# *Science of Music / Musical Instruments*

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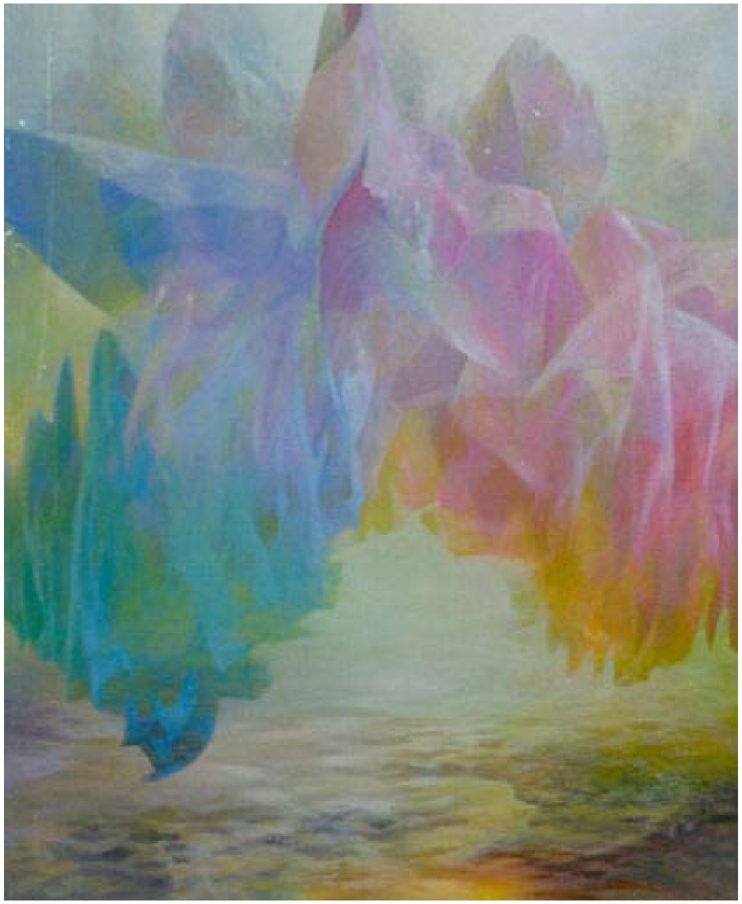


“**Music of the Spheres**” *Michail Spiridonov*, 1997-8

**Orpheum Children’s Science Museum Talk**

Chancellor Inn

Oct. 21, 2004

# What is Sound?

Sound describes two different physical phenomena:

• Sound = A disturbance in a physical medium (gas/liquid/solid) which propagates in that medium. What is this exactly? How does this happen?

• Sound = An auditory sensation in one's ear(s)/in one's brain - what is this exactly??? How does this happen?

• Humans & other animal species have developed the ability to hear sounds - because sound(s) exist in the natural environment.

• All of our senses are a direct consequence of the existence of stimuli in the environment - eyes/light, ears/sound, tongue/taste, nose/smells, touch/sensations, balance/gravity, migratorial navigation/earth's magnetic field.

• Why do we have *two* ears? Two ears are the *minimum* requirement for spatial *location* of a sound source.

• Ability to locate a sound is very beneficial - e.g. for locating food & also for avoiding becoming food….

# Acoustics

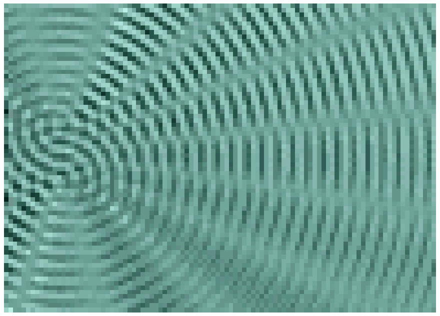
• Scientific study of sound

• Broad interdisciplinary field - involving physics, engineering, psychology, speech, music, biology, physiology, neuroscience, architecture, etc….

•Different branches of acoustics:

- Physical Acoustics
- Musical Acoustics
- Psycho-Acoustics
- Physiological Acoustics
- Architectural Acoustics
- Etc...

# Sound Waves



Sound propagates in a physical medium (gas/liquid/solid) as a wave, or as a sound pulse ( = a collection/superposition of traveling waves)

• An acoustical disturbance propagates as a collective excitation (i.e. vibration) of a group of atoms and/or molecules making up the physical medium.

• Acoustical disturbance, e.g. sound wave carries *energy*, ***E*** and *momentum*, ***P***

• For a homogeneous (i.e. uniform) medium, disturbance propagates with a constant speed, ***v***

• Longitudinal waves - atoms in medium are displaced longitudinally from their equilibrium positions by acoustic disturbance - i.e. along/parallel to direction of propagation of wave.

• Transverse waves - atoms in medium are displaced transversely from their equilibrium positions by acoustic disturbance - i.e. perpendicular to direction of propagation of wave.

• Speed of sound in air: $v_{air} = \sqrt{(B_{air}/\rho_{air})}$ ~ 344 m/s (~ 1000 ft/sec) at sea level, 20 degrees Celsius.

• Speed of sound in metal, e.g. aluminum: $v_{Al} = \sqrt{(Y_{Al}/\rho_{Al})}$ ~ 1080 m/s.

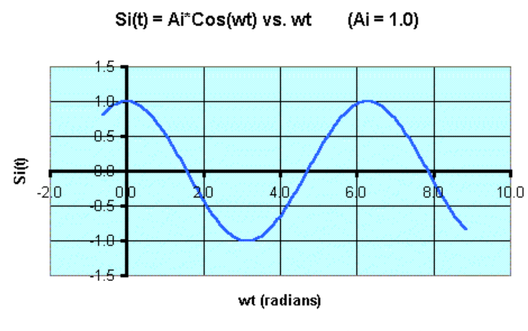
• Speed of transverse waves on a stretched string: $v_{string} = \sqrt{(T_{string}/\mu_{string})}$ where mass per unit length of string, $\mu_{string} = M_{string}/L_{string}$

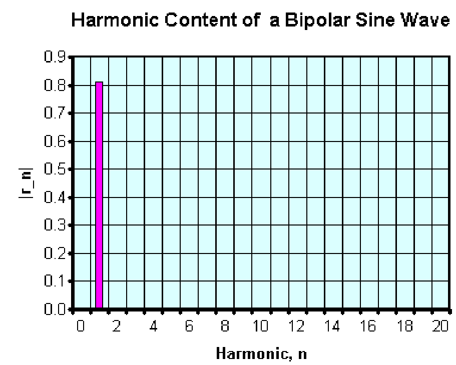
# <u>Harmonic Content of Complex Waveforms</u>

From mathematical work (1804-1807) of Jean Baptiste Joseph Fourier (1768-1830), the spatial/temporal shape of any *periodic* waveform can be shown to be due to linear combination of fundamental & higher harmonics!

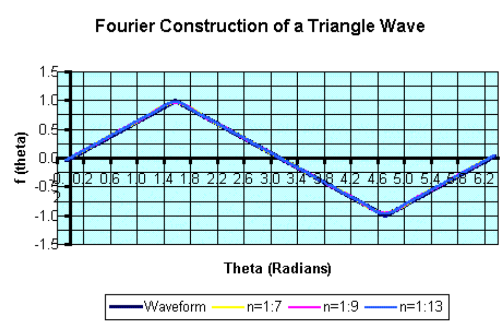
Sound Tonal Quality - *Timbre* - harmonic content of sound wave

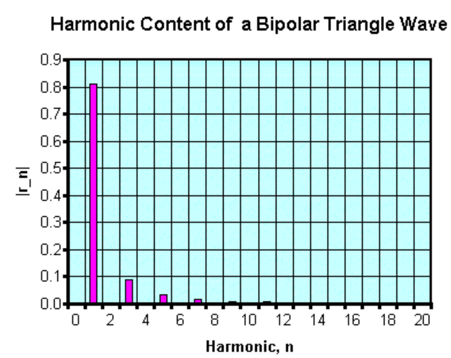
<u>Sine/Cosine Wave</u>: Mellow Sounding – fundamental, no higher harmonics



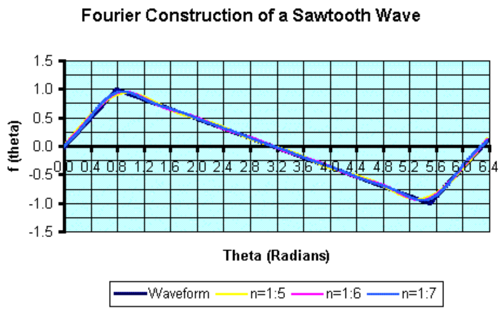



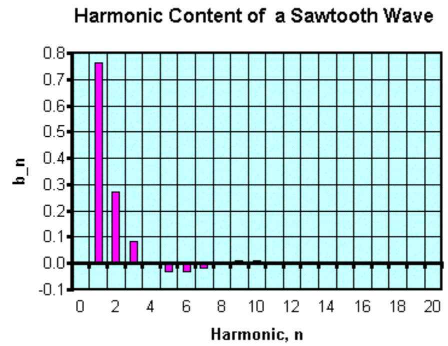

<u>Triangle Wave</u>: A Bit Brighter Sounding – has higher harmonics!



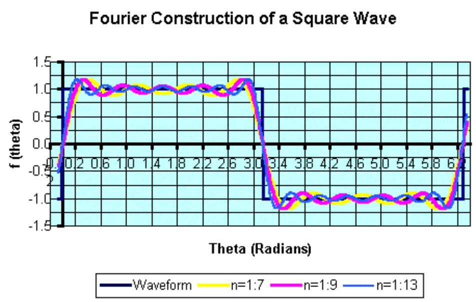



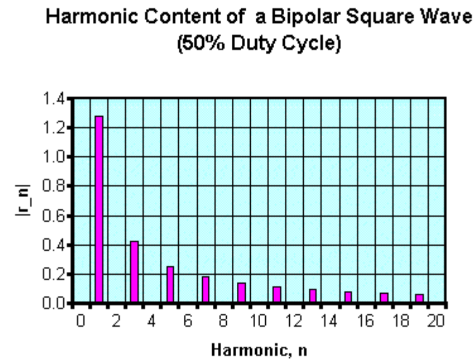
Asymmetrical Sawtooth Wave: Even Brighter Sounding – even more harmonics!




Square Wave: Brighter Sounding – has the most harmonics!

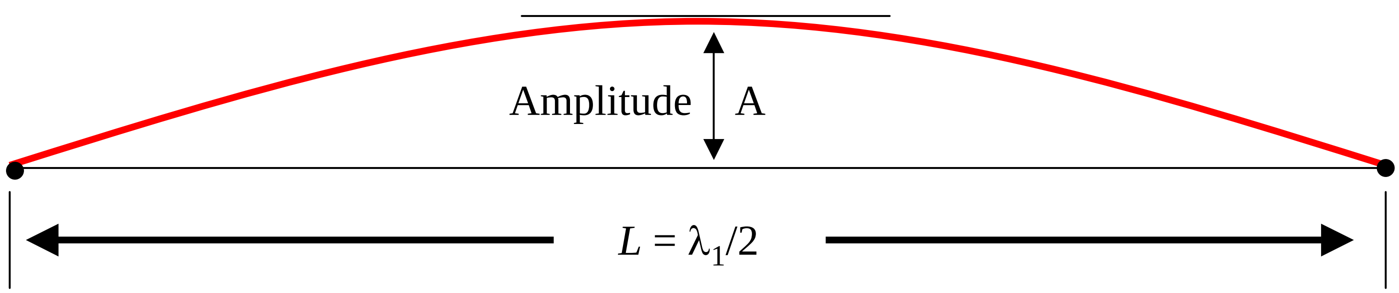


# Standing Waves on a Stretched String

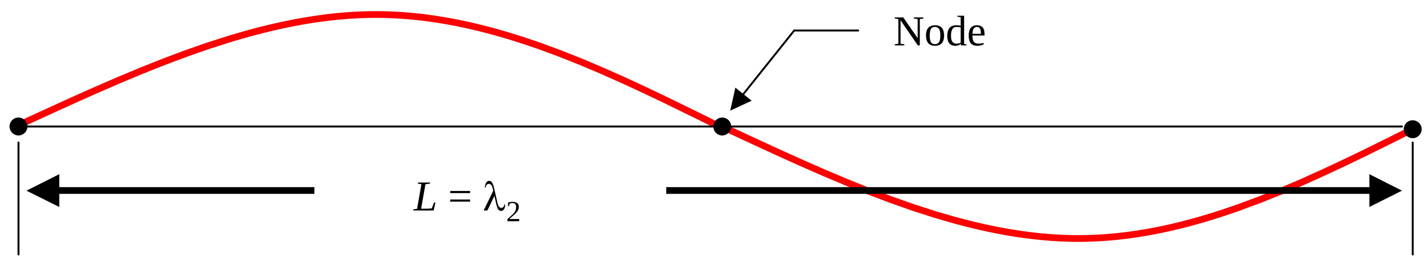
Standing wave = superposition of left- and right-going traveling waves

- Left & right-going traveling waves reflect off of end supports
- Polarity flip of traveling wave occurs at fixed end supports. No polarity flip for free ends.
- Different modes of string vibrations - resonances occur!
- For string of length L with fixed ends, the lowest mode of vibration has frequency $f_1 = v/2L$ ($v = f_1\lambda_1$) (f in cycles per second, or Hertz (Hz))
- Frequency of vibration, $f = 1/\tau$, where $\tau$ = period = time to complete 1 cycle
- Wavelength, $\lambda_1$ of lowest mode of vibration has $\lambda_1 = 2L$ (in meters)
- Amplitude of wave (maximum displacement from equilibrium) is A - see figure below - snapshot of standing wave at one instant of time, t:

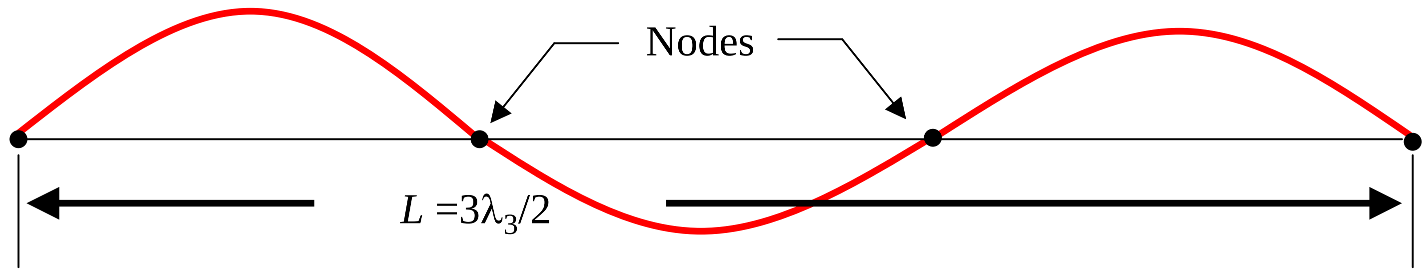


String can also vibrate with higher modes:

• Second mode of vibration of standing wave has $f_2 = 2v/2L = v/L$ with $\lambda_2 = 2L/2 = L$

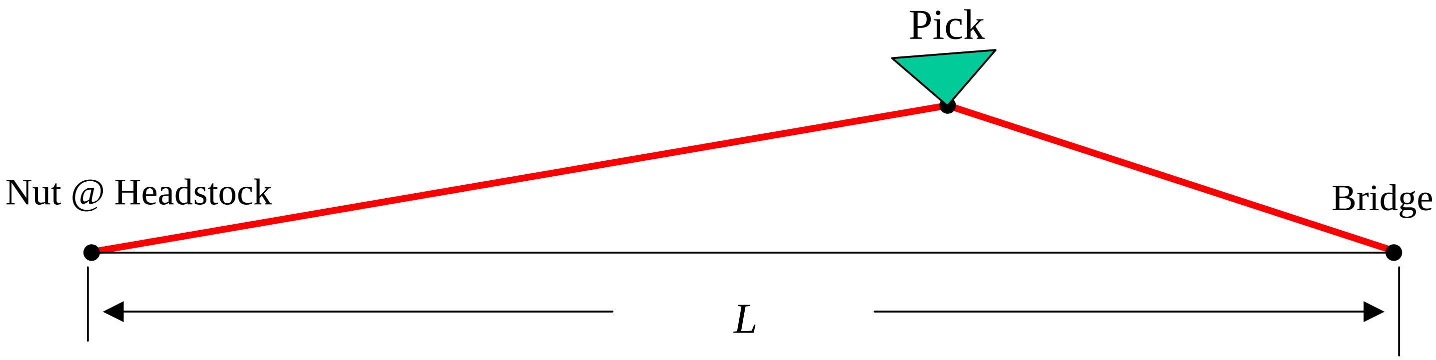



• Third mode of vibration of standing wave has $f_3 = 3v/2L$ with $\lambda_3 = 2L/3$

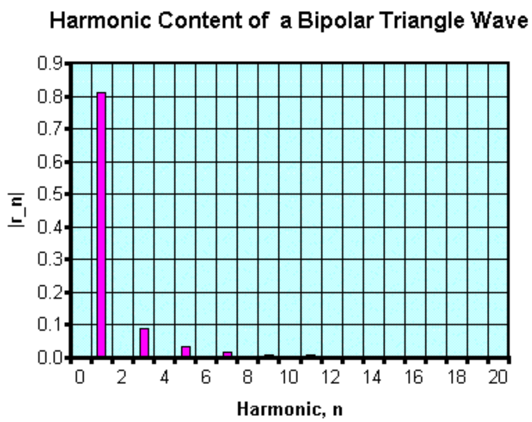


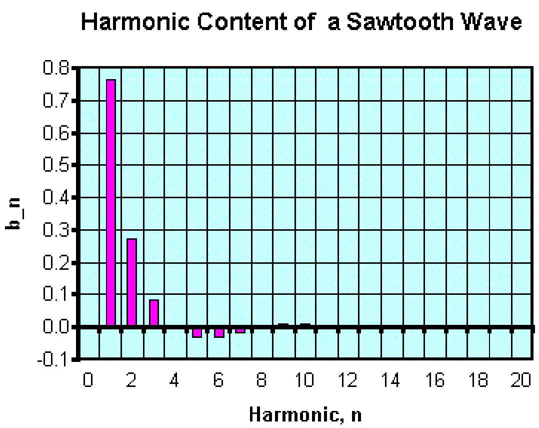

• The n$^{th}$ mode of vibration of standing wave on a string, where $n$ = integer = 1,2,3,4,5,…. has frequency $f_n = n(v/2L) = n\,f_1$, since $v = f_n\lambda_n$ and thus the $n^{th}$ mode of vibration has a wavelength of $\lambda_n = (2L)/n = \lambda_1/n$

When we e.g. pick (i.e. pluck) the string of a guitar, initial waveform is a triangle wave:




The geometrical shape of the string (a triangle) at the instant the pick releases the string can be shown mathematically (using Fourier Analysis) to be due to a linear superposition of standing waves consisting of the fundamental plus higher harmonics of the fundamental! Depending on where pick along string, harmonic content changes. Pick near the middle, mellower (lower harmonics); pick near the bridge - brighter - higher harmonics emphasized!



## What *is* Music?

- An aesthetically pleasing sequence of tones?
- *Why* is music pleasurable to humans?
- Music has always been part of human culture, as far back as we can tell
- Music important to human evolution?
- *Memory* of music much better (stronger/longer) than “normal” memory! Why? How?
- Music shown to *stimulate* human brain activity
- Music *facilitates* brain development in young children and in *learning*
- Music/song is also important to other living creatures - birds, whales, frogs, etc.
- Many kinds of animals utilize sound to communicate with each other
- What is it about music that does all of the above ???

## Human Development of Musical Instruments

- Emulate/mimic human voice (some instruments much more so than others)!
- Sounds from musical instruments can evoke powerful emotional responses - happiness, joy, sadness, sorrow, shivers down your spine, raise the hair on back of neck, etc.

# Musical Instruments

- Each musical instrument has its own characteristic sounds - quite complex!
- Any note played on an instrument has fundamental + harmonics of fundamental.
- Higher harmonics - brighter sound
- Less harmonics - mellower sound
- Harmonic content of note can/does change with time:
    - Takes time for harmonics to develop - "attack" (leading edge of sound)
    - Harmonics don't decay away at same rate (trailing edge of sound)
    - Higher harmonics tend to decay more quickly
- Sound output of musical instrument is not uniform with frequency
    - Details of construction, choice of materials, finish, etc. determine *resonant structure* (formants) associated with instrument - mechanical vibrations!
- See harmonic content of guitar, violin, recorder, singing saw, drum, cymbals, etc.
- See laser interferogram pix of vibrations of guitar, violin, handbells, cymbals, etc.

# Vibrational Modes of a Violin

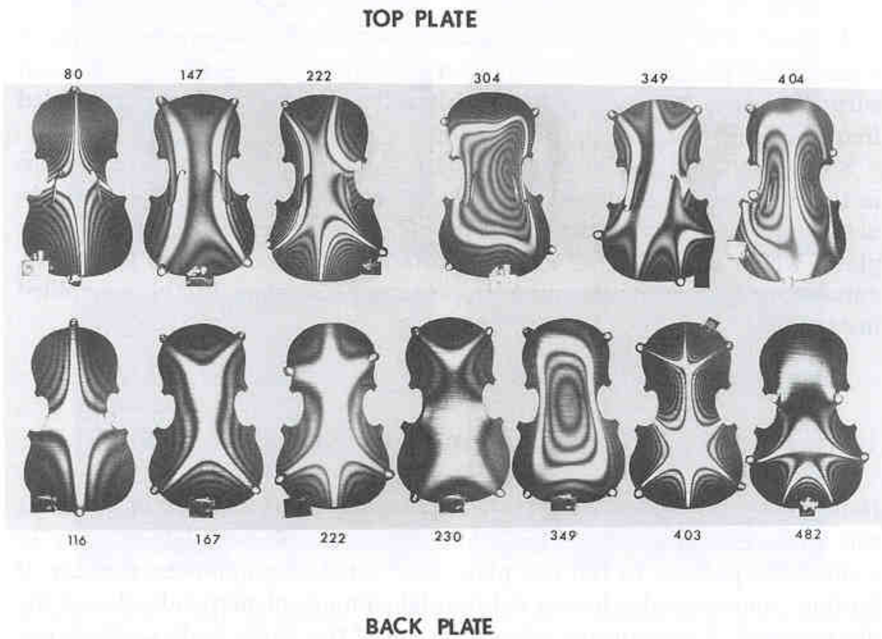
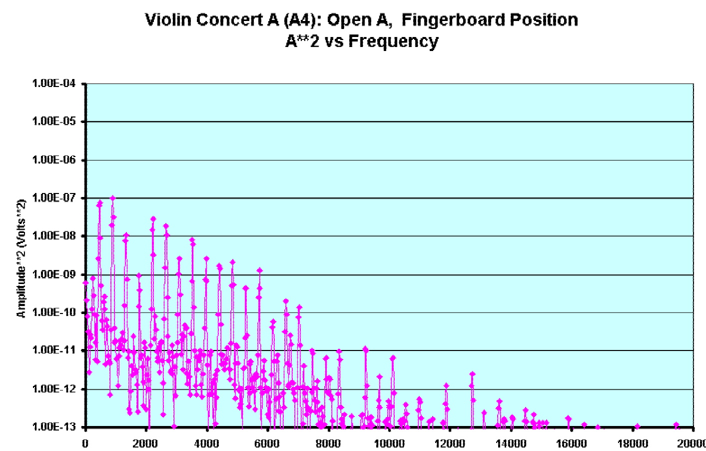
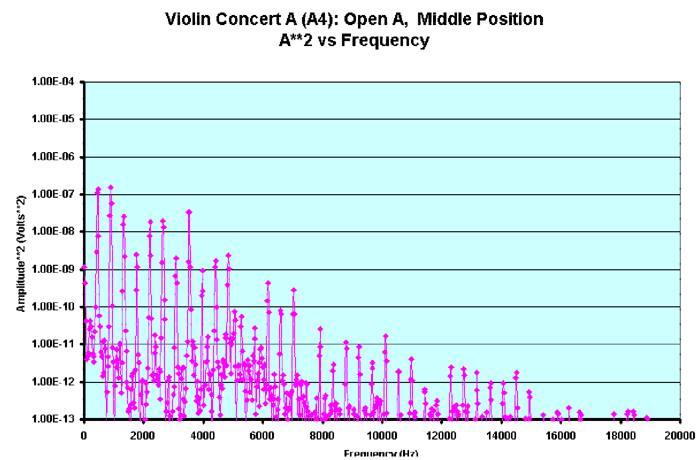


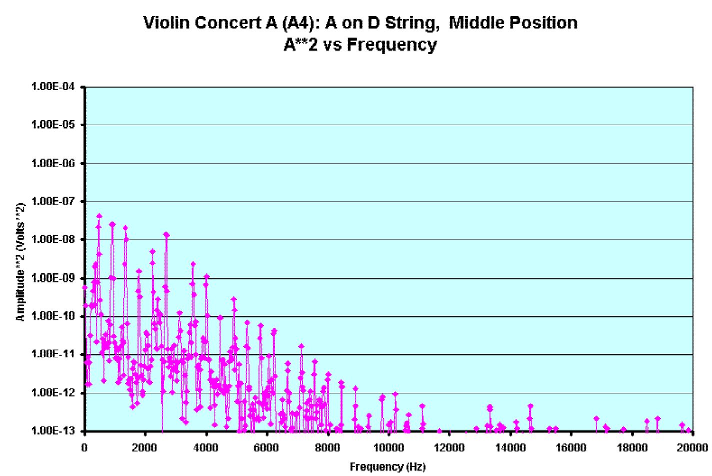

FIGURE 10.14. Time-average holographic interferograms of a free violin top plate and back plate (Hutchins et al., 1971).

