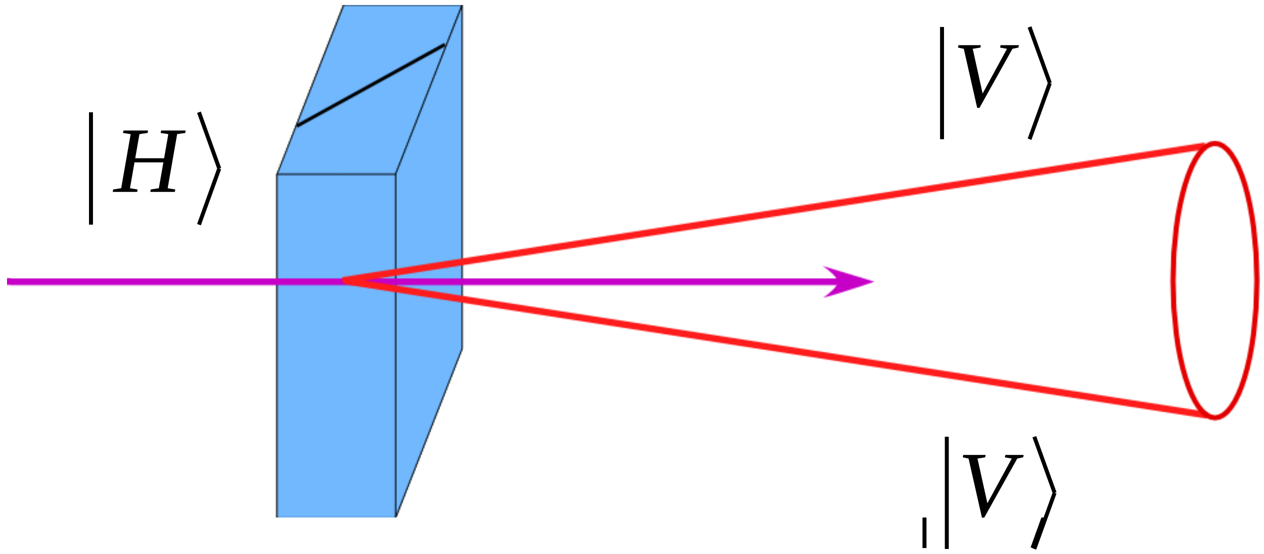


Applications



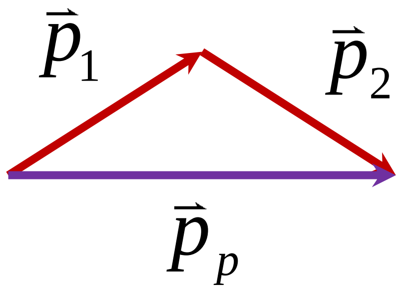


Spontaneous Parametric Down Conversion



$$\omega_p = \omega_1 + \omega_2$$

$$\vec{p}_p = \vec{p}_1 + \vec{p}_2$$



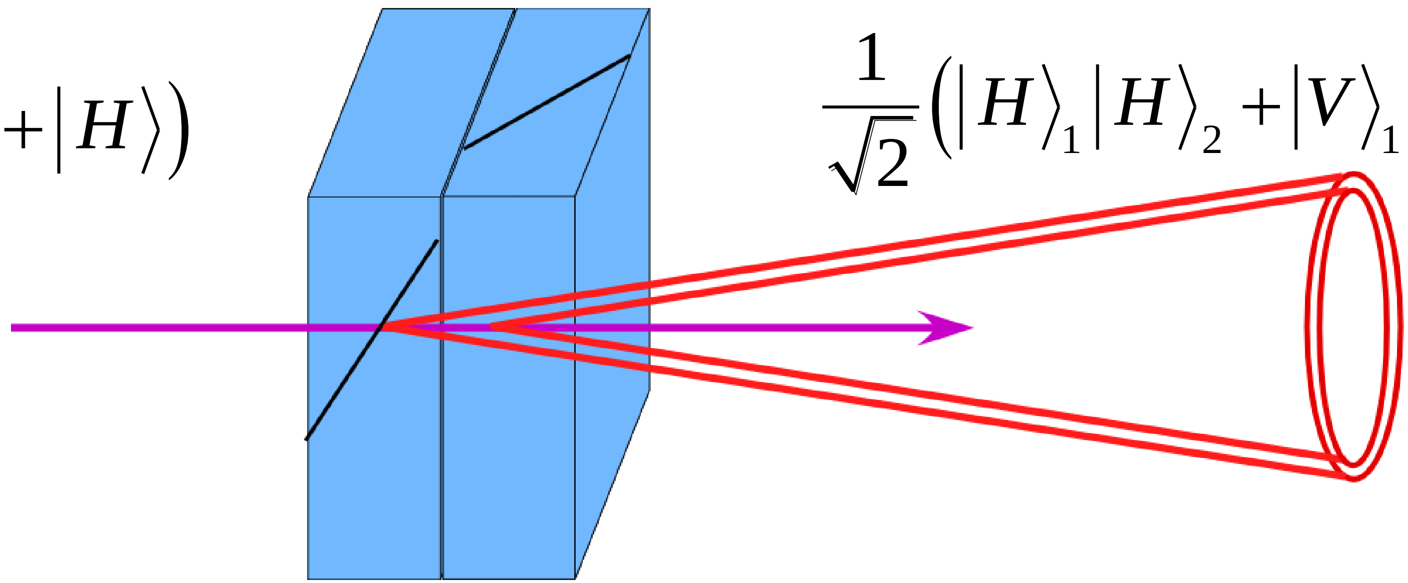
- Nonlinear crystals (such as BBO) have a small chance of splitting a high-energy photon into two low-energy photons*.
- Daughter photons diverge in a cone from the source because of energy and momentum conservation.

