Sound waves propagate in a physical medium:

* gas, liquid or solid (and/or a plasma – the 4th state of matter @ very high temperature!)

* mass density of the medium $\rho$ (= mass per unit volume) is important

* sound wave ("disturbance" = energy pulse) propagates in the medium with a characteristic speed of propagation $v$ in that medium.

* propagation speed, $v$ depends on density & elastic properties of the medium.

**Simple model of (one-dimensional) medium:**

![Diagram of one-dimensional medium](image1.png)

**Crystalline metal – sound waves propagating in a 3-D lattice of atoms:**

![Crystal lattice examples](image2.png)

- Fe, V, Nb, Cr
- Al, Ni, Ag, Cu, Au
- Ti, Zn, Mg, Cd
**Longitudinal Sound Waves Propagating in 1-Dimension:**

Displacement of atoms from equilibrium positions – *i.e.* compression/rarefaction is in/along/parallel to direction of propagation of soundwave

Longitudinal sound waves – in gases, liquids and solids (*i.e.* bulk materials)

**Propagation of Longitudinal Waves in One Dimension as a Function of Time:**

![Diagram of longitudinal waves propagating in one dimension](image)

*Fig. 7. A longitudinal wave moving in the one-dimensional medium.*
**Tuning Fork Used to Generate 1-Dimensional Longitudinal Waves:**

Displacement of atoms from equilibrium positions is perpendicular (i.e. transverse) to direction of propagation of wave.

Waves in a solid – e.g. a vibrating string (1-D), or a vibrating rectangular, triangular or circular sheet/membrane (2-D), a vibrating hollow box, pyramid, cylinder or sphere (3-D)!

**Transverse Sound Waves Propagating in 1-Dimension:**

**Transverse Sinusoidal Traveling Wave in 1-Dimension:**
Propagation of Transverse Waves on a Stretched String as a Function of Time:

Fig. 8. A transverse wave in a string.

Fig. 5. A transverse wave moving in the one-dimensional medium.
Propagation of Longitudinal Waves in an Air Column:

Fig. 9. Tuning fork attached to piston at the end of an air column.

FIG. 10. A longitudinal wave in an air column.

(displacement of air molecules from their equilibrium positions) & (over-pressure “pressure displacement” from equilibrium pressure – 1 Atm.)

n.b. Over-pressure and displacement of air molecules are 90° out of phase with each other!
$A$ = displacement amplitude of wave = max displacement from equilibrium position (meters, m)

$f$ = frequency of wave = number of complete cycles per second of wave
  (cycles per second = cps = Hertz, or Hz).

$\lambda$ = “lambda” = spatial wavelength of wave – distance to complete one oscillation cycle 
  (meters, m)

$\tau$ = “tau” = period of wave = time to complete one oscillation cycle = “temporal wavelength”
  (seconds (secs, or s)):

\[
\tau = \frac{1}{f}
\]

$v$ = speed of propagation of wave (meters/second = m/s):

\[
v = f \lambda
\]

$\omega$ = “omega” = angular frequency (radians/second = rads/sec, or rads/s):

\[
\omega = 2\pi f, \quad f = \frac{\omega}{2\pi}
\]

$k$ = spatial wave number (inverse meters, i.e. 1/meters = 1/m):

\[
k = \frac{2\pi}{\lambda}
\]

Hence, we also see that:

\[
v = f \lambda = 2\pi f \cdot \lambda / 2\pi = \omega / k
\]

Sinusoidal longitudinal wave propagation {in the +ve z-direction} (relevant e.g. for sound propagation in air/water) is mathematically described by:

\[
Z(z, t) = Z_o \sin \left( \omega t - kz \right)
\]

where $z$ = listener’s position, $Z_o$ = longitudinal displacement amplitude (i.e. along the direction of propagation) and $Z(z, t)$ is the instantaneous longitudinal displacement (i.e. longitudinal deviation) from the wave’s equilibrium position at the point $z$ at time $t$.

Note that the argument of the sine function \((\omega t - kz)\) = constant. Thus, as time $t$ increases, the position $z$ must also increase – hence the name traveling wave. Hence, we also see that an argument of the sine function of the form \((\omega t + kz)\) = constant mathematically represents a traveling longitudinal displacement plane wave propagating in the −z direction: $Z(z, t) = Z_o \sin (\omega t + kz)$. 