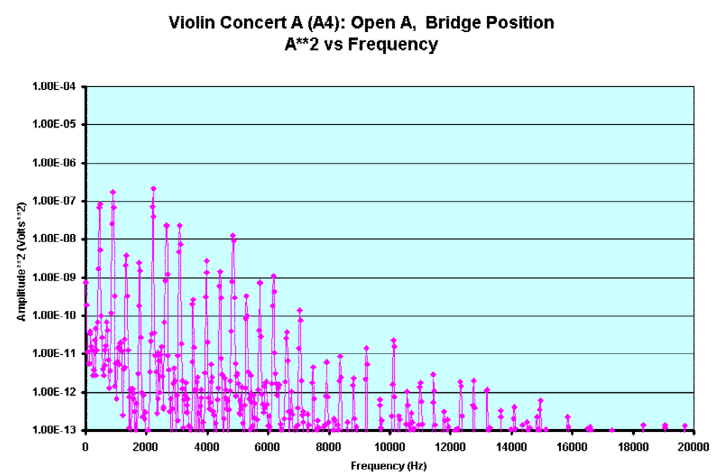
# Harmonic Content of a Violin:

## Freshman Students, UIUC Physics 199 POM Course, Fall Semester, 2003



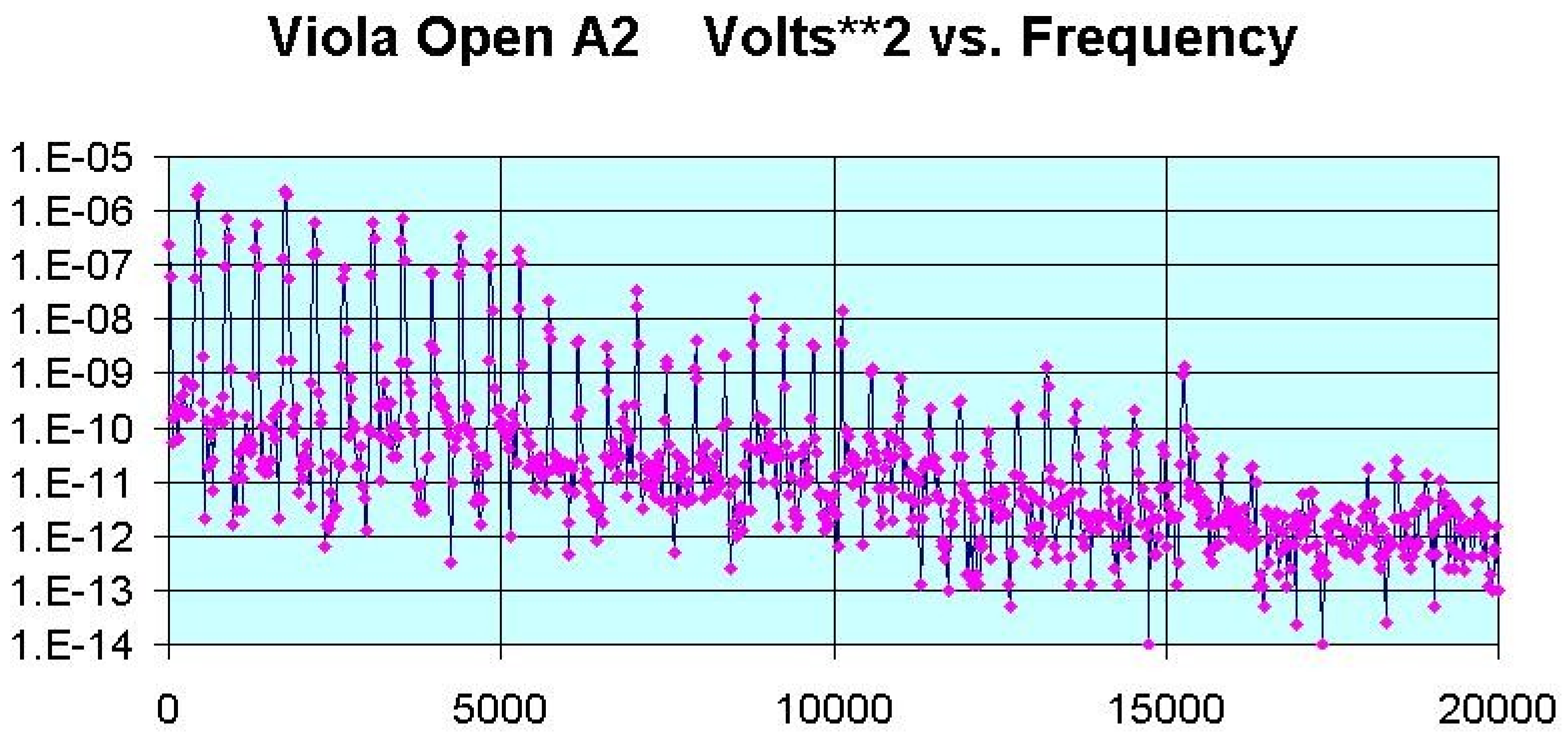



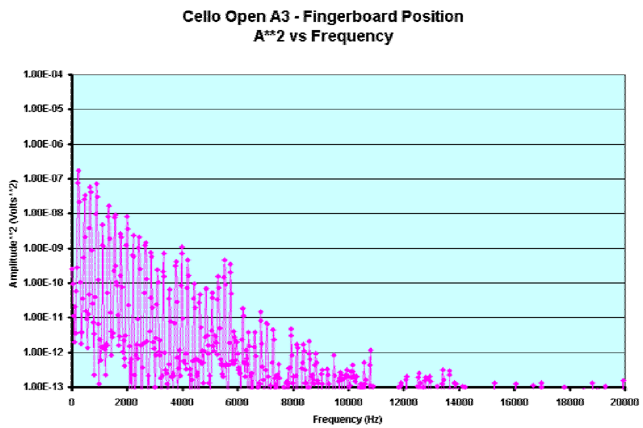
# Harmonic Content of a Viola – Open A2

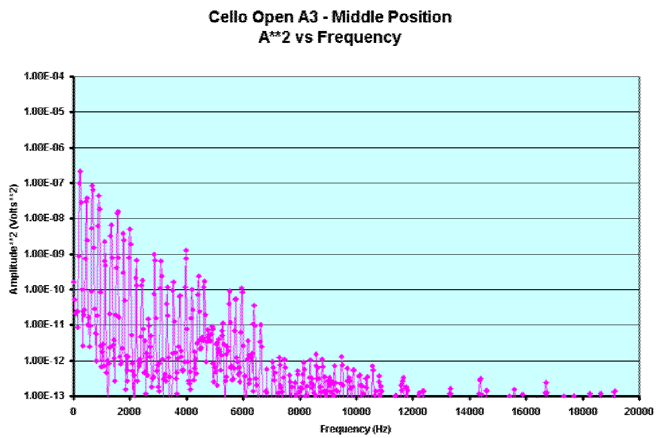
**Laura Book (Uni High, Spring Semester, 2003)**

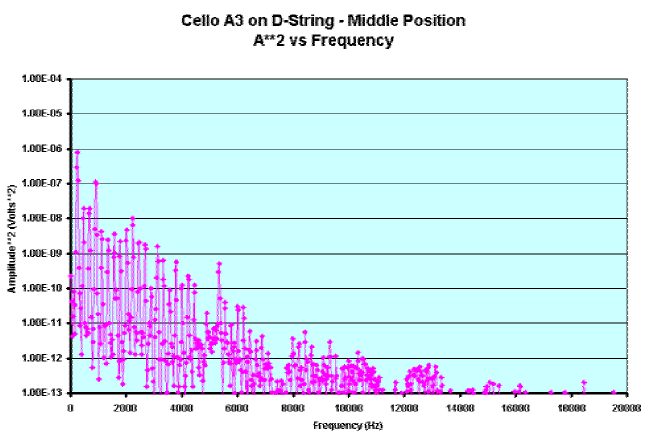


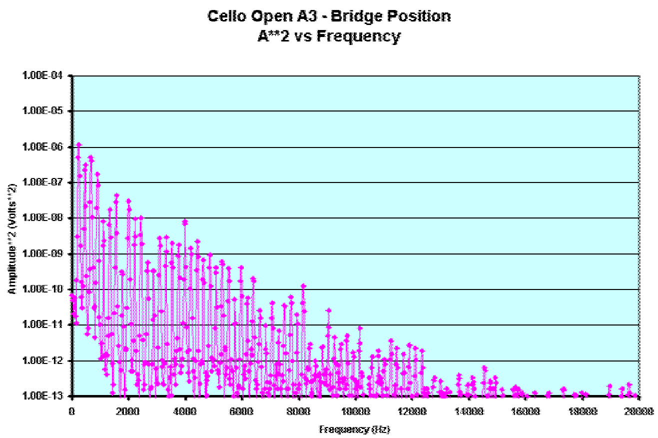
# Harmonic Content of a Cello:

## Freshman Students, UIUC Physics 199 POM Course, Fall Semester, 2003



# Vibrational Modes of an Acoustic Guitar

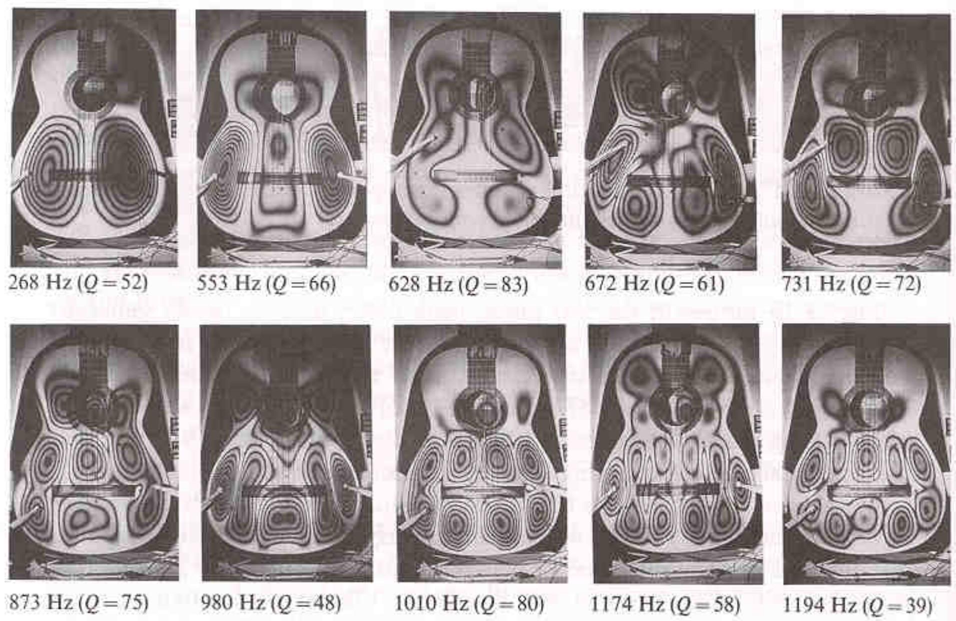



FIGURE 9.16. Time-averaged holographic interferograms of top-plate modes of a guitar (Guitar BR11). The resonant frequencies and $Q$ values of each mode are shown below the interferograms (Richardson and Roberts, 1985).

# Resonances of an Acoustic Guitar

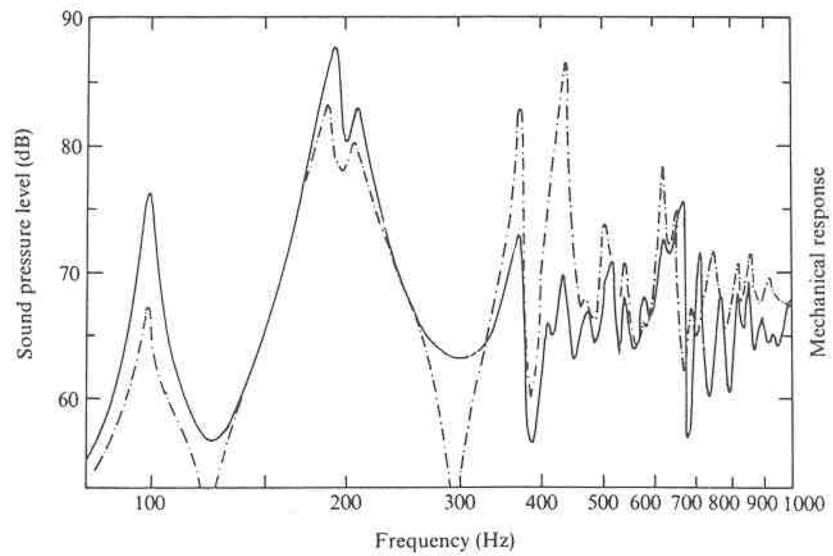
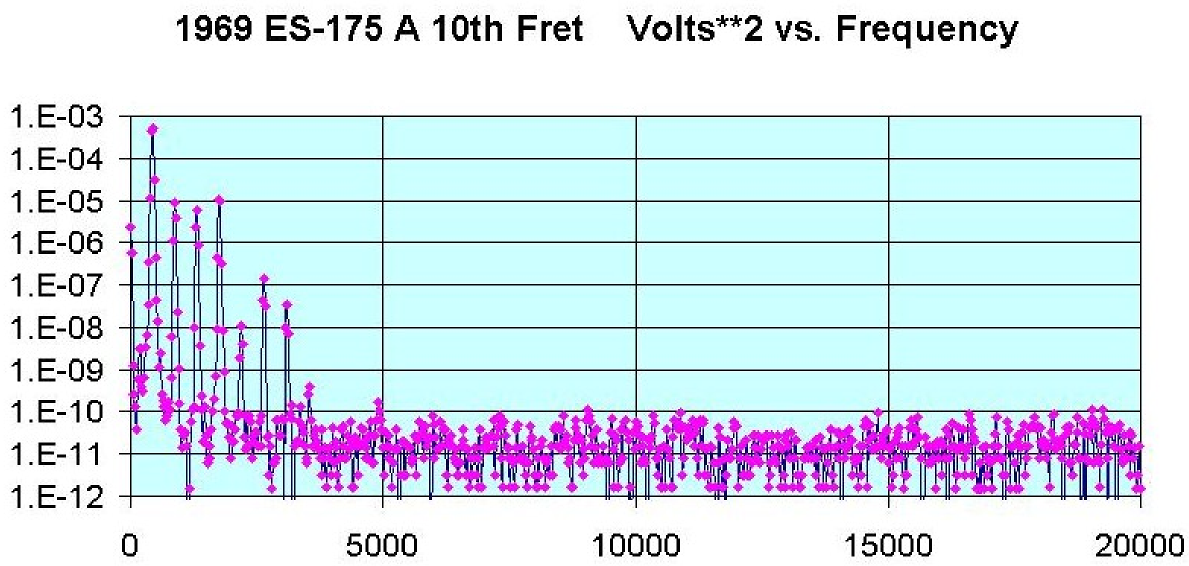



FIGURE 9.20. Mechanical frequency response and sound spectrum 1 m in front of a Martin D-28 folk guitar driven by a sinusoidal force of 0.15 N applied to the treble side of the bridge. Solid curve, sound spectrum; dashed curves, acceleration level at the driving point.

# Harmonic Content of 1969 Gibson ES-175 Electric Guitar

**Jacob Hertzog (Uni High, Spring Semester, 2003)**



**Musical Properties of a 1954 Fender Stratocaster, S/N 0654 (August, 1954):**

**Measuring Mechanical Vibrational Modes of 1954 Fender Stratocaster:**

## Mechanical Vibrational Modes of 1954 Fender Stratocaster:

**E4 = 329.63 Hz (High E)**

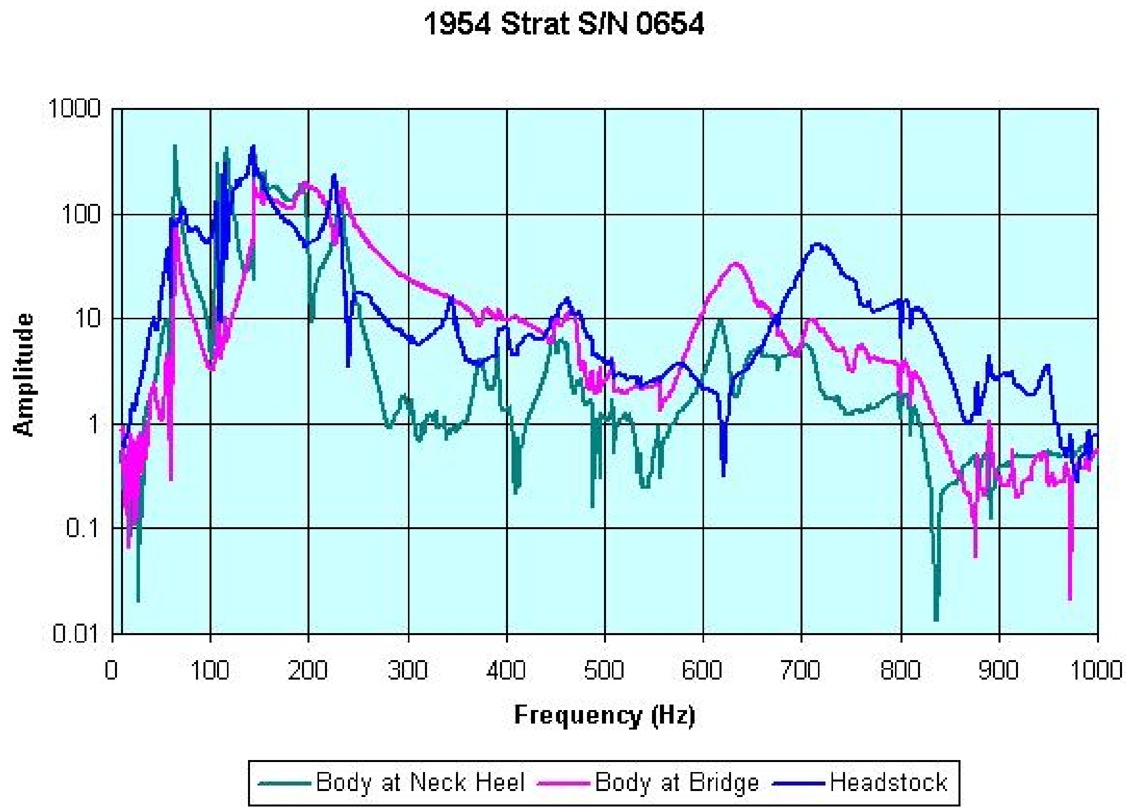
**B3 = 246.94 Hz**

**G3 = 196.00 Hz**

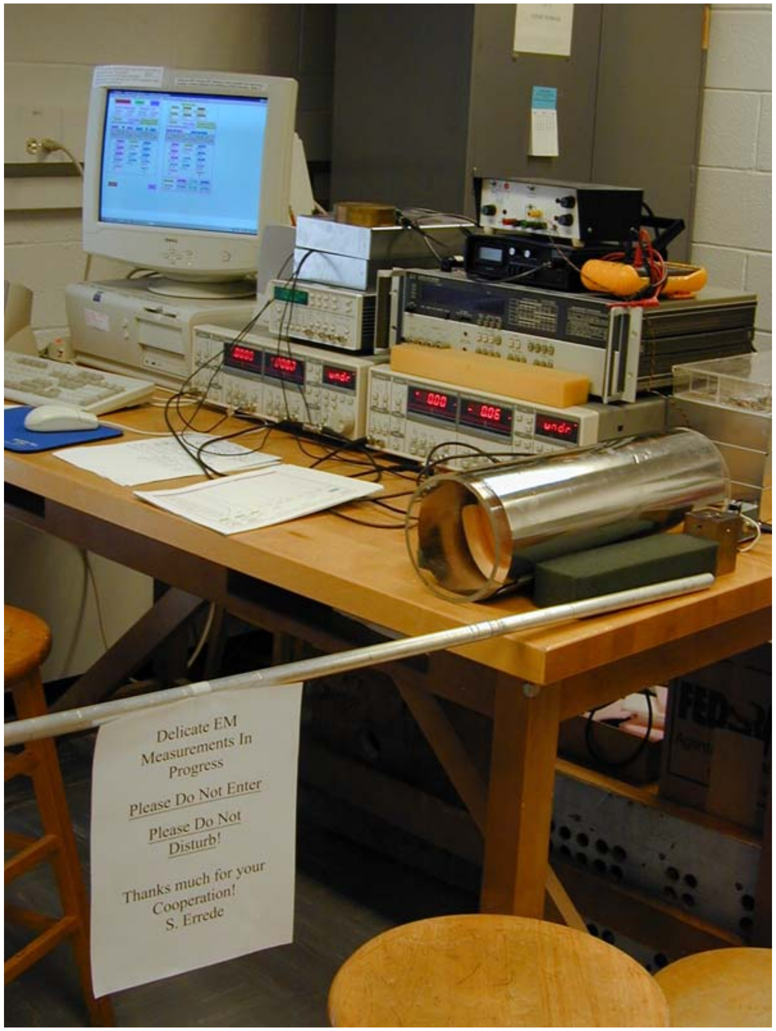
**D3 = 146.83 Hz**

**A2 = 110.00 Hz**

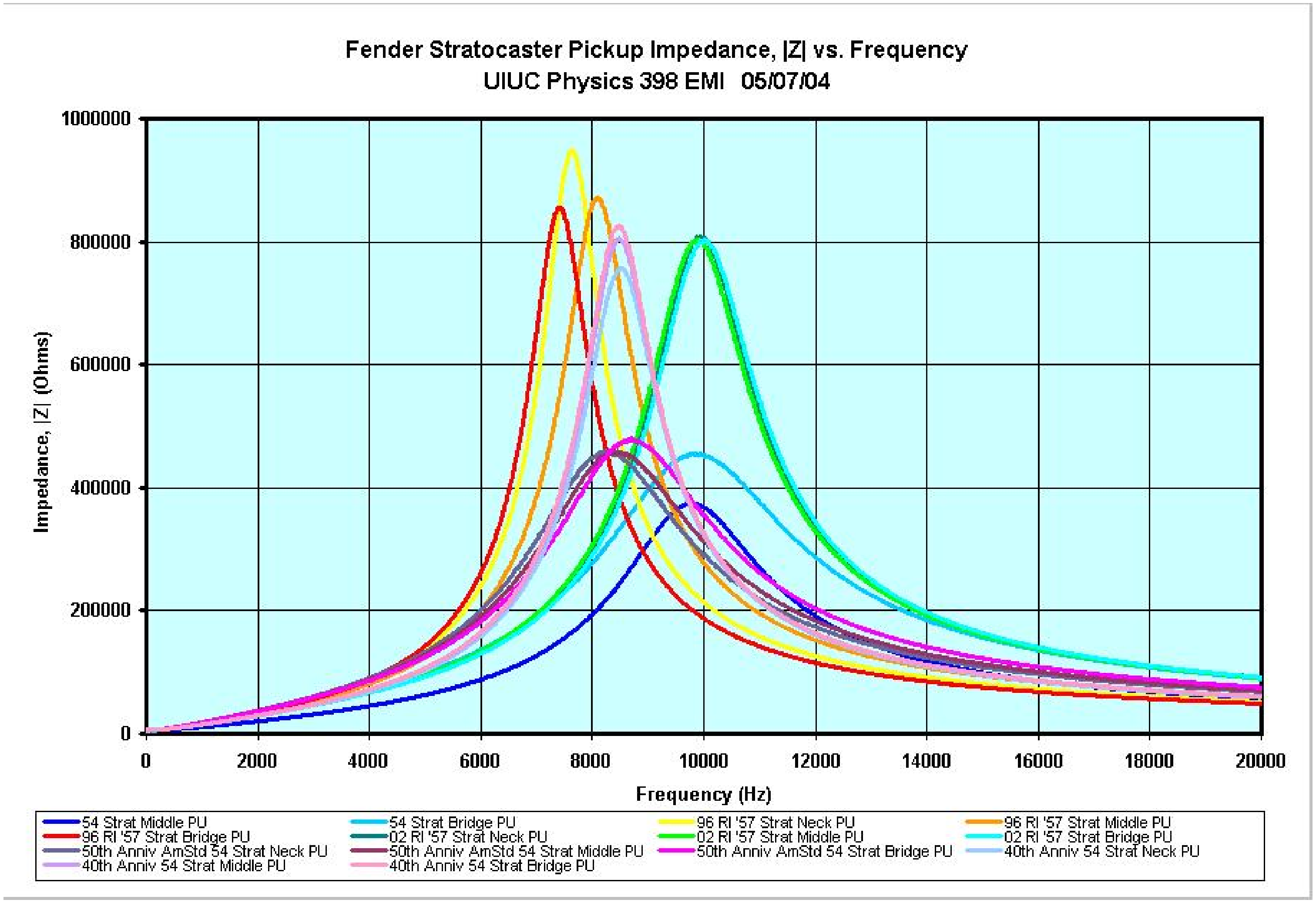
**E2 = 82.407 Hz (Low E)**



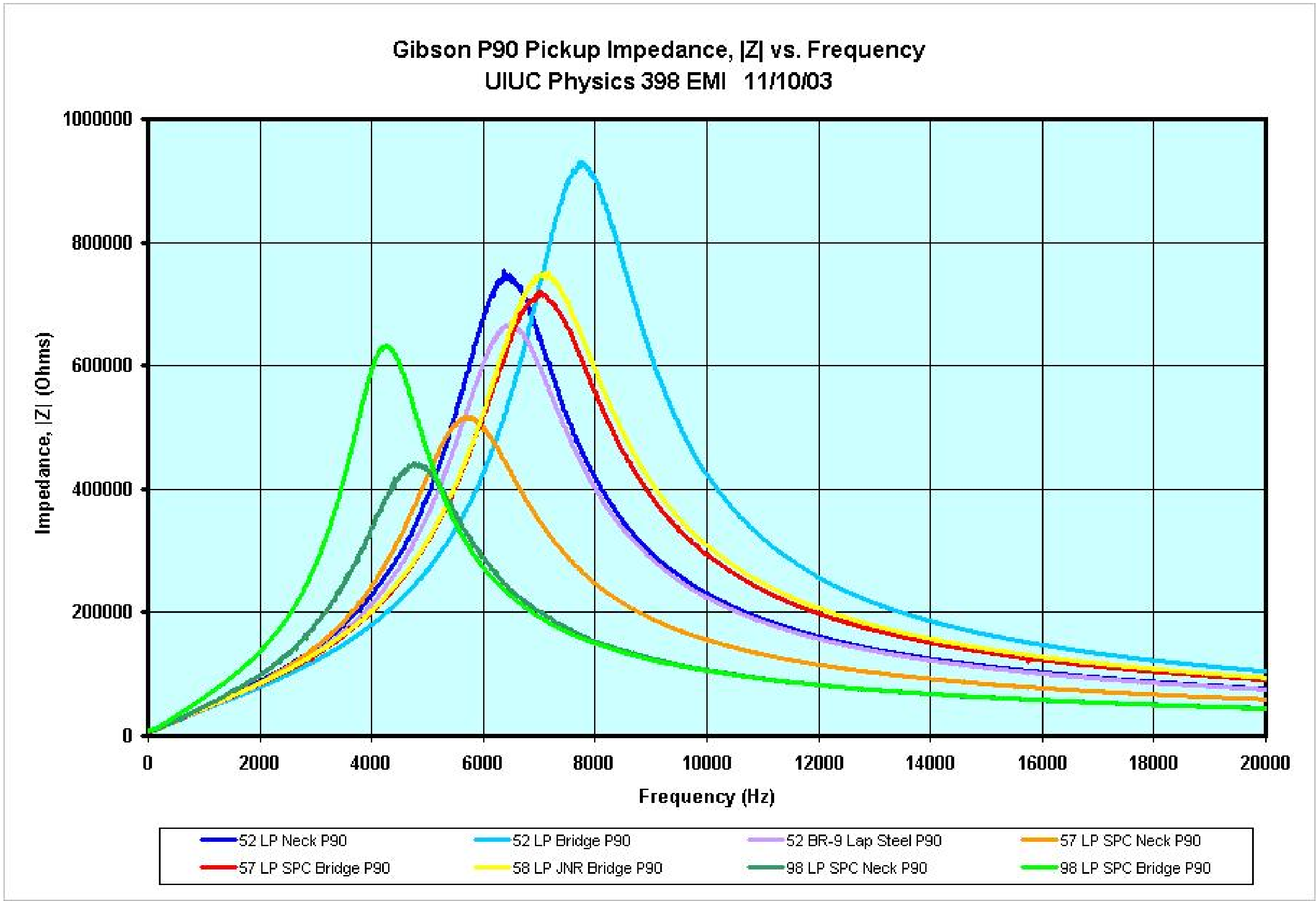
## UIUC Physics 398EMI Test Stand for Measurement of Electric Guitar Pickup Properties:




## Comparison of Vintage (1954's) vs. Modern Fender Stratocaster Pickups:

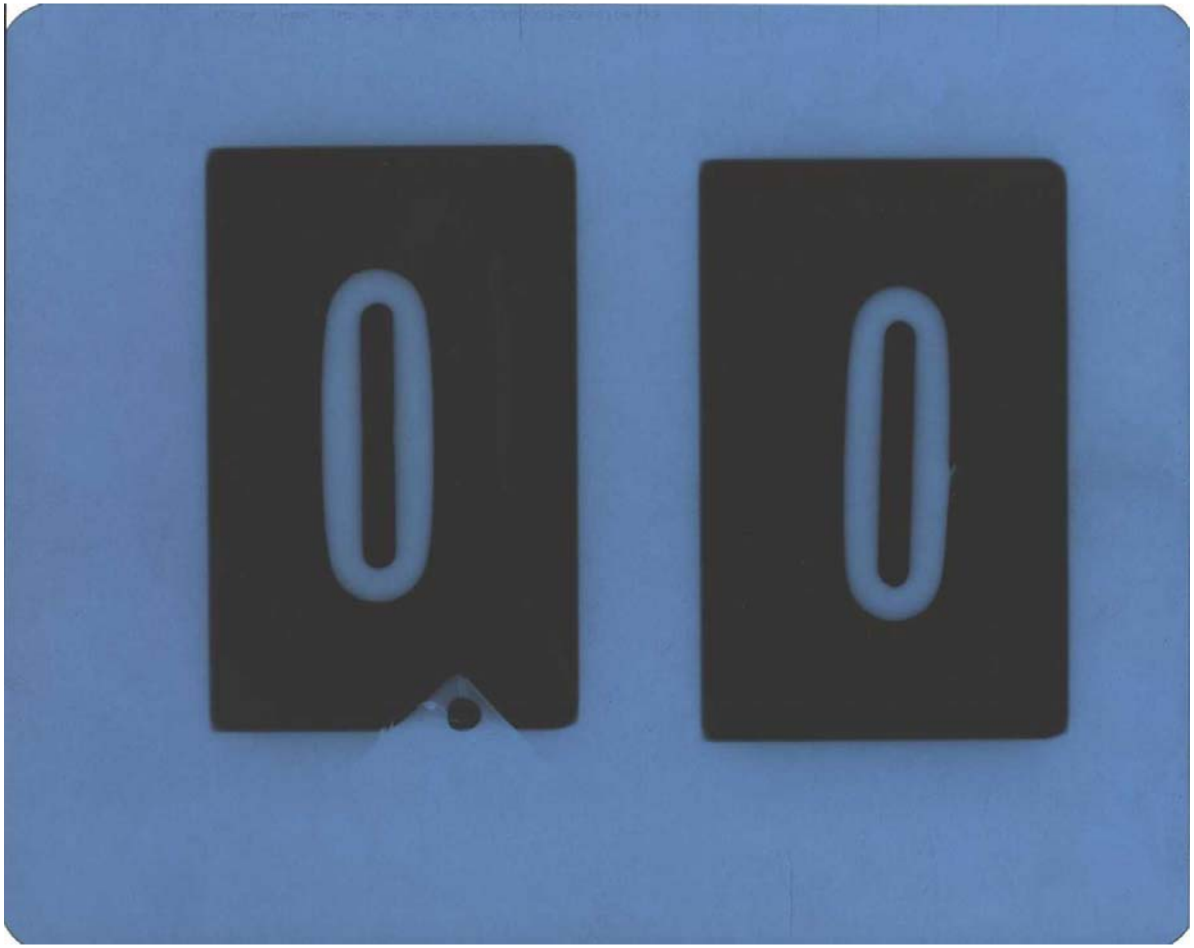


# Comparison of Vintage (1950's) vs. Modern Gibson P-90 Pickups:



## X-Ray Comparison of 1952 Gibson Les Paul Neck P90 Pickup vs. 1998 Gibson Les Paul Neck P90 Pickup

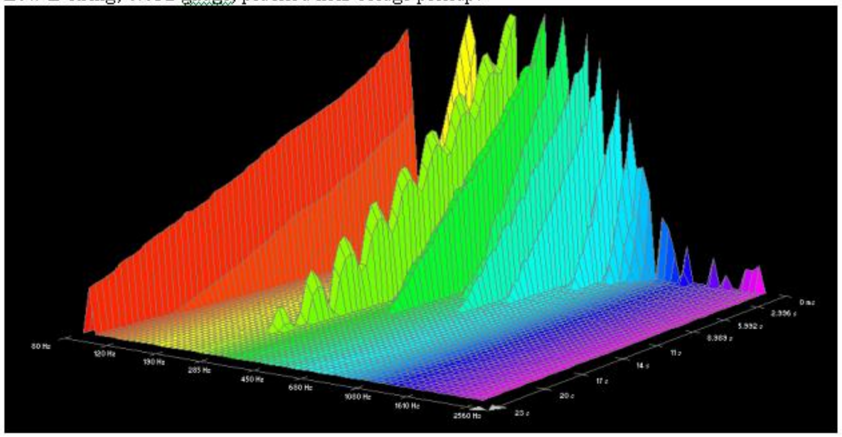
**SME & Richard Keen, UIUC Veterinary Medicine, Large Animal Clinic**



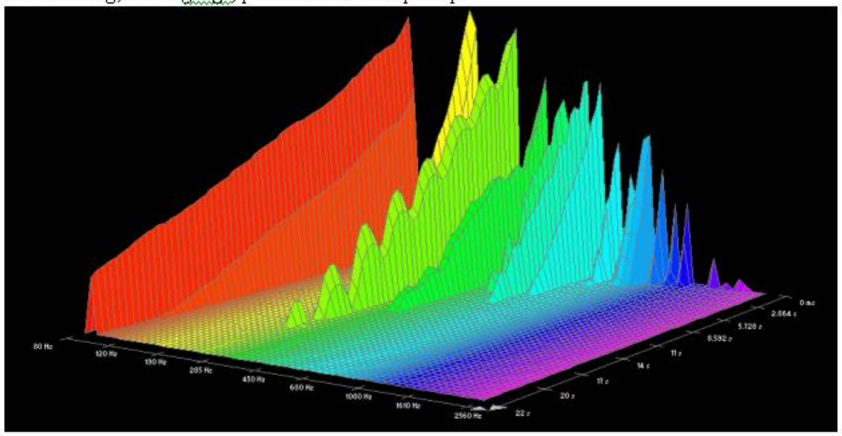
## Study/Comparison of Harmonic Properties of Acoustic and Electric Guitar Strings

