ILLINOIS

Quantum Entanglement, Beyond Quantum Mechanics, and Why Quantum Mechanics

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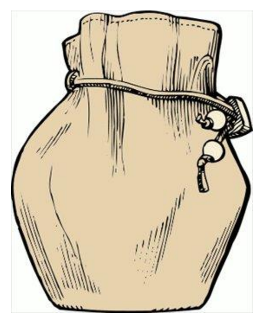


Physics 403 talk: July 28, 2014

Entanglement is a feature of compound quantum systems

- States that can be written $|\Psi\rangle_{AB} = |\varphi^1\rangle_A |\varphi^2\rangle_B$ are separable
- States that cannot be written this way are entangled

Example: the *Bell states* are inseparable

$$\left|\Phi^{\pm}\right\rangle = \frac{1}{\sqrt{2}}\left(|0\rangle\,|0\rangle \pm |1\rangle\,|1\rangle\right)$$

$$\left|\Psi^{\pm}\right\rangle = \frac{1}{\sqrt{2}}\left(|0\rangle\,|1\rangle \pm |1\rangle\,|0\rangle\right)$$

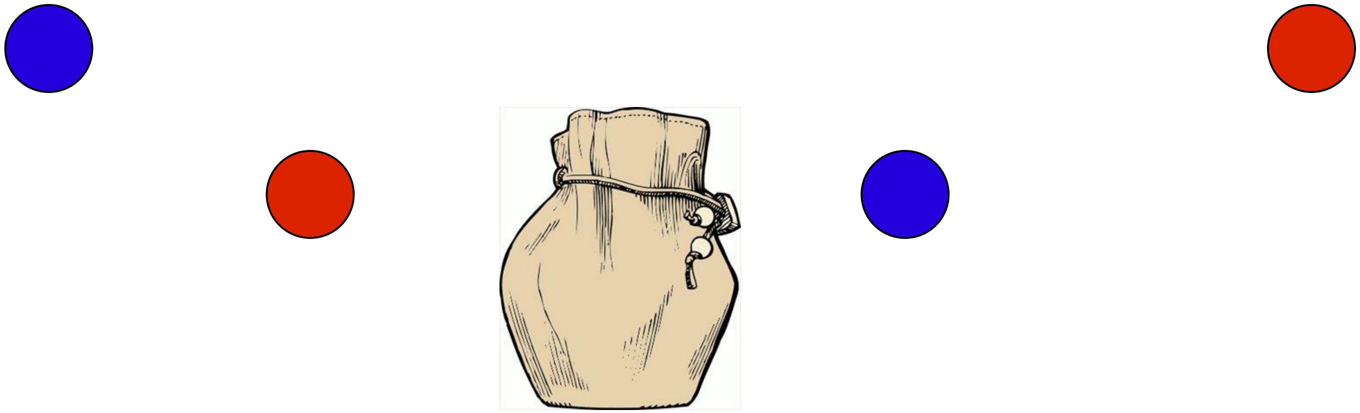
$$\begin{aligned}\left|\Phi'\right\rangle &= \left(\alpha\,|0\rangle + \beta\,|1\rangle\right)\left(\gamma\,|0\rangle + \delta\,|1\rangle\right)\\ &= \alpha\gamma\,|0\rangle\,|0\rangle + \alpha\delta\,|0\rangle\,|1\rangle + \beta\gamma\,|1\rangle\,|0\rangle + \beta\delta\,|1\rangle\,|1\rangle\end{aligned}$$

No solution!

Measurement outcomes are random and correlated

Classical “entanglement”?

