

# Entanglement



*“About your cat, Mr. Schrödinger—I have  
good news and bad news.”*

Virginia O. Lorenz, Paul Kwiat, Brad Christensen



PHYS403 Summer 2020

illinois.edu

# Entanglement

- A quantum object can be in a superposition of two states
- The cat is a quantum object
- It can be awake and asleep
- If we check, it will be in only one of the states



- If we have two objects, we can entangle the states such that knowing about one object affects the other



# Entanglement

- An Entangled State:

$$\left[ \left( \begin{array}{c} \text{red double arrow} \\ |V\rangle \end{array} \begin{array}{c} \text{red double arrow} \\ |V\rangle \end{array} \right) + \left( \begin{array}{c} \text{blue double arrow} \\ |H\rangle \end{array} \begin{array}{c} \text{blue double arrow} \\ |H\rangle \end{array} \right) \right]$$

- If I measure one object, it will end up in just one state, causing the other object to also be in just one state
- E.g. photons whose polarizations are entangled:  $\begin{array}{c} \text{red double arrow} \\ |V\rangle \end{array} \begin{array}{c} \text{red double arrow} \\ |V\rangle \end{array} + \begin{array}{c} \text{blue double arrow} \\ |H\rangle \end{array} \begin{array}{c} \text{blue double arrow} \\ |H\rangle \end{array}$   
 $|VV\rangle + |HH\rangle$



# Properties of Entanglement

*at least*  
“It takes <sup>V</sup>two to tangle.”  
J. Eberly, 2015

$$\psi_{pair} \propto |HH\rangle + |VV\rangle \quad \text{Entangled}$$

1935: Entanglement is  
“*the* characteristic trait of quantum mechanics,  
the one that enforces its entire departure from  
classical lines of thought”

—E. Schrödinger

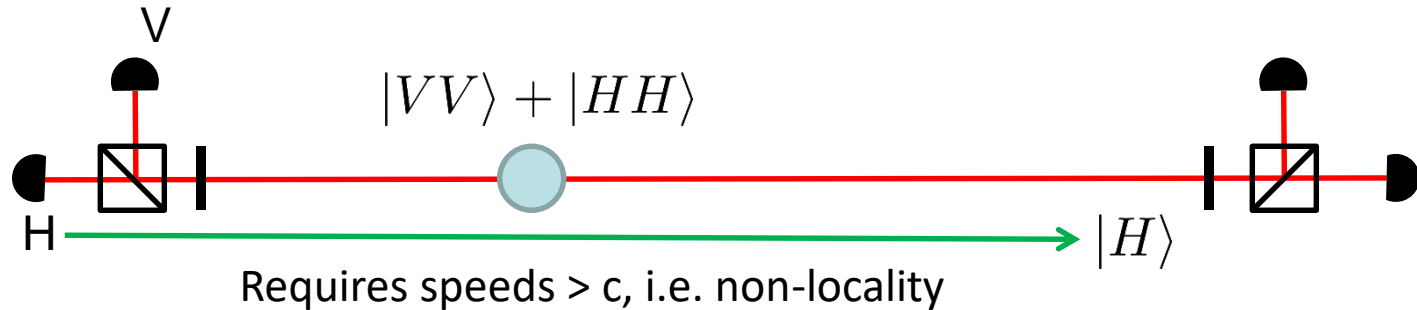
$$\psi_{12} = \psi_1\psi_2 \propto |HH\rangle + |VV\rangle + |HV\rangle + |VH\rangle \quad \text{Not Entangled}$$

In an **entangled** state, neither particle has definite properties alone.

⇒ All the information is stored in the *joint* properties.



# 1935: Einstein, Podolsky, Rosen (EPR) Paradox



*spooky*

EPR: Action at a distance (non-locality) is spooky.

Is Quantum Mechanics wrong?

Maybe correlations are due to some local element of reality ("local hidden variable" model)?

A. Einstein, B. Podolsky, and N. Rosen, Phys. Rev. **47**, 777 (1935).



# 1964: Bell's theorem

- Bell's theorem shows Quantum Mechanics gives different statistical predictions than any local realistic model
  - Certain inequalities are violated if non-local correlations exist, tested by measuring statistical correlations between spatially separated entangled systems

*“If [a hidden variable theory] is local it will not agree with quantum mechanics, and if it agrees with quantum mechanics it will not be local.”*

*– John Bell, 1975*

J.S. Bell, Physics **1**, 195-200 (1964)





## Strong Loophole-Free Test of Local Realism\*

Lynden K. Shalm,<sup>1,†</sup> Evan Meyer-Scott,<sup>2</sup> Bradley G. Christensen,<sup>3</sup> Peter Bierhorst,<sup>1</sup> Michael A. Wayne,<sup>3,4</sup> Martin J. Stevens,<sup>1</sup> Thomas Gerrits,<sup>1</sup> Scott Glancy,<sup>1</sup> Deny R. Hamel,<sup>5</sup> Michael S. Allman,<sup>1</sup> Kevin J. Coakley,<sup>1</sup> Shellee D. Dyer,<sup>1</sup> Carson Hodge,<sup>1</sup> Adriana E. Lita,<sup>1</sup> Varun B. Verma,<sup>1</sup> Camilla Lambrocco,<sup>1</sup> Edward Tortorici,<sup>1</sup> Alan L. Migdall,<sup>4,6</sup> Yanbao Zhang,<sup>2</sup> Daniel R. Kumor,<sup>3</sup> William H. Farr,<sup>7</sup> Francesco Marsili,<sup>7</sup> Matthew D. Shaw,<sup>7</sup> Jeffrey A. Stern,<sup>7</sup> Carlos Abellán,<sup>8</sup> Waldimar Amaya,<sup>8</sup> Valerio Pruneri,<sup>8,9</sup> Thomas Jennewein,<sup>2,10</sup> Morgan W. Mitchell,<sup>8,9</sup> Paul G. Kwiat,<sup>3</sup> Joshua C. Bienfang,<sup>4,6</sup> Richard P. Mirin,<sup>1</sup> Emanuel Knill,<sup>1</sup> and Sae Woo Nam<sup>1,‡</sup>

<sup>1</sup>*National Institute of Standards and Technology, 325 Broadway, Boulder, Colorado 80305, USA*

<sup>2</sup>*Institute for Quantum Computing and Department of Physics and Astronomy, University of Waterloo, 200 University Avenue West, Waterloo, Ontario, Canada, N2L 3G1*

<sup>3</sup>*Department of Physics, University of Illinois at Urbana-Champaign, Urbana, Illinois 61801, USA*

<sup>4</sup>*National Institute of Standards and Technology, 100 Bureau Drive, Gaithersburg, Maryland 20899, USA*

<sup>5</sup>*Département de Physique et d'Astronomie, Université de Moncton, Moncton, New Brunswick E1A 3E9, Canada*

<sup>6</sup>*Joint Quantum Institute, National Institute of Standards and Technology and University of Maryland, 100 Bureau Drive, Gaithersburg, Maryland 20899, USA*

<sup>7</sup>*Jet Propulsion Laboratory, California Institute of Technology, 4800 Oak Grove Drive, Pasadena, California 91109, USA*

<sup>8</sup>*ICFO-Institut de Ciències Fòniques, The Barcelona Institute of Science and Technology, 08860 Castelldefels (Barcelona), Spain*

<sup>9</sup>*ICREA-Institució Catalana de Recerca i Estudis Avançats, 08015 Barcelona, Spain*

<sup>10</sup>*Quantum Information Science Program, Canadian Institute for Advanced Research, Toronto, Ontario, Canada*

