# Experimental Measurements of Complex Sound Fields:

As stated previously in P406 Lecture Notes 11 part 2 (*p.* 10), in order to uniquely determine the nature of a complex 3-D sound field $\tilde{S}(\vec{r},t)$ at the space-time point $(\vec{r},t)$, one needs to carry out **_two_** physical measurements – one of the complex scalar over-pressure $\tilde{p}(\vec{r},t)$ $(Pascals)$ and the complex 3-D vector particle velocity $\vec{\tilde{u}}(\vec{r},t) = \tilde{u}_x(\vec{r},t)\hat{x} + \tilde{u}_y(\vec{r},t)\hat{y} + \tilde{u}_z(\vec{r},t)\hat{z}$ (m/s).

In order to carry out measurements of these two physical quantities, generally speaking, we need to use pressure/particle velocity **_transducers_** of some kind, that: *a.*) intrinsically respond to/ are sensitive to over-pressure/particle velocity, respectively and *b.*) convert (transduce) the over-pressure/particle velocity to a usable signal – *e.g.* an electrical voltage (or an electrical current).

Transducers Used for the Measurement of Over-Pressure:

For the measurement of over-pressure $\tilde{p}(\vec{r},t)$ at a given listener's space-time point $(\vec{r},t)$, fortunately human-kind has now spent well over a century in the development of **_microphones_** – nearly all of which (but not all) are directly sensitive/respond to over-pressure, just as our human ears do. It is therefore no accident that the technology for the measurement of over-pressure $\tilde{p}(\vec{r},t)$ is far more advanced today than that for the measurement of particle velocity $\vec{\tilde{u}}(\vec{r},t)$. However recent advances *e.g.* in MEMS (Micro-Electrical Mechanical System) and nano-fabrication technology have in turn enabled the development of novel, compact transducers that produce an electrical signal in direct response to the local particle velocity $\vec{\tilde{u}}(\vec{r},t)$.

There are many different kinds of microphones, the two most popular types are **_dynamic_** microphones and **_condenser_** microphones. Dynamic microphones consist of a {very} thin diaphragm to which a small coil of wire is attached, which in turn is immersed in a strong magnetic field, provided by a permanent magnet, as shown in the figure below:


Cross section of a dynamic mic

-1-
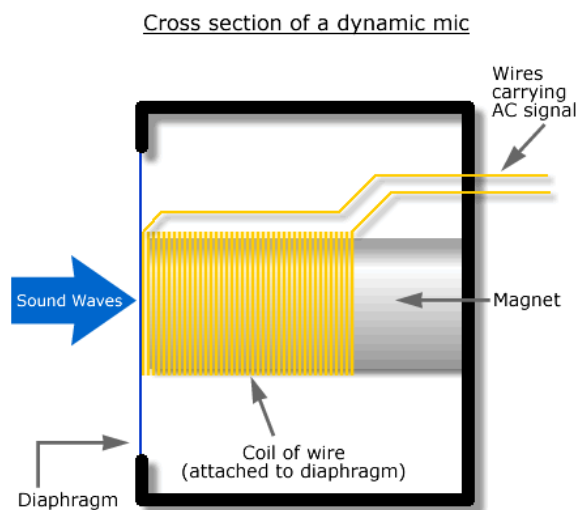
When an over-pressure $\tilde{p}(\vec{r},t) = \tilde{p}_o(\vec{r})e^{i\omega t}$ is present at the diaphragm of the microphone, located at the listener point $\vec{r}$, if the characteristic size of the microphone – e.g. the radius of the microphone diaphragm $a_{dia} \ll \lambda$, the variation of the over-pressure amplitude $\tilde{p}_o(\vec{r})$ over the surface of the microphone diaphragm is ~ negligible, i.e. $\tilde{p}_o(\vec{r}) \simeq \tilde{p}_o$ for $a_{dia} \ll \lambda$. The {net} force acting on the diaphragm {+ coil} of mass m_{dia} is $\vec{F}(t) = \int_{A_{dia}} -\tilde{p}(\vec{r},t)\overrightarrow{da} \simeq -\tilde{p}_o \vec{A}_{dia}e^{i\omega t}$ (N) {where $\vec{A}_{dia} = A_{dia}\hat{n}$ and $\hat{n}$ is the outward-pointing normal to the diaphragm}, accelerating it {by Newton's 2nd law: $\vec{F}(t) = m_{dia}\vec{a}_{dia}(t)$ if no other forces acting on the diaphragm are present - which, in general, there are – see Appendix to these lecture notes...} thereby causing it to vibrate/oscillate, which in turn induces a voltage signal in the coil due to the time rate of change of magnetic flux $\tilde{\Phi}_m(t) = \vec{B}(t)\bullet\vec{A}_{coil}$ threading the coil as it vibrates along its axis. The induced EMF in the coil of the dynamic microphone is: $\tilde{\varepsilon}_{coil}(t) = -N_{coil}\, d\tilde{\Phi}_m(t)/dt = -N_{coil}\left(d\vec{B}(t)/dt\right)\bullet\vec{A}_{coil}$ $(Volts)$.

Note that the basic physics of how a dynamic microphone works is simply the time-reversed physics of how a dynamic loudspeaker works – arising from the manifest time-reversal invariant nature of the EM interaction at the microscopic level!

Because of the need to attach a coil (usually copper wire) to the diaphragm, the diaphragm + coil assembly of a dynamic microphone is considerably heavier than the pressure-sensitive elements associated e.g. with condenser microphones. Thus, dynamic microphones often do not have as flat a frequency and phase response as condenser microphones for this (and other) reason(s) – hence dynamic microphones are not often thought of as "research/lab-grade" quality… However, the intrinsic output impedance of dynamic microphones is (relatively speaking) quite low (the industry standard is 600 Ω) which is very appealing e.g. for use in live performances of music, from the {important} perspective of noise immunity…

If the microphone element is completely **_sealed_** (as in the above figure), the over-pressure sensed by the device is the instantaneous **_difference_** between pressure on the front vs. back side of the diaphragm. This is an **_omni-directional_** microphone – for frequencies f with wavelengths $\lambda = v/f$ that are **_large_** in comparison the physical size a_{dia} of the microphone, it responds uniformly to sound coming from any/all directions, as shown in the polar plot below:
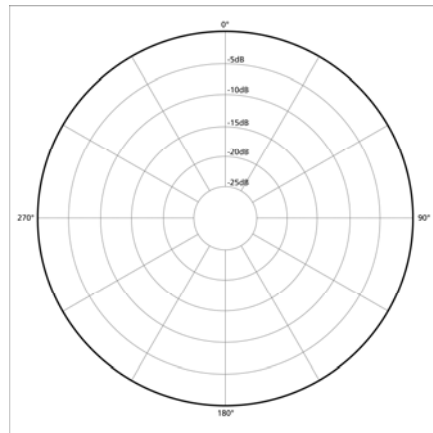
Because the {large} ambient pressure P_{amb} varies from day-to-day with the weather, it is necessary to have a small hole in the body of the sealed microphone element in order to allow equalization of the static pressures on either side of the diaphragm in order to avoid damaging it. The existence of the small pressure-equalization hole/port in the microphone body only affects extremely low frequency response $f \ll 20\,Hz$, well below the audio band of human hearing.

Condenser microphones also have a (very low mass) diaphragm – which has an extremely thin layer of conductor deposited on its surface. The diaphragm is in proximity to a planar electrode, thus forming the plates of a parallel-plate capacitor, as shown in the figure below.
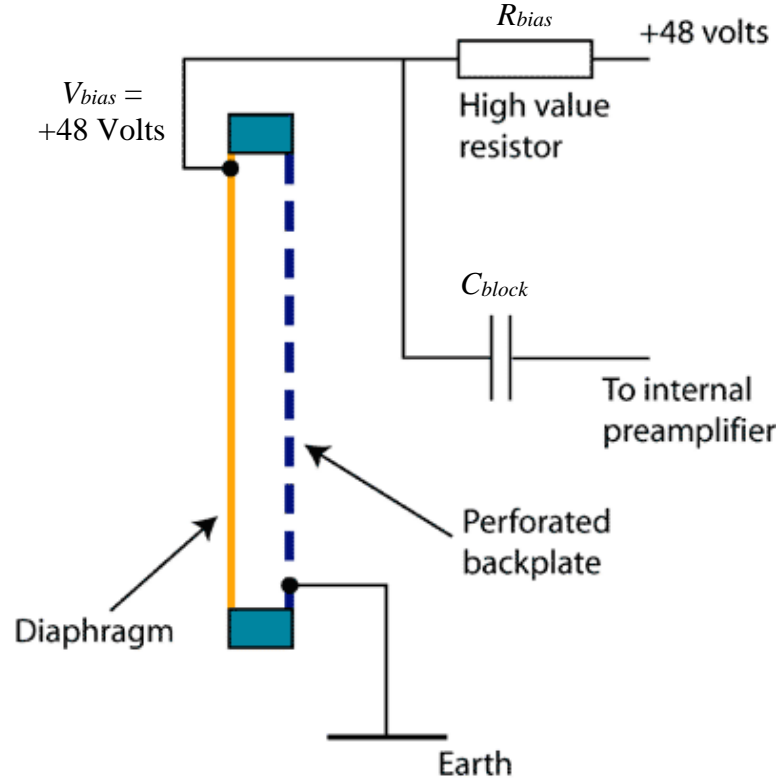

A constant/DC voltage V_{bias} is placed across the plates of this capacitor {no DC current flows across the air-gap between the plates of this capacitor}. When an over-pressure $\tilde{p}(\vec{r},t)$ is present at the {metalized} diaphragm of the condenser mic, then for $a_{dia} \ll \lambda$, it exerts a {net} force $\vec{\tilde{F}}(t) \simeq -\tilde{p}_o \vec{A}_{dia} e^{i\omega t}$ (N) on the diaphragm, accelerating it, thereby causing it to vibrate in response to the over-pressure exerted on it. The {very} small gap d_{gap} between the diaphragm and electrode thus varies in time $d_{gap}(t) \equiv d_{gap} + x_{dia}(t)$, where $x_{dia}(t) \ll d_{gap}$ is the displacement of the diaphragm, and since the capacitance of this parallel plate capacitor is
$C(t) = \varepsilon_o A_{dia}/d_{gap}(t) = \varepsilon_o A_{dia}/\left[d_{gap} + x_{dia}(t)\right] \simeq \varepsilon_o A_{dia}\left[1 - \left(x_{dia}(t)/d_{gap}\right)\right]/d_{gap}$ for
$x_{dia}(t) \ll d_{gap}$, the capacitance also varies in time, which in turn causes the charge present on the planar electrode to vary in time, since $Q(t) = C(t) \cdot V_{bias}$.

If the planar electrode is connected to ground (earth) and the diaphragm of the condenser mic held at potential V_{bias} by a very high resistance R_{bias}, then a time-varying AC voltage signal $V_R(t) = I(t) \cdot R_{bias}$ appears across this resistor due to the time-varying AC current $I(t) = dQ(t)/dt = V_{bias} \cdot dC(t)/dt = -\left(\varepsilon_o A_{dia} V_{bias}/d_{gap}^2(t)\right) \cdot \left(dd_{gap}(t)/dt\right)$ flowing through this resistor and the planar capacitor to ground. The capacitor C_{block} blocks the DC voltage V_{bias} present on the metalized diaphragm, but allows the AC voltage signal $V_R(t)$ to pass through it – which is then amplified by some kind of voltage amplifier – *e.g.* a vacuum triode voltage amplifier {back in the hey-day of vacuum tubes} or a high-input impedance FET (Field-Effect Transistor) amplifier or *e.g.* a low-noise high-input impedance FET-input op-amp (operational amplifier).

Condenser microphones intrinsically have {very} high output impedance, and require a bias voltage {*aka* "phantom power"} for them to operate, as well as a high input impedance preamplifier of some kind. If designed properly, the condenser mic + mic preamp system together will have very good, flat frequency and phase response and also have low intrinsic noise.

