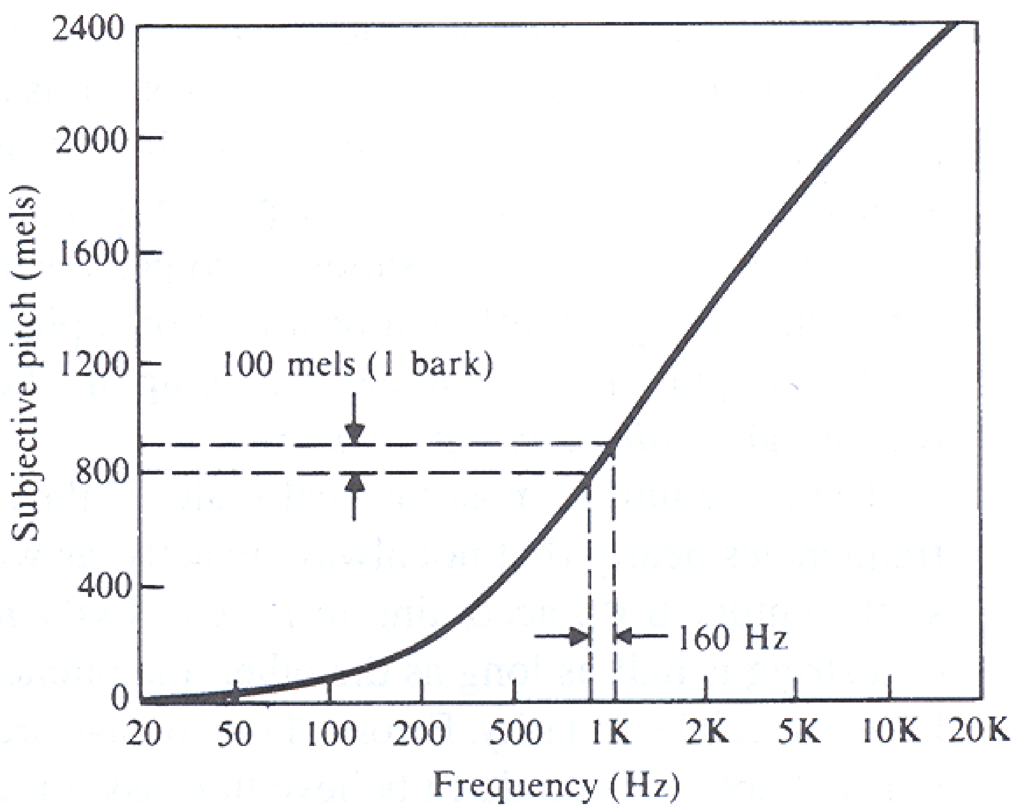
Pitch *vs*. Frequency:

Pitch = human ear's ***perception*** of frequency of a sound vibration

Low pitch ⇔ low frequency of vibration/oscillation
High pitch ⇔ high frequency of vibration/oscillation

Q: Is the relation between {perceived} pitch *vs*. frequency ***linear***?
e.g. a straight line $y = mx + b$ relation? A: **No**… See figure below:

Define frequency "units" associated with ***subjective*** pitch = ***mels*** ⇔ analogous to ***Hz***.



<u>The Audible Frequency Range of Human Hearing (when young)</u>:

$$20\ Hz < f < 20\ KHz \qquad (\simeq 3 \text{ orders of magnitude})$$

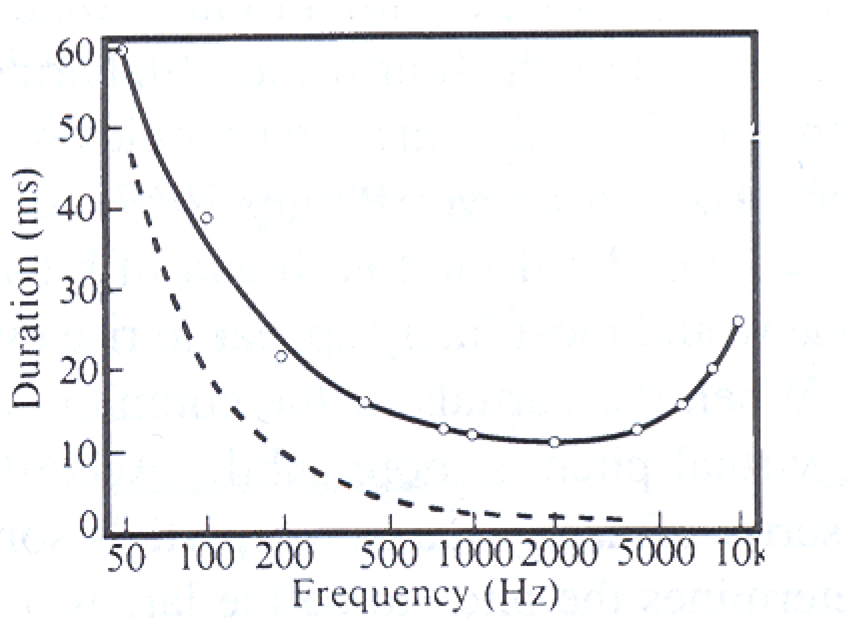
As we grow older, the ange of frequencies that we can hear decreases (both high and low frequencies – mostly on the high frequency end…)

Frequency ranges of musical instruments typically $\sim 100\ Hz$ to $\sim$ few KHz

e.g. guitar Low E = 82 Hz
High E = 330 Hz

Piano highest note is ~ 4200 Hz
Very little above ~ 10 KHz (squeals & scrapes)

The human ear needs to be able to perceive a sound for <u>minimum</u> length of time Δt. In order to determine a pitch – *i.e.* pure/single-frequency tone – the minimum duration time Δt of the pure tone depends on its frequency:



For $f \sim 100\ Hz$ $(\tau \sim 10\ msec)$: $t_{\min} \sim 40\ msec$ $(\sim 4 \text{ cycles})$

For $f \sim 1000\ Hz$ $(\tau \leq 1\ msec)$: $t_{\min} \sim 13\ msec$ $(\sim 13 \text{ cycles})$

The minimum duration time Δt for human perception of a pitch is certainly in part due our ear & brain processing, but for low frequencies especially, minimum time duration is ***<u>also</u>*** due to the ***<u>uncertainty</u>*** ***<u>principle</u>*** $\Delta f \Delta t = 1$, which tells us that a pure tone/single-frequency sine wave signal of ***<u>finite</u>*** duration Δt in fact has a ***<u>finite</u>*** frequency spread Δf ! Only as the time duration $\Delta t \to \infty$ does $\Delta f \to 0$.

This can be seen by taking the Fourier transform of a finite-length {time duration Δt_o} pure-tone/single frequency {$f = f_o$} ***time domain*** sinusoidal signal $p(t) = p_o \sin\omega_o t$ to the ***frequency domain*** $p(f)$:

If $p(t) = p_o \sin\omega_o t = p_o \sin 2\pi f_o t$ for $|t| \le \tfrac{1}{2}\Delta t_o$, and: $p(t) = 0$ for $|t| > \tfrac{1}{2}\Delta t_o$, and defining the {rectangular} ***window function*** $\mathrm{w}(t) = 1\ (=0)$ for $|t| \le \tfrac{1}{2}\Delta t_o$ $\left(|t| > \tfrac{1}{2}\Delta t_o\right)$, respectively, then:

$$p(f) = \int_{t=-\infty}^{t=+\infty} p(t)\cdot\sin\omega t\, dt = p_o \int_{t=-\infty}^{t=+\infty} \mathrm{w}(t)\cdot\sin\omega_o t\cdot\sin\omega t\, dt = p_o \int_{t=-\frac{1}{2}\Delta t_o}^{t=+\frac{1}{2}\Delta t_o} \sin\omega_o t\cdot\sin\omega t\, dt$$