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The longitudinal speed of propagation $v$ of transverse displacement traveling waves, where the displacement (from equilibrium position) is e.g. in the $y$-direction, perpendicular ($i.e.$ transverse) to the direction of propagation, e.g. in the $\pm z$-direction is mathematically described by:

$$Y(z,t) = Y_0 \sin \left( \omega t \mp kz \right)$$

for a transverse traveling plane wave propagating in the $\pm z$-direction.

For transverse waves propagating on a stretched string having tension, $T$ ($n.b.$ the SI / metric units of tension $T$ (= force) is Newtons, $N = kg \cdot m/s^2$), the string has mass $M$ and length $L$, the longitudinal speed of propagation of transverse traveling waves on a string is given by:

$$v = \sqrt{\frac{T}{\mu}}$$

where: $\mu = \frac{M}{L}$ = mass per unit length of string (SI units: $kg/m$)

**Example: Tension and Transverse {Standing} Waves on the High-E String of a Guitar:**

$n.b.$ A standing wave = superposition of two traveling waves propagating in opposite directions!

The high-E string on a guitar has diameter, $D = 0.009" (\sim 230\mu m)$ ($0.001" = 1/1000" = 25.4\mu m$)

$L = \text{string length} = 63.5 \, \text{cm} = 0.635m$ = scale length e.g. of a Fender electric guitar

$\rho = \text{density of string} = 7.9 \, \text{gms/cm}^3 = 7900 \, \text{kg/m}^3$ for steel.

$A_\perp = \text{cross-sectional area of string} = \pi R^2 = \pi (D^2/4) = 4.104 \times 10^{-3} \, \text{cm}^2 = 4.104 \times 10^{-9} \, \text{m}^2$.

$V = \text{volume of string} = A_\perp \cdot L = \pi R^2 \cdot L = \pi (D^2/4) \cdot L = \pi D^2L/4 = 0.026 \, \text{cm}^3 = 0.026 \times 10^{-6} \, \text{m}^3$

Mass of string: $M = \rho V = \rho (A_\perp \cdot L) = \rho \left( \pi D^2L/4 \right) = 0.206 \, \text{gms} = 0.206 \times 10^{-3} \, \text{kg}$

$$\mu = \frac{M}{L} = \frac{\rho V}{L} = \frac{\rho (A_\perp \cdot L)}{L} = \rho \cdot A_\perp = \frac{0.206 \times 10^{-3} \, \text{kg}}{0.635m} = 3.242 \times 10^{-4} \, \text{kg/m}$$

Now $f_{\text{Hi-E}} = 332 \, \text{Hz}$ for open Hi-E string.

Thus, the longitudinal speed of transverse traveling waves on the Hi-E string of a guitar is:

$$v_{\text{Hi-E}} = f_{\text{Hi-E}} \cdot \lambda_{\text{Hi-E}} = 332 \, \text{Hz} \cdot 1.27 \, m = 421.6 \, \text{m/s}$$
Since \( v = \sqrt{T/\mu} \), the string tension \( T \) on the Hi-E string of the guitar is:

\[
T = \mu v^2 = (3.242 \times 10^{-4} \text{ kg/m}) \times (421.6 \text{ m/s})^2 = 57.6 \text{ kg-m/s}^2 = 57.6 \text{ N} \approx 12.95 \text{ lbs of force}.
\]

The typical string tension on a steel-stringed acoustic and/or electric guitar is \( T \sim 50 – 60 \text{ N} \).

\[\Rightarrow\] For steel 6 (12)-string guitar, total string tension is \( \sim 300 – 360 \text{ (600 – 720) } N \) !!!

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**PROPAGATION OF SOUND WAVES IN 2 & 3 DIMENSIONS**

SOUNDWAVES EMANATING FROM A POINT SOURCE

A sound wave freely propagating in a gas = **longitudinal** compression/rarefaction of the gas.

