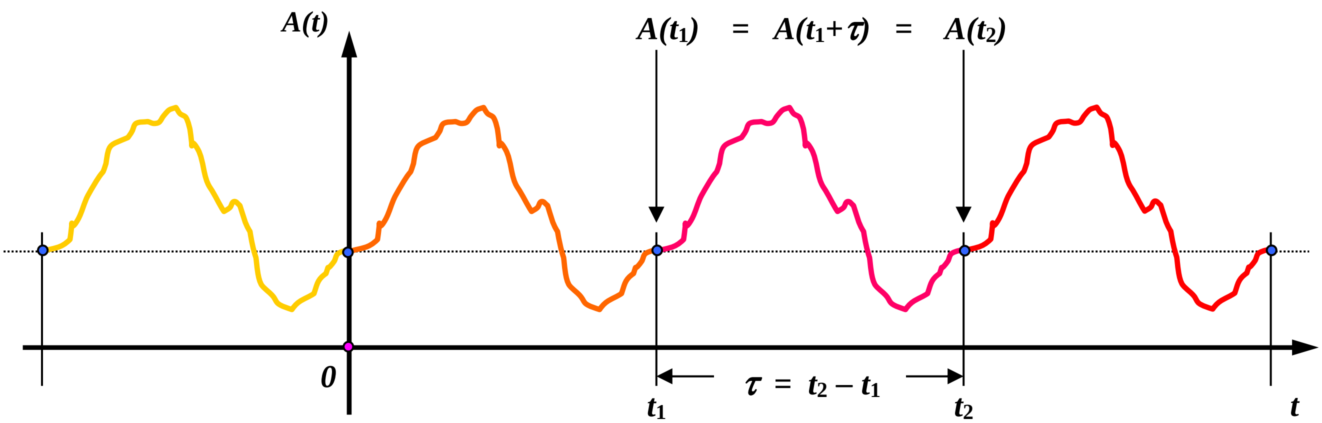
Tone Quality — Timbre

A ***pure*** tone (*aka* ***simple*** tone) consists of a ***single*** frequency, *e.g.* $f = 100\ Hz$.

Pure tones are rare in nature – natural sounds are often ***complex*** tones, consisting of/having more than one frequency – often many!

A ***complex*** tone = a ***superposition*** (*aka* linear combination) of several/many frequencies, each with its own amplitude and phase.

Musical instruments with a ***steady*** tone (*i.e.* a tone that doesn't change with time) create a periodic complex acoustical waveform (periodic means that it repeats every so often in time, *e.g.* with repeat period, τ):



Fourier analysis (*aka* harmonic) analysis — mathematically can represent ***any*** periodic waveform by an infinite, linear superposition of sine & cosine waves – integer harmonics of fundamental/lowest frequency:

$$A_{tot}(t) = a_o + \sum_{n=1}^{\infty} a_n \cos(n\omega_1 t) + \sum_{n=1}^{\infty} b_n \sin(n\omega_1 t)$$

$\boxed{\omega_1 = 2\pi f_1}$ f_1 = fundamental frequency, repeat period $\tau = 1/f_1$

Please see UIUC Physics 406 Lecture Notes – Fourier Analysis I, II, III & IV for more details… http://courses.physics.illinois.edu/phys406/406pom_lectures.html

A complex tone - *e.g.* plucking a single string on a guitar - is perceived as a single note, but consists of the fundamental frequency f_1, plus integer *harmonics* of the fundamental frequency: $f_2 = 2f_1,\ f_3 = 3f_1,\ f_4 = 4f_1,\ f_5 = 5f_1,$ *etc.*

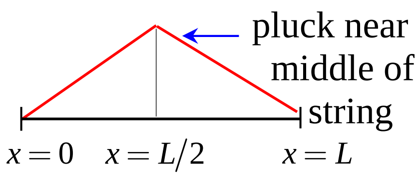
Harmonics of the fundamental also known as ***partials***

The fundamental = 1st harmonic/partial
The 2nd harmonic/partial has $f_2 = 2f_1$ (aka 1st overtone)
The 3rd harmonic/partial has $f_3 = 3f_1$ (aka 2nd overtone) …. *etc.*

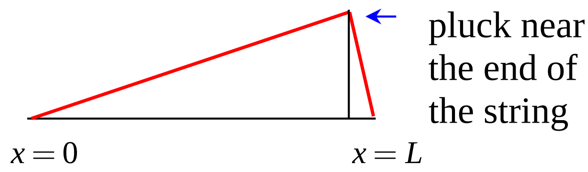
A vibrating string (guitar/violin/piano) contains many harmonics = complex tone.

The detailed ***shape*** of a plucked string on a guitar (or violin) uniquely determines its harmonic content! Please see/hear/touch Physics 406POM **Guitar.exe** demo!

"mellow" – less high harmonics



"bright" – more high harmonics



The geometrical ***shape*** of the string at the instant (t = 0) that the string is plucked **defines** the ***amplitudes*** (& ***phases***) of the harmonics associated with standing wave on the string:

Transverse Displacement of String: $y(x,t) = \sum_{n=1}^{\infty} c_n \sin(nk_1 x)\cos(n\omega_1 t + \phi_n)$

where: $k_1 = 2\pi/\lambda_1$ and: $\omega_1 = 2\pi f_1$ with: $v = f_1\lambda_1 = \omega_1 / k_1$

T = string tension

$\lambda_1 = 2L$ $\quad$ $v = \lambda_1 f_1 = \sqrt{\dfrac{T}{\mu}}$ $\quad$ $\mu = \dfrac{M}{L}$ $\quad$ M = mass of string

$k_n = nk_1 = \dfrac{2\pi}{\lambda_n}$ $\quad$ $v = \lambda_n f_n = \omega_n / k_n$ $\quad$ L = length of string

$\lambda_n = \lambda_1 / n$ and: $f_n = nf_1$ where: $n = 1, 2, 3, 4, \ldots.$

Hierarchy of tones/harmonics = harmonic series;

$$e.g.\quad y(x,t) = \sum_{n=1}^{\infty} b_n \sin(nk_1 x)\cos(n\omega_1 t)$$

= superposition of waves of frequencies $f_n = nf_1$ on a vibrating string

Note that $f_2 = 2f_1$ means that f_2 is one ***octave*** higher than f_1.

