

The Acoustic Impedance of Bb Clarinet

Marguerite Brown
Department of Physics
University of Chicago

ABSTRACT

To better understand the unique tone of both a plastic and wooden clarinet, an analysis of sound production was taken by several methods. The sound output from the clarinet was recorded and run through a phase-sensitive wave analysis program to determine the phase relations, frequencies, and intensities of the most prominent harmonics. Using a piezoelectric transducer, phase-sensitive measurements of the complex pressure and particle velocity at the input and output were taken across a range of frequencies. These measurements were then used to determine the complex impedance and sound intensity at the input and output. The phase results indicate some of the effects of the material and structure of the instrument on tone.

I. Background

The clarinet is part of the class of wind instruments excited by reeds, including bassoon, oboe, and brass instruments, which use the lips as a sort of “reed”.¹ The reed acts as a flow-control device, creating oscillations by altering the rate at which air enters the instrument², and effectively closes one end of the tubing. The other end (the bell) is open. Another class of wind instruments, including flutes and recorders, are excited by a stream of air across one end of the air column¹. The latter instruments are effectively open at both ends.

When an instrument is played, we hear a mixture of harmonics. The specific sound of the instrument is defined partly by the ratios of output intensity amplitudes between different harmonics. The time-averaged complex longitudinal sound intensity, $\tilde{I}(r)$, is defined as

$$\tilde{I}(r) = \tilde{p}(r) \cdot \tilde{u}_z^*(r)$$

where $\tilde{p}(r)$ is the complex pressure amplitude (Pascals) and $\tilde{u}_z(r)$ is the complex longitudinal particle velocity (m/s). The SI units of sound intensity are Watts/m²

The resonant harmonics of a certain pitch correlate with the points of maximum impedance. The complex specific acoustic longitudinal impedance, $\tilde{Z}(r)$, is defined as

$$\tilde{Z}(r) = \frac{\tilde{p}(r)}{\tilde{u}_z(r)}$$

This has units of acoustical ohms (= kg/m²-s).

The open end of a standing wave tube has a very low \tilde{p} and a high \tilde{u}_z , corresponding to a very low impedance. Similarly, the closed end of a standing wave tube has a high \tilde{p} and a low \tilde{u}_z . The cylindrical standing wave tube with both ends open or both ends closed resonates at a lowest, fundamental frequency f_0 , defined as

$$f_0 = \frac{c}{2L}$$

where c is the speed of sound and L is the length of the tube.

Higher resonant frequencies are defined as

$$f_n = nf_0 = \frac{nc}{2L}$$

where $n=1, 2, 3, \dots$

If one end is open and one end is closed, the fundamental pitch is defined as

$$f_0 = \frac{c}{4L}$$

and the higher frequencies as

$$f_{2n-1} = (2n-1)f_0 = \frac{(2n-1)c}{4L}$$

In other words, only the odd harmonics resonate in a closed-open cylinder.

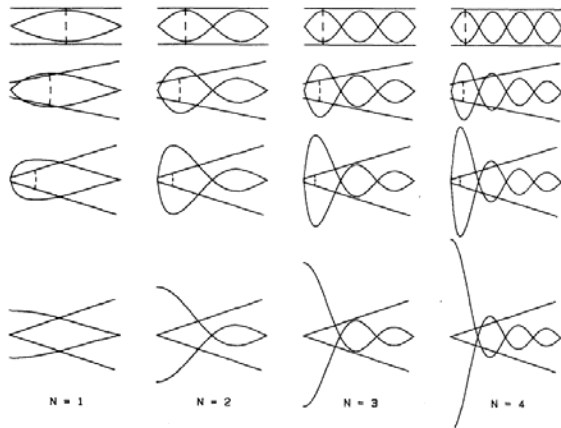


Figure 1: The pressure waveform for resonant frequencies of a cylindrical bore and conical bores at various stages of completion. If a conical bore is nearly complete, it will resonate at the same frequencies as an open-open cylinder of the same length even if the small end is closed.³

The expected resonant frequencies of an idealized instrument derived from these equations do not exactly match what we measure in most real situations because of the effects of damping and dissipation that are disregarded in the ideal case. We define the inharmonicity to be the difference between the actual and expected values of n . In other words,

$$\Delta n = n_{\text{actual}} - n_{\text{ideal}}$$

The clarinet is the only instrument of its class with a cylindrical bore, and these same harmonic patterns are not true in a conical bore. A tube resembling a complete or nearly complete cone will in fact have the same resonant frequencies as the open-open tube, as shown in figure 1. If a cone is nearly complete and closed at the small end, it resonates in the same way as a complete cone.

For both conical and cylindrical bores, the fundamental pitch is determined by the length of the instrument. Tone holes in the instrument effectively shorten the length of the tube and allow for more notes to be played. However, an instrument with open tone holes does not resonate with the same harmonic range as an instrument with the same effective length and no tone holes. Above a certain critical frequency, a pipe with open tone holes stops resonating².

In most cases, the fundamental frequency will have the highest input impedance of the harmonics present, so the instrument will most strongly resonate at the fundamental pitch. It is nonetheless possible to manipulate the vibrations of the air column so that it will resonate most strongly at a higher harmonic.

Register holes are a special type of tone hole that force a frequency other than the fundamental to have the highest

impedance. A register hole is typically placed about a third of the way down the instrument. Opening the register hole creates a pressure node that lowers the input impedance at the fundamental frequency while leaving a higher harmonic relatively unaffected. The second or third harmonic is then most likely to sound when the instrument is played. Creating the pressure node also causes the wavelength of the fundamental pitch to shorten, making the fundamental frequency slightly higher².

The sound wave produced is thus a combination of waves with different frequencies. Often, the components of the overall sound wave are not in phase with the fundamental wave. The patterns to these phase shifts are also unique to each instrument, and can be observed in the complex harmonics, pressure, particle velocity, intensity, and impedance. It has been argued that it is these phase relations that creates a unique tone, rather than the amplitudes of the components⁴.

Backus performed some measurements of the input impedance for clarinet and other reed instruments by applying a constant acoustic current to the instrument and measuring the pressure changes over frequency, and observed several tendencies in the different ranges on the clarinet. It is useful here to define the three registers: the chalumeau, which is the lowest register; the clarion, which begins at the lowest note making use of the register key (concert A4 on a Bb clarinet) and ends on concert Bb5; and the altissimo register, consisting of all notes above that.

