## **The Fractal Nature of Human Music**

At the very beginning of/first day's lecture for this course, I posed the question: What <u>is</u> human music? Is it "just" an aesthetically pleasing <u>sequence</u> of tones, with some kind of rhythm/beat to it? If so, precisely <u>why</u> is a rhythmic <u>sequence</u> of musical tones aesthetically pleasing to our ears? Or, is our human music just some kind of "<u>auditory cheesecake</u>"?

We have discussed <u>some</u> of the underlying aspects of human music over the course of the semester – that our music is "anthropocentric" in nature (since we humans are primarily interested in our own species – just as <u>all</u> creatures living on this planet are primarily interested in <u>their</u> own species) – and that the sequences of musical notes associated with human music, with its consonant {and dissonant} tonal combinations, our musical scale(s) with their associated frequency intervals between successive notes do indeed reflect (and are derived from) the integer-related  $f_n = nf_1$  harmonic content of the <u>human voice</u>, which, in turn arises from the 1-D mechanical vibrational nature of the eigen-modes of the human vocal chords. The musical instruments that we humans have developed over the millennia artistically mimic the human voice (some instruments to a greater degree than others...); the temporal aspects of human music anthropocentrically and artistically reflect our own internal <u>human rhythms</u> – e.g. heartbeat / pulse, breathing, running/walking, etc. via artistic use of the percussion family of musical instruments in our music...

However, the nature of human music goes even *deeper* than just these aspects...

Over ~ the past century or so, there has been a "quiet" (*i.e.* under-appreciated) revolution in our understanding of the nature of a wide range of physical phenomena in our universe. There currently exists an already lengthy and steadily-growing list of processes that exhibit non-trivial temporal correlations – *i.e.* that the instantaneous state of a system in the <u>here-and-now</u> is dependent on what happened in the <u>past</u>, and whatever happened in the <u>past</u> will also indeed have an effect on the system in the <u>future</u>. However, such dynamical processes are not <u>purely</u> deterministic, but instead have intrinsic "noise" fluctuations associated with them.

Some examples of physical systems exhibiting 1/f noise are numerous electrical components, from vacuum tubes, carbon-composition resistors, op-amps, thin films, to giant magneto-resistance sensors/transducers, as well as terrestrial weather patterns (*e.g.* rainfall, annual flooding of the Nile River, ocean surface temperatures...), astrophysical phenomena (*e.g.* sunspots, cosmic microwave background, *x*-ray emission from Seyfert galaxies (thought to contain super-massive black holes at their centers), the interplanetary magnetic field), the geophysical record of the earth, earthquakes, agriculture (*e.g.* fluctuations in annual crop yields), chemical reactions, phase transitions, radioactive decays, optical systems (*e.g.* photon counting, lasers), traffic flow, financial transactions, signals in *myelinated* nerves, heartbeat, EEG... as well as 1/f noise – long-range temporal correlations in human music – in pitch (frequency), amplitude (volume/loudness) as well as tempo/rhythm (phase)!

## Some Examples of Physical Systems Exhibiting 1/f Noise:



Log Frequency

### **Fractional Brownian Motion, Fractional Brownian Noise**

A dynamical system that obeys fractional Brownian noise (fBn), *e.g.* electrical noise in a carbon composition resistor, is a single-valued function of time V(t). The <u>increments</u> of the dynamical system from one moment to the next  $\Delta V(\Delta t) = V(t_2) - V(t_1)$  obey a Gaussian probability distribution function (PDF):

$$f\left(\Delta V\left(\Delta t\right);\sigma_{\Delta V}\right) = \frac{1}{\sqrt{2\pi}\sigma_{\Delta V}}e^{-\Delta V^{2}(\Delta t)/2\pi\sigma_{\Delta V}^{2}}$$

The frequency-domain power spectral density (PSD) function  $S_V(f)$  associated with the time-domain fluctuating quantity V(t) is a measure of the mean squared variation (*i.e.* variance)  $\langle V^2(t) \rangle$  of V(t) in a unit bandwidth centered on the frequency f. Note that if the physical units of V(t) are *e.g. Volts*, the units of the PSD function  $S_V(f)$  are *Volts*<sup>2</sup>/Hz.