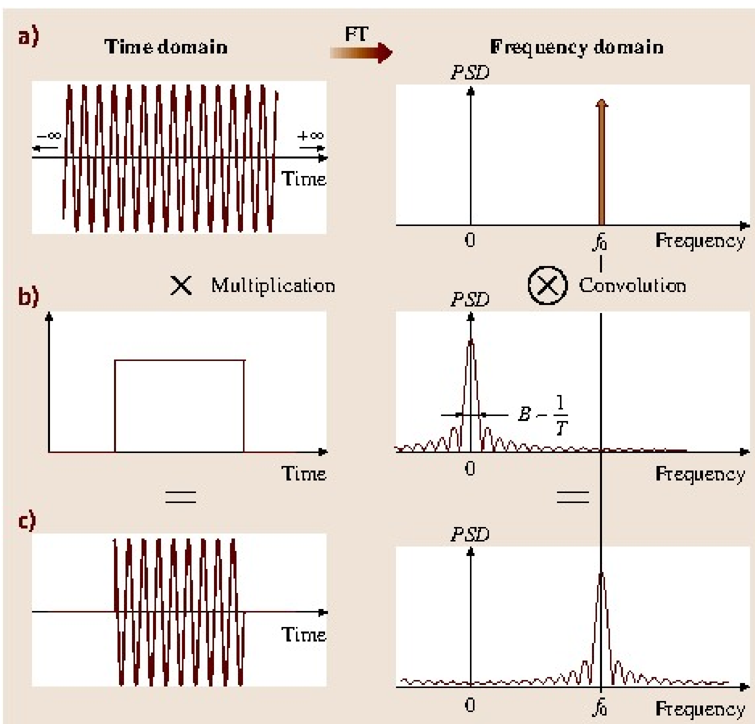
Using the trigonometric identity $\sin A \sin B = \tfrac{1}{2}\left[\cos(A-B) - \cos(A+B)\right]$:

$$p(f) = \tfrac{1}{2} p_o \int_{t=-\frac{1}{2}\Delta t_o}^{t=+\frac{1}{2}\Delta t_o} \left[\cos(\omega-\omega_o)t - \cos(\omega-\omega_o)t\right] dt = \tfrac{1}{2} p_o \Delta t_o \left\{ \frac{\sin\left[\tfrac{1}{2}(\omega-\omega_o)\Delta t_o\right]}{\left[\tfrac{1}{2}(\omega-\omega_o)\Delta t_o\right]} - \frac{\sin\left[\tfrac{1}{2}(\omega-\omega_o)\Delta t_o\right]}{\left[\tfrac{1}{2}(\omega-\omega_o)\Delta t_o\right]} \right\}$$

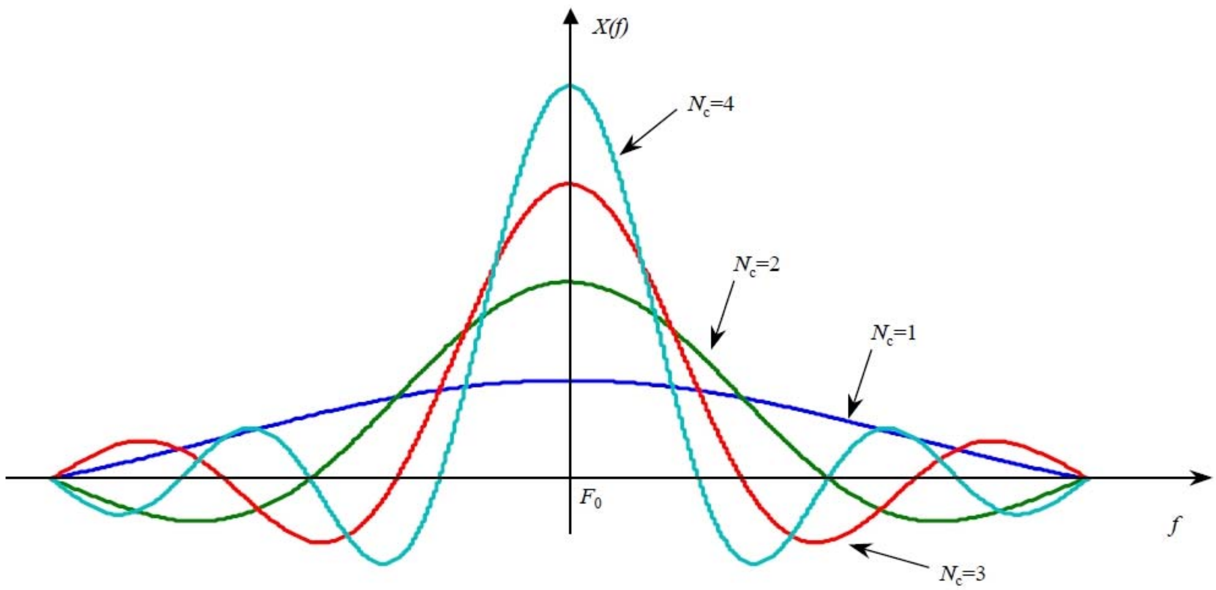
The ***sinc function*** $\mathrm{sinc}(x) \equiv \dfrac{\sin x}{x}$ {*n.b.* $\mathrm{sinc}(0) = 1$}, hence we can write $p(f)$ as:

$$p(f) = \tfrac{1}{2} p_o \Delta t_o \left\{ \mathrm{sinc}\left[\tfrac{1}{2}(\omega-\omega_o)\Delta t_o\right] - \mathrm{sinc}\left[\tfrac{1}{2}(\omega+\omega_o)\Delta t_o\right] \right\}$$

The ***power spectral density functions*** $S_{pp}(f) \propto |p(f)|^2$ (a ***frequency domain*** quantity) for infinite length and finite length sine-wave signals are shown below:

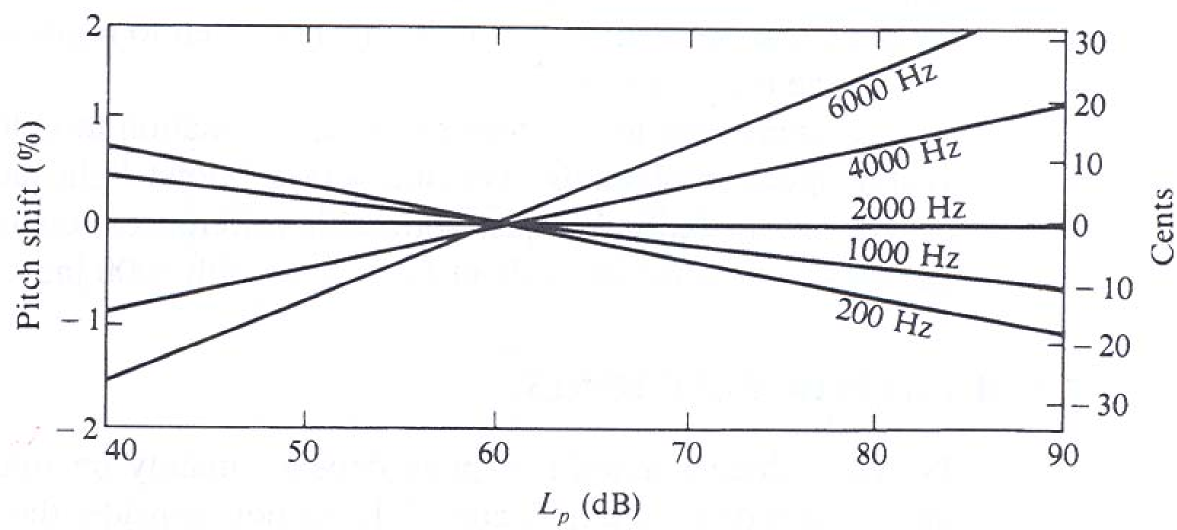


The sinc function $\mathrm{sinc}\left[\frac{1}{2}(\omega-\omega_o)\Delta t_o\right] \equiv \sin\left[\frac{1}{2}(\omega-\omega_o)\Delta t_o\right] / \left[\frac{1}{2}(\omega-\omega_o)\Delta t_o\right]$ for sine-wave signals of {short} time duration $\Delta t_o = 1\tau_o, 2\tau_o, 3\tau_o, 4\tau_o$ where $\tau_o = 1/f_o$ and the corresponding # of cycles of oscillation $N_c \equiv \Delta t_o/\tau_o = 1, 2, 3, 4$ are shown in the figure below. Note that the ***width*** Δf_o of the main peak (at $f = f_o$) depends ***inversely*** on the time duration Δt_o of the signal, due to the ***uncertainty principle*** $\Delta f \, \Delta t = 1$.



Human perception of pitch also depends {~ weakly} on the ***loudness*** of the sound.

