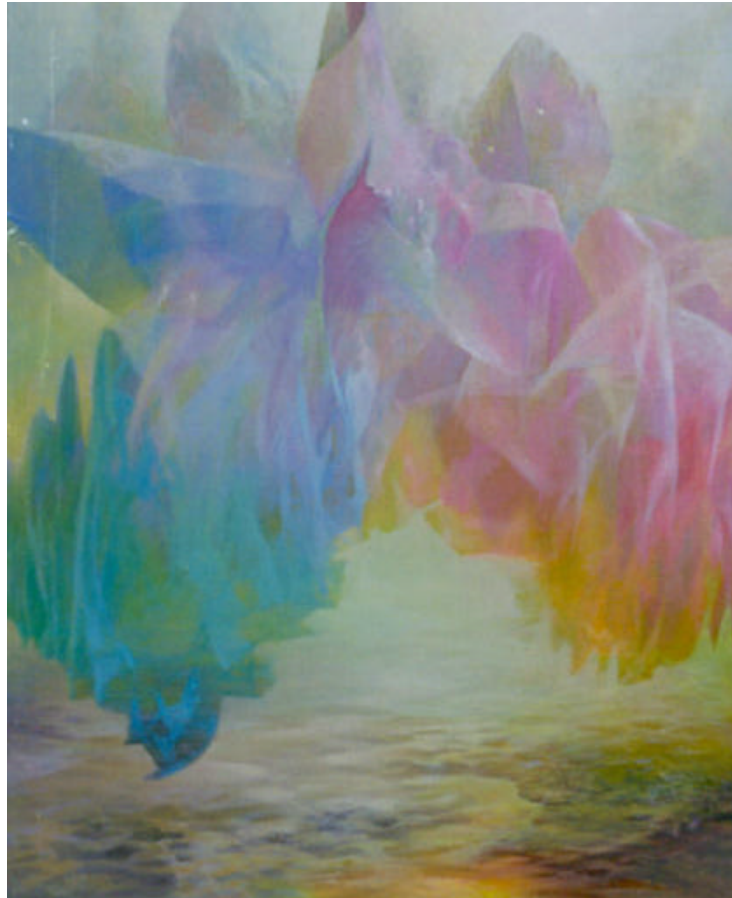# *Physics of Music / Physics of Musical Instruments*

Steven Errede

Professor of Physics

The University of Illinois at Urbana-Champaign

NSF REU Talk

UIUC Physics

July 14, 2003

"**Music of the Spheres**" *Michail Spiridonov*, 1997-8

What is Sound?

Sound describes two different physical phenomena:

• Sound = An auditory sensation in one's ear(s)/in one's brain…

• What is this exactly?

• Sound = A disturbance in a physical medium (gas/liquid/solid) which propagates in that medium. What is this exactly? How does this happen?

• Humans (& many other animal species) have developed the ability to hear sounds - because sound(s) exist in the natural environment.

• All of our senses are a direct consequence of the existence of stimuli in the environment - eyes/light, ears/sound, tongue/taste, nose/smells, touch/sensations, balance/gravity, migratorial navigation/earth's magnetic field.

• Why do we have two ears? Two ears are the minimum requirement for spatial location of a sound.

• Ability to locate a sound is very beneficial - e.g. for locating food & also for avoiding becoming food….

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Acoustics

• Scientific study of sound

• Broad interdisciplinary field - involving physics, engineering, psychology, speech, music, physiology, neuroscience, architecture, etc….

•Different branches of acoustics:

> • Physical Acoustics
>
> • Musical Acoustics
>
> • Psycho-Acoustics
>
> • Physiological Acoustics
>
> • Architectural Acoustics
>
> • Etc...

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Sound Waves
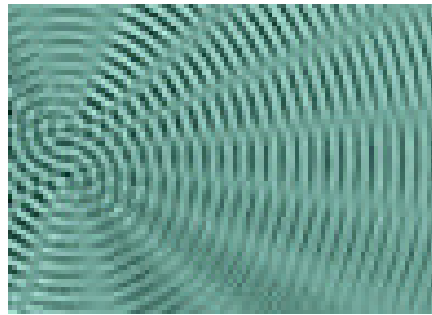
Sound propagates in a physical medium (gas/liquid/solid) as a wave, or as a sound pulse (= a collection/superposition of traveling waves)

• An acoustical disturbance propagates as a collective excitation (i.e. vibration) of a group of atoms and/or molecules making up the physical medium.

• Acoustical disturbance, e.g. sound wave carries energy, E and momentum, P

• For a homogeneous (i.e. uniform) medium, disturbance propagates with a constant speed, v

• Longitudinal waves - atoms in medium are displaced longitudinally from their equilibrium positions by acoustic disturbance - i.e. along/parallel to direction of propagation of wave.

• Transverse waves - atoms in medium are displaced transversely from their equilibrium positions by acoustic disturbance - i.e. perpendicular to direction of propagation of wave.

• Speed of sound in air: $v_{air} = \sqrt{(B_{air}/\rho_{air})} \sim 344$ m/s (~ 1000 ft/sec) at sea level, 20 degrees Celsius.

• Speed of sound in metal, e.g. aluminum: $v_{Al} = \sqrt{(Y_{Al}/\rho_{Al})} \sim 1080$ m/s.

• Speed of transverse waves on a stretched string: $v_{string} = \sqrt{(T_{string}/\mu_{string})}$ where mass per unit length of string, $\mu_{string} = M_{string}/L_{string}$

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Standing Waves on a Stretched String

Standing wave = superposition of left- and right-going traveling waves

• Left & right-going traveling waves reflect off of end supports

• Polarity flip occurs at fixed end supports. No polarity flip for free ends.

• Different modes of string vibrations - resonances occur!

• For string of length L with fixed ends, the lowest mode of vibration has frequency $f_1 = v/2L$ ($v = f_1\lambda_1$) (f in cycles per second, or Hertz (Hz))

• Frequency of vibration, $f = 1/\tau$, where τ = period = time to complete 1 cycle

• Wavelength, λ_1 of lowest mode of vibration has $\lambda_1 = 2L$ (in meters)

• Amplitude of wave (maximum displacement from equilibrium) is A - see figure below - snapshot of standing wave at one instant of time, t:
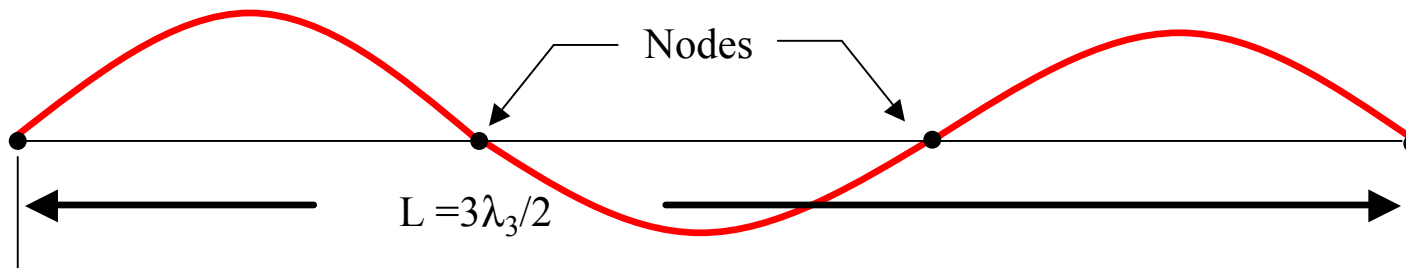
Amplitude | A

$L = \lambda_1/2$

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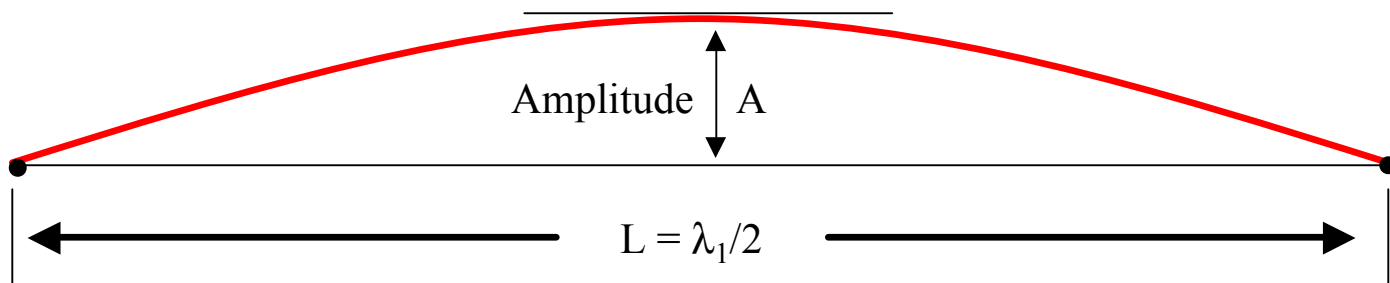
String can also vibrate with higher modes:

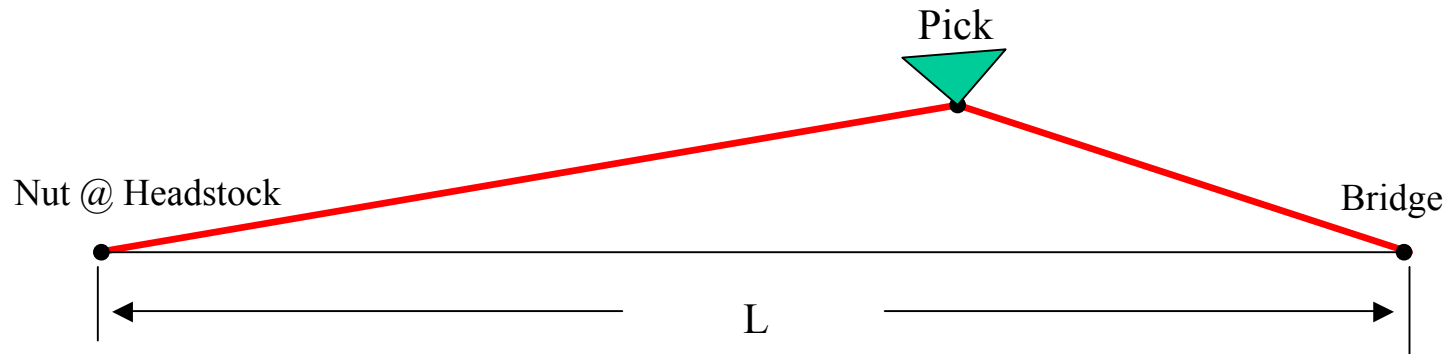• Second mode of vibration of standing wave has $f_2 = 2v/2L = v/L$ with $\lambda_2 = 2L/2 = L$