- Classical things can be random and correlated, too…



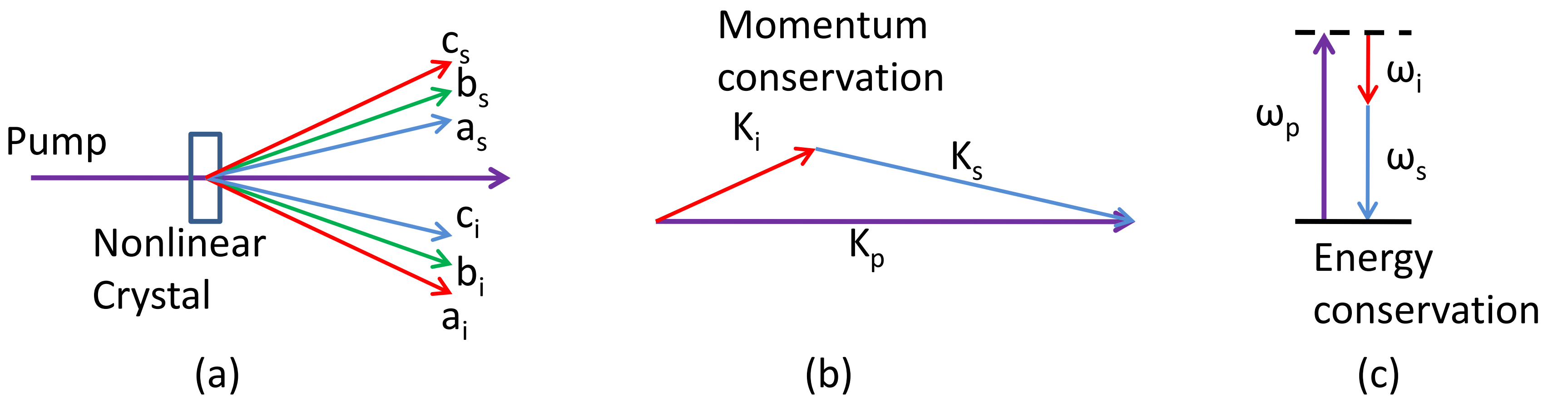
- … but not entangled!

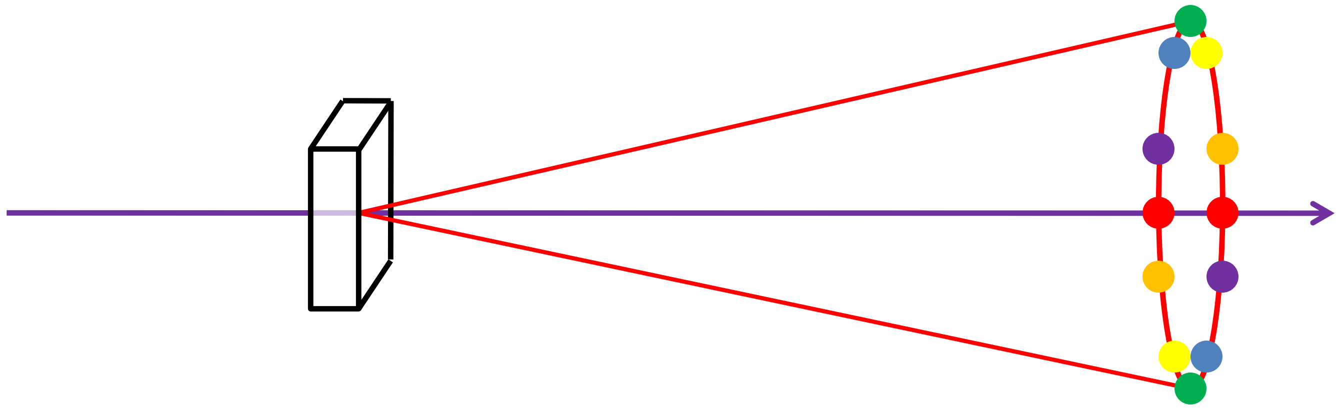
How is this different from an entangled state?

- Each marble has a defined color from the beginning (local hidden variable)
- The processes are <u>distinguishable</u> in principle
- There is no conjugate measurement basis