**Ryan Lee (UIUC Physics P398EMI, Fall 2002)**

Low E-string, 0.052 gauge, plucked near bridge pickup:

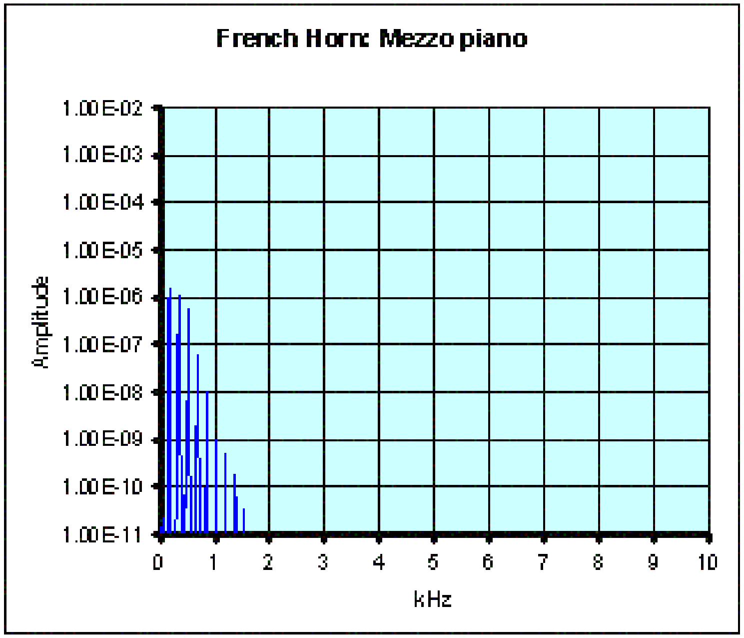


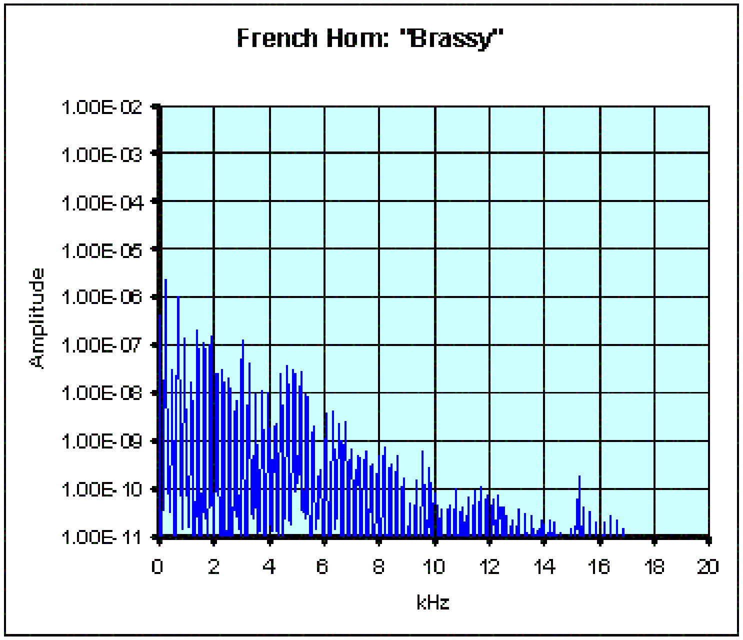

Low E-string, 0.052 gauge, plucked near neck pickup:



## Harmonic Content of a Conn 8-D French Horn:  Middle-C (C4)

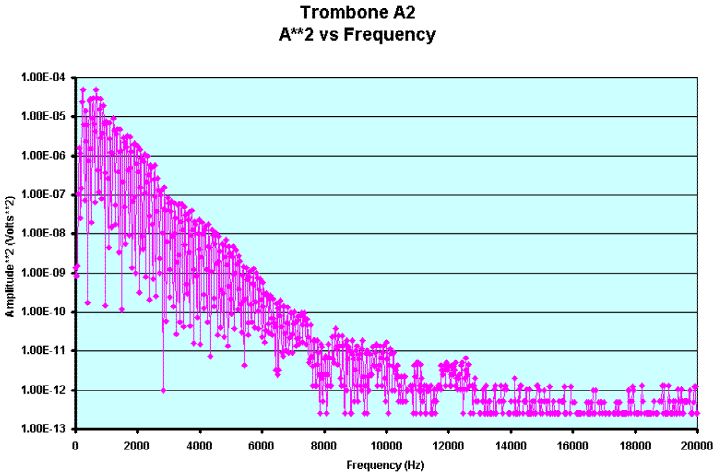
**Chris Orban UIUC Physics Undergrad, Physics 398EMI Course, Fall Semester, 2002**

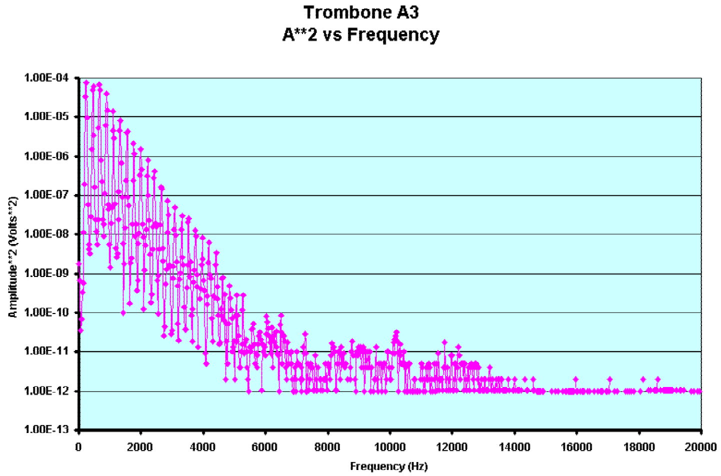


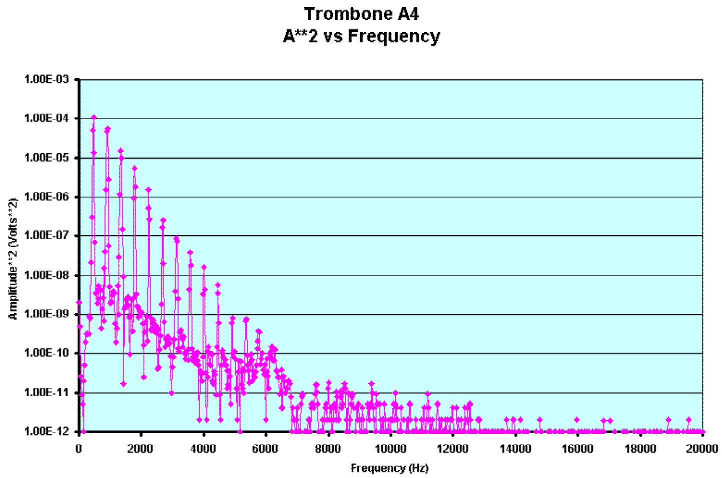



# Harmonic Content of a Trombone:

**Freshman Students in UIUC Physics 199 POM Class, Fall Semester**

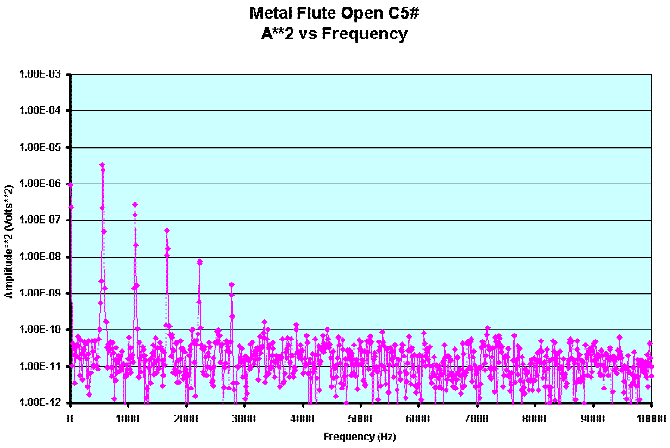


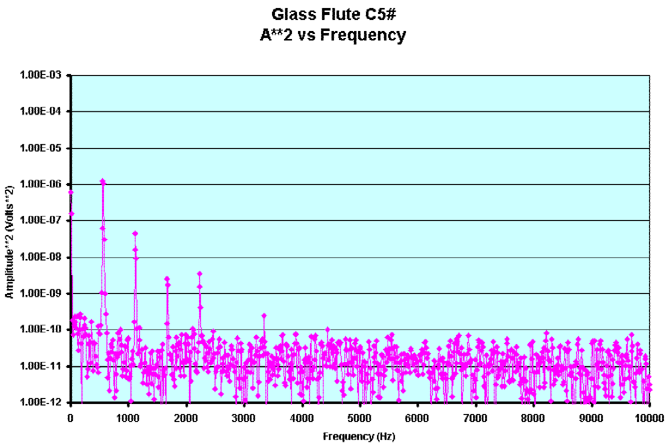



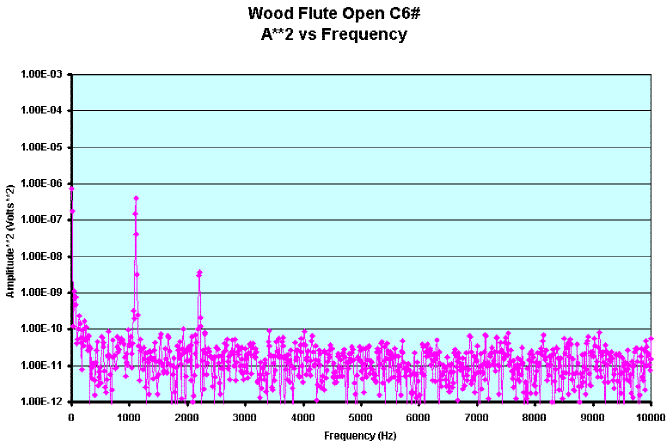



## Comparison of Harmonic Content of Metal, Glass and Wooden Flutes:

**Freshman Students in UIUC Physics 199 POM Class, Fall Semester, 2003**

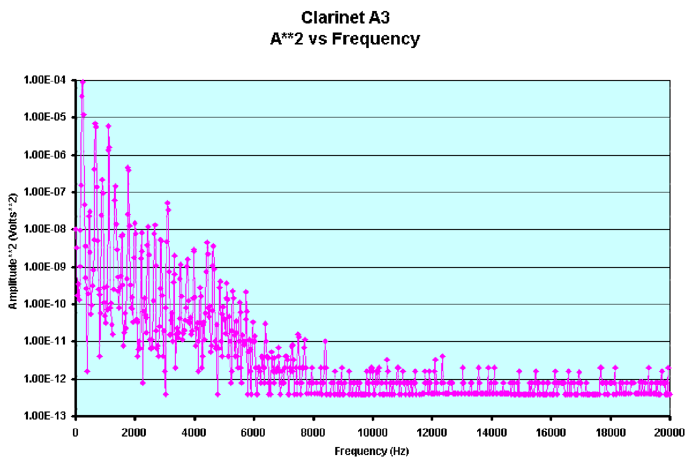


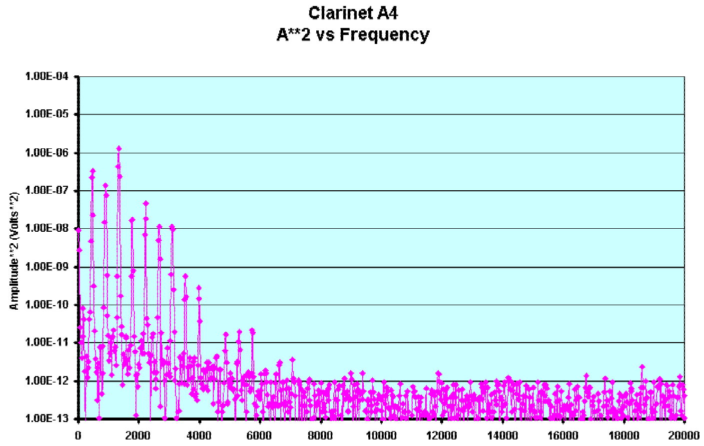



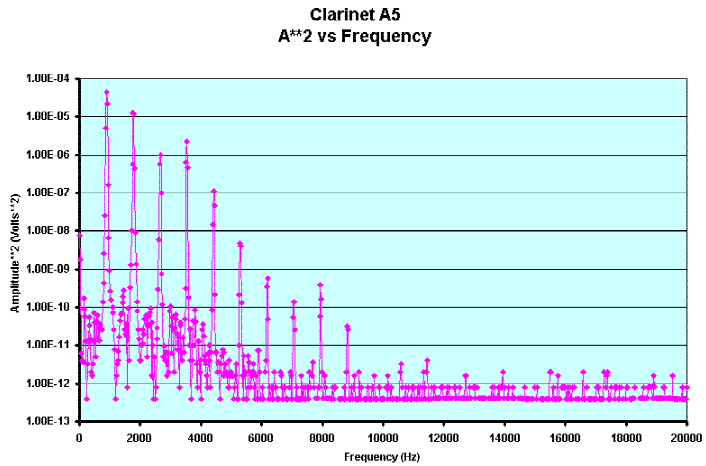



# Harmonic Content of a Clarinet:

**Freshman Students in UIUC Physics 199 POM Class, Fall Semester**

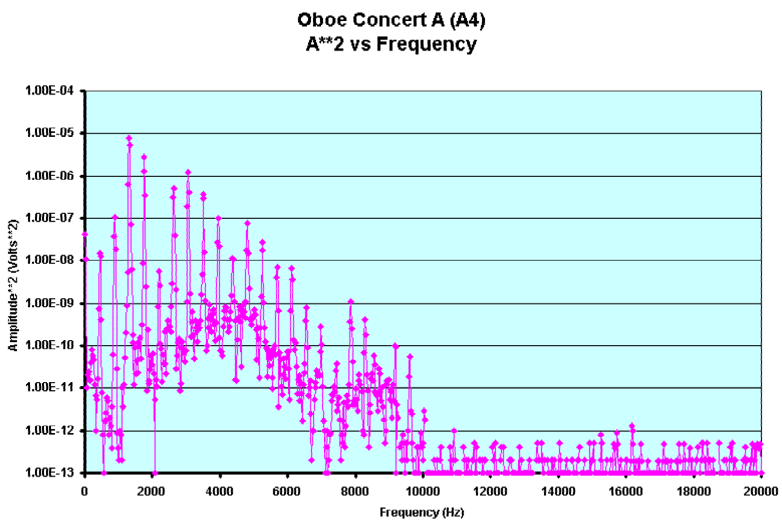


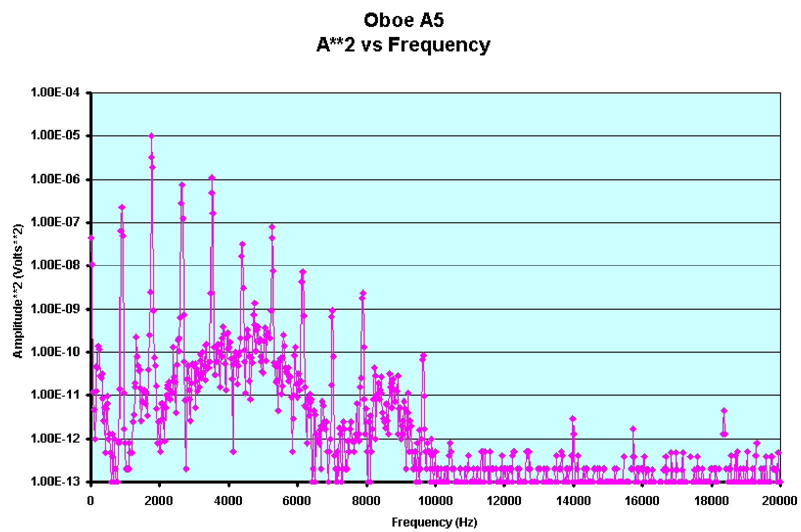



# Harmonic Content of an Oboe:

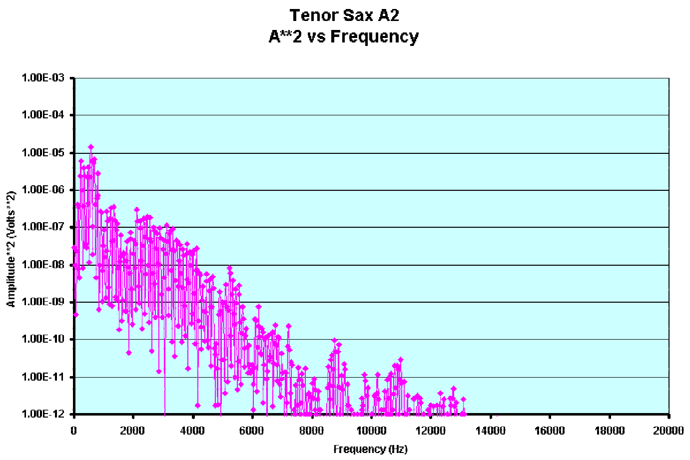
**Freshman Students in UIUC Physics 199 POM Class, Fall Semester, 2003**

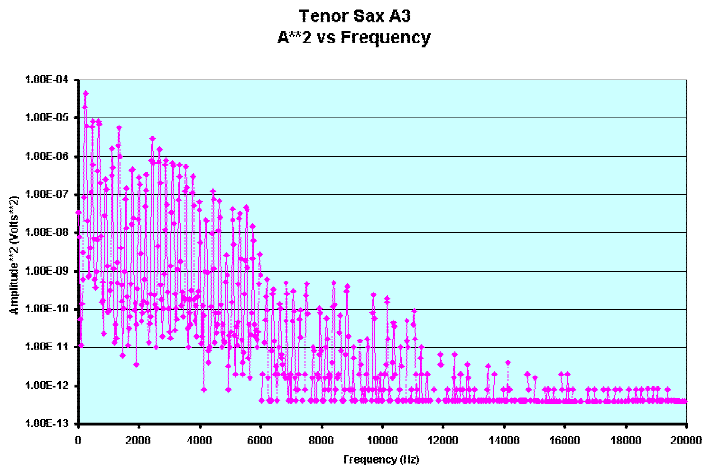


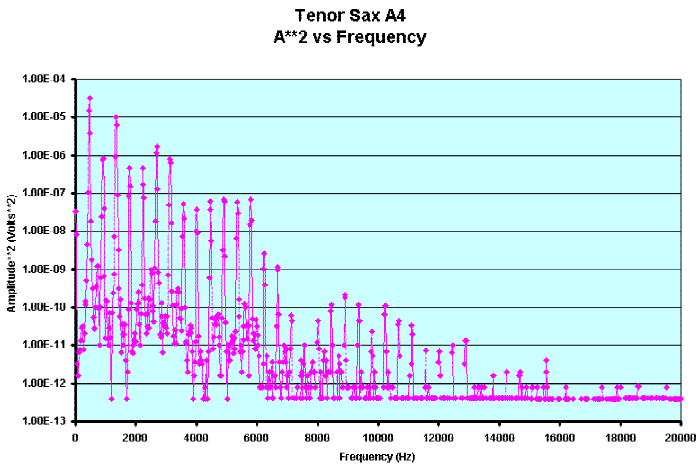



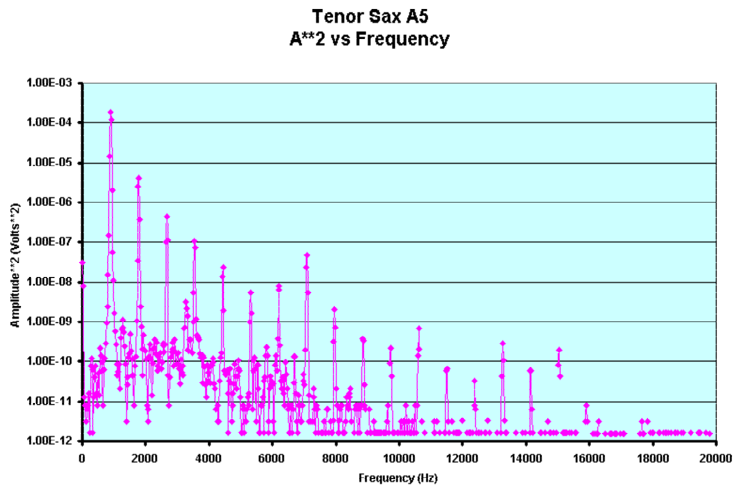
# Harmonic Content of a Tenor Sax:

## Freshman Students in UIUC Physics 199 POM Class, Fall Semester, 2003



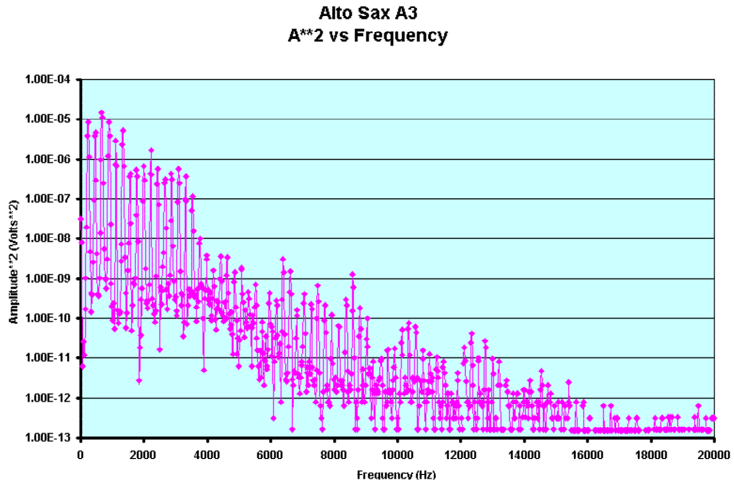



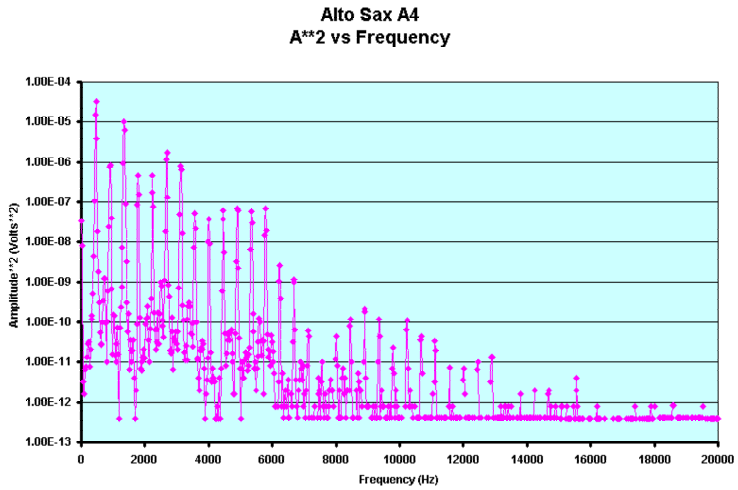



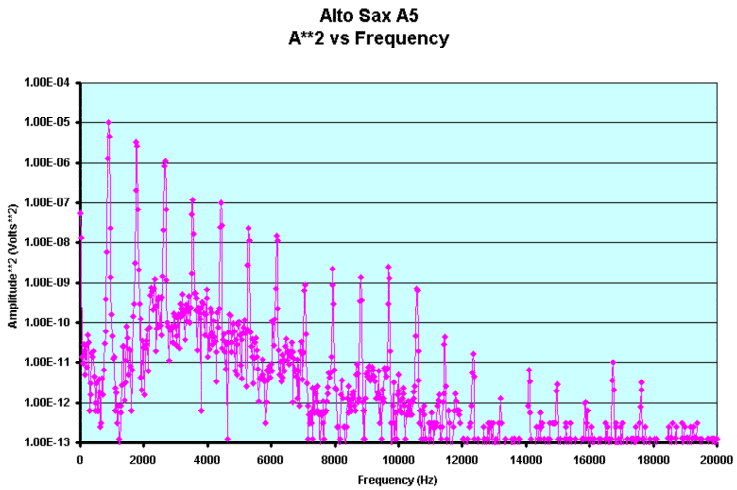



# Harmonic Content of an Alto Sax:

**Freshman Students in UIUC Physics 199 POM Class, Fall Semester, 2003**

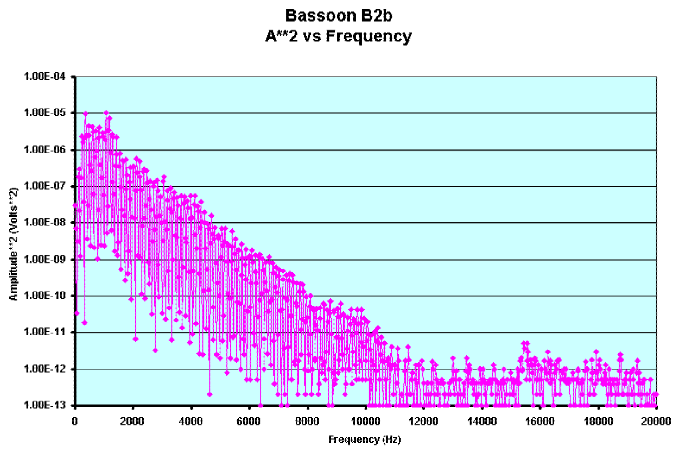


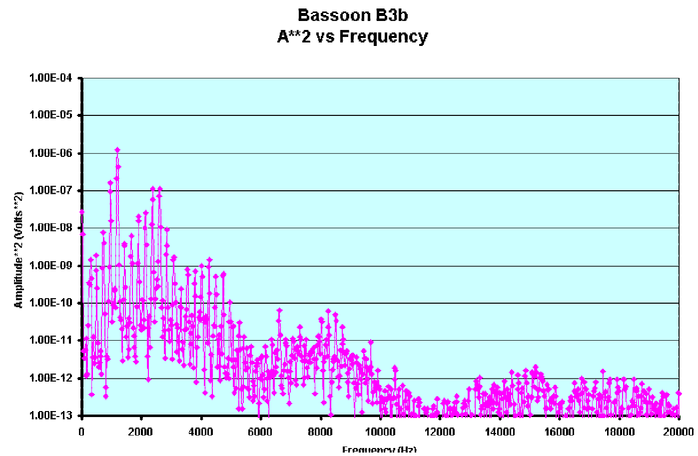



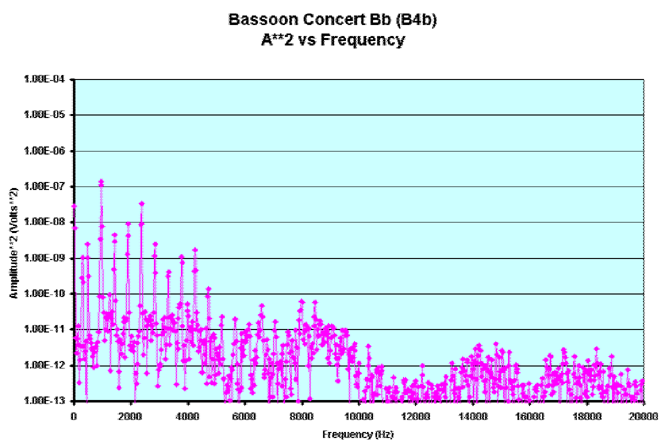



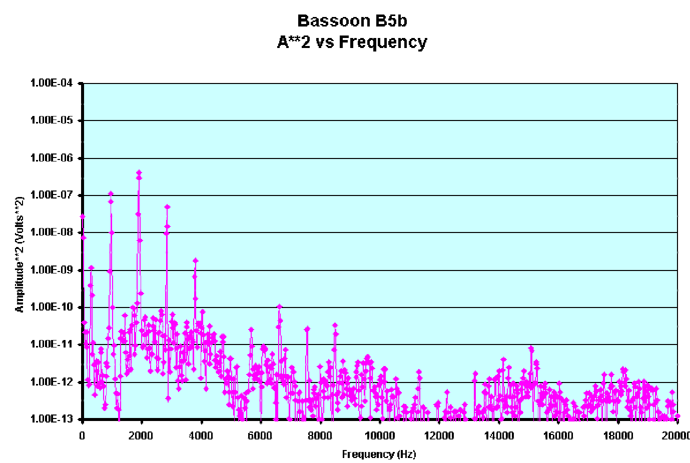
# Harmonic Content of the Bassoon:

**Prof. Paul Debevec, SME, UIUC Physics Dept. Fall Semester, 2003**



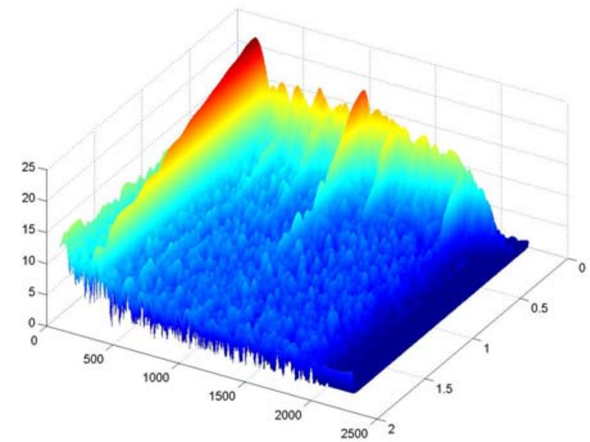



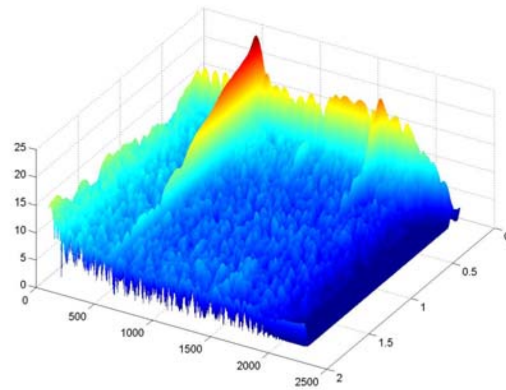



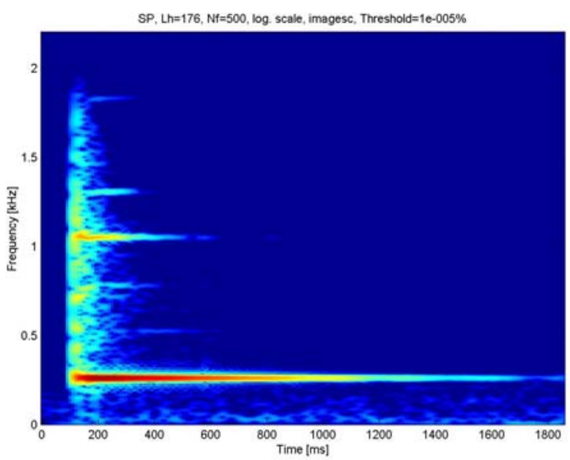



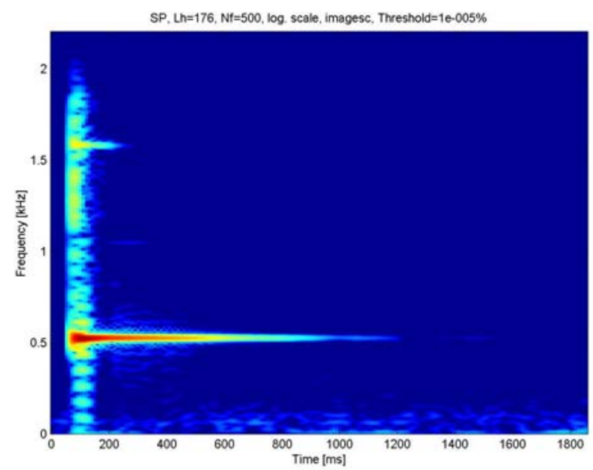
# Time-Dependence of the Harmonic Content of Marimba and Xylophone:

**Roxanne Moore, Freshman in UIUC Physics 199 POM Course, Spring Semester, 2003**







# Vibrational Modes of Membranes and Plates

# (Drums and Cymbals)

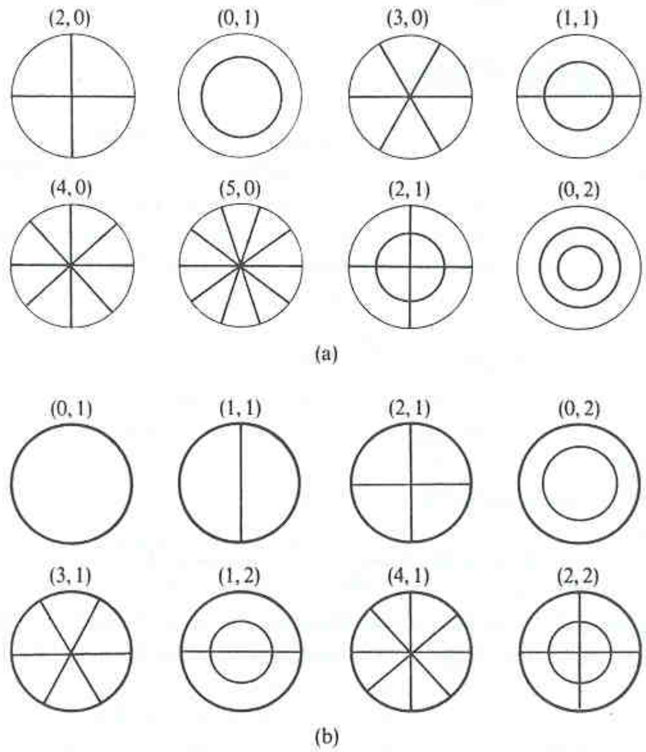



FIGURE 3.8. Vibrational modes of circular plates: (a) free edge and (b) clamped or simply supported edge. The mode number $(n, m)$ gives the number of nodal diameters and circles, respectively.