* Effect arises due to non-linearities in the *f* & *I* response of the human ear.
* Pitch (perceived *f*) ***changes*** as loudness increases – see graph below...
* Effect exists only for pure/simple tones (!!!)
* Complex tones show ***no*** perceived pitch changes with loudness! (why??)



Two ears of same person may ***NOT*** perceive sound of a given frequency as having the same pitch!!! = DIPLACUSIS – happens ***only*** for diseased, and/or injured ears.

For ***normal*** musical purposes, frequency and pitch are synonymous (usually)
n.b. applies ***only*** to ***periodic*** sounds.

Sound ***pulses*** are made up of a ***continuum*** of frequencies, sound ***pulses*** are thus anharmonic and hence have ***no*** characteristic frequency and/or pitch.

The human ear can discriminate ***changes*** in sound intensity levels/sound pressure levels/loudnesses of ***JND*** $= \Delta L = |L_1 - L_2|$ ~1/2 ***dB***; Our ability to do so also depends on frequency and sound pressure level/loudness:

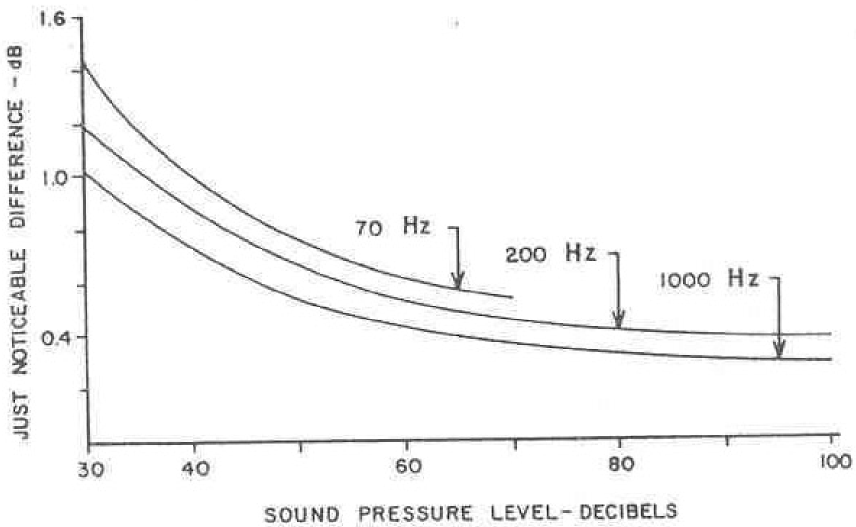
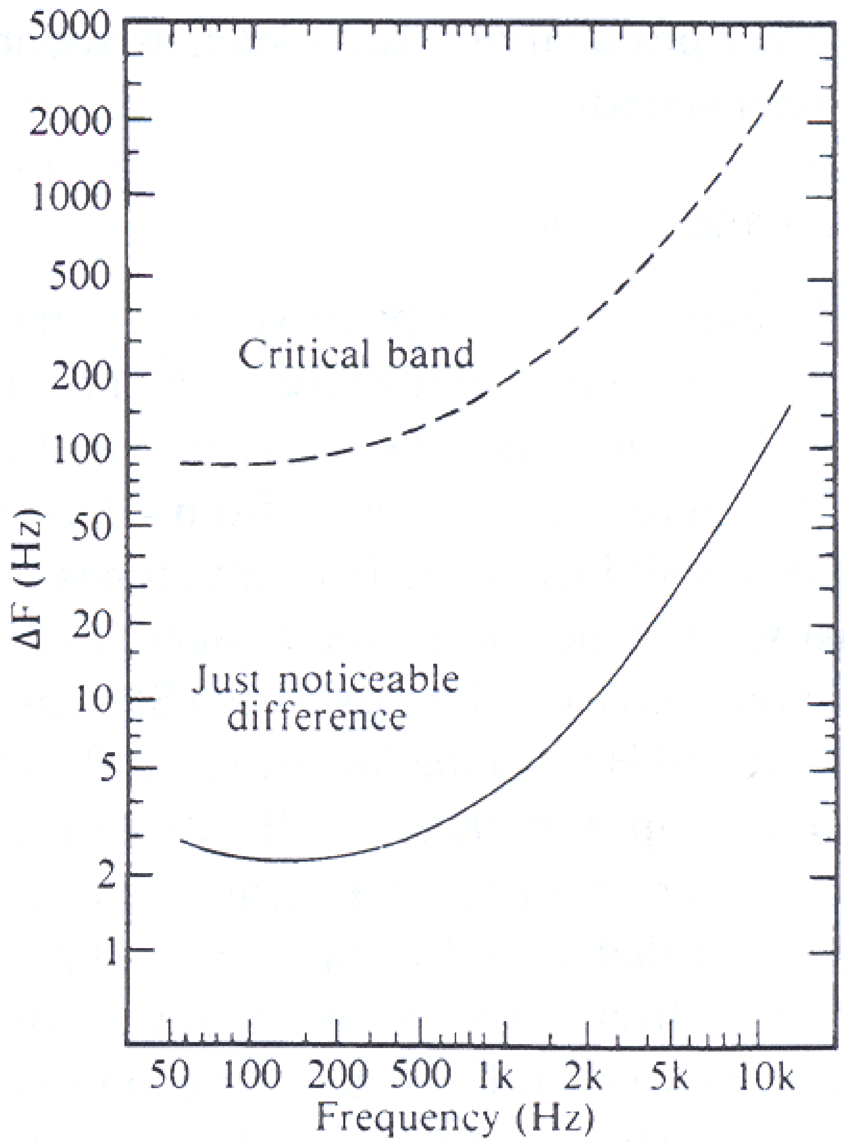