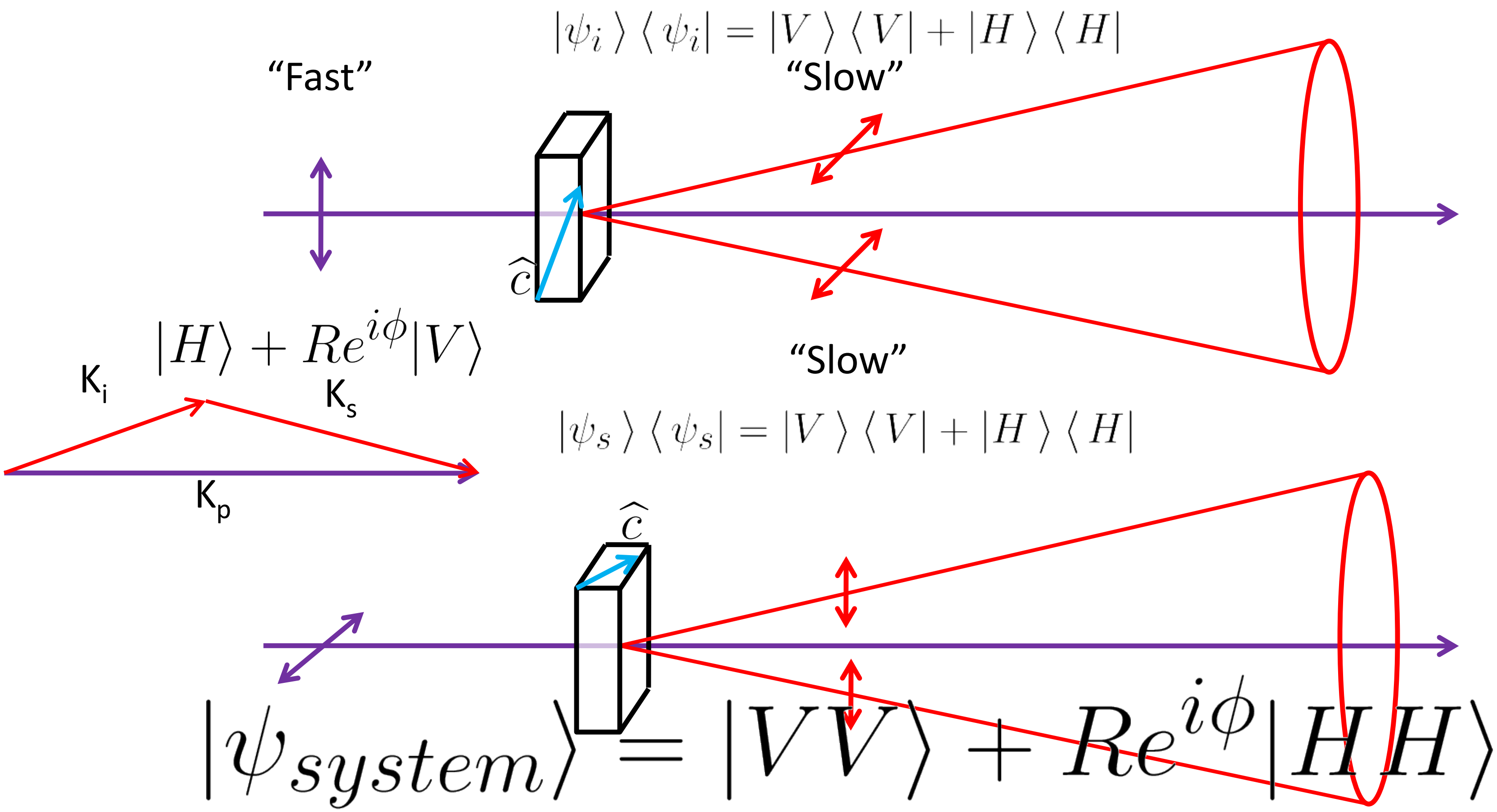
Entangled systems give
random and correlated measurement outcomes
<u>in every measurement basis!</u>