Again, condenser microphone elements with omni-directional response are completely sealed (except for a small ambient pressure equalization port) as shown in the figure below:

-4-
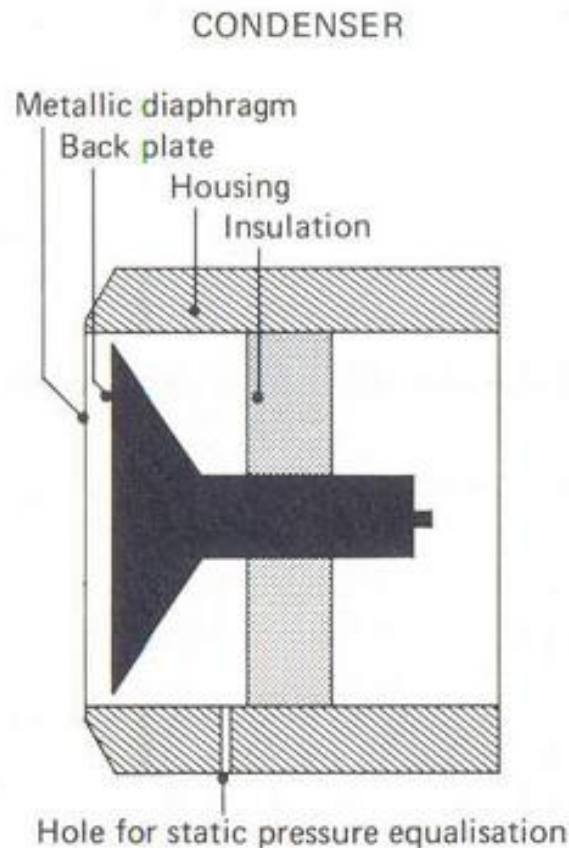
A more modern/high-tech and much more compact version of the condenser microphone uses a permanently-polarized electret-film type material to provide the electrostatic $\vec{E}$-field in the small gap d_{gap} between the diaphragm (with metalized surface) and the electret film (also with a metalized surface). An electret film consists of a material (*e.g. PVDF* – polyvinylidene fluoride – a piezo-electric material), the molecules of which have a permanent electric dipole moment associated with them, analogous to permanent magnetic materials. Due to the permanent electric polarization $\vec{P}$ $\left(Coulombs/m^2 \right)$ associated with the molecules making up the electret material, the **_surface_** of the electret film has a **_bound_** surface electric charge density, $\sigma_b \equiv \vec{P} \cdot \hat{n}$ $\left(Coulombs/m^2 \right)$ associated with it, which produces a constant/uniform electric field $\vec{E}_{gap}$ between the diaphragm and the electret material, of magnitude $\left| \vec{E}_{gap} \right| = \sigma_b/\varepsilon_o$ with potential difference across the plates of this capacitor of $V_{gap} = \left| \vec{E}_{gap} \right| d_{gap} = \left(\sigma_b/\varepsilon_o \right) d_{gap}$. When an over-pressure $\tilde{p}(\vec{r},t)$ is present on the diaphragm of this microphone, the voltage across the gap between plates of this capacitor also becomes time-dependent $V_{gap}(t) = \left| \vec{E}_{gap} \right| \cdot d_{gap}(t) = \left(\sigma_b/\varepsilon_o \right) \cdot d_{gap}(t) = \left(\sigma_b/\varepsilon_o \right) \cdot \left[d_{gap} + x_{dia}(t) \right]$. If this voltage signal is amplified, *e.g.* with a high input impedance FET, it makes for a wonderfully pressure-sensitive microphone, one with excellent frequency/phase response and intrinsically low noise characteristics. A schematic diagram of an electret condenser microphone is shown in the figure below.
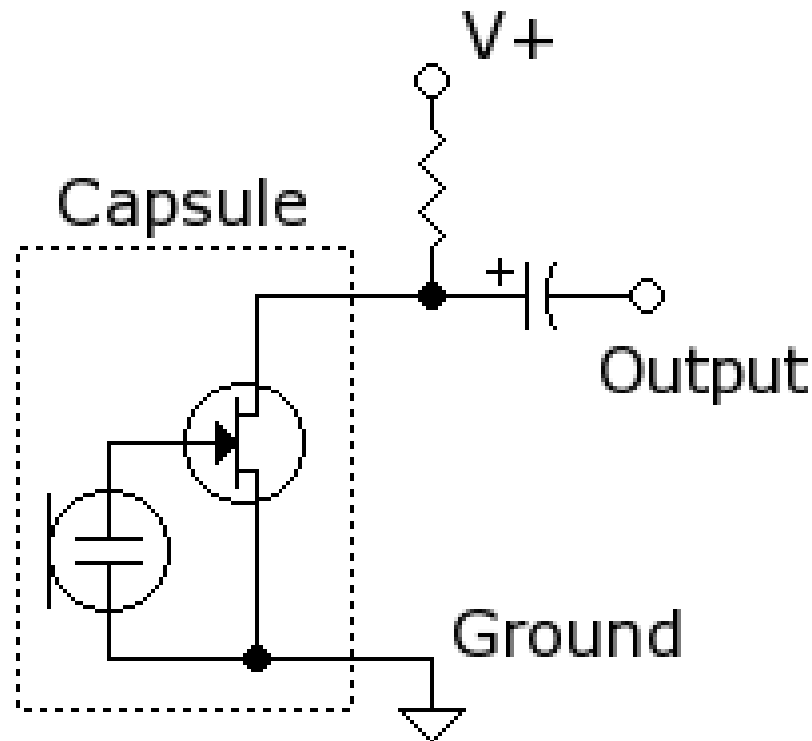
A cross-sectional/cutaway view of a back-electret type condenser microphone is shown in the figure below:
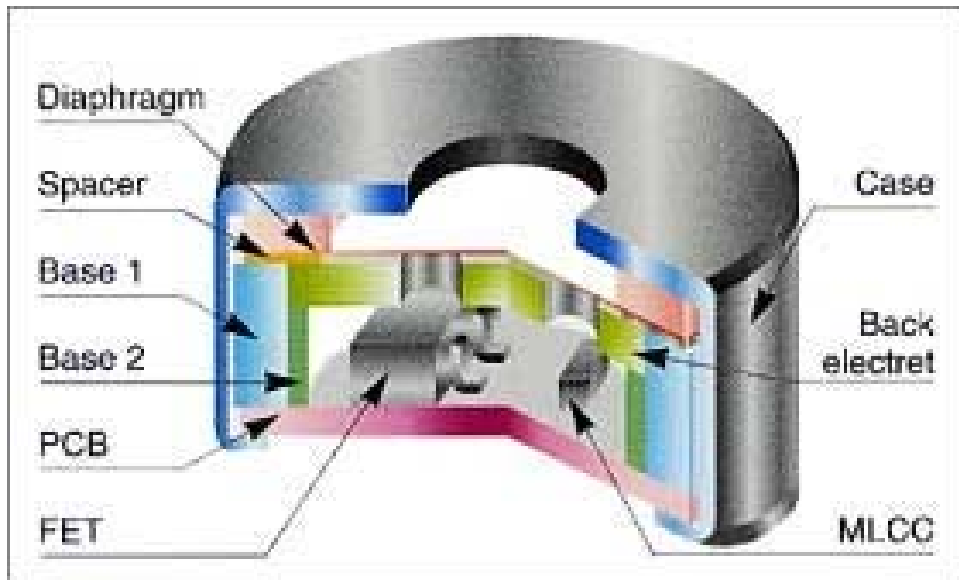
Various types of high-performance sub-miniature electret condenser microphones made by Knowles Electronics (of Itasca, Illinois http://www.knowles.com/) are shown in the figure below.

The electret condenser mic in the bottom left-hand corner FG-23329 is the world's smallest – it is only 1/10" in diameter, and has a *flat* frequency and phase response out to $f \sim 10\ KHz$:
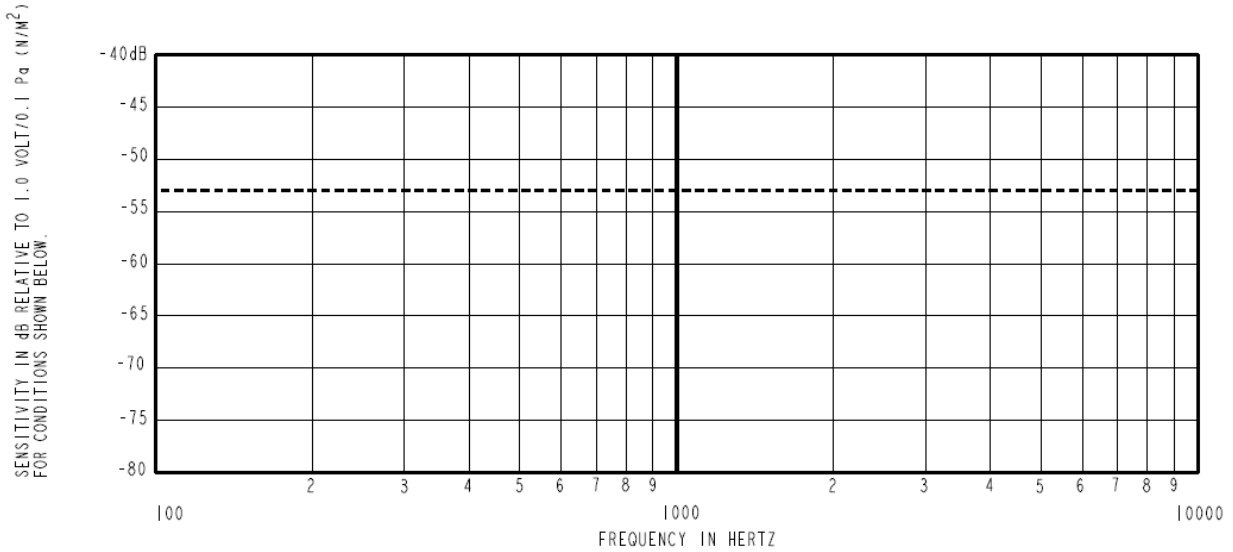
From the graph of the frequency response of this electret condenser microphone, note that the sensitivity of this microphone is given as −53 *dB* referenced to 1.0 *Volt*/0.1 *Pascal* (N/m^2).

This means that for an over-pressure amplitude of $p = 0.1\ Pascals$, the voltage amplitude V_{p-mic} output from the device is $-53\ dB = 20\log_{10}\left(V_{p-mic}/1.0\right) = 10\log_{10}\left(V_{p-mic}/1.0\right)^2$ or $V_{p-mic} = 1.0 \times 10^{-53/20} = 10^{-2.65} = 2.24\ mV$.

For an over-pressure amplitude of $p = 1.0\ Pascals$ (corresponding to a sound pressure level $SPL = 94.0\ dB$), this corresponds to a 10× larger voltage output from this device, of $V_{p-mic} = 22.4\ mV$. The **_sensitivity_** of this pressure microphone is thus:

$$S_{p-mic} \equiv V_{p-mic}/p = 2.24\ mV/0.1Pa = 22.4\ mV/1.0Pa = 22.4\ mV/Pa$$

Absolute calibration of a pressure microphone/measurement of the sensitivity of the pressure microphone S_{p-mic} is carried out by placing it *e.g.* in a monochromatic (*e.g.* $f = 1.0\ KHz$ sine-wave) **_free-air_** sound field $\tilde{S}(\vec{r},t)$ at *NTP* with a $SPL = 94.0\ dB$ {set using *e.g.* a NIST-calibrated *SPL* meter (*C*-weighting) in proximity to the microphone}. This *SPL* corresponds to an over-pressure amplitude of $p = 1.0\ Pascals$, since $SPL(dB) = 20\log_{10}(p/p_0)$ where $p_0 = 2 \times 10^{-5}\ Pa$ is the reference pressure at the (average) threshold of human hearing. The *AC* voltage amplitude V_{p-mic} output from the pressure microphone immersed in a $SPL = 94.0\ dB$ free-air sound field can be easily measured *e.g.* on an oscilloscope or a true *RMS* digital multi-meter.

In the UIUC Physics 406 POM lab, we use the Knowles Electronics FG-23329 subminiature electret condenser microphone capacitively coupled to a 11× gain low-noise non-inverting op-amp preamplifier, the circuit for which is shown in the figure below:

KA FET Mic Preamp
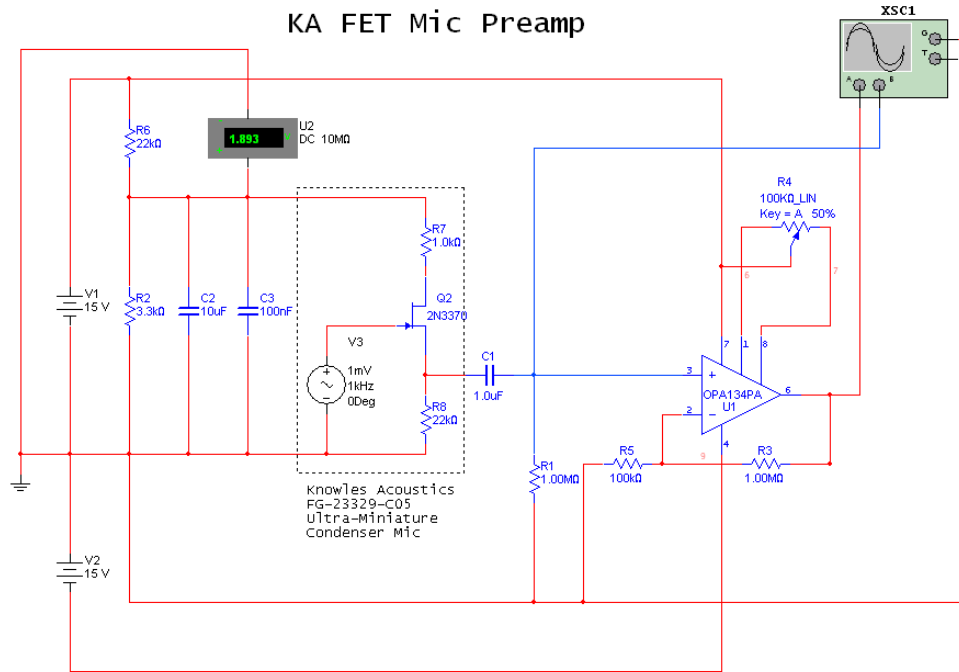
The **_high-pass_** filter time constant τ_{RC} associated with the pressure mic preamp's $C = 1\,\mu F$ input coupling capacitor (*n.b.* which also blocks the ~ +1.5 V$_{DC}$ quiescent *DC* voltage output from the mic's internal FET) and $R = 1\,\text{M}\Omega$ is $\tau_{RC} \equiv RC = 1.0\,sec$, corresponding to a $-3\,dB$ corner frequency of $f_{-3\,dB} \equiv 1/(2\pi \cdot \tau_{RC}) = 1/(2\pi RC) = 1/2\pi \simeq 0.16\,Hz$. The frequency and phase response of this pressure mic's preamp circuit is shown in the figure below:
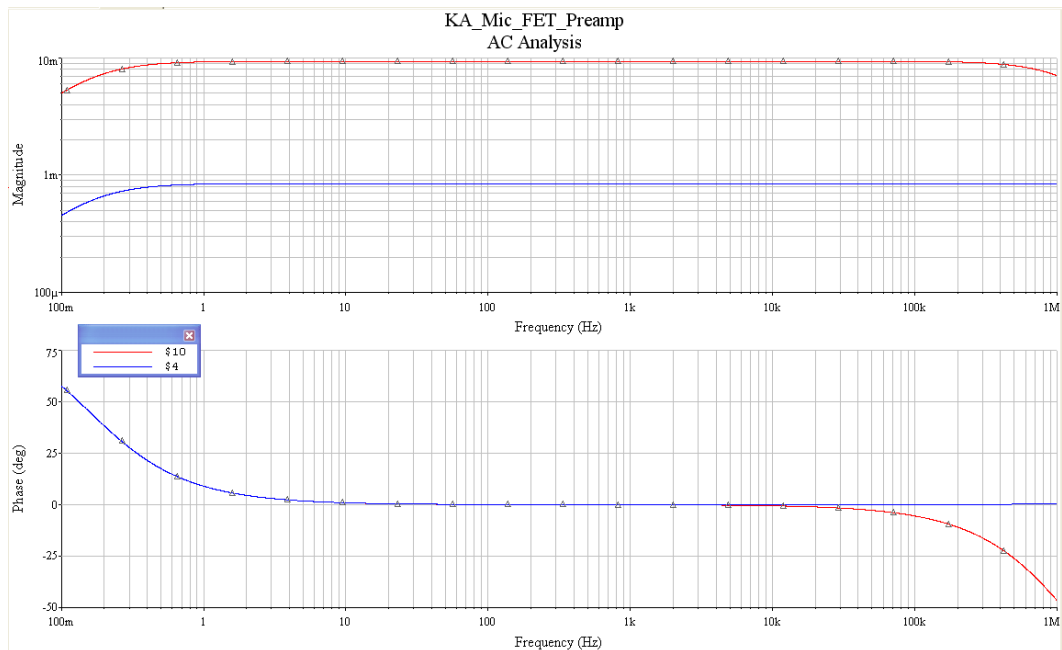
Transducers Used for the Measurement of Particle Velocity:

How can one measure particle velocity $\vec{\tilde{u}}(\vec{r},t)$? There are two ways – one is to build a device which is **_directly_** sensitive to particle velocity, the other is to exploit what the {linearized} Euler equation for inviscid fluid flow tells us:

$$\frac{\partial \vec{\tilde{u}}(\vec{r},t)}{\partial t} = -\frac{1}{\rho_o}\vec{\nabla}\tilde{p}(\vec{r},t)$$

If we focus, for the moment on **_one_** spatial component of this equation, *e.g.* in the $\hat{z}$-direction, this equation in Cartesian coordinates becomes simply:

$$\frac{\partial \tilde{u}_z(\vec{r},t)}{\partial t} = -\frac{1}{\rho_o}\frac{\partial \tilde{p}(\vec{r},t)}{\partial z} \;\Rightarrow\; -\frac{1}{\rho_o}\frac{\Delta_z \tilde{p}(\vec{r},t)}{\Delta z}$$

Euler's equation tells us that if we measure the pressure **_gradient_** {an over-pressure **_difference_**} $\Delta_z \tilde{p}(\vec{r},t) \equiv \tilde{p}(z_2,t) - \tilde{p}(z_1,t)$ over a **_small_** separation distance $\Delta z \equiv z_2 - z_1$ (*n.b.* with $\Delta z \ll \lambda$), then Euler's equation tells us that the pressure gradient is proportional to the time rate of change of the *z*-component of the particle velocity $\partial \tilde{u}_z(\vec{r},t)/\partial t$! Thus, if we integrate the pressure gradient $\Delta_z \tilde{p}(\vec{r},t)/\Delta z$ with respect to time, the integral is **_proportional_** to the *z*-component of the particle velocity:

$$\tilde{u}_z(\vec{r},t) = -\frac{1}{\rho_o \Delta z}\int_{t'=-\infty}^{t'=t}\Delta_z \tilde{p}(\vec{r},t')dt' = -\frac{1}{\rho_o(z_2-z_1)}\int_{t'=-\infty}^{t'=t}\left[\tilde{p}(z_2,t')-\tilde{p}(z_1,t')\right]dt'$$

Likewise, if we then also measure the instantaneous pressure gradients $\Delta_x \tilde{p}(\vec{r},t)\;(\Delta_y \tilde{p}(\vec{r},t))$ over small distances $\Delta x\;(\Delta y)$ in the $\hat{x}\;(\hat{y})$-directions, respectively, and then integrate these two signals, then we also obtain measurements of:

$$\tilde{u}_x(\vec{r},t) = -\frac{1}{\rho_o \Delta x}\int_{t'=-\infty}^{t'=t}\Delta_x \tilde{p}(\vec{r},t')dt' = -\frac{1}{\rho_o(x_2-x_1)}\int_{t'=-\infty}^{t'=t}\left[\tilde{p}(x_2,t')-\tilde{p}(x_1,t')\right]dt'$$

and: $\quad \tilde{u}_y(\vec{r},t) = -\frac{1}{\rho_o \Delta y}\int_{t'=-\infty}^{t'=t}\Delta_y \tilde{p}(\vec{r},t')dt' = -\frac{1}{\rho_o(y_2-y_1)}\int_{t'=-\infty}^{t'=t}\left[\tilde{p}(y_2,t')-\tilde{p}(y_1,t')\right]dt'$

To measure a pressure gradient $\Delta_r \tilde{p}(\vec{r},t) = \tilde{p}(\vec{r}_2,t) - \tilde{p}(\vec{r}_1,t)$ between two points one can use *e.g.* two {hopefully} identical/perfectly frequency/phase-matched microphones (with $S_{mic1} = S_{mic2}$), and then use (or build) a (precision) low-noise **differential** preamp to take the voltage difference $\Delta \tilde{V}(t) = \tilde{V}_{mic2}(\vec{r}_2,t) - \tilde{V}_{mic1}(\vec{r}_1,t)$ between the two signals, since:

$$\Delta_r \tilde{p}(\vec{r},t) = \tilde{p}(\vec{r}_2,t) - \tilde{p}(\vec{r}_1,t) = \frac{\tilde{V}_{mic2}(\vec{r}_2,t)}{S_{mic2}} - \frac{\tilde{V}_{mic1}(\vec{r}_1,t)}{S_{mic1}}$$

The over-pressure ***difference*** signal is input to an ***integrating*** op-amp, which electronically carries out the above integral operation. The voltage signal output from the integrating op-amp is thus ***proportional*** to the 1-D particle velocity $\tilde{u}_r(\vec{r},t) = \tilde{u}_r\left(\frac{1}{2}(\vec{r}_1 + \vec{r}_2),t\right)$! Using 3 such microphone pairs (in orthogonal *x-y-z* orientation) thus yields the full 3-D particle velocity $\vec{\tilde{u}}(\vec{r},t)$!

The use of matched pairs of omni-directional pressure microphones separated by a small distance $\Delta z \ll \lambda$ to measure the local pressure gradient $\Delta_z \tilde{p}(\vec{r},t)/\Delta z$ and thereby infer (*i.e.* compute) the local 1-D particle velocity $\partial \tilde{u}_z(\vec{r},t)/\partial t$ is known as the so-called ***p-p method***.
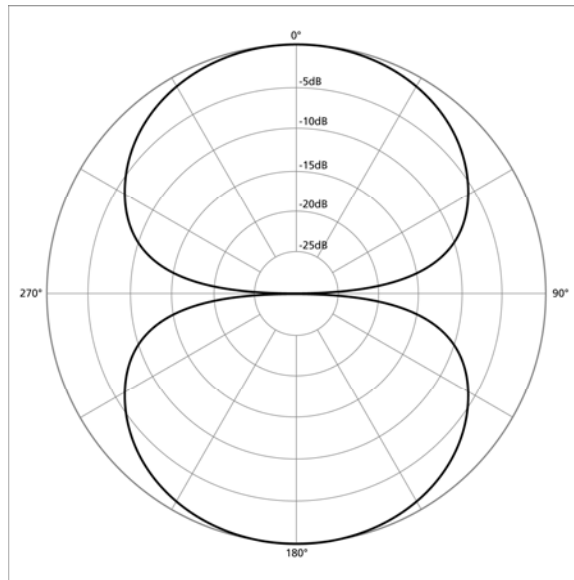
If the diaphragm of the pressure microphone is ***fully-open*** (*i.e.* not enclosed), such a microphone {of characteristic longitudinal/*z*-dimension *d*} measures the local over-pressure ***difference*** $\Delta \tilde{p}_{dia}(\vec{r},t) = \tilde{p}_{dia}^{front}(\vec{r},t) - \tilde{p}_{dia}^{rear}(\vec{r},t)$ for $d \ll \lambda$ – *i.e.* it becomes a ***differential-pressure*** microphone. Furthermore, it is obvious that differential pressure microphones have a vectorial-type directional response, since the voltage signal output from this microphone will reverse its sign if the differential microphone is rotated about its symmetry axis by 180°. The angular response of a differential pressure microphone is therefore vectorially described by $\Delta \tilde{p}_{dia}(\vec{r},t)\hat{n}$ where $\hat{n}$ is defined as the ***outward*** pointing unit vector from the front surface of the microphone diaphragm. For a sound wave with vector wavenumber $\vec{k} = |\vec{k}|\hat{k} = k\hat{k}$ ($\hat{k} \parallel$ to the propagation direction), the ***angular*** response of the differential pressure microphone is $\Delta \tilde{p}_{dia}(\vec{r},t)\hat{n} \cdot \hat{k} = \Delta \tilde{p}_{dia}(\vec{r},t)\cos\Theta$ where Θ is the 3-D opening angle between $\hat{n}$ and $\hat{k}$.

Thus when: $\Theta = 0^o$, $\hat{n} \cdot \hat{k} = \cos\Theta = +1$ and: $\Delta \tilde{p}_{dia}(\vec{r},t)\hat{n} \cdot \hat{k} = +\Delta \tilde{p}_{dia}(\vec{r},t)$.

When: $\Theta = 90^o$, $\hat{n} \cdot \hat{k} = \cos\Theta = 0$ and: $\Delta \tilde{p}_{dia}(\vec{r},t)\hat{n} \cdot \hat{k} = 0$.

When: $\Theta = 180^o$, $\hat{n} \cdot \hat{k} = \cos\Theta = -1$ and: $\Delta \tilde{p}_{dia}(\vec{r},t)\hat{n} \cdot \hat{k} = -\Delta \tilde{p}_{dia}(\vec{r},t)$.