* Burnham and Weinberg, PRL 25, 84 (1970)

Polarization Entanglement

$$\frac{1}{\sqrt{2}}\left(|V\rangle + |H\rangle\right)$$

$$\frac{1}{\sqrt{2}}\left(|H\rangle_1 |H\rangle_2 + |V\rangle_1 |V\rangle_2\right)$$

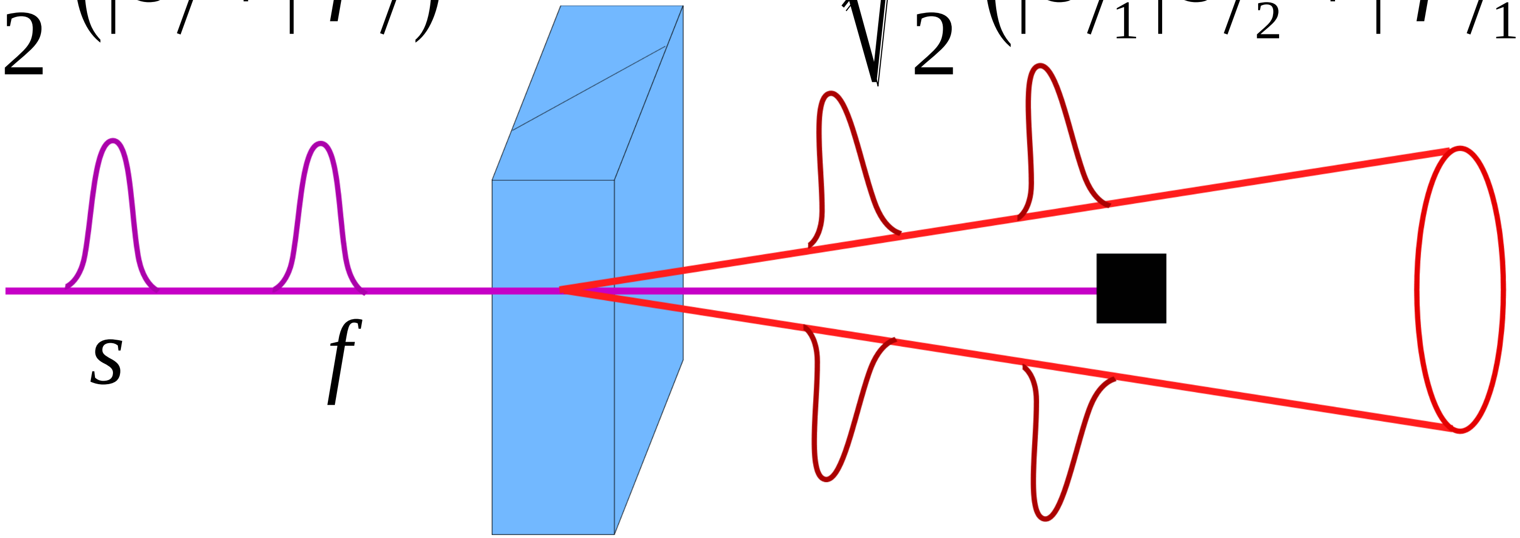


- A polarization entangled state can be created by pumping two orthogonally oriented crystals with a superposition of H and V polarization*.
- The two-photon state can be controlled by manipulating the pump polarization state.