# <u>Study/Comparison of Acoustic Properties of Tom Drums</u>

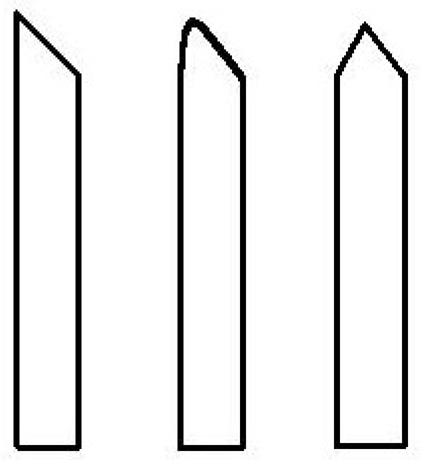
**Eric Macaulay (*Illinois Wesleyan University*), Nicole Drummer, SME (*UIUC*) Dennis Stauffer (Phattie Drums)**

**Eric Macaulay**
**(*Illinois Wesleyan University*)**

**NSF REU Summer Student @ UIUC Physics, 2003**

**Investigated/Compared Bearing Edge Design – Energy Transfer from Drum Head => Shell of Three “identical” 10” Diameter Tom Drums**

**Differences in Bearing Edge Design of Tom Drums (Cutaway View):**



**Single $45^o$**

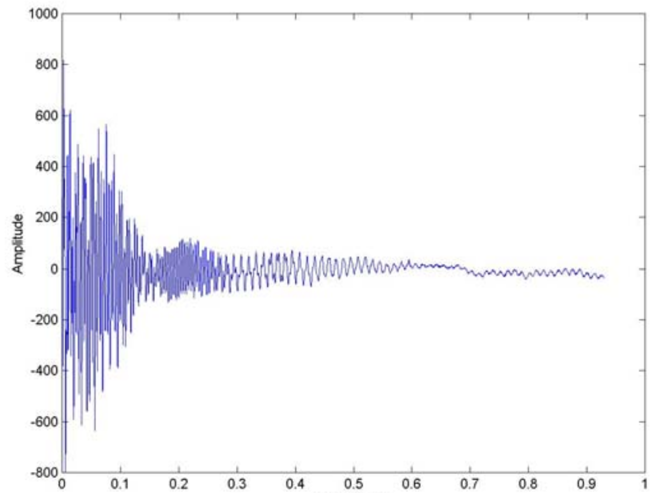
**Rounded $45^o$ (Classic)**

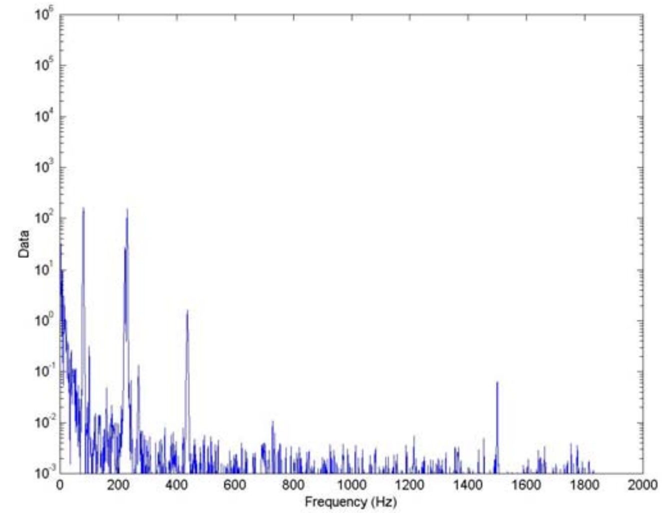
**and Double $45^o$ (Modern)**

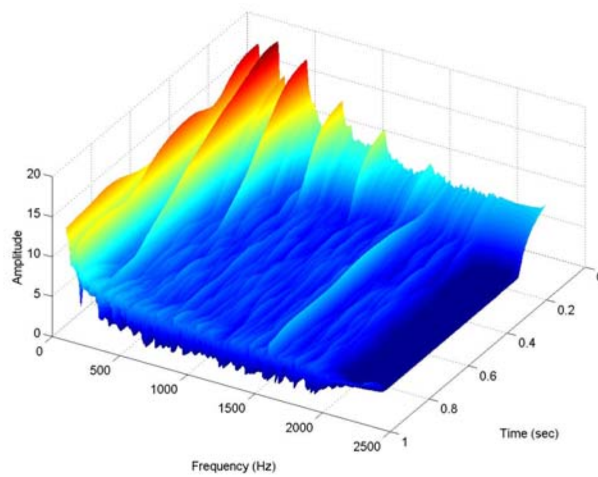
**Recording Sound(s) from Drum Head vs. Drum Shell:**

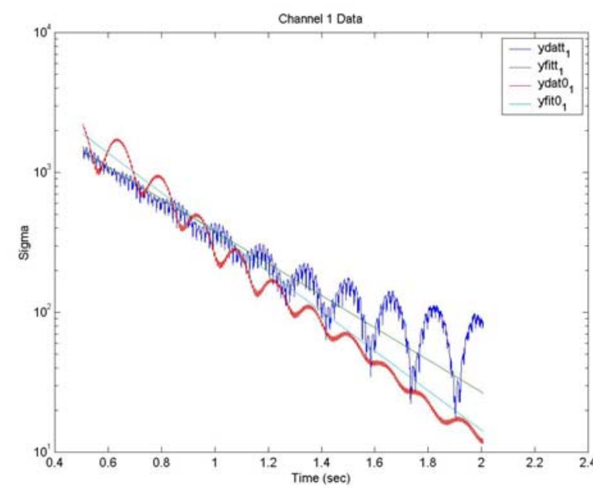
# <u>Analysis of Recorded Signal(s) From 10” Tom Drum(s):</u>

**{Shell Only Data (Shown Here)}**

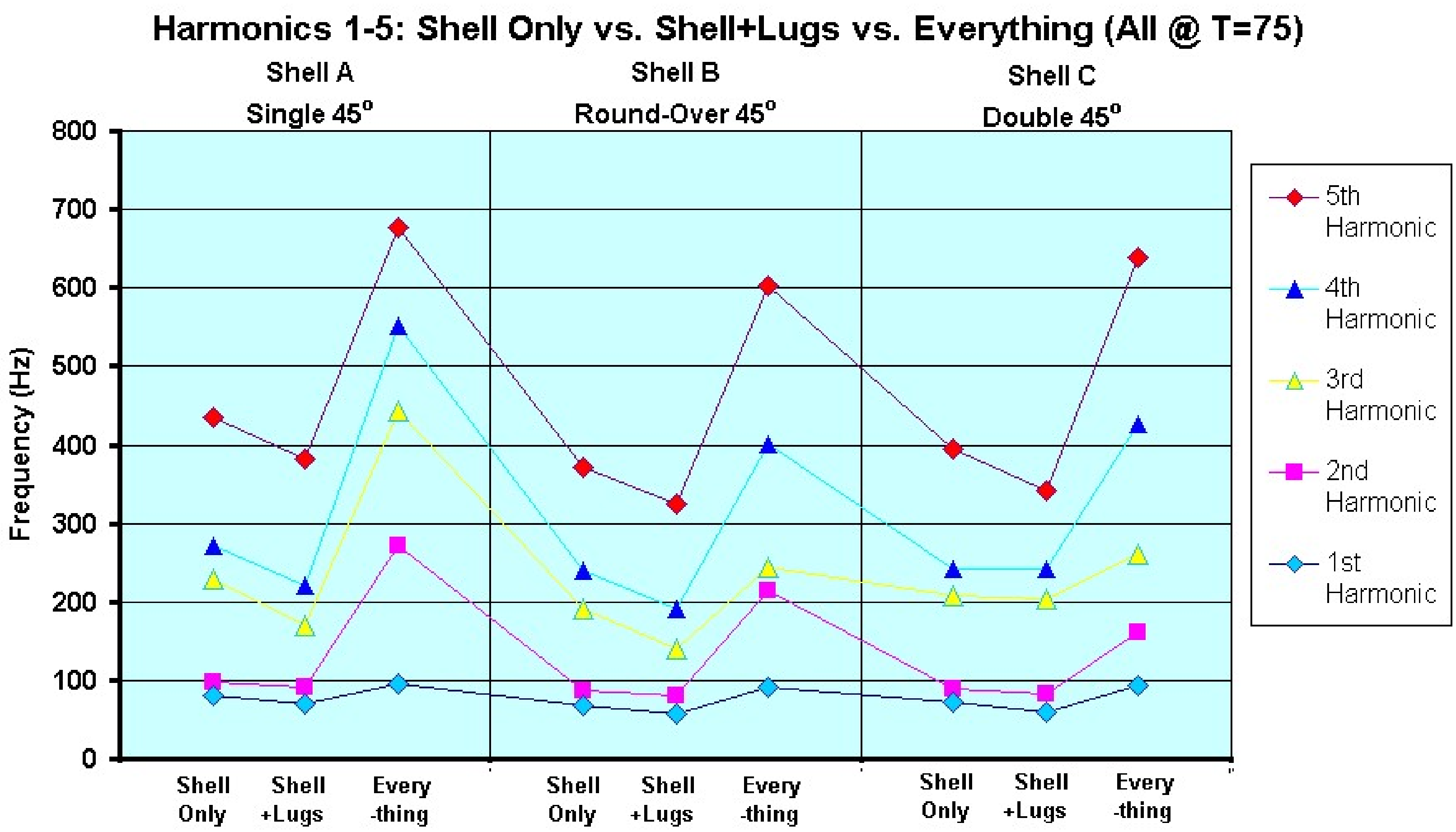


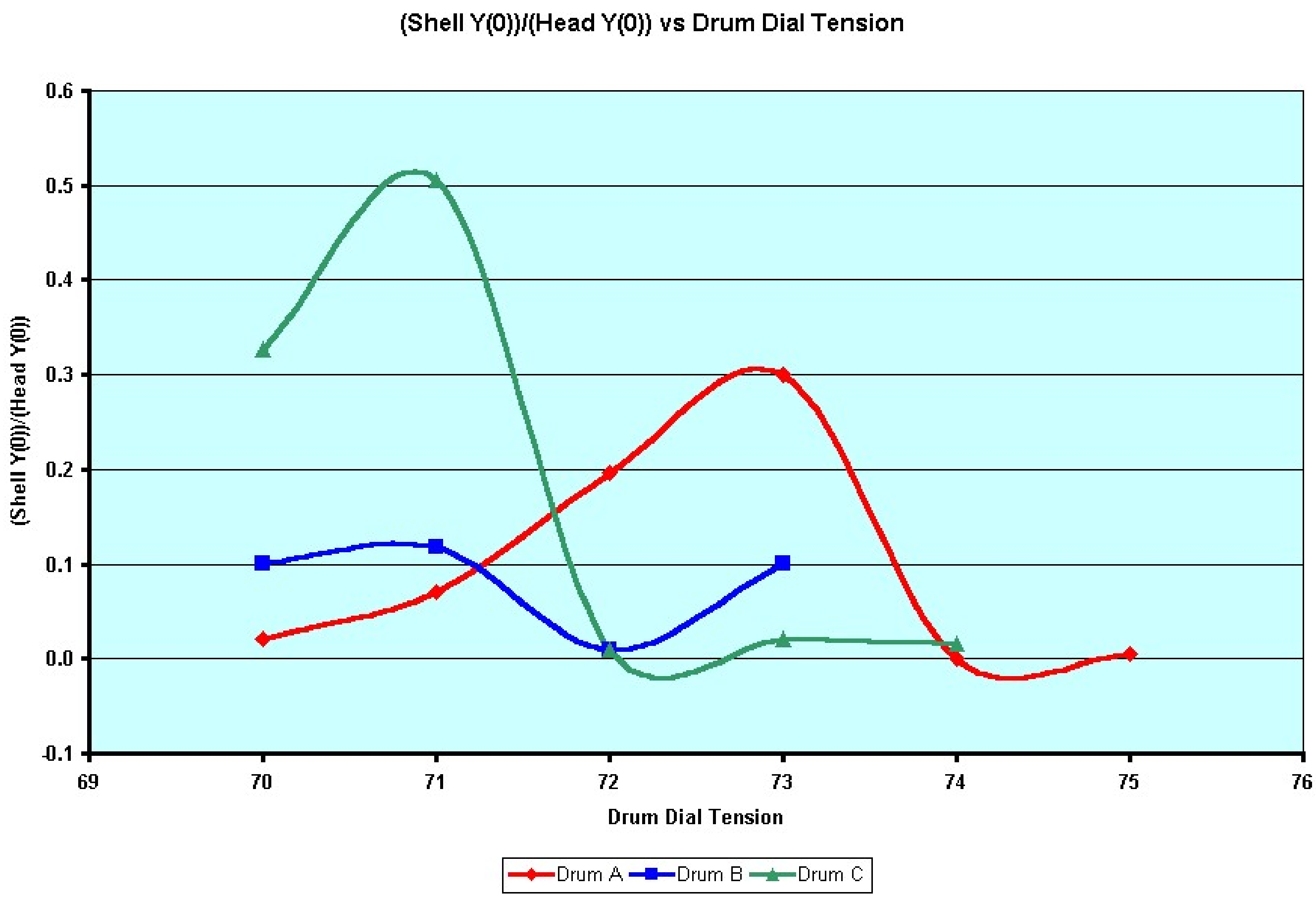
# Progression of Major Harmonics for Three 10” Tom Drums



**Ratio of Initial Amplitude(s) of Drum Shell/Drum Head vs. Drum Head Tension.**

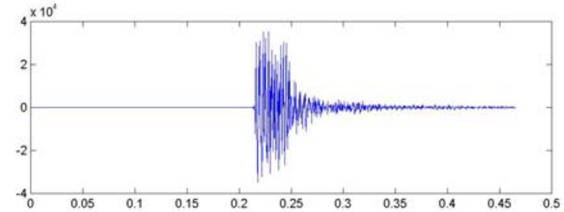
**Drum A = Single 45°, Drum B = Round-Over 45°, Drum C = Double-45°.**

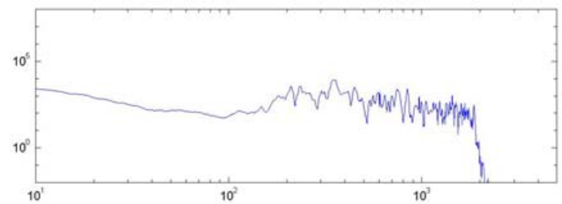
**At Resonance, the Double-45° 10" Tom Drum transferred more energy from drum head => drum shell. Qualitatively, it sounded best of the three.**

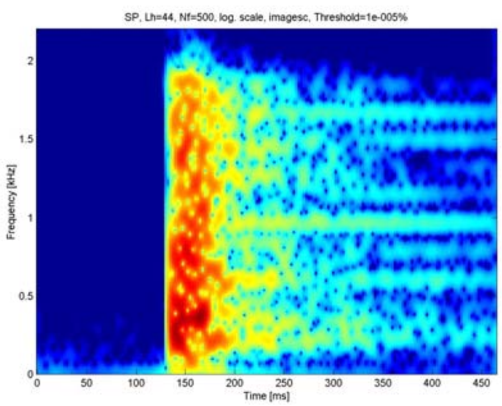


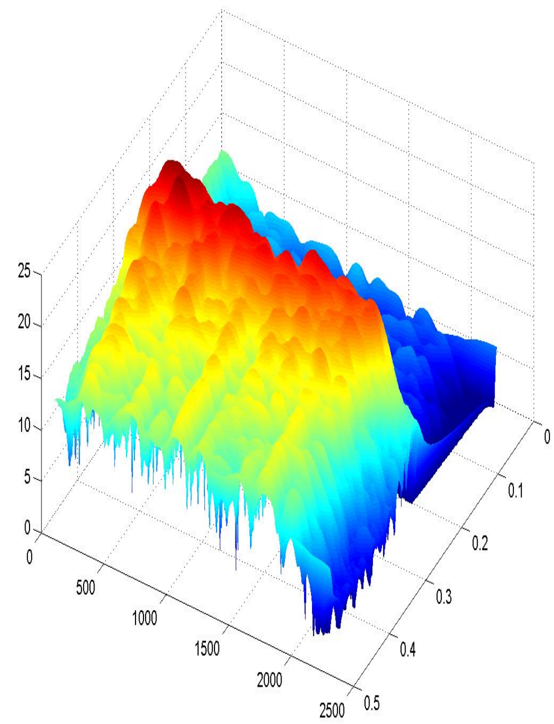
# Harmonic Content vs. Time of a Snare Drum

**Eric Macaulay (Illinois Wesleyan University), Lee Holloway, Mats Selen, SME (UIUC), Summer 2003**



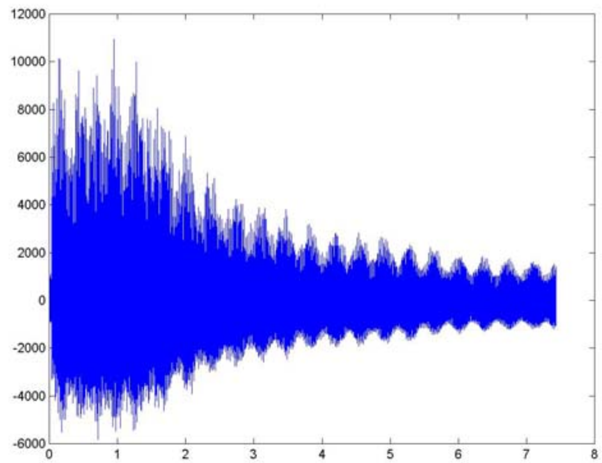


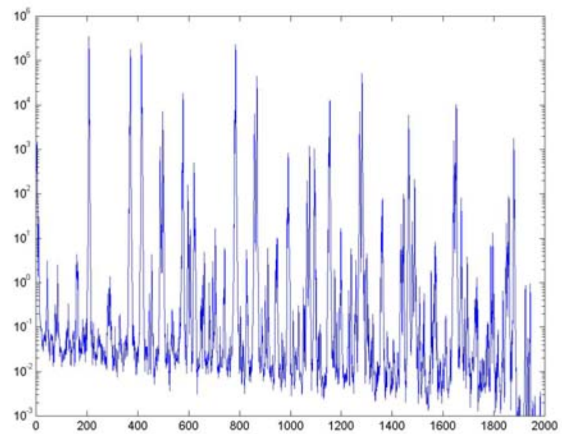


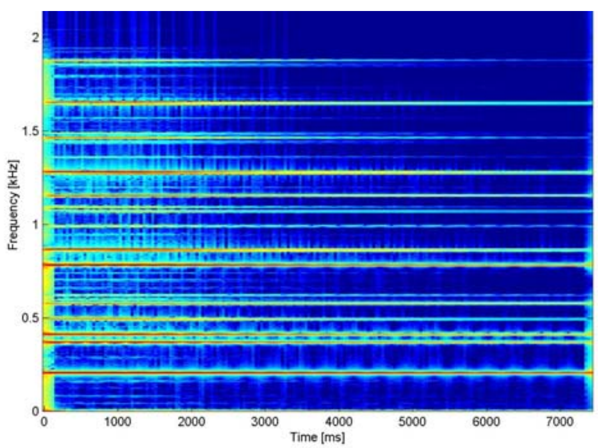



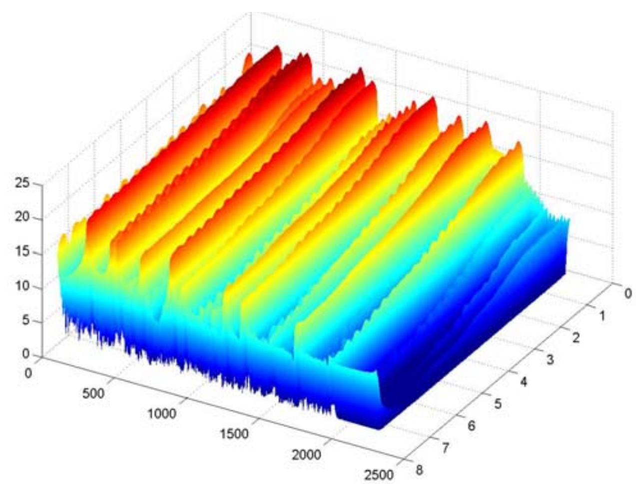
# Harmonic Content vs. Time of Tibetan Bowl

