Absolute Calibration of Pressure and Particle Velocity Microphones

A. Absolute Calibration of Pressure Microphones

The Sound Pressure Level (*SPL*): $SPL = L_P \equiv 10\log_{10}\left(p_{rms}^2/p_0^2\right) = 20\log_{10}\left(p_{rms}/p_0\right)\ (dB)$

where p_0 = reference sound over-pressure amplitude = 2.0×10^{-5} *rms Pascals* = 20 *rms* μPa
= the {average} threshold of human hearing (at f = 1 *KHz*).

Useful conversion factor(s): *p* = 1.0 *rms Pascal* ↔ *SPL* = 94.0 *dB* (in a free-air sound field)
n.b. 1.0 *rms Pascal* = 1.0 *rms Pa* = 1.0 *rms* N/m^2.

We can determine (*i.e.* measure) the ***absolute*** sensitivity of a pressure microphone *e.g.* in a free-air sound field – such as the great wide-open or in an anechoic room{at an industry-standard reference frequency *f* = 1 *KHz*} side-by-side with a {NIST-absolutely calibrated} *SPL* Meter a distance of a few meters away from a sound source – *e.g.* a loudspeaker. We turn up the volume of the sound source until the *SPL* Meter *e.g.* reads a steady *SPL* of 94.0 *dB* (quite loud!) which corresponds to a *p* = 1.0 *rms Pascal* acoustic over-pressure amplitude. We then measure the *rms AC* voltage amplitude of the output of the pressure microphone $V_{p\text{-}mic}$ (in *rms Volts*) *e.g.* using a Fluke 77 DMM on *RMS AC* volts. Thus, the ***absolute*** sensitivity of the pressure microphone is:

$$S_{p\text{-}mic}\left(V/Pa, @\ f = 1KHz, SPL = 94\ dB\right) = V_{p\text{-}mic}\left(rms\ Volts\right)/1.0\left(rms\ Pa\right)$$

n.b. The Knowles Acoustics 1/10" diameter FG-23329-C05 omni-directional electret condenser pressure microphone + op-amp preamp that we routinely use in the UIUC Physics of Music/ Musical Instruments Lab has a *flat* frequency response from 100 *Hz* to 10 *KHz*, with a nominal sensitivity of $S_{mic} = -53\,dB$ relative to 1.0 *rms Volt*/0.1 *rms Pascal*. The KA omni-directional pressure microphone op-amp preamp has a voltage gain of 10× (flat in frequency over the audio frequency range {20 *Hz* – 20 *KHz*} to within ± 0.1 *dB*), thus the nominal sensitivity of the KA omni-directional pressure microphone + op-amp preamp is $S_{p\text{-}mic+preamp} = -43\,dB$ over the audio frequency range.

The typical mean/average sensitivity of the Knowles Acoustics omni-directional electret-condenser pressure microphone + preamp that we routinely use in the UIUC Physics of Music/Musical Instruments Lab, determined at *f* = 1 *KHz* and *SPL* = 94.0 *dB* using an *EXTEC* 407768 *SPL* meter (*C*-weighted) and a Fluke 77 DMM on *RMS AC* volts is:

$$\left\langle S_{p\text{-}mic}^{KA}\right\rangle \simeq 280\ rms\ mV/rms\ Pa$$

B. Absolute Calibration of Particle Velocity Microphones

The Sound Particle Velocity Level (*SUL*): $SUL = L_u \equiv 10\log_{10}\left(u_{rms}^2/u_0^2\right) = 20\log_{10}\left(u_{rms}/u_0\right)\ (dB)$
where u_0 = reference particle velocity amplitude = $4.84\times10^{-8}\ rms\ m/s$ = the {average} threshold of human hearing (at f = 1 *KHz*).

Useful conversion factor: $u = 2.42\ rms\ mm/s \leftrightarrow SUL = SPL = 94.0\ dB$ (in a free-air sound field)

We can determine (*i.e.* measure) the ***absolute*** sensitivity of a particle velocity microphone in a free-air sound field {*e.g.* at f = 1 *KHz*}, side-by-side with a {NIST-absolutely calibrated} *SPL* Meter. We again turn up the volume until the *SPL* Meter *e.g.* reads 94.0 *dB*, which corresponds to *u* = 2.42 *rms mm/s* particle velocity amplitude, also known as $1.0\ rms\ Pa^*$. We then measure the *RMS AC* voltage amplitude of the output of particle velocity microphone $V_{u\text{-}mic}$ (in *rms Volts*) *e.g.* using a Fluke 77 DMM on *RMS AC* Volts. Thus, the ***absolute*** sensitivity of the particle velocity microphone (at f = 1 *KHz*) is: $S_{u\text{-}mic}\left(@\,SPL = 94dB\right) = V_{u\text{-}mic}/1.0\ \ \left(rms\,V/rms\ Pa^*\right)$

where in a free-air sound field: $1.0\ rms\ Pa^* \equiv \dfrac{1.0\left(rms\ Pascal\right)}{\rho_o^{air}\left(kg/m^3\right)\bullet c_o^{air}} = 2.42\left(rms\ mm/s\right)$.