In the chalumeau register, the impedance peaks were compressed, resulting in a negative inharmonicity of greater amplitude in higher resonances. The impedance of the lowest resonance of notes in the chalumeau is typically

between 800 and 1000 Ohms, and the next resonance (about three times the fundamental) has an impedance between 500 and 700 Ohms.

For most notes on the clarinet, the highest impedance peak corresponds to the note played with that fingering. In the altissimo register and the higher notes of the clarion, though, the highest impedance peak tends to occur at a different harmonic, usually one of the lower ones.

In the clarion register, Backus observes the resonances as randomly distributed relative to the harmonics.¹

II. Experimental Apparatus and Method

We first recorded several notes being played on both a wooden Buffet R13 clarinet and a plastic Vito clarinet at the bell of the instrument with a Peavey PVM-45 dynamic microphone and a Marantz PMD671 24-bit digital recorder. The sound produced from the mouthpiece alone was also recorded. Each recording was analyzed for the frequencies and phase relations of the harmonics using a MATLAB-based computer program written by Joseph Yasi⁶.

We also attempted to measure pressure and particle velocity as the instrument was played with microphones in the mouthpiece, but unfortunately the amplitude of sound in the mouthpiece was over 130 dB even at very soft dynamics, so the microphones were not able to take data.

A phase-sensitive apparatus was developed to measure the complex input and output impedances and sound intensities across a range of frequencies by exciting the instrument with a piezoelectric transducer. In this apparatus, an Agilent 33220A frequency generator produces a sine wave at a specified

frequency and sends it through a voltage amplifier to increase the voltage output by a factor of 10, and through a negative impedance converter (NIC) to produce a constant current with a piezoelectric transducer. The NIC causes a -90° phase shift in the signal. The piezoelectric transducer is attached to the reed opening of the mouthpiece with cyanoacrylate glue. As a reference for the phase, the sine wave is also sent to 4 SRS-830 lock-in amplifiers measuring the complex pressure and particle velocity at the input and output of the instrument.

In order to measure the input impedance, a clarinet mouthpiece was modified to fit a particle velocity microphone and a differential pressure microphone, as seen in figure 2. The mouthpiece was then sealed off with an apiezon sealing compound. Pressure and particle velocity microphones also were placed at the bell of the instrument to measure the output impedance.

The microphones used to measure pressure were omni-directional 1/10" diameter Knowles Acoustics FG-23329 high-performance microphones, placed perpendicular to the airflow. To measure longitudinal particle velocity, we modified

Knowles Acoustics EK-23132 high performance microphones to work as differential pressure microphones by removing the back cover plate. Because inertial forces dominate over dissipative forces or viscous effects in air, we can then find the particle velocity using the one-dimensional Euler's equation for inviscid fluid flow,

$$\rho_0 \frac{\partial u_z(z,t)}{\partial t} = - \frac{\partial p(z,t)}{\partial z}$$

where ρ_0 is the density of air. To solve for $u_z(z,t)$, we integrate Euler's equation,

$$u_z(z,t) = - \frac{1}{\rho_0} \int_{-\infty}^t \frac{\partial p(z,t')}{\partial z} dt'$$

This equation is physically solved using a custom-built integrating preamp for the differential pressure microphone signal. The data-taking program starts at 29.5 Hz and continues to take data in 1 Hz steps up to 4030.5 Hz. Because of background noise from the ventilation system in the room, we sometimes have to start at 39.5Hz or 49.5Hz⁵.

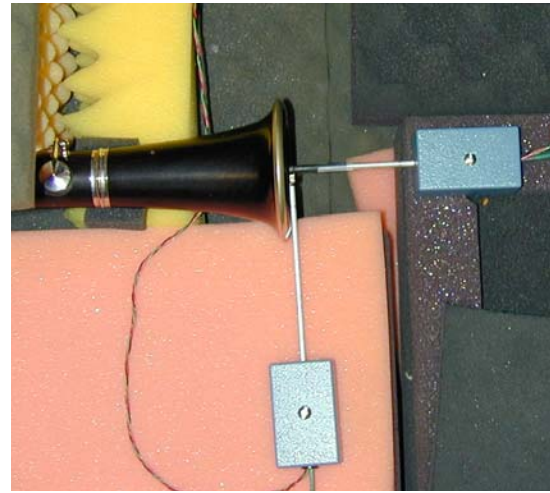
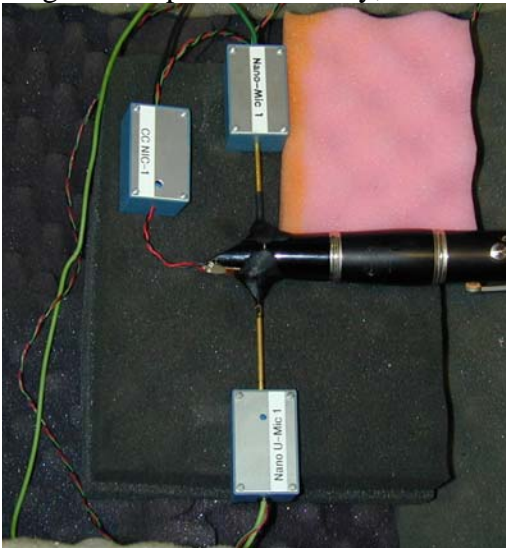


Figure 2: the set up for microphones in the mouthpiece (left) and at the bell of the clarinet.

The absolute microphone sensitivities were calibrated using an Extech Sound Level Calibrator (SLC) model 407766 and an Extech Sound Pressure Level (SPL) Meter model 407768. Using the SPL meter, we confirm that the sound from the SLC is produced at 94dB. We then calibrate the sensitivities of the pressure and particle velocity microphones by placing them in the opening of the SLC and measuring the AC rms output voltage with a Fluke digital multimeter. The average pressure microphone sensitivity was about 280 mV(rms)/ Pa(rms) and the average particle velocity microphone sensitivity was about 80 mV(rms)/ Pa*(rms), where 1 Pa* is 2.4 mm/second.