Ratio $f_2/f_1 = 2:1$ The musical interval between harmonics 1 and 2 is an ***octave***.
Ratio $f_3/f_2 = 3:2$ The musical ***interval*** between harmonics 2 and 3 is a ***fifth***, *etc.*

Tone Structure:

We can build up/construct a complex waveform by linear superposition/linear combination of the harmonics:

$$\begin{aligned} A_{tot}(t) &= a_o + \sum_{n=1}^{\infty} a_n \cos(n\omega_1 t) + \sum_{n=1}^{\infty} b_n \sin(n\omega_1 t) \\ &= a_o + (a_1 \cos\omega_1 t + a_2 \cos 2\omega_1 t + a_3 \cos 3\omega_1 t + a_4 \cos 4\omega_1 t + \ldots) \\ &\quad + (b_1 \sin\omega_1 t + b_2 \sin 2\omega_1 t + b_3 \sin 3\omega_1 t + b_4 \sin 4\omega_1 t + \ldots) \end{aligned}$$

⇒ See/try out the UIUC P406's **Fourisim.exe** and/or **Guitar.exe** computer demo programs to learn/see/hear more about complex waveforms…

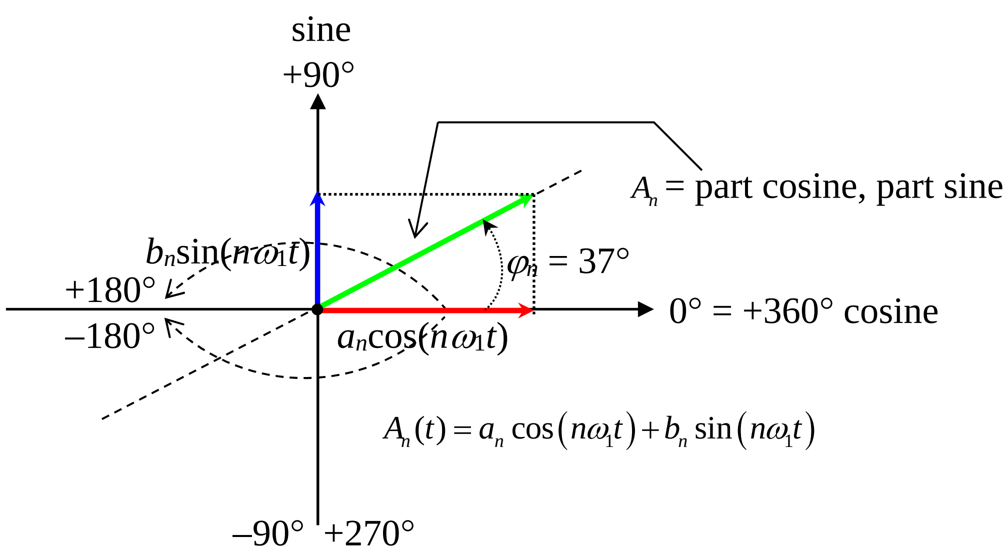
Harmonic Synthesis: ***Adding*** harmonics together to produce a complex waveform.

⇒Please see & hear the Hammond Organ harmonic synthesis demo…⇐

Harmonic Analysis: ***Decomposing*** a complex waveform into constituent harmonics.

Any complex periodic waveform can be analyzed into its constituent harmonics *i.e.* harmonic amplitudes and phases (*e.g.* relative to the fundamental).

Pure sine $\{b_n \sin(n\omega_1 t)\}$ and cosine $\{a_n \cos(n\omega_1 t)\}$ waves have a 90° phase relation with respect to each other, *e.g.* at a given time, *t*:



From the above phasor diagram, note that we can equivalently rewrite $A_n(t)$ as:

$$A_n(t) = a_n \cos(n\omega_1 t) + b_n \sin(n\omega_1 t) = A_n \cos(n\omega_1 t - \varphi_n)$$

From trigonometry, we see that: $a_n = A_n \cos\varphi_n$ and $b_n = A_n \sin\varphi_n$, and since:

$\cos(A-B) = \cos A\cos B + \sin A\sin B$, hence we see that:

$$A_n(t) = A_n\cos\varphi_n\cos(n\omega_1 t) + A_n\sin\varphi_n\sin(n\omega_1 t) = A_n\cos(n\omega_1 t - \varphi_n)$$

We also see that: $A_n = \sqrt{A_n^2\cos^2\varphi_n + A_n^2\sin^2\varphi_n} = \sqrt{a_n^2 + b_n^2}$ and that: $\varphi_n = \tan^{-1}(b_n/a_n)$.

Hence, we can equivalently write the Fourier series expression:

$$\boxed{A_{tot}(t) = a_o + \sum_{n=1}^{\infty} a_n\cos(n\omega_1 t) + \sum_{n=1}^{\infty} b_n\sin(n\omega_1 t)}$$

as:

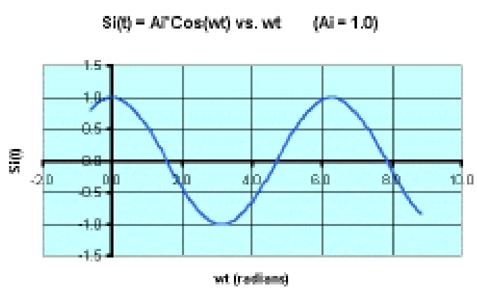
$$\boxed{A_{tot}(t) = a_o + \sum_{n=1}^{\infty} A_n\cos(n\omega_1 t + \varphi_n)}$$

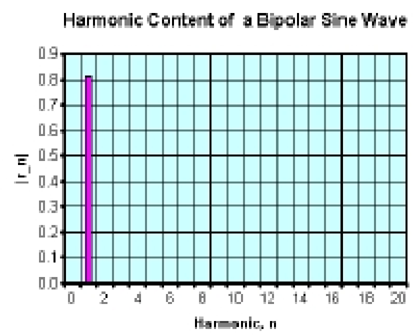
with: $A_n = \sqrt{a_n^2 + b_n^2}$ and: $\varphi_n = \tan^{-1}(b_n/a_n)$.

Fourier analysis applies to any/all kinds of complex periodic waveforms – electrical signals, optical waveforms, *etc*. - ***<u>any</u>*** periodic waveform (temporal, spatial, *etc*.). Please see/read Physics 406 Series of Lecture Notes on Fourier Analysis I-IV for much more details/info…

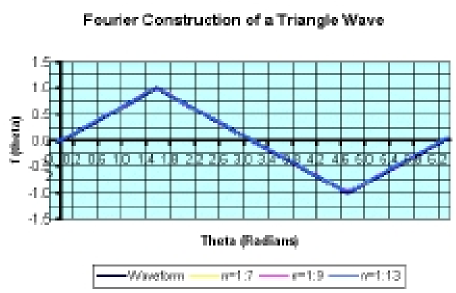
Basic Musical Waveforms

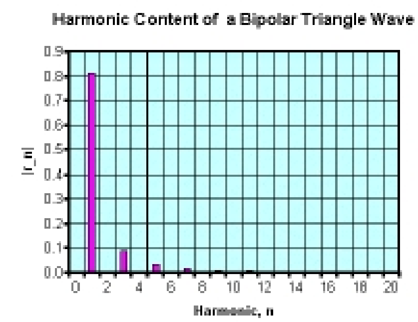
1. Sine/Cosine Wave: Mellow Sounding – No High Harmonics



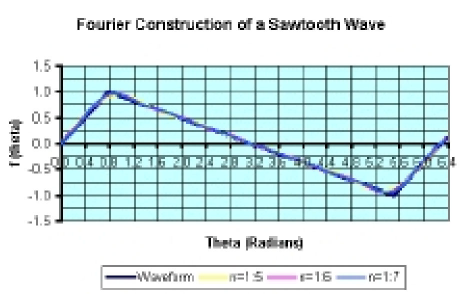


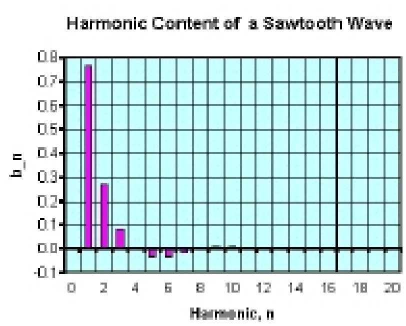
2. Triangle Wave: A Bit Brighter Sounding – Has Harmonics!



