

Entanglement



“About your cat, Mr. Schrödinger—I have good news and bad news.”

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with adapted PHYS403 content from Paul Kwiat, Brad Christensen

PHYS403, July 26, 2017



Entanglement

- A quantum object can be in a superposition of two states
- ~~The cat is a quantum object~~
- It can be awake and asleep
- If we check, it will be in only one of the states

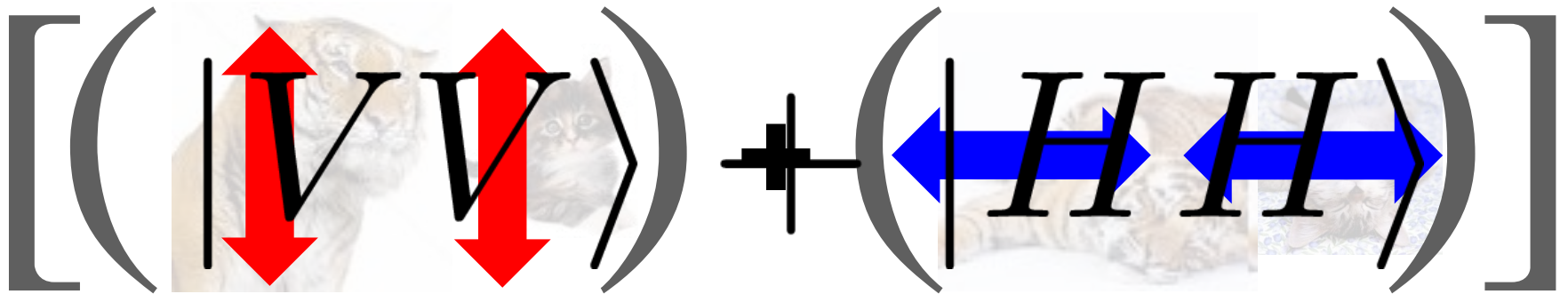


- If we have two objects, we can entangle the states such that knowing about one object affects the other



Entanglement

- An Entangled State:



- If I measure one object, it will end up in just one state, causing the other object to also be in just one state

- E.g. photons whose polarizations are entangled: $|\uparrow\uparrow\rangle + |\leftrightarrow\leftrightarrow\rangle$
 $|VV\rangle + |HH\rangle$



Properties of Entanglement

at least
“It takes ^Vtwo to tangle.”
J. Eberly, 2015

$$\psi_{pair} \propto |HH\rangle + |VV\rangle \quad \text{Entangled}$$

1935: Entanglement is
“*the characteristic trait of quantum mechanics,*
the one that enforces its entire departure from
classical lines of thought”

—E. Schrödinger

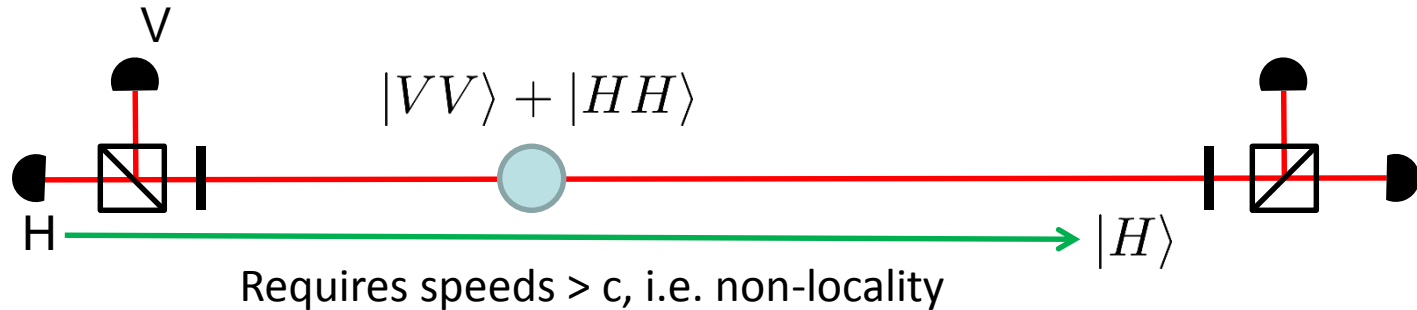
$$\psi_{12} = \psi_1\psi_2 \propto |HH\rangle + |VV\rangle + |HV\rangle + |VH\rangle \quad \text{Not Entangled}$$

In an **entangled** state, neither particle has definite properties alone.

⇒ All the information is stored in the *joint* properties.



1935: Einstein, Podolsky, Rosen (EPR) Paradox



spukhafte Fernwirkung

EPR: Action at a distance (non-locality) is spooky.

Is Quantum Mechanics wrong?

Maybe correlations are due to some local element of reality (“local hidden variable” model)?

A. Einstein, B. Podolsky, and N. Rosen, Phys. Rev. **47**, 777 (1935).



1964: Bell's theorem

- Bell's theorem shows Quantum Mechanics gives different statistical predictions than any local realistic model
 - Certain inequalities are violated if non-local correlations exist, tested by measuring statistical correlations between spatially separated entangled systems

“If [a hidden variable theory] is local it will not agree with quantum mechanics, and if it agrees with quantum mechanics it will not be local.”

– John Bell, 1975

J.S. Bell, *Physics* **1**, 195-200 (1964)