Downconversion



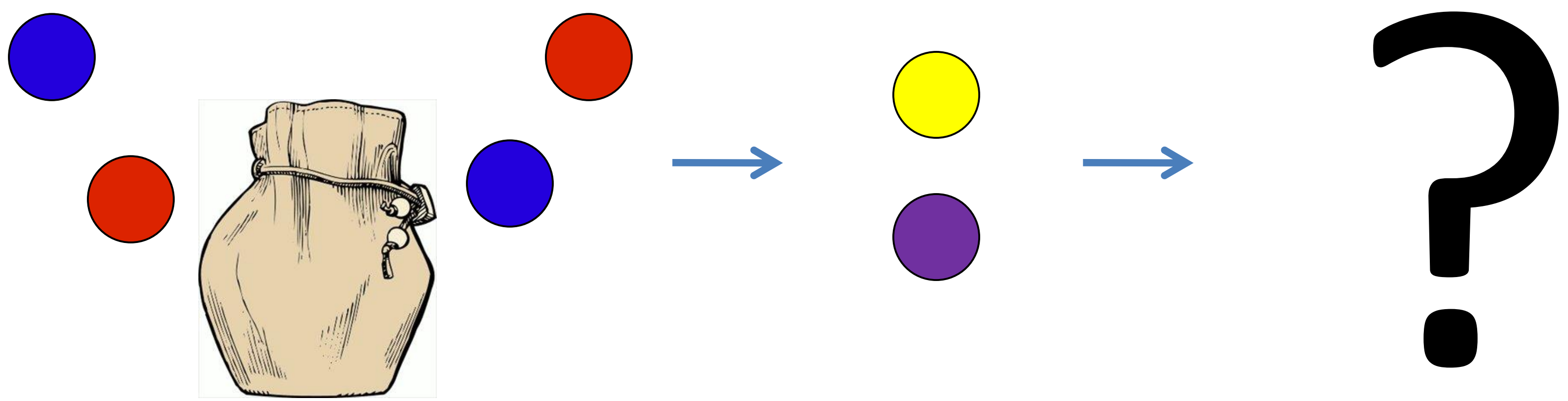


Polarization Entanglement

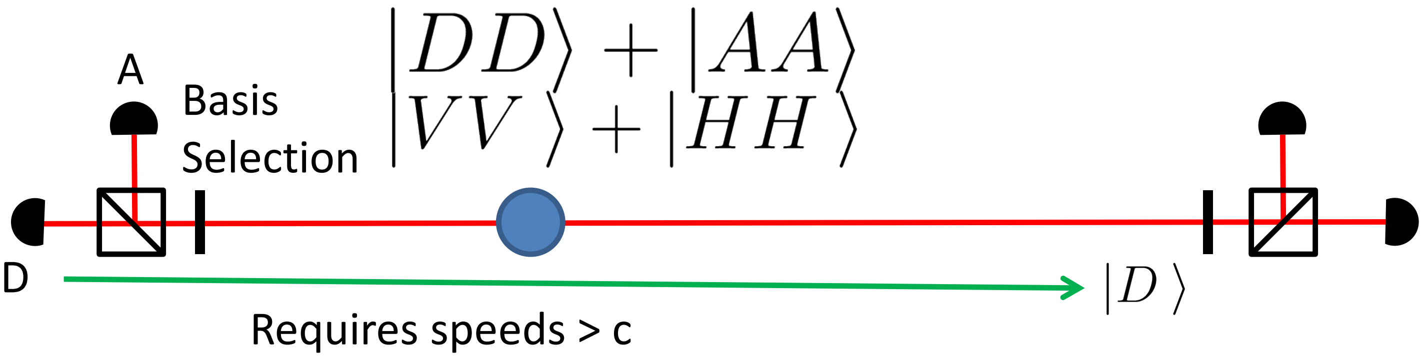


$$|\psi_{system}\rangle = |VV\rangle + Re^{i\phi}|HH\rangle$$

$$|HH\rangle + |VV\rangle \rightarrow \begin{matrix} |H\rangle = |D\rangle + |A\rangle \\ |V\rangle = |D\rangle - |A\rangle \end{matrix} \rightarrow |DD\rangle + |AA\rangle$$