Node

$L = \lambda_2$
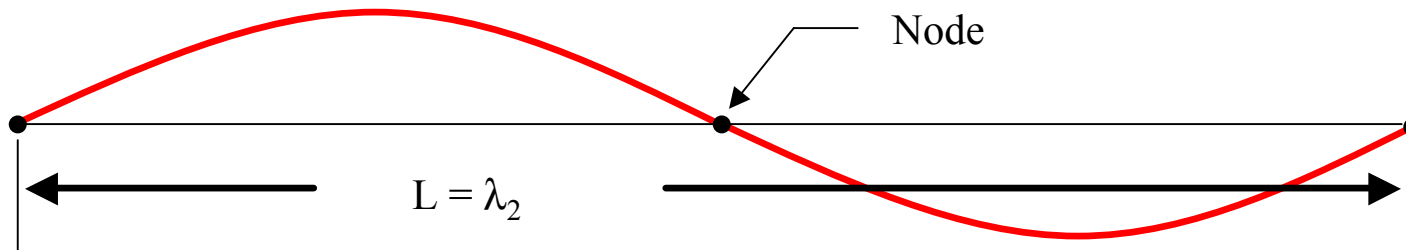
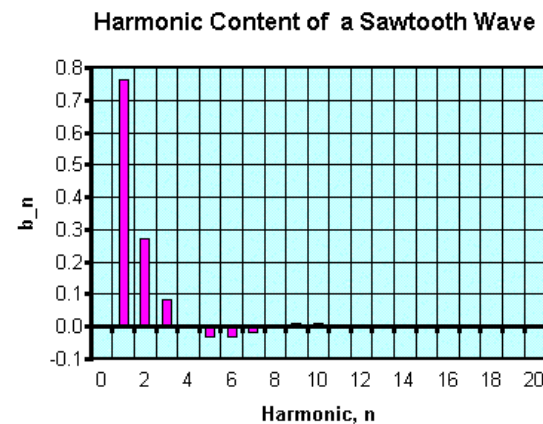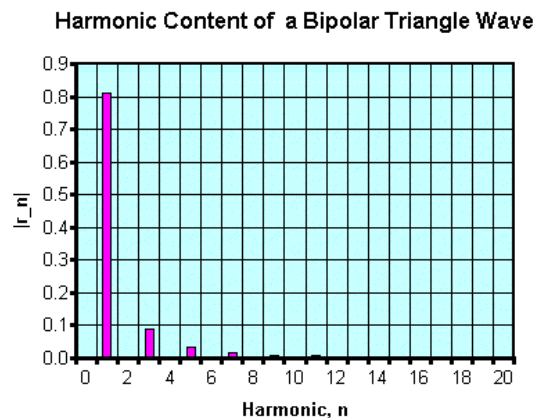• Third mode of vibration of standing wave has $f_3 = 3v/2L$ with $\lambda_3 = 2L/3$

Nodes

$L = 3\lambda_3/2$

• The nth mode of vibration of standing wave on a string, where n = integer = 1,2,3,4,5,….
has frequency $f_n = n(v/2L) = n\,f_1$, since $v = f_n\lambda_n$ and thus the nth mode of vibration has a
wavelength of $\lambda_n = (2L)/n = \lambda_1/n$

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When we e.g. pick (i.e. pluck) the string of a guitar, initial waveform is a triangle wave:

Pick

Nut @ Headstock   Bridge

L

The geometrical shape of the string (a triangle) at the instant the pick releases the string can be shown mathematically (using Fourier Analysis) to be due to a linear superposition of standing waves consisting of the fundamental plus higher harmonics of the fundamental! Depending on where pick along string, harmonic content changes. Pick near the middle, mellower (lower harmonics); pick near the bridge - brighter - higher harmonics emphasized!

Harmonic Content of a Bipolar Triangle Wave

Harmonic Content of a Sawtooth Wave

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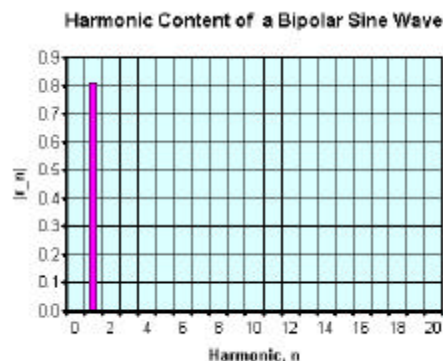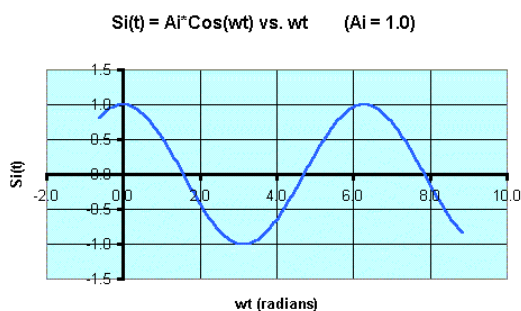NSF REU Talk, July 14, 2003

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Harmonic Content of Complex WaveForms

In fact, geometrical/mathematical shape of any *periodic* waveform can be shown to be due to linear combination of fundamental & higher harmonics!

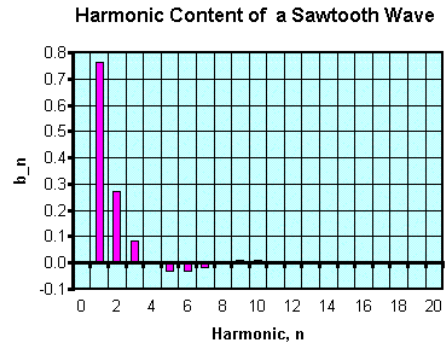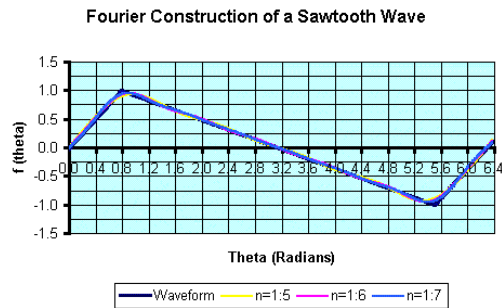Sound Tonal Quality - *Timbre* - harmonic content of sound wave

Sine/Cosine Wave: Mellow Sounding – fundamental, no higher harmonics

Triangle Wave: A Bit Brighter Sounding – has higher harmonics!

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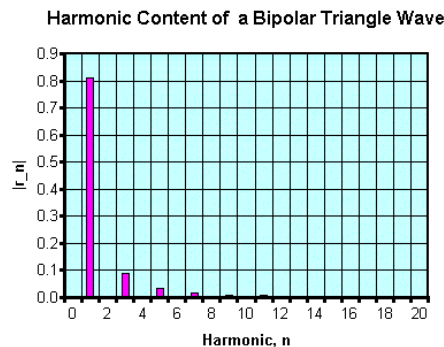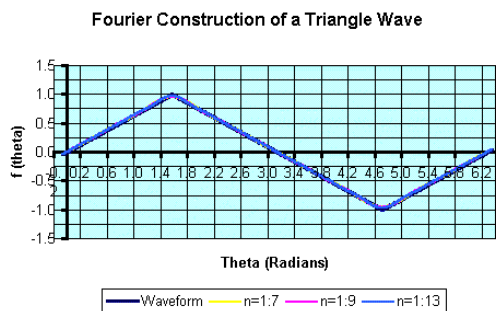
Asymmetrical Sawtooth Wave: Even Brighter Sounding – even more harmonics!

Fourier Construction of a Sawtooth Wave

Harmonic Content of a Sawtooth Wave

Square Wave: Brighter Sounding – has the most harmonics!

Fourier Construction of a Square Wave

Harmonic Content of a Bipolar Square Wave (50% Duty Cycle)

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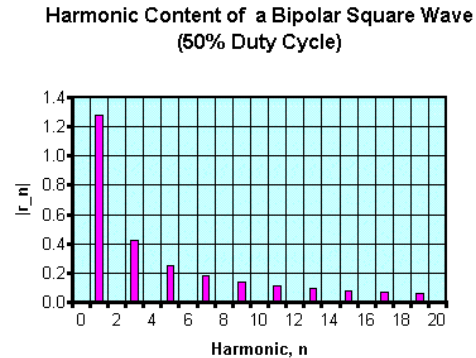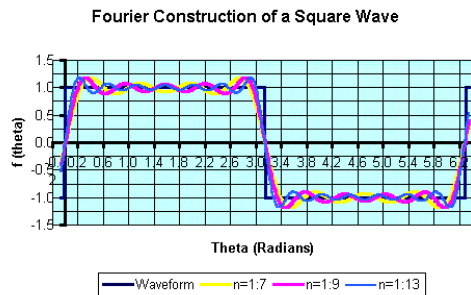
What *is* Music?

• An aesthetically pleasing sequence of tones?

• *Why* is music pleasurable to humans?

• Music has always been part of human culture, as far back as we can tell

• Music important to human evolution?

• Music shown to *stimulate* human brain

• Music facilitates brain development in young children and in learning

• Music is also important to other living creatures - birds, whales, frogs, etc.

• Many kinds of animals utilize sound to communicate with each other

• What is it about music that does all of the above ???

Human Development of Musical Instruments

• Emulate/mimic human voice (some instruments much more so than others)!

• Sounds from musical instruments can evoke powerful emotional responses - happiness, joy, sadness, sorrow, shivers down your spine, raise the hair on back of neck, etc.

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Musical Instruments

• Each musical instrument has its own characteristic sounds - quite complex!

• Any note played on an instrument has fundamental + harmonics of fundamental.

• Higher harmonics - brighter sound

• Less harmonics - mellower sound

• Harmonic content of note can/does change with time:

> • Takes time for harmonics to develop - "attack" (leading edge of sound)

> • Harmonics don't decay away at same rate (trailing edge of sound)

> • Higher harmonics tend to decay more quickly

• Sound output of musical instrument is not uniform with frequency

> • Details of construction, choice of materials, finish, etc. determine *resonant structure* (formants) associated with instrument - mechanical vibrations!

• See harmonic content of guitar, violin, recorder, singing saw, drum, cymbals, etc.

• See laser interferogram pix of vibrations of guitar, violin, handbells, cymbals, etc.