Using another MATLAB-based computer program, the complex impedance and sound intensity at the input and output are calculated from the corresponding complex pressures and particle velocities. Over the frequency range of interest, the program then separately graphs the real and imaginary components, the phase offsets, and the value in the complex plane for pressure, particle velocity, impedance, and intensity.

However, the raw-data results for both pressure and particle velocity are affected by several phase shifts. We determined these phase shifts both by performing the same measurements on a standing wave tube and by attaching the piezoelectric transducer to a baffle board, both of which have known phase results. The phase shift in the circuit from the frequency generator to the piezo driver totals to about -180° in the range of interest. The particle velocity mic preamp has an additional phase shift of -90° , while the pressure mic preamp does not have any shift. There are also frequency-dependent phase shifts for both results.

To get phase-corrected results, we must also apply a frequency dependent rotation matrix, defined as

$$\begin{bmatrix} X_{corr}(f) \\ Y_{corr}(f) \end{bmatrix} = \begin{bmatrix} \cos[\varphi_c(f)] & \sin[\varphi_c(f)] \\ -\sin[\varphi_c(f)] & \cos[\varphi_c(f)] \end{bmatrix} \begin{bmatrix} X_{obs}(f) \\ Y_{obs}(f) \end{bmatrix}$$

for $\varphi_c < 0$.

We then analyze the data taken on the instrument with phase corrections and absolute microphone sensitivities applied⁵.

III. Results and Discussion

The notes recorded on the clarinet were D3, Eb4, Ab4, A4, Bb5, and G6. D3, Eb4, and Ab4 are the notes in the chalumeau register, A4 and Bb5 are in the clarion register, and G6 is in the altissimo. D3 is the note with all tone holes closed.

We also recorded the sound produced from the Vandoren M13 lyre mouthpiece used in these recordings, and from the modified Selmer Bundy mouthpiece also used to measure impedance. We can see from the results in Figure 3 that the modifications done to the clarinet mouthpiece to take impedance measurements did not have a significant impact on the results.

From the analysis of each waveform on both the wooden and plastic instruments (Figures 4-5), we can see several trends across the ranges. One of these is the formant, a fixed frequency region at which the harmonics of any tone played on the instrument are emphasized. The two formant regions on the clarinet occur around 1500-1700 Hz and 3700-4300 Hz⁴.

Many notes also had a peak near 0 Hz, likely because of a non-linear distortion resulting that occurs because of the mix of frequencies in the clarinet's tone⁷. Lower notes, such as the D3, tend to

have a greater number of clearly defined harmonic peaks. The frequency correlating to the intended note is typically the first harmonic measured when the note is played.

Contrasting between the Vito and Buffet clarinets, we can see that for lower notes, the Vito tends to have more prominent harmonics at higher frequencies (Figures 4-5). In higher notes such as the G6, the Buffet clarinet tends to have more

prominent and clearly defined harmonics (figure 5). It is likely because of this trend that the Buffet instrument tends to play more easily at higher frequencies.

Looking at inharmonicity (figure 6), we also see that the harmonics on the Buffet R13 clarinet tended to occur at frequencies closer to the expected value than those on the Vito. This also could contribute to the better quality of sound produced from the Buffet.

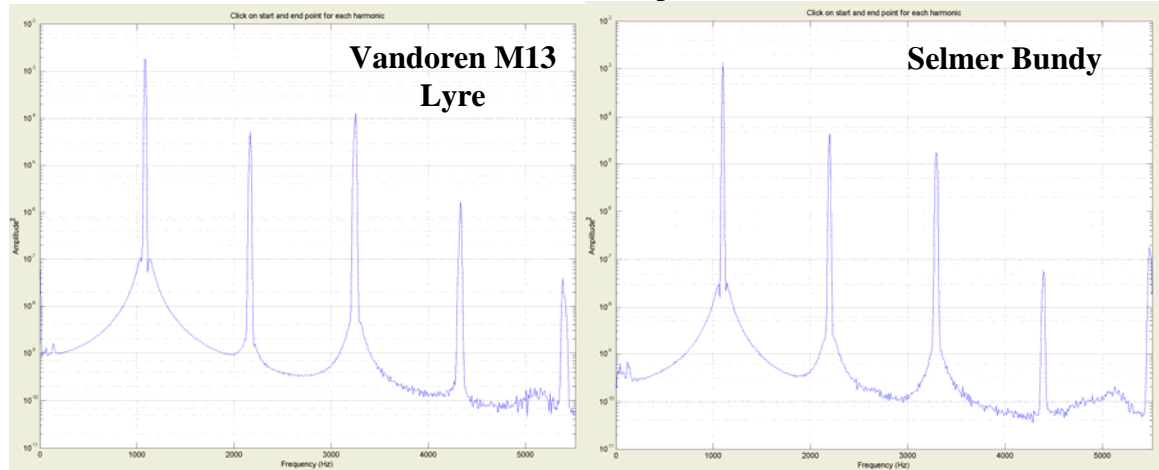


Figure 3: The comparison of the harmonics of different clarinet mouthpieces. LEFT: Vandoren M13 Lyre, which was used when the instrument was played. RIGHT: Modified Selmer Bundy Mouthpiece, also used in the impedance measurements

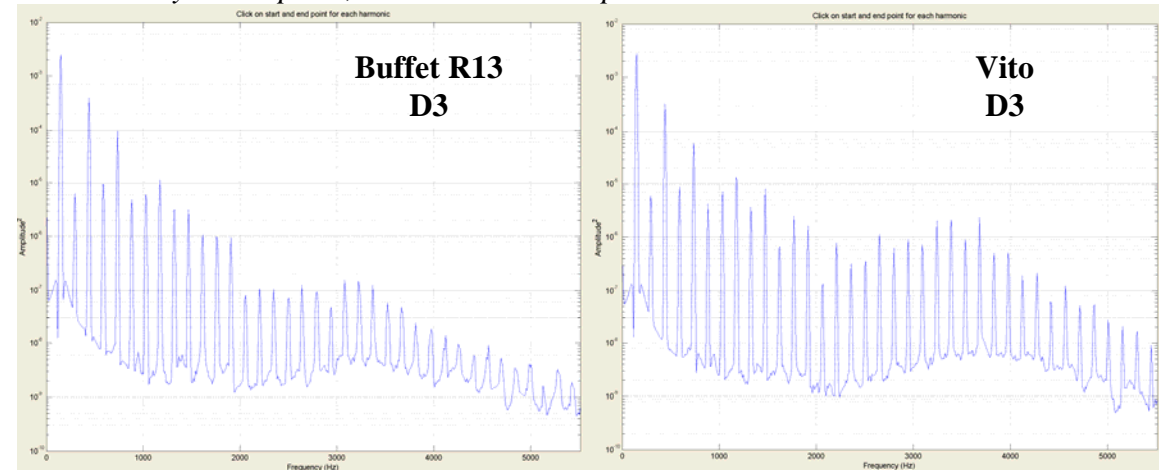


Figure 4: The harmonics measured for D3 on the Buffet R13 (left) and the Vito.