Another way to characterize the average behavior of V(t) is via the use of the time-domain auto-correlation (*aka* self-correlation) function  $\langle V(t)V(t+\Delta t)\rangle$ , which is a measure of how the fluctuating quantities V(t) and  $V(t+\Delta t)$  are related to each other as a function of the delay time difference  $\Delta t \equiv t_2 - t_1$ . For a stationary (*i.e.* time-independent) process, the autocorrelation function  $\langle V(t)V(t+\Delta t)\rangle$  is independent of t and depends only on the time difference  $\Delta t$ .

The frequency-domain PSD function  $S_V(f)$  and time-domain auto-correlation function  $\langle V(t)V(t+\Delta t)\rangle$  are related to each other by the Wiener-Khintchine relations:

$$\langle V(t)V(t+\Delta t)\rangle = \int_{f=0}^{f=\infty} S_V(f)\cos(2\pi f\Delta t)df$$

and:

$$S_{V}(f) = 4 \int_{\Delta t=0}^{\Delta t=\infty} \left\langle V(t)V(t+\Delta t) \right\rangle \cos\left(2\pi f \Delta t\right) d(\Delta t)$$

Many (but by no means all) fluctuating quantities V(t) can be characterized by a <u>single</u> correlation time constant  $\tau_c$ . The classic example of Brownian "random walk" motion of pollen grains in water <u>is</u> characterized by a single time constant  $\tau_c$ , the mean time between successive collisions. The fluctuating quantity V(t) is thus <u>correlated</u> with  $V(t + \Delta t)$  for <u>short</u> time differences  $\Delta t \ll \tau_c$ , but is *independent* of  $V(t + \Delta t)$  for *long* time differences  $\Delta t \gg \tau_c$ . This in turn implies that the PSD function  $S_V(f)$  is "white" (*i.e.* flat, independent of frequency) in the frequency range over which V(t) is independent, *i.e.*  $f = 1/\Delta t \ll 1/(2\pi\tau_c)$ . The PSD function  $S_{V}(f)$  decreases rapidly with increasing frequency (typically as  $1/f^{2}$ ) in the frequency range  $f = 1/\Delta t \gg 1/(2\pi\tau_c)$  over which V(t) is correlated with  $V(t + \Delta t)$ . Hence, a fluctuating quantity V(t) with e.g. a 1/f PSD function  $S_V(f)$  <u>cannot</u> be characterized by a <u>single</u> correlation time constant – a 1/f PSD function  $S_V(f)$  instead implies correlations in V(t) over <u>all</u> time scales that correspond to the frequency range for which the PSD function  $S_{V}(\omega)$ exhibits the 1/f behavior. Note that in general, the negative slope of  $S_{v}(f)$  implies some degree of correlation. A steep (shallow) slope of  $S_{V}(f)$  implies a strong (weak) degree of correlation, respectively. Hence a fluctuating quantity V(t) with a Brownian  $1/f^2$  PSD function  $S_{v}(f)$  is strongly correlated, whereas one with a "white noise"  $1/f^{0}$  (*i.e.* flat) PSD function  $S_{V}(f)$  has <u>*no*</u> temporal correlations.

# $1/f^0$ White Noise vs. $1/f^1$ Pink Noise vs. $1/f^2$ Brown Noise:



White Noise - <u>flat</u> frequency-domain spectral distribution - all frequencies equally probable per unit time - fluctuations have <u>no</u> correlations in the time-domain (*i.e.* temporal correlations exist only for/at <u>infinite</u> time intervals).

Pink 1/f Noise – spectral slope of  $S_v(f)$  vs. f graph is –1 on log-log graph (–10 *dB*/decade) – 1/f noise fluctuations have <u>long-range</u> temporal (time-domain) correlations.

Brown  $1/f^2$  Noise - spectral slope of  $S_v(f)$  vs. f graph is -2 on log-log graph (-20 dB/decade) –  $1/f^2$  noise fluctuations have <u>short-range</u> temporal (time-domain) correlations.