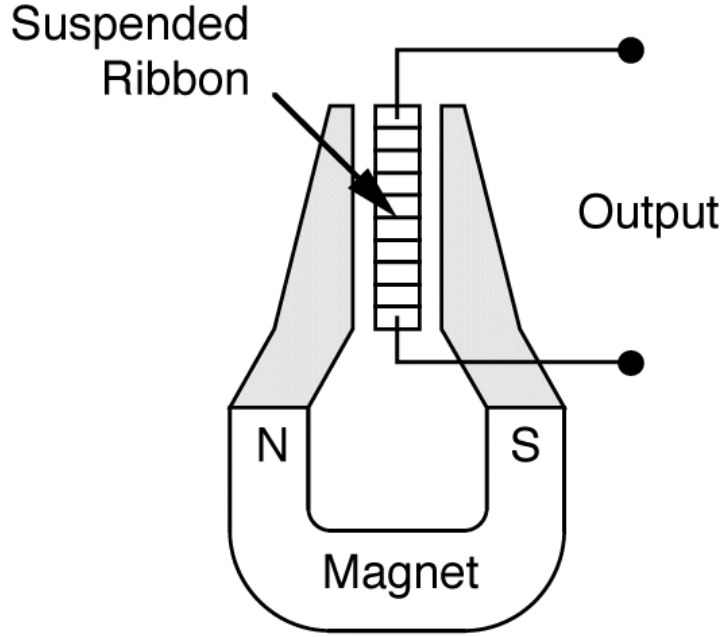The polar response of a ***differential*** pressure microphone is a ***figure-8 pattern***, as shown below:

Another type of {early} differential pressure microphone is the so-called ***ribbon*** microphone, as shown in the figure below:

The very thin {corrugated} metal ribbon diaphragm is open to the ambient pressure on both sides of it. The thin metal ribbon diaphragm is corrugated to suppress excitation of low-frequency mechanical vibrational modes of the ribbon. Since the metal ribbon diaphragm is conducting and immersed in a strong, transverse magnetic field $\vec{B}_o \, (Tesla)$, a differential over-pressure $\Delta\tilde{p}_{ribbon}(\vec{r},t)$ exerts a net force on the ribbon diaphragm of this microphone of magnitude $\left|\vec{F}(\vec{r},t)\right| = \Delta\tilde{p}_{ribbon}(\vec{r},t) A_{ribbon} \, \hat{n}\cdot\hat{k} = \Delta\tilde{p}_{ribbon}(\vec{r},t) A_{ribbon} \cos\Theta$, causing the ribbon of mass m_{ribbon} to accelerate (again by Newton's 2nd law – if no other forces act on the microphone diaphragm) $\vec{a}_{ribbon}(\vec{r},t) = \vec{F}(\vec{r},t)/m_{ribbon}$ and thus move back-and-forth in response to the differential over-pressure. Since the thin corrugated metal ribbon diaphragm is immersed in the static transverse magnetic field $\vec{B}_o$, a magnetic/Lorentz force acts on the free electrons in the metal ribbon $\vec{F}_m = -e\vec{v}_e \times \vec{B}_o = -e\vec{E}$, where $\vec{v}_e(\vec{r},t) = \vec{v}_{ribbon}(\vec{r},t)$ is the velocity vector of the free electrons associated with the macroscopically vibrating metal ribbon. Since $\vec{v}_{ribbon}(\vec{r},t) \perp \vec{B}_o$ then:

$$\vec{v}_{ribbon}(\vec{r},t)\times\vec{B}_o = \left|\vec{v}_{ribbon}(\vec{r},t)\right|\cdot\left|\vec{B}_o\right| \underbrace{\sin\theta}_{=1} = \left|\vec{v}_{ribbon}(\vec{r},t)\right|\cdot\left|\vec{B}_o\right|$$

A time-varying *EMF* (*i.e.* a voltage) $\tilde{\varepsilon}(t) = \left|\vec{v}_{ribbon}(\vec{r},t)\right|\cdot\left|\vec{B}_o\right|\cdot\ell_{ribbon}$ {where $\ell_{ribbon}(m)$ is the length of the metal ribbon immersed in the *B*-field} is induced across the top/bottom of the corrugated metal ribbon diaphragm due to the differential over-pressure $\Delta\tilde{p}_{ribbon}(\vec{r},t)$ acting on it.

Due to the intrinsically larger mass {and physical size} associated with the thin corrugated metal ribbon, the frequency and phase response of a ribbon-type differential pressure microphone is usually not as good as *e.g.* the modern, compact high-tech electret-type differential microphones; ribbon microphones are therefore not often thought of as laboratory / research-grade quality devices. Still, ribbon mics have many storied uses in the now ~ century long history of sound recording.

Note that the frequency response of a ***differential*** pressure microphone of characteristic size d is ***not*** flat. The (complex) response function of the differential microphone as a function of (angular) frequency ω and wavenumber-differential mic axis opening angle Θ is given by:

$$\tilde{H}_{diff-mic}(\omega,\Theta) = 1 - e^{-i\vec{k}\cdot\vec{d}} = 1 - e^{-ikd(\hat{k}\cdot\hat{n})} = 1 - e^{-ikd\cos\Theta} = 1 - e^{i(\omega/c)d\cos\Theta}$$

$$= 1 - \cos\left[(\omega/c)d\cos\Theta\right] + i\sin\left[(\omega/c)d\cos\Theta\right]$$

For $kd = (\omega/c)d \ll 1$ (*i.e.* $\omega \ll c/d$) then:

$$\tilde{H}_{diff-mic}(\omega,\Theta) \simeq 1 - \cos 0 + i(\omega/c)d\cos\Theta = +i(\omega/c)d\cos\Theta$$

Thus, for $kd = (\omega/c)d \ll 1$ (*i.e.* $\omega \ll c/d$) the frequency response of a ***differential*** pressure microphone is such that it increases ***linearly*** with frequency (*n.b.* its response $\tilde{H}_{diff-mic}(\omega,\Theta) = 0$ at $\omega = 2\pi f = 0$). We also see that for $kd = (\omega/c)d \ll 1$ a ***differential*** pressure microphone has a ***frequency-independent*** phase shift of +90° relative to the incident sound wave (for $0 \leq \Theta \leq 90°$).

As discussed above, it is necessary to ***integrate*** the signal output from a ***differential*** pressure microphone in order to obtain a signal that is proportional to the component of the particle velocity parallel to the $\hat{n}$-axis of the device. This can be achieved electronically using a simple integrating op-amp preamplifier circuit, as shown in the figure below:


KA FET Diff Mic Preamp
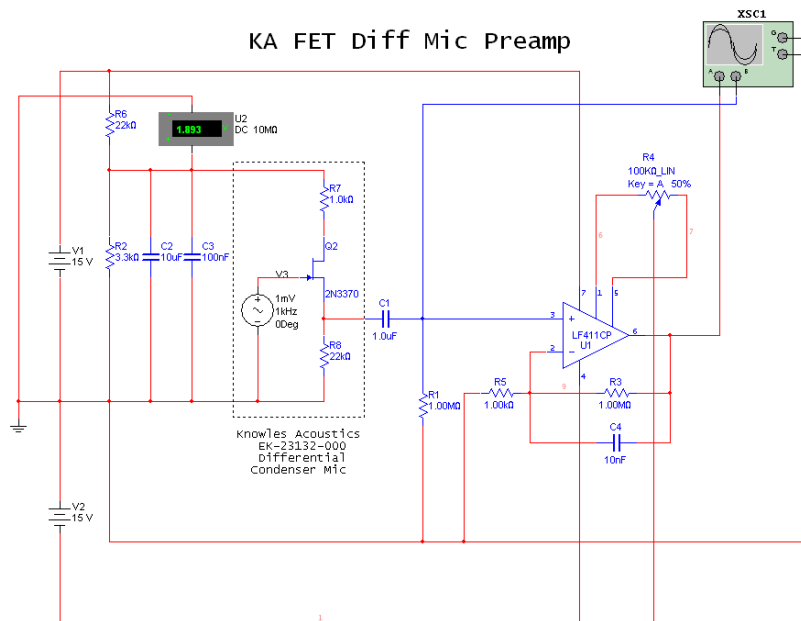
The (complex) frequency response of the simple op-amp integrator preamplifier circuit is:

$$\tilde{H}_{oai}(\omega) = (R_{fb}/R_1) \cdot \frac{1}{1+i\omega R_{fb}C} = (R_{fb}/R_1) \cdot \frac{1}{1+i\omega\tau_{fb}} \quad \left\{ \text{using: } \tau_{fb} \equiv R_{fb}C \ (sec) \right\}$$

At low frequencies, such that $\omega \ll 1/R_{fb}C = 1/\tau_{fb}$: $\tilde{H}_{oai}(\omega \ll 1/R_{fb}C) \simeq (R_{fb}/R_1)$ is a purely **_real_**

quantity – *i.e.* the output of the op-amp integrator is **_in-phase_** with the input. At high frequencies,

such that $\omega \gg 1/R_{fb}C = 1/\tau_{fb}$ then: $\tilde{H}_{oai}(\omega \gg 1/R_{fb}C) \simeq (R_{fb}/R_1) \cdot \frac{1}{i\omega R_{fb}C} = -i\frac{(R_{fb}/R_1)}{\omega R_{fb}C}$ and thus

we see that the high-frequency output response of the op-amp integrator is proportional to $1/\omega$
and is -90° **_out-of-phase_** with the input signal. The frequency and phase response of the simple
op-amp integrator circuit (alone) is shown in the figure below.
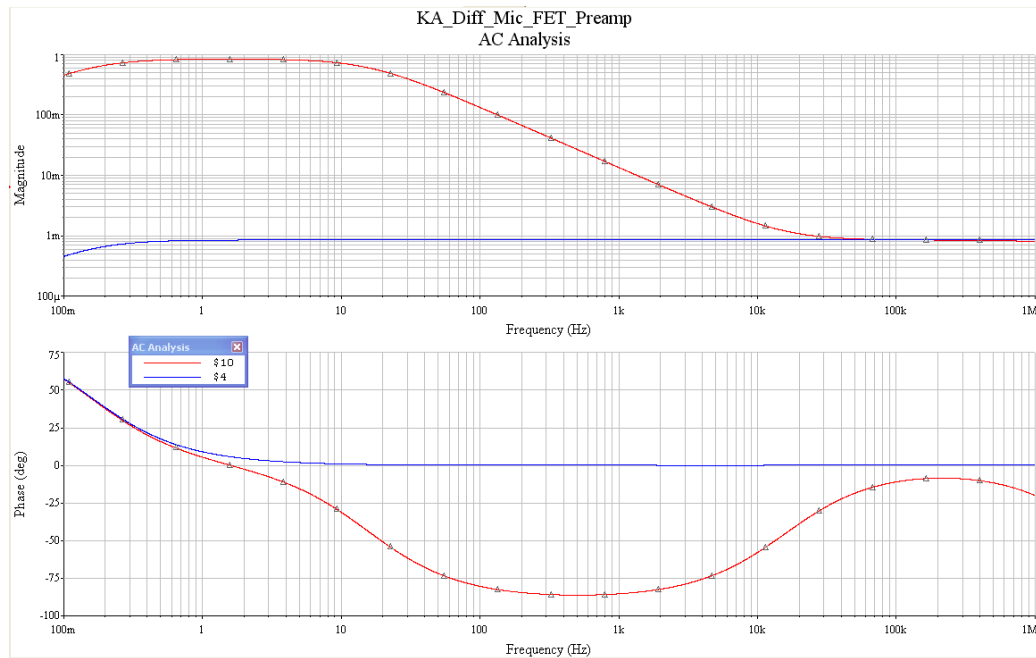
Note that the phase response of this op-amp integrator circuit is ~ constant in the frequency
range ~ $50\ Hz < f < ~4\ KHz$, which is of primary interest for most musical instruments.

This simple op-amp integrator circuit is used in conjunction with a modified version of the
Knowles Electronics EK-23132 subminiature electret condenser microphone (with back plate
removed and replaced with a very fine-mesh copper screen {electrically connected to mic case
using conductive epoxy paint for RFI/EMI suppression}) for a 1-D particle velocity microphone.
Note that well above the -3 dB low-frequency pole of the op-amp integrator circuit (*i.e.*
$\omega \gg 1/R_f C$) but still well below $\omega \ll c/d$, the combined response of the differential pressure
microphone + op-amp integrator circuit is constant, independent of (angular) frequency ω:

$$\tilde{H}_{u-mic}(\omega, \Theta) \Big|_{1/R_f C \ll \omega \ll c/d} = \tilde{H}_{diff-mic}(\omega, \Theta) \cdot \tilde{H}_{oai}(\omega) \Big|_{1/R_f C \ll \omega \ll c/d} \simeq \left(\frac{d}{c} \right) \left[\frac{R_{fb}/R_1}{R_{fb}C} \right] \cos\Theta$$

Note that because of the integrating op-amp's high gain $\left(R_{fb}/R_1 \right)$ at very low frequencies, these types of particle velocity microphones are quite sensitive to wind/drafts/convection currents and also low-frequency ventilation/room noise…