Strong Loophole-Free Test of Local Realism*

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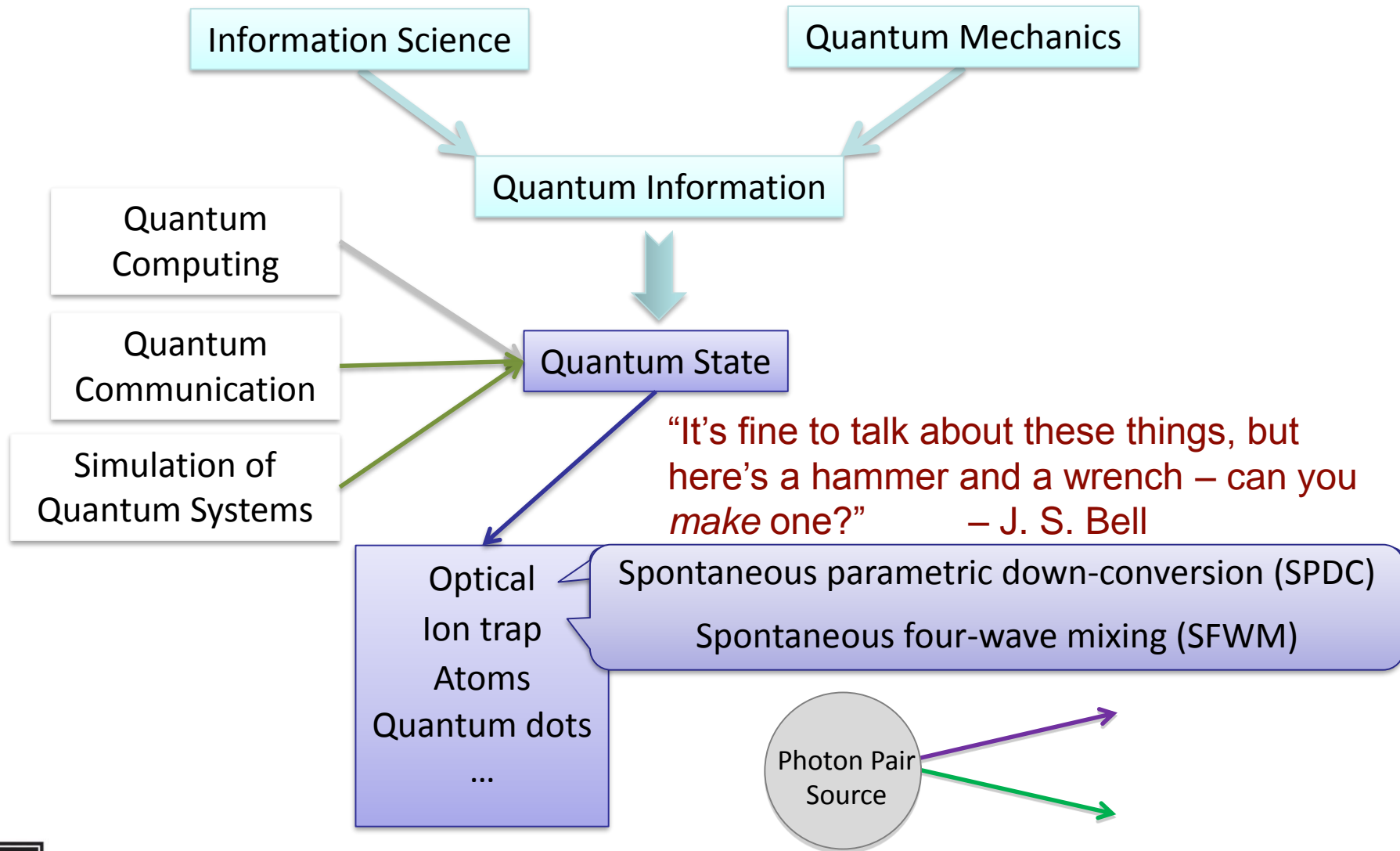
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We present a loophole-free violation of local realism using entangled photon pairs. We ensure that all relevant events in our Bell test are spacelike separated by placing the parties far enough apart and by using fast random number generators and high-speed polarization measurements. A high-quality polarization-entangled source of photons, combined with high-efficiency, low-noise, single-photon detectors, allows us to make measurements without requiring any fair-sampling assumptions. Using a hypothesis test, we compute p values as small as 5.9×10^{-9} for our Bell violation while maintaining the spacelike separation of our events. We estimate the degree to which a local realistic system could predict our measurement choices. Accounting for this predictability, our smallest adjusted p value is 2.3×10^{-7} . We therefore reject the hypothesis that local realism governs our experiment.

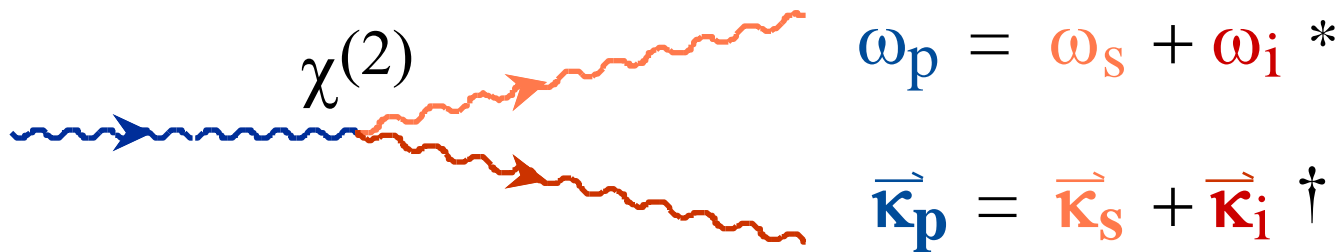


The last 50 years: Quantum Information



1970: Spontaneous Parametric Down-Conversion

- Burnham & Weinberg, PRL **25**, 84 (1970):