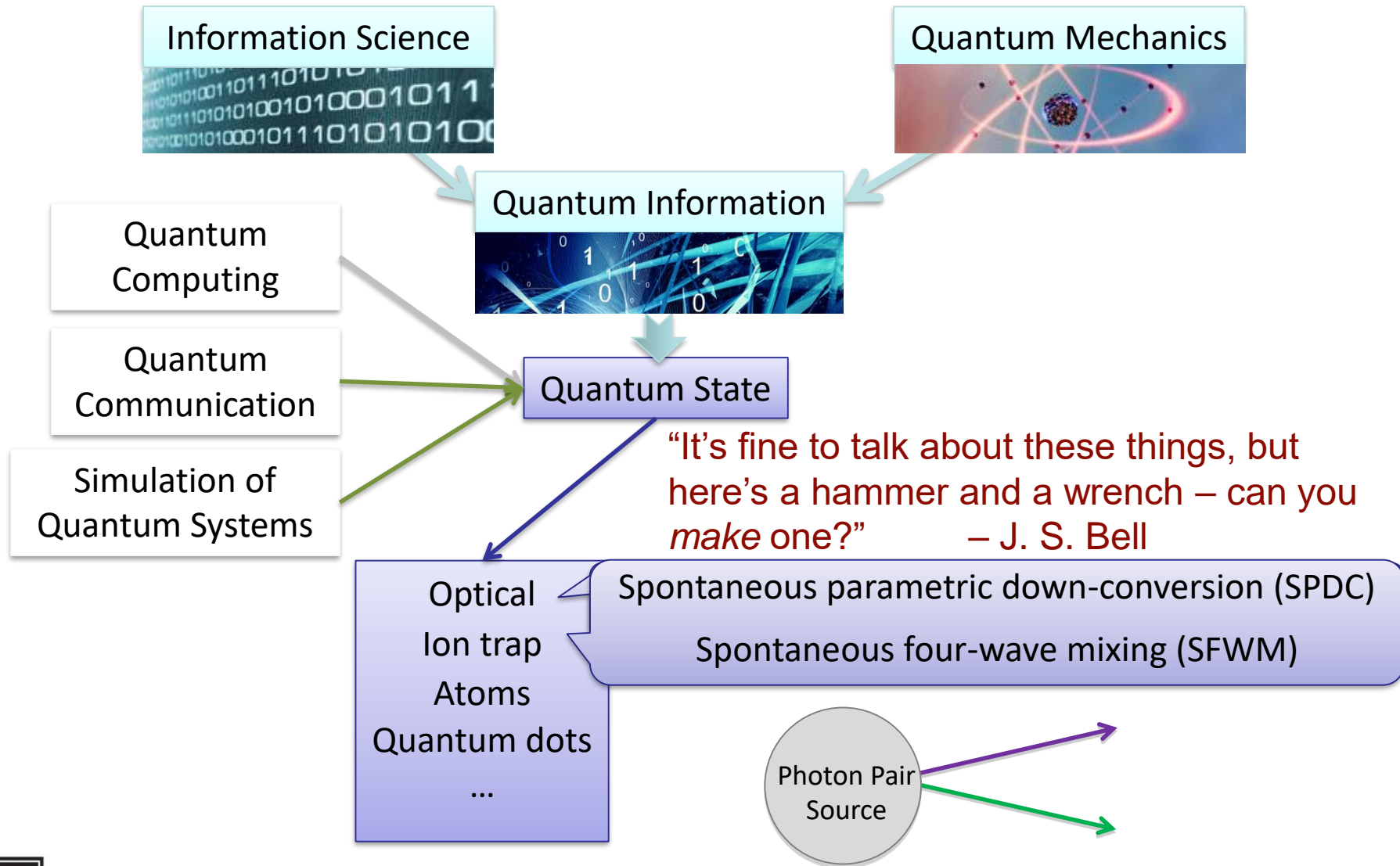
(Received 10 November 2015; published 16 December 2015)

We present a loophole-free violation of local realism using entangled photon pairs. We ensure that all relevant events in our Bell test are spacelike separated by placing the parties far enough apart and by using fast random number generators and high-speed polarization measurements. A high-quality polarization-entangled source of photons, combined with high-efficiency, low-noise, single-photon detectors, allows us to make measurements without requiring any fair-sampling assumptions. Using a hypothesis test, we compute  $p$  values as small as  $5.9 \times 10^{-9}$  for our Bell violation while maintaining the spacelike separation of our events. We estimate the degree to which a local realistic system could predict our measurement choices. Accounting for this predictability, our smallest adjusted  $p$  value is  $2.3 \times 10^{-7}$ . We therefore reject the hypothesis that local realism governs our experiment.





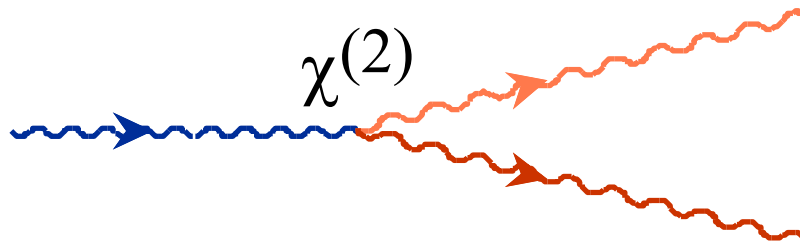
# The last 50 years: Quantum Information





# 1970: Spontaneous Parametric Down-Conversion

- Burnham & Weinberg, PRL **25**, 84 (1970):



A diagram illustrating the SPDC process. A blue wavy line with an arrow pointing right represents the pump beam. It enters a region labeled  $\chi^{(2)}$ . From this region, two orange wavy lines with arrows pointing away from each other represent the signal and idler beams.

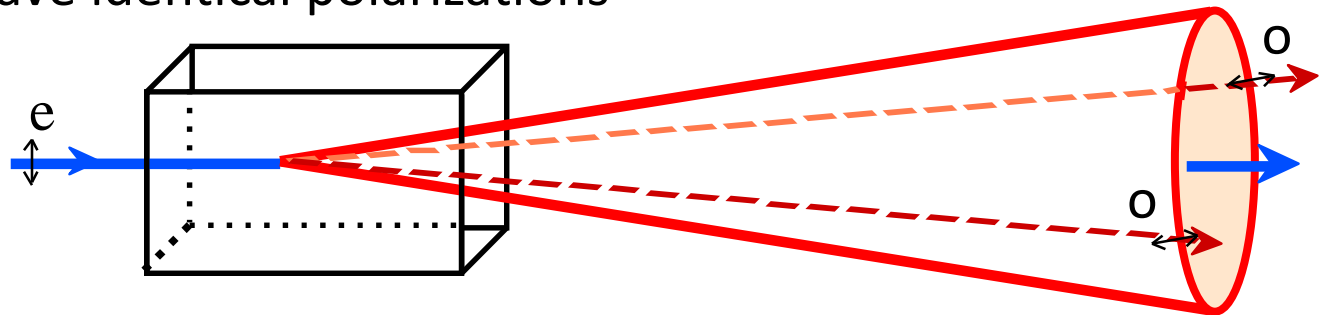
$$\omega_p = \omega_s + \omega_i^*$$
$$\vec{k}_p = \vec{k}_s + \vec{k}_i^\dagger$$