The **_Microflown_** is a *MEMS* device (first developed at the University of Twente, in the Netherlands in 1994) that responds directly to particle velocity. The heart of the device consists of two parallel, very small-diameter platinum nano-wires separated by $\sim 100 \ \mu m$, heated to a temperature of $T \sim 200$ ºC by passing a small electrical current $I \sim$ few *mA* through them, as shown in the scanning electron microscope (SEM) image below:
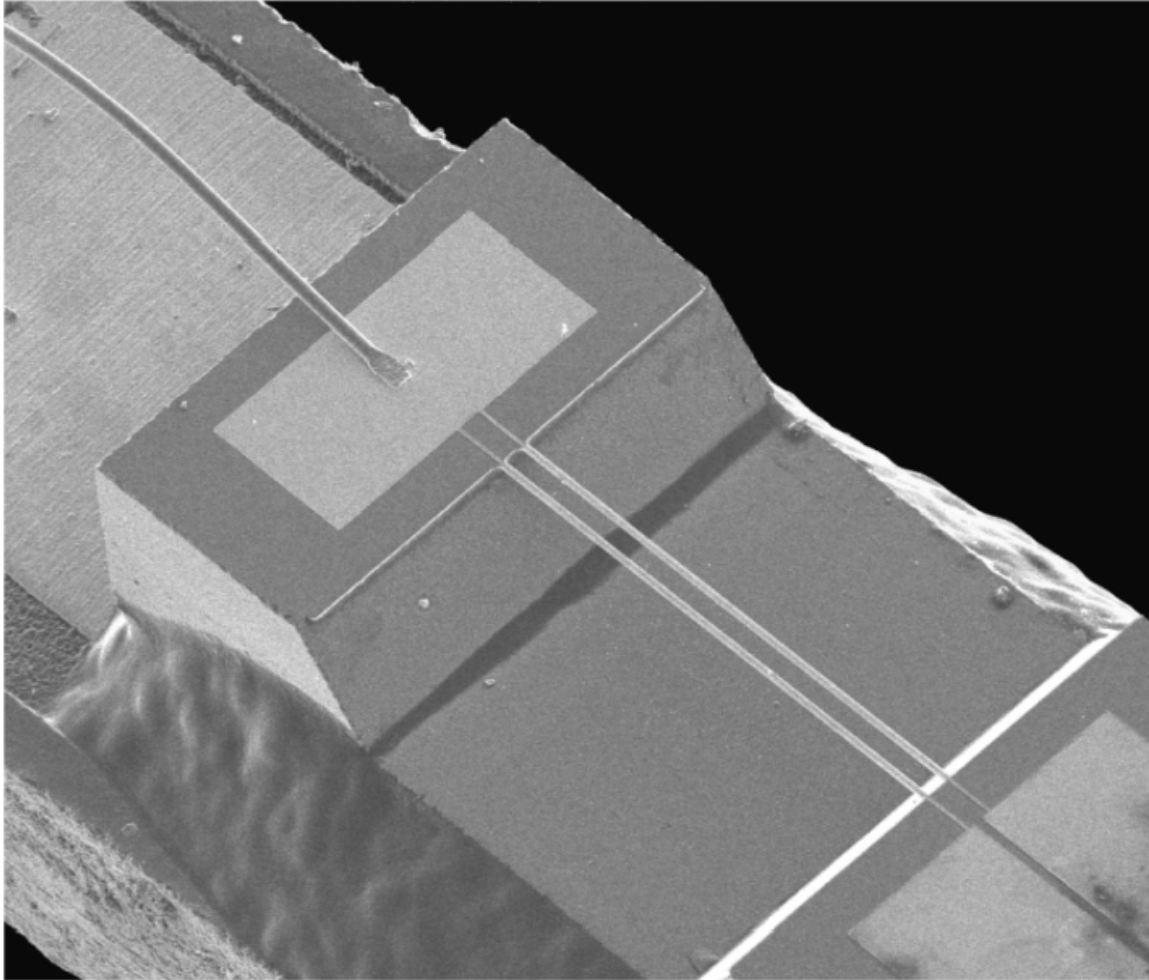
In a sound field $\tilde{S}(\vec{r},t)$ the flow of air in the local vicinity of the two wires of the Microflown produces a small ***differential*** cooling of the two wires (this effect is similar, but not identical to the principle of how a so-called hot-wire anemometer works), as shown in the figure below:
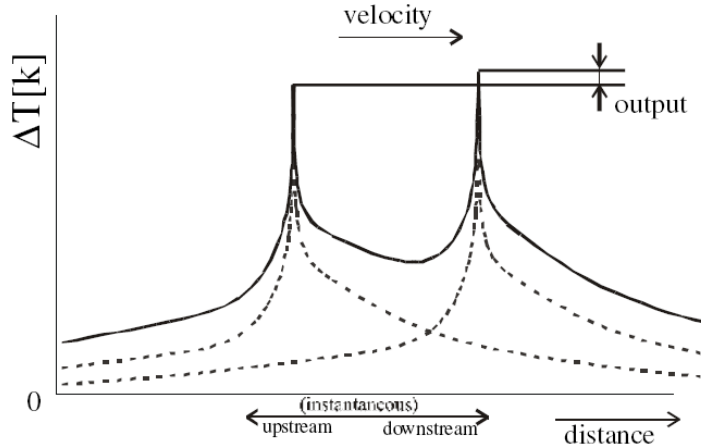
Fig. 3.4: Dotted line: temperature distribution due to convection for two heaters. Both heaters have the same temperature. Solid line: sum of two single temperature functions: a temperature difference occurs.

The Microflown operates over an eight orders-of-magnitude flow range of particle velocities, from ~ 10 *nm/s* up to ~ 1 *m/s*. Two forms of heat transport play a major role in the operation of the Microflown – heat diffusion and heat convection (*EM* radiative transport of heat is negligible in comparison). Due to convective heat transfer, the upstream platinum nano-wire is cooled more than the downstream one, and because of this the Microflown can distinquish between positive and negative air flow velocity directions, as shown in the figure below, for a harmonic sound field:
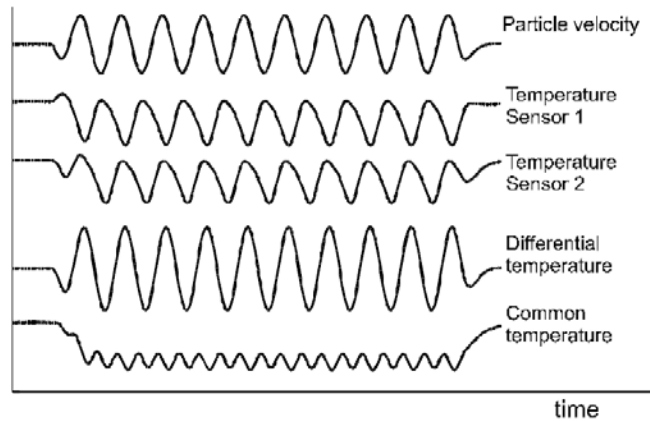

Fig. 3.5: Measured temperatures of the Microflown due to an acoustic disturbance. Due to particle velocity both temperature sensors cool down but in a different manner. The temperature difference is proportional to the particle velocity. The sum signal (common signal) represents a sort of hot wire anemometer and measures the common temperature drop. It has an illustrative double frequency that is common for hot wire anemometry.

Since the resistance of the platinum nano-wires is proportional to their temperature $\{R_w = \rho(T)\ell_w/A_\perp\}$, the differential cooling of the platinum nano-wires results in a small differential change in their resistance, $\Delta R_w(T)$. The two heated platinum nano-wires of the Microflown are connected together as half of a Wheatstone bridge, in which the junction point of the two platinum wires is connected to the base of an *NPN* bipolar junction transistor (BJT) operating as an (inverting) voltage amplifier in the so-called common emitter (CE) configuration, as shown in the figure below:
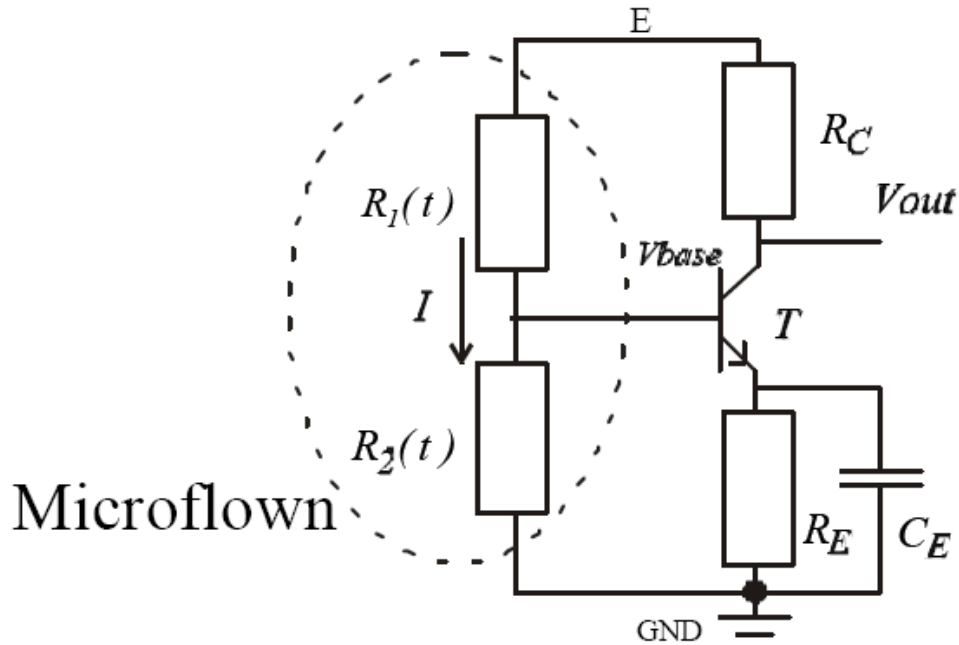

Fig. 13: CE configuration with a half Wheatstone bridge as input.

The voltage on the base of the BJT is: $V_b(t) = \frac{1}{2}E + I\Delta R_w(t)$. The BJT is used in this circuit as a voltage amplifier; the *AC* component of the output voltage on the collector of the BJT is $V_{out}^{AC}(t) \simeq -(R_C/R_E)\bullet I\Delta R_w(t)$, where the − sign denotes the 180° phase shift of the inverting nature of the BJT CE voltage amplifier circuit.

The frequency and phase response of the Microflown are not flat. At low frequencies ($f < f_{bl} \simeq 100 Hz$), the sensitivity increases 6 *dB*/octave due to the effect of the thermal boundary layer on each of the platinum nano-wires. Between $100\,Hz < f < 1\,KHz$, the frequency response of the device is ~ relatively flat. The response at higher frequencies decreases due to heat diffusion effects (*i.e.* the time heat takes to travel from one wire to the other), with a corner frequency at $f_{difn} \sim 1\,KHz$, and a second high-frequency roll-off at higher frequencies occurs due to the heat capacity (*i.e.* thermal mass) of the platinum nano-wires with a corner frequency on the order of $f_{htcp} \sim 8-20\,KHz$ (depending on the device). The overall frequency and phase response of a Microflown device is shown in the figure below:
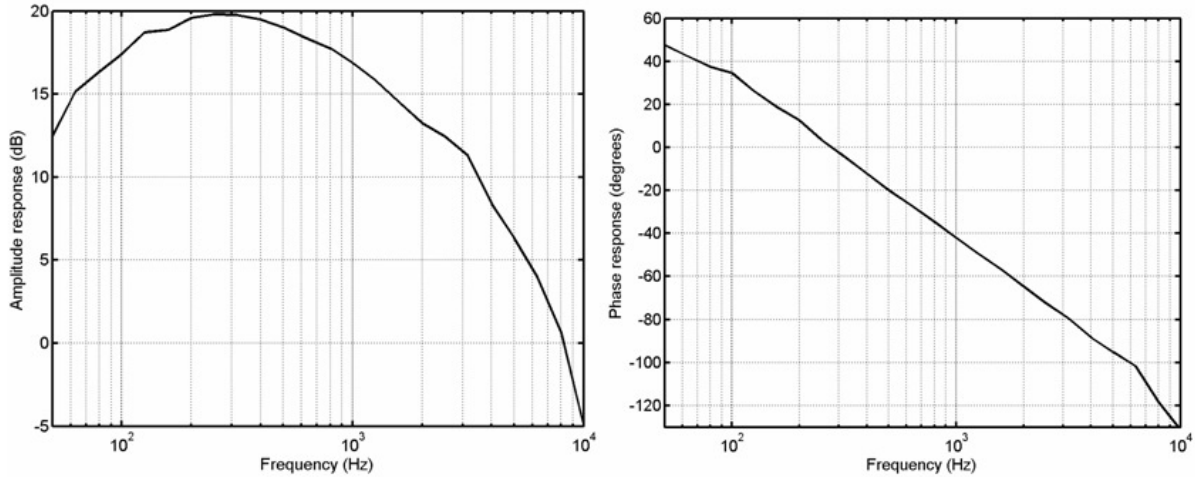
Fig. 3.12: Amplitude and phase response of a Titan element in a ½" package (from [3]).

The frequency and phase response of a Microflown can represented in terms of an electronic model of the device, with the output of an "ideal" Microflown {having flat frequency and phase reponse} subsequently altered by three passive *RC*-type networks to emulate the low frequency and high frequency behavior of the device, as shown in the figure below:
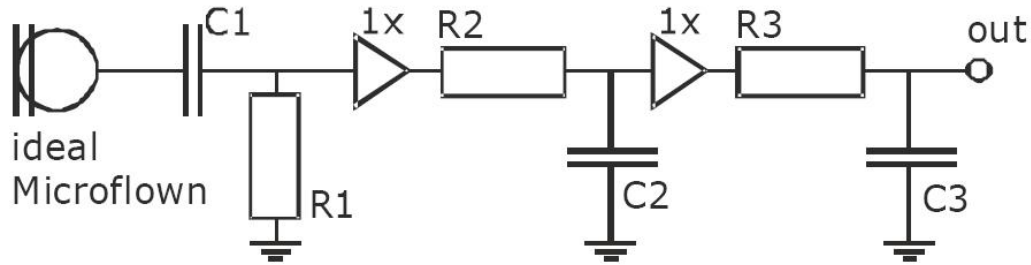

Fig. 3.6: Electrical model of the frequency dependent behaviour of a Microflown.