**Eric Macaulay (Illinois Wesleyan University), Lee Holloway, Mats Selen, SME (UIUC)**
**Summer, 2003**

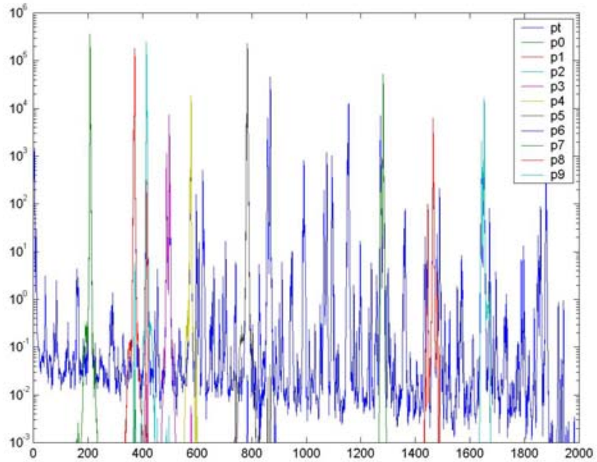


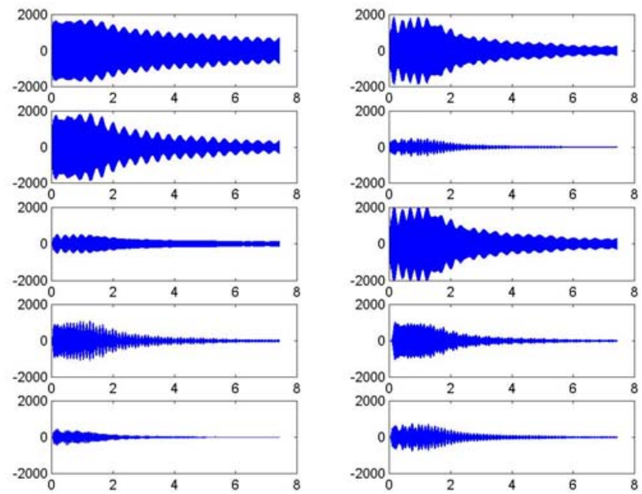



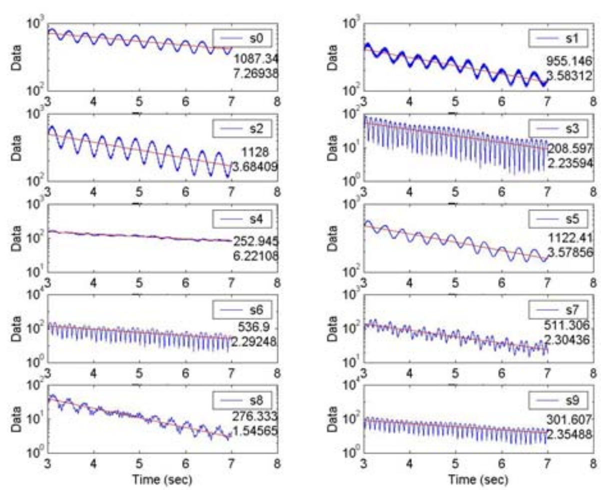



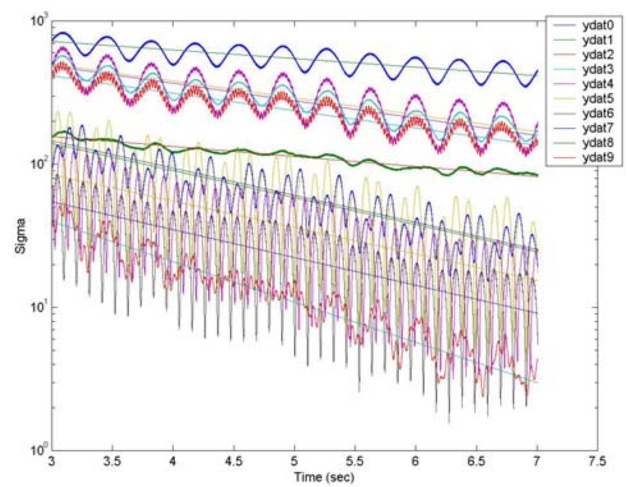



# Tibetan Bowl Studies – Continued:



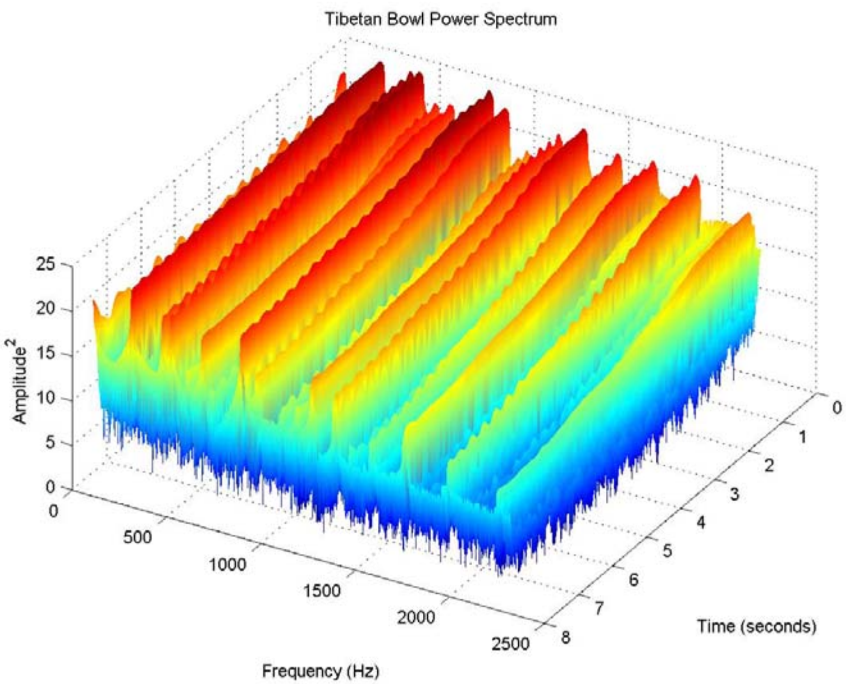



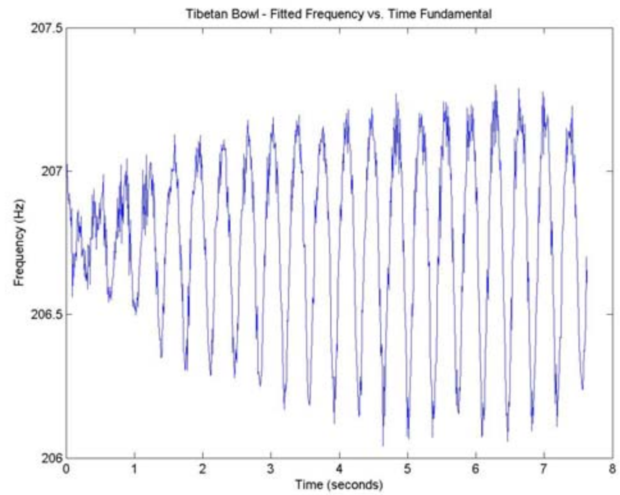


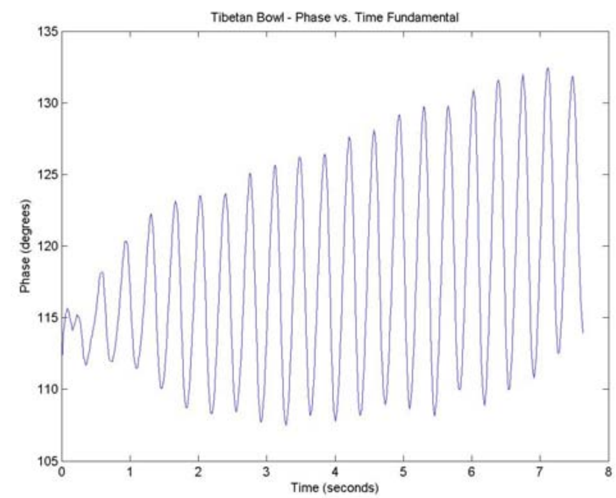



**<u>Tibetan Bowl Studies II:</u> Joseph Yasi (Rensselaer Polytechnic), SME (UIUC)**

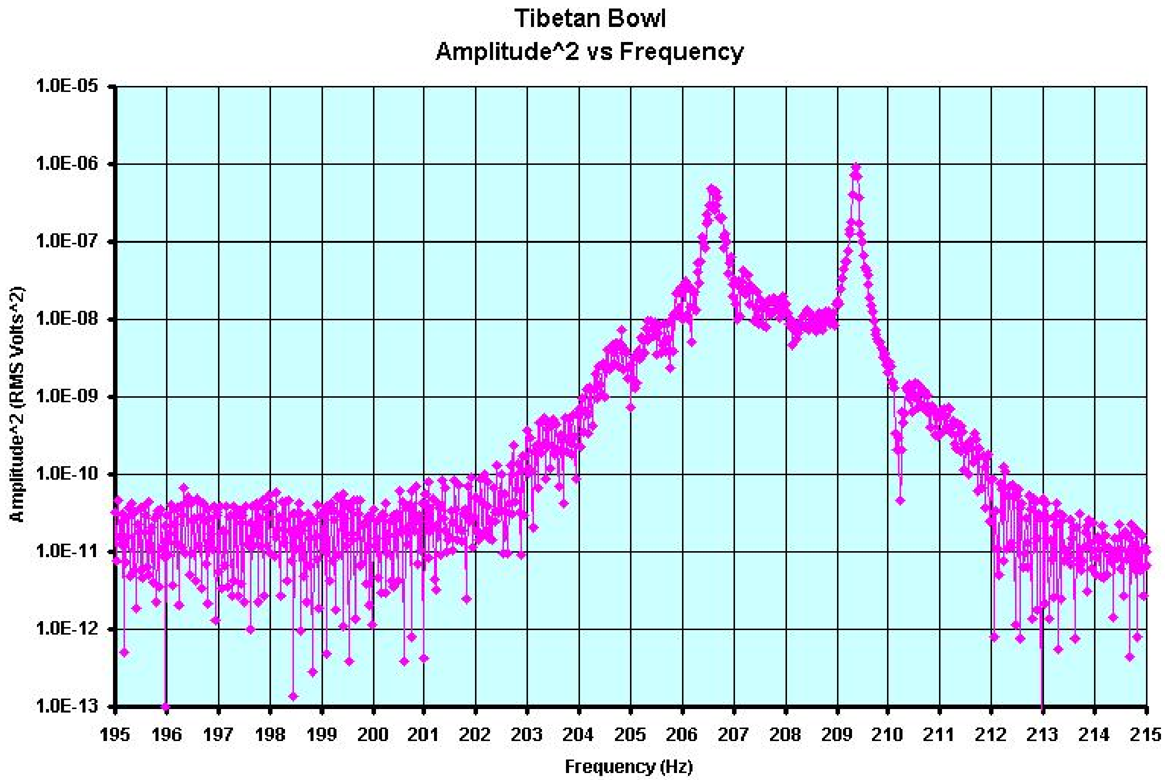
**Summer, 2004 => Preliminary Results <=**




**Frequency & phases of harmonics (even the fundamental) of Tibetan Bowl are not constants !!!**



Tibetan Bowl
Amplitude^2 vs Frequency
Amplitude^2 (RMS Volts^2)
1.0E-05
1.0E-06
1.0E-07
1.0E-08
1.0E-09
1.0E-10
1.0E-11
1.0E-12
1.0E-13
195 196 197 198 199 200 201 202 203 204 205 206 207 208 209 210 211 212 213 214 215
Frequency (Hz)

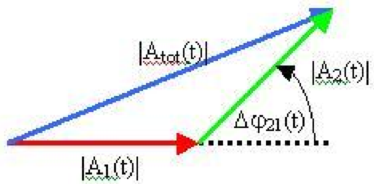
# Beats Phenomenon

Linearly superpose (i.e. add) two signals $A_1(t)$ and $A_2(t)$ with similar frequencies, $\omega_2 \sim \omega_1$.

$$A_1(t) = A_{10}Cos(\omega_1(t)t) \qquad A_2(t) = A_{20}Cos(\omega_2(t)t + \Delta\varphi_{21}(t))$$

$$A_{tot}(t) = A_1(t) + A_2(t) = A_{10}Cos(\omega_1(t)t) + A_{20}Cos(\omega_2(t)t + \Delta\varphi_{21}(t))$$

Phasor Relation:




Intensity:

$$I_{tot}(t) = |A_{tot}(t)|^2 = |A_1(t) + A_2(t)|^2$$

$$I_{tot}(t) = A_{10}^2 Cos^2(\omega_1(t)t) + A_{20}^2 Cos^2(\omega_2(t)t + \Delta\varphi_{21}(t)) + 2A_{10}A_{20}Cos(\omega_1(t)t)Cos(\omega_2(t)t + \Delta\varphi_{21}(t))$$

Define:

$$\vartheta_1(t) \equiv \omega_1(t)t \qquad \vartheta_2(t) \equiv (\omega_2(t)t + \Delta\varphi_{21}(t))$$

Identity:

$$Cos\,\vartheta_1 Cos\,\vartheta_2 \equiv \tfrac{1}{2}\left[Cos(\vartheta_2 + \vartheta_1) + Cos(\vartheta_2 - \vartheta_1)\right]$$

Thus:

$$I_{tot}(t) = A_{10}^2 Cos^2(\omega_1(t)t) + A_{20}^2 Cos^2(\omega_2(t)t + \Delta\varphi_{21}(t))$$
$$+ A_{10}A_{20}\left[Cos((\omega_2(t) + \omega_1(t))t + \Delta\varphi_{21}(t)) + Cos((\omega_2 - \omega_1)t - \Delta\varphi_{21}(t))\right]$$

Define:

$$\Omega_{21}(t) \equiv (\omega_2(t) + \omega_1(t)) \qquad \Delta\omega_{21}(t) \equiv |\omega_2(t) - \omega_1(t)|$$

Then:

$$\boxed{I_{tot}(t) = A_{10}^2 Cos^2(\omega_1(t)t) + A_{20}^2 Cos^2(\omega_2(t)t + \Delta\varphi_{21}(t)) + A_{10}A_{20}\left[Cos(\Omega_{21}(t)t + \Delta\varphi_{21}(t)) + Cos(\Delta\omega_{21}(t)t - \Delta\varphi_{21}(t))\right]}$$

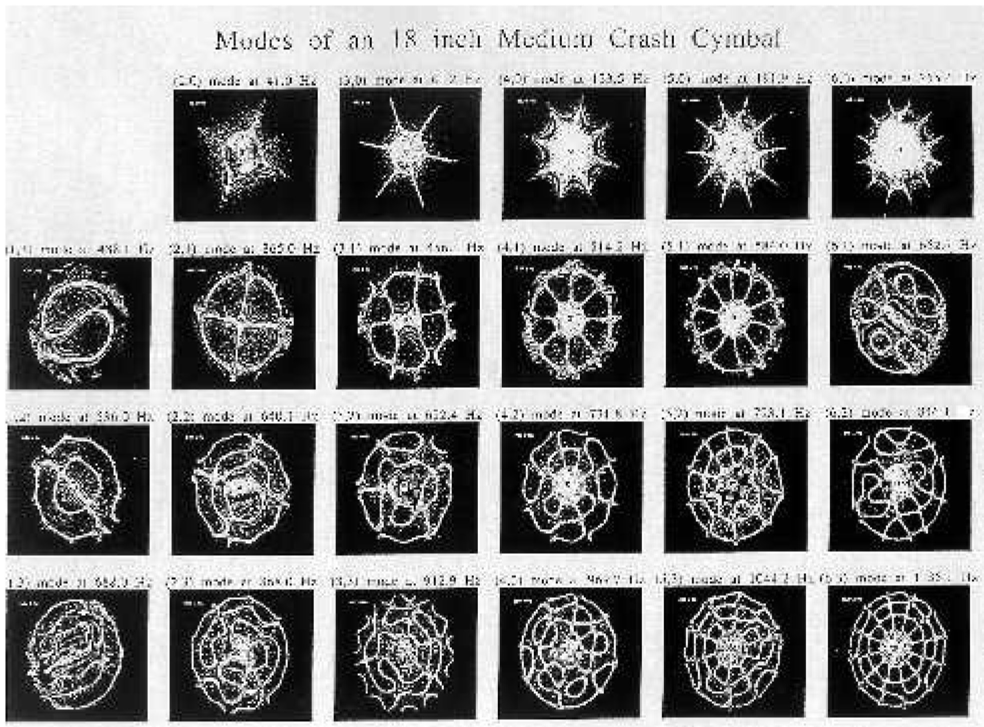
Identity:

$$Cos^2\vartheta = Cos\,\vartheta Cos\,\vartheta \equiv \tfrac{1}{2}\left[Cos0 + Cos2\vartheta\right] = \tfrac{1}{2}\left[1 + Cos2\vartheta\right]$$

Then:

$$I_{tot}(t) = \tfrac{1}{2}A_{10}^2\left[1 + Cos^2 2(\omega_1(t)t)\right] + \tfrac{1}{2}A_{20}^2\left[1 + Cos^2 2(\omega_2(t)t + \Delta\varphi_{21}(t))\right]$$
$$+ A_{10}A_{20}\left[Cos(\Omega_{21}(t)t + \Delta\varphi_{21}(t)) + Cos(\Delta\omega_{21}(t)t - \Delta\varphi_{21}(t))\right]$$

# Vibrational Modes of Cymbals:



# Vibrational Modes of Handbells:

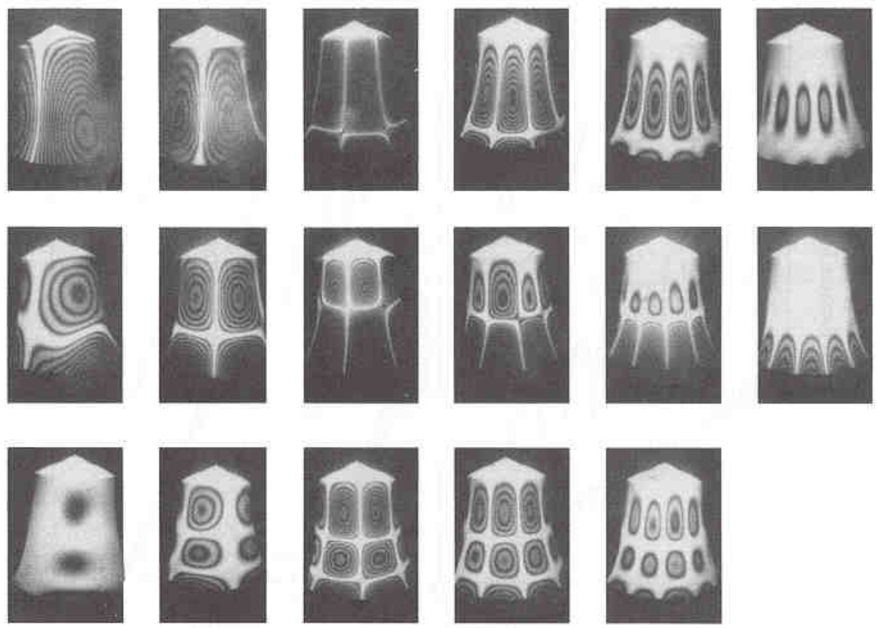
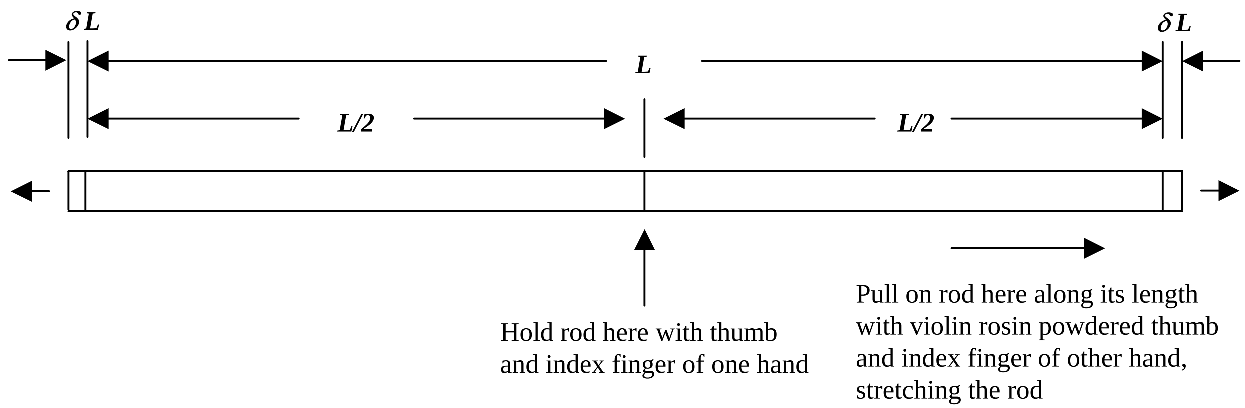
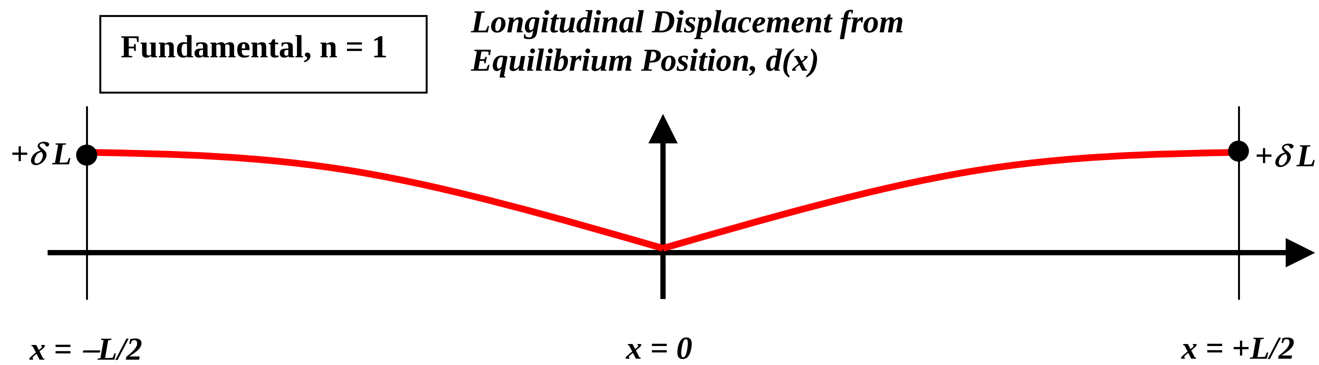


FIGURE 21.16. Time-average hologram interferograms of vibrational modes in a $C_5$ handbell (Rossing et al., 1984).

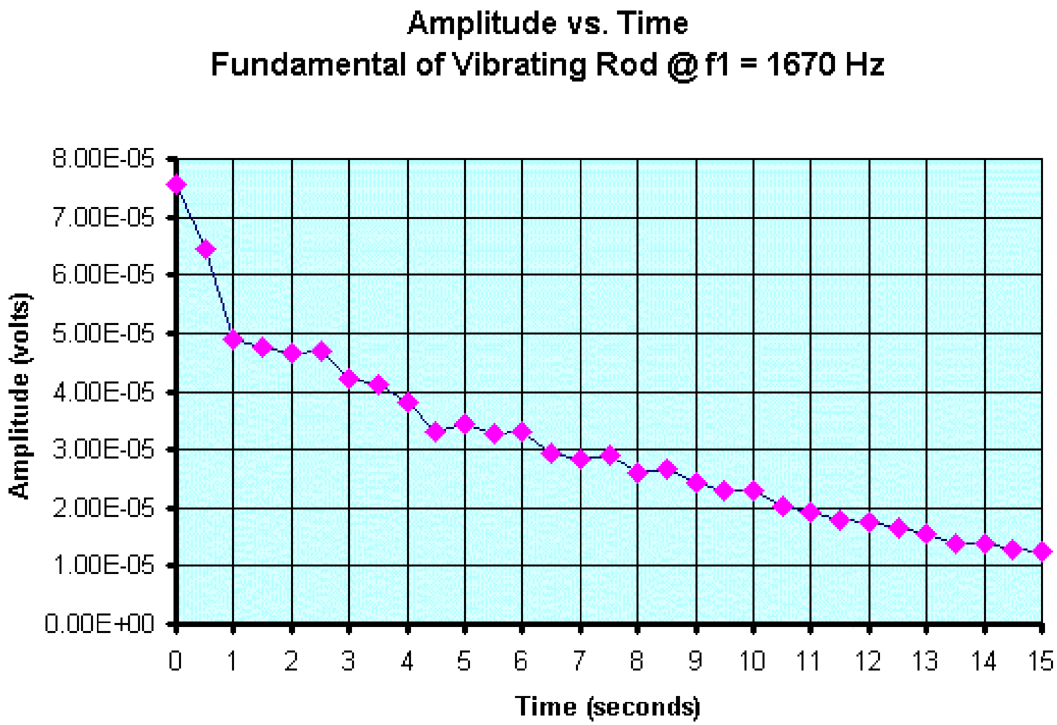
## Modal Vibrations of a "Singing" Rod:

A metal rod (e.g. aluminum rod) a few feet in length can be made to vibrate along its length – make it "sing" at a characteristic, resonance frequency by holding it precisely at its mid-point with thumb and index finger of one hand, and then pulling the rod along its length, toward one of its ends with the thumb and index finger of the other hand, which have been dusted with crushed violin rosin, so as to obtain a good grip on the rod as it is pulled.

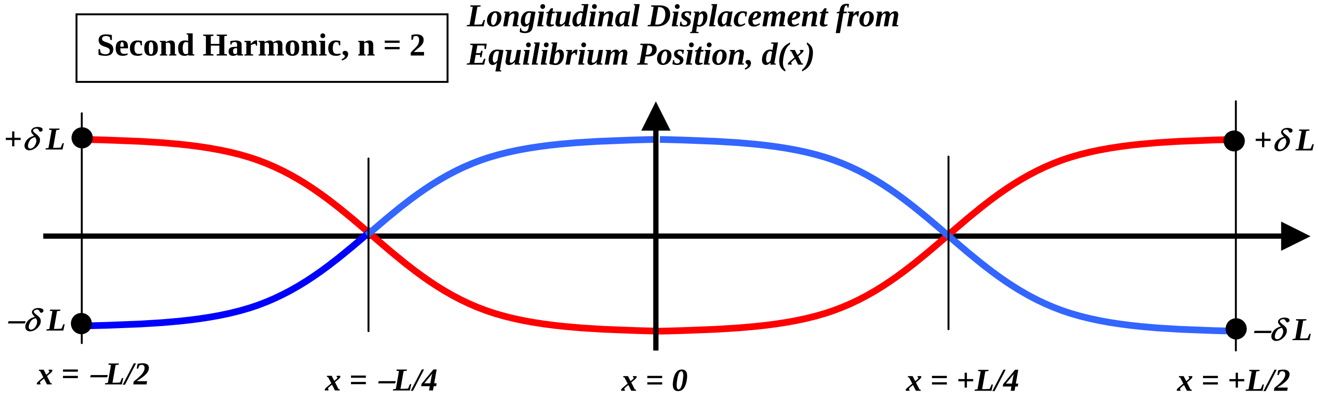


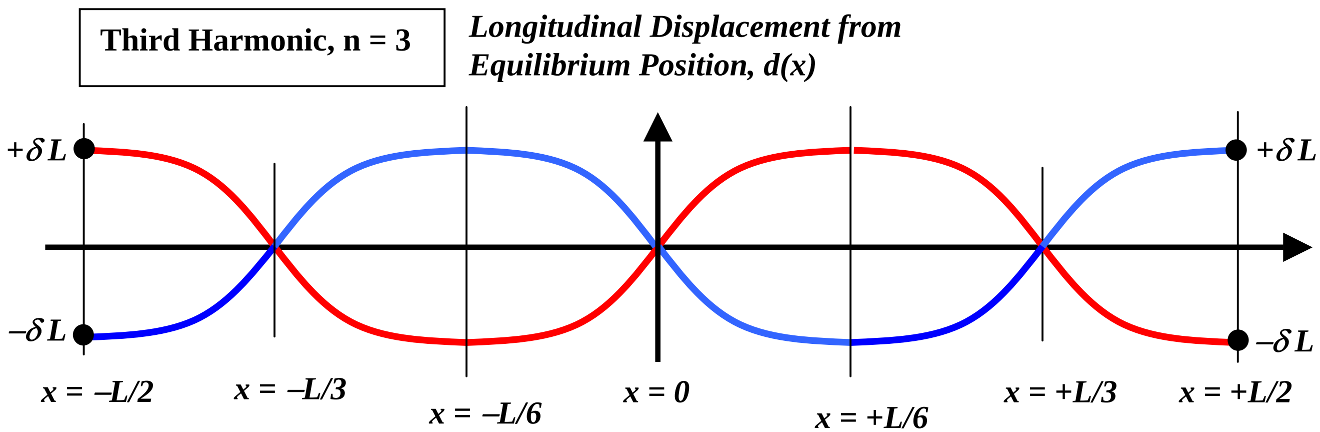



## Decay of Fundamental Mode of Singing Rod:



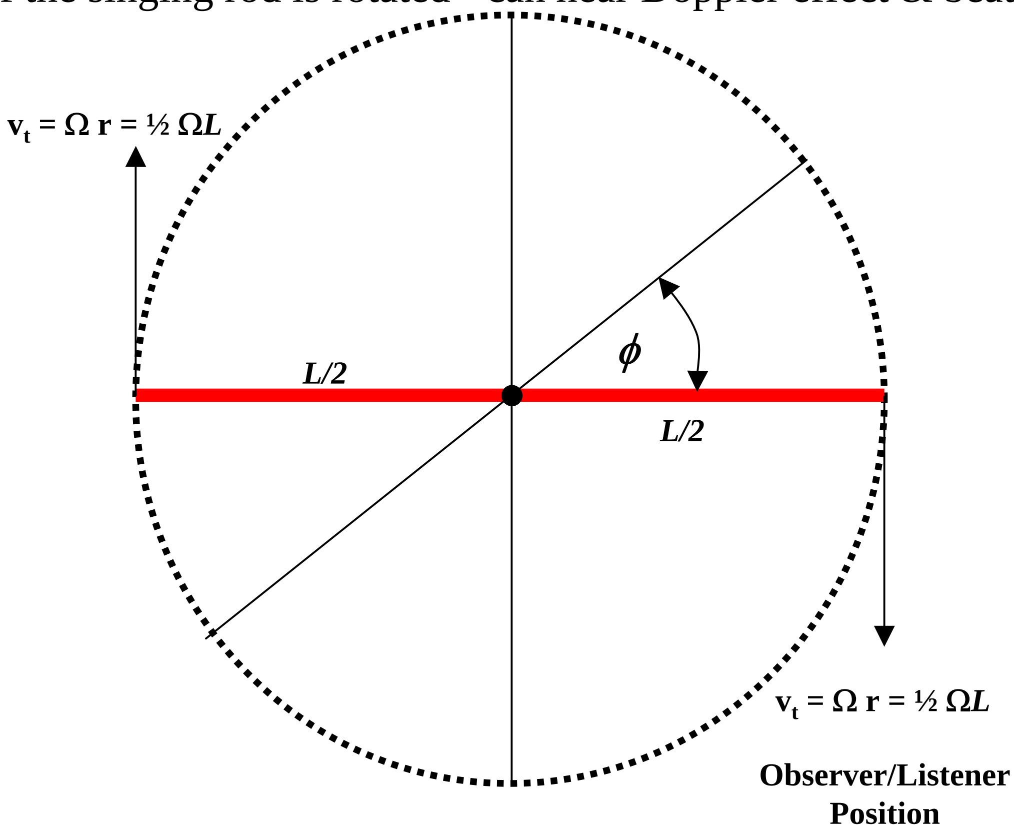
Of course, there also exist higher modes of vibration of the singing rod:



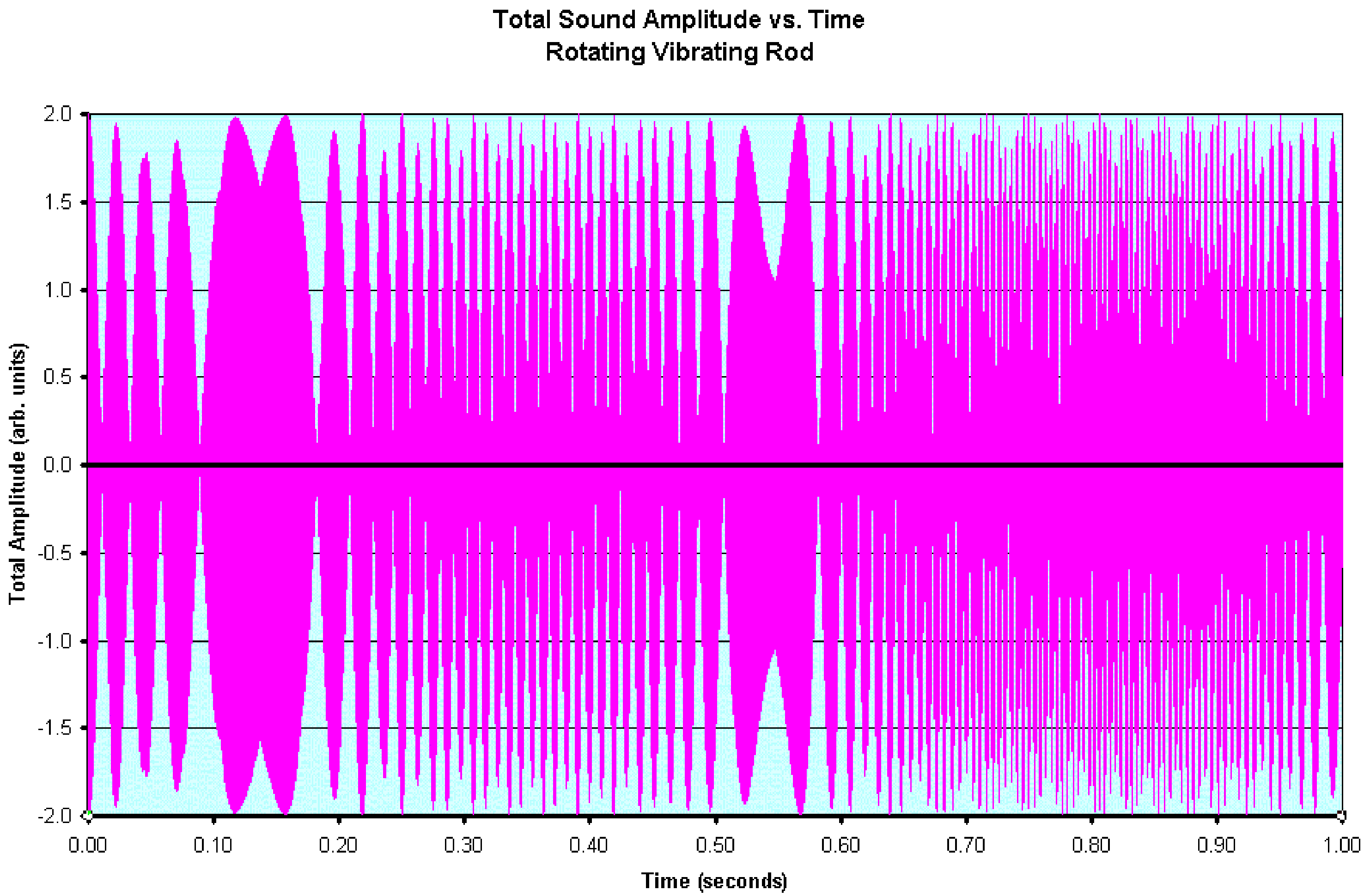

• See singing rod demo...

