Uncertainties in Measurements

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uncertainties

- any measurement must include uncertainties
  - any report must include a discussion of the uncertainties
- two types:
  - statistical:
    - uncertainties based on the number of observations
    - uncertainty usually goes like the sqrt(N), these describe 1σ uncertainties
  - systematic:
    - uncertainties inherent in the methods, equipment, stability, external conditions …
    - these are typically more challenging to identify and to quantify
    - focus on the most important sources
  - measurements are often limited by one or the other
    - if your measurement is statistics limited, try to take more data, if you can significantly improve uncertainties (doubling data, improves uncertainties by 40%)
    - if your measurement is systematics limited, taking more data won't help

Search for the Standard Model Higgs Boson in the Diphoton Decay Channel with 4.9 fb^{-1} of pp Collision Data at $\sqrt{s} = 7$ TeV with ATLAS

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(ATLAS Collaboration)
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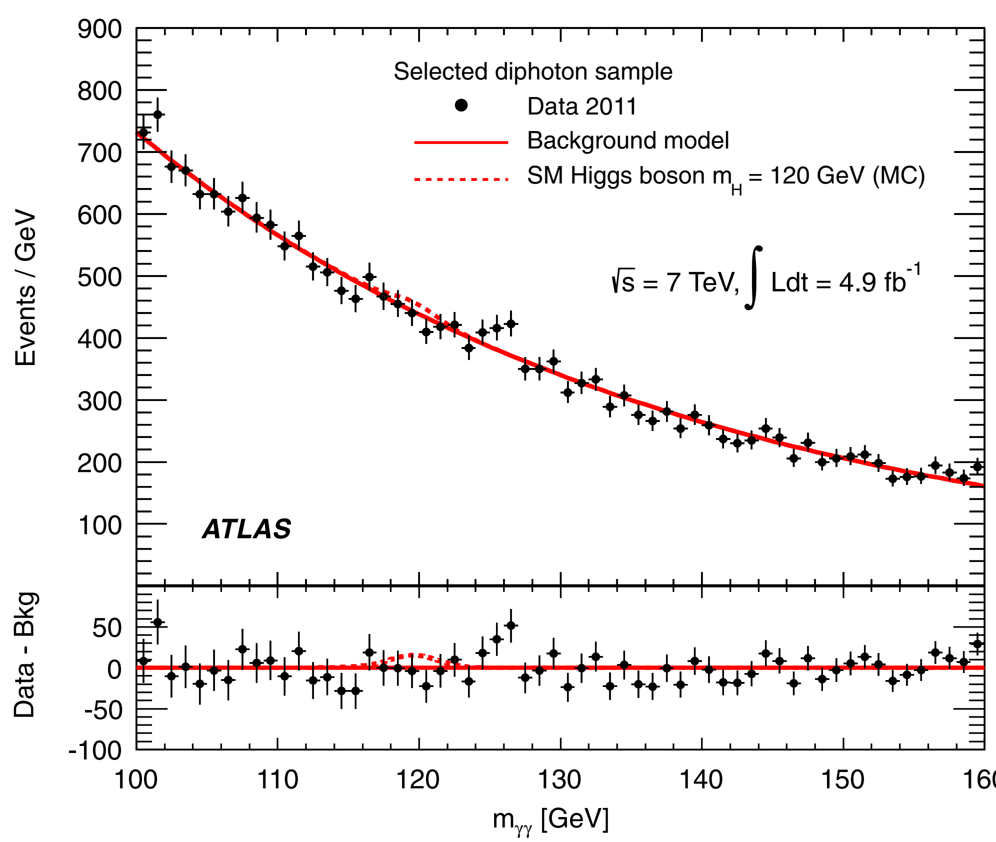
A search for the standard model Higgs boson is performed in the diphoton decay channel. The data used correspond to an integrated luminosity of 4.9 fb^{-1} collected with the ATLAS detector at the Large Hadron Collider in proton-proton collisions at a center-of-mass energy of $\sqrt{s} = 7$ TeV. In the diphoton mass range 110–150 GeV, the largest excess with respect to the background-only hypothesis is observed at 126.5 GeV, with a local significance of 2.8 standard deviations. Taking the look-elsewhere effect into account in the range 110–150 GeV, this significance becomes 1.5 standard deviations. The standard model Higgs boson is excluded at 95% confidence level in the mass ranges of 113–115 GeV and 134.5–136 GeV.