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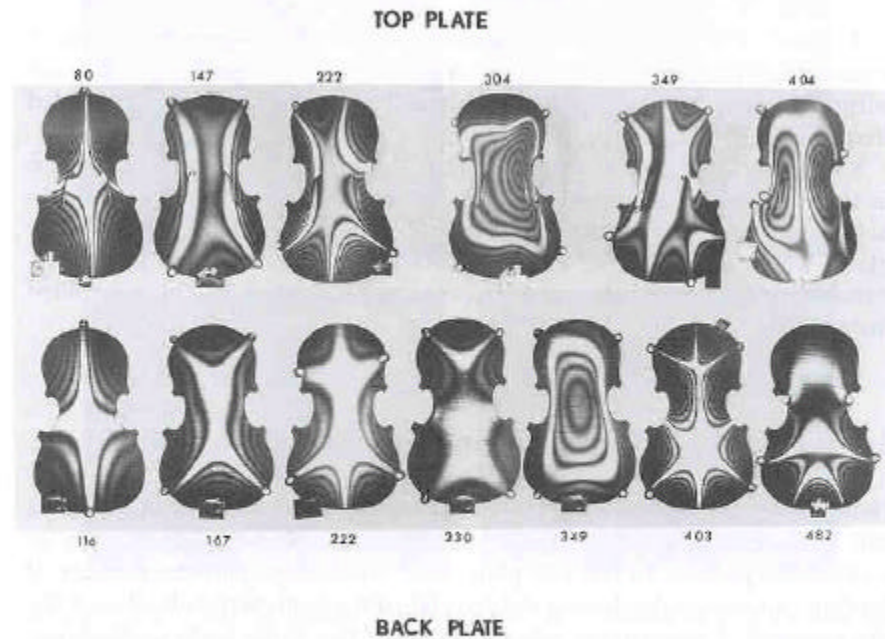
11

Vibrational Modes of a Violin

TOP PLATE

BACK PLATE

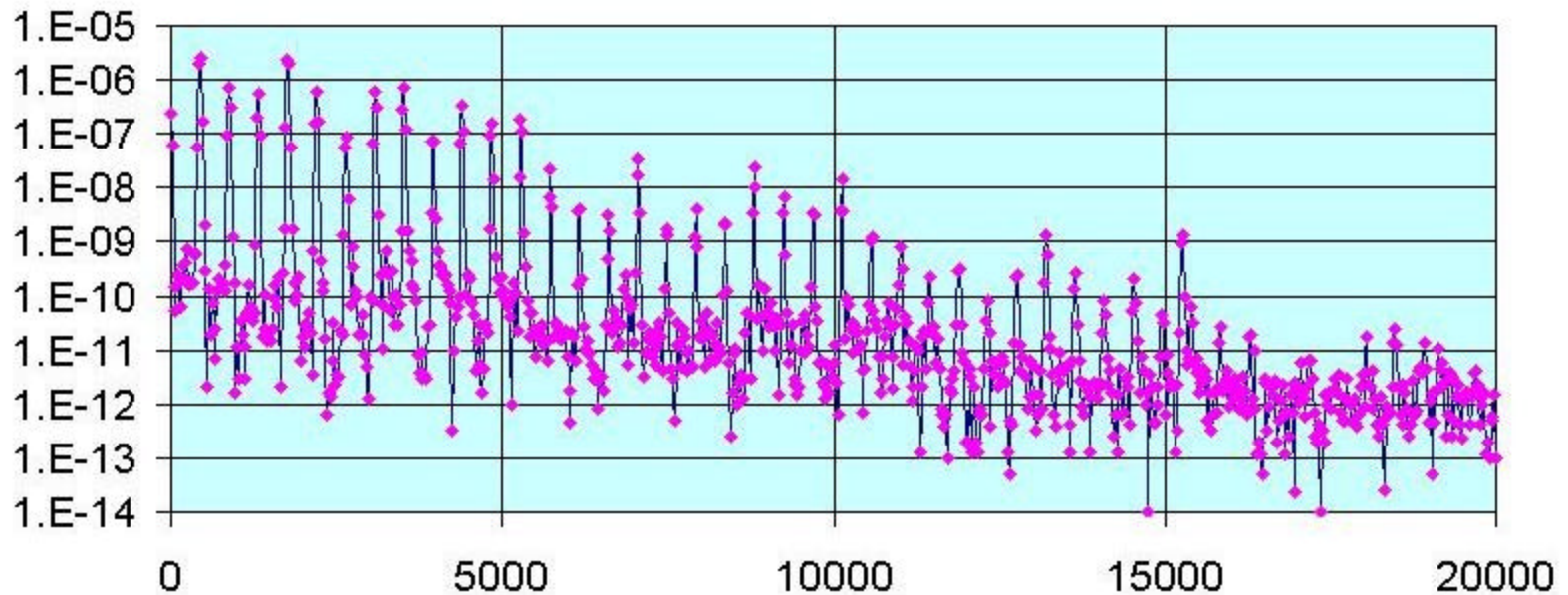FIGURE 10.14. Time-average holographic interferograms of a free violin top plate and back plate (Hutchins et al., 1971).

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Harmonic Content of a Viola – Open A2

Laura Book (Uni High, Spring Semester, 2003)

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Vibrational Modes of an Acoustic Guitar

268 Hz ($Q=52$)   553 Hz ($Q=66$)   628 Hz ($Q=83$)   672 Hz ($Q=61$)   731 Hz ($Q=72$)

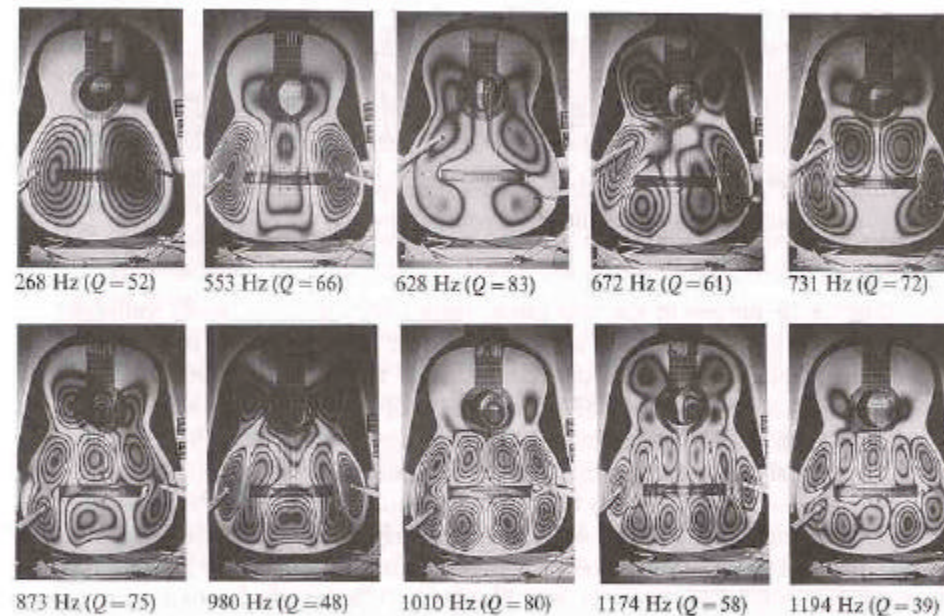873 Hz ($Q=75$)   980 Hz ($Q=48$)   1010 Hz ($Q=80$)   1174 Hz ($Q=58$)   1194 Hz ($Q=39$)

FIGURE 9.16. Time-averaged holographic interferograms of top-plate modes of a guitar (Guitar BR11). The resonant frequencies and Q values of each mode are shown below the interferograms (Richardson and Roberts, 1985).

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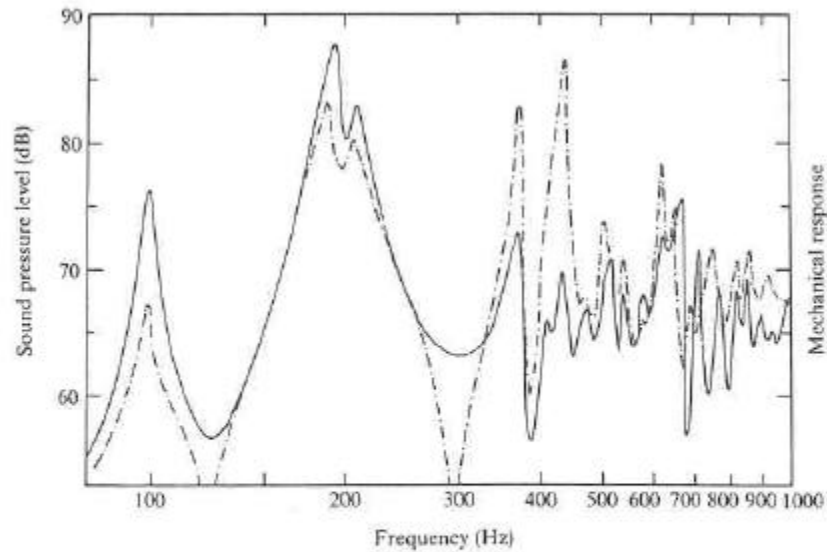
14

Resonances of an Acoustic Guitar

FIGURE 9.20. Mechanical frequency response and sound spectrum 1 m in front of a Martin D-28 folk guitar driven by a sinusoidal force of 0.15 N applied to the treble side of the bridge. Solid curve, sound spectrum; dashed curves, acceleration level at the driving point.