Gives rise to a **longitudinal** displacement wave. If harmonic/sinusoidal in nature, far away from the sound source we have traveling plane waves, e.g. propagating in the + z-direction:

\[
\xi_z(z,t) = \xi_o \cos \left( \omega t - kz \right) = \text{mean (or avg.) longitudinal (i.e. z-) displacement of air molecules}
\]

and a corresponding **over-pressure** wave of the form: \( p(z,t) = P(z,t) - P_{am} \), where \( P(z,t) \) is the instantaneous absolute pressure, and \( P_{am} \) = atmospheric pressure = constant, typically \( \sim 14.7 \text{ psi} \approx 1.03 \times 10^5 \text{ Pascals (Pa)} \) at NTP (i.e. sea level and temperature \( T = 20 ^\circ \text{C} \)).

For a harmonic traveling plane wave, the instantaneous over-pressure is:

\[
p(z,t) = p_o \sin \left( \omega t - kz \right)
\]

Sound waves can propagate through elastic, compressible media as **longitudinal** waves.
The over-pressure \( p = P - P_{\text{atm}} = \Delta P \) required to compress a gas of initial volume \( V \) to \( V - \Delta V \) is:

\[
p = P - P_{\text{atm}} = \Delta P = -B \left( \frac{\Delta V}{V} \right)
\]

(Adiabatic) bulk modulus, \( B \) of fluid (here, a gas)

\( B \) is the so-called adiabatic bulk modulus, \( B = \frac{1}{\kappa} \) where \( \kappa = \text{compressibility of the fluid (liquid or gas)} \) – n.b. \( B \) has same SI units as pressure, \( p \) (from dimensional analysis of above formula).

Thus, we see that the adiabatic bulk modulus \( B \) of a fluid (liquid or gas) is the (negative) of the change in the \{over-pressure\} divided by the fractional change in the volume of the fluid due to the change in the over-pressure:

\[
B = \frac{-\Delta P}{(\Delta V/V)} \quad \text{(Pascals)}
\]

Now, for so-called adiabatic (i.e. slow) compression of a gas due to e.g. propagation of sound waves in the gas:

\( \gamma \) = “gamma” \( \equiv \frac{C_v}{C_p} = \text{Ratio of: specific heat of gas @ constant volume}{\text{specific heat of gas @ constant pressure}} \)

\( \gamma = 5/3 \) for monatomic gases (e.g. helium, neon, argon & xenon)

\( \gamma = 7/5 \) for diatomic gases (e.g. oxygen & nitrogen molecules – \( O_2 \) & \( N_2 \))

The Ideal Gas Law:

\[
P V = N R T
\]

Absolute temperature (degrees Kelvin)

Pressure Volume # Moles \( R = \text{universal gas constant} = 8.3145 \text{ (Joules/mole/deg.K)} \)
\( N/m^2 \) \( m^3 \) \( \text{of gas} \)

\( e.g. \) Carbon atom has 12 atomic mass units (amu’s), and thus 1 mole (mol) of carbon {having Avogadro’s number, \( N_A = 6.022 \times 10^{23} \text{ atoms/mole} \)} weighs 12 grams.

Now air @ NTP (a mixture of oxygen & nitrogen molecules, traces of argon, etc.) is NOT a perfect ideal gas – but is close to an ideal gas.

For the so-called adiabatic condition: \( P V^\gamma = \text{constant} = K \), thus: \( P = K V^{-\gamma} \)
Then for small pressure variations \( p = P - P_{\text{atm}} = \Delta P \ll P_{\text{atm}} \):  
\( \Rightarrow (\Delta P / P) = -\gamma (\Delta V / V) \)

However, from above:  
\( B = -\frac{\Delta P}{(\Delta V / V)} \).  
Thus, we see that:  
\( B = \gamma P = \gamma P_{\text{atm}} \) for \( p = P - P_{\text{atm}} \ll P_{\text{atm}} \).

For 1-D traveling plane waves:  
\( p = \Delta P = -B (\Delta V / V) = -B \frac{\partial \xi_z(z,t)}{\partial z} \).

Thus, we see that there is a relation between overpressure \( p(z,t) \) and the local slope (i.e. the spatial gradient) of the longitudinal displacement \( \partial \xi_z(z,t) / \partial z \):

If longitudinal displacement: \( \xi_z(z,t) = \xi_o \cos(\omega t - kz) \) then:  
\( p(z,t) = -B \frac{\partial \xi_z(z,t)}{\partial z} \) gives:

\[
p(z,t) = p(z,t) - P_{\text{atm}} = \Delta P(z,t) = -B \frac{\partial \xi_z(z,t)}{\partial z} = -Bk \xi_o \sin(\omega t - kz) = p_o \sin(\omega t - kz)
\]

Hence, we see that the over-pressure \( p(z,t) \) and the longitudinal displacement of air molecules from their equilibrium positions \( Z(z,t) \) are 90° out-of-phase with each other.