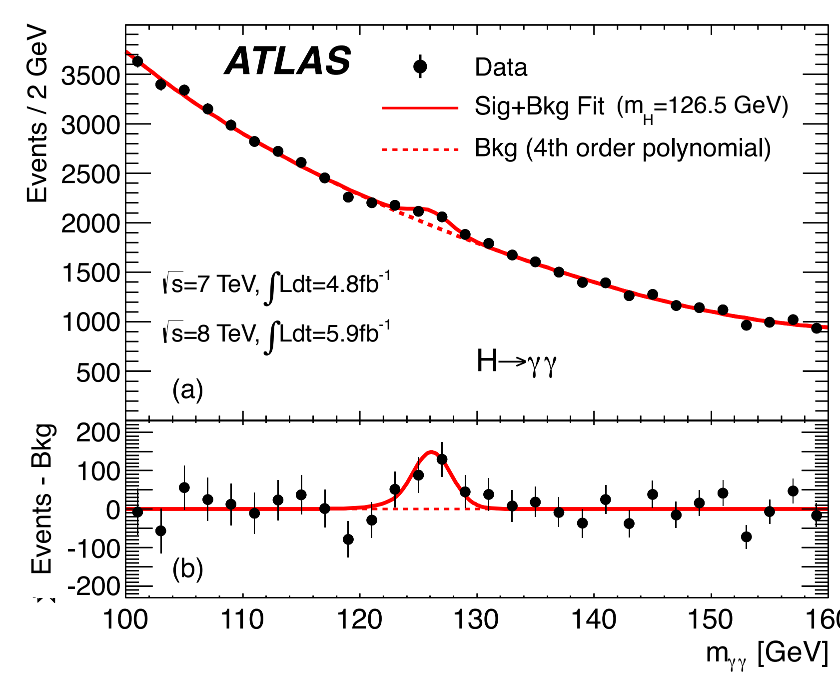
Observation of a New Particle in the Search for the Standard Model Higgs Boson with the ATLAS Detector at the LHC

The ATLAS Collaboration

Physics Letters B716 (2012) 1

Abstract

A search for the Standard Model Higgs boson in proton-proton collisions with the ATLAS detector at the LHC is presented. The datasets used correspond to integrated luminosities of approximately 4.8 fb^{-1} collected at $\sqrt{s} = 7$ TeV in 2011 and 5.8 fb^{-1} at $\sqrt{s} = 8$ TeV in 2012. Individual searches in the channels $H \to ZZ^{(*)} \to 4\ell$, $H \to \gamma\gamma$ and $H \to WW^{(*)} \to e\nu\mu\nu$ in the 8 TeV data are combined with previously published results of searches for $H \to ZZ^{(*)}$, $WW^{(*)}$, $b\bar{b}$ and $\tau^+\tau^-$ in the 7 TeV data and results from improved analyses of the $H \to ZZ^{(*)} \to 4\ell$ and $H \to \gamma\gamma$ channels in the 7 TeV data. Clear evidence for the production of a neutral boson with a measured mass of 126.0 ± 0.4 (stat) ± 0.4 (sys) GeV is presented. This observation, which has a significance of 5.9 standard deviations, corresponding to a background fluctuation probability of 1.7×10^{-9}, is compatible with the production and decay of the Standard Model Higgs boson.