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Harmonic Content of 1969 Gibson ES-175 Electric Guitar

Jacob Hertzog (Uni High, Spring Semester, 2003)

1969 ES-175 A 10th Fret   Volts**2 vs. Frequency

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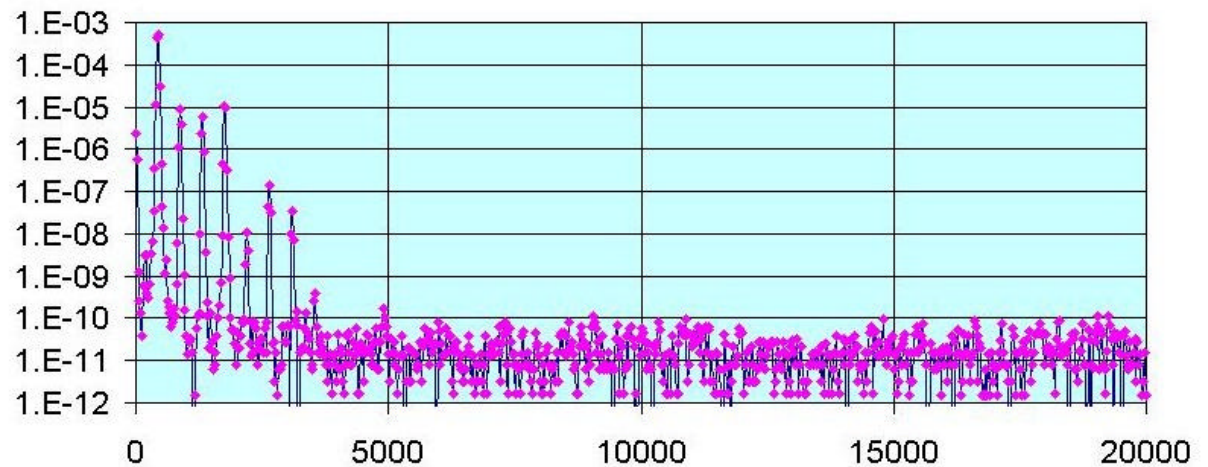
Musical Properties of a 1954 Fender Stratocaster, S/N 0654 (August, 1954)

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Measuring Mechanical Vibrational Modes of 1954 Fender Stratocaster

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Mechanical Vibrational Modes of 1954 Fender Stratocaster

1954 Strat S/N 0654

E4 = 329.63 Hz (High E)

B3 = 246.94 Hz

G3 = 196.00 Hz

D3 = 146.83 Hz

A2 = 110.00 Hz

E2 = 82.407 Hz (Low E)

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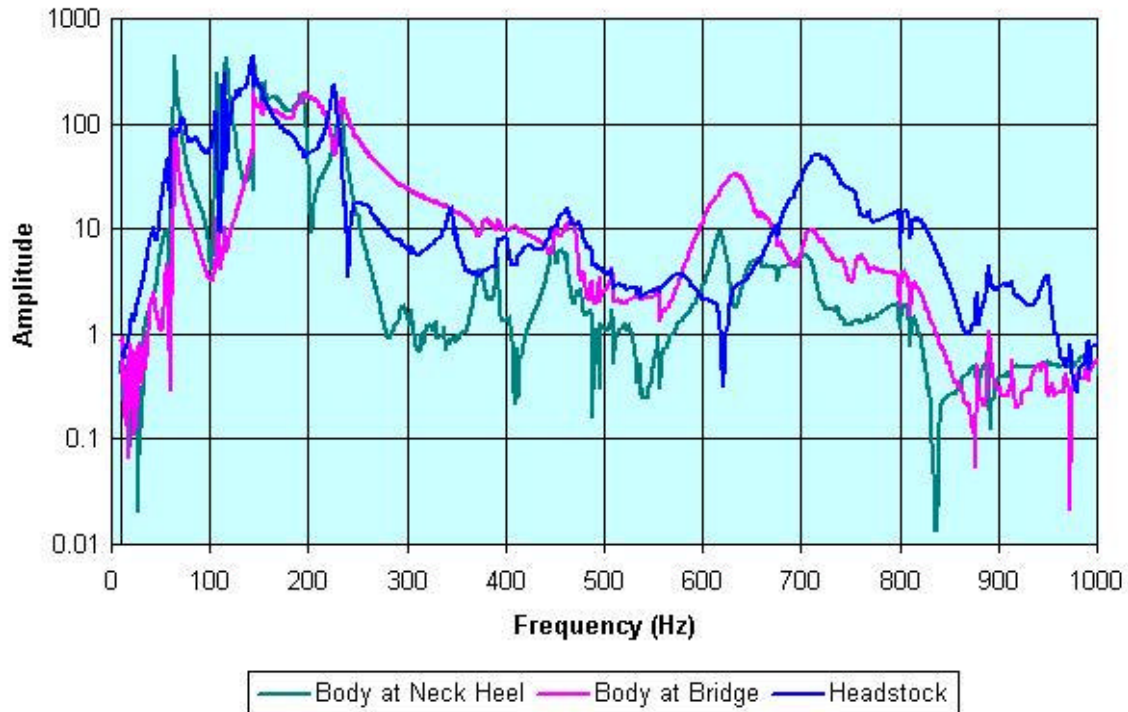
UIUC Physics 398EMI Test Stand for Measurement of Electric Guitar Pickup Properties

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Impedance, |Z| vs. Frequency for Bridge Pickup of 1954 Fender Stratocaster

1954 Strat S/N 0654 Bridge Pickup
|Z| vs. Frequency

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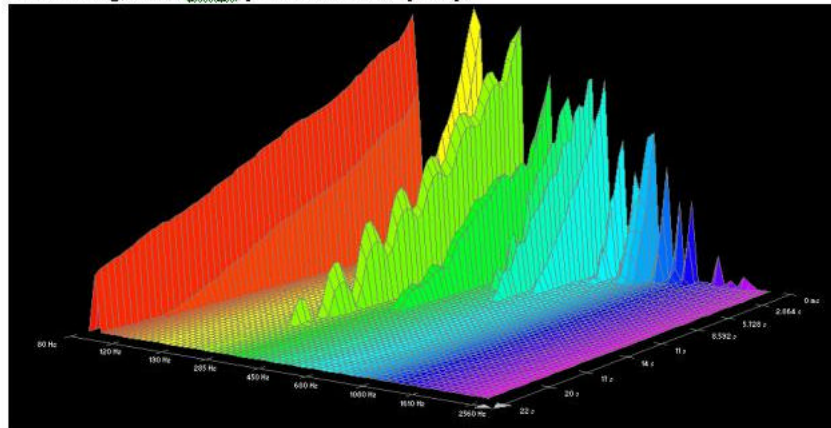
21
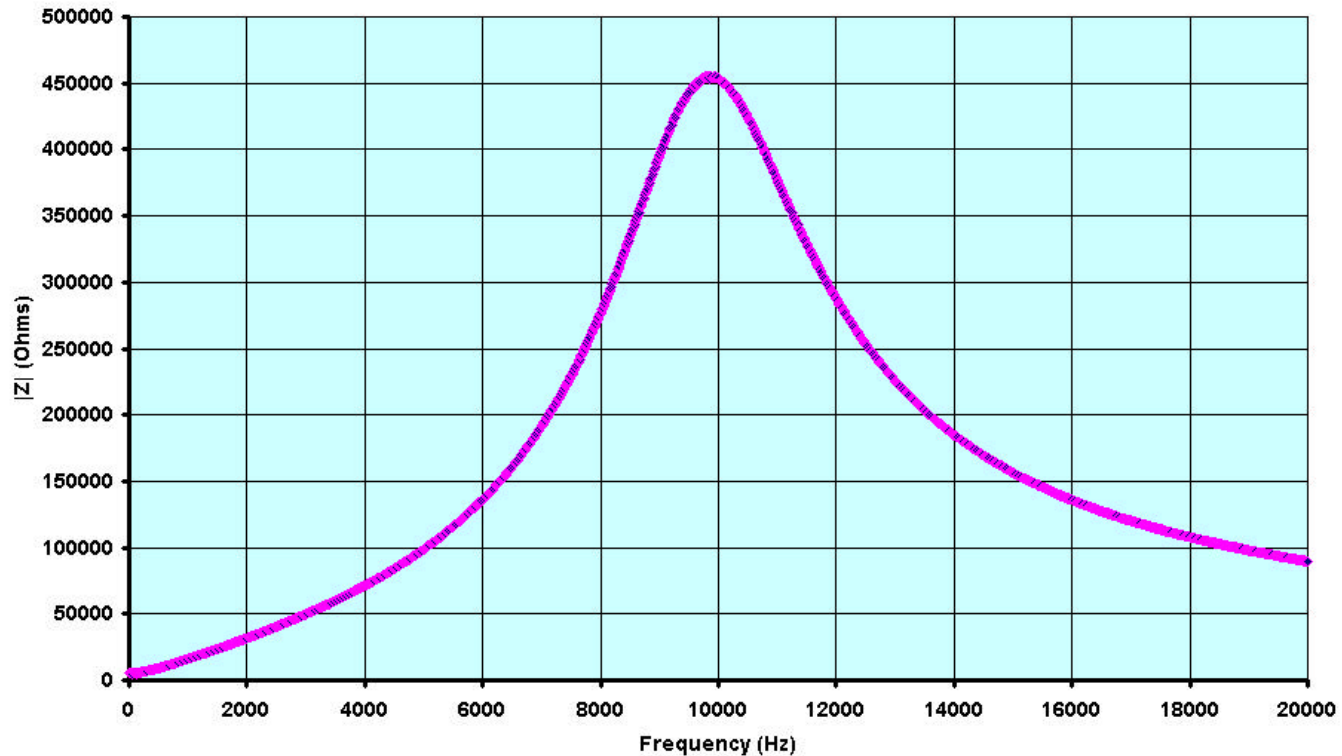
Study/Comparison of Harmonic Properties of Acoustic and Electric Guitar Strings
Ryan Lee (UIUC Physics P398EMI, Fall 2002)

Low E-string, 0.052 gauge, plucked near bridge pickup:
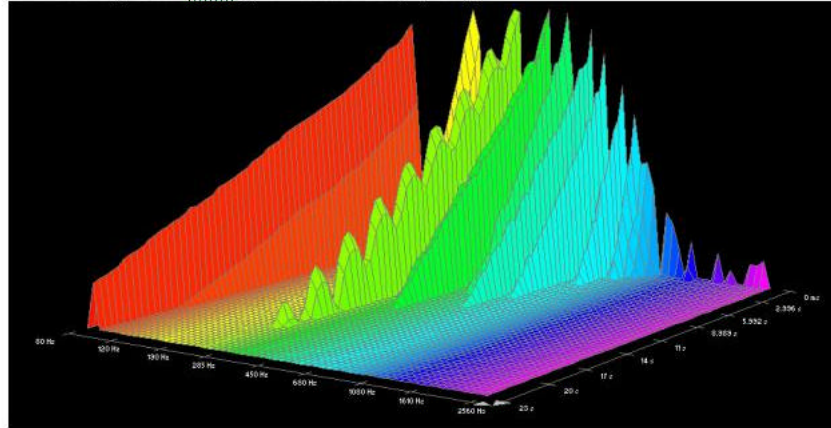
Low E-string, 0.052 gauge, plucked near neck pickup:

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Vibrational Modes of Handbells
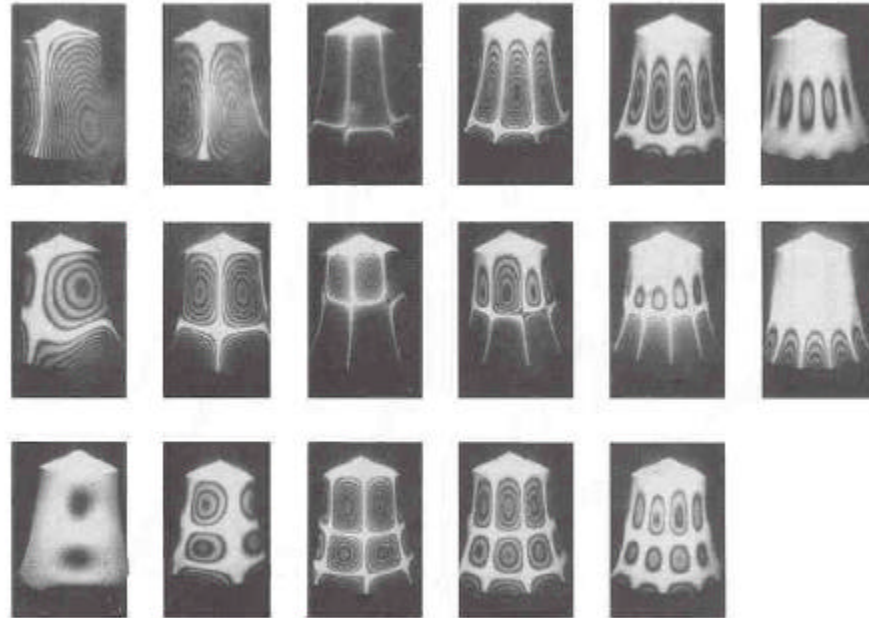

FIGURE 21.16. Time-average hologram interferograms of vibrational modes in a C_5 handbell (Rossing et al., 1984).