\*Energy conservation  $\rightarrow$  energy entanglement

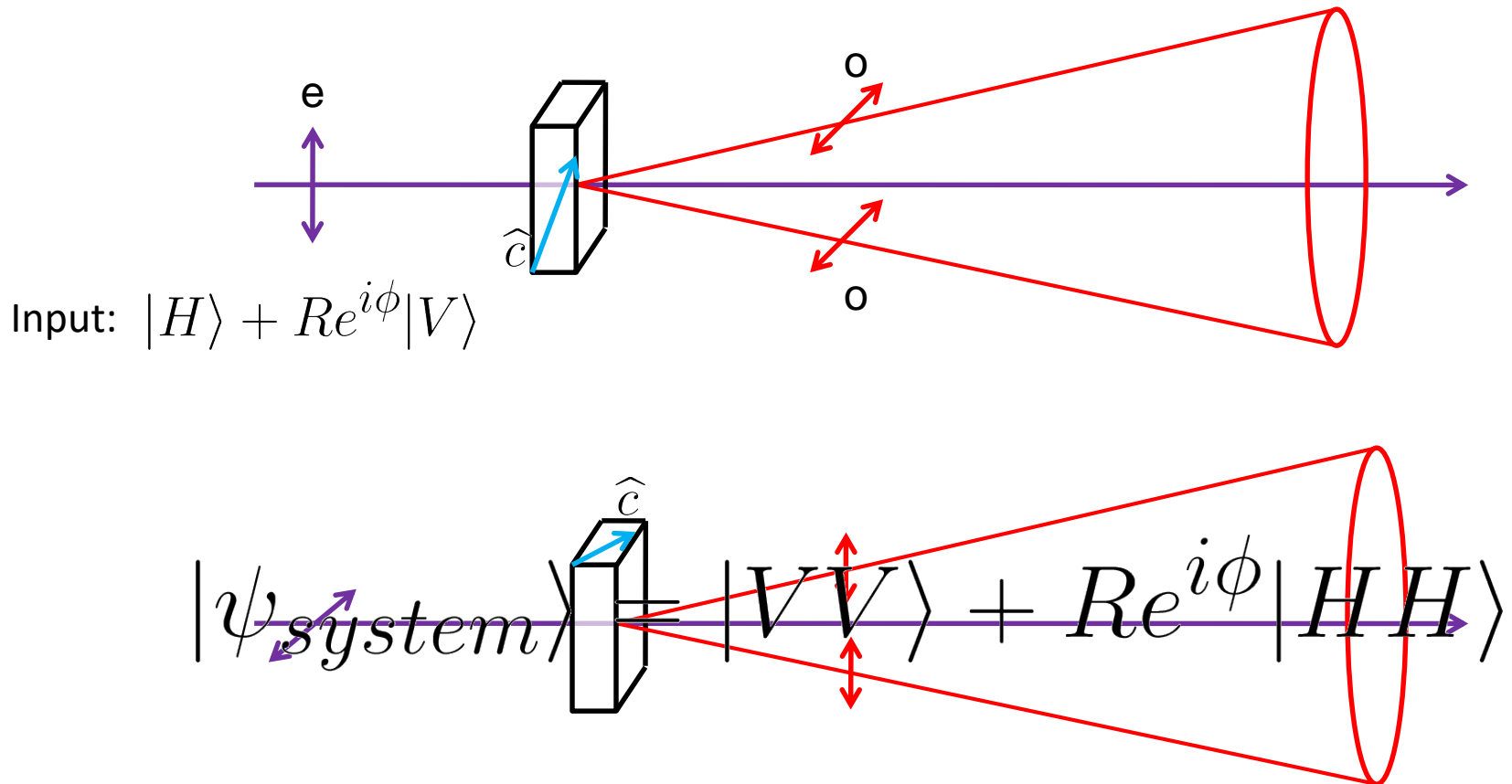
†Momentum conservation  $\rightarrow$  momentum entanglement