#### **Amplitude (Volume/Loudness) Fluctuations in Human Music:**

The instantaneous acoustic power  $P_{ac}(t)$  output *e.g.* from a loudspeaker is related to the instantaneous electrical power input to the loudspeaker  $P_{em}(t)$  by  $P_{ac}(t) = \varepsilon_{ls}P_{em}(t)$  where  $\varepsilon_{ls}$  is the loudspeaker's efficiency for converting electrical power into acoustical power, typically ~ O(1-few %). Using Ohm's law, the instantaneous electrical power input to the loudspeaker is proportional to the <u>square</u> of the instantaneous voltage V(t) across the terminals of the loudspeaker:  $P_{em}(t) \approx V^2(t)/R_{ls}$ , where  $R_{ls}$  is the resistance of the loudspeaker.

The on-axis, <u>direct</u> sound pressure level associated with the sound coming from the loudspeaker, heard by a listener located a distance  $r_{\perp}$  away from, but along the axis of the loudspeaker is:

$$SPL_{direct}(r_{\perp},t) = L_{p}^{direct}(r_{\perp},t) = L_{Pwr}(t) + 10\log_{10}(Q/4\pi r_{\perp}^{2})(dB)$$

where the loudness level  $L_{Pwr}(t) \equiv 10 \log_{10} (P_{ac}(t)/P_{ac}^{o}) (dB)$ , the <u>reference</u> acoustic power level  $P_{em}^{o} \equiv 10^{-12}$  Watts and Q is the directivity factor of the loudspeaker. Thus, we see that:

$$SPL_{direct}(r_{\perp},t) = 10\log_{10}(P_{ac}(t)/P_{ac}^{o}) + 10\log_{10}(Q/4\pi r_{\perp}^{2})(dB)$$
  
=  $10\log_{10}(\varepsilon_{ls}P_{em}(t)/P_{ac}^{o}) + 10\log_{10}(Q/4\pi r_{\perp}^{2})(dB)$   
=  $10\log_{10}(\varepsilon_{ls}V^{2}(t)/R_{ls}P_{ac}^{o}) + 10\log_{10}(Q/4\pi r_{\perp}^{2})(dB)$ 

Recall that in a free-field acoustics situation, the Loudness = Sound *Intensity* Level:

$$L_{I}^{direct}\left(r_{\perp},t\right) \equiv 10\log_{10}\left(I\left(r_{\perp},t\right)/I_{o}\right) \approx L_{p}^{direct}\left(r_{\perp},t\right) \equiv 20\log_{10}\left(p\left(r_{\perp},t\right)/p_{o}\right) \text{ to within } \sim 0.1 \text{ } dB.$$

Thus, we see that Loudness is proportional to {the base-10 log} of  $V^2(t)$ . Hence, the moment-to-moment fluctuations in the Loudness associated with human music can be obtained *e.g.* by squaring the instantaneous electrical voltage associated with a music signal and obtaining the corresponding PSD function  $S_{V^2}(f)$  associated with  $V^2(t)$ , as shown below in the bottom left & right figures 2 & 3, taken from the seminal paper: "1/f Noise in Music: Music from 1/f Noise", R.F. Voss and J. Clarke, J. Acoust. Soc. Am. **63**, *p.* 258-263 (1978). The instantaneous music voltage signal V(t) was first band-pass filtered in the 100 – 10 KHz frequency range, squared and then sent through a 20 Hz low-pass filter to observe the moment-to-moment Loudness correlations in human music. The log-log plot of the audio PSD function  $S_{V^2}(f)$  in the bottom right-hand figure 3 clearly shows  $1/f^1$  loudness fluctuations associated with Bach's  $1^{\text{st}}$  Brandenburg Concerto. Figure 4 shows the audio PSD function  $S_{V^2}(f)$  associated with audio signals from different radio stations. Figure 5 shows the audio PSD function  $S_{V^2}(f)$ associated with different musical pieces/musical composers, both plots clearly show  $1/f^1$ loudness fluctuations!

©Professor Steven Errede, Department of Physics, University of Illinois at Urbana-Champaign, Illinois 2002 - 2017. All rights reserved.

- 5 -



FIG. 2. Bach's First Brandenburg Concerto (linear scales). (a) Spectral density of audio signal,  $S_V(f) vs f$ ; (b) spectral density of audio power fluctuations,  $S_V 2(f) vs f$ .



FIG. 3. Bach's First Brandenburg Concerto (log scales). (a)  $S_{V}(f) vs f$ ; (b)  $S_{V2}(f) vs f$ .