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Vibrational Modes of Membranes and Plates
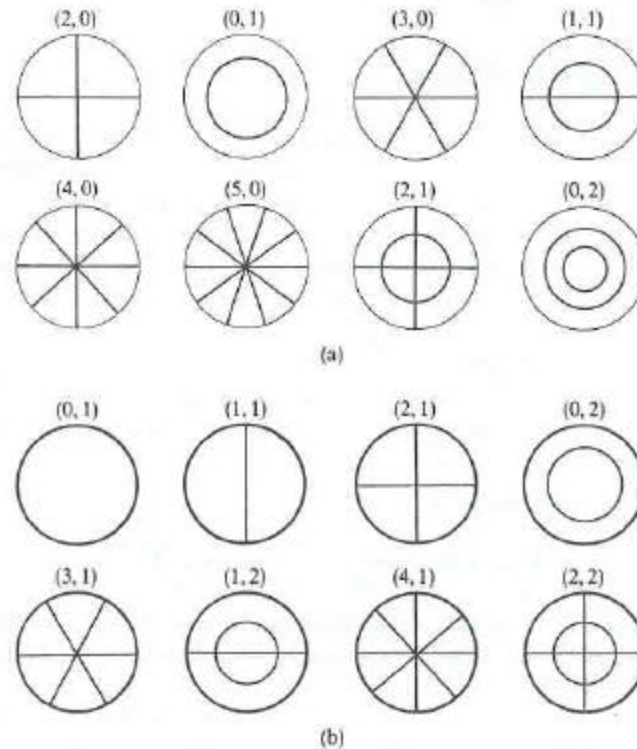
(Drums and Cymbals)

FIGURE 3.8. Vibrational modes of circular plates: (a) free edge and (b) clamped or simply supported edge. The mode number (n, m) gives the number of nodal diameters and circles, respectively.

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Harmonic Content vs. Time of Tibetan Bowl and Snare Drum

Eric Macaulay (Illinois Wesleyan University), Lee Holloway, Mats Selen, SME (UIUC)

Tibetan Bowl

Snare Drum

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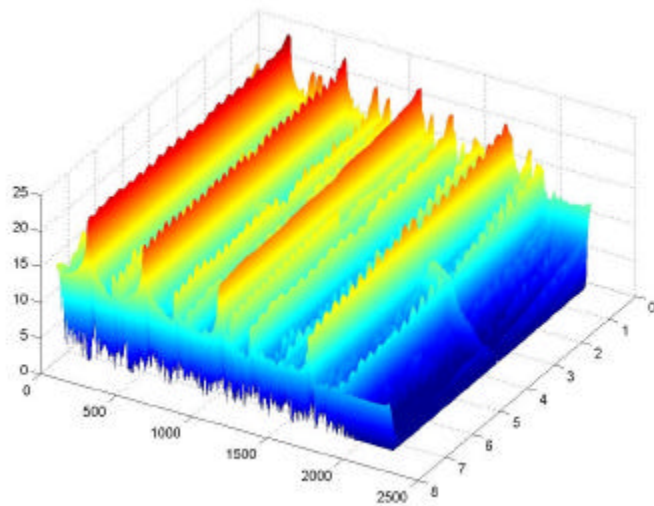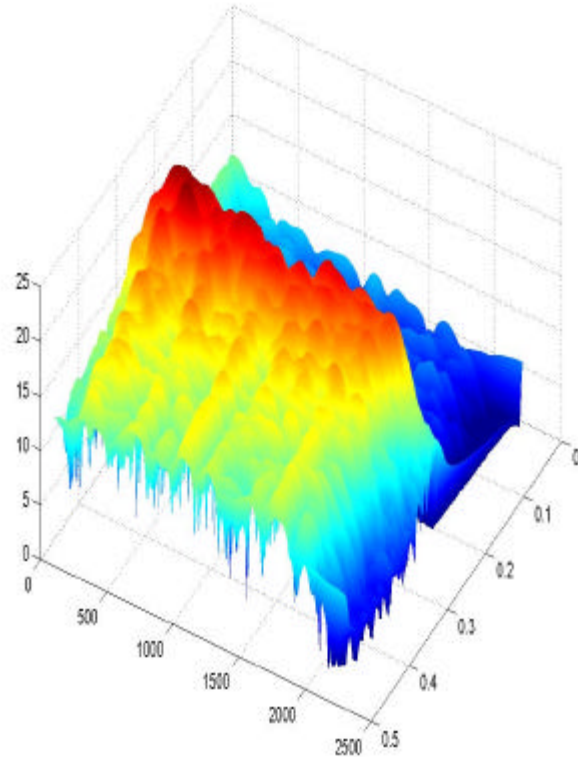
Study/Comparison of Acoustic Properties of Tom Drums