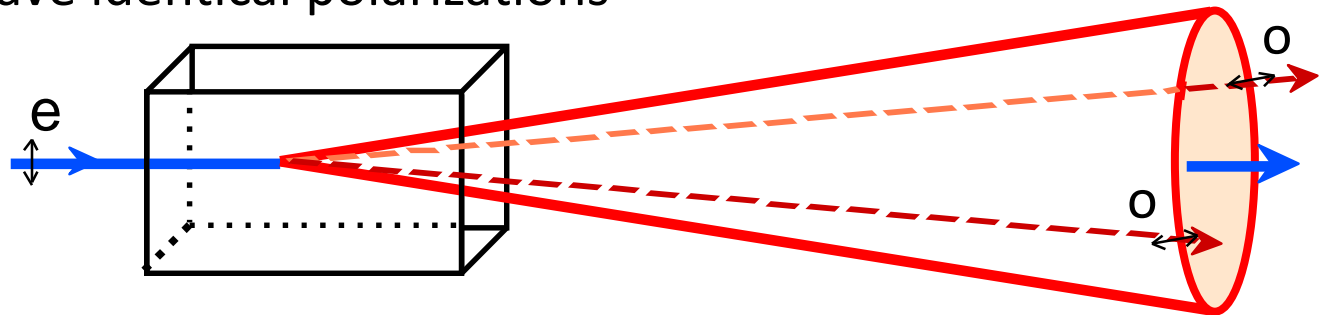


*Energy conservation \rightarrow energy entanglement

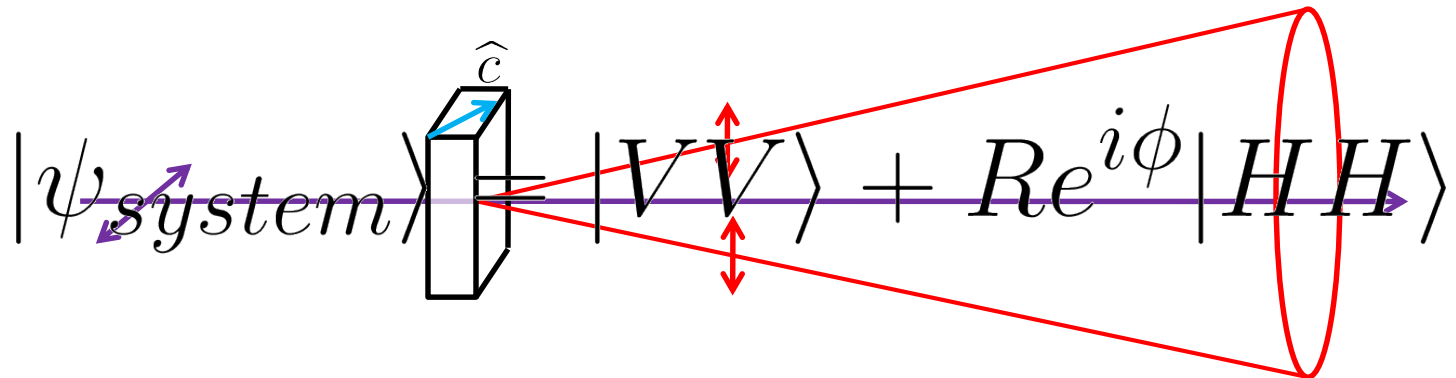
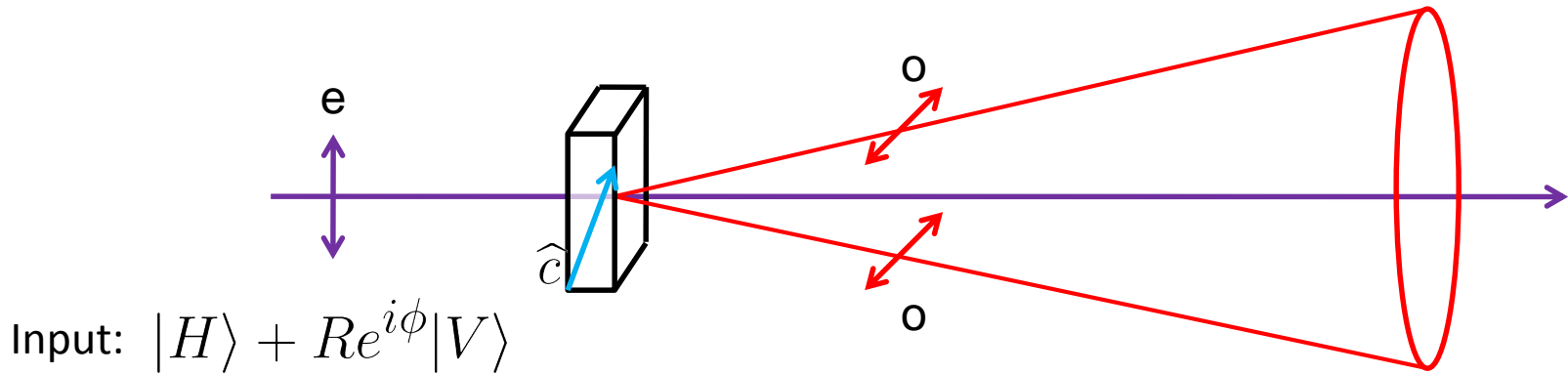
†Momentum conservation \rightarrow momentum entanglement