## Type-I phase-matching

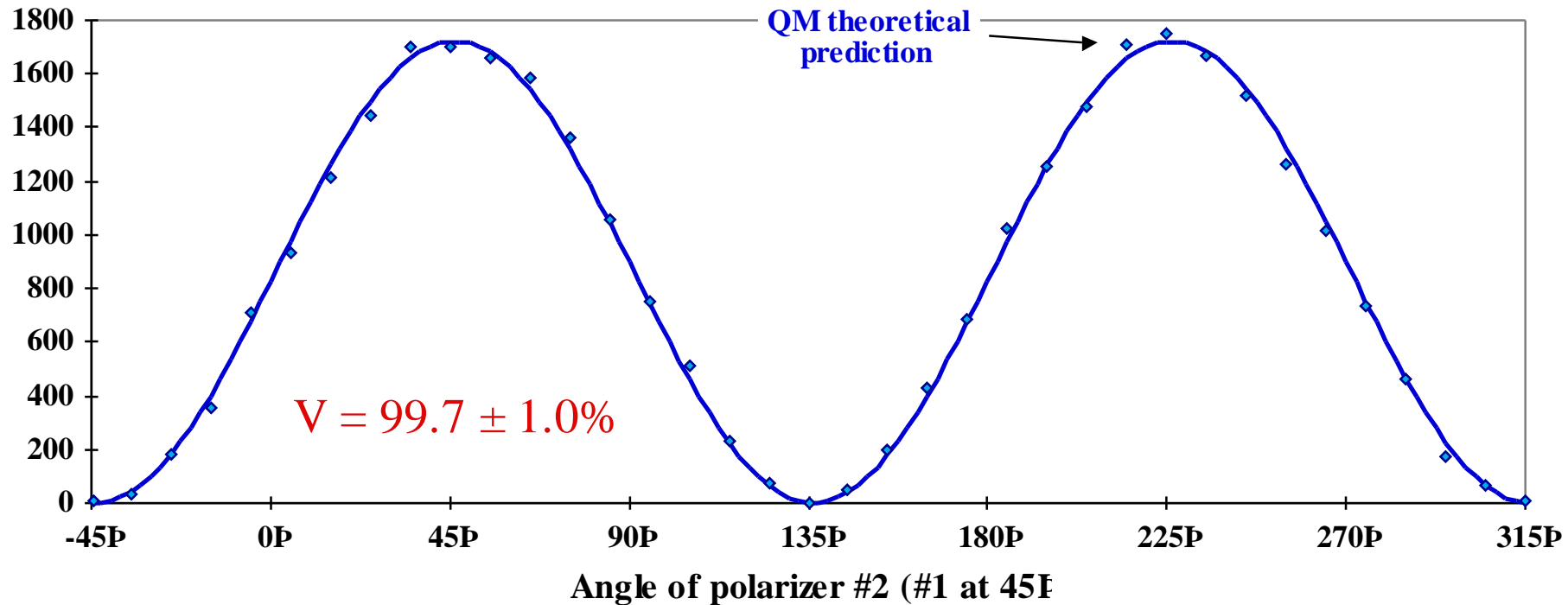
Photons have identical polarizations



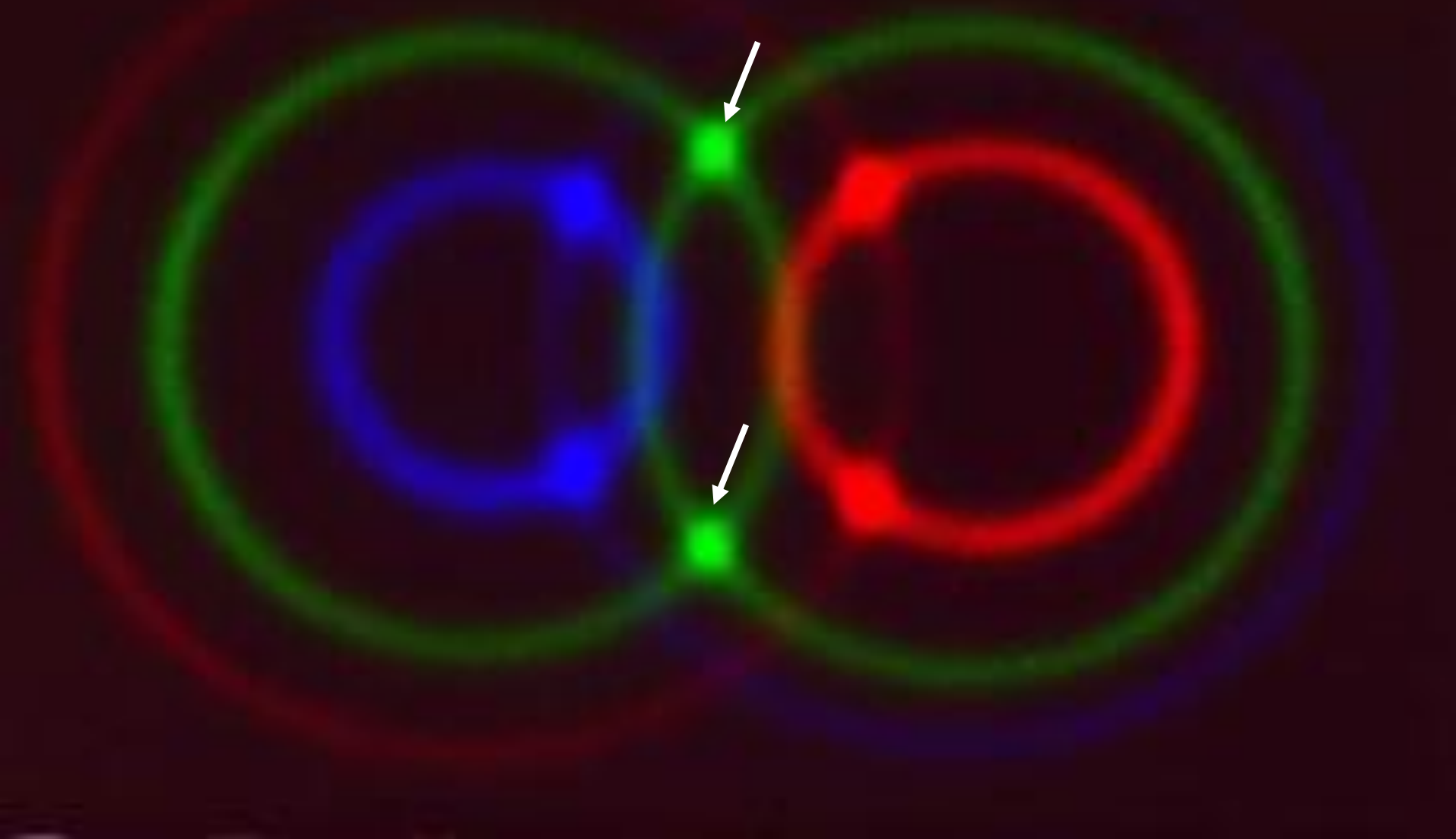
# Polarization Entanglement



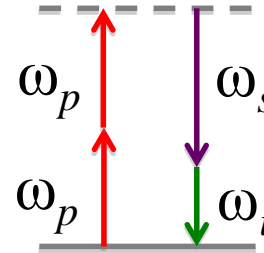
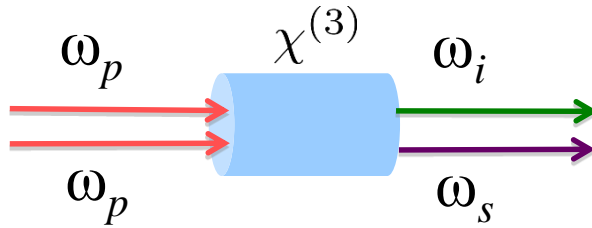
# Proof of Quantum Correlations



$$|\psi_{system}\rangle = |VV\rangle + Re^{i\phi}|HH\rangle$$



# Spontaneous four-wave mixing



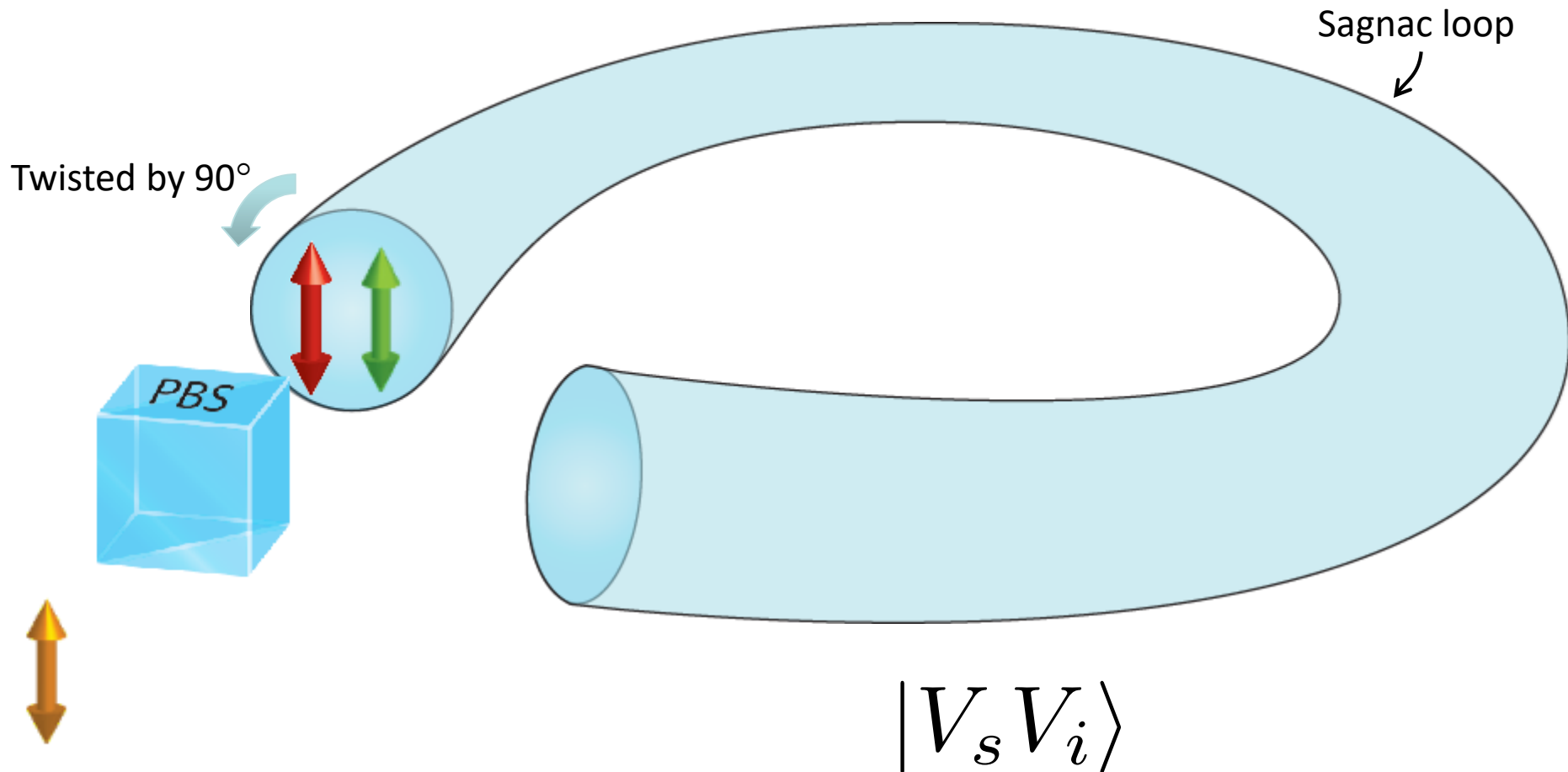
Conservation of energy

- Spontaneous four-wave mixing in polarization-maintaining optical fiber:



- *Birefringent* phase-matching:  $\Delta k = 2k(\omega_p) - k(\omega_s) - k(\omega_i) + 2\Delta n \frac{\omega_p}{c} = 0$

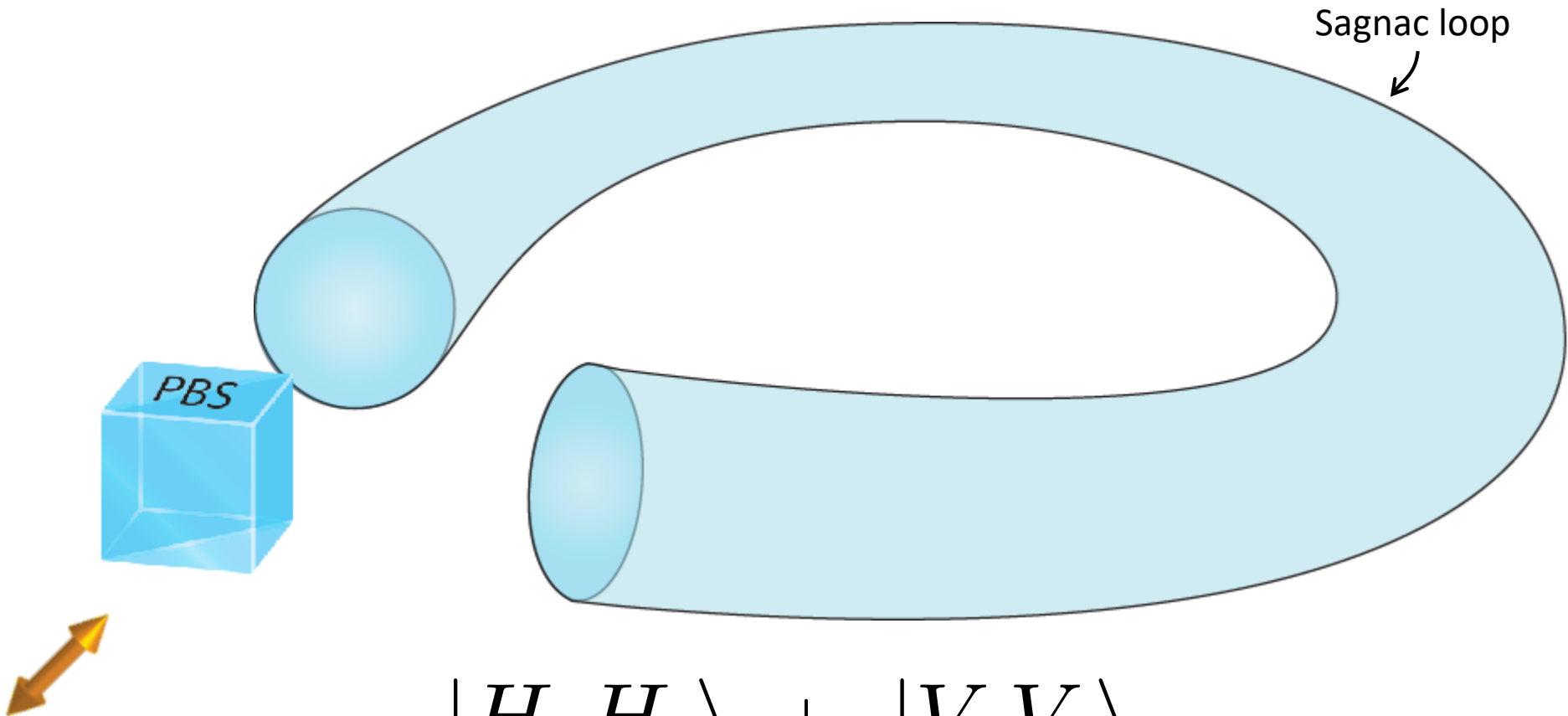
# Generation of polarization entanglement



Pump travels on slow axis. Signal and idler travel on fast axis.  
One end of the fiber is twisted by 90° relative to the other end.



# Generation of polarization entanglement



$$|H_s H_i\rangle + |V_s V_i\rangle$$

Pump travels on slow axis. Signal and idler travel on fast axis.  
One end of the fiber is twisted by 90° relative to the other end.

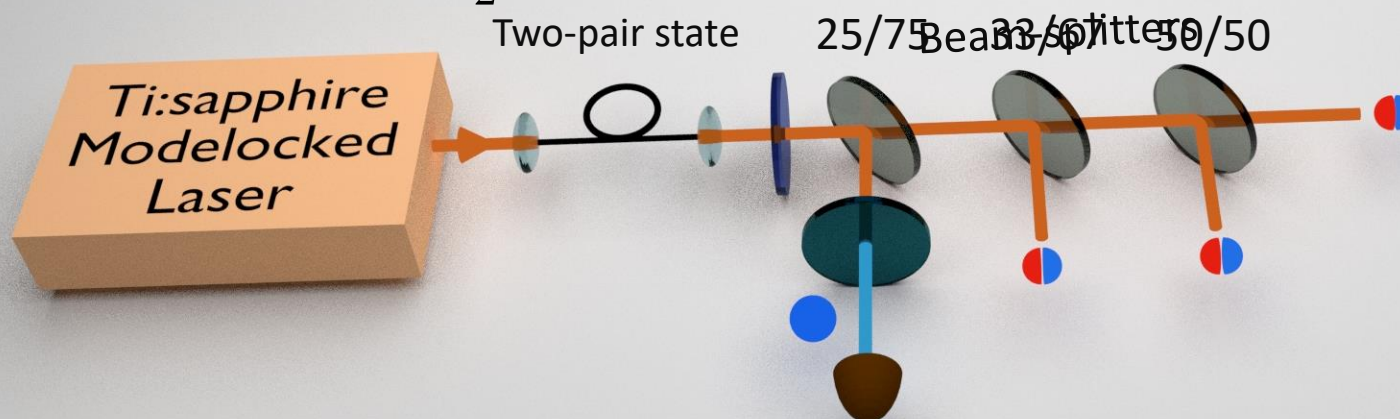




# Three-photon discrete-energy-entangled W-state

$$|W\rangle = \frac{1}{\sqrt{3}} (|RRB\rangle + |RBR\rangle + |BRR\rangle)$$

$$|\psi\rangle = \frac{\beta^2}{2} (a_{\omega_B}^\dagger a_{\omega_R}^\dagger)^2 |0\rangle$$



$$|W\rangle = \frac{1}{\sqrt{3}} (|RRB\rangle + |RBR\rangle + |BRR\rangle)$$

- Test non-locality of quantum mechanics
- Quantum communication protocols
- Robust against loss & decoherence



# Why are entangled states important?

- Responsible for quantum measurements and decoherence
- Central to demonstrations of quantum nonlocality (e.g., Bell's inequalities, GHZ, Hardy, etc.)
- **Quantum cryptography** – separated particles' correlations allow sharing of secret random key
- **Quantum teleportation** – transmit unknown quantum state via 2 classical bits + EPR pair
- **Quantum computation** – intermediate states are all complex entangled states



# Classical Cryptography

USA TODAY:



**RSA Algorithm (1978):** Generate random prime numbers  $p$  &  $q$ .  
Compute  $n = pq$ ,  $\phi(n) = (p-1)(q-1)$ ,  $e$  co-prime with  $\phi$ ,  $d = e^{-1} \bmod \phi(n)$   
Release  $e$ ,  $n$  as public key. Encrypt:  $c = \text{message}^e \bmod n$   
Keep  $d$  as private key. Decrypt:  $\text{message} = c^d \bmod n$

9AFGJI4JT09RKSP

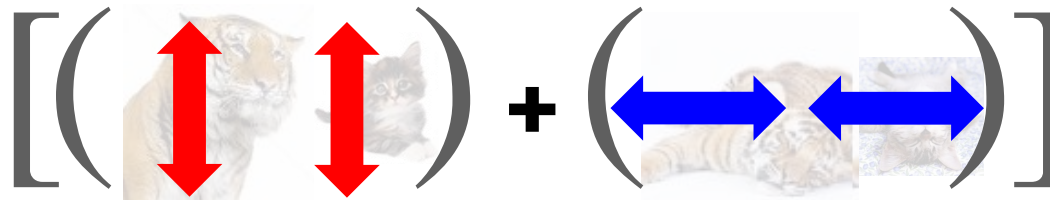
**From:** UIUC  
**Sent:** Friday, March 1, 2014 11:40 AM  
**To:** 'Virginia Lorenz'  
**Subject:** Physics

Hi Virginia,  
...