* Kwiat et al., PRA 60, R773 (1999)

Temporal-Mode Entanglement*

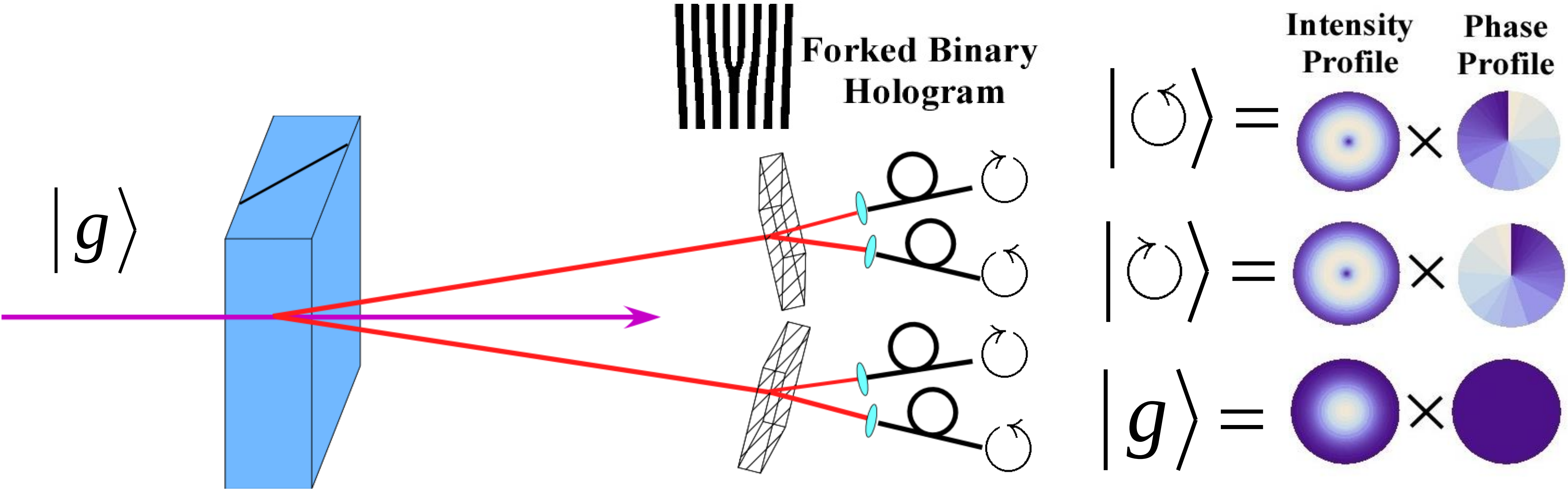
$$\sqrt{\frac{1}{2}}\left(|s\rangle + |f\rangle\right) \qquad \sqrt{\frac{1}{2}}\left(|s\rangle_1|s\rangle_2 + |f\rangle_1|f\rangle_2\right)$$



- A temporal-mode entangled state can be created by pumping a nonlinear crystal with a coherent superposition of pulses.

* Brendel et al., PRL 82, 2594 (1999)