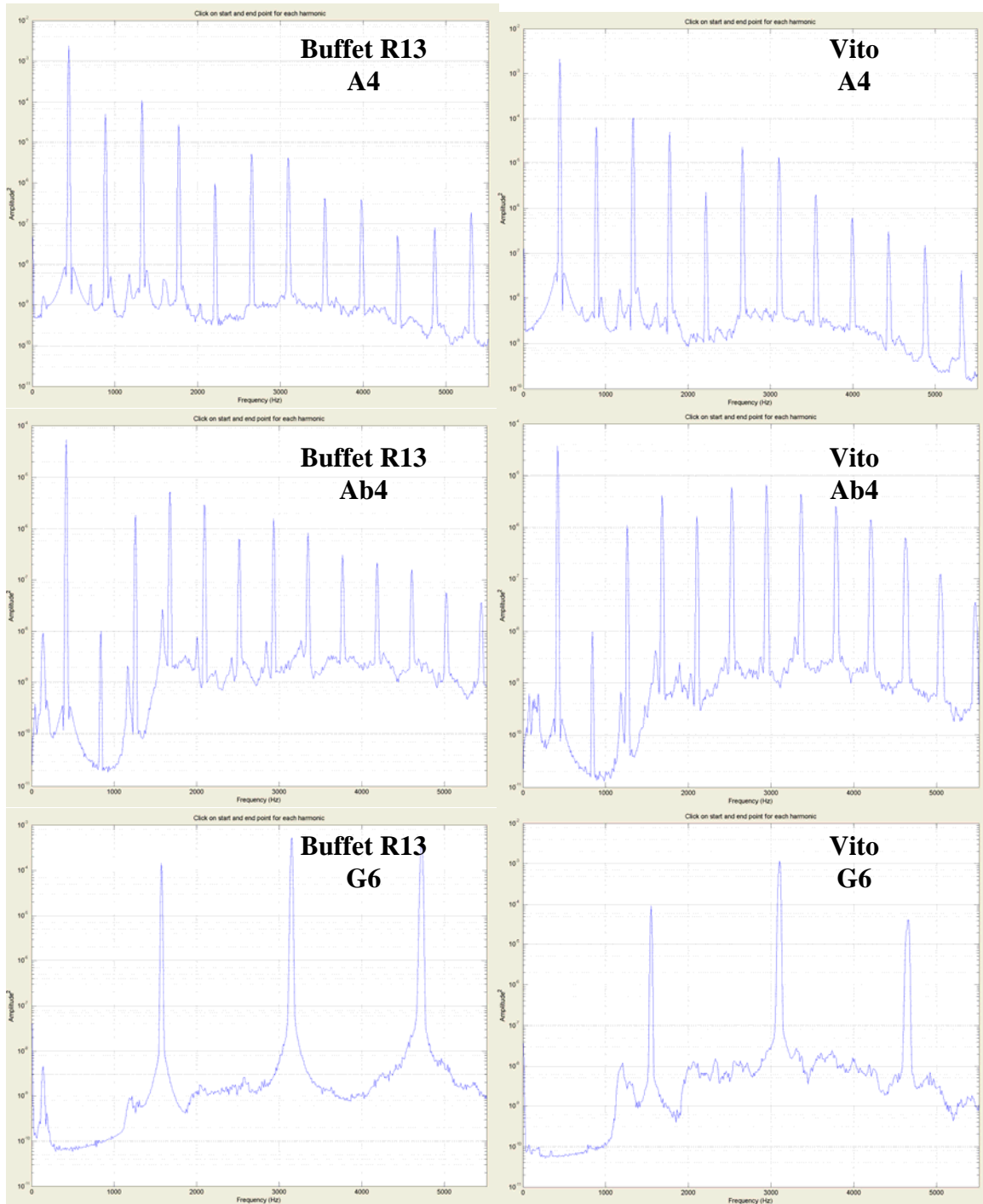


Figure 5: the harmonics extracted from the analysis of the waveform recording for (from top) A4, Ab4, and G6. The graphs on the left are from the Buffet R13 clarinet, and those on the right are from the Vito. We can see in the graphs of Ab4, which is typically considered the weakest note on clarinet, a distinct dip in amplitude around 1000 Hz, the location of the second harmonic. For the G6, the frequency corresponding to G6 is actually the first peak, but the second peak, corresponding to G7, is stronger. We hear G6 because of the nonlinear response of the human ear, which causes us to hear the first harmonic more strongly⁷.