Type-I phase-matching

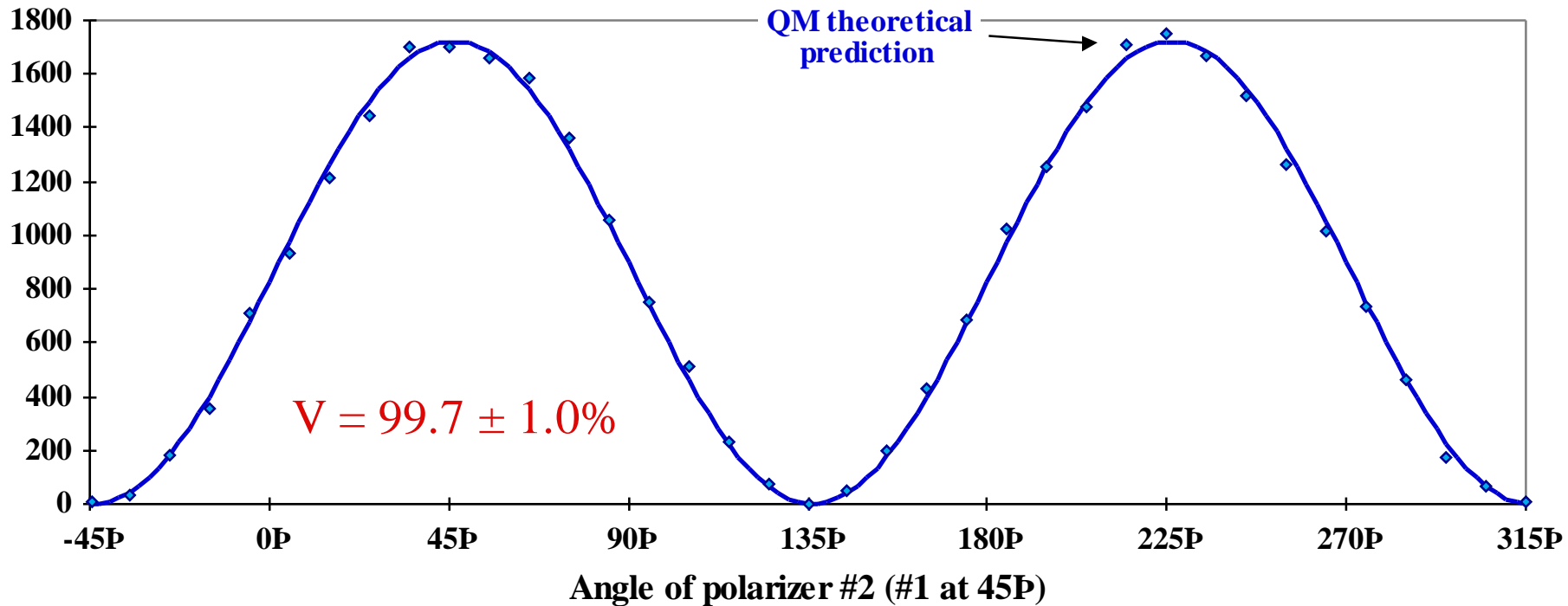
Photons have identical polarizations



Polarization Entanglement



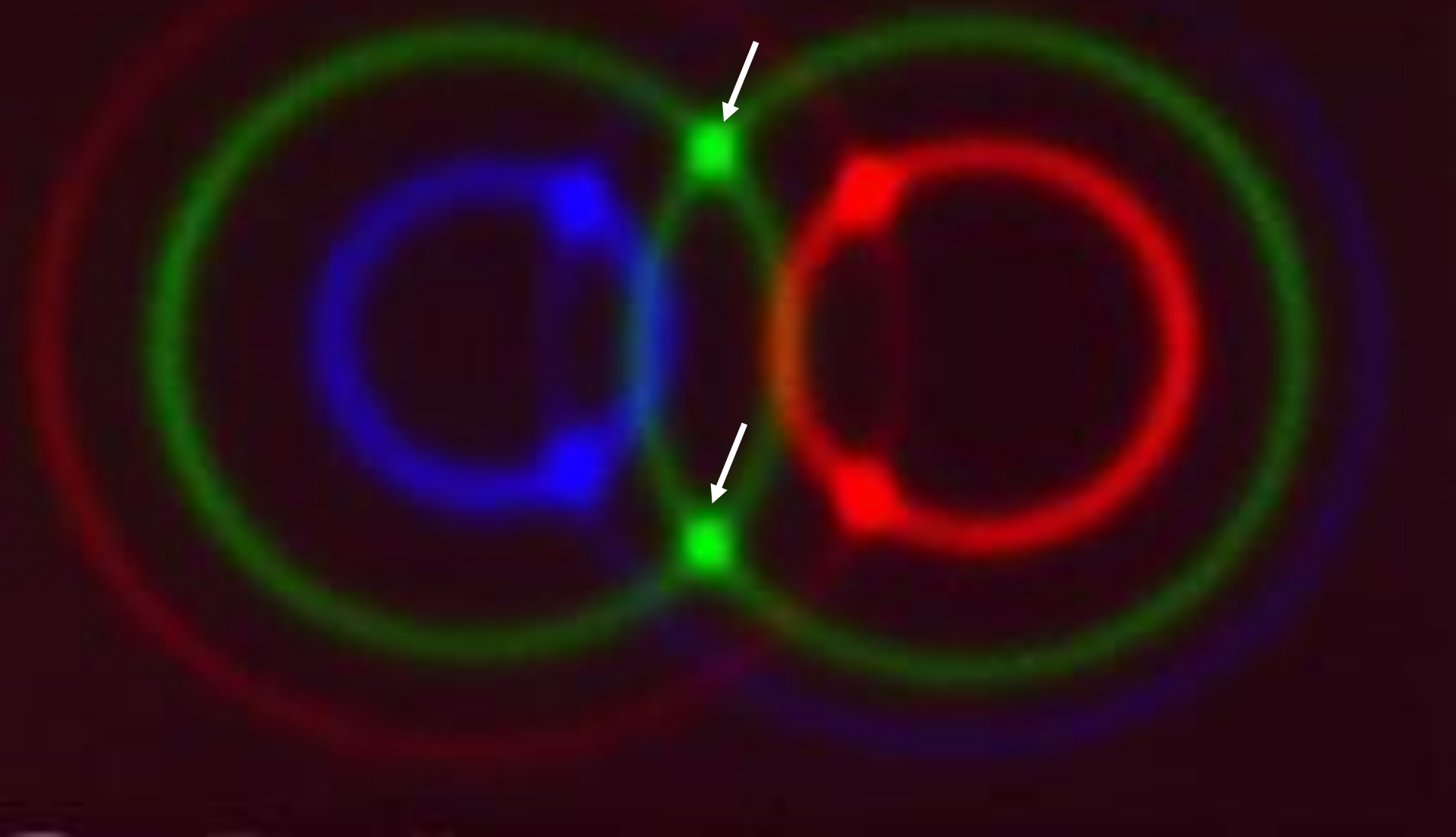
Proof of Quantum Correlations



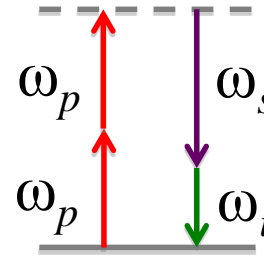
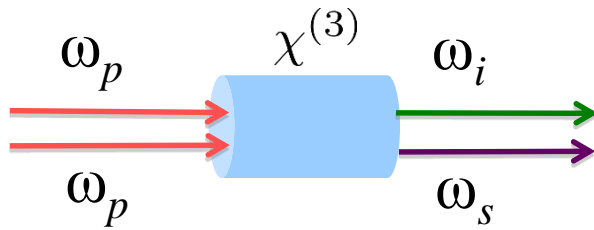
Near-perfect quantum behavior



$$|\psi_{system}\rangle = |VV\rangle + Re^{i\phi}|HH\rangle$$



Spontaneous four-wave mixing



Conservation of energy

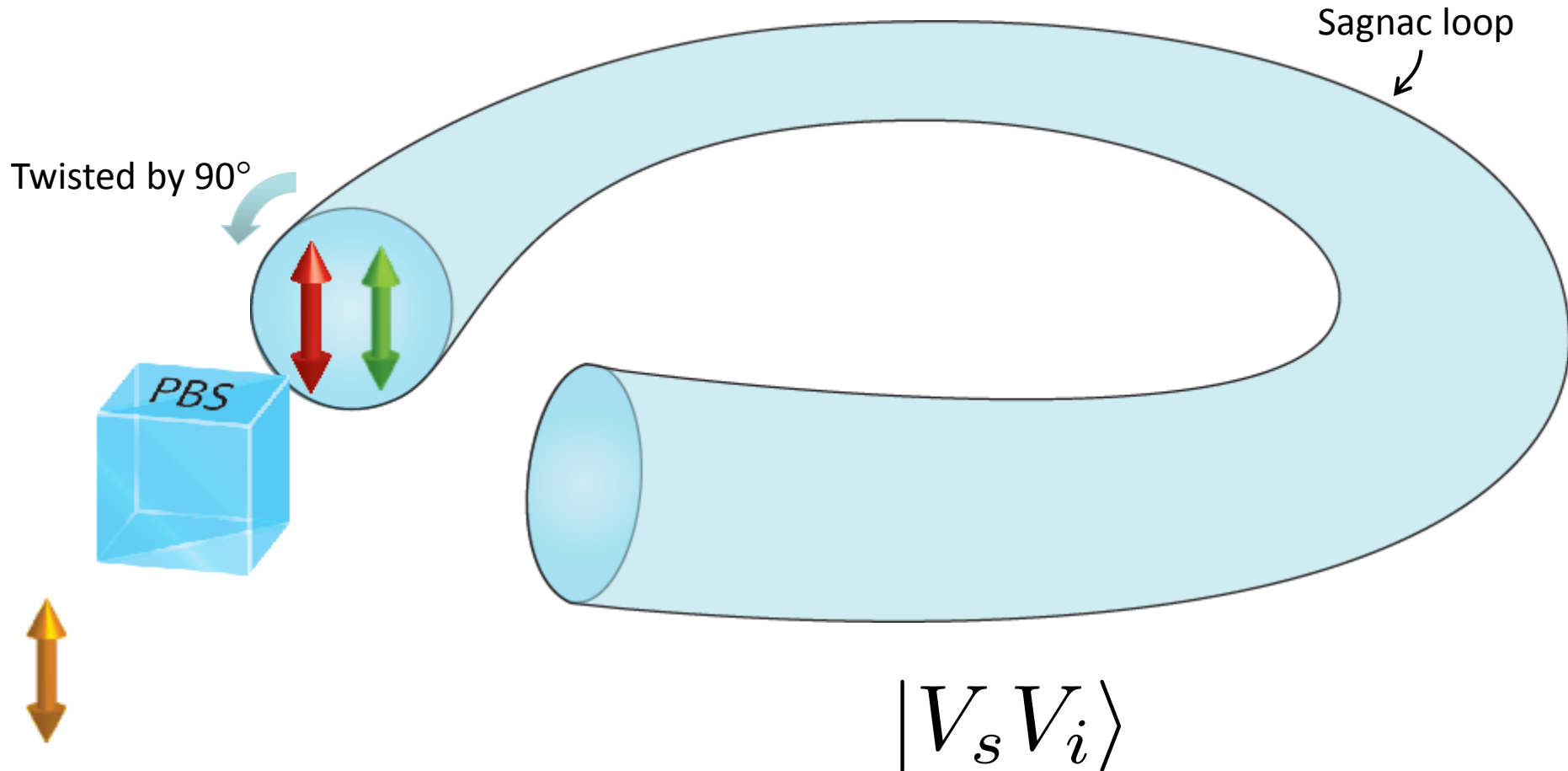
- Spontaneous four-wave mixing in polarization-maintaining optical fiber:



- Birefringent phase-matching: $\Delta k = 2k(\omega_p) - k(\omega_s) - k(\omega_i) + 2\Delta n \frac{\omega_p}{c} = 0$