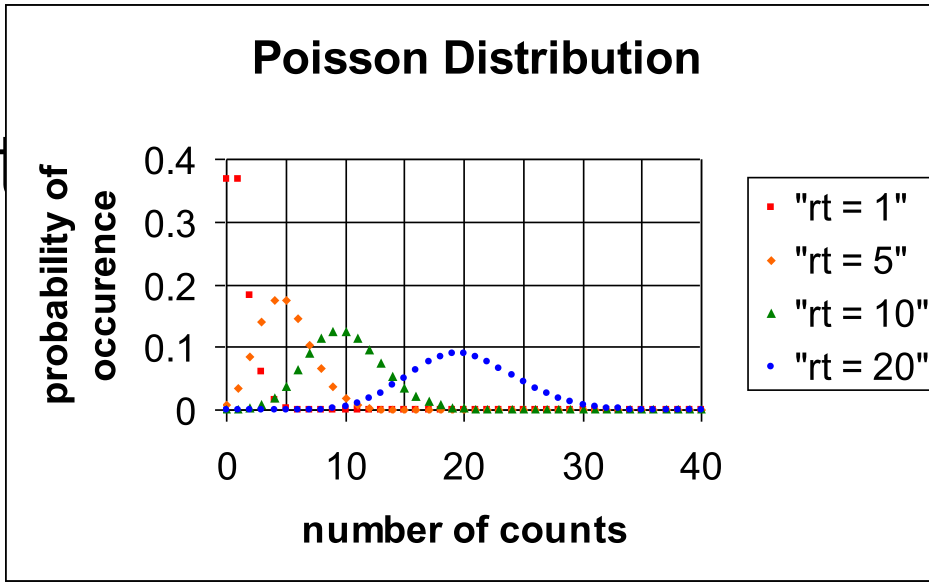


counting experiments

$$P_n(rt) = \frac{(rt)^n}{n!} e^{-rt}, \quad n = 0, 1, 2, \ldots$$

rt = (decay rate)(time) = number of counts

- random processes follow Poisson distribution
- nuclear decay is one such process, but this applies t counting experiments
- asymmetric distribution at small number of counts
    - you can't observe negative counts
    - becomes Gaussian as rt increases
- distribution is a probability distribution, not the number of counts
- $\sigma/\mu = 1/\sqrt{rt}$ → larger rt, smaller uncertainty on μ



$$\sum_{n=0}^{\infty} P_n(rt) = 1$$, probabilities sum to 1

$$< n > = \sum_{n=0}^{\infty} n \cdot P_n(rt) = rt$$, the mean

$$\sigma = \sqrt{\sum_{n=0}^{\infty} (n - < n >)^2 P_n(rt)} = \sqrt{rt}$$, standard deviation

accuracy and precision

- precision:
  - measurements close together
- accuracy:
  - measurements that contain the true value inside the uncertainty
- want to be both accurate and precise!
- in this class you will try to be accurate, but other measurements will typically be more precise than we can do with this equipment

$$\Delta z = \sqrt{(\Delta x)^2 + (\Delta y)^2 + \ldots}$$

addition/subtraction

$$\frac{\Delta z}{z} = \sqrt{\left(\frac{\Delta x}{x}\right)^2 + \left(\frac{\Delta y}{y}\right)^2 + \ldots}$$

multiplication/division

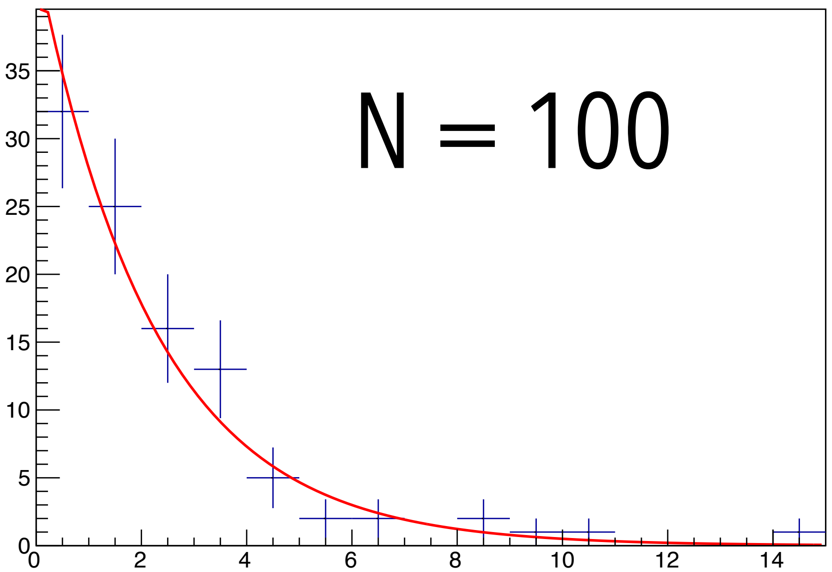
- these formula are true if x and y are *independent* of each other
  - if you have correlated measurements then you must deal with the covariance
- many automated programs will do this for you, but *you* must figure out if you have correlated measurements
- think about a measurement with a lot of background:
  - if Δx & Δy are large then Δz will be large when $z = x - y$
  - clear why minimizing background is very important for many measurements!

fitting data

$$\chi^2 = \sum_{i=1}^{n} \frac{(x_i - \mu_i)^2}{\sigma_i^2}$$

- fitting:
    - you provide the functional form—the fit should be meaningful
    - many implementations of chi2 minimization fitting around
    - need to understand how well the fit describes your data
        - this will only take into account statistical uncertainties, not systematics