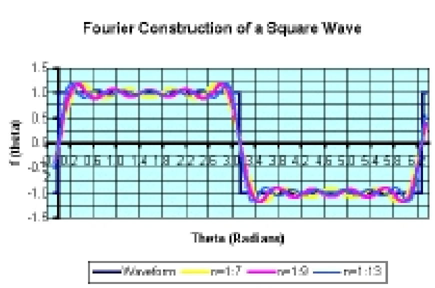


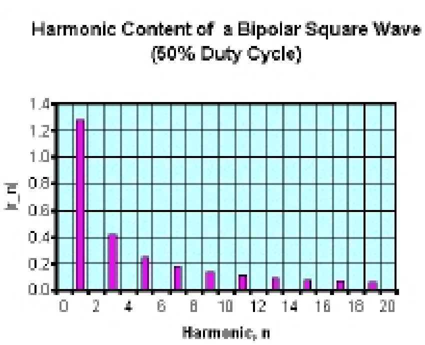
3. Sawtooth Wave: Even Brighter Sounding – Even More Harmonics





4. Square Wave: Brightest Sounding – Has the Most Harmonics





Effect of {Relative} Phase on Tone Quality:

Human ears ***are*** sensitive to phase information in the $\sim 100 \leq f \leq 1500$ Hz range.

In a complex tone, there also exists subtle sound change(s) associated with the phase of higher harmonics ***relative*** to the fundamental. Due in part to ***non-linear*** response(s) in the ear (& auditory processing in brain) - *i.e.* the non-linear response associated with the firing of auditory nerves/firing of hair cells due to vibrations on the basilar membrane in the cochlea, from overall sound wave incident on one's ears. This is ***especially*** true for ***loud*** sounds!!! Non-linear auditory response(s) also become ***increasingly*** important with increasing sound pressure levels.

Please see/read Physics 406 Lecture Notes on "Theory of Distortion (I & II)" for details on how a non-linear system responds to pure and complex periodic signals.

Harmonic Spectrum:

Please see above figure(s) for harmonic content associated with:

a.) a pure sine wave
b.) a symmetrical triangle wave
c.) a sawtooth (= asymmetrical triangle) wave
d.) a bipolar square wave

Musical instruments have ***transient*** response(s) – *i.e.* the harmonic content of the sounds produced by musical instruments changes/evolves in time.

How harmonics evolve in time is important.
How the harmonics build up to their steady-state values is important for overall tone quality, *e.g.* at the beginning of each note.
How the harmonics decay at the end of each note is also important - very often the higher harmonics decay more rapidly than lower-frequency harmonics, due to frequency-dependent dissipative processes.

Formants:

Nearly all musical instruments have frequency regions that emphasize certain notes moreso than others – these are known in musical parlance as ***formants*** – *i.e.* ***resonances*** – due to ***constructive*** interference of sound waves in those frequency regions. If resonances (constructive interference) exists within a given musical instrument for certain frequency range(s), there will also exist ***anti-resonances*** (destructive interference) for certain other frequency ranges, *e.g.* in between successive formants.

The physical consequence of such facts is that the sound level output from many musical instruments is ***not*** constant (*i.e*. flat) with frequency. See following plot of harmonic amplitude(s) *vs*. frequency for a hypothetical musical instrument:

Formants/Resonances (& Anti-Resonances):

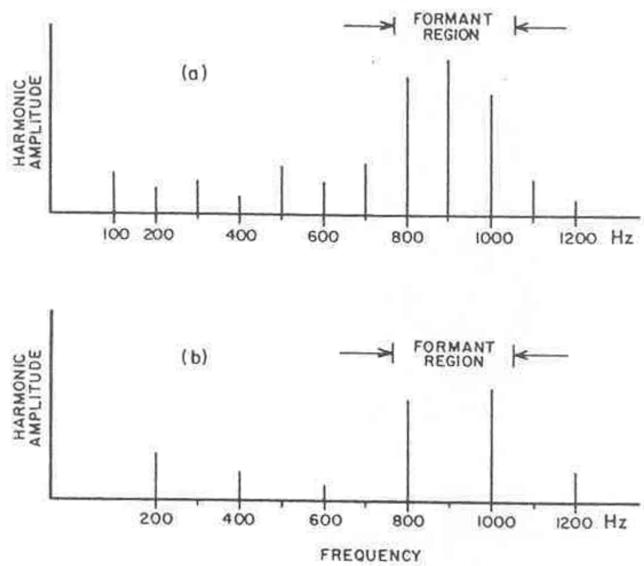


FIG. 8. Example of hypothetical tone produced by an instrument having a formant in the region 800–1000 hertz. (a) Fundamental of 100 hertz. (b) Fundamental of 200 hertz.

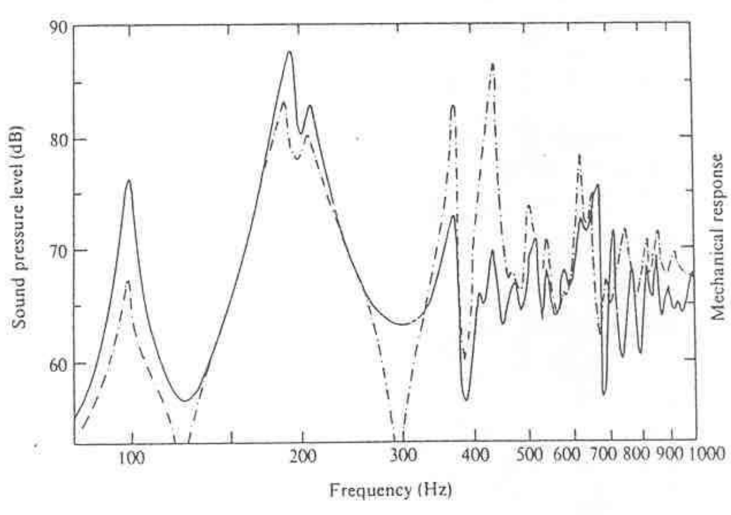
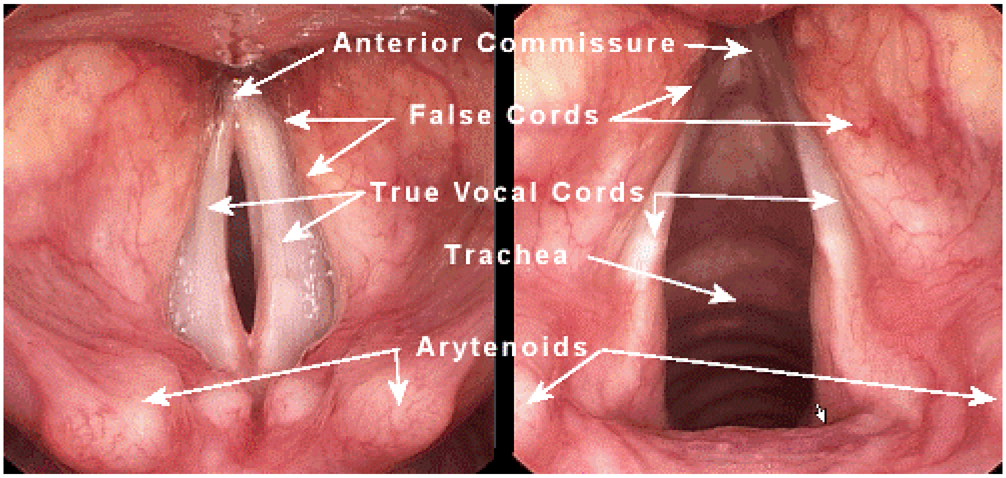