• If the singing rod is rotated - can hear Doppler effect & beats:



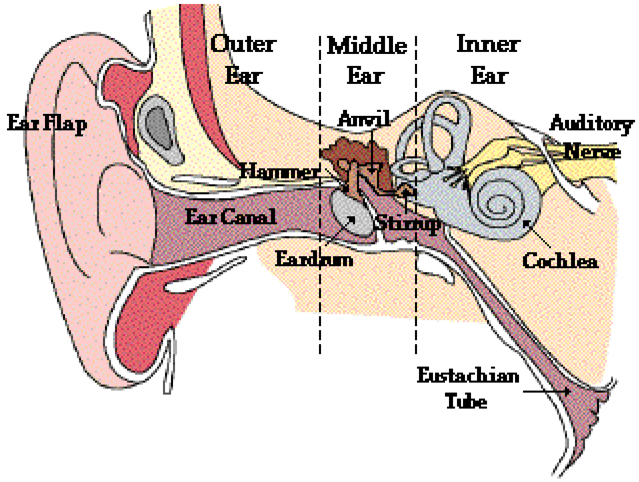

• Frequency of vibrations raised (lowered) if source moving toward (away from) listener, respectively

• Hear Doppler effect & beats of rotating "singing" rod...

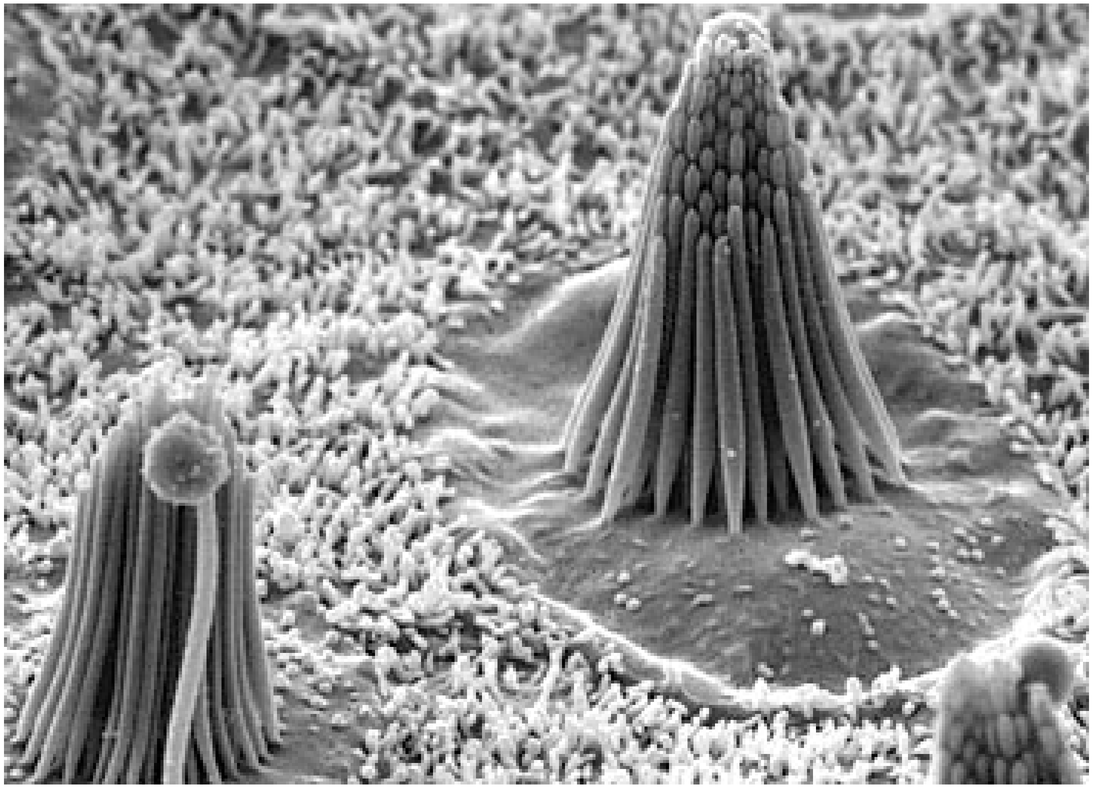

Total Sound Amplitude vs. Time
Rotating Vibrating Rod
Total Amplitude (arb. units)
2.0
1.5
1.0
0.5
0.0
-0.5
-1.0
-1.5
-2.0
0.00
0.10
0.20
0.30
0.40
0.50
0.60
0.70
0.80
0.90
1.00
Time (seconds)

• Would Mandi Patrick (UIUC Feature Twirler) be willing to lead the UI Singing Rod Marching Band at a half-time show ???

# How Do Our Ears Work?




- Sound waves are focussed into the ear canal via the ear flap (aka pinna), and impinge on the ear drum. Folds in pinna for enhancing determination of *location* of sound source!
- Ossicles in middle ear - hammer/anvil/stirrup - transfer vibrations to oval window - membrane on cochlea, in the inner ear.
- Cochlea is filled with perilymph fluid, which transfers sound vibrations into Cochlea.
- Cochlea contains basilar membrane which holds ~ 30,000 hair cells in Organ of Corti.
- Sensitive hairs respond to the sound vibrations – preserve both amplitude and phase information – send signals to brain via auditory nerve.
- Brain processes audio signals from *both* ears - you *hear* the “sound”
- Human hearing response is ~ logarithmic in sound intensity/loudness.



**Scanning Electon Micrograph of Clusters of (Bullfrog) Hair Cells**

## Our Hearing – Pitch-Wise is not Perfectly Linear, Either:

### Deviation of Tuning from Tempered Scale Prediction

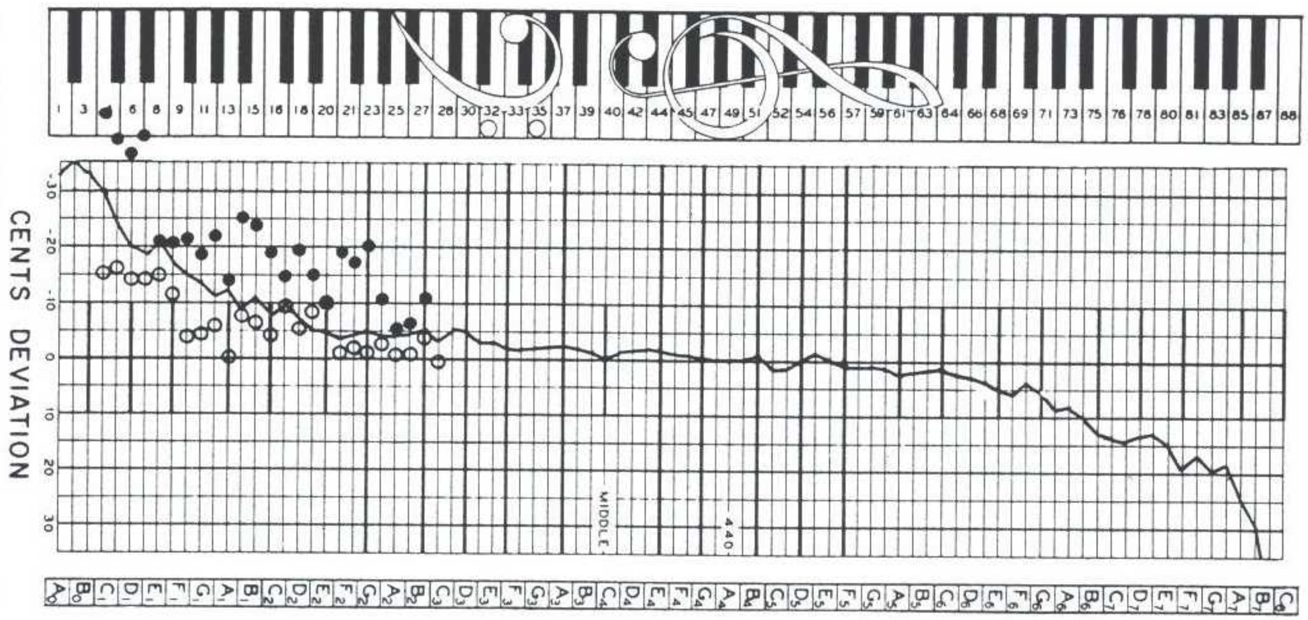



Fig. 3. Stretched tuning on the piano; average of sixteen pianos.

A perfectly tuned piano (tempered scale)
would sound *flat* in the upper register
and sound *sharp* in the lower register

# Consonance & Dissonance

Ancient Greeks - Aristotle and his followers - discovered using a *Monochord* that certain combinations of sounds with *rational number* (*n/m*) frequency ratios were pleasing to the human ear, for example (in *Just Diatonic Scale*):

• **Unison** - 2 simple-tone sounds of *same* frequency, i.e. $f_2 = (1/1)\ f_1 = f_1$ (= e.g. 300 Hz)

• **Minor Third** - 2 simple-tone sounds with $f_2 = (6/5)\ f_1 = 1.20\ f_1$ (= e.g. 360 Hz)

• **Major Third** - 2 simple-tone sounds with $f_2 = (5/4)\ f_1 = 1.25\ f_1$ (= e.g. 375 Hz)

• **Fourth** - 2 simple-tone sounds with $f_2 = (4/3)\ f_1 = 1.333\ f_1$ (= e.g. 400 Hz)

• **Fifth** - 2 simple-tone sounds with $f_2 = (3/2)\ f_1 = 1.50\ f_1$ (= e.g. 450 Hz)

• **Octave** - one sound is $2^{nd}$ harmonic of the first - i.e. $f_2 = (2/1)\ f_1 = 2\ f_1$ (= e.g. 600 Hz)

• Also investigated/studied by Galileo Galilei, mathematicians Leibnitz, Euler, physicist Helmholtz, and many others - debate/study is still going on today...

• These 2 simple-tone sound combinations are indeed very special!

• The resulting, overall waveform(s) are *time-independent* – they create *standing* waves on basilar membrane in cochlea of our inner ears!!!

• The human brain's signal processing for these special 2 simple-tone sound consonant combinations is especially easy!!!

**Example:**

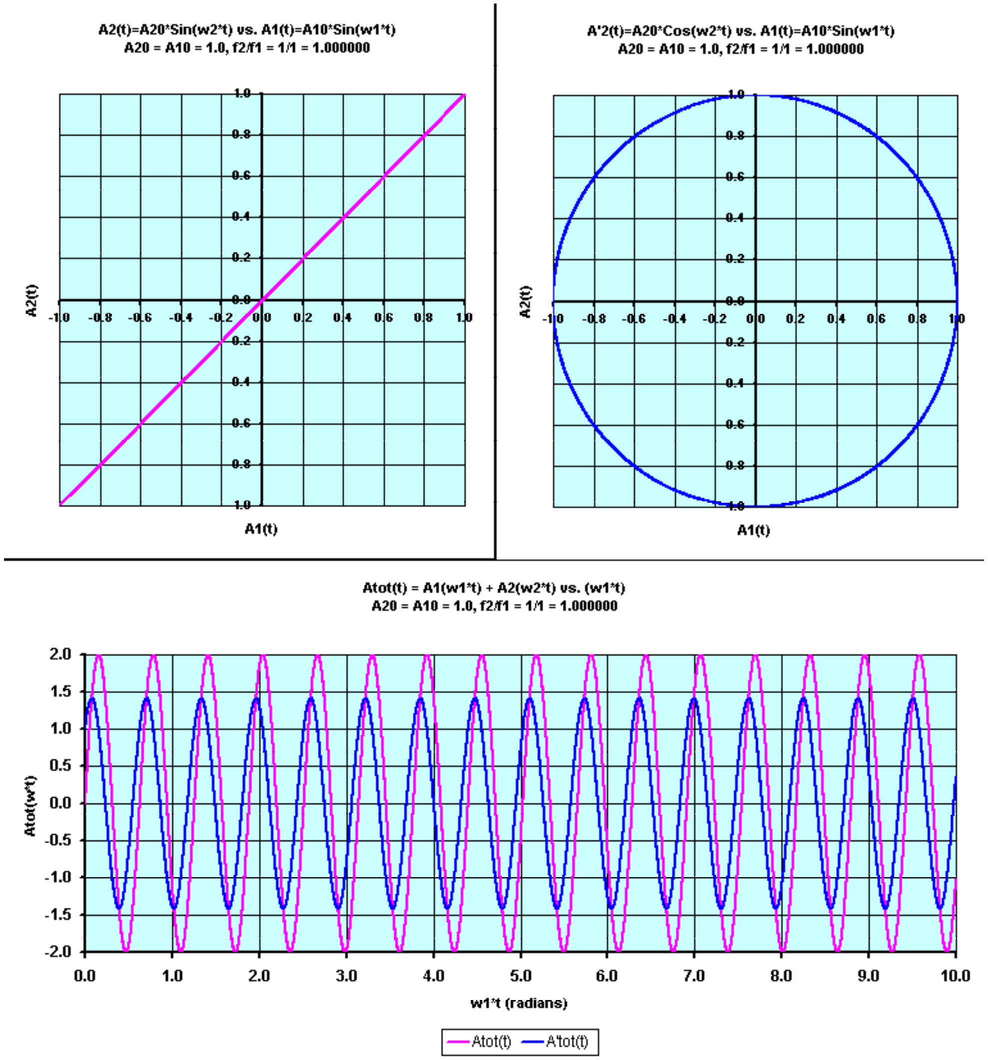
**Consonance of Unison**

Two simple-tone signals with:

$f_2 = (1/1)\ f_1 = 1\ f_1$

(e.g. $f_1$ = 300 Hz

and $f_2$ = 300 Hz)



**Example:**

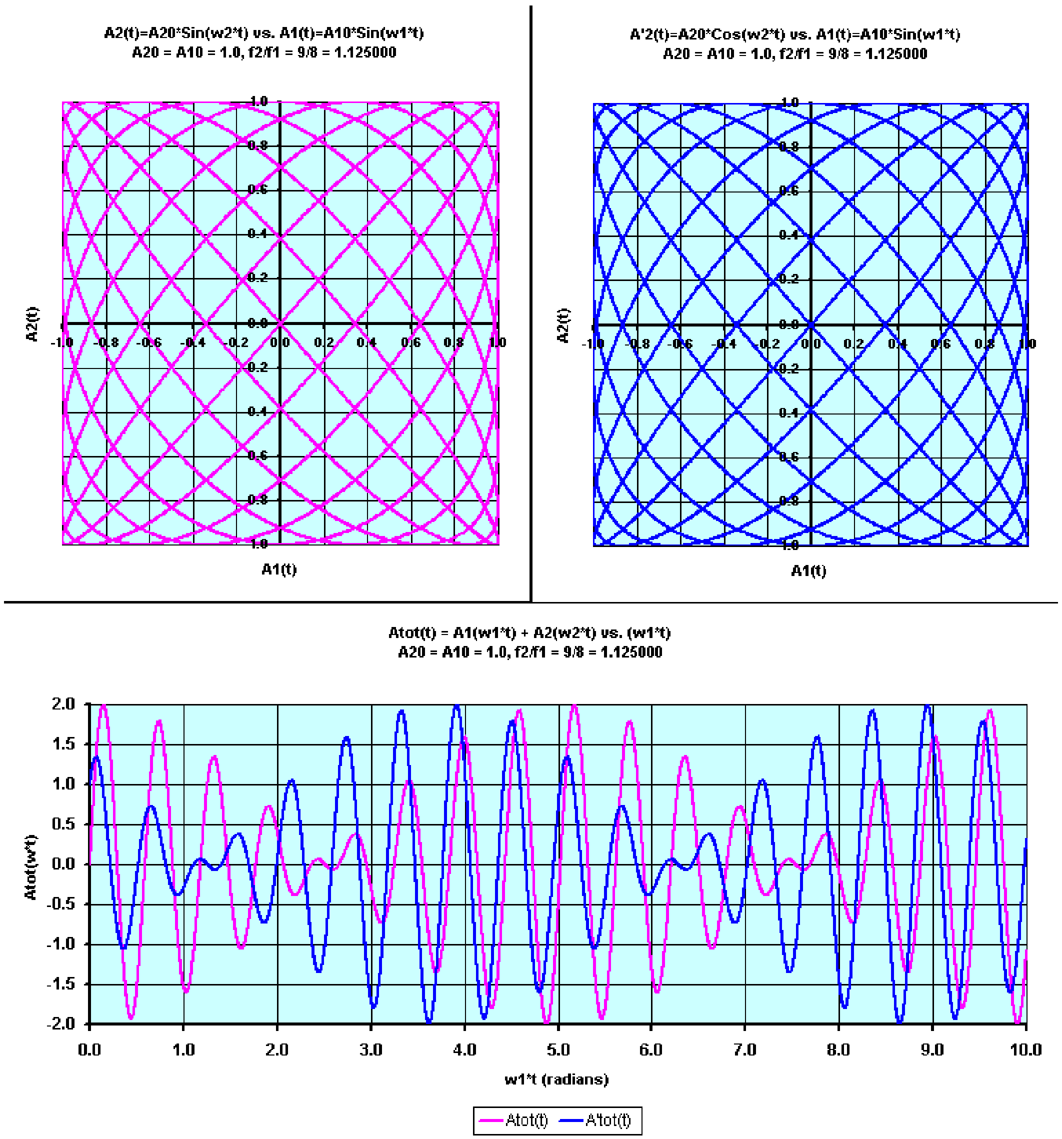
**Consonance of Second**

Two simple-tone signals with:

$f_2 = (9/8)\, f_1 = 1.125\, f_1$

(e.g. $f_1$ = 300 Hz

and $f_2$ = 337.5 Hz)



**Example:**

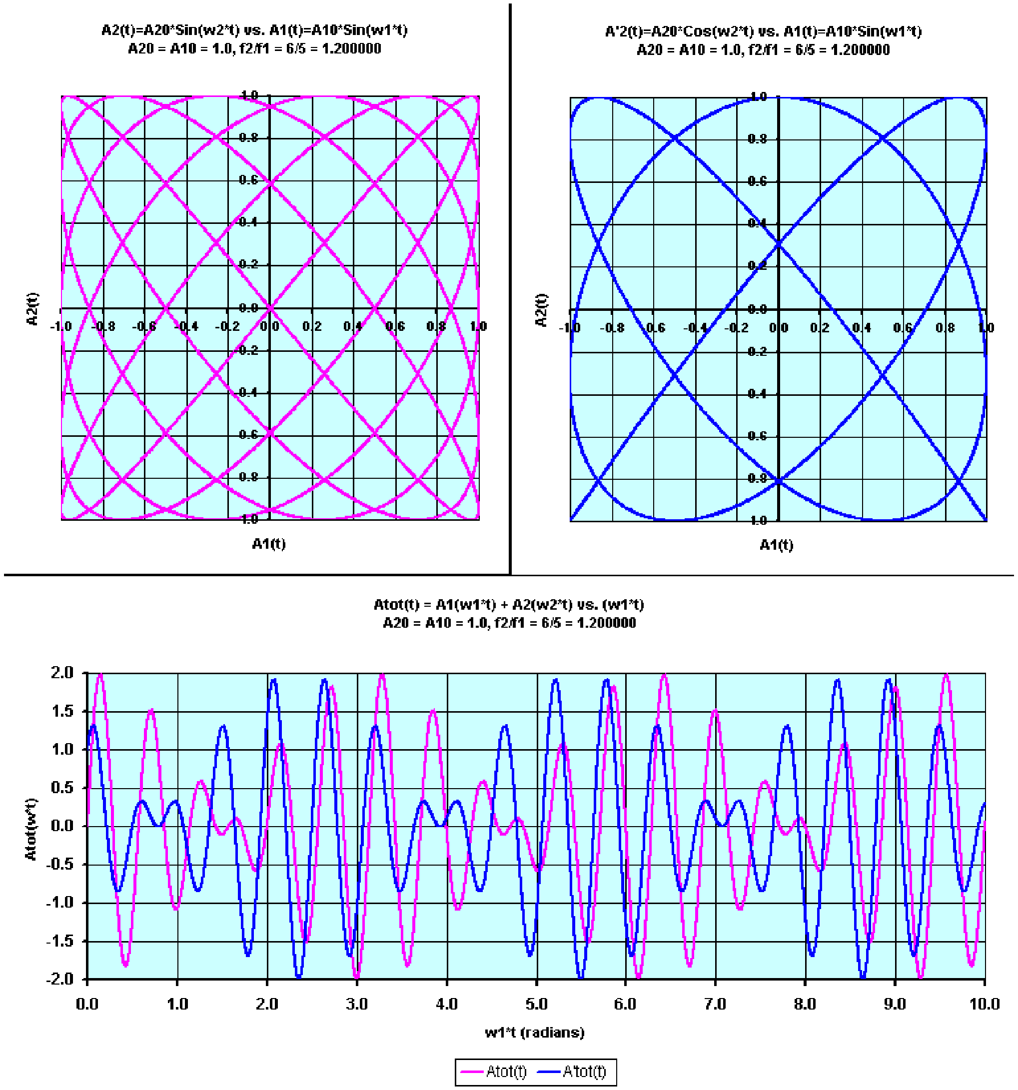
**Consonance of Minor 3rd**

Two simple-tone signals with:

$f_2 = (6/5)\, f_1 = 1.20\, f_1$

(e.g. $f_1$ = 300 Hz

and $f_2$ = 360 Hz)



**Example:**

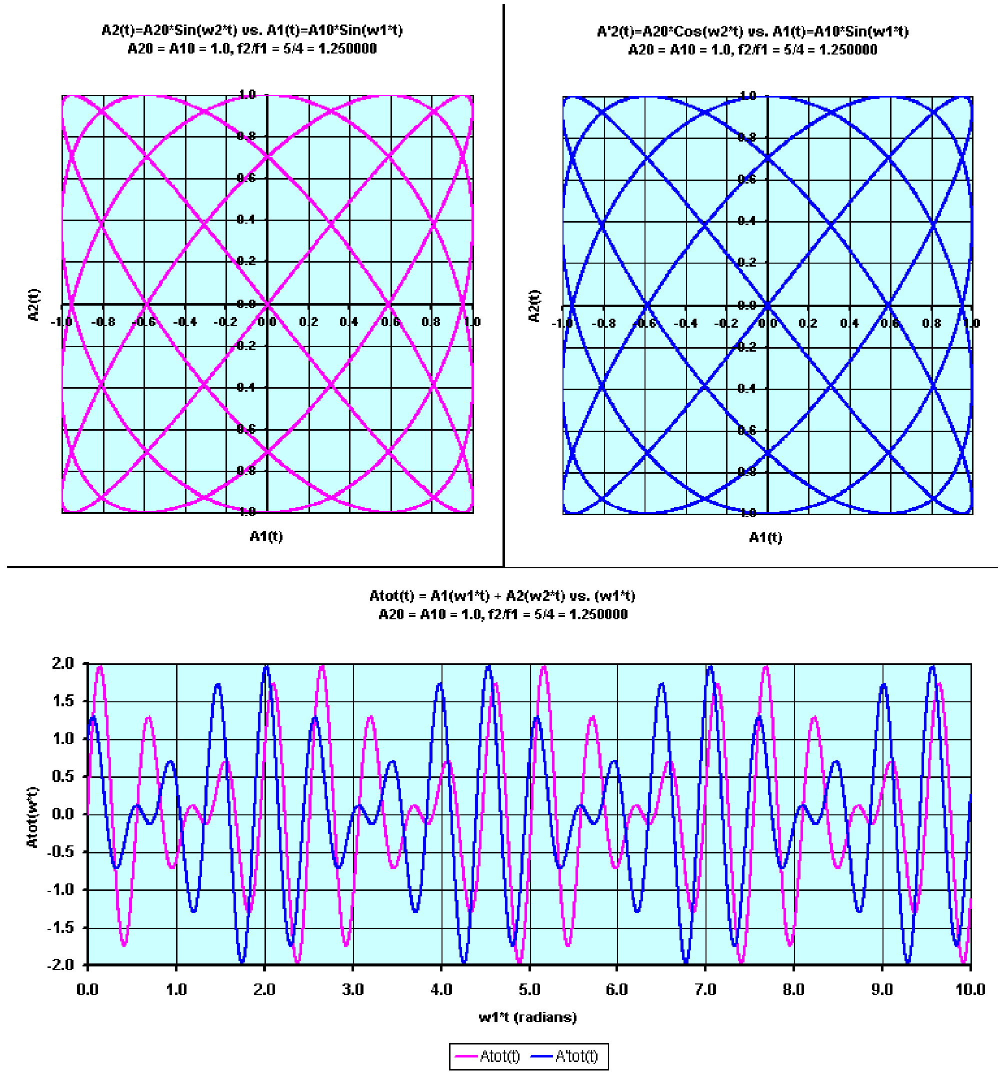
**Consonance of Major 3rd**

Two simple-tone signals with:

$f_2 = (5/4)\, f_1 = 1.25\, f_1$

(e.g. $f_1$ = 300 Hz

and $f_2$ = 375 Hz)



**Example:**

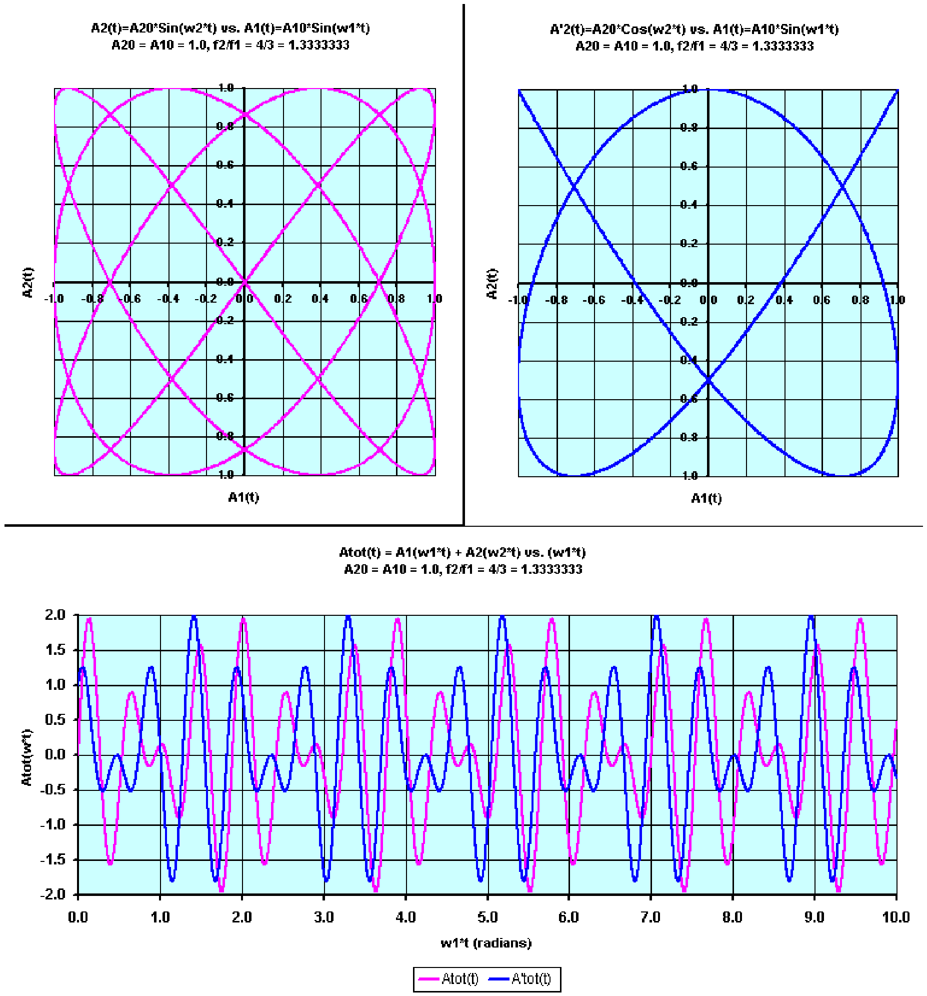
**Consonance of Fourth**

Two simple-tone signals with:

$f_2 = (4/3)\, f_1 = 1.333\, f_1$

(e.g. $f_1$ = 300 Hz

and $f_2$ = 400 Hz)



**Example:**

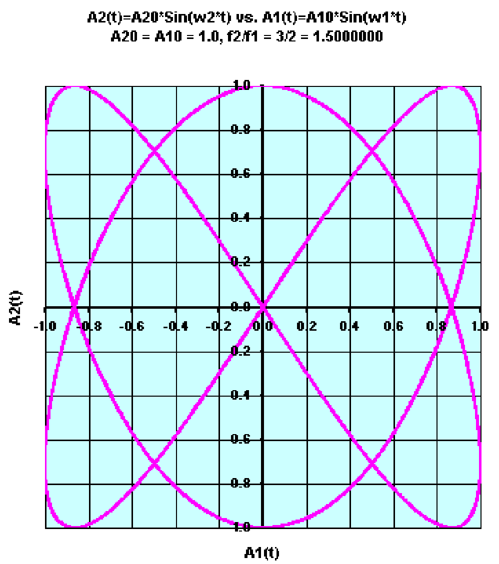
**Consonance of Fifth**

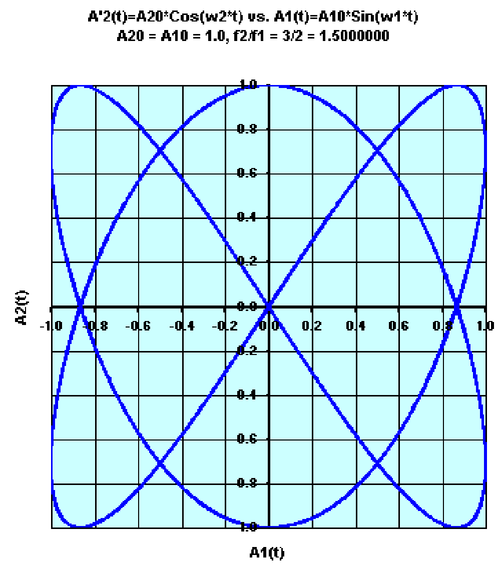
Two simple-tone signals with:

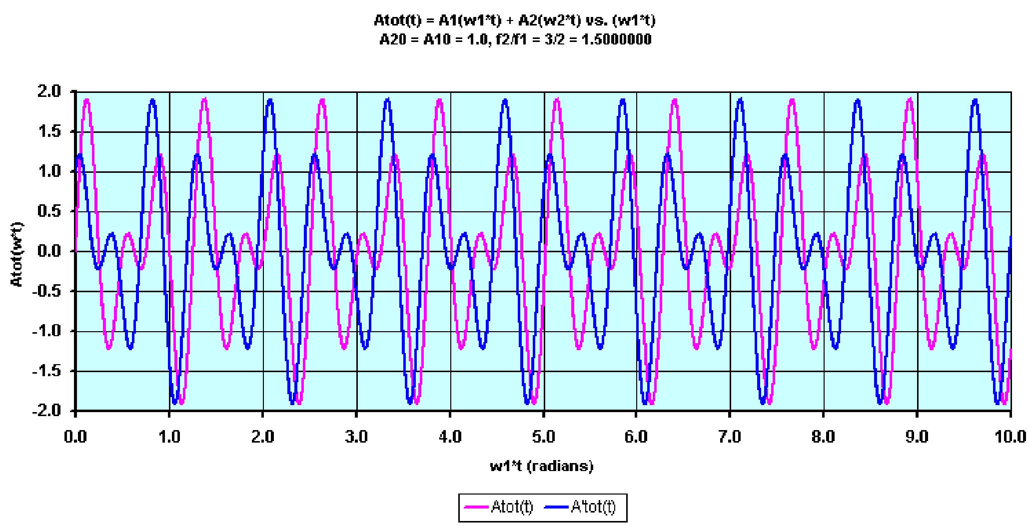
$f_2 = (3/2)\, f_1 = 1.5\, f_1$

(e.g. $f_1 = 300$ Hz

and $f_2 = 450$ Hz)



**Example:**

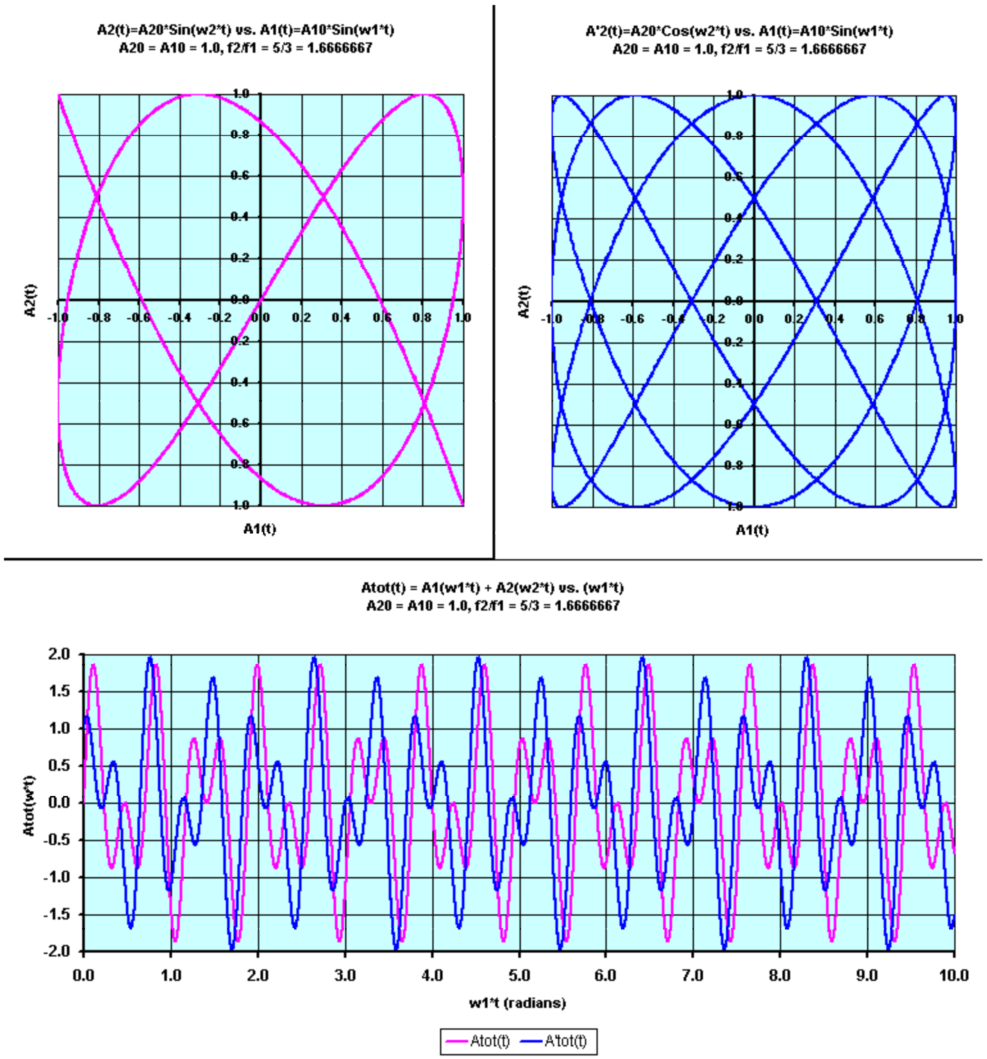
**Consonance of Sixth**

Two simple-tone signals with:

$f_2 = (5/3)\ f_1 = 1.666\ f_1$

(e.g. $f_1 = 300$ Hz

and $f_2 = 500$ Hz)



**Example:**

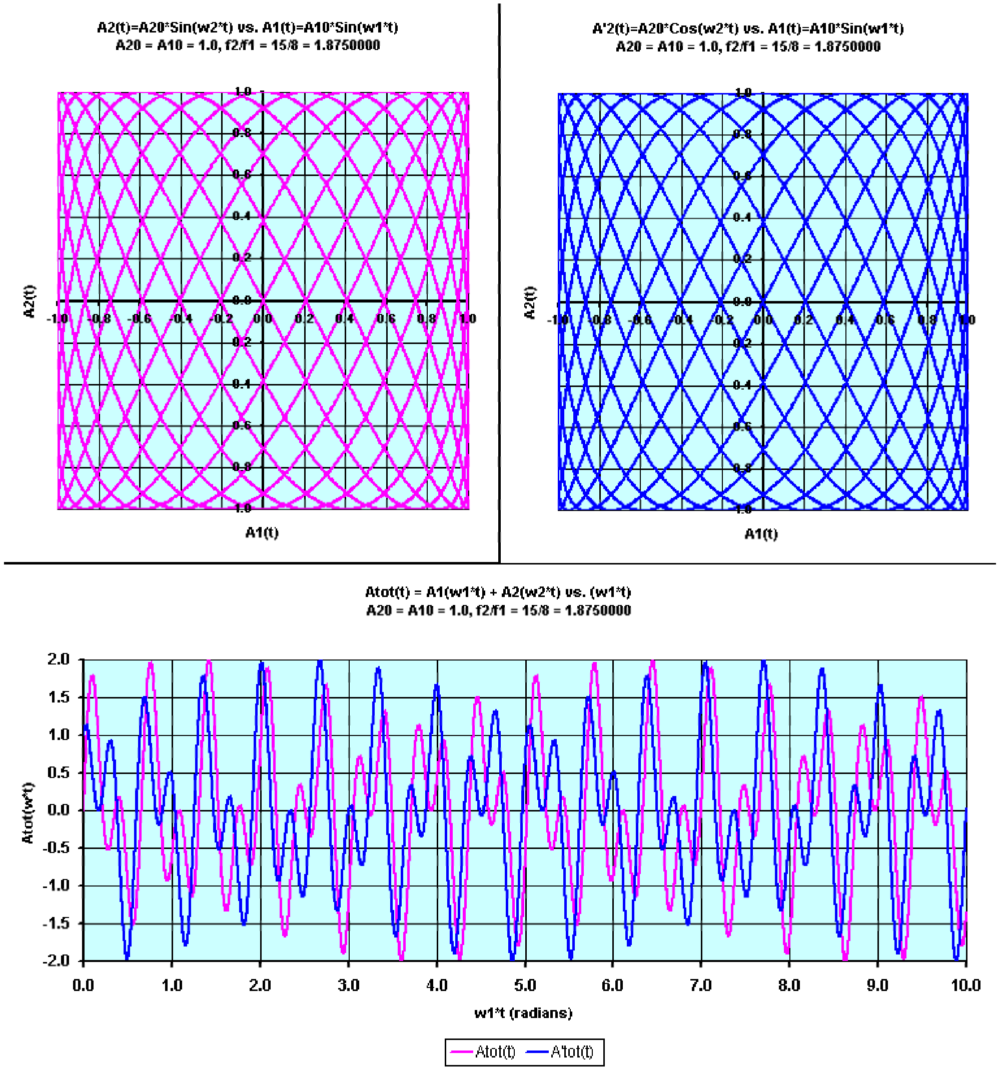
**Consonance of Seventh**

Two simple-tone signals with:

$f_2 = (15/8)\, f_1 = 1.875\, f_1$

(e.g. $f_1$ = 300 Hz

and $f_2$ = 562.5 Hz)



**Example:**

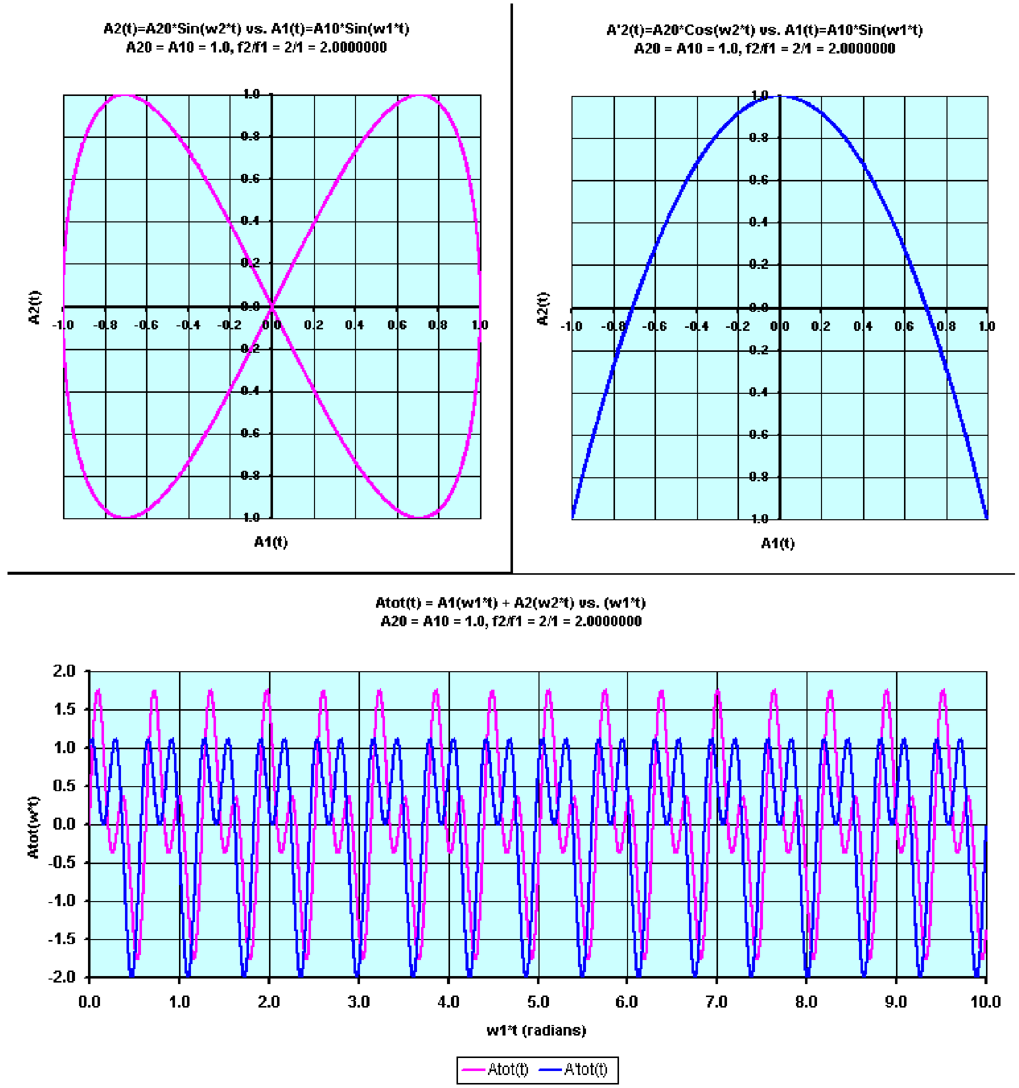
**Consonance of Octave**

Two simple-tone signals with:

$f_2 = (2/1)\, f_1 = 2\, f_1$

(e.g. $f_1 = 300$ Hz

and $f_2 = 600$ Hz)



**Example:**

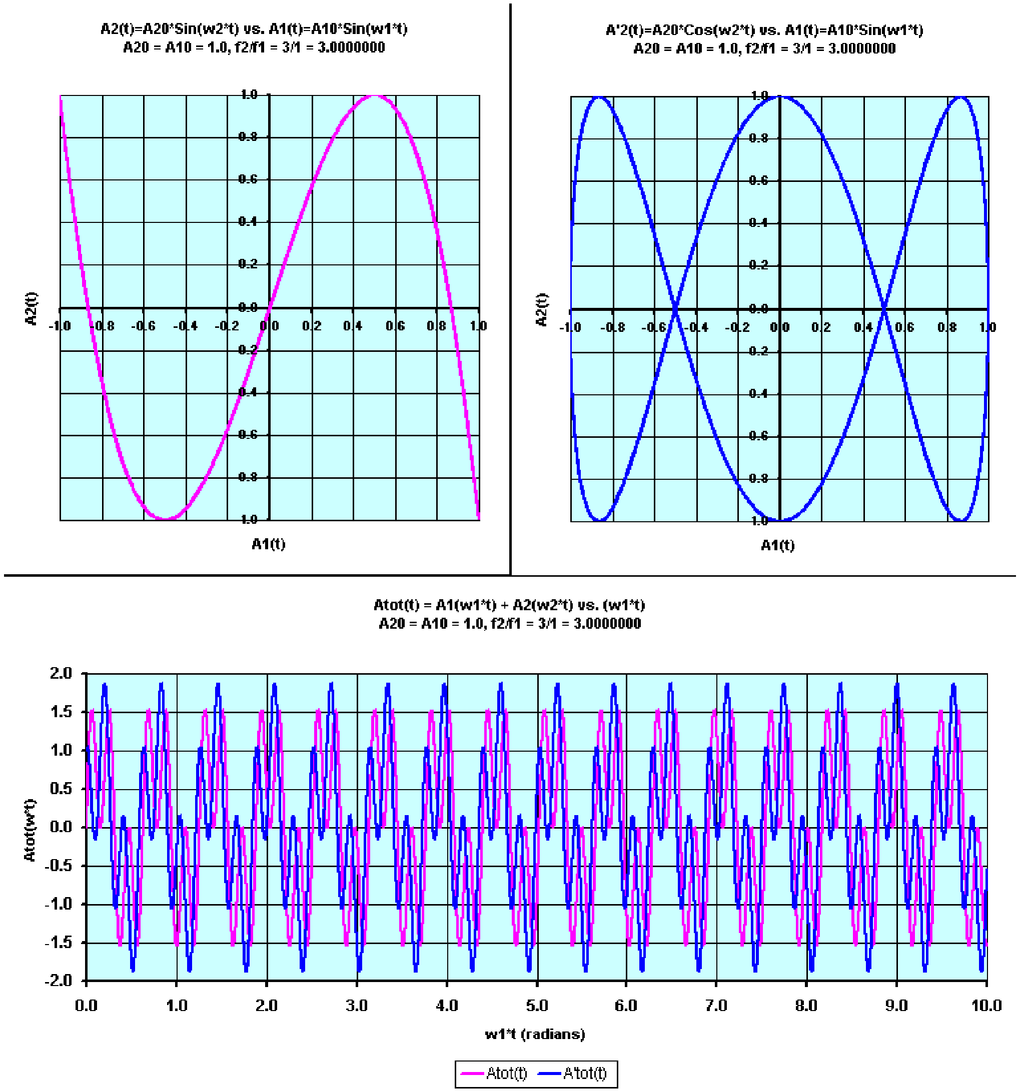
**Consonance of 1st & 3rd Harmonics**

Two simple-tone signals with:

$f_2 = (3/1)\, f_1 = 3\, f_1$

(e.g. $f_1$ = 300 Hz

and $f_2$ = 900 Hz)



**Example:**

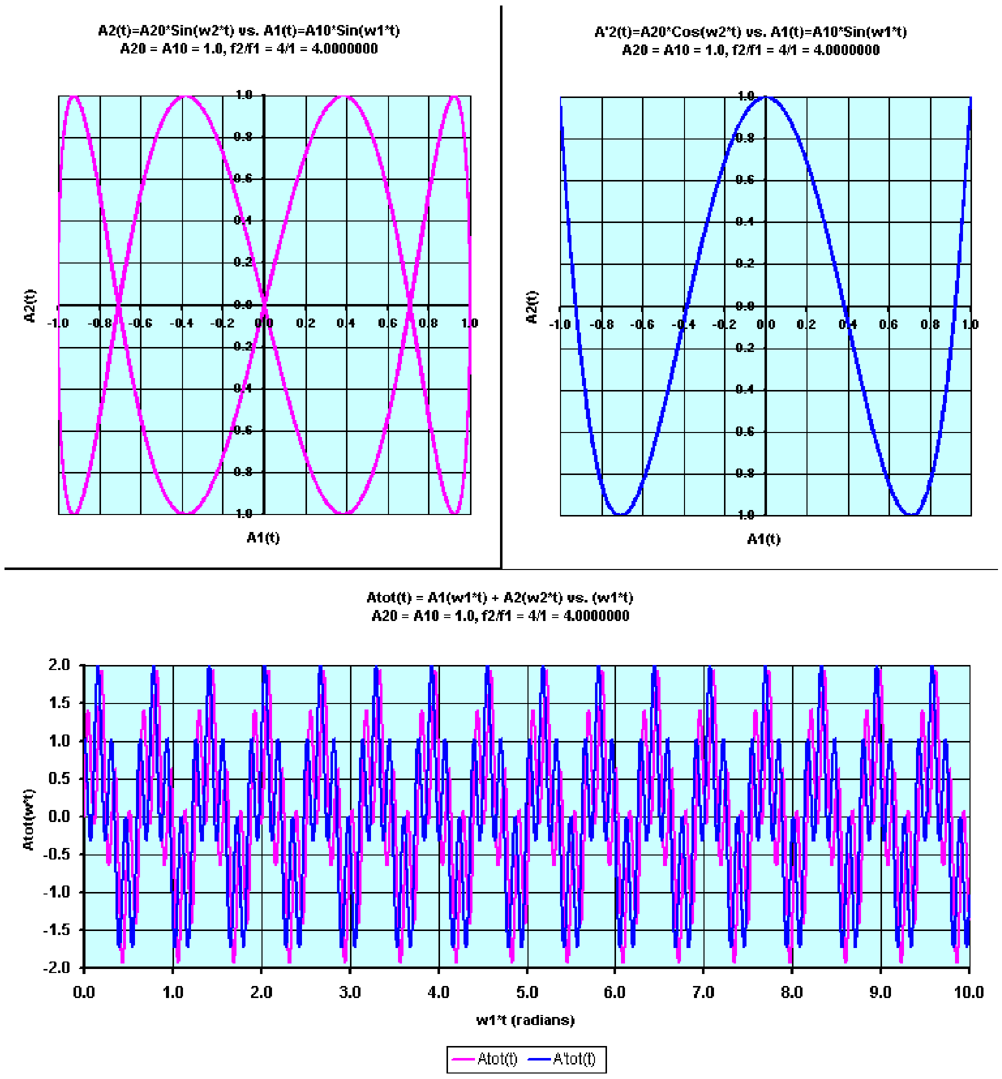
**Consonance of 1st & 4th Harmonics**

Two simple-tone signals with:

$f_2 = (4/1)\, f_1 = 4\, f_1$

(e.g. $f_1 = 300$ Hz

and $f_2 = 1200$ Hz)



**Example:**

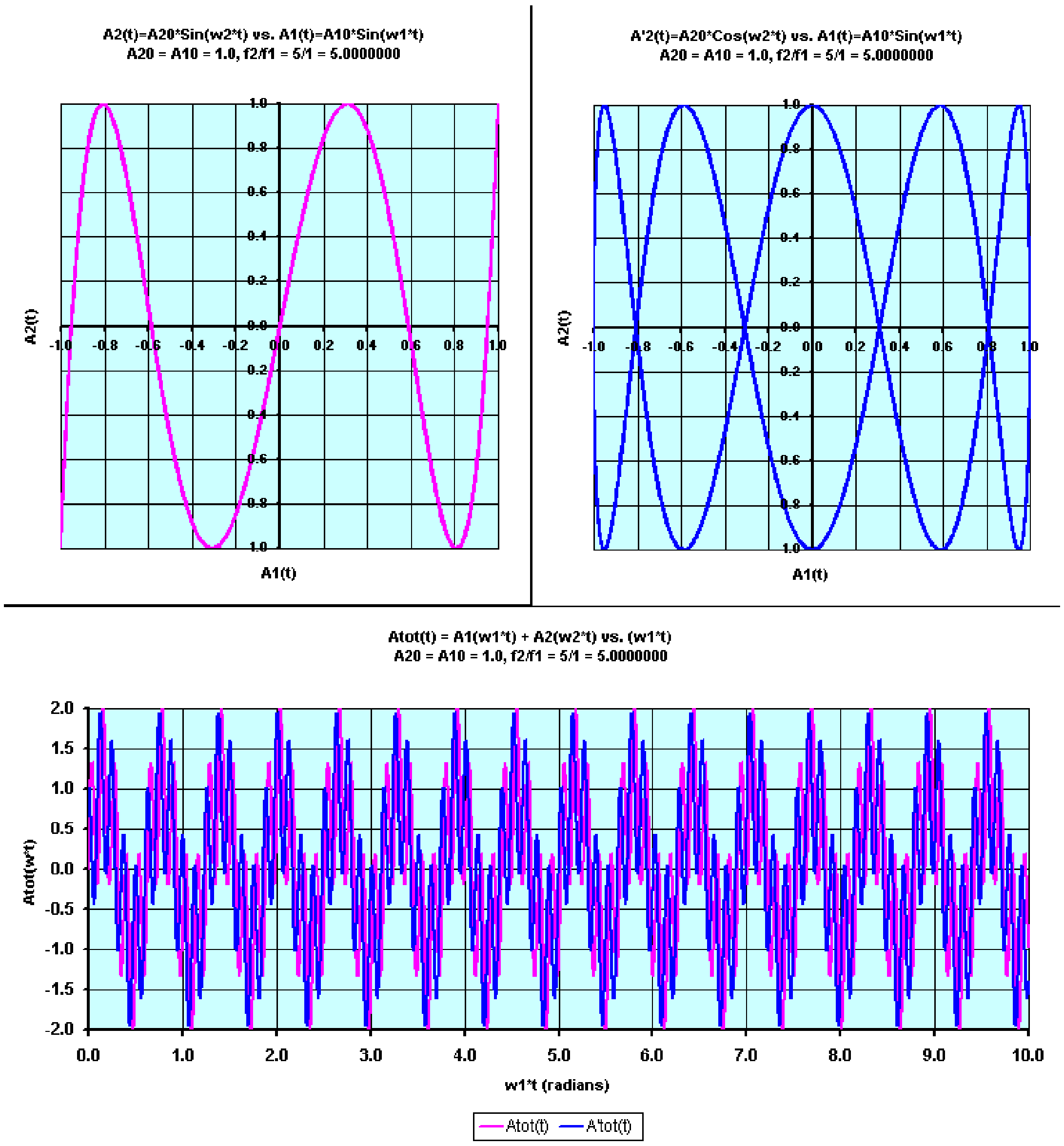
**Consonance of 1st & 5th Harmonics**

Two simple-tone signals with:

$f_2 = (5/1)\, f_1 = 5\, f_1$

(e.g. $f_1 = 300$ Hz

and $f_2 = 1500$ Hz)



# Consonance of Harmonics

# Just Diatonic Scale

**Fundamental Frequency,**

**$f_o$ = 100 Hz**

| | Unison | Second | ??? | ??? | Minor 3rd | Major 3rd | Fourth | Fifth | Sixth | Seventh | Octave |
|---|---|---|---|---|---|---|---|---|---|---|---|
| Harmonic | C-C | D-C | ???-C | ???-C | $E_b$-C | E-C | F-C | G-C | A-C | B-C | C-C |
| # | 1:1 | 9:8 | 8:7 | 7:6 | 6:5 | 5:4 | 4:3 | 3:2 | 5:3 | 15:8 | 2:1 |
| 1 | 100.0 | 112.5 | 114.3 | 116.7 | 120.0 | 125.0 | 133.3 | 150.0 | 166.7 | 187.5 | 200.0 |
| 2 | 200.0 | 225.0 | 228.6 | 233.3 | 240.0 | 250.0 | 266.7 | 300.0 | 333.3 | 375.0 | 400.0 |
| 3 | 300.0 | 337.5 | 342.9 | 350.0 | 360.0 | 375.0 | 400.0 | 450.0 | 500.0 | 562.5 | 600.0 |
| 4 | 400.0 | 450.0 | 457.1 | 466.7 | 480.0 | 500.0 | 533.3 | 600.0 | 666.7 | 750.0 | 800.0 |
| 5 | 500.0 | 562.5 | 571.4 | 583.3 | 600.0 | 625.0 | 666.7 | 750.0 | 833.3 | 937.5 | 1000.0 |
| 6 | 600.0 | 675.0 | 685.7 | 700.0 | 720.0 | 750.0 | 800.0 | 900.0 | 1000.0 | 1125.0 | 1200.0 |
| 7 | 700.0 | 787.5 | 800.0 | 816.7 | 840.0 | 875.0 | 933.3 | 1050.0 | 1166.7 | 1312.5 | 1400.0 |
| 8 | 800.0 | 900.0 | 914.3 | 933.3 | 960.0 | 1000.0 | 1066.7 | 1200.0 | 1333.3 | 1500.0 | 1600.0 |
| 9 | 900.0 | 1012.5 | 1028.6 | 1050.0 | 1080.0 | 1125.0 | 1200.0 | 1350.0 | 1500.0 | 1687.5 | 1800.0 |
| 10 | 1000.0 | 1125.0 | 1142.9 | 1166.7 | 1200.0 | 1250.0 | 1333.3 | 1500.0 | 1666.7 | 1875.0 | 2000.0 |
| 11 | 1100.0 | 1237.5 | 1257.1 | 1283.3 | 1320.0 | 1375.0 | 1466.7 | 1650.0 | 1833.3 | 2062.5 | 2200.0 |
| 12 | 1200.0 | 1350.0 | 1371.4 | 1400.0 | 1440.0 | 1500.0 | 1600.0 | 1800.0 | 2000.0 | 2250.0 | 2400.0 |
| 13 | 1300.0 | 1462.5 | 1485.7 | 1516.7 | 1560.0 | 1625.0 | 1733.3 | 1950.0 | 2166.7 | 2437.5 | 2600.0 |
| 14 | 1400.0 | 1575.0 | 1600.0 | 1633.3 | 1680.0 | 1750.0 | 1866.7 | 2100.0 | 2333.3 | 2625.0 | 2800.0 |
| 15 | 1500.0 | 1687.5 | 1714.3 | 1750.0 | 1800.0 | 1875.0 | 2000.0 | 2250.0 | 2500.0 | 2812.5 | 3000.0 |
| 16 | 1600.0 | 1800.0 | 1828.6 | 1866.7 | 1920.0 | 2000.0 | 2133.3 | 2400.0 | 2666.7 | 3000.0 | 3200.0 |
| 17 | 1700.0 | 1912.5 | 1942.9 | 1983.3 | 2040.0 | 2125.0 | 2266.7 | 2550.0 | 2833.3 | 3187.5 | 3400.0 |
| 18 | 1800.0 | 2025.0 | 2057.1 | 2100.0 | 2160.0 | 2250.0 | 2400.0 | 2700.0 | 3000.0 | 3375.0 | 3600.0 |
| 19 | 1900.0 | 2137.5 | 2171.4 | 2216.7 | 2280.0 | 2375.0 | 2533.3 | 2850.0 | 3166.7 | 3562.5 | 3800.0 |
| 20 | 2000.0 | 2250.0 | 2285.7 | 2333.3 | 2400.0 | 2500.0 | 2666.7 | 3000.0 | 3333.3 | 3750.0 | 4000.0 |
| 21 | 2100.0 | 2362.5 | 2400.0 | 2450.0 | 2520.0 | 2625.0 | 2800.0 | 3150.0 | 3500.0 | 3937.5 | 4200.0 |
| 22 | 2200.0 | 2475.0 | 2514.3 | 2566.7 | 2640.0 | 2750.0 | 2933.3 | 3300.0 | 3666.7 | 4125.0 | 4400.0 |
| 23 | 2300.0 | 2587.5 | 2628.6 | 2683.3 | 2760.0 | 2875.0 | 3066.7 | 3450.0 | 3833.3 | 4312.5 | 4600.0 |
| 24 | 2400.0 | 2700.0 | 2742.9 | 2800.0 | 2880.0 | 3000.0 | 3200.0 | 3600.0 | 4000.0 | 4500.0 | 4800.0 |
| 25 | 2500.0 | 2812.5 | 2857.1 | 2916.7 | 3000.0 | 3125.0 | 3333.3 | 3750.0 | 4166.7 | 4687.5 | 5000.0 |
| 26 | 2600.0 | 2925.0 | 2971.4 | 3033.3 | 3120.0 | 3250.0 | 3466.7 | 3900.0 | 4333.3 | 4875.0 | 5200.0 |
| 27 | 2700.0 | 3037.5 | 3085.7 | 3150.0 | 3240.0 | 3375.0 | 3600.0 | 4050.0 | 4500.0 | 5062.5 | 5400.0 |
| 28 | 2800.0 | 3150.0 | 3200.0 | 3266.7 | 3360.0 | 3500.0 | 3733.3 | 4200.0 | 4666.7 | 5250.0 | 5600.0 |
| 29 | 2900.0 | 3262.5 | 3314.3 | 3383.3 | 3480.0 | 3625.0 | 3866.7 | 4350.0 | 4833.3 | 5437.5 | 5800.0 |
| 30 | 3000.0 | 3375.0 | 3428.6 | 3500.0 | 3600.0 | 3750.0 | 4000.0 | 4500.0 | 5000.0 | 5625.0 | 6000.0 |
| 31 | 3100.0 | 3487.5 | 3542.9 | 3616.7 | 3720.0 | 3875.0 | 4133.3 | 4650.0 | 5166.7 | 5812.5 | 6200.0 |
| 32 | 3200.0 | 3600.0 | 3657.1 | 3733.3 | 3840.0 | 4000.0 | 4266.7 | 4800.0 | 5333.3 | 6000.0 | 6400.0 |