FIG. 4. Spectral density of audio power fluctuations,  $S_V 2(f)$  vs f for (a) Scott Joplin piano rags; (b) classical radio station; (c) rock station; and (d) news and talk station.

FIG. 6. Audio power fluctuation spectra densities,  $S_V^2(f)$  vs f for (a) Davidovsky's Synchronism I, II, and III, (b) Babbit's String Quartet number 3; (c) Jolas' Quartet number 3; (d) Carter's Piano concerto in two movements; and (e) Stock-hausen's Momente.

- 6 -

©Professor Steven Errede, Department of Physics, University of Illinois at Urbana-Champaign, Illinois 2002 - 2017. All rights reserved.

#### **Frequency (Pitch) Fluctuations in Human Music:**

A proxy for the instantaneous frequency/frequencies present in human music (and/or human speech) is the instantaneous *rate* Z(t)(#/s) of *zero crossings* associated with an audio signal V(t). A low frequency signal will have a small number of zero crossings per second, whereas a high frequency signal will have a large number of zero crossings per second associated with it. For human music, Z(t) approximately follows the melody. Again, temporal correlations in frequency arising from moment-to-moment fluctuations in the frequencies of successive notes of the melody of a song can be obtained via the time-domain autocorrelation function  $\langle Z(t)Z(t+\Delta t)\rangle$ , which via the Wiener-Khintchine theorem is related to the frequency-domain PSD function  $S_Z(f) = 4 \int_{\Delta t=0}^{\Delta t=\infty} \langle Z(t)Z(t+\Delta t)\rangle \cos(2\pi f \Delta t) d(\Delta t)$ . The PSD function(s)  $S_Z(f)$  associated with frequency/pitch fluctuations in various kinds/types of human music, as well as for different composers & musical genres are shown below in the following four figures.



FIG. 5. Spectral density of frequency fluctuations,  $S_Z(f)$  vs f for four radio stations (a) classical; (b) jazz and blues; (c) rock; and (d) news and talk.



FIG. 7. Frequency fluctuation spectral densities,  $S_Z(f)$  vs f for (a) Davidovsky's Synchronism I, II, and III; (b) Babbit's String Quartet number 3; (c) Jolas' Quartet number 3; (d) Carter's Piano concerto in two movements; and (e) Stockhausen' Momente.

- 7 -©Professor Steven Errede, Department of Physics, University of Illinois at Urbana-Champaign, Illinois 2002 - 2017. All rights reserved.



f (Hz)

### **Frequency (Pitch) Fluctuations in Music:**

Fig. 4. Melody fluctuation spectral densities.

## **Tempo/Beat (Phase) Fluctuations in Music:**

Analyzing the tempo/beat of human music for evidence of  $1/f^{\beta}$  fluctuations is more difficult to achieve, however very recently an incredibly nice paper was published on this subject: "Musical Rhythm Spectra from Bach to Joplin Obey a 1/f Power Law", D.J. Levitin, P. Chordia, V. Menon, Proc. Nat. Acad. Sci. **109** (10) *p*. 3716-3720 (2012). The tempo/beat/rhythmic aspects of statistically large samples of human music were analyzed using the onset of notes from {digitized} sheet music {in Humdrum kern data format files, see *e.g.* Kern Scores http://kern.humdrum.org/}, rasterizing the rhythm as shown in the figure below:



The authors analyzed fluctuations in the tempo/rhythm of music written by several different composers, and for many different musical genres. As shown in the figures below, the authors discovered significant variation in the values of the exponent ~  $0.5 \le \beta \le \sim 1.1$  for tempo/rhythm moment-to-moment fluctuations, whereas amplitude / loudness and/or frequency/pitch moment-to-moment fluctuations in human music have considerably less variation, being centered close to  $\beta \sim 1.0$ .