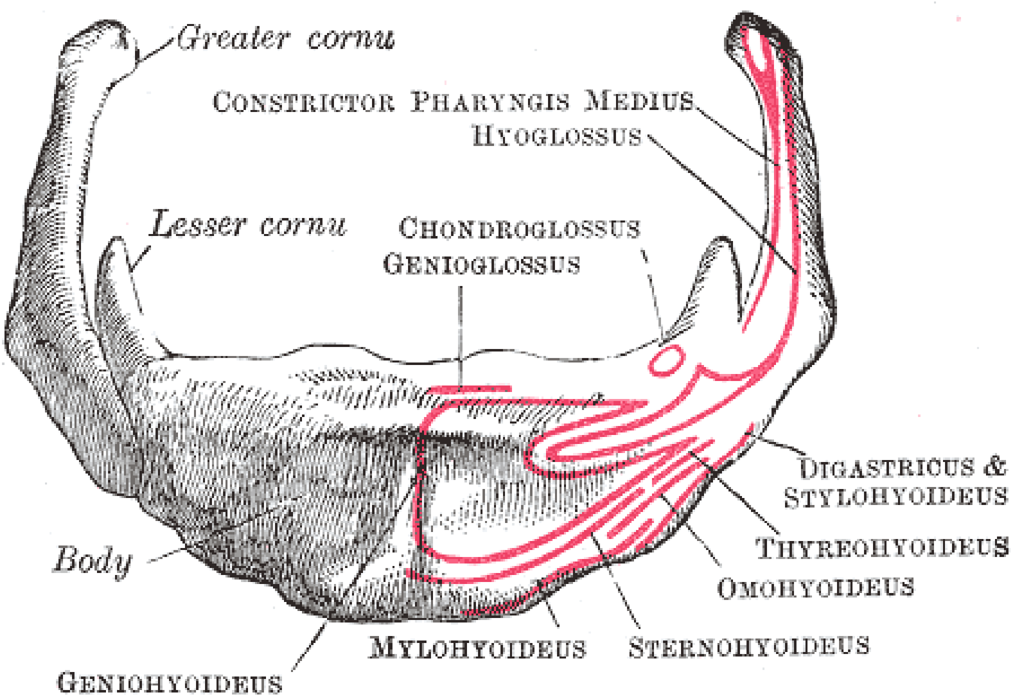
FIGURE 9.20. Mechanical frequency response and sound spectrum 1 m in front of a Martin D-28 folk guitar driven by a sinusoidal force of 0.15 N applied to the treble side of the bridge. Solid curve, sound spectrum; dashed curves, acceleration level at the driving point.

The Uniqueness of the Human Voice:

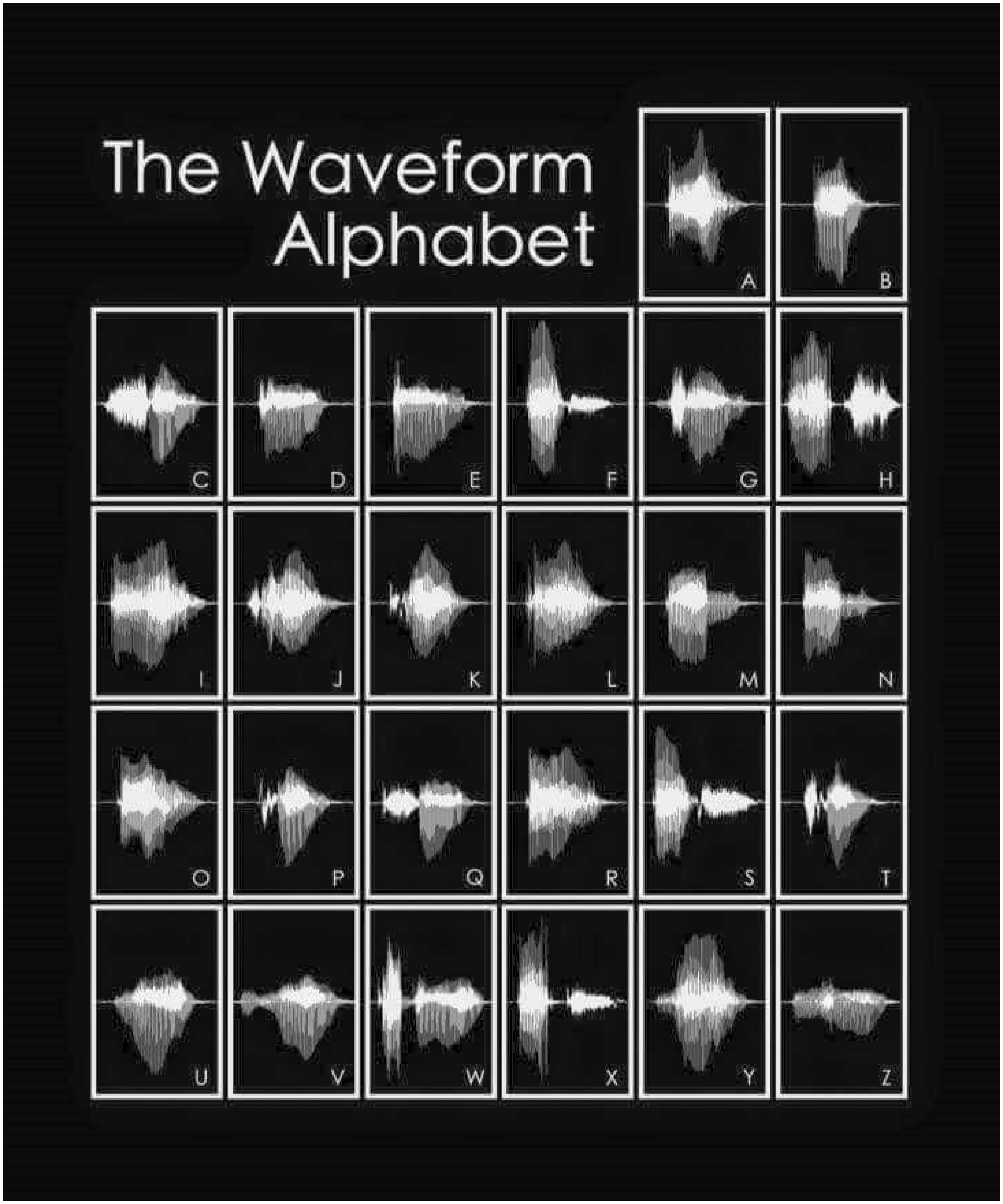
The human voice – larnyx (voice box) + hyoid bone (& attendant musculature) + lungs/throat/mouth/nasal cavities enable a rich pallet of sounds to be produced!