Orbital Angular Momentum Entanglement*

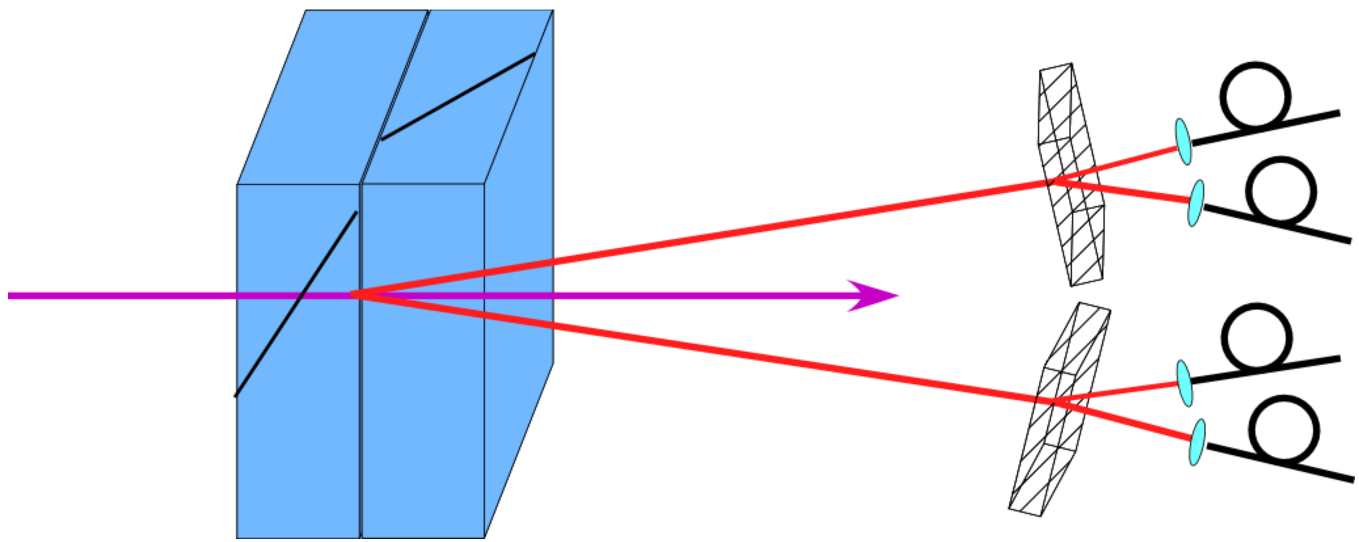


$$N\left(|\circlearrowleft\rangle_1|\circlearrowright\rangle_2+|\circlearrowright\rangle_1|\circlearrowleft\rangle_2 \sout{+\alpha|g\rangle_1|g\rangle_2+\ldots}\right)$$

- Conservation of orbital angular momentum → photons are always correlated in spatial mode.
- A combination of fibers and holograms are used to filter out all but the ±1 orbital angular momentum modes.

* Barreiro et al., PRL 95, 260501 (2005)

Hyperentanglement



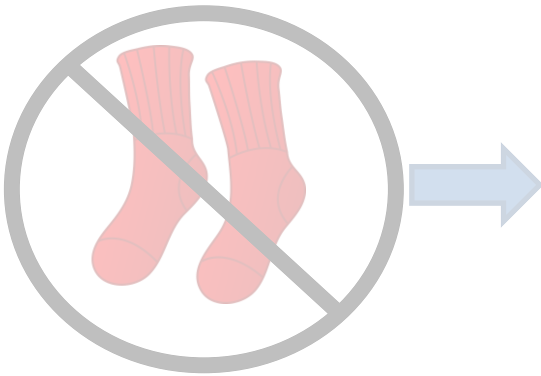
$$\frac{1}{2}\left(|HH\rangle + |VV\rangle\right) \otimes \left(|\circlearrowright\circlearrowleft\rangle + |\circlearrowleft\circlearrowright\rangle\right)$$

- By combining these techniques, we create states simultaneously entangled in multiple degrees of freedom*.

* Barreiro et al., PRL 95, 260501 (2005)