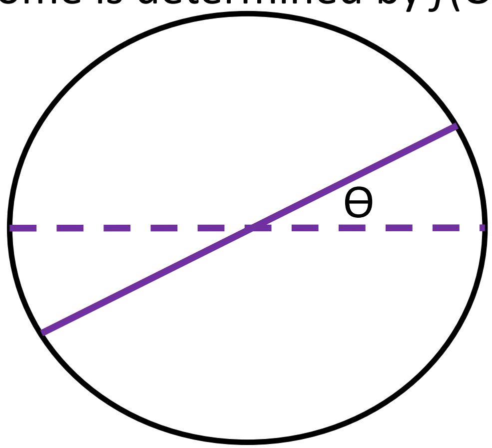


Hidden-Variables

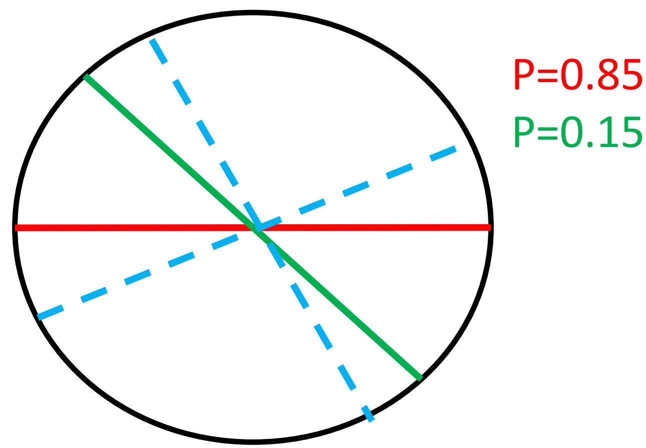


"If, without in any way disturbing a system, we can predict with certainty (i.e., with probability equal to unity) the value of a physical quantity, then there exists an element of physical reality corresponding to this physical quantity."

If measured in the Θ basis, then the outcome is determined by *f*(Θ) = {0,1}



Problem:



A. Einstein, B. Podolsky, and N. Rosen, Phys. Rev. **47,** 777 (1935).

Quantifying a “nonlocal resource”

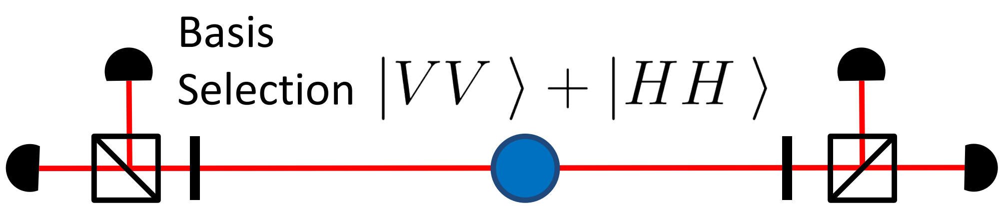
- Consider the CHSH Bell inequality:

  $$S = E(a,b) - E(a,b') + E(a',b) + E(a',b')$$

  - Classically, $S \leq 2$
  - Quantum mechanically, $S \leq 2\sqrt{2}$
  - Algebraically, $S \leq 4$
- What sort of theory could achieve the algebraic bound?
  - A theory only limited by causality!

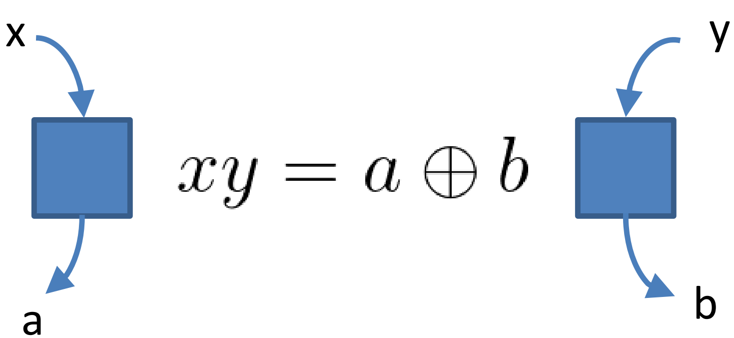
Quantifying a "nonlocal resource"

- For QM, S = $2\sqrt{2}$ by using maximally entangled particles



Correlated if the same basis is chosen, partially correlated else

- S = 4 is achieved using "PR Boxes"



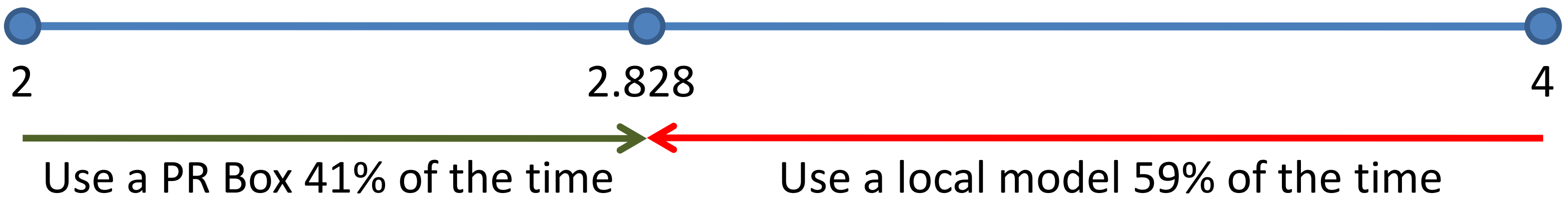
X	Y	a/b	E(x,y)
0	0	0/0 or 1/1	Correlated = 1
0	1	0/0 or 1/1	Correlated = 1
1	0	0/0 or 1/1	Correlated = 1
1	1	1/0 or 0/1	Anti-Correlated = -1