Human hyoid bone - unique to our species - homo sapiens - other primates do not have!



Time-Domain Waveforms Associated with Speaking Letters of the Alphabet:



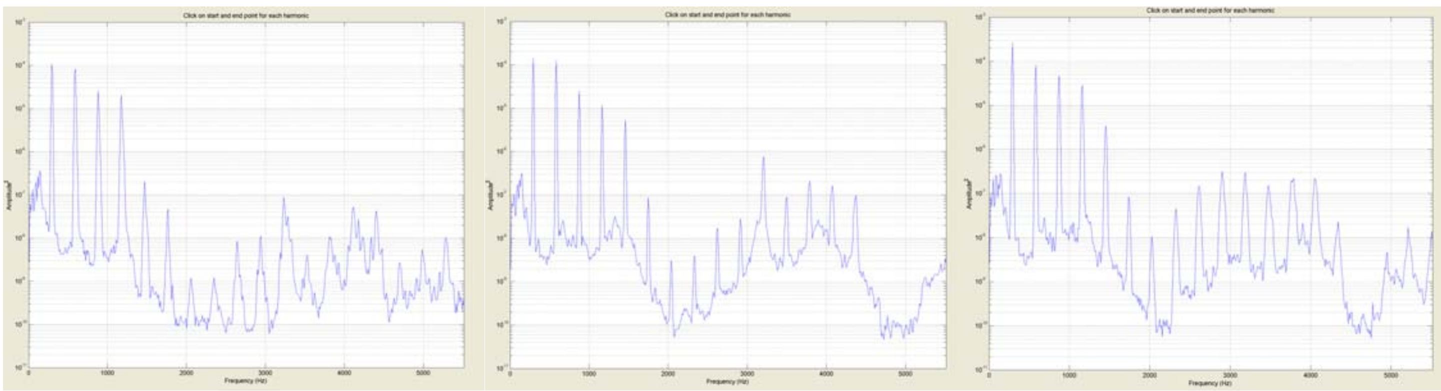
The Uniqueness of the Human Voice:

The harmonic content associated with musical notes sung by three women UIUC undergraduates were analyzed using the Matlab-based wav_analysis program {Please see Abby Ekstrand's Physics 193POM Final Report, Spring Semester, 2007: http://courses.physics.illinois.edu/phys193/193_student_projects_spring07.html}

e.g. for the note D4 (f_{D4} = 293.7 *Hz*):

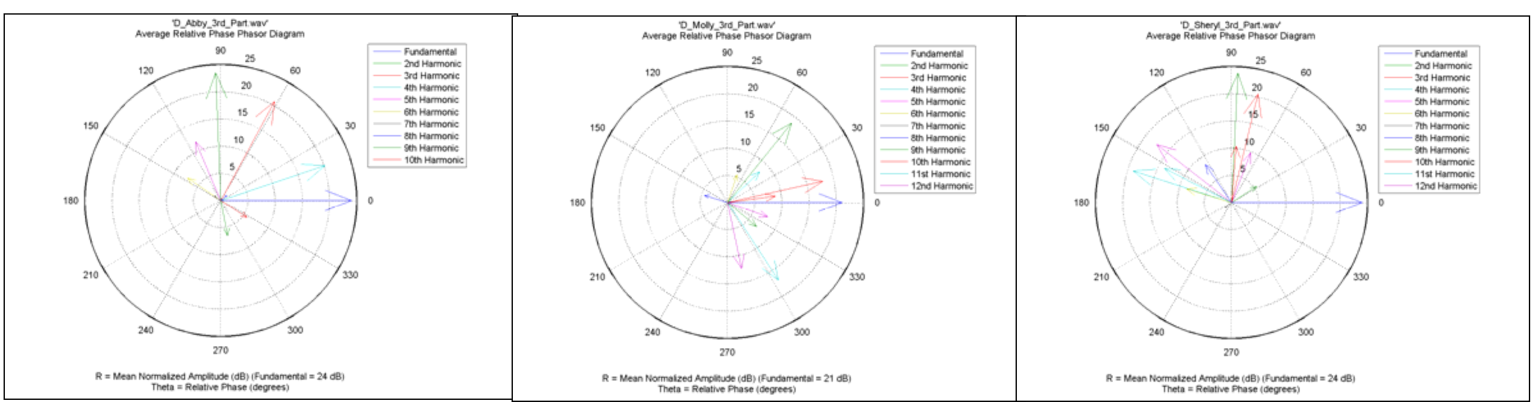
Amplitude² *vs.* Frequency:

Abby Molly Cheryl



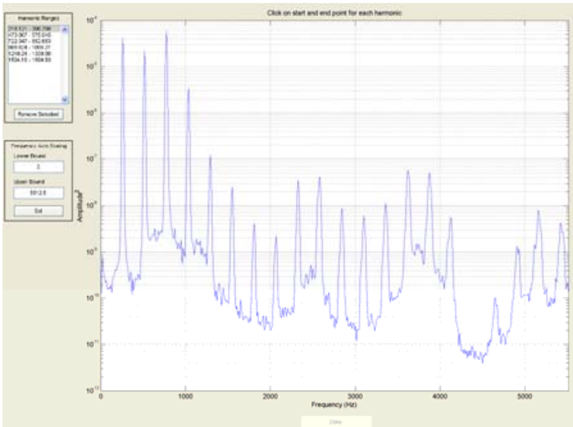
Note the differences in formant/resonance regions in the above frequency spectra!

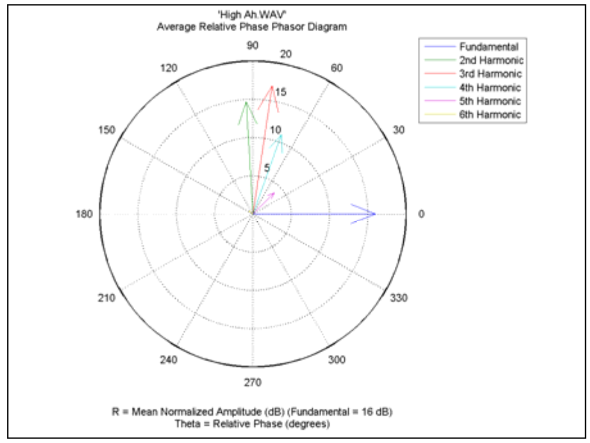
2-D Harmonic Amplitude/Phase Diagrams – the time-averaged SPL (dB) for each harmonic is represented by the ***length*** of each arrow & the time-averaged relative phase of each harmonic is represented by the ***angle*** of each arrow, relative to the horizontal axis, for each of the higher harmonics relative to the fundamental. The fundamental is always the blue arrow on the horizontal axis oriented @ 0 degrees.