Fig. 2. Musical rhythm spectra obey a 1/f power law. (A) Rasterized rhythm representation (Lower) showing note onsets extracted from Beethoven's Quartet Op. 18. No. 1 (score, Upper). The representation shown is schematic: actual durations were extracted from the Humdrum kern format (Materials and Methods). (B) (Left) The spectrum of the rhythm raster from A has power that decays linearly (in a log-scale) with frequency as 1/f (gray dots). The slope of the spectrum (spectral exponent or  $\beta$ ) is 0.8. Colored segments show the sequence of durations (internote intervals). Black line represents the linear fit to the spectrum in the frequency range of 0.01 to 1 Hz (delineated by dotted vertical gray lines). Dashed line represents extrapolation of the linear fit to other frequencies. (Right) The spectrum of a sequence with the note onsets shuffled randomly, keeping durations intact. The shuffled spectrum is flat ( $\beta = 0.0$ ). Other conventions are as shown (*Left*). (*C*) Distribution of rhythm spectral exponents pooled across genres (black) obtained by linear fits to individual pieces across the population of 1,788 pieces analyzed. Gray: spectral exponent distribution for the corresponding shuffled rhythms. Inverted triangles represent the distribution median. Dashed vertical line:  $\beta = 0$ .

#### **Tempo/Beat Fluctuations – Different Types/Genres of Music:**



Fig. 3. The 1/f rhythm spectra are ubiquitous across genres. (A) Rhythm spectra for quartets. Average spectra (dark blue points) and linear fit (dark blue) to average spectrum in the frequency range of 0.01 to 1 Hz. Faded blue lines represent spectra of individual pieces. Gray data represent spectra of shuffled rhythms. Other conventions are as in Fig. 2*B*. (*B*) Distribution of rhythm spectral exponents obtained by linear fits to individual pieces (blue), and for the corresponding shuffled rhythms (gray). Inverted triangle represents median exponents. Dashed vertical line:  $\beta = 0$ . (C) Rhythm spectra for sonatas (red) and corresponding shuffled rhythms (gray). Other conventions are as in *A*. (*D*) Distribution of rhythm spectral exponents for musical genres ordered from largest mean exponent to smallest. Larger exponents indicate correlations over longer timescales, and hence more predictable rhythms (vertical gray arrow). Circles are mean exponents, and error bars are 95% CI. Disjoint intervals indicate significantly different mean exponents (Tukey–Kramer HSD).

#### **Tempo/Beat Fluctuations – Different Composers of Music:**



Fig. 4. Composers exhibit distinct 1/f rhythm spectra. (A) Average rhythm spectra for Beethoven (dark green), Haydn (violet), and Mozart (olive green): contemporary composers belonging to the Classical era (1750–1820). Other conventions are as in Fig. 3A. (B) Distribution of rhythm spectral exponents for compositions of Beethoven, Haydn, and Mozart. Color conventions are as in A. Other conventions are as in Fig. 3B. (C) Average rhythm spectra for Monteverdi (blue) and Joplin (green): composers separated by nearly three centuries of compositions. Other conventions are as in Fig. 3B. (D) Distribution of spectral exponents for compositions of Monteverdi and Joplin (Joplin. Color conventions are as in C. Other conventions are as in Fig. 3B. (E) Distribution of spectral exponents for composers ordered from largest mean exponent to smallest. Spectral exponents of Haydn, for example (dotted vertical lines, 95% CI), are significantly different from those of Beethoven and Mozart (P < 0.05, Tukey–Kramer HSD). Other conventions are as in Fig. 3E.

- 11 ©Professor Steven Errede, Department of Physics, University of Illinois at Urbana-Champaign, Illinois 2002 - 2017. All rights reserved.



# $1/f^{\beta}$ Noise in Drummer's Timing of Beat!

Figure 1. Demonstration of the presence of temporal deviations and LRC in a simple drum recording. A professional drummer (inset) was recorded tapping with one hand on a drum trying to synchronize with a metronome at 180 beats per minute (A). An excerpt of the recorded audio signal is shown over the beat index *n* at sampling rate 44.1 kHz. The beats detected at times  $S_n$  (green lines, see Methods) are compared with the metronome beats (red dashed lines). (B) The deviations  $d_n = S_n - M_n$  fluctuate around a mean of -16.4 ms, i.e. on average the subject slightly anticipates the ensuing metronome clicks. Inset: The probability density function of the time series is well approximated by a Gaussian distribution (standard deviation 15.6 ms). Our main focus is on more complex rhythmic tasks, however (see Table 1). A detrended fluctuation analysis of  $\{d_n\}$  is shown in Fig. 2C (middle curve). doi:10.1371/journal.pone.0026457.g001

- 12 -

©Professor Steven Errede, Department of Physics, University of Illinois at Urbana-Champaign, Illinois 2002 - 2017. All rights reserved.