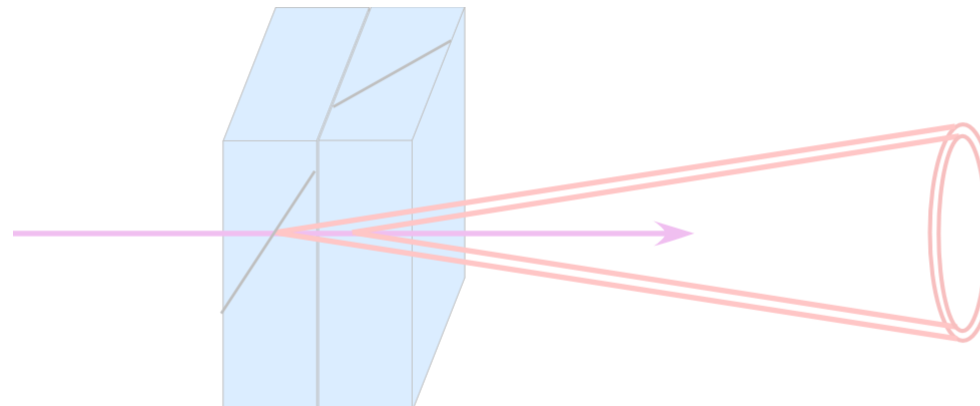
Contents

What is Entanglement

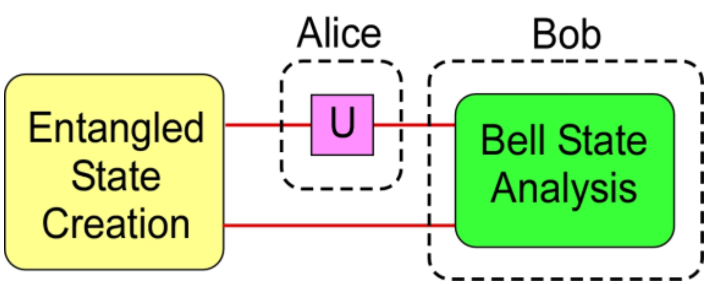


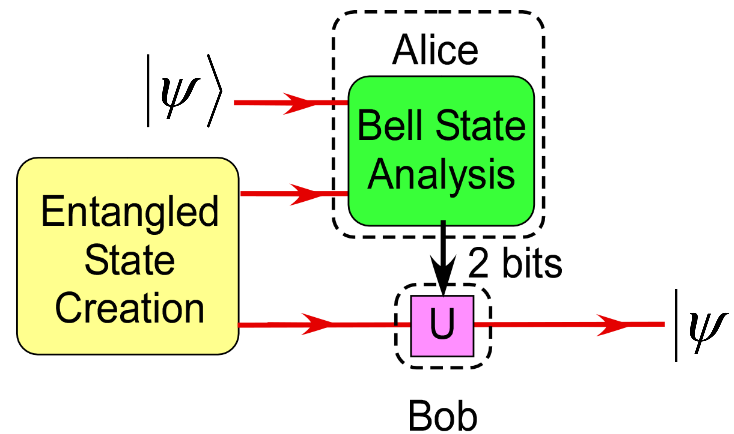
$$|\Phi^{\pm}\rangle = \frac{1}{\sqrt{2}}(|0\rangle|0\rangle \pm |1\rangle|1\rangle)$$