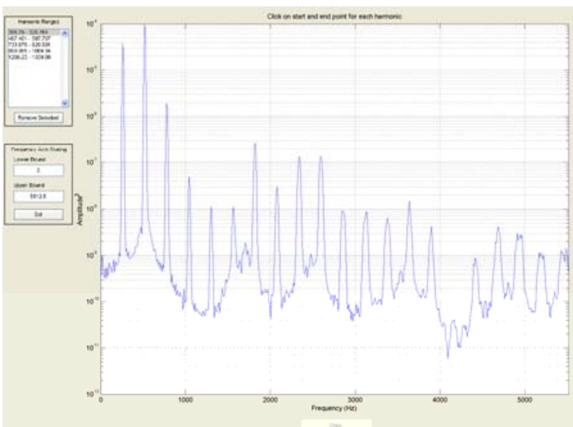
Harmonic Content of Vowels – John Nichols (P406 Spring, 2010):

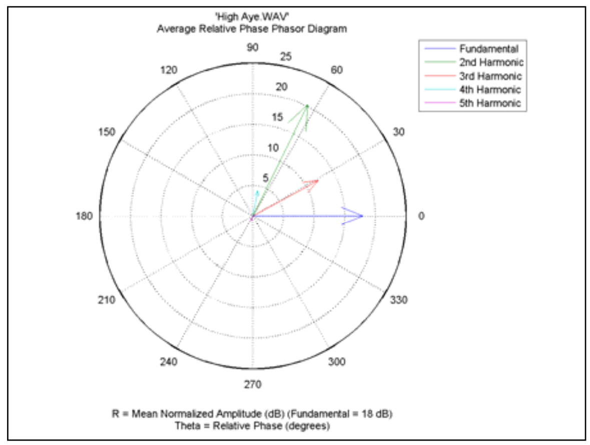
High
“Aaaah”



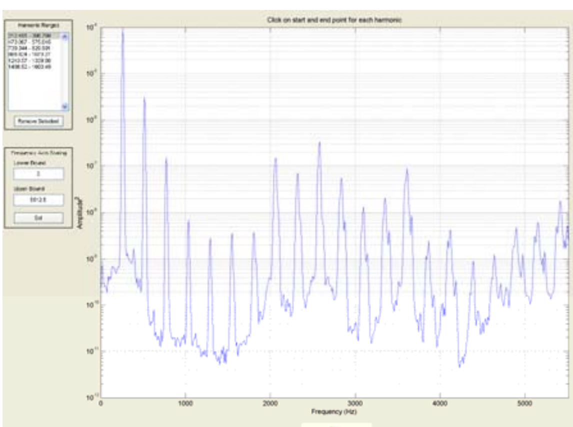


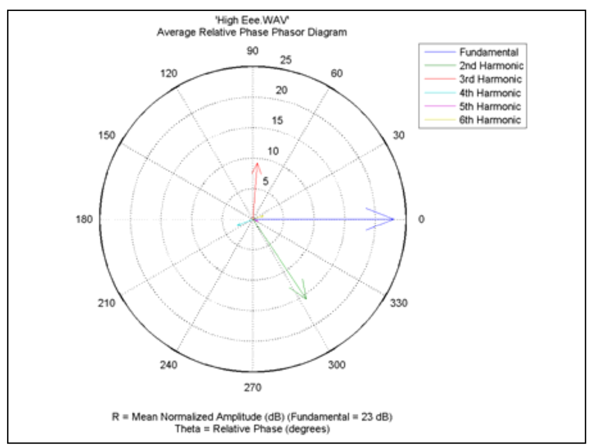
High
“Aaaay”





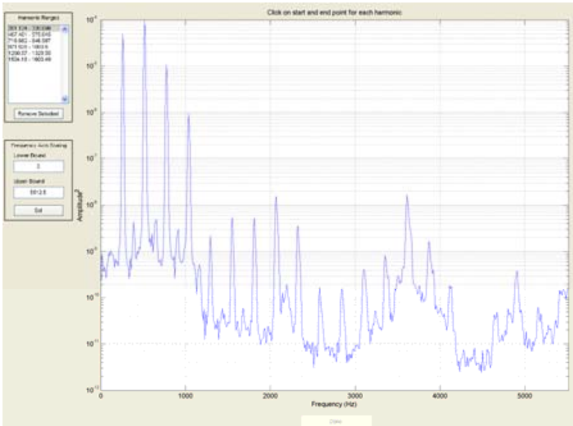
High
“Eeee”

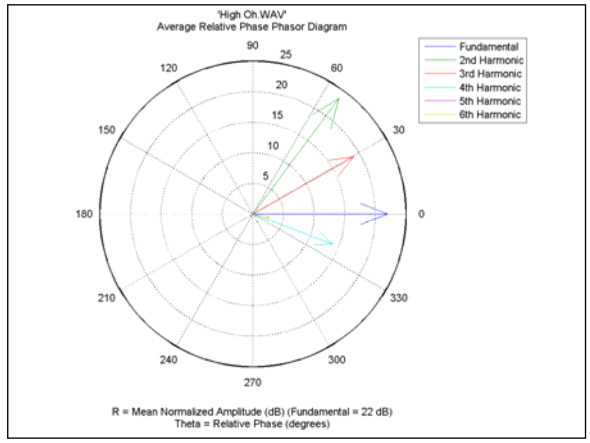




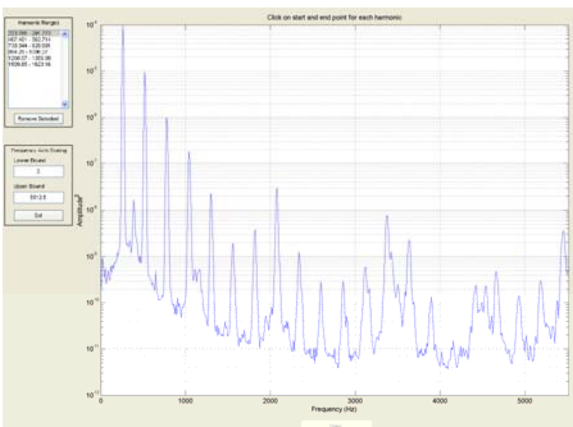
Harmonic Content of Vowels – Cont. – John Nichols (P406 Spring, 2010):