In a free-air sound field (such as the great wide-open), *e.g.* for monochromatic traveling plane waves, the 1-D particle velocity amplitude *u* is related to the over-pressure amplitude *p* by: $u = p/\rho_o^{air}c_o^{air}$ where $\rho_o^{air} = 1.204\,kg/m^3$ and speed of sound in air is $c_o^{air} \simeq 344\,m/s\ @\ NTP$.

How does one physically measure particle velocity – *i.e.* how can we design & build a particle velocity transducer?

The Euler equation for inviscid fluid flow (*i.e.* neglecting any/all dissipation) is:

$$\frac{\partial\vec{u}\left(\vec{r},t\right)}{\partial t} = -\frac{1}{\rho_o}\vec{\nabla}p\left(\vec{r},t\right)$$

For propagation of 1-D longitudinal sound waves *e.g.* in the *z*-direction, the 1-D Euler equation is:

$$\frac{\partial u_z\left(\vec{r},t\right)}{\partial t} = -\frac{1}{\rho_o}\frac{\partial p\left(\vec{r},t\right)}{\partial z} \quad\Rightarrow\quad \frac{\partial u_z\left(\vec{r},t\right)}{\partial t} = -\frac{1}{\rho_o}\frac{\Delta p\left(\vec{r},t\right)}{\Delta z} \text{ for } \Delta z \ll \lambda$$

This equation physically tells us that if we measure the ***differential*** pressure amplitude $\Delta p\left(\vec{r}\right)$ across a small longitudinal distance $\Delta z \ll \lambda$ at a point in space $\vec{r}$, the differential over-pressure amplitude $\Delta p\left(\vec{r}\right)$ is linearly proportional to the time derivative of the longitudinal particle velocity amplitude $u_z\left(\vec{r}\right)$ at that point. Integrating the above 1-D Euler equation with respect to time:

$$u_z\left(\vec{r},t\right) = \int_{t'=-\infty}^{t'=t}\left(\frac{\partial u_z\left(\vec{r},t'\right)}{\partial t'}\right)dt' = -\frac{1}{\rho_o\Delta z}\int_{t'=-\infty}^{t'=t}\Delta p\left(\vec{r},t'\right)dt' \text{ for } \Delta z \ll \lambda$$

This equation tells us that the time-dependent 1-D particle velocity $u_z(\vec{r},t)$ is proportional to the integral of the time-dependent differential pressure $\Delta p(\vec{r},t)$ at the point $\vec{r}$.

For a ***harmonic/periodic***, ***single***-***frequency*** sound field, the instantaneous differential pressure at the point $\vec{r}$ at time *t* is: $\Delta p(\vec{r},t) = \Delta p(\vec{r})\cos(\omega t + \varphi)$. Integrating this expression over time:

$$\int_{t'=-\infty}^{t'=t} \Delta p(\vec{r},t')\,dt' = \Delta p(\vec{r})\int_{t'=-\infty}^{t'=t} \cos(\omega t' + \varphi)\,dt' = \left(\Delta p(\vec{r})/\omega\right)\sin(\omega t + \varphi)$$

Thus, we see that the 1-D particle velocity ***amplitude*** is proportional to the differential pressure ***amplitude*** at the point $\vec{r}$ via the relation: $\boxed{u_z(\vec{r}) = \Delta p(\vec{r})/\omega\rho_o\Delta z}$.

Note also that a –90° phase relation exists between the instantaneous 1-D particle velocity $u_z(\vec{r},t) = u_z(\vec{r})\sin(\omega t + \varphi)$ and the instantaneous differential pressure $\Delta p(\vec{r},t) = \Delta p(\vec{r})\cos(\omega t + \varphi)$.

It is possible to convert an omni-directional pressure microphone to a ***differential*** pressure microphone by {carefully} removing the rear/back cover of the microphone.

In the UIUC Physics of Music/Musical Instruments Lab, we use a so-modified, rectangular-shaped Knowles Acoustics EK-23132 omni-directional electret condenser differential pressure microphone, which has $\Delta z \sim 2\,mm$. The voltage output from this so-modified differential pressure mic is input to a simple op-amp integrator preamp circuit to carry out the above time integral operation. Hence, we obtain a signal output from this differential pressure mic + op-amp integrator preamp which is proportional to the 1-D particle velocity, $u_z(\vec{r},t)$!

Note that such a differential pressure mic only measures the component of the vector particle velocity $\vec{u}$ normal to the plane of the microphone element, *i.e.* $\vec{u}\bullet\hat{n} = u\cos\theta$. Thus, ***three*** such 1-D particle velocity microphones are needed to measure vector $\vec{u}$ at a specific (*x*,*y*,*z*) point in space.