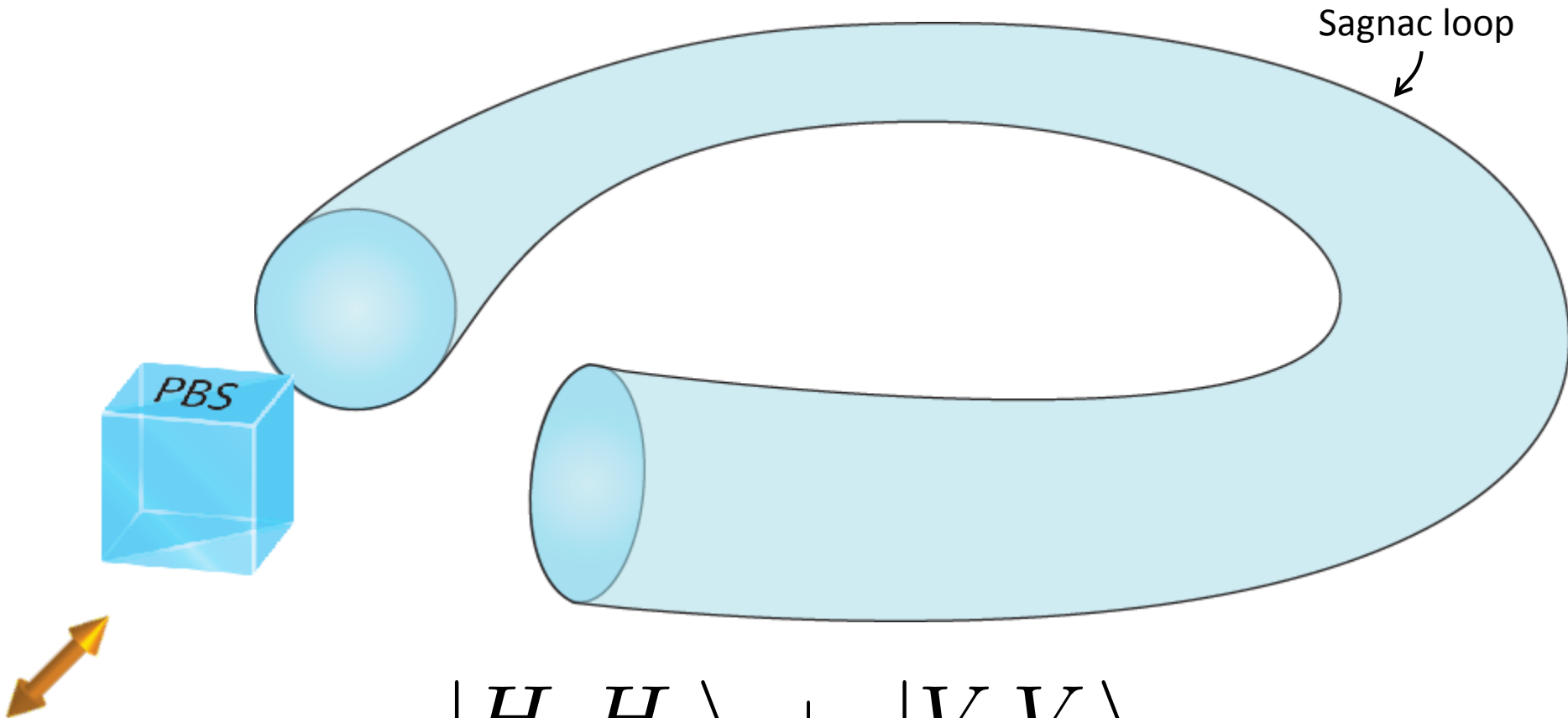
Generation of polarization entanglement



Pump travels on slow axis. Signal and idler travel on fast axis.
One end of the fiber is twisted by 90° relative to the other end.



Generation of polarization entanglement



$$|H_s H_i\rangle + |V_s V_i\rangle$$

Pump travels on slow axis. Signal and idler travel on fast axis.
One end of the fiber is twisted by 90° relative to the other end.



Why are entangled states important?

- Responsible for quantum measurements and decoherence
- Central to demonstrations of quantum nonlocality (e.g., Bell's inequalities, GHZ, etc.)
- **Quantum cryptography** – separated particles' correlations allow sharing of secret random key
- **Quantum teleportation** – transmit unknown quantum state via 2 classical bits + EPR pair
- **Quantum computation** – intermediate states are all complex entangled states



Classical Cryptography

USA TODAY:



RSA Algorithm (1978): Generate random prime numbers p & q .
Compute $n = pq$, $\phi(n) = (p-1)(q-1)$, e co-prime with ϕ , $d = e^{-1} \pmod{\phi(n)}$
Release e , n as public key. Encrypt: $c = \text{message}^e \pmod{n}$
Keep d as private key. Decrypt: $\text{message} = c^d \pmod{n}$



9AFGJI4JT09RKSP

From: UIUC
Sent: Friday, March 14, 2014 11:40 AM
To: 'Virginia Lorenz'
Subject: Physics

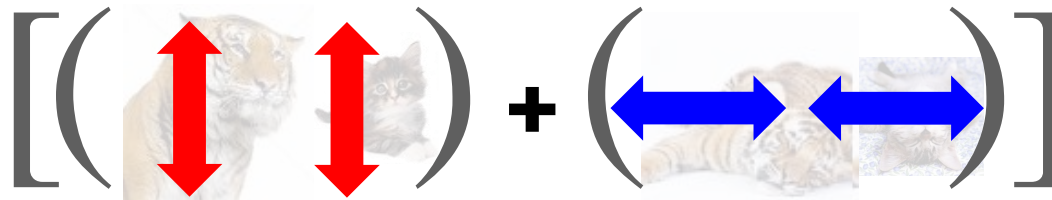
Hi Virginia,
...



- There is no indication of eavesdropping
- Security relies on computational difficulty of factoring the public key

B

Quantum Key Distribution



Security is guaranteed by the laws of quantum physics!

Ekert Protocol (1991): Generate entangled photon pair.

Cerberis QKD Server



Cerberis from IDQ is a standalone rack-mountable QKD server; providing secure quantum keys based on the BB84 and SARG protocols. Integrated with IDQ's Centaurus Ethernet and Fiber Channel encryptors, Cerberis has been deployed by governments, enterprises and financial institutions since 2007.

<http://www.idquantique.com/quantum-safe-crypto/>



- Eavesdropping without being detected is impossible because measurement changes the correlations

Everitt Laboratory
Electrical and...

Engineering
Hall

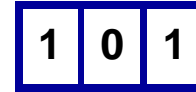
Laboratory
Of Physics,
University...

B

Entanglement, and the scaling that results, is the key to the power of quantum computing

- Classically, information is stored in a bit register:

- A 3-bit register can store **one** number, from 0-7



- Quantum Mechanically, a register of 3 qubits can store all of these numbers in superposition:

$$|000\rangle + |001\rangle + |010\rangle + |011\rangle + |100\rangle + |101\rangle + |110\rangle + |111\rangle = |0\rangle + |1\rangle + \dots |7\rangle$$

Result:

- Classical: one N-bit number
- Quantum: 2^N (all possible) N-bit numbers
 - N.B. A 300-qubit register can simultaneously store more combinations than there are particles in the universe.
- Acting on the qubits simultaneously affects all the numbers:

$$(|0\rangle + |1\rangle + \dots |7\rangle) \otimes |f(x)\rangle \Rightarrow |0\rangle |f(0)\rangle + |1\rangle |f(1)\rangle + \dots |7\rangle |f(7)\rangle$$

- Some important problems benefit from this entanglement, enabling solutions of otherwise insoluble problems.