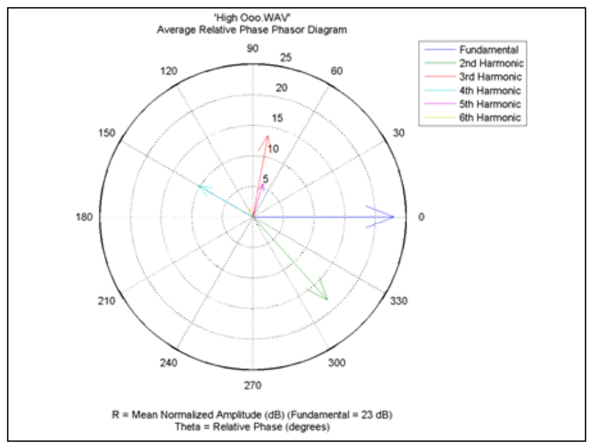
High "Oh"



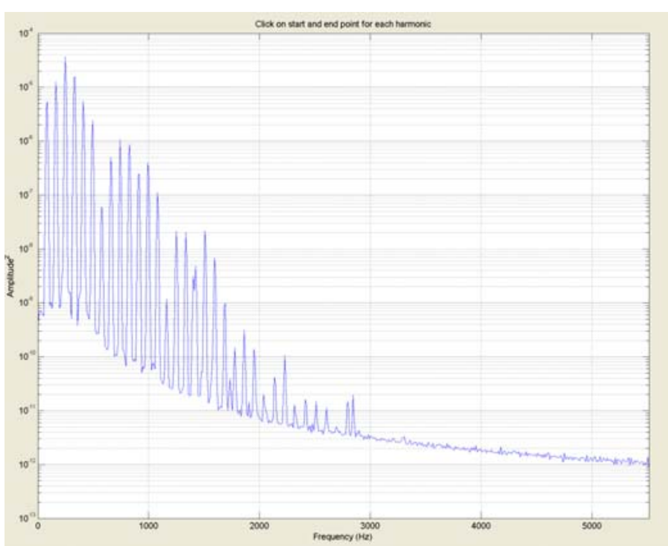


High "Ooooh"





Harmonic Content of a Martin D16 Acoustic Guitar – Low-E String (82 *Hz*):



Sound Spectrum of a Tawa-Tawa Gong (as a function of time):

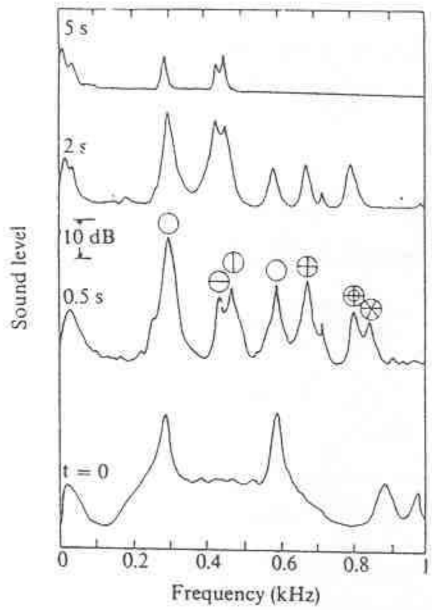


FIGURE 20.11. Sound spectrum of a tawa tawa gong. The initial sound ($t = 0$) comes mainly from two prominent axisymmetric modes, but after 0.5 s many modes of vibration have been excited, which decay at varying rates. Some of the modes are identified at the peaks (Rossing and Shepherd, 1982).

Sound Spectrum of a Large Gamelan Gong:

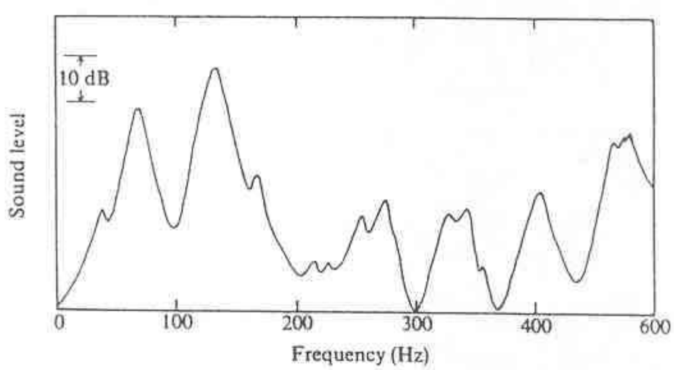


FIGURE 20.12. Sound spectrum of a large gamelan gong. The principal modes of vibration have frequencies of 67 Hz and 135 Hz, and their corresponding partials are about an octave apart (Rossing and Shepherd, 1982).

The Build-Up and Decay of Harmonics from a Tam-Tam Gong:

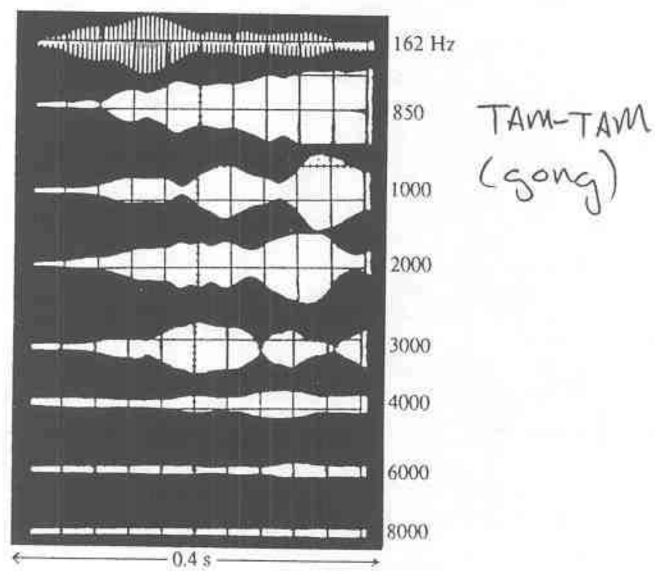
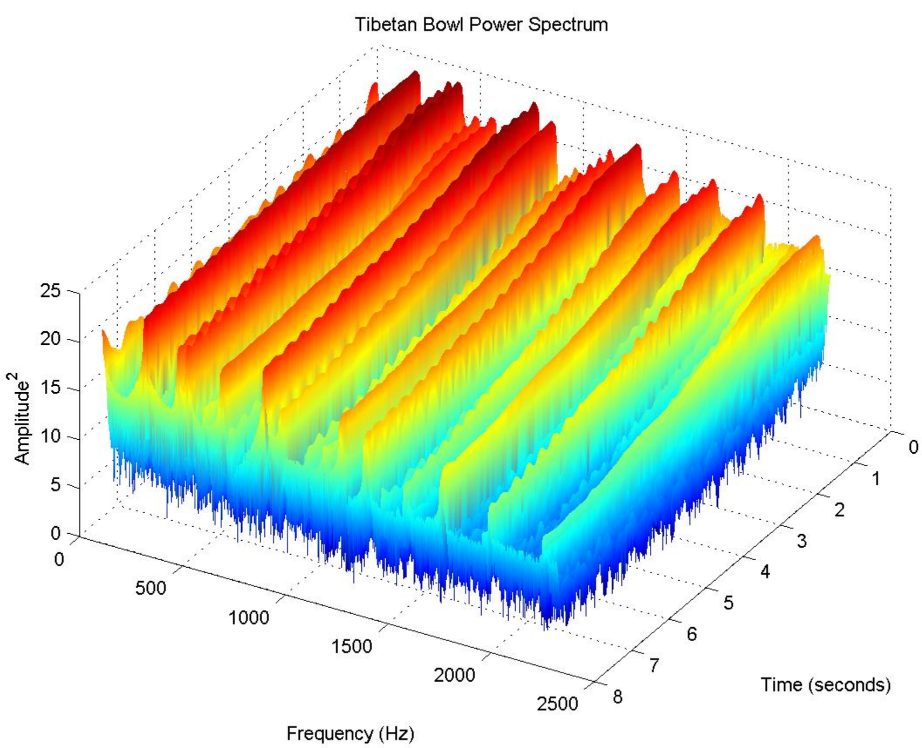


FIGURE 20.8. Buildup and decay of vibrations in different frequency bands during the first 0.4 s (Rossing and Fletcher, 1982).