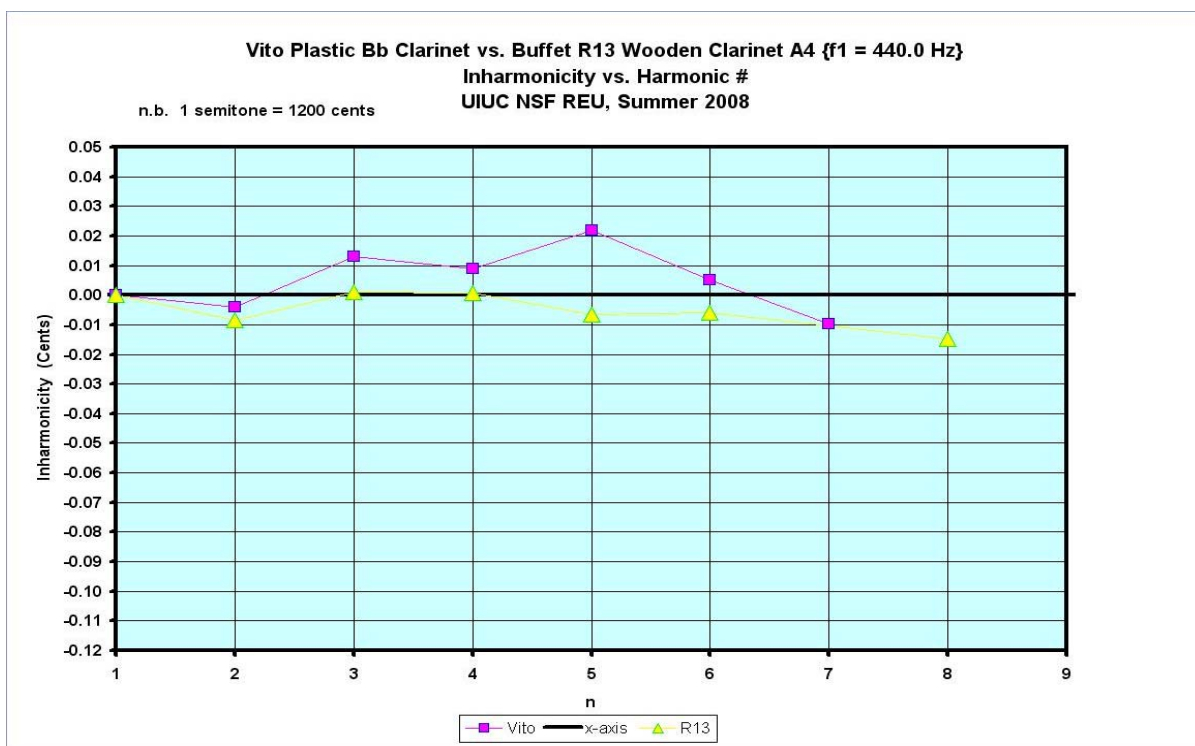


Figure 6: The inharmonicity of the Buffet R13 and Vito for the note A4. We can see that the harmonics are closer to the ideal value on the Buffet R13.

When we measured the complex input impedance of both clarinets, the amplitude of impedance peaks generally occurred at a slightly higher frequency than they did for a person playing the instrument. The presence of the reed in the clarinet lowers the frequencies of resonances when the instrument is played², so this shift was likely caused by the use of a piezoelectric transducer instead of a reed to excite the air column.

The effects of the register key on the instrument became far more noticeable when we measured input impedance. The fingerings for D3 and A4 differ only by the opening of a register hole for A4. Contrasting the input impedance for the two (Figure 7), we see that the third harmonic in A4 is the most prominent, whereas the fundamental is strongest in D3. Another less obvious difference is the frequency of the fundamental resonance,

which occurs at 151.5 Hz on the note D3, but at 212.5 Hz for A4. This demonstrates that the pressure node must shorten the wavelength of the fundamental pitch while leaving the 3rd harmonic relatively unchanged.

Another pair of notes that are related by the register key is Eb4 and Bb5 (Figure 8). The fundamental pitch was also raised from 330.5 Hz for the Eb4 to 376.5 Hz for Bb5. However, the third harmonic was also raised from 975.5 Hz to 995.5 Hz.

From this, we can see the problem that results from the clarinet having only one register key used with a number of tone hole combinations. D3 has every tone hole closed, while Eb4 has most of the tones holes open, making the instrument about half as long. For both of these notes, the register key is not quite placed at the ideal position 1/3rd of the way down the

instrument, but the difference in distance is more extreme for Eb4.

For notes at the higher end of the chalumeau range, however, the input impedance of the third harmonic is close enough to that of the fundamental that a player could practically reach both notes

without the use of the register key. The use of only a single register key on Bb clarinet is only possible in practice because of the player's ability to manipulate the pitch with his lips².

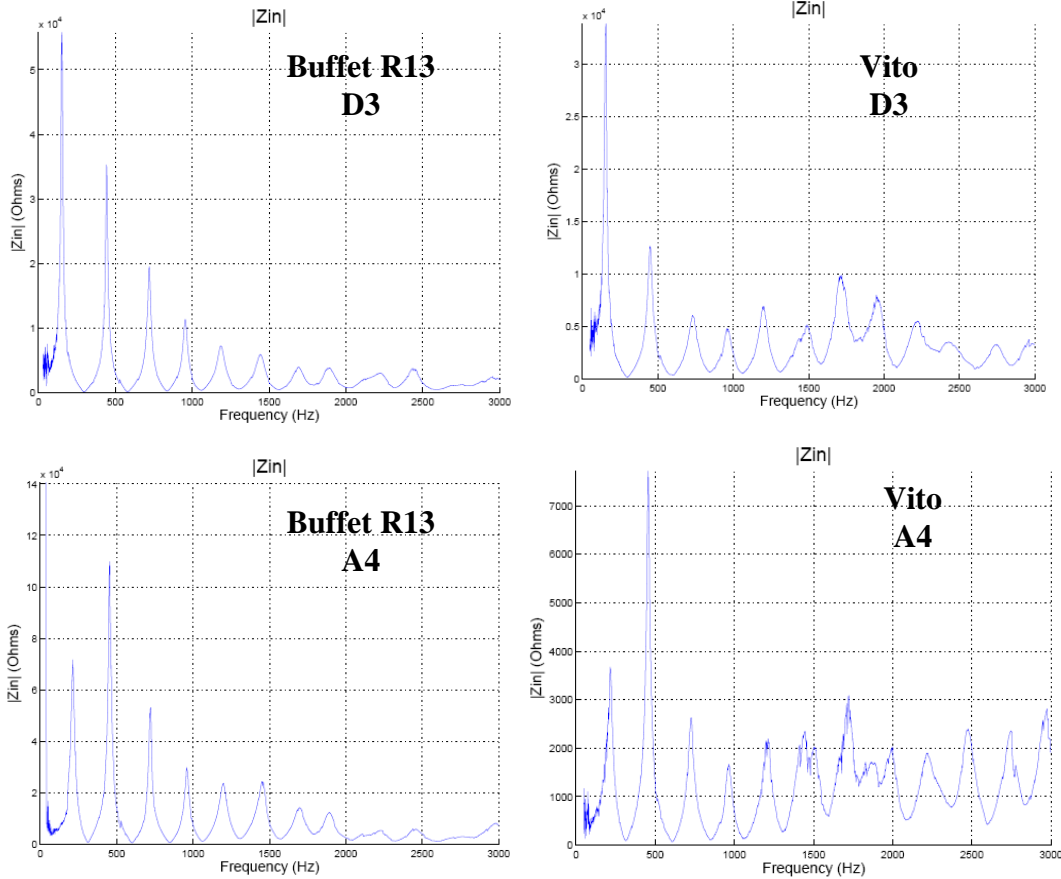


Figure 7: Input Impedance magnitude for D3 (top) and A4. Buffet results are on the left.