Note also that for $\lambda \gg \Delta z$, the response of a differential pressure mic ***increases linearly*** with frequency, *i.e.* $\Delta p(\vec{r}) \propto f$. However, as discussed above, the time integral of the differential pressure is inversely proportional to frequency, *i.e.* $\int_{t'=-\infty}^{t'=t} \Delta p(\vec{r},t')\,dt' \propto 1/f$. The output of the ***integrating*** op-amp preamp has an *RC* time constant designed such that the output of the integrating op-amp ***decreases linearly*** with frequency above *f* = 20 *Hz*. Hence, we see that overall frequency response of the KA particle velocity mic, which is the ***product*** of the frequency response of the differential pressure mic × the frequency response of the ***integrating*** op-amp has a ***flat*** frequency response above *f* = 20 *Hz*!

The typical mean/average sensitivity of the Knowles Acoustics EK-23132 particle velocity/ differential pressure mic + integrating op-amp preamp that we routinely use in the UIUC Physics of Music/Musical Instruments Lab, determined at $f = 1\,KHz$ and *SPL* = 94.0 *dB* using an *EXTEC* 407768 *SPL* meter (*C*-weighted) and a Fluke 77 DMM on *RMS AC* volts is:

$$\boxed{\left\langle S_{u\text{-}mic}^{KA}\right\rangle = 90\,rms\ mV/rms\ Pa^{*} = 90\,rms\ mV/2.42\,rms\ mm/s = 3.72\times10^{4}\ rms\ mV/rms\ m/s}$$

C. <u>The Longitudinal Specific Acoustic Impedance, *Z*</u>

In one dimension, the 1-D/longitudinal specific acoustic impedance (*n.b.* a property of ***medium*** in which sound waves propagate) is defined as:

$$\boxed{Z \equiv p/u \quad \left(Pa\text{-}s/m = N\text{-}s/m^3 \equiv Acoustic\ Ohms, \Omega_a\right)} \qquad (\vec{Z} \equiv p/\vec{u} \text{ in 3-D})$$

In order to determine the 1-D/longitudinal specific acoustic impedance *Z*, we use the raw rms voltage amplitudes associated with the signals output from the pressure and particle velocity mics and their respective absolutely-calibrated microphone sensitivities:

$$\boxed{Z\left(Pa\text{-}s/m\right) = \frac{p\left(rms\ Pa\right)}{u_z\left(rms\ m/s\right)} = \frac{V_{p\text{-}mic}^{KA}\left(rms\ mV\right)/S_{p\text{-}mic}^{KA}\left(rms\ mV/rms\ Pa\right)}{V_{u\text{-}mic}^{KA}\left(rms\ mV\right)/S_{u\text{-}mic}^{KA}\left(rms\ mV/rms\ m/s\right)} = K_z \frac{V_{p\text{-}mic}^{KA}}{V_{u\text{-}mic}^{KA}}}$$

Thus, for 1-D/longitudinal specific acoustic impedance measurements using the Knowles Acoustics *p*- and 1-D *u*-mics, the typical mean/average overall *Z*-conversion factor K_z is:

$$\boxed{K_z \equiv \frac{\left\langle S_{u\text{-}mic}^{KA}\right\rangle}{\left\langle S_{p\text{-}mic}^{KA}\right\rangle} = \frac{3.72\times10^4\ rms\ mV/rms\ m/s}{280\ \ rms\ mV/rms\ Pa} \simeq 132.8\,Pa\text{-}s/m\left(=132.8\,\Omega_a\right)}$$

D. <u>Time-Averaged Sound Intensity</u>

For a harmonic/periodic single-frequency sound field, the time-averaged {vector/3-D} sound intensity at the point $\vec{r}$ is: $\boxed{\left\langle \vec{I}\left(\vec{r}\right)\right\rangle = \lim_{T\to\infty}\frac{1}{T}\int_0^T \vec{u}\left(\vec{r},t\right)p\left(\vec{r},t\right)dt \quad \left(rms\,W/m^2\right)}$

The Sound Intensity Level (SIL), *aka* Loudness: $\boxed{SIL = L_I \equiv 10\log_{10}\left(I/I_0\right)\ (dB)}$

where I_0 = reference sound intensity level = 10^{-12} Watts (@ *f* = 1 *KHz*) and $I = \left\langle\left|\vec{I}\right|\right\rangle$.

The following table gives some useful correspondences of *p*, *u* and *I* for various *SPL*'s:

Sound Type	***SPL* (*dB*)**	***p* (*rms Pa*)**	***u* (*rms m/s*)**	**I (*rms W/m*^{2})**
Artillery Fire	**140**	**200**	**0.48**	**100**
Rock Concert	**120**	**20**	**0.048**	**1**
Motorcycle	**100**	**2.0**	**0.0048**	10^{-2}
Reference Level	**94**	**1.0**	2.4×10^{-3}	2.8×10^{-3}
Vacuum Cleaner	**80**	**0.2**	4.8×10^{-4}	10^{-4}
Normal Conversation	**60**	**0.02**	4.8×10^{-5}	10^{-6}
Whispering	**40**	**0.002**	4.8×10^{-6}	10^{-8}
Empty Theater	**20**	2×10^{-4}	4.8×10^{-7}	10^{-10}
Threshold of Hearing	**0**	2×10^{-5}	4.8×10^{-8}	10^{-12}