Harmonic Spectrum *vs*. Time of a Tibetan Bowl:



The bassoon has a pronounced formant/resonance in the *f* ~ 440 – 500 *Hz* region and a weaker one at *f* ~ 1220 – 1280 *Hz*. See table below for some brass and woodwind instruments:

TABLE I

Formant frequencies in hertz for woodwind and brass instruments

INSTRUMENT	FORMANT I	FORMANT II
Flute	800	
Oboe	1400	3000
English Horn	930	2300
Clarinet	1500–1700	3700–4300
Bassoon	440–500	1220–1280
Trumpet	1200–1400	2500
Trombone	600–800	
Tuba	200–400	
French Horn	400–500	

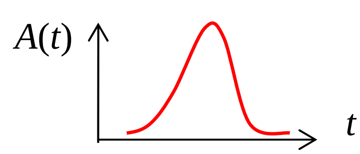
<u>Sound Effects</u>: (Create enhanced/richer musical structure to sound(s) from musical instruments)

<u>Vibrato Effect</u> — periodic, slow rhythmical variation/fluctuation of <u>frequency</u> of complex tone.
— <u>frequency modulation</u>

<u>Tremelo Effect</u> — periodic, slow rhythmical variation/fluctuation of <u>amplitude</u> of complex tone.
— <u>amplitude modulation</u>

<u>Chorus Effect</u> — Two or more instruments (of same type) simultaneously playing the same music.
— not at exactly same frequency
— not perfectly in phase – slight vibrato with respect to each other – beat against each other in musically pleasing way.

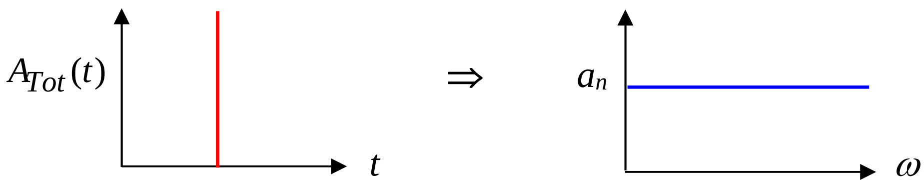
<u>Non-Periodic Sounds</u> — *e.g.* sound <u>pulses</u>



Some sounds produced by certain musical instruments (*e.g.* percussion instruments) are ***<u>not</u>*** periodic. Non-periodic sounds - sound pulses - can be fully described mathematically as a superposition (linear combination) of a ***<u>continuum</u>*** (or ***<u>spectrum</u>***) of frequencies, with certain amplitudes.

<u>Example</u>: A noise "spike" (of infinitely short duration) consists of a linear combination of ALL frequencies – with ***<u>equal</u>*** amplitudes!!

A noise ***<u>spike</u>*** in time has a ***<u>flat</u>*** frequency spectrum!



<u>Human Perception of Tone Quality</u> - "Subjective Tones"

The human ear/brain ***<u>are</u>*** systems with non-linear responses. For example, when two ***<u>loud</u>*** ***<u>pure</u>*** tones (frequency f_1 & f_2) are simultaneously sounded together, a third difference tone $|f_2 - f_1|$ can be heard!! (Actually two additional tones $(f_1 \& f_2)$ and $|f_2 - f_1|$ can be heard). This can ***<u>only</u>*** happen if there exist non-linear response(s) in the human ear/brain!

<u>Example</u>: If one sounds two loud pure-tone notes together, one sound with frequency $f_1 = 300\ Hz$, the other with frequency $f_2 = 400\ Hz$ the human ear also hears $(f_1 \& f_2)$ and $|f_2 - f_1|$ sum and difference tones:

<u>Summation</u> <u>tone</u>: $f_1 + f_2 = 300\ Hz + 400\ Hz = 700\ Hz$ ← *n.b.* harder to hear
<u>Difference</u> <u>tone</u>: $|f_1 - f_2| = |f_2 - f_1| = |300 - 400| = 100\ Hz$

These <u>sum</u> and <u>difference</u> frequencies arise solely due to <u>non</u>-<u>linear</u> response(s) of the human ear/brain. Linear <u>sum</u> and <u>difference</u> frequencies $(f_1 \& f_2)$ and $|f_2 - f_1|$ arise primarily from <u>quadratic</u> non-linear response terms. Cubic, quartic, quintic, *etc*. (non-linear response) terms give high order frequency effects! *e.g.* $2f_1 - f_2, 3f_1 - 2f_2, 2f_1 + f_2, \ldots$ }. When many frequencies/harmonics are present, the non-linear response of the human ear/brain produces <u>inter-modulation</u> <u>distortion</u> (many such sum and difference frequencies) – giving rise to perception of a complicated set of combination tones. Please see/read UIUC Physics 406 Lecture Notes on Theory of Distortion I & II for more details…

<u>Related Phenomenon</u>:

The perceived harmonic content of a complex tone changes with loudness level!!

e.g. triangle and square waves sound ***<u>brighter</u>*** at 100 *dB* than *e.g.* @ 60 *dB*

This is simply due to fact that the human ear has an ~ ***<u>logarithmic</u>*** response to sound intensity, which indeed is a ***<u>non</u>***-***<u>linear</u>*** response to sound intensity.

$$\text{Loudness, } L = 10\log_{10}\left(I/I_o\right)$$

Compare the ***<u>ratio</u>*** of loudnesses *e.g.* for the $3^{rd} \leftrightarrow 1^{st}$ harmonics of a square wave @ 100 *dB* to that for $3^{rd} \leftrightarrow 1^{st}$ harmonic loudness ***<u>ratio</u>*** for a square wave @ 60 *dB*:

$$\left(\frac{L_3}{L_1}\right)\Bigg|_{\text{square wave @ 100 } dB} = 90.5\%$$

Not the same fractional amount!!!

$$\left(\frac{L_3}{L_1}\right)\Bigg|_{\text{square wave @ 60 } dB} = 84.1\%$$

Loud complex sounds are thus perceived to be brighter-sounding than the same complex sounds at reduced loudness! See UIUC Physics 406 Lecture Notes on Fourier Analysis for more details…

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