Quantum Logic

Controlled-Not Gate:

$$|0\rangle_c |0\rangle_t \rightarrow |0\rangle_c |0\rangle_t$$

$$|0\rangle_c |1\rangle_t \rightarrow |0\rangle_c |1\rangle_t$$

$$|1\rangle_c |0\rangle_t \rightarrow |1\rangle_c |1\rangle_t$$

$$|1\rangle_c |1\rangle_t \rightarrow |1\rangle_c |0\rangle_t$$

$$\left(|0\rangle_c + |1\rangle_c \right) |0\rangle_t \xrightarrow{CNOT} |0\rangle_c |0\rangle_t + |1\rangle_c |1\rangle_t$$

2-Qubit interactions lead to entangled states.

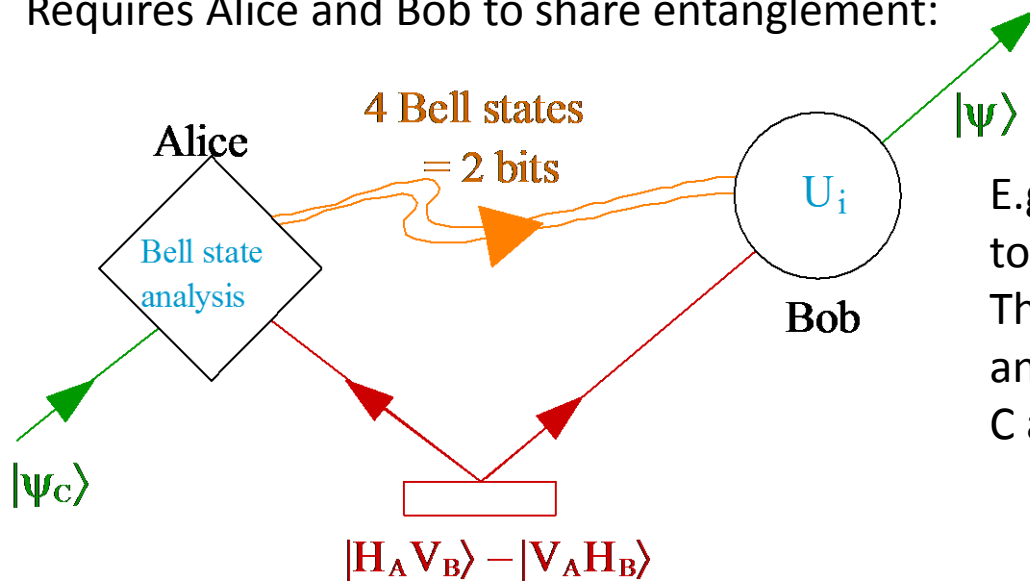


Quantum Teleportation

Bennett et al., PRL **70**, 1895 (1993)

The basic idea: transfer the (infinite) amount of information in a qubit from Alice to Bob without sending the qubit itself.

Requires Alice and Bob to share entanglement:



E.g. Alice measures photons C and A to be in a singlet state. Then since C and A are perpendicular, and since A and B are perpendicular, C and B must be identical!

Remarks:

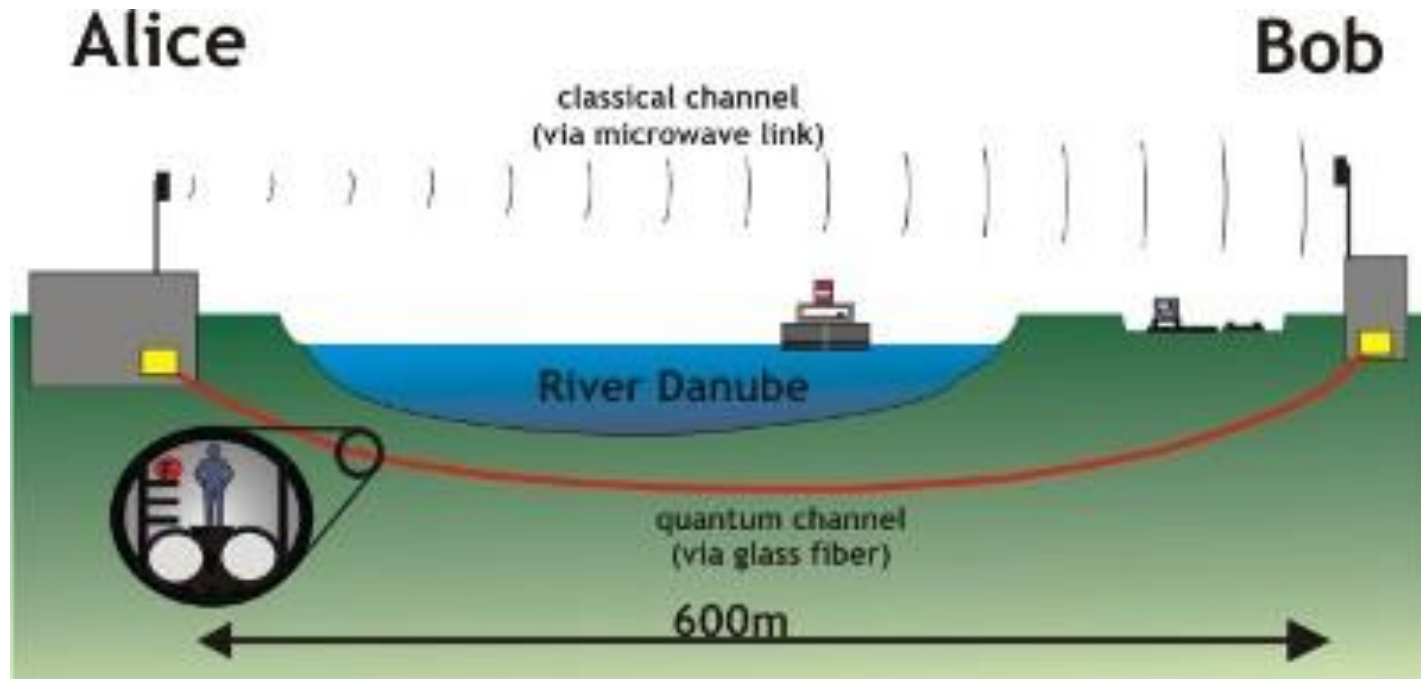
- The original state is gone.
- Neither Alice nor Bob know what it was.
- Requires classical communication – no superluminal signaling.
- Bell state analysis is hard.