The (mean, or avg.) longitudinal speed of air molecules \( u_z(z,t) \) is the time rate of change of the (mean, or avg.) longitudinal displacement of air molecules, i.e. the time derivative \( \partial \xi_z(z,t) / \partial t \).

Thus, for a harmonic/sinusoidal sound wave in air, the instantaneous particle velocity is:

\[
u_z(z,t) = \frac{\partial \xi_z(z,t)}{\partial t} = -\omega \xi_o \sin(\omega t - kz) = u_o \sin(\omega t - kz)
\]

Thus, we also see that the longitudinal speed of air molecules \( u_z(z,t) \) and the overpressure \( p(z,t) \) are in-phase with each other for sound waves propagating in “free” air:

\[
p(z,t) = p_o \sin(\omega t - kz) = -B \frac{\partial \xi_z(z,t)}{\partial z} = -Bk \xi_o \sin(kz - \omega t) = -\frac{Bk}{\omega} \omega \xi_o \sin(kz - \omega t) = \frac{Bk}{\omega} u_z(z,t)
\]

A medium has a so-called **characteristic specific acoustic impedance** \( z_a(\vec{r}) \) associated with it at the listener point \( \vec{r} \) – which physically is a measure of how easy (or difficult) it is for (acoustic) energy to **flow** from one point to another in the medium. For **longitudinal** traveling acoustic plane waves propagating **freely** in a gas, the **characteristic longitudinal specific acoustic impedance** \( z_o^l(\vec{r}) \) is defined as (using \( v = \omega / k = f \lambda \), and \( B = \rho v^2 \) {see below...}):

\[
z_o^l(\vec{r}) \equiv p(\vec{r})/u_z(\vec{r}) = Bk / \omega = B / v = \rho v
\]

(IS units of \( z_o(\vec{r}) \): Pa-s/m = acoustic Ohms).

The **characteristic longitudinal specific acoustic impedance** for **air** is:

\[
z_o^{l,air}(\vec{r}) = \rho_o^{air} v_o^{air} = 1.204 \text{ kg} / \text{m}^3 \cdot 345 \text{ m/s} = 415 \text{ Pa-s/m} = 415 \Omega_{ac} = 415 \text{ Rayls} \@ \text{NTP}.
\]

Aka Rayls, in honor of Lord Rayleigh (John William Strutt)
Because $z_a^{\text{air}}(\vec{r}) = \rho^{\text{air}} v^{\text{air}}$, since both the mass density of air $\rho^{\text{air}}$ and the longitudinal speed of propagation $v^{\text{air}}$ have a slight temperature (and pressure-) dependences (as well as slight humidity dependences), $z_a^{\text{air}}(\vec{r})$ also has a slight temperature (and pressure-) dependence (as well as a slight humidity dependence) – see below….

The wave equation for describing propagation of longitudinal sound waves in a gas relates the speed of longitudinal sound propagation in the gas $v$ (m/s) to the adiabatic bulk modulus of the gas $B$ (in N/m$^2$ = Pascals) and the gas density, $\rho = M/V$ (in kg/m$^3$) by the following formula:

$$v = \sqrt{B/\rho}$$

From dimensional analysis:

\[ SI \text{ units: } B/\rho = \frac{N}{m^2} \frac{kg/m^3}{kg/m^3} = \frac{m^2/s^2}{m^2/s^2} \text{ note: } 1 \text{ N} = 1 \text{ kg-m/s}^2 \]

Thus: $$v = \sqrt{B/\rho} = \sqrt{m^2/s^2} = m/s = \text{meters/second} = \text{dimensions of speed}$$

Now, since $B = \gamma P$ then: $$v = \sqrt{B/\rho} = \sqrt{\gamma P/\rho}$$

The gas density $\rho = \frac{M}{V} = \frac{M_{\text{molar}} n_{\text{mole}}}{V} = \frac{M_{\text{molar}} P}{RT}$ \{for an ideal gas, and using the ideal gas law\}

where $M_{\text{molar}} = \text{molar mass} (\text{kg/mol})$, and $n_{\text{mole}} = \# \text{ of moles of gas.}$

Then for an ideal gas: $$v = \sqrt{B/\rho} = \sqrt{\gamma P/\rho} = \sqrt{\gamma RT/M_{\text{molar}}}$$

Hence, e.g. the speed of sound in air has a temperature-dependence!