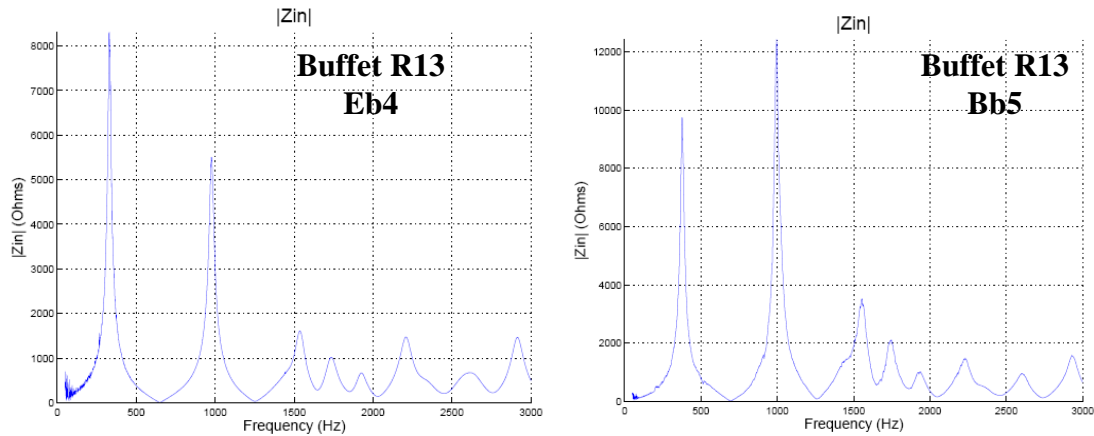


Figure 8: The input impedance magnitude on the Buffet R13 for Eb4 (left) and Bb5.

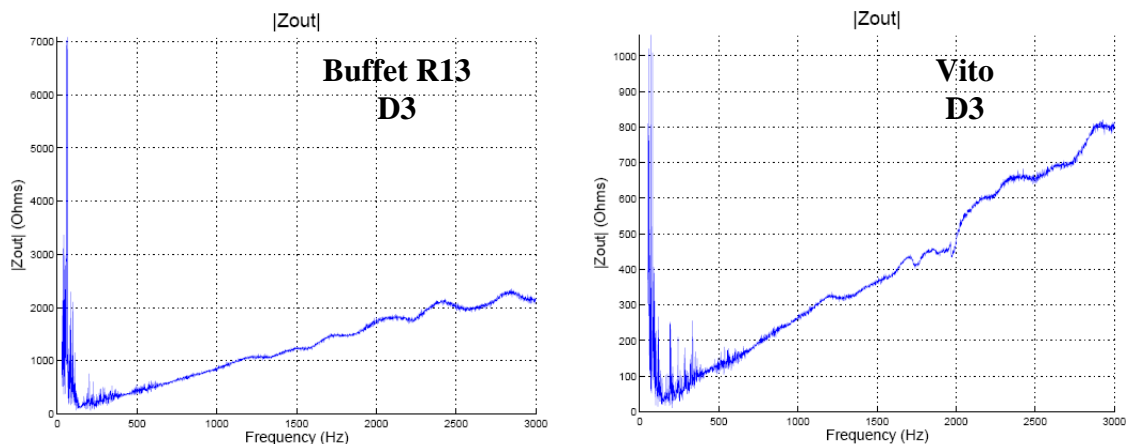


Figure 9: The output impedance amplitudes for D3. Buffet results are on the left, Vito on the right.