Using this electrical model to represent an actual Microflown, the frequency and phase response of the device are approximately given by:

$$V_{out}(f) \simeq V_{out}(250Hz) \cdot \frac{1}{\sqrt{1+\left(f_{blay}/f\right)^2}} \cdot \frac{1}{\sqrt{1+\left(f/f_{difn}\right)^2}} \cdot \frac{1}{\sqrt{1+\left(f/f_{htcp}\right)^2}}$$

and:

$$\varphi(f) \simeq \tan^{-1}\left(f_{blay}/f\right) - \tan^{-1}\left(f/f_{difn}\right) - \tan^{-1}\left(f/f_{htcp}\right)$$

A signal conditioner is used on the Microflown output signal to correct (*i.e.* undo/invert) the frequency and phase response such that both are flat at the output of the signal conditioner.

The angular/polar response of the Microflown is the same as that associated with a **_differential_** pressure microphone – *i.e.* a figure-8 pattern, as shown in the figure below for
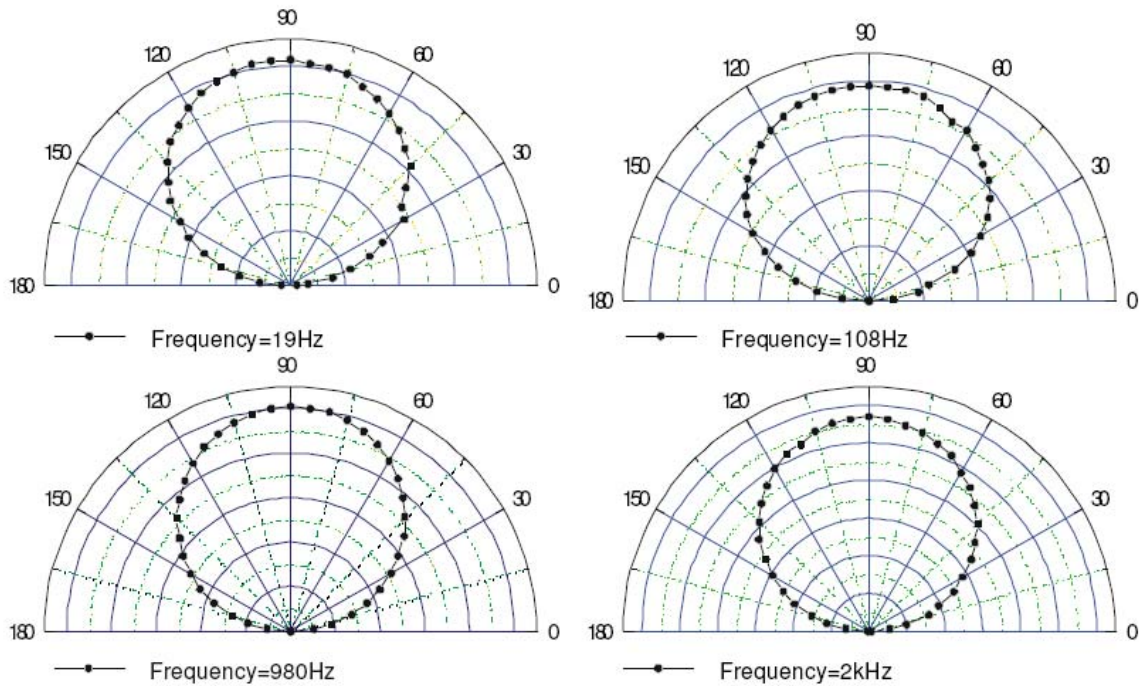
Fig. 3.21: Polar patterns of a ½" ICP Microflown at different frequencies (linear scale, only half the response is measured).

various frequencies:

   Three orthogonal Microflowns can be used to measure the local 3-D vector particle velocity $\vec{\tilde{u}}(\vec{r},t)$. Used in conjunction with a small electret condenser pressure microphone to measure the local over-pressure $p(\vec{r},t)$, the Ultimate Sound Probe (USP) was developed by Microflown, as
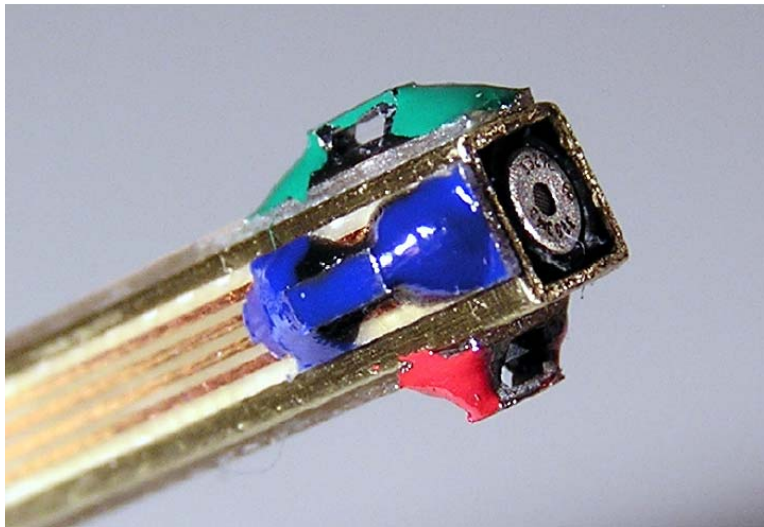
shown in the figure below:

More information about the Microflown devices, their uses and applications as well as many interesting and useful educational/technical documents can be found on Microflown's website: http://www.microflown.com.

   __Absolute__ calibration of a particle velocity microphone/measurement of the sensitivity of the particle velocity microphone S_{mic} is again carried out by placing it *e.g.* in a monochromatic ($e.g. f = 1.0$ *KHz* sine-wave) *__free-air__* sound field $\tilde{S}(\vec{r},t)$ at *NTP* with a *SPL* = 94.0 *dB* {set using *e.g.* a NIST-calibrated *SPL* meter (*C*-weighting) in proximity to the microphone}. As mentioned above, this *SPL* corresponds to an over-pressure amplitude of $p = 1.0$ *Pascals* , since $SPL(dB) = 20\log_{10}(p/p_0)$ where $p_0 = 2\times10^{-5} Pa$ is the reference pressure at the (average) threshold of human hearing. However, for a free-air sound field with *__purely-real__* longitudinal *__specific__* *__characteristic__* acoustic impedance $z_o = \rho_o c = p/u \simeq 413\,\Omega_a$, a *SPL* = 94.0 *dB* free-air sound field has a *__purely-real__* over-pressure amplitude of $p = 1.0$ *Pascals* and also has a *__purely-real__* particle velocity amplitude of $u = p/z_o \simeq 1.0/413 = 2.42\times10^{-3}\,m/s = 2.42\,mm/s$. The *AC* voltage amplitude V_{u-mic} output from the particle velocity microphone immersed in a *SPL* = 94.0 *dB* free-air sound field can be easily measured *e.g.* using an oscilloscope or a true *RMS* digital multi-meter.

   If the measured *AC* voltage amplitude output from a particle velocity microphone immersed in a *SPL* = 94.0 *dB* free-air sound field is $V_{u-mic} = xx.x\,mV$, the *__sensitivity__* of this particle velocity microphone is thus: $S_{u-mic} \equiv V_{u-mic}/u = xx.x\,mV/2.42\,mm/s = xx.x/2.42\;mV/(mm/s)$

Appendix A: Forces Acting on a Microphone Diaphragm:

   There is almost always more than *__just__* the force associated with an over-pressure $\vec{\tilde{F}}_{pressure}(t) \simeq \tilde{p}_o \vec{A}_{dia} e^{i\omega t} = \tilde{p}_o A_{dia} e^{i\omega t}\hat{n}$ acting on the diaphragm of a microphone. In general, there can be (*a*.) *__velocity-dependent__* force(s) associated with dissipative/viscous/frictional processes – *i.e.* force term(s) of the form $\vec{\tilde{F}}_{dissipate}(t) = -m_{dia}\gamma_{dia}\vec{\tilde{v}}_{dia}(t)$ that oppose the driving motion, where the so-called damping "constant" γ_{dia} has *SI* units of sec^{-1}. In addition, there can be (*b*.) "spring-like" restoring forces $\vec{\tilde{F}}_{restore}(t) = -k_{dia}\vec{\tilde{x}}_{dia}(t)$ that are linearly proportional to the *__displacement__* of the diaphragm $\vec{\tilde{x}}_{dia}(t) = \tilde{x}_{dia}(t)\hat{n}$ along its axis $\hat{n}$, where $k_{dia}(N/m)$ is the spring constant associated with the restoring force. Thus, a somewhat more realistic situation associated with forces acting on the diaphragm of a microphone, Newton's 2nd law becomes:

$$\vec{\tilde{F}}_{tot}(t) = \vec{\tilde{F}}_{pressure}(t) + \vec{\tilde{F}}_{dissipate}(t) + \vec{\tilde{F}}_{restore}(t) = m_{dia}\vec{\tilde{a}}_{dia}(t)$$

We can rewrite this as: $m_{dia}\vec{\tilde{a}}_{dia}(t) - \vec{\tilde{F}}_{dissipate}(t) - \vec{\tilde{F}}_{restore}(t) = \vec{\tilde{F}}_p(t)$

Then: $m_{dia}\vec{\tilde{a}}_{dia}(t) + m_{dia}\gamma_{dia}\vec{\tilde{v}}_{dia}(t) + k_{dia}\vec{\tilde{x}}_{dia}(t) = \tilde{p}(t)\vec{A}_{dia}$

Since all forces act along/against the same direction (*i.e.* the $\hat{n}$-axis = the outward pointing normal to the plane of the microphone diaphragm), then using the relations:

$$\vec{v}_{dia}(t)=\partial\vec{x}_{dia}(t)/\partial t\equiv\dot{\vec{x}}_{dia}(t)\ \ \text{and:}\ \ \vec{a}_{dia}(t)=\partial\vec{v}_{dia}(t)/\partial t=\partial^2\vec{x}_{dia}(t)/\partial t^2\equiv\ddot{\vec{x}}_{dia}(t),$$

and dropping the vector arrows, we arrive at the 2nd order ***inhomogeneous*** differential equation of 1-D motion of the microphone diaphragm:

$$m_{dia}\ddot{\tilde{x}}_{dia}(t)+m_{dia}\gamma_{dia}\dot{\tilde{x}}_{dia}(t)+k_{dia}\tilde{x}_{dia}(t)=\tilde{p}(t)A_{dia}$$

The general solution to this equation for steady-state driven harmonic motion of the diaphragm is of the form $\tilde{x}_{dia}(t)=\tilde{x}_o(\omega)e^{i\omega t}$. For un-damped, non-driven (*i.e.* $\gamma_{dia}=0$ and $\tilde{p}(t)=0$) steady-state motion of the diaphragm, the natural resonant angular frequency is $\omega_o\equiv\sqrt{k_{dia}/m_{dia}}$. For damped and driven steady-state harmonic motion (*i.e.* $\gamma_{dia}\neq0$ and $\tilde{p}(t)=p_oe^{i\omega t}\neq0$):

$$-\omega^2m_{dia}\tilde{x}_o(\omega)e^{i\omega t}+i\omega m_{dia}\gamma_{dia}\tilde{x}_o(\omega)e^{i\omega t}+k_{dia}\tilde{x}_o(\omega)e^{i\omega t}=p_oe^{i\omega t}A_{dia}$$

or: $$\left[-\omega^2m_{dia}+i\omega m_{dia}\gamma_{dia}+k_{dia}\right]\tilde{x}_o(\omega)=p_oA_{dia}$$

Dividing though by m_{dia}: $\left[-\omega^2+i\omega\gamma_{dia}+(k_{dia}/m_{dia})\right]\tilde{x}_o(\omega)=p_oA_{dia}/m_{dia}$

Using the natural angular resonant frequency relation $\omega_o^2\equiv(k_{dia}/m_{dia})$:

$$\left[-\omega^2+i\omega\gamma_{dia}+\omega_o^2\right]\tilde{x}_o(\omega)=p_oA_{dia}/m_{dia}$$

or:

$$\tilde{x}_o(\omega)=\left(\frac{p_oA_{dia}}{m_{dia}}\right)\frac{1}{\left[(\omega_o^2-\omega^2)+i\omega\gamma_{dia}\right]}=\left(\frac{p_oA_{dia}}{m_{dia}}\right)\frac{1}{\left[(\omega_o^2-\omega^2)+i\omega\gamma_{dia}\right]}\cdot\frac{\left[(\omega_o^2-\omega^2)-i\omega\gamma_{dia}\right]}{\left[(\omega_o^2-\omega^2)-i\omega\gamma_{dia}\right]}$$

$$=\left(\frac{p_oA_{dia}}{m_{dia}}\right)\frac{\left[(\omega_o^2-\omega^2)-i\omega\gamma_{dia}\right]}{(\omega_o^2-\omega^2)^2+\omega^2\gamma_{dia}^2}$$

The ***real/in-phase component*** of the complex displacement amplitude $\tilde{x}_o(\omega)$ of the microphone diaphragm is:

$$x_{o\,r}(\omega)\equiv\text{Re}\{\tilde{x}_o(\omega)\}=\left(\frac{p_oA_{dia}}{m_{dia}}\right)\frac{(\omega_o^2-\omega^2)}{(\omega_o^2-\omega^2)^2+\omega^2\gamma_{dia}^2}$$