A professional drummer was asked to drum 180 beats per minute ( $\Delta t = 1/3$ =0.333 sec per beat) in sync with a metronome, but slightly anticipated the metronome's clicking by {a mean value of} < $\delta t$ > = -16.4 ms.

Additionally, the drummer had a Gaussian-distributed width  $\sigma_{\Delta t} \sim 15.6$  ms about his mean time between beats, with ~ 1/f type fluctuations in the beat! Thus, we see from the above example that our human music also has temporal 1/f noise fluctuations intrinsic to the *playing/performance* of live music! Why? Because the nerve signals associated with the totality of playing a musical instrument (whether alone/solo, or in a band / ensemble, or whole orchestra) in going to/from our brains, traveling along myelinated nerve fibers also intrinsically exhibit temporal 1/f noise fluctuations! Thus, in this sense, it is not at all surprising that human music indeed reflects this fact, with its own temporal 1/f noise fluctuations in amplitude/loudness, frequency/pitch and beat/tempo/rhythm!

Humans also <u>do</u> much appreciate/enjoy <u>complexity</u> and <u>richness</u> in music – e.g. vibrato, the chorusing effect of superposing individual sounds from multiple identical instruments – as in an orchestra – each with their own temporal 1/f noise fluctuations...

It has often been said that human music is "universal" in nature, in that it transcends all human cultures; our music communicates something (*i.e.* emotions/feelings) to all humans. The above offers another window/perspective on the "universal" nature of human music, with its intrinsic forms of temporal 1/f noise fluctuations, which are manifest/operative in the everyday world, all around us!

## Turn the Physics Around – *Generate* Music from $1/f^{\beta}$ Noise!

It should be clear to the reader that since human music <u>does</u> have temporal  $1/f^{\beta}$  "noise" fluctuations in amplitude/loudness, frequency/pitch and beat/tempo/rhythm, that it is indeed possible to generate a "new" kind of music – <u>fractal music</u> e.g. via computer programs using  $1/f^{\beta}$  random "noise" generator(s) for these parameters!

We stumbled on this ourselves some years back, in the process of developing the Chaotic Water Drop experiment for the UIUC Advanced/Modern Physics Lab:



A "leaky" water faucet most of the time exhibits a periodic rate of water drops falling from/dripping off of the faucet. A 2-D scatterplot of successive time differences between adjacent water drops  $t_{n+2} - t_{n+1}$  vs.  $t_{n+1} - t_n$  in the periodic regime exhibits a linear y vs. x correlation in the scatterplot as the flow/leak rate in the precision needle valve is slowly changed, or a 2-D Gaussian distribution for *fixed* flow/leak rate. However, for certain very specific flow rates thru the precision needle valve, chaotic/strange attractor behavior in the scatterplot of successive time differences between adjacent water drops  $t_{n+2} - t_{n+1}$  vs.  $t_{n+1} - t_n$  occurs, as shown below in the two scatterplot figures. The first scatterplot was obtained slowly scanning the flow rate over a large range; the second scatterplot was obtained at a fixed flow rate.



©Professor Steven Errede, Department of Physics, University of Illinois at Urbana-Champaign, Illinois 2002 - 2017. All rights reserved.

Experimentally, it is quite difficult, *e.g.* by using only the <u>visual</u> information of successive water drop time differences  $t_{n+1} - t_n$ , as displayed on an oscilloscope trace to "find"/locate the flow-regimes of non-linear/chaotic dynamics associated with a "leaky" water faucet. However, we discovered that if we real-time converted the successive water drop time differences  $t_{n+1} - t_n$  to audio musical frequencies via  $f = 1/\tau = 1/(t_{n+1} - t_n)$  {*i.e.* low (high) pitches = long (short) time differences, respectively}, that by <u>listening</u> to the sequence of notes (time differences), it became <u>extremely</u> easy to "home" in/determine the chaotic drip regimes! For the periodic drip regime, successive notes were the same pitch – the "music" of the periodic drip regime, the sequence of notes (time differences) amazingly sounded <u>very</u> musical – very much like <u>jazz</u> music! However, from the above discussion(s), one can easily understand that this is indeed <u>no</u> accident – the fractal  $1/f^{\beta}$  "noise" nature of our music is indeed <u>intimately</u> related to the fractal  $1/f^{\beta}$  nature of a chaotically dripping/leaky water faucet! The PSD function  $S_v(f)$  shows a ~  $1/f^1$  behavior associated with the fluctuations in the leaky water faucet time differences, indicating that long-range temporal correlations do indeed exist:



FIG. 4. Power spectra S(f) for the interval increments for the time series presented in Fig. 1. A straight line corresponding to the  $\beta = -1$  curve is presented for comparison.