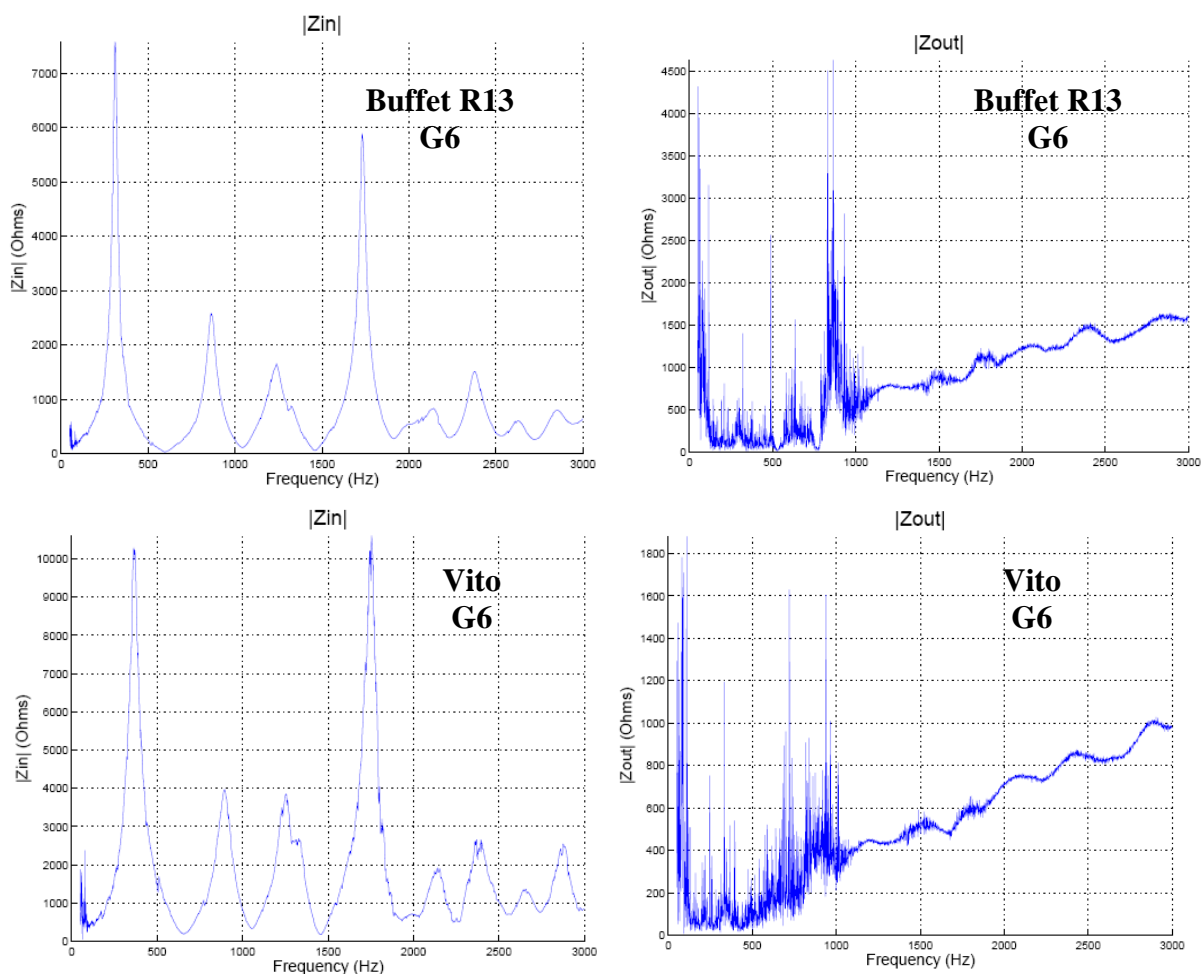


Figure 10: The input and output impedance for G6 on the Buffet (top) and Vito clarinets. The Vito clarinet does not have as clearly defined peaks in this range.

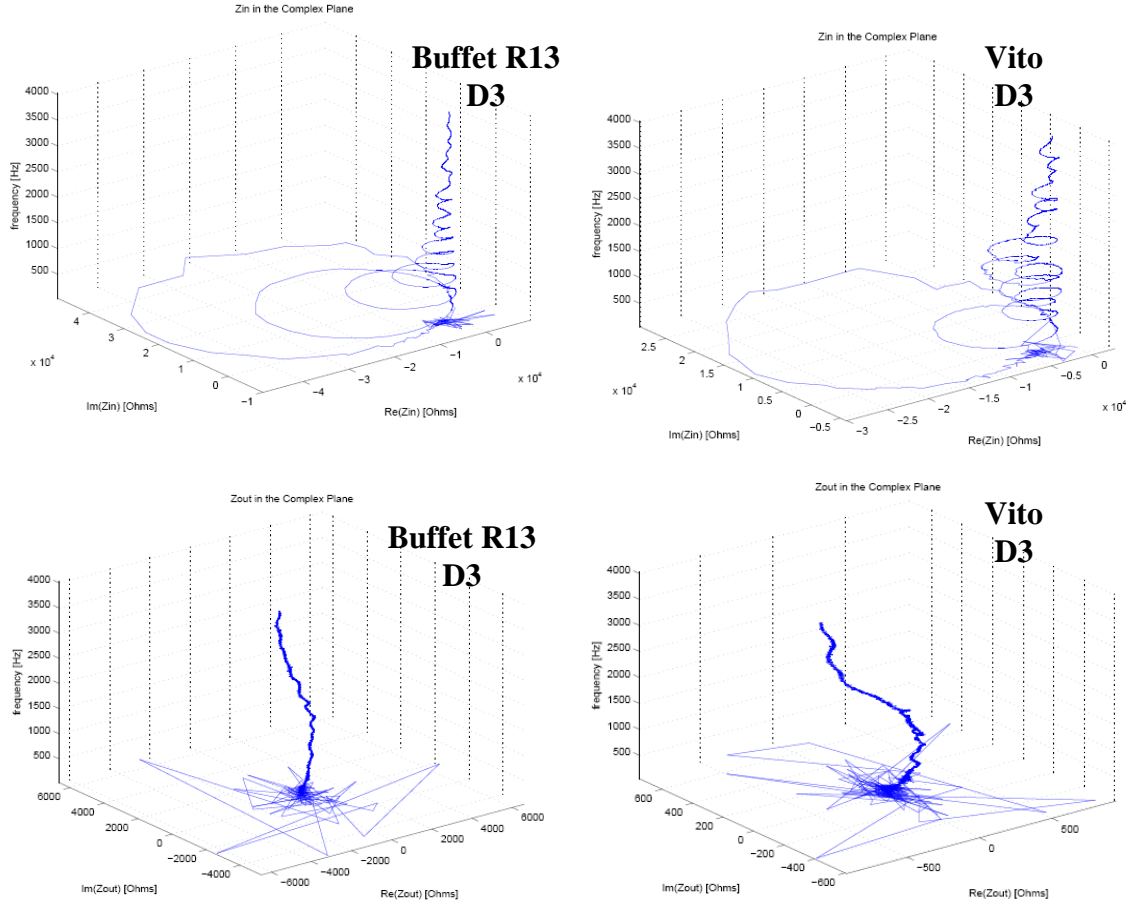


Figure 11: Complex input and output impedance for D3 on the Buffet (left) and Vito (right). Over the frequency range of interest, the phase pattern for impedance on the Buffet tends to be more centered about a certain point than those on the Vito.