FIG. 2. Just noticeable difference in sound pressure level for three frequencies.

A ***JND*** ~1/2 ***dB*** change in sound intensity level corresponds to a fractional change in sound intensity of $\Delta I / I$ ~ 12%. Thus, due to the ~ logarithmic response of the human ear, it is not terribly sensitive to ***changes*** in the loudness of sounds.

The typical human ear can discern ***changes*** in pitch/frequency at the $\Delta f \sim 3\ Hz$ level in the frequency range $\sim 30\ Hz \le f \le 1000\ Hz$. Again, has frequency dependence:



Note that:

At very low frequencies: $\Delta f / f \simeq 3/30 \quad = 10\% \ (\simeq \text{ 2 semitones})$,
Whereas at higher frequencies: $\Delta f / f \simeq 3/1000 = 0.3\% \ (\simeq \text{ 0.1 semitones})$

A good musician can discern frequency changes ***significantly*** smaller than this – *e.g.* above $\boldsymbol{f} \ge 500\ Hz$: ≈ 0.03 semitone (*i.e.* $\Delta f / f \simeq 1/1000 = 0.1\%$)!!!

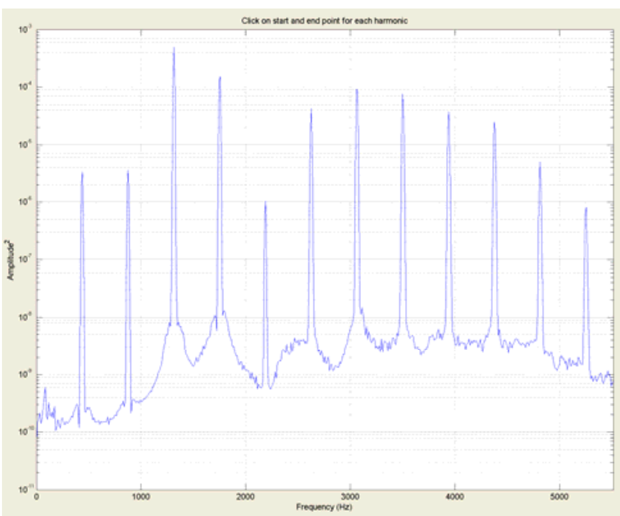
∴ The human ear/brain is capable of detecting small changes in frequency!!!