S = E(0,0) – E(1,1) + E(1,0) + E(0,1) = 4

S. Popescu, D. Rohrlich, Found. Phys. **24**, 379 (1994).

PR Box as a nonlocal resource

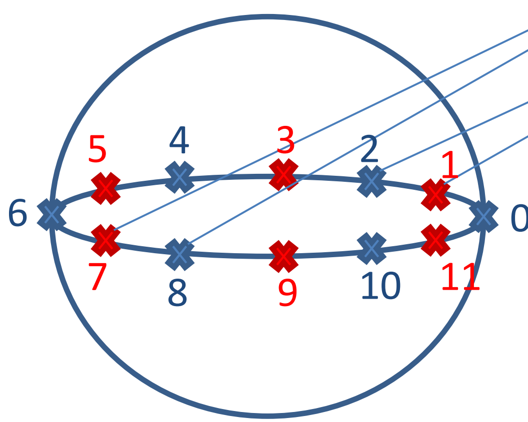
- A PR box could simulate the QM maximum CHSH value



- If a PR box is used, it is completely nonlocal and cannot be predicted
- If a local model is used, it is a classical result and can be predicted perfectly
- Beyond-QM theories could predict the outcomes of a CHSH Bell test with a 41%/2 + 59% = 79% probability
- Can we design a Bell inequality where nature would need to use a PR box 100% of the time?

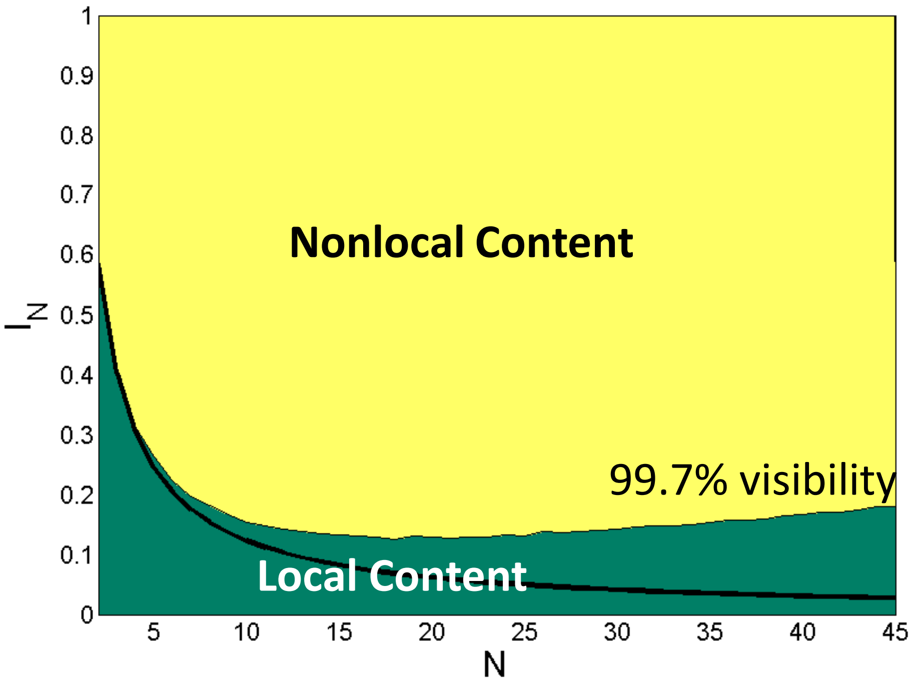
Chained Bell Inequality

$$I_N = \sum_{a,b} P(a=b|0,2N-1) + \sum_{|x-y|=1} \left(1 - \sum_{a,b} P(a=b|x,y)\right)$$



$I_\infty = 0$ (i.e., the maximal nonlocal value)

I_2 is the CHSH Bell inequality



$I_{18} = 0.126 \pm 0.001$

$I_{45} = 0.180 \pm 0.001$

Requires 360 measurements!