The ***imaginary/quadrature component*** of the complex displacement amplitude $\tilde{x}_o(\omega)$ of the microphone diaphragm is:

$$x_{o\,i}(\omega)\equiv\text{Im}\{\tilde{x}_o(\omega)\}=-\left(\frac{p_oA_{dia}}{m_{dia}}\right)\frac{\omega\gamma_{dia}}{(\omega_o^2-\omega^2)^2+\omega^2\gamma_{dia}^2}$$

The **_phase_** of the complex displacement amplitude $\tilde{x}_o(\omega)$ of the microphone diaphragm is:

$$\varphi_x(\omega) \equiv \tan^{-1}\left(\frac{\text{Im}\{\tilde{x}_o(\omega)\}}{\text{Re}\{\tilde{x}_o(\omega)\}}\right) = \tan^{-1}\left(\frac{-\omega\gamma_{dia}}{(\omega_o^2 - \omega^2)}\right) = -\tan^{-1}\left(\frac{\omega\gamma_{dia}}{(\omega_o^2 - \omega^2)}\right)$$

The **_magnitude_** of the complex displacement amplitude $\tilde{x}_o(\omega)$ of the microphone diaphragm is:

$$\left|\tilde{x}_o(\omega)\right| \equiv \sqrt{x_{or}^2(\omega) + x_{oi}^2(\omega)} = \left(\frac{p_o A_{dia}}{m_{dia}}\right)\frac{\sqrt{(\omega_o^2 - \omega^2)^2 + \omega^2\gamma_{dia}^2}}{(\omega_o^2 - \omega^2)^2 + \omega^2\gamma_{dia}^2} = \left(\frac{p_o A_{dia}}{m_{dia}}\right)\frac{1}{\sqrt{(\omega_o^2 - \omega^2)^2 + \omega^2\gamma_{dia}^2}}$$
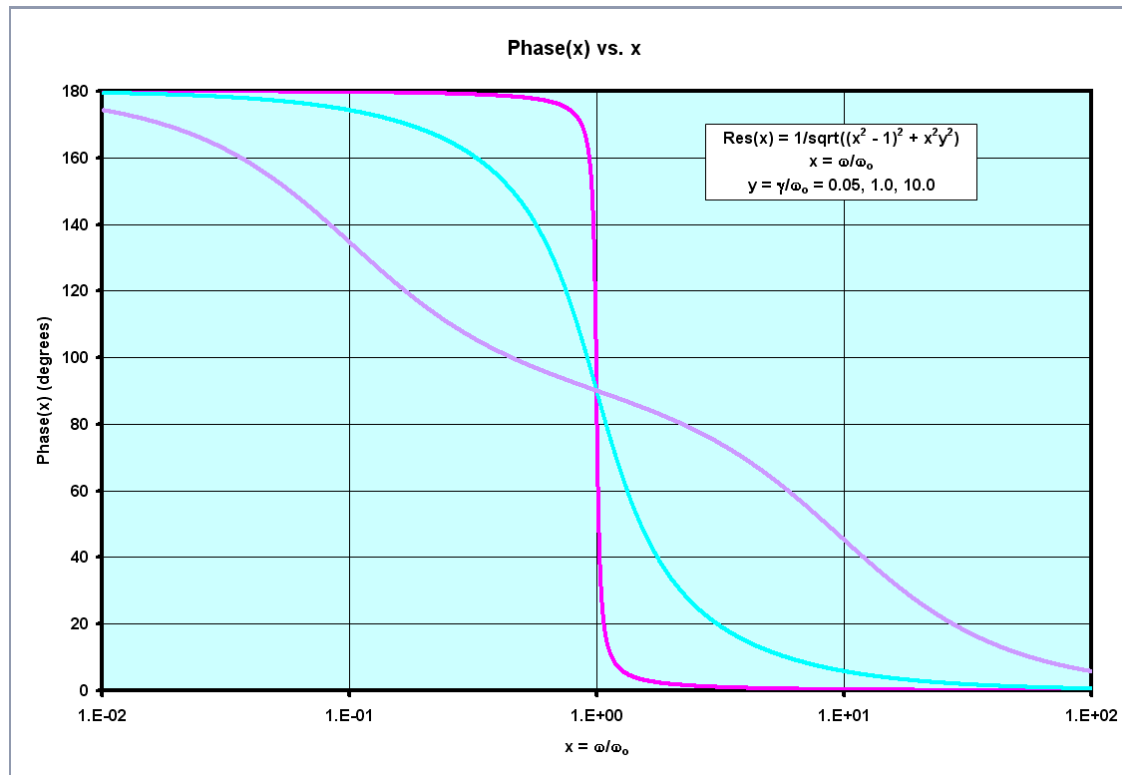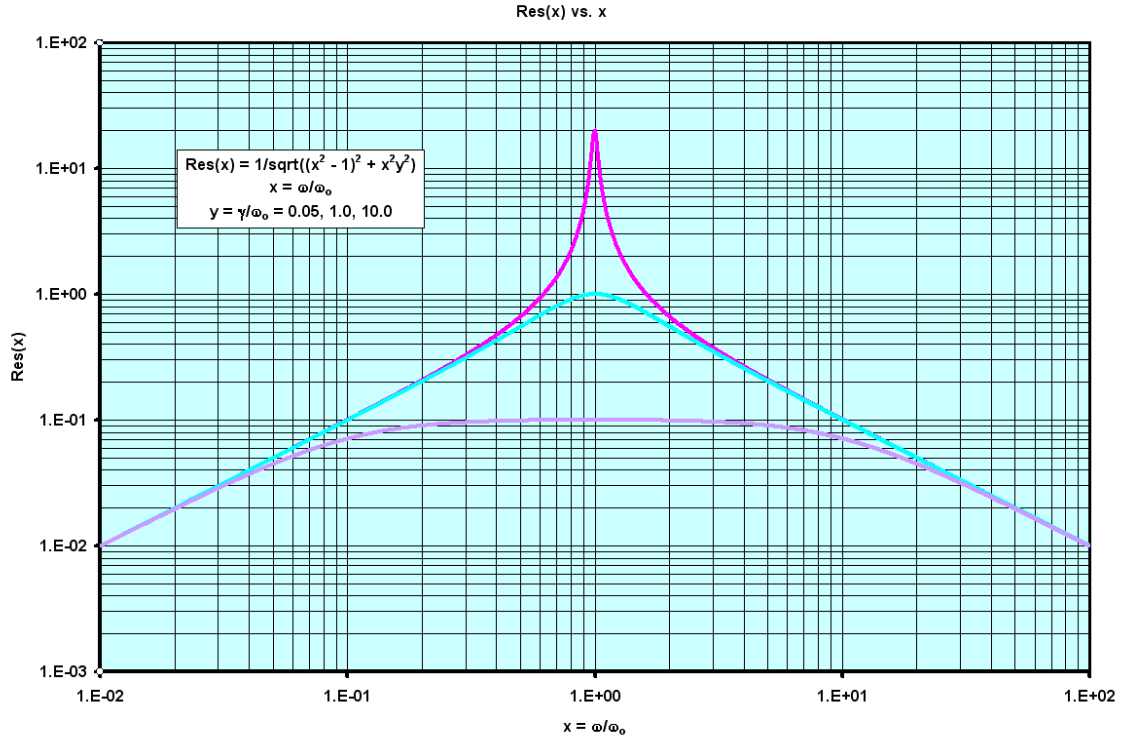
   In order for a microphone to be effective as a pressure transducer, the face of the diaphragm of the microphone must (*a.*) be nearly massless – *i.e.* $m_{dia} \sim 0$ and (*b.*) have great stiffness, so that the diaphragm acts like a piston. On the other hand, the supporting edge of the diaphragm, where it is attached to the rigid structural body of the microphone must be reasonably compliant, in order to allow the back-and-forth motion of the diaphragm along its $\hat{n}$-axis *e.g.* in response to a harmonic over-pressure amplitude. Let us see what is required in terms of damping the motion of the diaphragm.

   If we define $x(\omega) \equiv \omega/\omega_o$ and $y \equiv \gamma_{dia}/\omega_o$ then we can rewrite the {normalized} magnitude of the complex displacement amplitude $\tilde{x}_o(\omega)$ and phase as:

$$\text{Res}(x(\omega)) \equiv \frac{\omega_o^2 \left|\tilde{x}_o(\omega)\right|}{\left(\frac{p_o A_{dia}}{m_{dia}}\right)} = \frac{1}{\sqrt{\left[1 - \left(\frac{\omega}{\omega_o}\right)^2\right]^2 + \left(\frac{\omega}{\omega_o}\right)^2\left(\frac{\gamma_{dia}}{\omega_o}\right)^2}} = \frac{1}{\sqrt{(1 - x^2)^2 + x^2 y^2}}$$

$$\varphi_x(\omega) = -\tan^{-1}\left(\frac{\omega\gamma_{dia}}{(\omega_o^2 - \omega^2)}\right) = -\tan^{-1}\left(\frac{\left(\frac{\omega}{\omega_o}\right)\left(\frac{\gamma_{dia}}{\omega_o}\right)}{1 - \left(\frac{\omega}{\omega_o}\right)^2}\right) = -\tan^{-1}\left(\frac{x \cdot y}{1 - x^2}\right)$$

   We have the usual three conditions of **_underdamped_**, **_critically damped_** and **_overdamped_** motion for $y \equiv \gamma_{dia}/\omega_o \ll 1$, $y \equiv \gamma_{dia}/\omega_o = 1$ and $y \equiv \gamma_{dia}/\omega_o \gg 1$, respectively. The following two color-coded figures show normalized Res(*x*) *vs.* *x* and Phase(*x*) *vs.* *x* for *y* = 0.05 (pink), 1.0 (cyan) and 10.0 (violet):

Res(x) vs. x

$$\text{Res}(x) = 1/\text{sqrt}((x^2 - 1)^2 + x^2 y^2)$$
$$x = \omega/\omega_o$$
$$y = \gamma/\omega_o = 0.05, 1.0, 10.0$$

Phase(x) vs. x

$$\text{Res}(x) = 1/\text{sqrt}((x^2 - 1)^2 + x^2 y^2)$$
$$x = \omega/\omega_o$$
$$y = \gamma/\omega_o = 0.05, 1.0, 10.0$$

From the above two figures, we see that if we **_overdamp_** the diaphragm of the microphone ($y \equiv \gamma_{dia}/\omega_o = 10$, violet curves), we achieve a region of flat frequency response, albeit with **_non-constant_** phase shift over the {fractional} frequency range $0.01 \leq x(\omega) \equiv \omega/\omega_o \leq 100$. These response curves are relevant for the design of microphones whose output signal response $\tilde{V}_{mic}(t)$ to an over-pressure $\tilde{p}(\vec{r},t)$ that is linearly proportional to the **_displacement_** of the diaphragm, $\tilde{x}_{dia}(t)$ - *e.g* **_electret condenser-type_** microphones.

The output responses of **_dynamic_** and **_ribbon-type_** microphones to an over-pressure $\tilde{p}(t)$ are linearly proportional to the **_velocity_** of the diaphragm of the microphone, $\tilde{v}_{dia}(t) = \partial \tilde{x}_{dia}(t)/\partial t$, since *e.g.* a time-varying *EMF* (*i.e.* voltage) $\tilde{\varepsilon}_{mic}(t) = |\vec{\tilde{v}}_{dia}(t)| \cdot |\vec{B}_o| \cdot \ell_{ribbon}$ {where $\ell_{ribbon}(m)$ is the length of the metal ribbon} is produced across the top/bottom of the corrugated metal ribbon diaphragm due to the instantaneous differential over-pressure $\Delta \tilde{p}_{ribbon}(t)$ acting on it.