Experimental Teleportation

1997: **First demonstration** [Bouwmeester *et al.*, Nature **390**, 575 (1997)]

2004: **Quantum teleportation across the Danube** [Ursin *et al.*, Nature **430**, 849 (2004)]

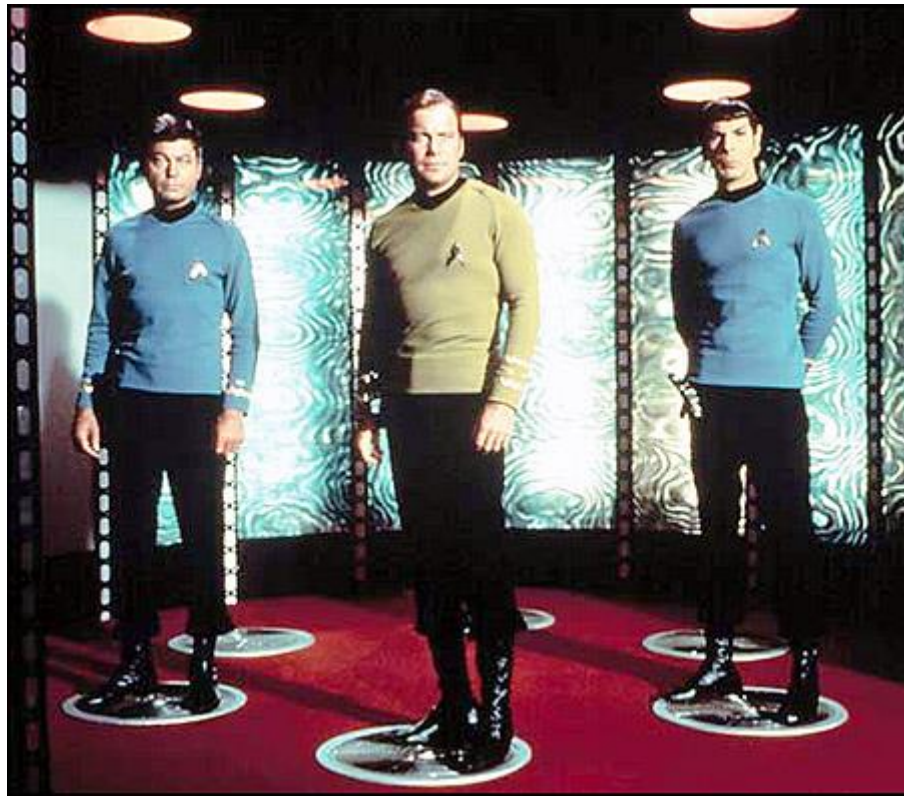


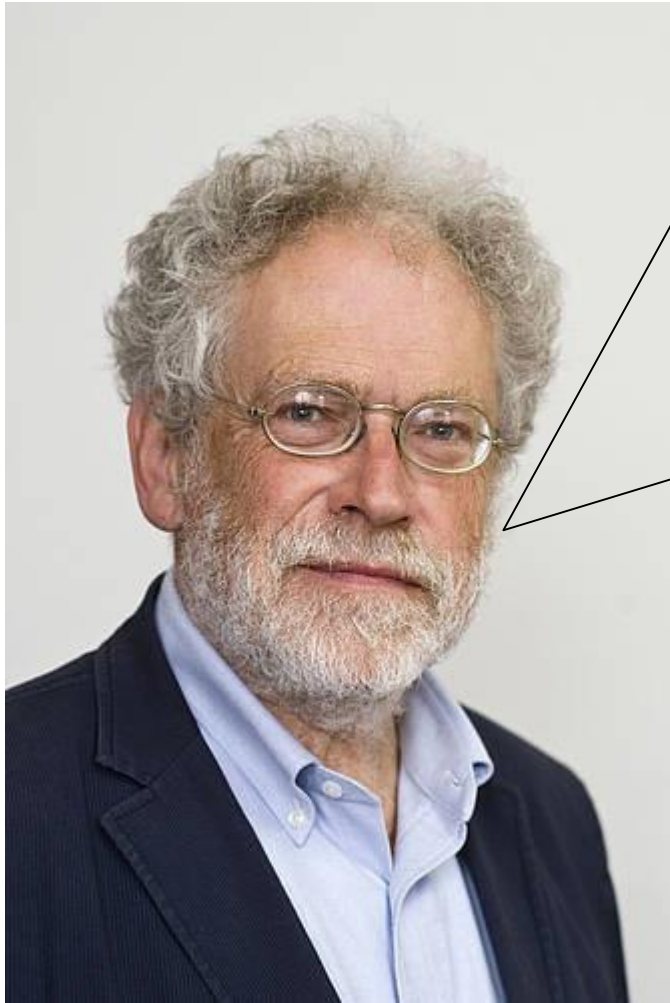
- Now demonstrated teleportation of entanglement, other degrees of freedom, continuous variables, energy states of ions, 2-qubits ...



But there's Quantum Teleportation, and then there's

Quantum Teleportation





Anton Zeilinger

Yes, but there are two major differences. Firstly, we **transfer properties**, not matter. And secondly, until now we have had more success with light particles and occasionally with atoms, not with larger objects.

And even if it was possible, the problems involved would be huge. Firstly: for physical reasons, the original has to be **completely isolated** from its environment for the transfer to work. There has to be a total vacuum for it to work. And it is a well-known fact that this is **not particularly healthy** for human beings. Secondly, you would take all the properties from a person and transfer them onto another. This means producing a being who no longer has any hair colour, no eye colour, nix. A man without qualities! This is not only unethical – it's so crazy that it's impossible to imagine.





Anton Zeilinger

The atoms in a human being are the equivalent to the information mass of about a **thousand billion billion billion bits**. Even with today's top technology, this means it would take about 30 billion years to transfer this mass of data. That's twice the age of the universe. So we'll need a number of major breakthroughs in technology first.

...

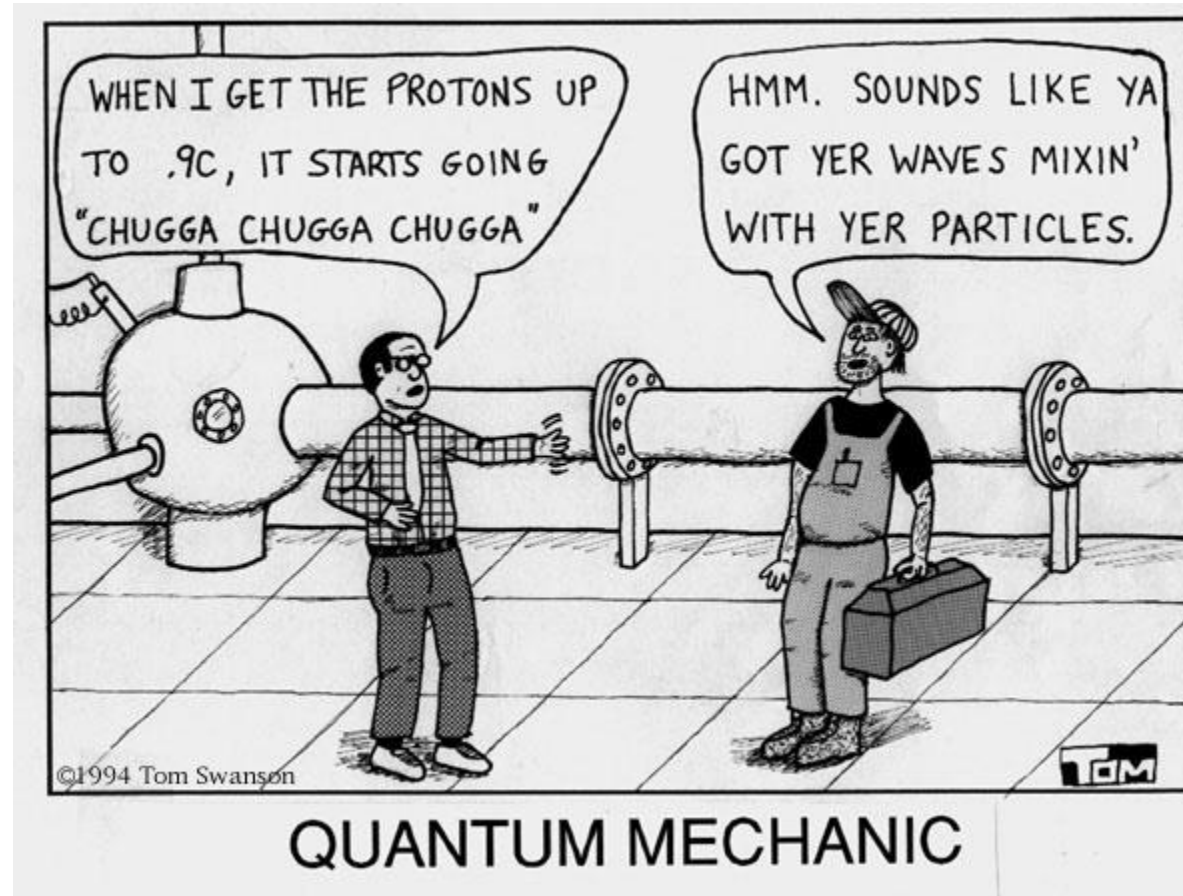
Who knows, perhaps in a thousand years we really will be able to **teleport a coffee cup**. But beware: even the tiniest interference can mean that the cup arrives without its handle. This method of transport would be far too dangerous for humans.