Entanglement Creation



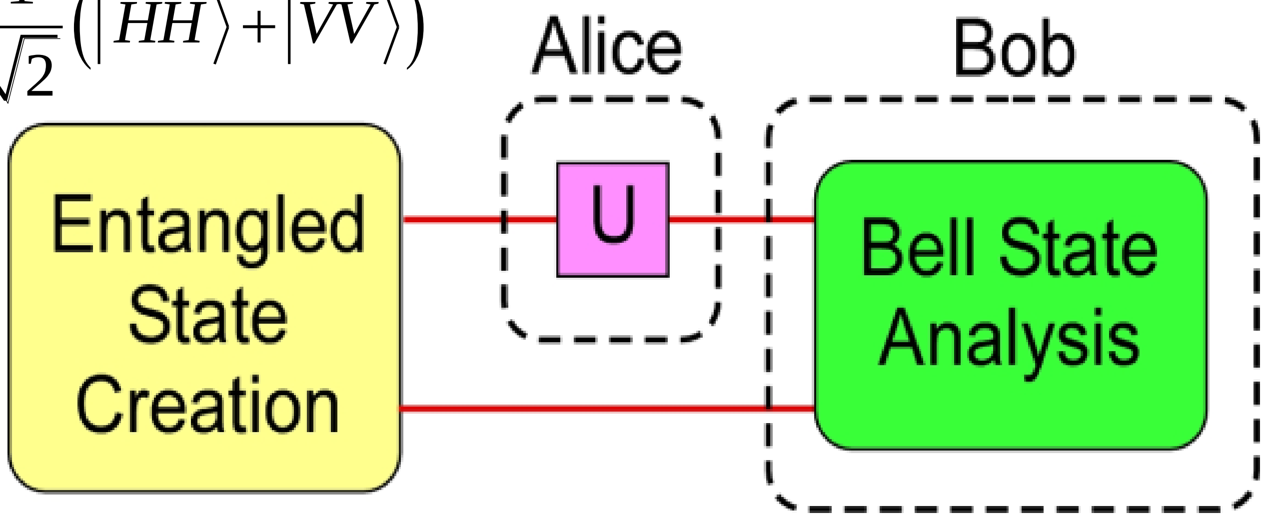
Applications





Super-Dense Coding

$$\frac{1}{\sqrt{2}}\left(|HH\rangle + |VV\rangle\right)$$



$$\Phi^{+} = \frac{1}{\sqrt{2}}\left(|HH\rangle + |VV\rangle\right)$$

$$\Phi^{-} = \frac{1}{\sqrt{2}}\left(|HH\rangle - |VV\rangle\right)$$

$$\Psi^{+} = \frac{1}{\sqrt{2}}\left(|HV\rangle + |VH\rangle\right)$$

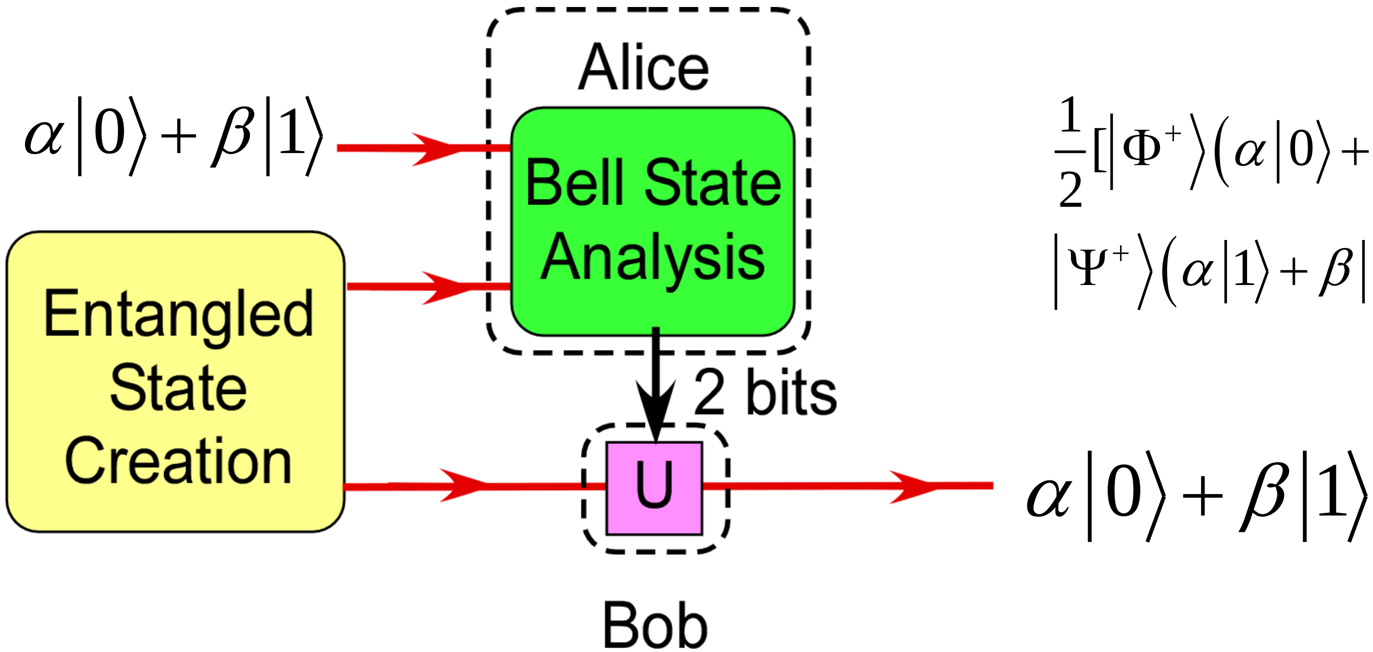
$$\Psi^{-} = \frac{1}{\sqrt{2}}\left(|HV\rangle - |VH\rangle\right)$$

- Super-dense coding may be used to communicate a 2-bit message with a single transmitted qubit*.
- Bob can only distinguish three Bell states limiting this technique to 1.59 bits/photon**.

* Bennett and Wiesner, PRL 69, 2881 (1992)

** Mattle et al., PRL 76, 4656–4659 (1996)

Quantum Teleportation



$$\frac{1}{2}[|\Phi^{+}\rangle(\alpha|0\rangle + \beta|1\rangle) + |\Phi^{-}\rangle(\alpha|0\rangle - \beta|1\rangle) + |\Psi^{+}\rangle(\alpha|1\rangle + \beta|0\rangle) + |\Psi^{-}\rangle(\alpha|1\rangle - \beta|0\rangle)]$$

- Quantum teleportation can be used to teleport a qubit with a 2-bit message over classical communication channel*.
- The utility of this technique is limited because of the required Bell state measurement**.

* Bennett and Wiesner, PRL 69, 2881 (1992)

** Vaidman and Yoran, PRA 59, 116 (1999)

Summary

- Entangled systems cannot be completely described independently
- Entanglement is a type of correlation between quantum systems that is stronger than any classical correlation
- Entanglement is fairly easy to create in the lab
- Entanglement plays a central role in quantum information applications