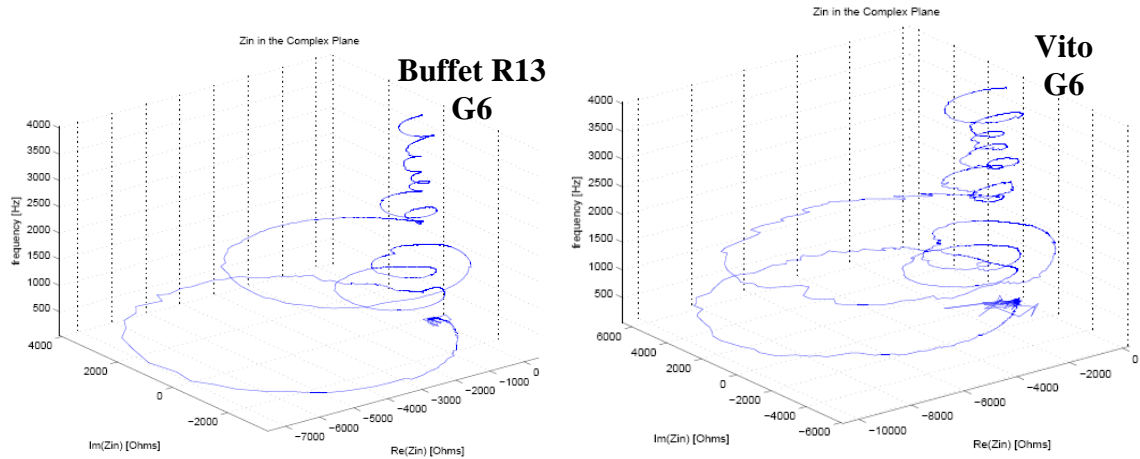


Figure 12: The complex input impedance for G6 on the Buffet (left) and Vito. We can see that in the range, the input impedance on the Vito is not as smooth as on the Buffet.

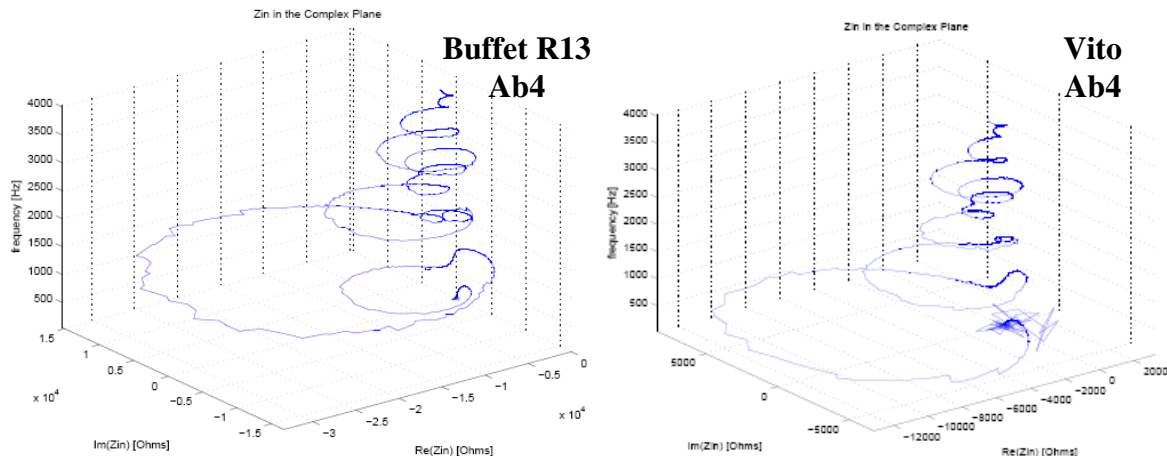


Figure 13: The input impedance for Ab4 on the Buffet (left) and Vito clarinets. Ab4 is the weakest sounding note on clarinet, and we can see that, even for the Buffet, the pattern of the phase across the range of interest is rather asymmetric.

As shown in figure 9, we can also see a difference in the output impedance amplitudes between the Buffet R13 and Vito. The Vito has noticeably higher impedance at the higher frequencies at any range. This is also true for higher notes such as G6 (figure 10), but not to as great a degree. For the G6, the main difference is in the input impedance. From this, we can see that the Vito has less clearly defined input impedance peaks.

From figures 11-13, we can also see how the phase relations can affect the changes in tone quality between different notes and different materials for constructing the instrument. For the clarinet, the tone hole combinations with the best tone quality tend to have relatively smooth input impedance curves centered around a certain point. On weaker notes such as Ab4, we see jagged curves and asymmetric patterns for the input impedance.

IV. Conclusions

With the data taken this summer, we began to explore the effects of different materials used to construct the clarinet. Finalized phase corrections can give us greater insight on the effects of phase

offsets on tone quality. With appropriately desensitized microphones, we could also begin to explore the changes in pressure and particle velocity along the inside of the instrument, both while the instrument is being played and when excited by a piezoelectric transducer. Taking measurements for more of the notes would also help determine some of the effects of specific tone hole lattices. Developing a means of measuring the effect of vocal tract resonances, pressure on the reed from the embouchure, and other such factors dependant on the musician can also help us understand the way the unique sound is produced from an instrument.

V. Acknowledgements

Thanks to Professor Steve Errede for his guidance with this research, and to Brendan Sullivan and Nicole Pfiester for their assistance throughout the project.

The REU program is supported by the National Science Foundation Grant PHY-0243675.

VI. Endnotes and References

¹ John Backus, J. Acoust. Soc. Am. 56 (4), 1266 (1974).

²Arthur H. Benade, *Fundamentals of Musical Acoustics* (Oxford University Press, London, 1976).

³R. Dean Ayers, Lowell J. Aliason, and Daniel Mahgerefteh, *Am J. Phys.* 53 (6), 520 (June 1985).

See also: J. Wolfe, “Pipes and Harmonics” (unpublished)

<<http://www.phys.unsw.edu.au/jw/pipes.html>>.

⁴John Backus, *The Acoustical Foundations of Music*, 2nd ed. (W. W. Norton & Company, New York, 1969), Ch 6, pp. 94-109.

⁵D. Pignotti, “Acoustic Impedance of a Bb Trumpet” (unpublished)

<http://online.physics.uiuc.edu/courses/phys199pom/NSF_REU_Reports/2007_reu/David_Pignotti_Senior_Thesis/David_Pignotti_Senior_Thesis.pdf>.

⁶J. Yasi, “An Algorithm for Extracting the Relative Phase of Harmonics from a Periodic Digital Signal” (unpublished)

<http://online.physics.uiuc.edu/courses/phys199pom/NSF_REU_Reports/2004_reu/Joe_Yasi_Final_Paper.pdf>.

⁷S. Errede, “Theory of Distortion I and II” (unpublished)

<http://online.physics.uiuc.edu/courses/phys498pom/Lecture_Notes/Distortion/PDF_Files/Theory_of_Distortion2.pdf>.