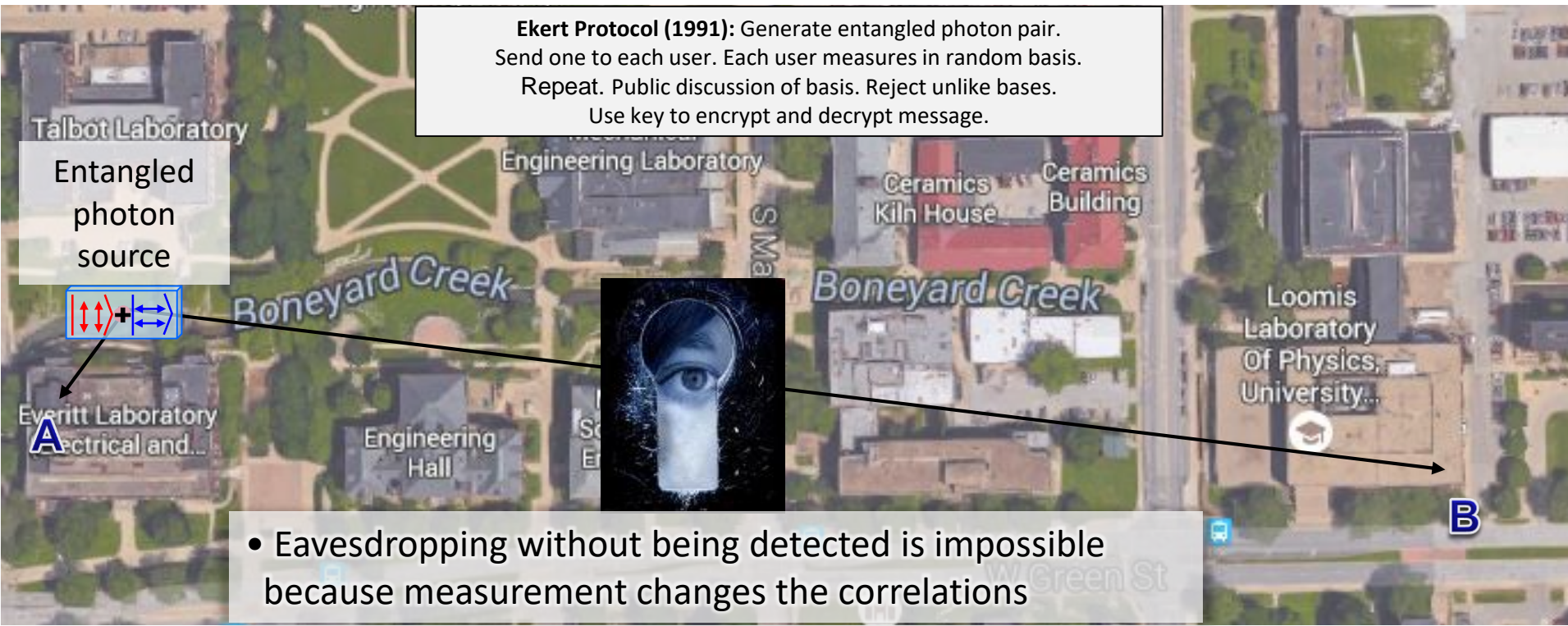
Security relies on computational difficulty of factoring the public key



# Quantum Key Distribution



Security is guaranteed by the laws of quantum physics!



# Entanglement, and the scaling that results, is the key to the power of quantum computing

- Classically, information is stored in a bit register:

- A 3-bit register can store **one** number, from 0-7



- Quantum Mechanically, a register of 3 qubits can store all of these numbers in superposition:

$$|000\rangle + |001\rangle + |010\rangle + |011\rangle + |100\rangle + |101\rangle + |110\rangle + |111\rangle = |0\rangle + |1\rangle + \dots |7\rangle$$

**Result:**

- Classical: one N-bit number
- Quantum:  $2^N$  (all possible) N-bit numbers
  - N.B. A 300-qubit register can simultaneously store more combinations than there are particles in the universe.

- Acting on the qubits simultaneously affects all the numbers:

$$(|0\rangle + |1\rangle + \dots |7\rangle) \otimes |f(x)\rangle \Rightarrow |0\rangle |f(0)\rangle + |1\rangle |f(1)\rangle + \dots |7\rangle |f(7)\rangle$$

- Some important problems benefit from this entanglement, enabling solutions of otherwise insoluble problems.



# Quantum Logic

## Controlled-Not Gate:

$$|0\rangle_c |0\rangle_t \rightarrow |0\rangle_c |0\rangle_t$$

$$|0\rangle_c |1\rangle_t \rightarrow |0\rangle_c |1\rangle_t$$

$$|1\rangle_c |0\rangle_t \rightarrow |1\rangle_c |1\rangle_t$$

$$|1\rangle_c |1\rangle_t \rightarrow |1\rangle_c |0\rangle_t$$

$$(|0\rangle_c + |1\rangle_c) |0\rangle_t \xrightarrow{CNOT} |0\rangle_c |0\rangle_t + |1\rangle_c |1\rangle_t$$

2-Qubit interactions lead to entangled states.

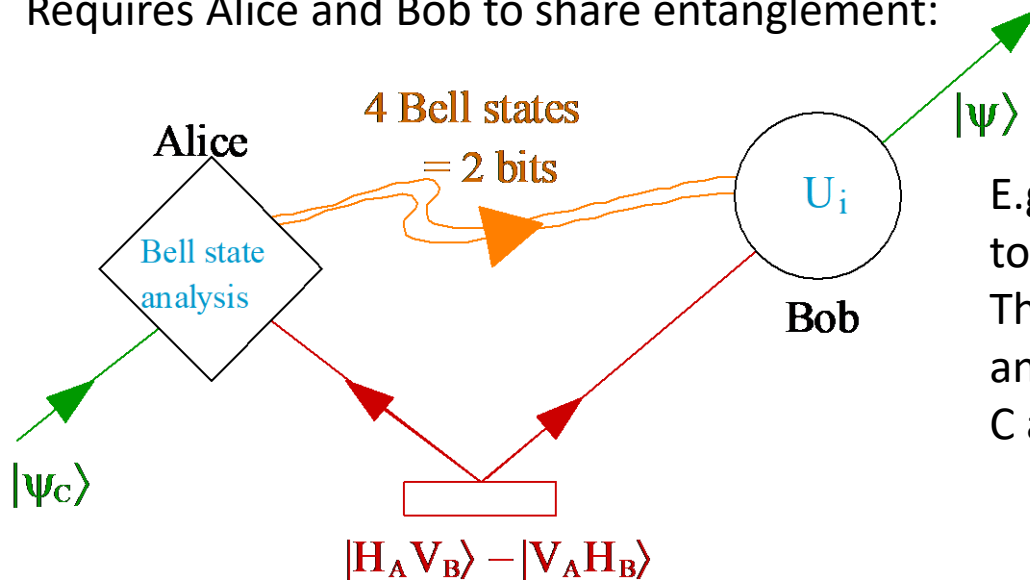


# Quantum Teleportation

Bennett et al., PRL **70**, 1895 (1993)

The basic idea: transfer the (infinite) amount of information in a qubit from Alice to Bob without sending the qubit itself.