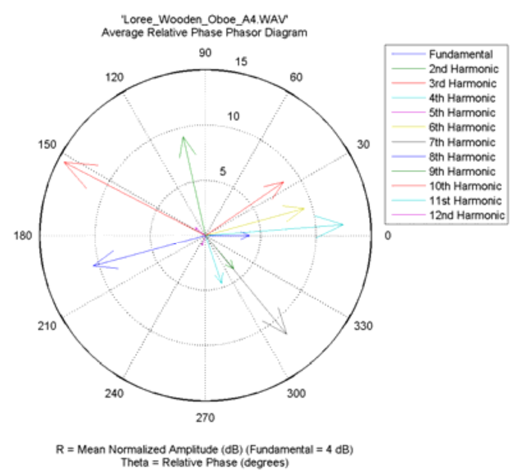
The human ear/brain is capable of perceiving a ***fundamental*** even when ***no*** fundamental is actually present!!! This is the so-called ***missing fundamental effect***.

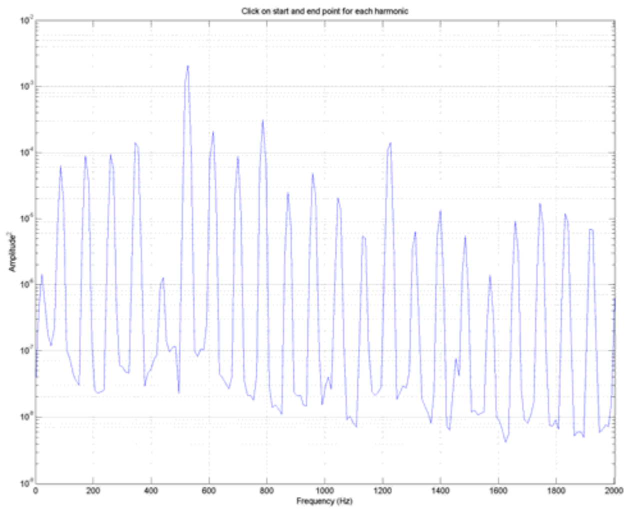
This effect is {again} a consequence of the non-linear response in/inside the human ear itself, and/or a non-linear response(s) in the human brain's ***processing*** of frequency information – whenever *e.g.* a ***quadratic*** non-linear response exists (in any system), if two signals *A* and *B* with frequencies $\boldsymbol{f_A}$ and $\boldsymbol{f_B}$ are input to that system, then sum and difference frequencies $(\boldsymbol{f_A + f_B})$ and $|\boldsymbol{f_A - f_B}|$ are produced! Thus, *e.g.* a 2nd harmonic $2\boldsymbol{f_1}$ and a 3rd harmonic $3\boldsymbol{f_1}$ can produce a "missing" fundamental from the difference frequency, $|3\boldsymbol{f_1} - 2\boldsymbol{f_1}| = \boldsymbol{f_1}$!!! For further details on distortion, read Physics 406 Lecture Notes on "Theory of Distortion I & II".

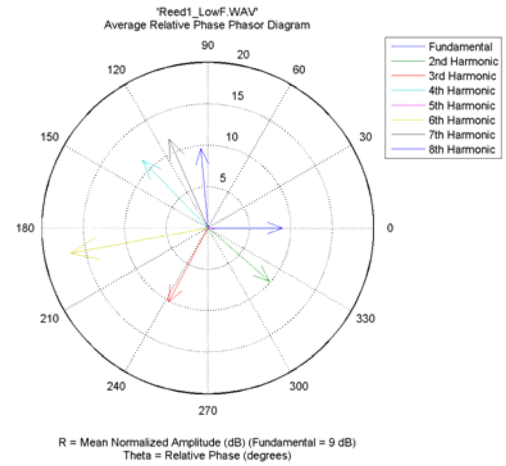
For some musical instruments – *e.g.* the trumpet, the oboe and/or the bassoon – the 2nd (or even 3rd and higher) harmonics can actually have a ***larger*** amplitude than that of the fundamental, however we perceive/hear the "note" that is played on the trumpet (and/or oboe, bassoon) as that of the fundamental!!!

The harmonic spectra – *aka* power spectral density functions $S_{pp}(f)\ vs.\ f$ and associated {time-averaged} relative phase harmonic phasor plots are shown below – *e.g.* for the steadily-played notes *A*4 (440.0 *Hz*) played on the oboe, and *F2* (87.3 *Hz*) played on the bassoon:









Note that the vertical axes of $S_{pp}(f)$ *vs.* f are displayed on a ***logarithmic*** scale.

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