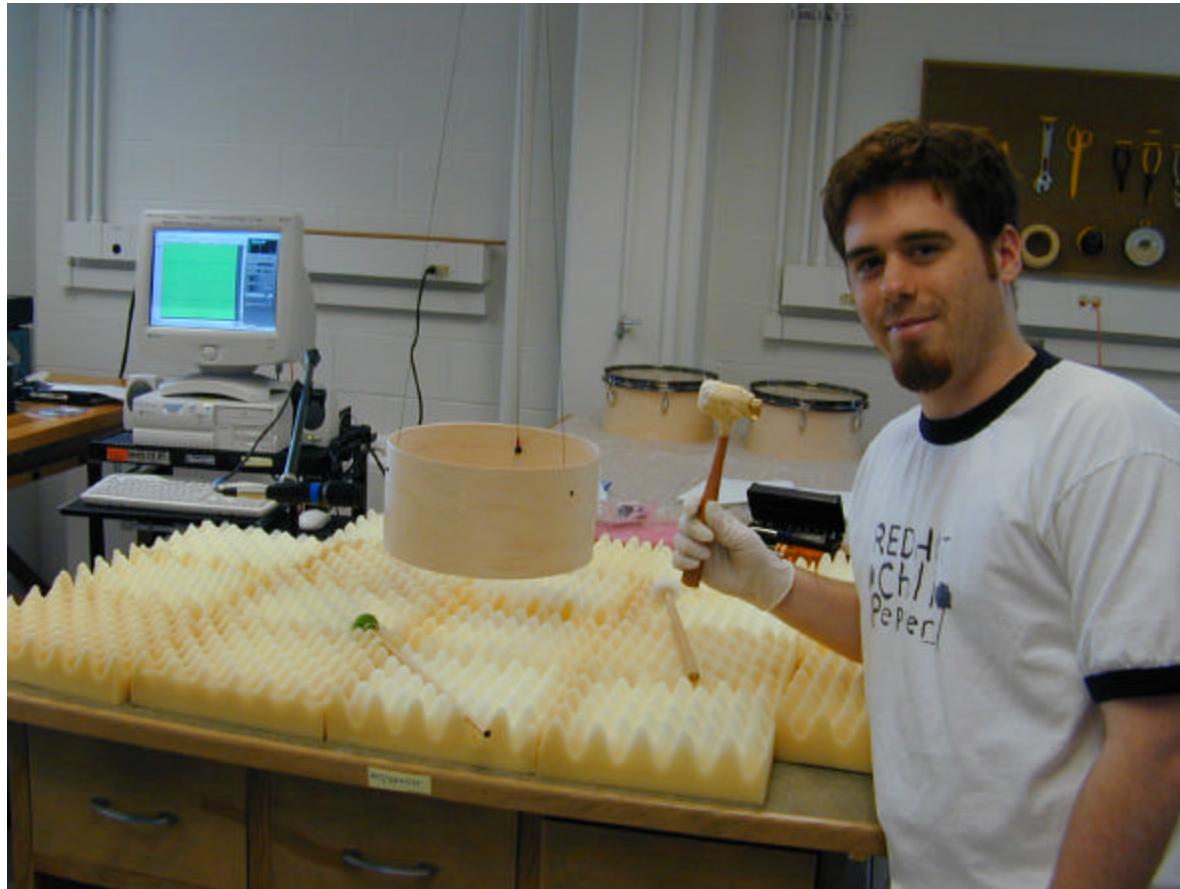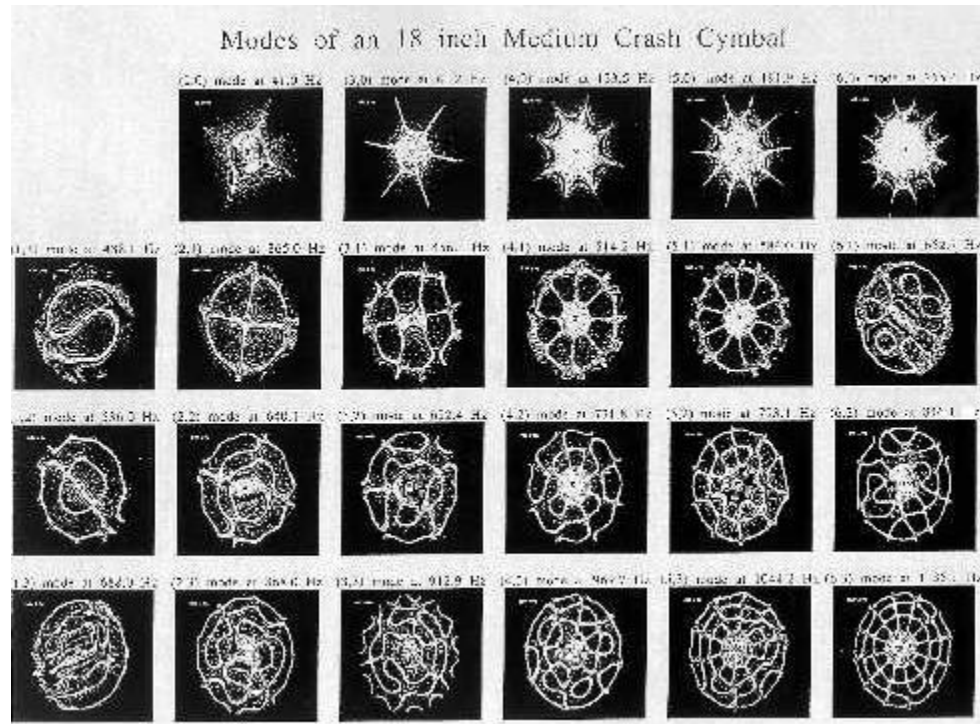**Eric Macaulay (Illinois Wesleyan University), Nicole Drummer, SME (UIUC)**
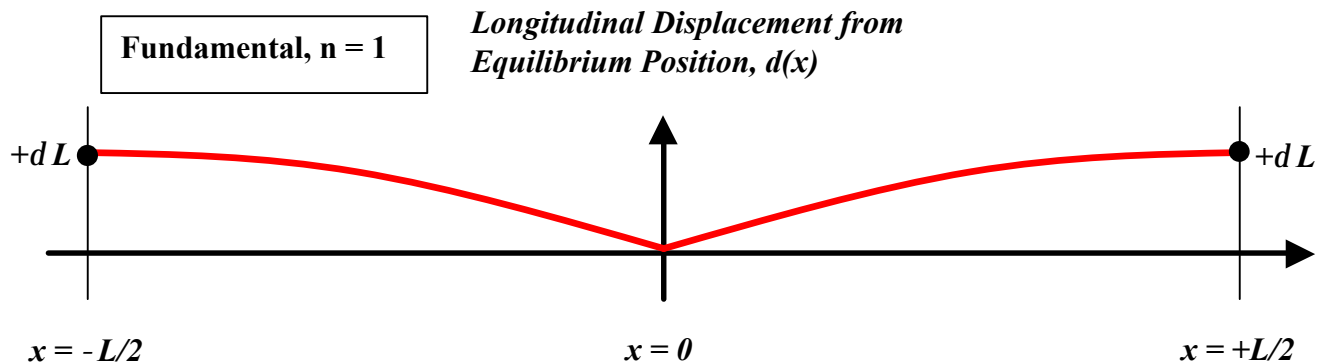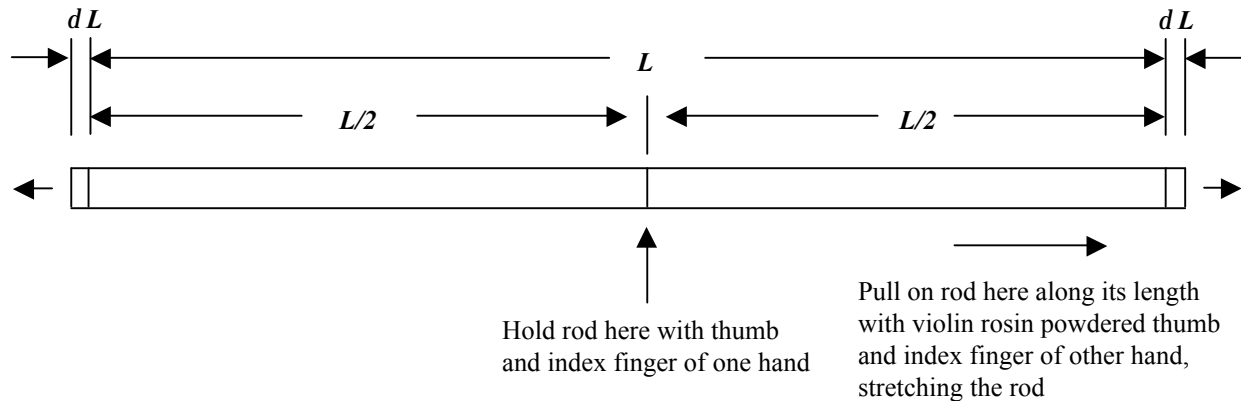
Dennis @ Phattie Drums

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Vibrational Modes of Cymbals

Modes of an 18 inch Medium Crash Cymbal

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Modal Vibrations of a "Singing" Rod

A metal rod (e.g. aluminum rod) a few feet in length can be made to vibrate along its length – make it "sing" at a characteristic, resonance frequency by holding it precisely at its mid-point with thumb and index finger of one hand, and then pulling the rod along its length, toward one of its ends with the thumb and index finger of the other hand, which have been dusted with crushed violin rosin, so as to obtain a good grip on the rod as it is pulled.

dL

L

L/2

L/2

dL

Hold rod here with thumb and index finger of one hand

Pull on rod here along its length with violin rosin powdered thumb and index finger of other hand, stretching the rod

Fundamental, n = 1

Longitudinal Displacement from Equilibrium Position, d(x)

$+dL$

$+dL$

$x = -L/2$

$x = 0$

$x = +L/2$

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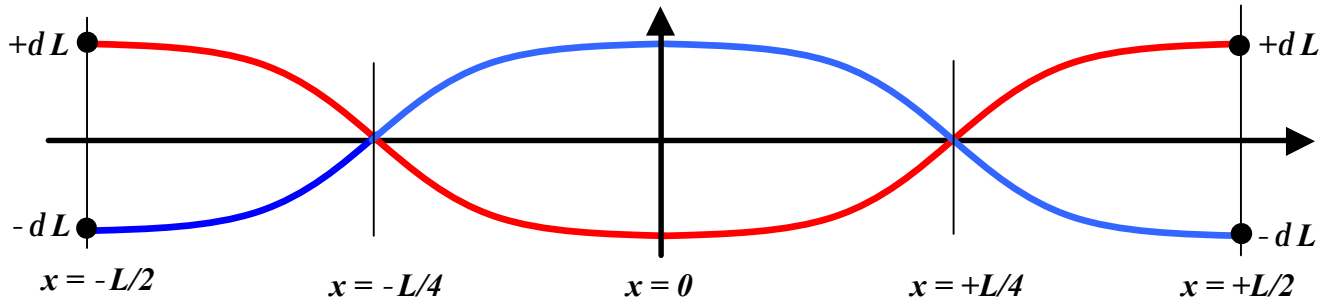# **Decay of Fundamental Mode of Singing Rod:**

Amplitude vs. Time
Fundamental of Vibrating Rod @ f1 = 1670 Hz

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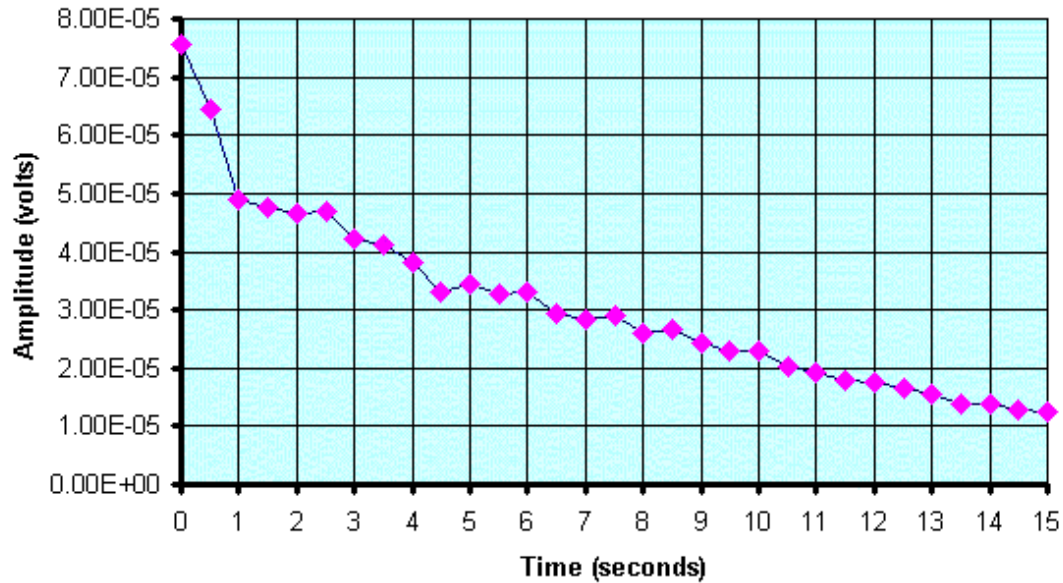
Of course, there also exist higher modes of vibration of the singing rod:

Second Harmonic, n = 2
Longitudinal Displacement from Equilibrium Position, d(x)

$+dL$ ... $-dL$

$x = -L/2$ $x = -L/4$ $x = 0$ $x = +L/4$ $x = +L/2$

Third Harmonic, n = 3
Longitudinal Displacement from Equilibrium Position, d(x)

$+dL$ ... $-dL$

$x = -L/2$ $x = -L/3$ $x = -L/6$ $x = 0$ $x = +L/6$ $x = +L/3$ $x = +L/2$

• See singing rod demo...