Requires Alice and Bob to share entanglement:



E.g. Alice measures photons C and A to be in a singlet state.  
Then since C and A are perpendicular, and since A and B are perpendicular, C and B must be identical!

Remarks:

- The original state is gone.
- Neither Alice nor Bob know what it was.
- Requires classical communication – no superluminal signaling.
- Bell state analysis is hard.

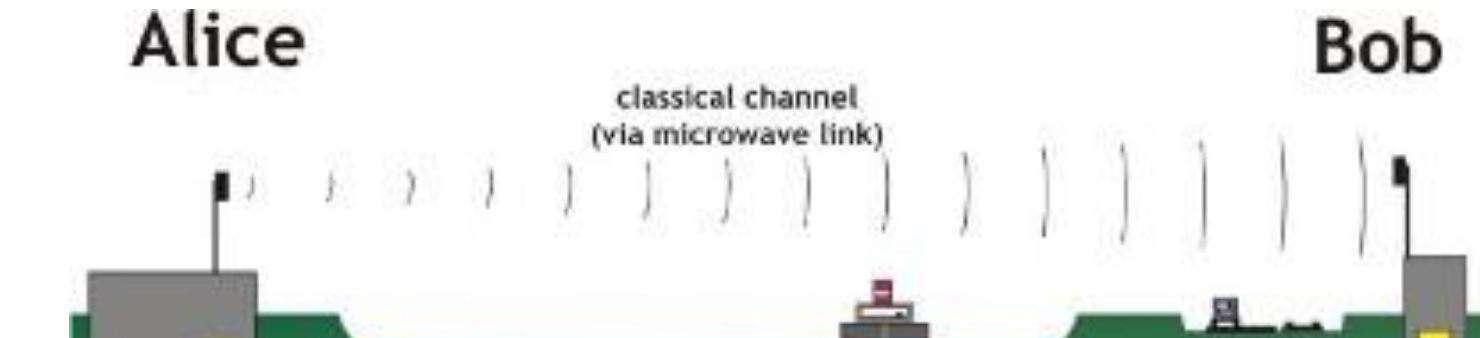




# Experimental Teleportation

1997: **First demonstration** [Bouwmeester *et al.*, Nature **390**, 575 (1997)]

2004: **Quantum teleportation across the Danube** [Ursin *et al.*, Nature **430**, 849 (2004)]



Cerberis QKD Server



Cerberis from IDQ is a standalone rack-mountable QKD server; providing secure quantum keys based on the BB84 and SARG protocols. Integrated with IDQ's Centauris Ethernet and Fiber Channel encryptors, Cerberis has been deployed by governments, enterprises and financial institutions since 2007.

<http://www.idquantique.com/quantum-safe-crypto/>

- Now demonstrated teleportation of entanglement, other degrees of freedom, continuous variables, energy states of ions, 2-qubits ...



# Satellite-to-ground QKD

**nature**  
International journal of science

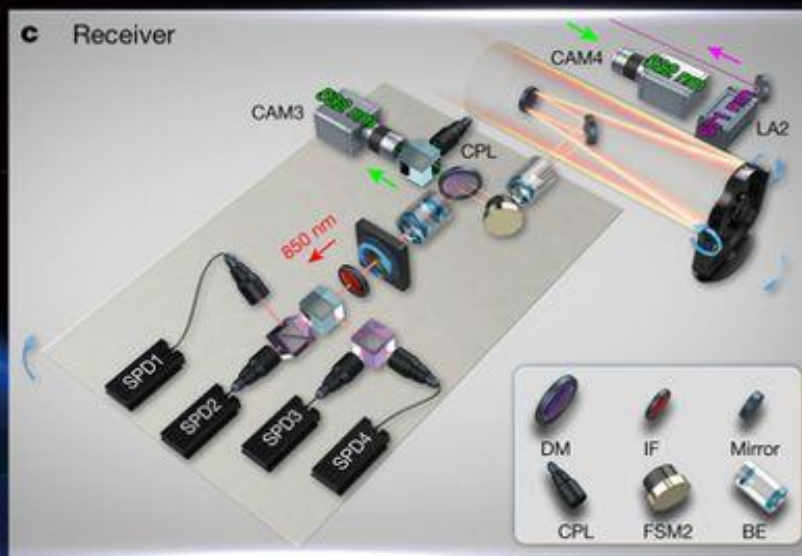
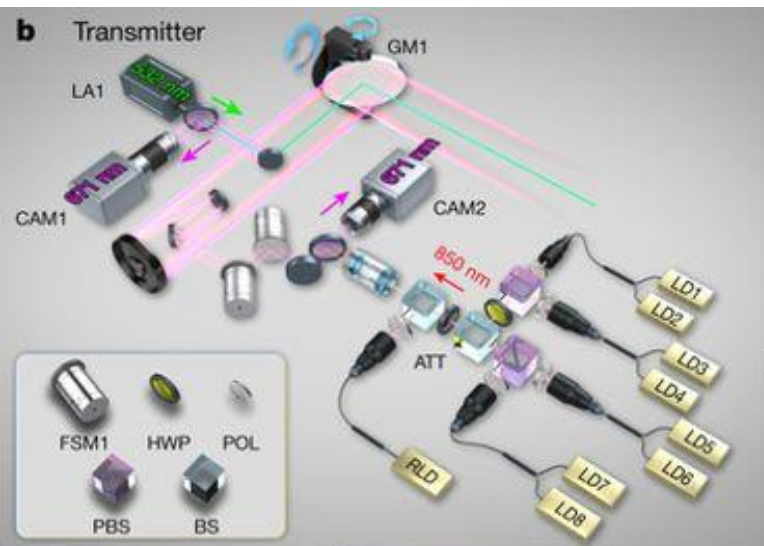
Article | Published: 09 August 2017