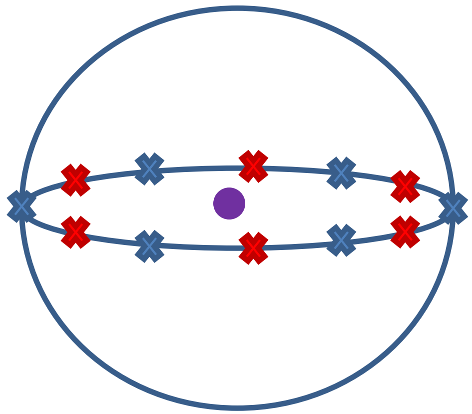
T. Stuart, J. Slater, R. Colbeck, R. Renner, W. Tittel, Phys. Rev. Lett. **109**, 020402 (2012).

Limiting the Predictive Power

$$\delta_N = \frac{\sum_{a,b} P(a=b|0,2N-1) + \sum_{|x-y|=1}(1-\sum_{a,b} P(a=b|x,y))}{2}$$

Bias term = 0.007

(Alice sees 0.5035|HH><HH|+ 4.965|VV><VV|)

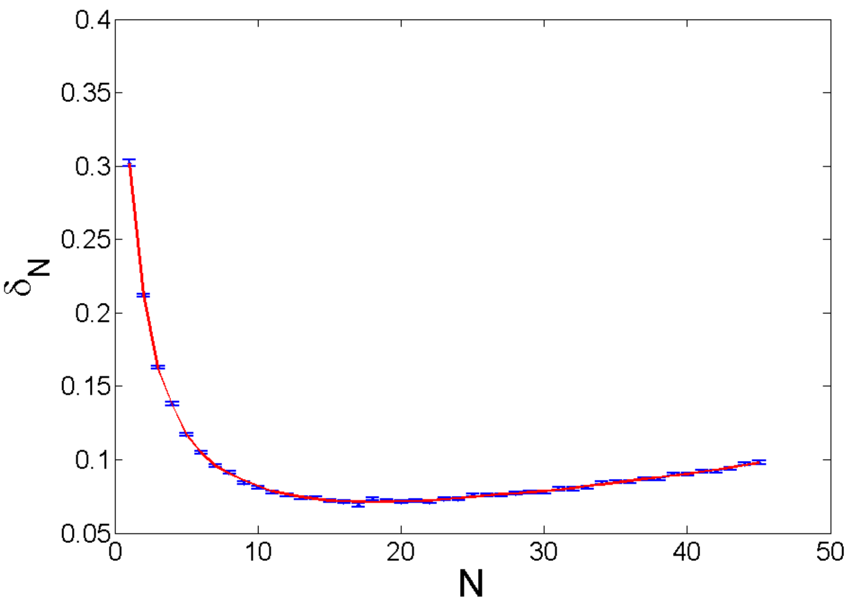


I_N = 0.126 → Predictive power = 0.126 + 0.874/2 + 0.007

Any further theory could only predict the results with 57% probability

δ_{18}=0.070±0.0005

δ_{45}=0.098±0.001



Quantum postulates aren't clean

Postulates of quantum mechanics

- Each physical system is associated with a separable complex Hilbert Space *H* with inner product. Rays (one-dimensional subspaces) in *H* are associated with states of the system.
- The Hilbert space of a composite system is the Hilbert space tensor product of the state spaces associated with the component systems.
- Physical symmetries act on the Hilbert space of quantum states unitarily or anti-unitarily due to Wigner's theorem.
- Physical observables are represented by Hermitian matrices on *H*. The expectation value of the observable *A* for the system in state represented by the unit vector $|\psi\rangle \in H$ is $\langle\psi|A|\psi\rangle$.

Postulates of special relativity

- The laws of physics are the same in all inertial frames of reference.
- The speed of light in free space has the same value *c* in all inertial frames of reference.

Can we re-derive quantum mechanics?

- Causality
- ???????

A1 to Q1 —1 bit→ Learns
A2 to Q2 A2 ***or*** A1

1 bit

1 bit

1 bit

- Currently we can only do a maximum bound
  - Quantum Mechanics has <u>Information causality</u>
    - Answering multiple one bit questions with one bit (but only one question can be found the answer to).
  - Quantum Mechanics has <u>Communication redundancy</u>
    - One bit answers only require one bit input