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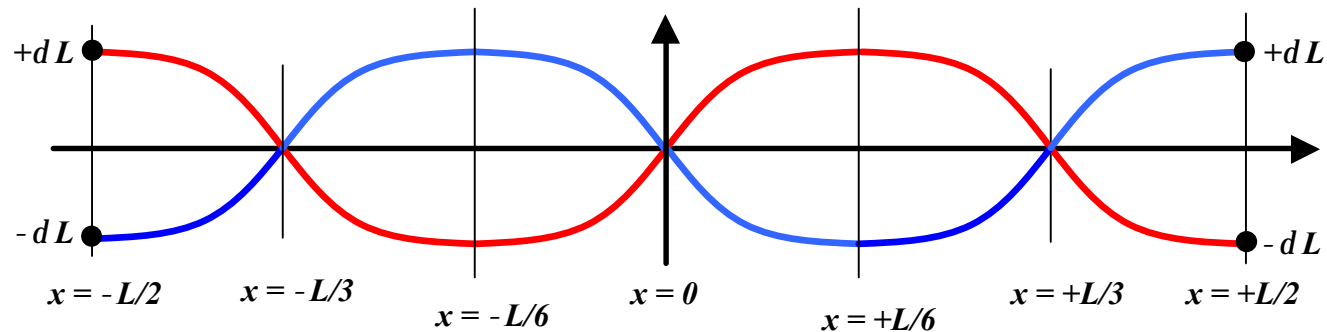
• If the singing rod is rotated - can hear Doppler effect & beats:

$v_t = \mathbf{W}r = \frac{1}{2}\,\mathbf{W}L$

$L/2$

$\boldsymbol{f}$

$L/2$

$v_t = \mathbf{W}r = \frac{1}{2}\,\mathbf{W}L$

Observer/Listener Position
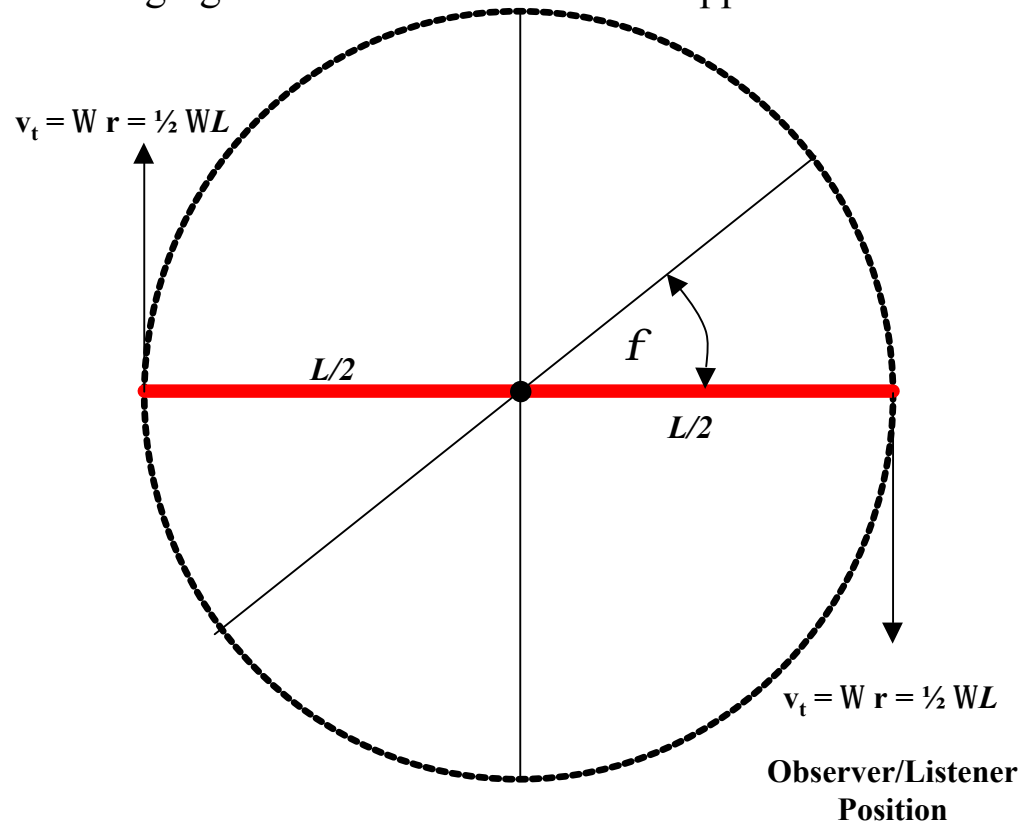
• Frequency of vibrations raised (lowered) if source moving toward (away from) listener, respectively

• Hear Doppler effect & beats of rotating "singing" rod...

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Total Sound Amplitude vs. Time
Rotating Vibrating Rod

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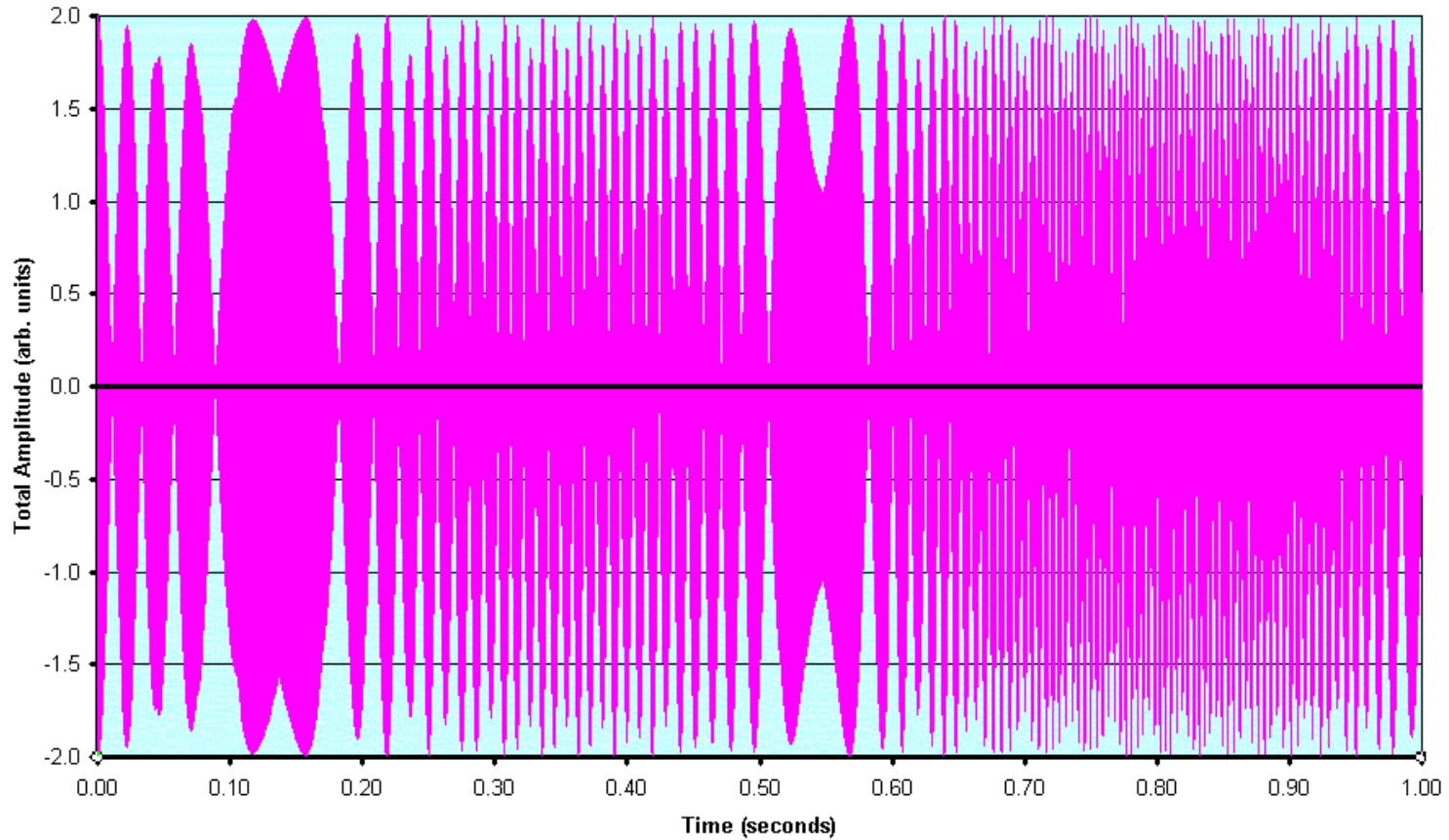
• Would Mandi Patrick (UIUC Feature Twirler) be willing to lead the UI Singing Rod Marching Band at a half-time show ???

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How Do Our Ears Work?
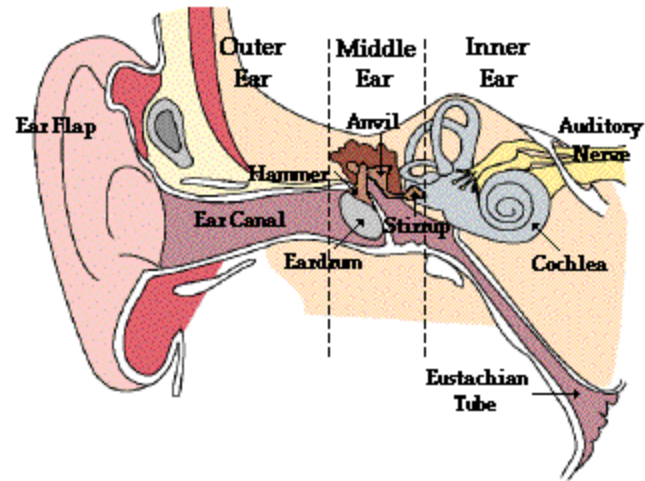
• Sound waves are focussed into the ear canal via the ear flap (aka pinna), and impinge on the ear drum.

• Ossicles in middle ear - hammer/anvil/stirrup - transfer vibrations to oval window - membrane on cochlea, in the inner ear.

• Cochlea is filled with perilymph fluid, which transfers sound vibrations into Cochlea.

• Cochlea contains basilar membrane which holds ~ 30,000 hair cells in Organ of Corti

• Sensitive hairs respond to the sound vibrations - send signals to brain via auditory nerve

• Brain processes audio signals from *both* ears - you *hear* the "sound"

• Human hearing response is ~ logarithmic.

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Consonance & Dissonance

Ancient Greeks - Aristotle and his followers - discovered using a *Monochord* that certain combinations of sound were pleasing to the human ear, for example:

• **Unison** - 2 sounds of *same* frequency, i.e. $f_2 = 1 f_1 = f_1$ (= e.g. 300 Hz)

• **Minor Third** - 2 sounds with $f_2 = (6/5) f_1 = 1.20 f_1$ (= e.g. 360 Hz)

•**Major Third** - 2 sounds with $f_2 = (5/4) f_1 = 1.25 f_1$ (= e.g. 375 Hz)

• **Fourth** - 2 sounds with $f_2 = (4/3) f_1 = 1.333 f_1$ (= e.g. 400 Hz)

• **Fifth** - 2 sounds with $f_2 = (3/2) f_1 = 1.50 f_1$ (= e.g. 450 Hz)

• **Octave** - one sound is 2nd harmonic of the first - i.e. $f_2 = (2/1) f_1 = 2 f_1$ (= e.g. 600 Hz)

• See Monochord Demo….

• Also investigated/studied by Galileo Galilei, mathematicians Leibnitz, Euler, physicist Helmholtz, and many others - debate/study is still going on today...