## Satellite-to-ground quantum key distribution

Sheng-Kai Liao, Wen-Qi Cai [...] Jian-Wei Pan

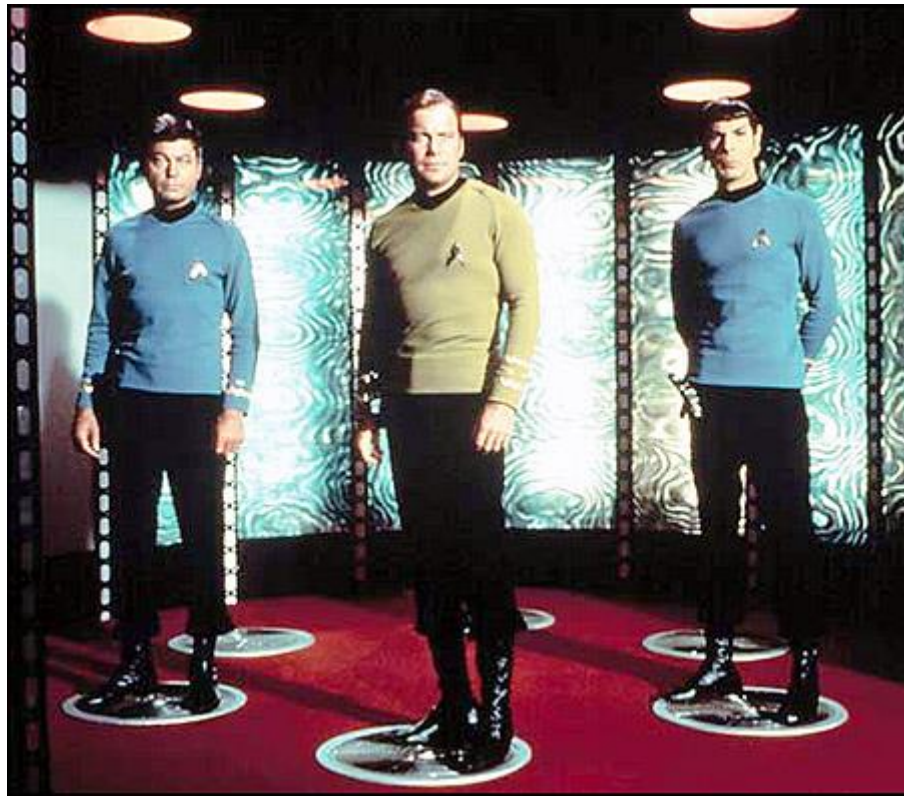
Nature 549, 43–47 (07 September 2017) | Download Citation

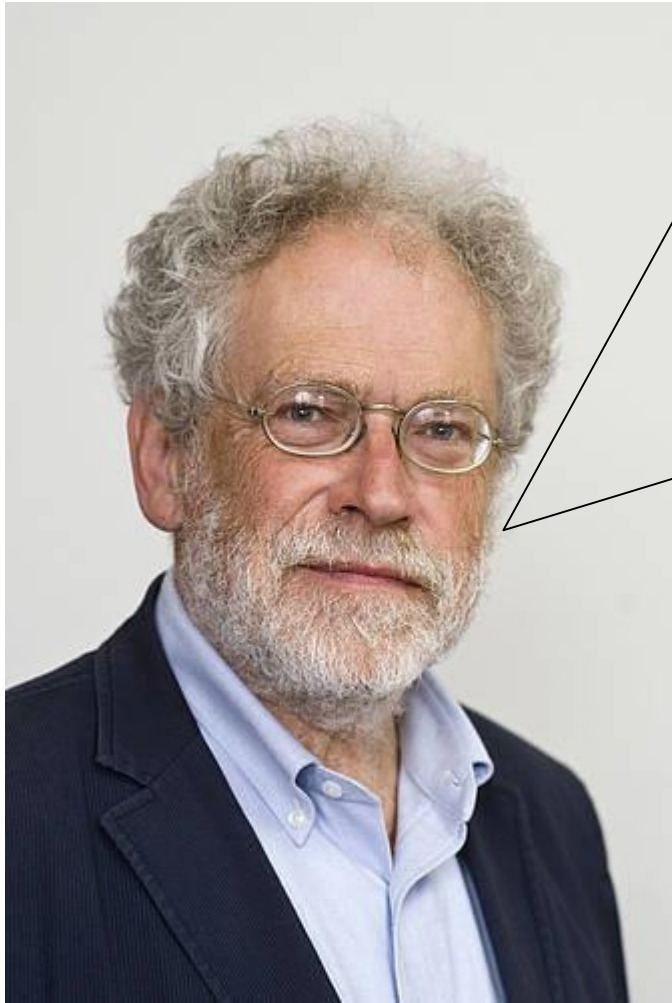
1,200 kilometers



But there's Quantum Teleportation, and then there's

# *Quantum Teleportation*





Anton Zeilinger

Yes, but there are two major differences. Firstly, we **transfer properties**, not matter. And secondly, until now we have had more success with light particles and occasionally with atoms, not with larger objects.

And even if it was possible, the problems involved would be huge. Firstly: for physical reasons, the original has to be **completely isolated** from its environment for the transfer to work. There has to be a total vacuum for it to work. And it is a well-known fact that this is **not particularly healthy** for human beings. Secondly, you would take all the properties from a person and transfer them onto another. This means producing a being who no longer has any hair colour, no eye colour, nix. A man without qualities! This is not only unethical – it's so crazy that it's impossible to imagine.







Anton Zeilinger

The atoms in a human being are the equivalent to the information mass of about a **thousand billion billion billion bits**. Even with today's top technology, this means it would take about 30 billion years to transfer this mass of data. That's twice the age of the universe. So we'll need a number of major breakthroughs in technology first.

...

Who knows, perhaps in a thousand years we really will be able to **teleport a coffee cup**. But beware: even the tiniest interference can mean that the cup arrives without its handle. This method of transport would be far too dangerous for humans.



But there is plenty we can do in the meanwhile...

[Create An ECAS Account](#) | [ECAS Login](#) | [About](#) | [Contact](#)



# DIGITAL SINGLE MARKET

## Digital Economy & Society

European Commission > European Commission will launch €1 billion quantum technologies flagship

[Home](#) | [The strategy](#) | [Economy](#) | [Society](#) | [Access & connectivity](#) | [Research & innovation](#) | [DG CONNECT](#)

### Research & innovation

- [Innovation](#)
- [Emerging Technologies](#)
  - [Brain Research](#)
  - [Future & Emerging Technologies](#)
    - [FET Open](#)
    - [FET Proactive](#)
    - [FET Flagships](#)
    - [Projects Portfolio](#)
- [European Open Science Cloud](#)
- [Digital Infrastructures](#)

## European Commission will launch €1 billion quantum technologies flagship

Published on 17/05/2016

Günther H. Oettinger, Commissioner for the Digital Economy and Society outlined the Commission's plan to launch a €1 billion flagship initiative on quantum technology.

Speaking at the [Quantum Europe Conference](#) organised by The Dutch presidency of the EU, the European Commission and the QuTech center in Delft, the Commissioner outlined his objective to reinforce European scientific leadership and excellence in quantum research and in quantum technologies.



 [Share](#)

# U.S. Department of Energy Unveils Blueprint for the Quantum Internet at 'Launch to the Future: Quantum Internet' Event

JULY 23, 2020



[Home](#) » U.S. Department of Energy Unveils Blueprint for the Quantum Internet at 'Launch to the Future: Quantum Internet' Event

## ***Nationwide Effort to Build Quantum Networks and Usher in New Era of Communications***

**CHICAGO, IL** – In a press conference today at the University of Chicago, the U.S. Department of Energy (DOE) unveiled a report that lays out a blueprint strategy for the development of a national quantum internet, bringing the United States to the forefront of the global quantum race and ushering in a new era of communications. This report provides a pathway to ensure the development of the [National Quantum Initiative Act](#), which was signed into law by President Trump in December of 2018.

Around the world, consensus is building that a system to communicate using quantum mechanics represents one of the most important technological frontiers of the 21st century. Scientists now believe that the construction of a prototype will be within reach over the next decade.

In February of this year, DOE National Laboratories, universities, and industry met in New York City to develop the blueprint strategy of a national quantum internet, laying out the essential research to be accomplished, describing the engineering and design barriers, and setting near-term goals.

"The Department of Energy is proud to play an instrumental role in the development of the national quantum internet," said U.S. Secretary of Energy Dan Brouillette. "By constructing this new and emerging technology, the United States continues with its commitment to maintain and expand





# Conclusion

- Quantum entanglement breaks local realism
- Generating entangled photons & reconstructing their state is relatively easy, but engineering for applications is still a challenge
- Entanglement is not just spooky, it's useful!