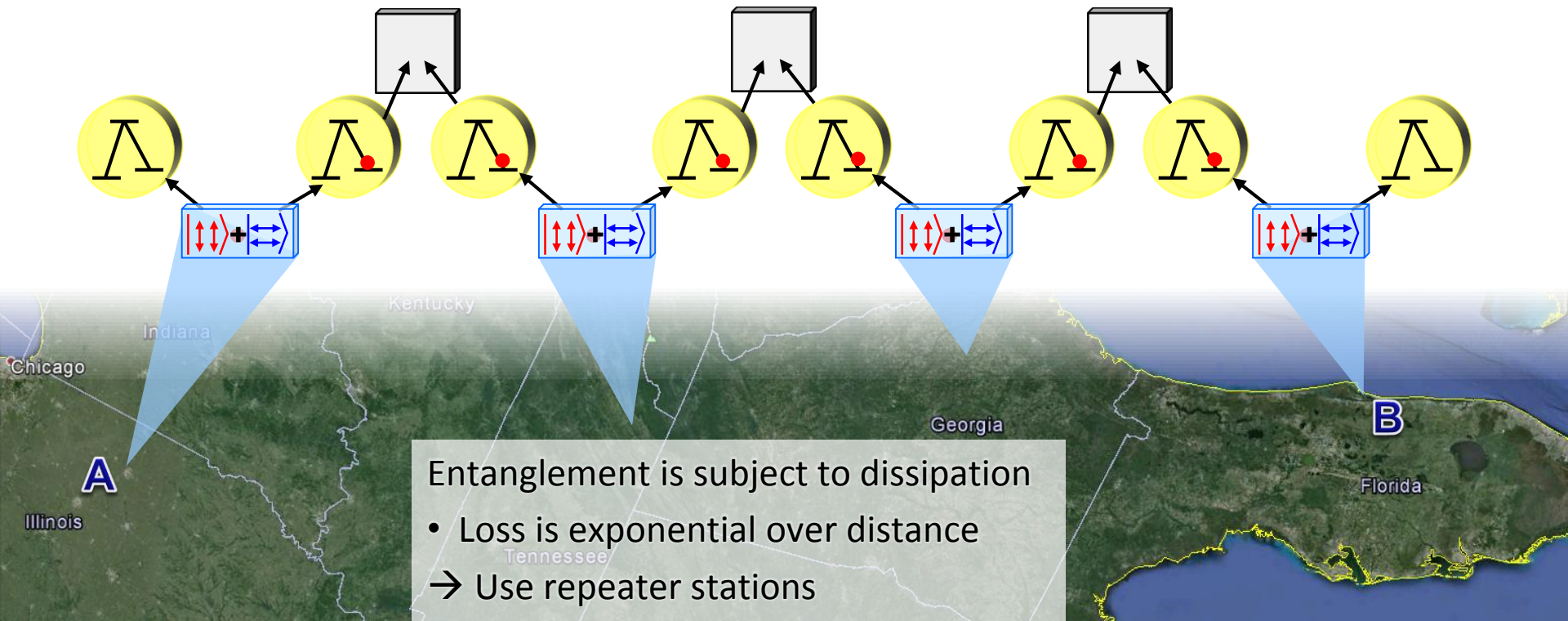
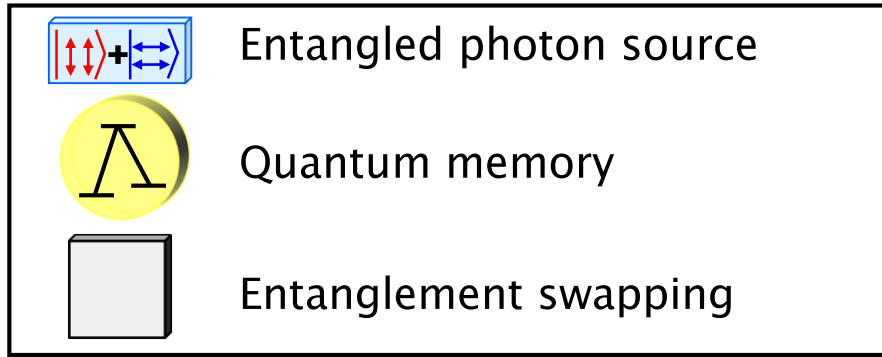
<http://www.signandsight.com/features/614.html>

Conclusion

- Quantum entanglement breaks local realism
- Generating entangled photons & reconstructing their state is relatively easy, but engineering for applications is still a challenge
- Entanglement is important and useful!



Long-Distance QKD



Binary digit -- "bit"

0, 1

copyable

Quantum bit -- "qubit"

$|0\rangle, |1\rangle, (|0\rangle + |1\rangle)/\sqrt{2}$

unclonable

Physical realization of qubits \rightarrow any 2 level system

2-level atom: $|g\rangle, |e\rangle$

spin-1/2: $|\uparrow\rangle, |\downarrow\rangle$

polarization: $|H\rangle, |V\rangle$

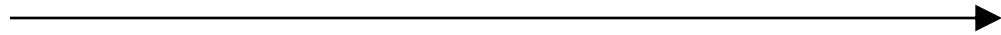
All 2-level systems are created equal, but some are more equal than others!

Quantum communication \rightarrow photons

Quantum storage \rightarrow atomic vapors, spins

Scaleable circuits \rightarrow ions, solid state systems

"Quantum" phenomena



Superposition

Interference

Wave-particle duality

Intrinsic randomness in measurement

Entanglement