Normalizing to the so-called standard temperature (300K) and pressure (1 atm):

$$v_{\text{air}} \approx 330 \sqrt{T(\degree C) + 273} \frac{1}{273} = 330 + 0.6T(\degree C) \text{ (m/s)}$$

The latter relation was obtained using the Taylor-series expansion for $\sqrt{1+\varepsilon} = 1 + \frac{1}{2} \varepsilon$ for $\varepsilon << 1$ and keeping only the first non-trivial term in the Taylor-series expansion, which is linear in $\varepsilon = T(\degree C)/273K << 1$.

$v_{\text{air}} \approx 330 \text{ m/s} @ T = 0 \degree C$ (\@ $p = 1$ atm (i.e. at sea level))

$v_{\text{air}} \approx 340 \text{ m/s} @ T = 20 \degree C$ (\@ $p = 1$ atm (i.e. at sea level))
Thus, we see that the speed of sound in air, $v_{air}$, increases with increasing temperature:

$$v_{air} \propto \sqrt{T}.$$ 

Note also that: $v_{helium} \approx 3v_{air}$ since: \{M_{molar}(Helium) = 4 gms\} \ll \{M_{molar}(Air) = 28 gms\}.

**Longitudinal speed of sound propagation (in bulk)**

**Longitudinal characteristic specific acoustic impedance for water:**

$$v_{H_2O} = \frac{B_{H_2O}}{\rho_{H_2O}} \approx 1480 \text{ m/s} >> v_{air} \approx 330 \text{ m/s}$$

$$z_{||}^{H_2O} = \rho_{o}^{H_2O} v_{o}^{H_2O} = 1000 \text{ kg/m}^3 \cdot 1480 \text{ m/s} = 1.48 \times 10^6 \text{ Pa-s/m} = 1.48 \times 10^6 \Omega_{ac}$$

Compare: $z_{||}^{H_2O} = 1.5 \times 10^6 \Omega_{ac} \gg z_{||}^{air} = 415 \Omega_{ac}$.

**Longitudinal speed of sound propagation (in bulk)**

**Longitudinal characteristic specific acoustic impedance for an elastic solid:**

$$v_{Solid} = \sqrt{\frac{Y_{Solid}}{\rho_{Solid}}} \quad \text{where: } Y_{Solid} = \text{Young’s modulus (force/unit area, i.e. N/m}^2\text{) – i.e. Pascals!}$$

$$Y \equiv \frac{S}{\Delta L/L} (N/m^2) = \text{Ratio of compressive stress/compressive strain}$$

$$z_{||}^{solid} = \frac{\rho_{o}^{solid} v_{o}^{solid}}{\rho_{o}^{solid}}$$

Now, e.g. for steel: $Y_{steel} = 2 \times 10^{11} \text{ Pascals}$ and: $\rho_{steel} = 7.9 \text{ gm/cm}^3 = 7900 \text{ kg/m}^3$.

Thus: $v_{Steel} = \sqrt{\frac{Y_{Steel}}{\rho_{Steel}}} = \sqrt{\frac{2 \times 10^{11}}{7800}} \approx 5000 \text{ m/s}$

$$z_{||}^{Steel} = \rho_{o}^{Steel} v_{o}^{Steel} = 7900 \text{ kg/m}^3 \cdot 5000 \text{ m/s} = 3.95 \times 10^7 \text{ Pa-s/m} = 3.95 \times 10^7 \Omega_{ac}$$

Compare: $z_{||}^{Steel} = 4.0 \times 10^7 \Omega_{ac} \gg z_{||}^{H_2O} = 1.5 \times 10^6 \Omega_{ac} \gg z_{||}^{air} = 415 \Omega_{ac}$.
### Longitudinal Speed of Sound Propagation in Various Bulk Media

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<tr>
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