We have already seen that a harmonically-varying overpressure amplitude at the diaphragm of the microphone $\tilde{p}(t) = p_o e^{i\omega t}$ results in a harmonically-varying displacement of the microphone $\tilde{x}_{dia}(t) = \tilde{x}_o(\omega)e^{i\omega t}$. Thus $\tilde{v}_{dia}(t) = \partial \tilde{x}_{dia}(t)/\partial t = i\omega \tilde{x}_o(\omega)e^{i\omega t} = i\omega \tilde{x}_{dia}(t)$, *i.e.* the velocity of the diaphragm $\tilde{v}_{dia}(t)$ is $+90^o$ ahead in phase relative to the displacement of the diaphragm, $\tilde{x}_{dia}(t)$ and is also linearly proportional to the (angular) frequency $\omega = 2\pi f$. We also see that $\tilde{v}_{dia}(t) = \partial \tilde{x}_{dia}(t)/\partial t = i\omega \tilde{x}_o(\omega)e^{i\omega t} = i\omega \tilde{x}_{dia}(t) = \tilde{v}_o(\omega)e^{i\omega t}$ and hence that $\tilde{v}_o(\omega) = i\omega \tilde{x}_o(\omega)$. Thus the complex velocity amplitude of the diaphragm of a microphone is:

$$\tilde{v}_o(\omega) = i\omega \tilde{x}_o(\omega) = i\omega \left(\frac{p_o A_{dia}}{m_{dia}} \right) \frac{\left[(\omega_o^2 - \omega^2) - i\omega\gamma_{dia} \right]}{(\omega_o^2 - \omega^2)^2 + \omega^2\gamma_{dia}^2} = \omega \left(\frac{p_o A_{dia}}{m_{dia}} \right) \frac{\left[\omega\gamma_{dia} + i(\omega_o^2 - \omega^2) \right]}{(\omega_o^2 - \omega^2)^2 + \omega^2\gamma_{dia}^2}$$

The **_real/in-phase component_** of the velocity amplitude $\tilde{v}_o(\omega)$ of the microphone diaphragm is:

$$v_{or}(\omega) \equiv \text{Im}\{\tilde{v}_o(\omega)\} = \omega \left(\frac{p_o A_{dia}}{m_{dia}} \right) \frac{\omega\gamma_{dia}}{(\omega_o^2 - \omega^2)^2 + \omega^2\gamma_{dia}^2}$$

The **_imaginary/quadrature component_** of the velocity amplitude $\tilde{v}_o(\omega)$ of the microphone diaphragm is:

$$v_{oi}(\omega) \equiv \text{Re}\{\tilde{v}_o(\omega)\} = \omega \left(\frac{p_o A_{dia}}{m_{dia}} \right) \frac{(\omega_o^2 - \omega^2)}{(\omega_o^2 - \omega^2)^2 + \omega^2\gamma_{dia}^2}$$

The **_phase_** of the velocity amplitude $\tilde{v}_o(\omega)$ of the microphone diaphragm is:

$$\varphi_v(\omega) \equiv \tan^{-1}\left(\frac{\text{Im}\{\tilde{v}_o(\omega)\}}{\text{Re}\{\tilde{v}_o(\omega)\}} \right) = \tan^{-1}\left(\frac{(\omega_o^2 - \omega^2)}{\omega\gamma_{dia}} \right)$$

The ***magnitude*** of the complex velocity amplitude $\tilde{v}_o(\omega)$ of the microphone diaphragm is:

$$\left|\tilde{v}_o(\omega)\right| \equiv \sqrt{v_{or}^2(\omega)+v_{oi}^2(\omega)} = \omega\left(\frac{p_o A_{dia}}{m_{dia}}\right)\frac{1}{\sqrt{\left(\omega_o^2-\omega^2\right)^2+\omega^2\gamma_{dia}^2}}$$

If we again define $x(\omega)\equiv\omega/\omega_o$ and $y\equiv\gamma_{dia}/\omega_o$ then we can rewrite the {normalized} magnitude of the complex velocity amplitude $\tilde{v}_o(\omega)$ and v-phase as:

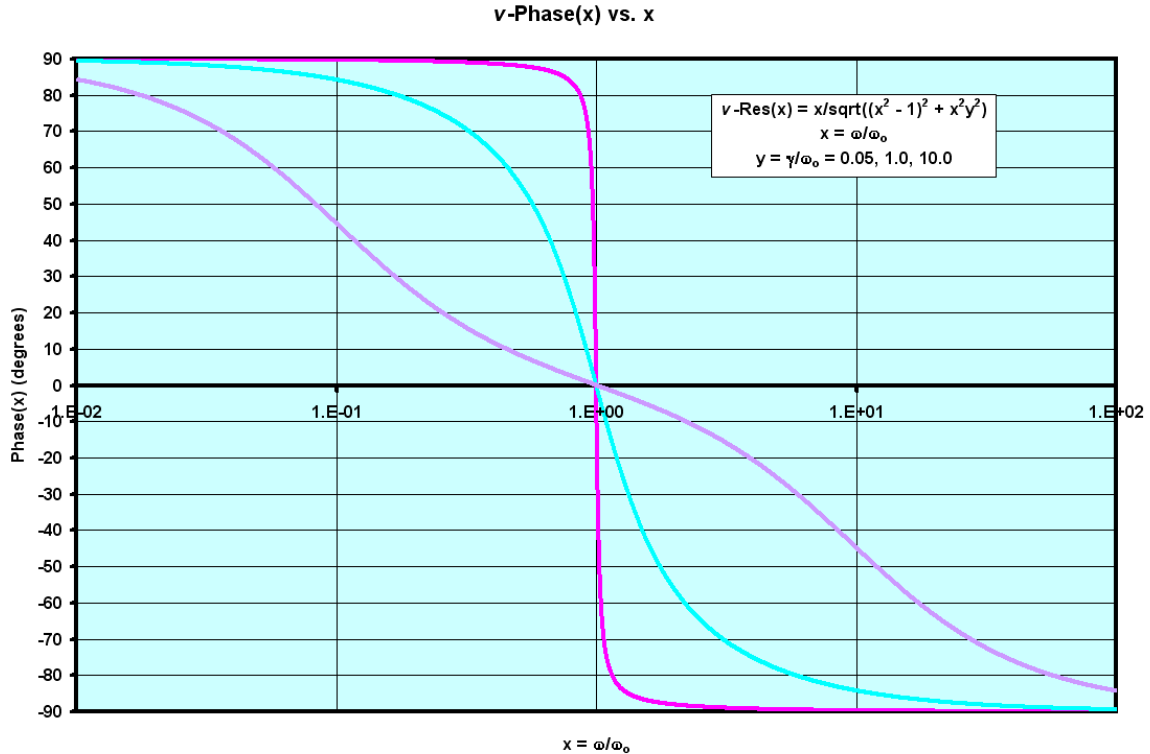$$v\text{-Res}(x(\omega)) \equiv \frac{\omega_o\left|\tilde{v}_o(\omega)\right|}{\left(\frac{p_o A_{dia}}{m_{dia}}\right)} = \frac{\left(\frac{\omega}{\omega_o}\right)}{\sqrt{\left[1-\left(\frac{\omega}{\omega_o}\right)^2\right]^2+\left(\frac{\omega}{\omega_o}\right)^2\left(\frac{\gamma_{dia}}{\omega_o}\right)^2}} = \frac{x}{\sqrt{\left(1-x^2\right)^2+x^2 y^2}}$$

$$\varphi_v(\omega) = \tan^{-1}\left(\frac{\left(\omega_o^2-\omega^2\right)}{\omega\gamma_{dia}}\right) = \tan^{-1}\frac{1-\left(\frac{\omega}{\omega_o}\right)^2}{\left(\frac{\omega}{\omega_o}\right)\left(\frac{\gamma_{dia}}{\omega_o}\right)} = \tan^{-1}\left(\frac{1-x^2}{x\cdot y}\right)$$

We again have the usual three conditions of ***underdamped***, ***critically damped*** and ***overdamped*** motion for $y\equiv\gamma_{dia}/\omega_o\ll1$, $y\equiv\gamma_{dia}/\omega_o=1$ and $y\equiv\gamma_{dia}/\omega_o\gg1$, respectively. The following two color-coded figures show normalized v-Res(x) vs. x and v-Phase(x) vs. x for y = 0.05 (pink), 1.0 (cyan) and 10.0 (violet):

_v_-Phase(x) vs. x
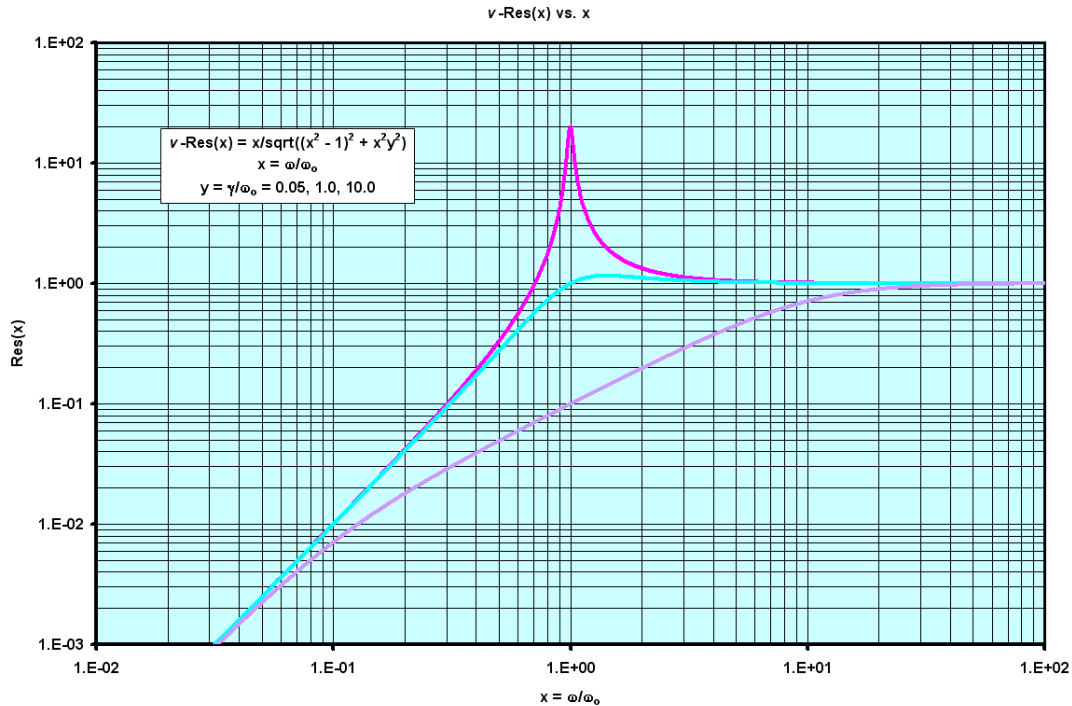
We see from the above two figures that for dynamic and/or ribbon microphones, whose output signal response $\tilde{\varepsilon}(t)$ to an over-pressure $\tilde{p}(t)$ is linearly proportional to the **_velocity_** of the diaphragm, $\tilde{v}_{dia}(t)$ if the diaphragm of these types of microphone (*a.*) has a **_sub_-_sonic_** resonance (*i.e.* $f_o \equiv \omega_o/2\pi < 20\ Hz$ and (*b.*) is **_critically_-_damped_** ($y \equiv \gamma_{dia}/\omega_o = 1.0$, cyan curve), then we are able to achieve a region of flat frequency response, albeit again with **_non_-_constant_** phase shift over the {fractional} frequency range $0.01 \le x(\omega) \equiv \omega/\omega_o \le 100$.

The above discussion(s) of damped simple harmonic oscillator-type descriptions of the frequency-dependent behavior of various types of microphone diaphragms are in fact quite simplistic – **_real_** microphones are considerably more complex than that given in the above discussion(s). Equivalent electronic circuit models are often used to analytically (and/or numerically) compute the frequency and phase response of such microphones. The figure below shows the equivalent electronic circuit used for modeling the behavior of a ribbon-type microphone:

Acoustical circuit (impedance analogy)
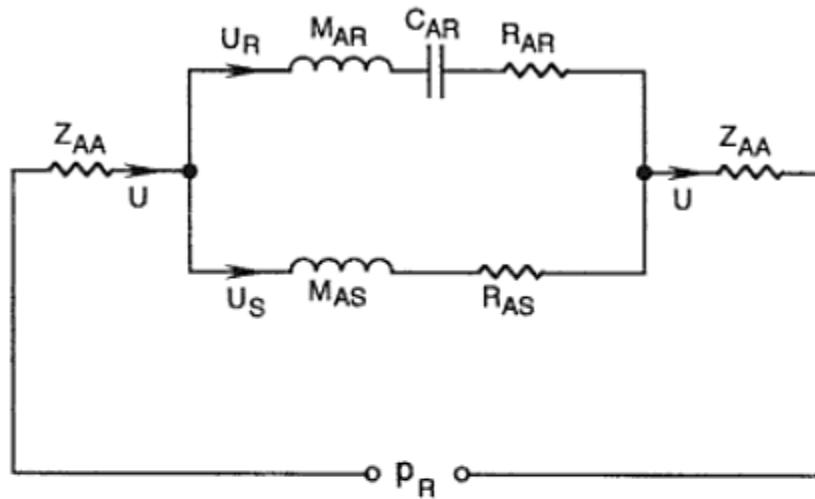
Where:

Z_{AA} = acoustical impedance of medium on one side of diaphragm
U = volume velocity through microphone
U_R = volume velocity of ribbon
U_S = volume velocity of air through slits at edges of ribbon
M_{AR} = acoustic mass of ribbon
C_{AR} = acoustic compliance of ribbon
R_{AR} = acoustic resistance of ribbon
M_{AS} = acoustic mass of slots
R_{AS} = acoustic resistance of slots
p_R = pressure gradient at ribbon

Much information on how microphones work, equivalent circuit modeling of microphones, *etc*. is discussed in **The Microphone Handbook**, 2*nd* Ed. by John Eargle, Elsevier, 2004.

Legal Disclaimer and Copyright Notice:

Legal Disclaimer:

The author specifically disclaims legal responsibility for any loss of profit, or any consequential, incidental, and/or other damages resulting from the mis-use of information contained in this document. The author has made every effort possible to ensure that the information contained in this document is factually and technically accurate and correct.

Copyright Notice:

The contents of this document are protected under both United States of America and International Copyright Laws. No portion of this document may be reproduced in any manner for commercial use without prior *written* permission from the author of this document. The author grants permission for the use of information contained in this document for *private*, *non-commercial* purposes only.