One other important take-home lesson to be learned here is that human hearing is amazingly astute at analyzing temporal correlations, whereas human vision is not very good at analyzing temporal correlations, even when portrayed in a visual format – e.g. 1-D pulse trains on an oscilloscope. On the other hand, human vision is amazingly good at analyzing 3-D spatial correlations – enabling us to get around in the world...

Fractal music is a rapidly growing activity, and industry! The figure shown below indicates the consequences of extremes in exponent  $0 \le \beta \le 2$  for (a)  $1/f^0$  white noise (no correlations), (b)  $1/f^1$  pink noise (some correlations) and (c)  $1/f^2$  brown noise (strong correlations). For human listeners, white noise music is found to be <u>too</u> random – very annoying to listen to after a short while... Likewise, brown noise is found to be boring – it is <u>too</u> predictable, it doesn't "go anywhere", musically. Pink noise is the most pleasing to our ears – it has some predictability, but also some surprises too – we humans do like complexity in our music – but not <u>too</u> much!

Early attempts at creating fractal music on a computer, *e.g.* generated with just temporal  $1/f^{\beta}$  "noise" fluctuations in amplitude/loudness and frequency/pitch (only) still sounded non-human, or "artificial" (*i.e.* computer-generated) – the addition of temporal  $1/f^{\beta}$  "noise" fluctuations in tempo/beat/rhythm are <u>also</u> needed in order for the fractal computer music to fully <u>convincingly</u> sound "human".



Fig. 5. Samples of stochastically composed fractal music based on the different types of noises shown in fig. 3. (a) "White" music is too random; (b) "1/f" music is the closest to actual music (and most pleasing) and (c) "brown" or  $1/f^2$  music is too correlated.

Benoit Mandelbrot, a mathematician who wrote the now famous book "The Fractal Geometry of Nature" (1982) showed that the temporal  $1/f^{\beta}$  fluctuations observed in many physical systems/many physical processes are but a special class of a broader, more general <u>fractal</u>, <u>self</u>-<u>similar</u> behavior of nature/our universe – which also includes *e.g.* 1D, 2-D and 3-D spatial "noise" fluctuations, such as the fractal nature of iterative, weathering/erosion processes associated with coastlines, mountains, cloud formations, as well as living systems, such as trees...

The following pix show examples of computer-generated fractal images of such things:





No generally recognized "universal" physical explanation of 1/f noise exists. Consequently, the ubiquity of 1/f noise in nature is one of the oldest puzzles of contemporary physics and science in general.

It would be very interesting to carry out spectral/correlation analyses *e.g.* on whale songs, to see if their music also has  $1/f^{\beta}$  fluctuations... One can also ask the question, since  $1/f^{\beta}$  fluctuations <u>are</u> widespread in nature/in the everyday world here on earth as well as extant elsewhere in many physics processes out there in the cosmos, <u>if</u> intelligent life exists elsewhere in the universe, <u>and</u> those life-forms had their own music, would <u>their</u> music also exhibit  $1/f^{\beta}$  fluctuations?

## Legal Disclaimer and Copyright Notice:

## Legal Disclaimer:

The author specifically disclaims legal responsibility for any loss of profit, or any consequential, incidental, and/or other damages resulting from the mis-use of information contained in this document. The author has made every effort possible to ensure that the information contained in this document is factually and technically accurate and correct.

## Copyright Notice:

The contents of this document are protected under both United States of America and International Copyright Laws. No portion of this document may be reproduced in any manner for commercial use without prior <u>written</u> permission from the author of this document. The author grants permission for the use of information contained in this document for <u>private</u>, <u>non-commercial</u> purposes only.