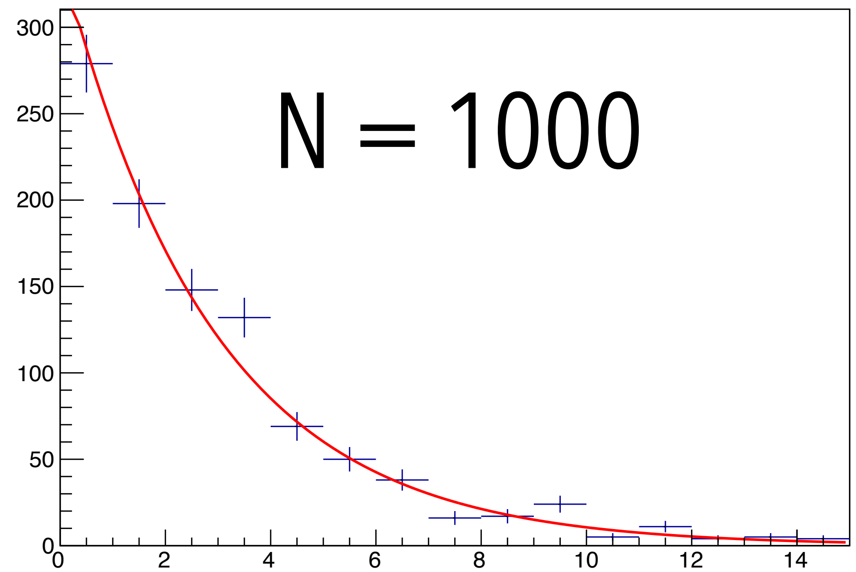
$Ae^{-x/\tau}$ $\tau = -3$



$\tau = -2.24 \pm 0.27$

$\chi^2/\mathrm{dof} = 5.5/9$

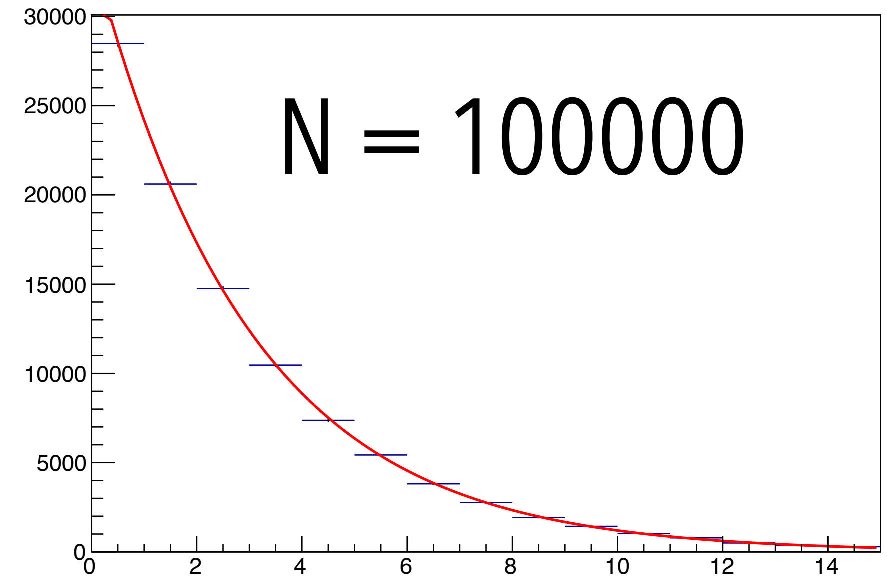
fit probability = 0.79



$\tau = -2.87 \pm 0.10$

$\chi^2/\mathrm{dof} = 25.3/13$

fit probability = 0.02



$\tau = -2.994 \pm 0.010$

$\chi^2/\mathrm{dof} = 15.1/13$

fit probability = 0.30

thinking about systematics

- stability:
  - if you repeat a measurement, will you get the same answer?
  - how uncertain is your calibration?
    - if you repeated it, would you get the same calibration
  - what happens if you take the same measurement on different days? do you get the same answer?

data rejection

- omitting data because it doesn't conform to your expectations isn't scientific
- if something looks off, try to understand why
  - what other things can you check?
  - are you getting results consistent with yesterday? is the data overall consistent?
  - can you go back to some control measurement where you know the answer?
- write everything down!
- if you need to omit data document why
- be aware of confirmation biases!

uncertainties

- uncertainties are inherent in all measurements
- it is typical in experimental physics that the majority of the time is spent on uncertainty analysis
- always question and think about your data
  - think of the questions you would ask if it was someone else's result
- use appropriate significant figures!
  - don't tell me you have measured $x = 3.948532 \pm 0.3$
  - $L = (1.979 \pm 0.012)$m or $L = (1.98 \pm 0.8)$m
    - the difference being if the first sign. digit of the uncertainty is small or large

summary

- many books written about uncertainty analysis
  - Bevington and Taylor are some of the most popular
- systematic uncertainties depend on the kind of measurement you are doing
- include in your report a discussion of how you evaluated your systematic uncertainties
- think critically about your data, but do not let your biases dictate which data you use
- write everything down so you know can know if there is something going on in your measurement