• These 2-sound combinations are indeed very special!

• The resulting, overall waveform(s) are *time-independent* – they create standing waves on basilar membrane in cochlea of our inner ears!!!

• The human brain's signal processing for these special 2-sound consonant combinations is especially easy!!!

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Fractal Music

Lorentz's Butterfly - Strange Attractor
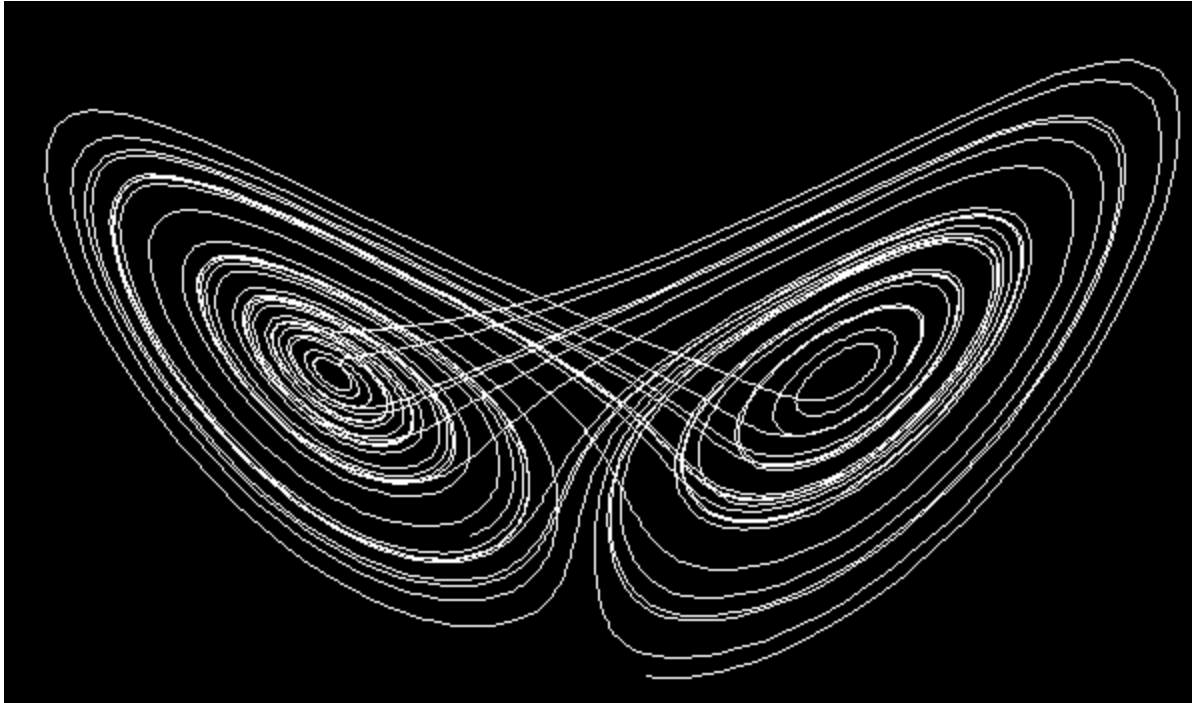
Iterative Equations:
dx/dt = 10(y - x)
dy/dt = x(28 - z) - y
dz/dt = xy - 8z/3.

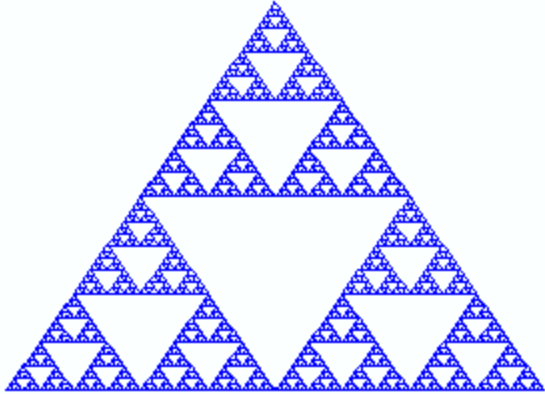Initial Conditions:
Change of t = 0.01 and
the initial values
x0 = 2, y0 = 3 and z0 = 5

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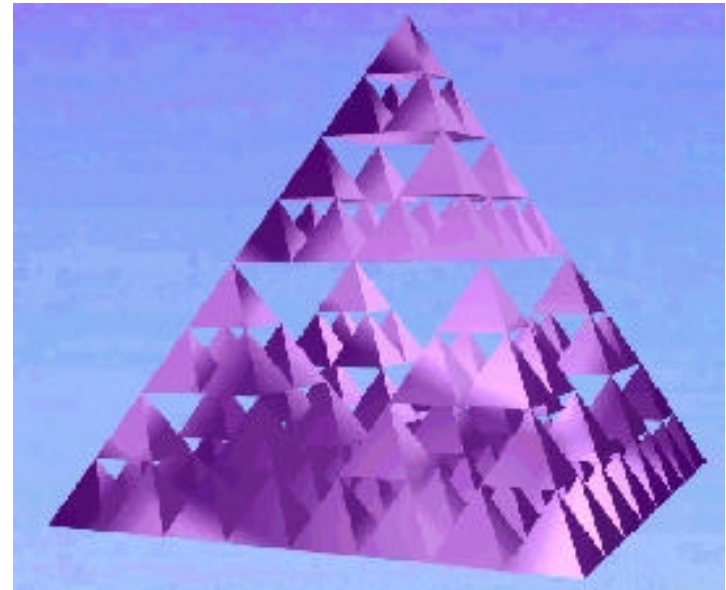NSF REU Talk, July 14, 2003

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Fractal Music

The Sierpinski Triangle

is a fractal structure with fractal dimension 1.584.

The area of a Sierpinski Triangle is ZERO!

3-D Sierpinski Pyramid

Beethoven's Piano Sonata no. 15, op. 28, 3rd Movement (Scherzo) is a combination of binary and ternary units iterating on diminishing scales, similar to the Sierpinski Structure !!!

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Fractal Music in Nature – chaotic dripping of a leaky water faucet!

Convert successive drop time differences and drop sizes to frequencies

Play back in real-time (online!) using FG – can *hear* the sound of chaotic dripping!

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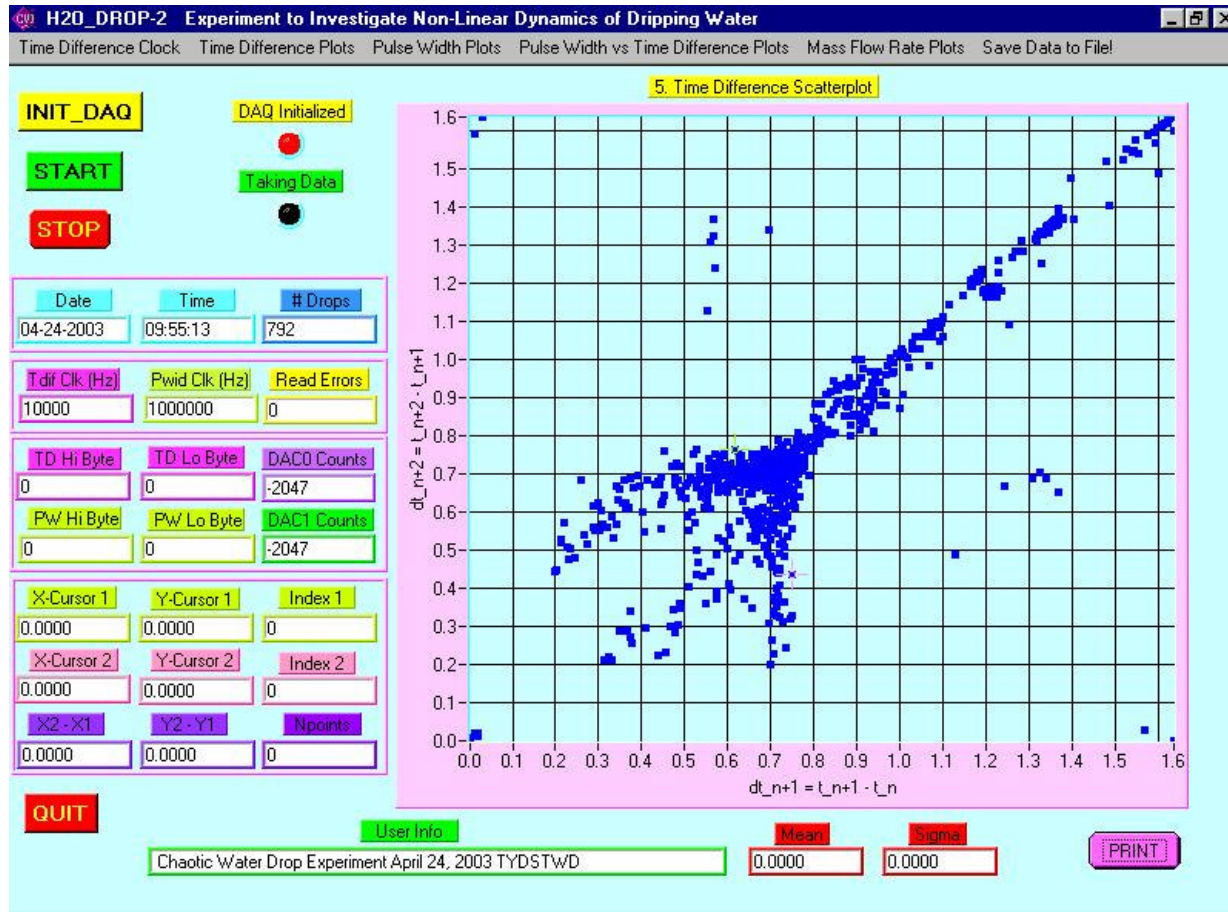
Conclusions and Summary:

• Music is an intimate, very important part of human culture

• Music is deeply ingrained in our daily lives - it's everywhere!

• Music constantly evolves with our culture - affected by many things

• Future: Develop new kinds of music...

• Future: Improve existing & develop totally new kinds of musical instruments...

• There's an immense amount of physics in music - much still to be learned !!!

• Huge amount of fun – combine physics & math with music – can hear/see/touch/feel/think!!

MUSIC

Be a Part of It - Participate !!!

Enjoy It !!!

Support It !!!

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For additional info on Physics of Music at UIUC - see e.g.

Physics 199 Physics of Music Web Page:

http://wug.physics.uiuc.edu/courses/phys199pom/

Physics 398 Physics of Electronic Musical Instruments Web Page:

http://wug.physics.uiuc.edu/courses/phys398emi/

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