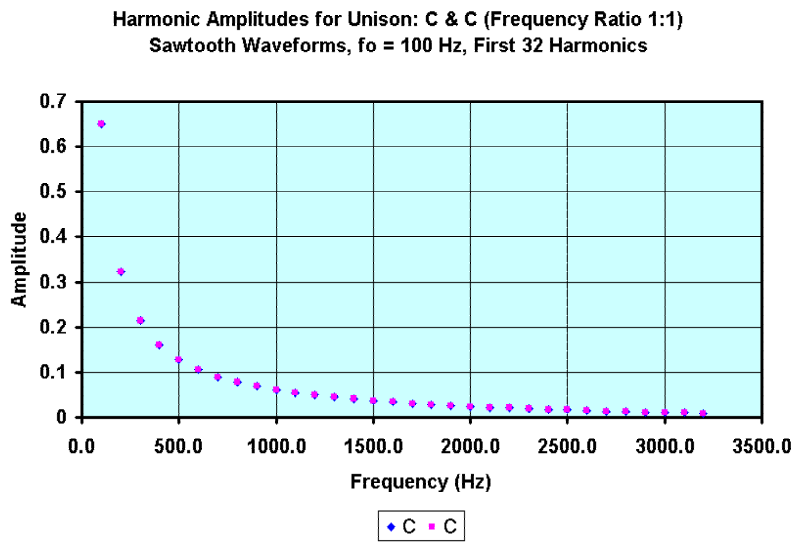
## Dissonance of Harmonics

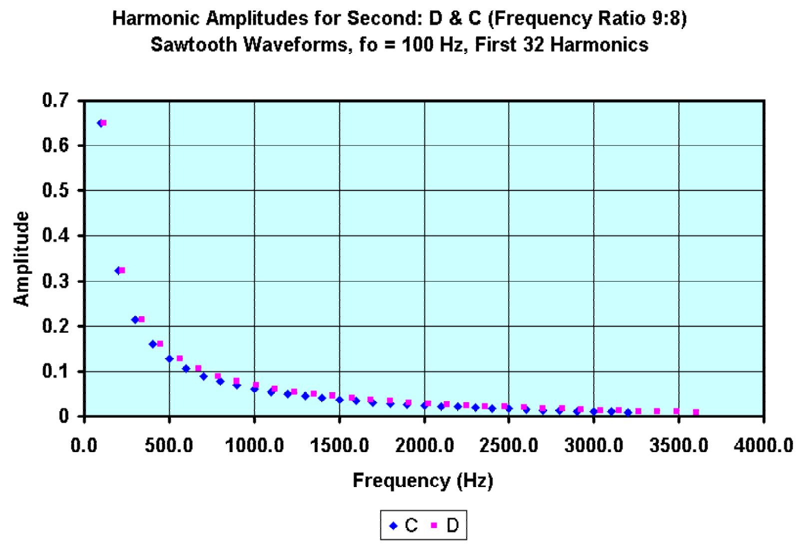
## Just Diatonic Scale

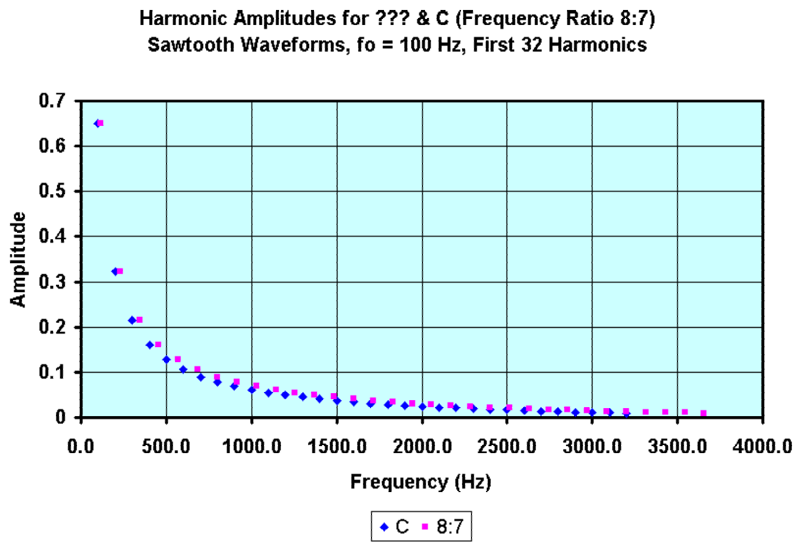
**Fundamental Frequency,**

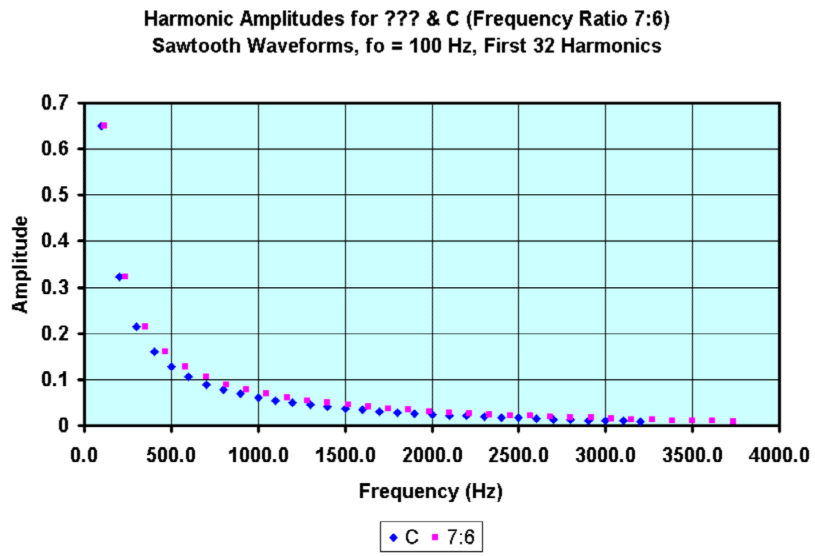
**$f_o$ = 100 Hz**

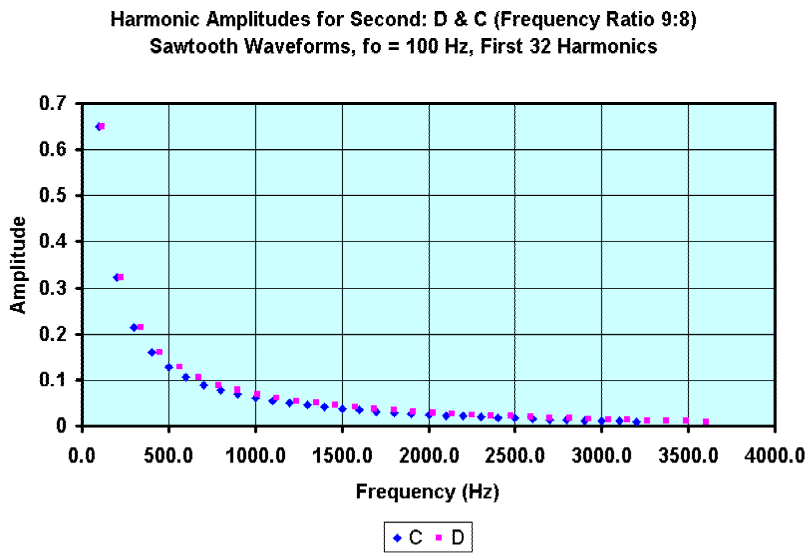
| Harmonic # | Unison C-C 1:1 | Second D-C 9:8 | ??? ???-C 8:7 | ??? ???-C 7:6 | Minor 3rd E♭-C 6:5 | Major 3rd E-C 5:4 | Fourth F-C 4:3 | Fifth G-C 3:2 | Sixth A-C 5:3 | Seventh B-C 15:8 | Octave C-C 2:1 |
|---|---|---|---|---|---|---|---|---|---|---|---|
| 1 | 100.0 | 112.5 | 114.3 | 116.7 | 120.0 | 125.0 | 133.3 | 150.0 | 166.7 | 187.5 | 200.0 |
| 2 | 200.0 | 225.0 | 228.6 | 233.3 | 240.0 | 250.0 | 266.7 | 300.0 | 333.3 | 375.0 | 400.0 |
| 3 | 300.0 | 337.5 | 342.9 | 350.0 | 360.0 | 375.0 | 400.0 | 450.0 | 500.0 | 562.5 | 600.0 |
| 4 | 400.0 | 450.0 | 457.1 | 466.7 | 480.0 | 500.0 | 533.3 | 600.0 | 666.7 | 750.0 | 800.0 |
| 5 | 500.0 | 562.5 | 571.4 | 583.3 | 600.0 | 625.0 | 666.7 | 750.0 | 833.3 | 937.5 | 1000.0 |
| 6 | 600.0 | 675.0 | 685.7 | 700.0 | 720.0 | 750.0 | 800.0 | 900.0 | 1000.0 | 1125.0 | 1200.0 |
| 7 | 700.0 | 787.5 | 800.0 | 816.7 | 840.0 | 875.0 | 933.3 | 1050.0 | 1166.7 | 1312.5 | 1400.0 |
| 8 | 800.0 | 900.0 | 914.3 | 933.3 | 960.0 | 1000.0 | 1066.7 | 1200.0 | 1333.3 | 1500.0 | 1600.0 |
| 9 | 900.0 | 1012.5 | 1028.6 | 1050.0 | 1080.0 | 1125.0 | 1200.0 | 1350.0 | 1500.0 | 1687.5 | 1800.0 |
| 10 | 1000.0 | 1125.0 | 1142.9 | 1166.7 | 1200.0 | 1250.0 | 1333.3 | 1500.0 | 1666.7 | 1875.0 | 2000.0 |
| 11 | 1100.0 | 1237.5 | 1257.1 | 1283.3 | 1320.0 | 1375.0 | 1466.7 | 1650.0 | 1833.3 | 2062.5 | 2200.0 |
| 12 | 1200.0 | 1350.0 | 1371.4 | 1400.0 | 1440.0 | 1500.0 | 1600.0 | 1800.0 | 2000.0 | 2250.0 | 2400.0 |
| 13 | 1300.0 | 1462.5 | 1485.7 | 1516.7 | 1560.0 | 1625.0 | 1733.3 | 1950.0 | 2166.7 | 2437.5 | 2600.0 |
| 14 | 1400.0 | 1575.0 | 1600.0 | 1633.3 | 1680.0 | 1750.0 | 1866.7 | 2100.0 | 2333.3 | 2625.0 | 2800.0 |
| 15 | 1500.0 | 1687.5 | 1714.3 | 1750.0 | 1800.0 | 1875.0 | 2000.0 | 2250.0 | 2500.0 | 2812.5 | 3000.0 |
| 16 | 1600.0 | 1800.0 | 1828.6 | 1866.7 | 1920.0 | 2000.0 | 2133.3 | 2400.0 | 2666.7 | 3000.0 | 3200.0 |
| 17 | 1700.0 | 1912.5 | 1942.9 | 1983.3 | 2040.0 | 2125.0 | 2266.7 | 2550.0 | 2833.3 | 3187.5 | 3400.0 |
| 18 | 1800.0 | 2025.0 | 2057.1 | 2100.0 | 2160.0 | 2250.0 | 2400.0 | 2700.0 | 3000.0 | 3375.0 | 3600.0 |
| 19 | 1900.0 | 2137.5 | 2171.4 | 2216.7 | 2280.0 | 2375.0 | 2533.3 | 2850.0 | 3166.7 | 3562.5 | 3800.0 |
| 20 | 2000.0 | 2250.0 | 2285.7 | 2333.3 | 2400.0 | 2500.0 | 2666.7 | 3000.0 | 3333.3 | 3750.0 | 4000.0 |
| 21 | 2100.0 | 2362.5 | 2400.0 | 2450.0 | 2520.0 | 2625.0 | 2800.0 | 3150.0 | 3500.0 | 3937.5 | 4200.0 |
| 22 | 2200.0 | 2475.0 | 2514.3 | 2566.7 | 2640.0 | 2750.0 | 2933.3 | 3300.0 | 3666.7 | 4125.0 | 4400.0 |
| 23 | 2300.0 | 2587.5 | 2628.6 | 2683.3 | 2760.0 | 2875.0 | 3066.7 | 3450.0 | 3833.3 | 4312.5 | 4600.0 |
| 24 | 2400.0 | 2700.0 | 2742.9 | 2800.0 | 2880.0 | 3000.0 | 3200.0 | 3600.0 | 4000.0 | 4500.0 | 4800.0 |
| 25 | 2500.0 | 2812.5 | 2857.1 | 2916.7 | 3000.0 | 3125.0 | 3333.3 | 3750.0 | 4166.7 | 4687.5 | 5000.0 |
| 26 | 2600.0 | 2925.0 | 2971.4 | 3033.3 | 3120.0 | 3250.0 | 3466.7 | 3900.0 | 4333.3 | 4875.0 | 5200.0 |
| 27 | 2700.0 | 3037.5 | 3085.7 | 3150.0 | 3240.0 | 3375.0 | 3600.0 | 4050.0 | 4500.0 | 5062.5 | 5400.0 |
| 28 | 2800.0 | 3150.0 | 3200.0 | 3266.7 | 3360.0 | 3500.0 | 3733.3 | 4200.0 | 4666.7 | 5250.0 | 5600.0 |
| 29 | 2900.0 | 3262.5 | 3314.3 | 3383.3 | 3480.0 | 3625.0 | 3866.7 | 4350.0 | 4833.3 | 5437.5 | 5800.0 |
| 30 | 3000.0 | 3375.0 | 3428.6 | 3500.0 | 3600.0 | 3750.0 | 4000.0 | 4500.0 | 5000.0 | 5625.0 | 6000.0 |
| 31 | 3100.0 | 3487.5 | 3542.9 | 3616.7 | 3720.0 | 3875.0 | 4133.3 | 4650.0 | 5166.7 | 5812.5 | 6200.0 |
| 32 | 3200.0 | 3600.0 | 3657.1 | 3733.3 | 3840.0 | 4000.0 | 4266.7 | 4800.0 | 5333.3 | 6000.0 | 6400.0 |

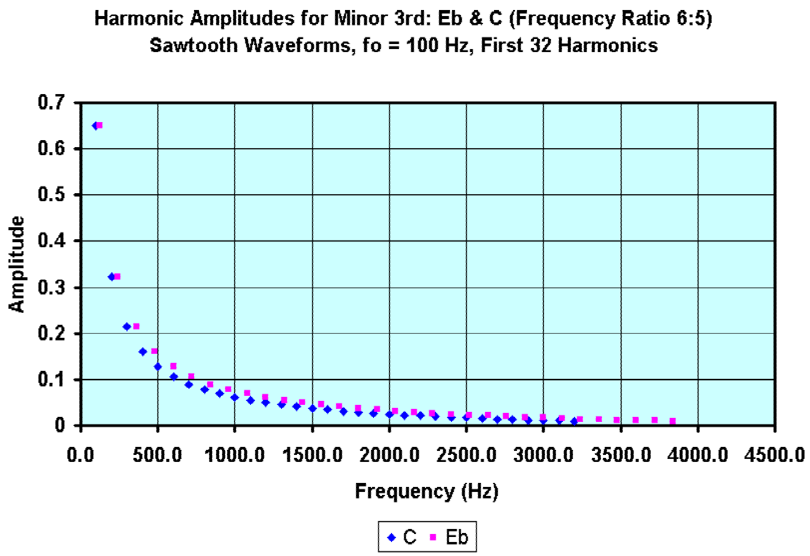

Harmonic Amplitudes for Unison: C & C (Frequency Ratio 1:1)
Sawtooth Waveforms, fo = 100 Hz, First 32 Harmonics
Amplitude
Frequency (Hz)
C C

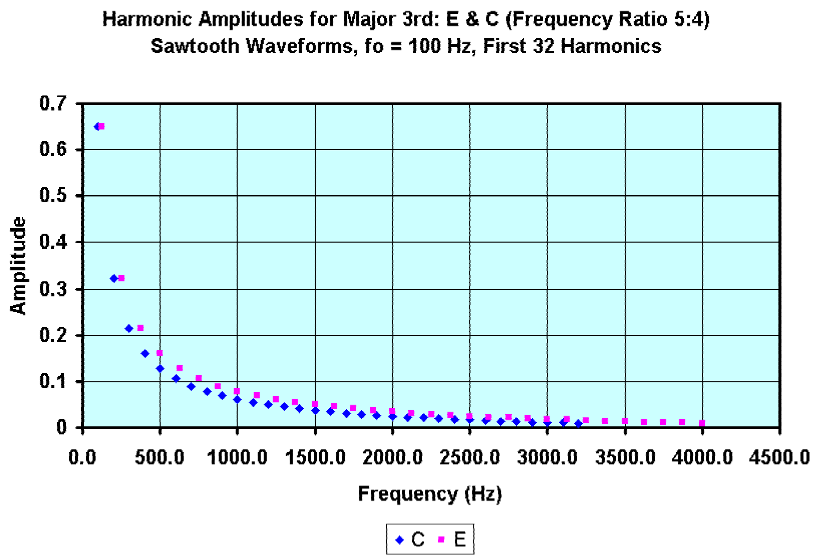

Harmonic Amplitudes for Second: D & C (Frequency Ratio 9:8)
Sawtooth Waveforms, fo = 100 Hz, First 32 Harmonics
Amplitude
Frequency (Hz)
C D

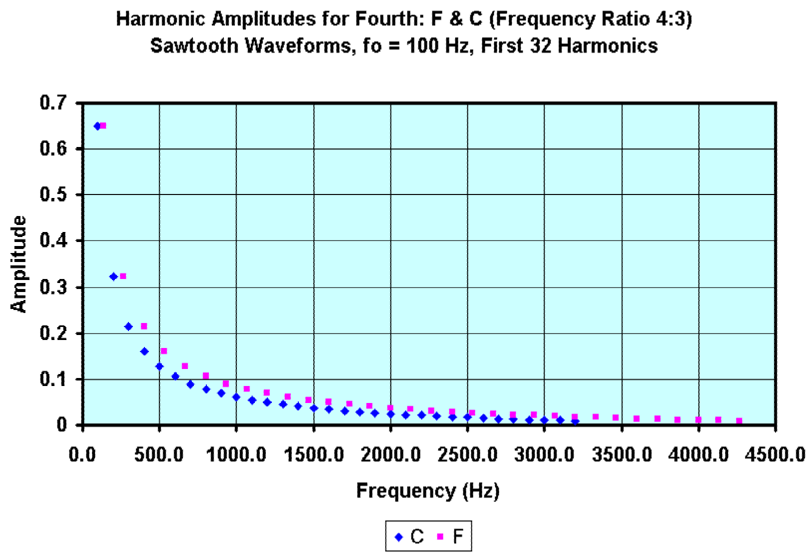

Harmonic Amplitudes for ??? & C (Frequency Ratio 8:7)
Sawtooth Waveforms, fo = 100 Hz, First 32 Harmonics
Amplitude
Frequency (Hz)
C 8:7

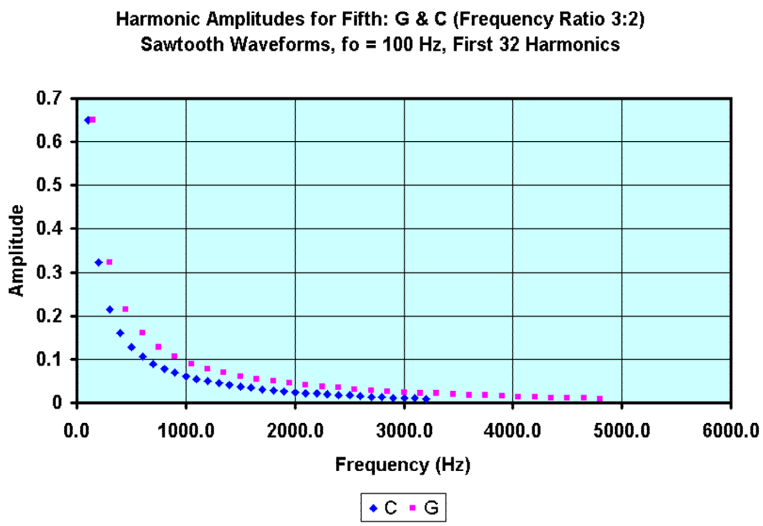

Harmonic Amplitudes for ??? & C (Frequency Ratio 7:6)
Sawtooth Waveforms, fo = 100 Hz, First 32 Harmonics
Amplitude
Frequency (Hz)
C 7:6

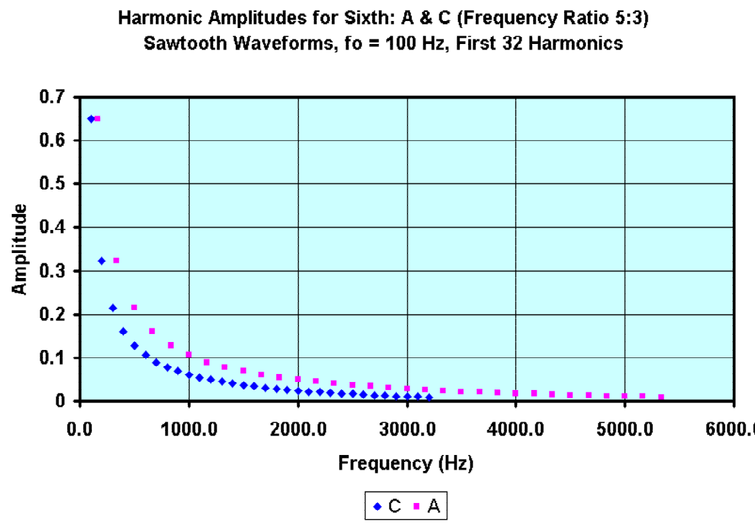

Harmonic Amplitudes for Second: D & C (Frequency Ratio 9:8)
Sawtooth Waveforms, fo = 100 Hz, First 32 Harmonics
Amplitude
Frequency (Hz)
C
D

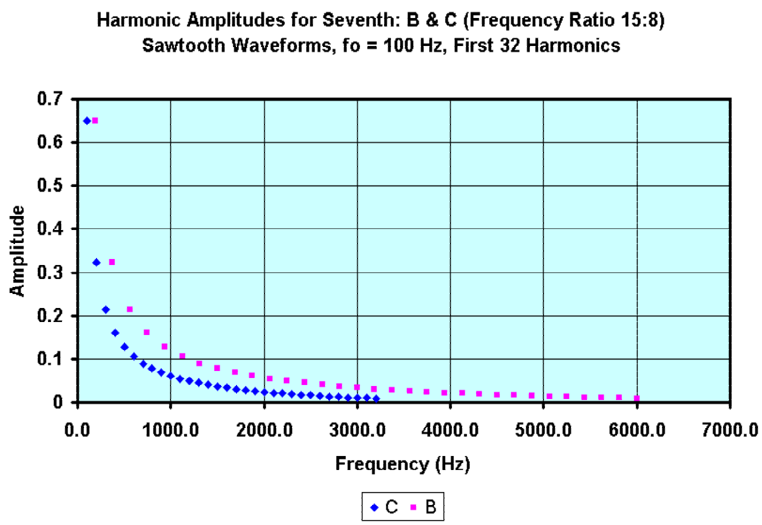

Harmonic Amplitudes for Minor 3rd: Eb & C (Frequency Ratio 6:5)
Sawtooth Waveforms, fo = 100 Hz, First 32 Harmonics
Amplitude
Frequency (Hz)
C
Eb

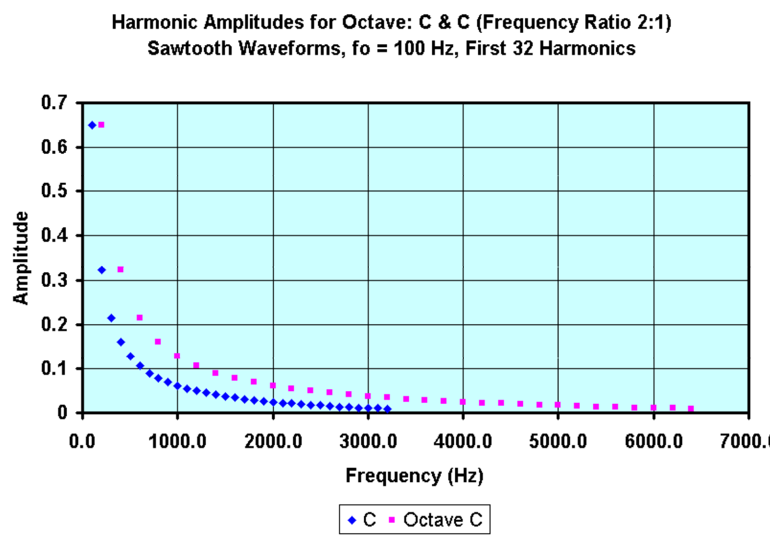

Harmonic Amplitudes for Major 3rd: E & C (Frequency Ratio 5:4)
Sawtooth Waveforms, fo = 100 Hz, First 32 Harmonics
Amplitude
Frequency (Hz)
C
E


Harmonic Amplitudes for Fourth: F & C (Frequency Ratio 4:3)
Sawtooth Waveforms, fo = 100 Hz, First 32 Harmonics
Amplitude
Frequency (Hz)
C
F


Harmonic Amplitudes for Fifth: G & C (Frequency Ratio 3:2)
Sawtooth Waveforms, fo = 100 Hz, First 32 Harmonics
Amplitude
Frequency (Hz)
C
G

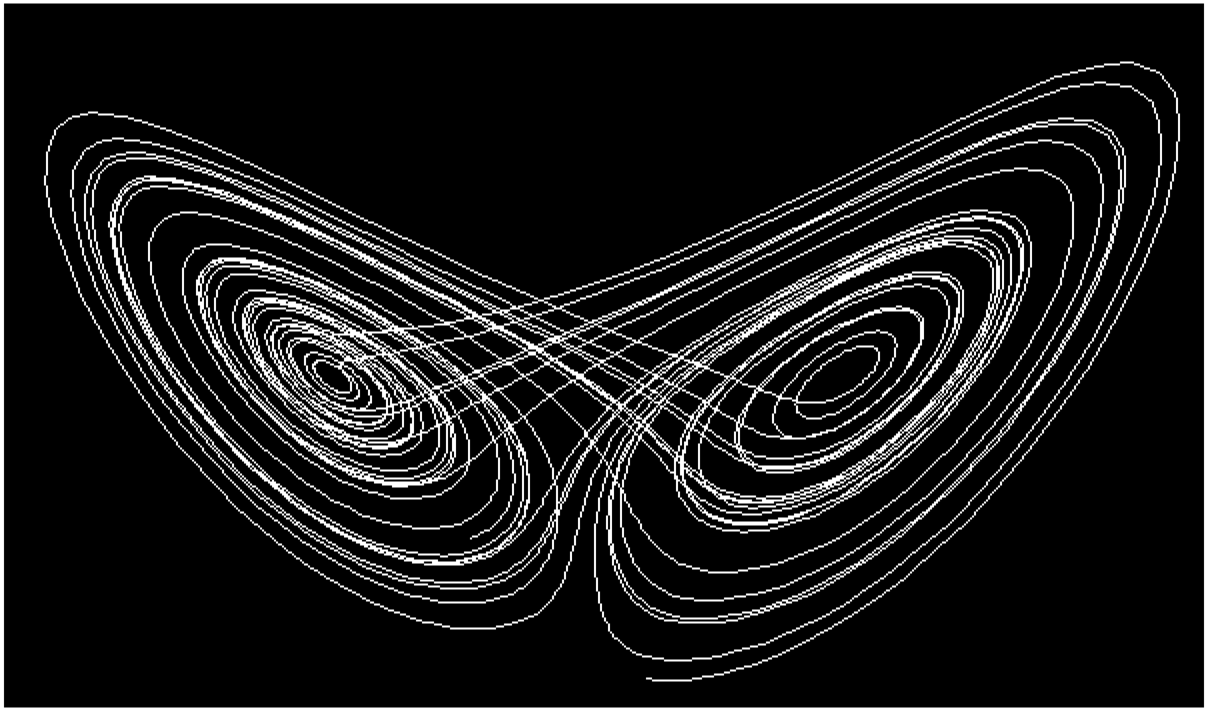

Harmonic Amplitudes for Sixth: A & C (Frequency Ratio 5:3)
Sawtooth Waveforms, fo = 100 Hz, First 32 Harmonics
Amplitude
Frequency (Hz)
C
A


Harmonic Amplitudes for Seventh: B & C (Frequency Ratio 15:8)
Sawtooth Waveforms, fo = 100 Hz, First 32 Harmonics
Amplitude
Frequency (Hz)
C
B


Harmonic Amplitudes for Octave: C & C (Frequency Ratio 2:1)
Sawtooth Waveforms, fo = 100 Hz, First 32 Harmonics
Amplitude
Frequency (Hz)
C
Octave C

# Fractal Music

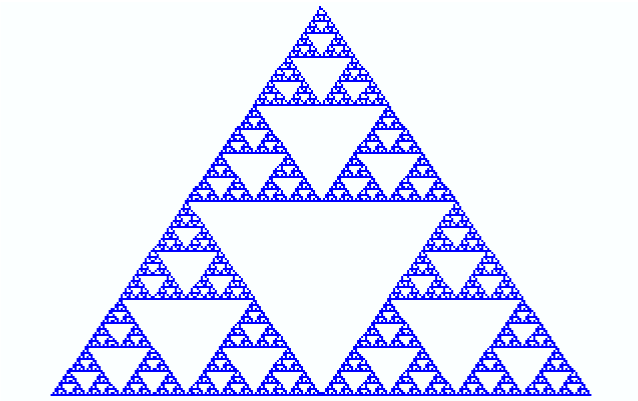
## Lorentz's Butterfly - Strange Attractor



Iterative Equations:
dx/dt = 10(y - x)
dy/dt = x(28 - z) - y
dz/dt = xy - 8z/3.

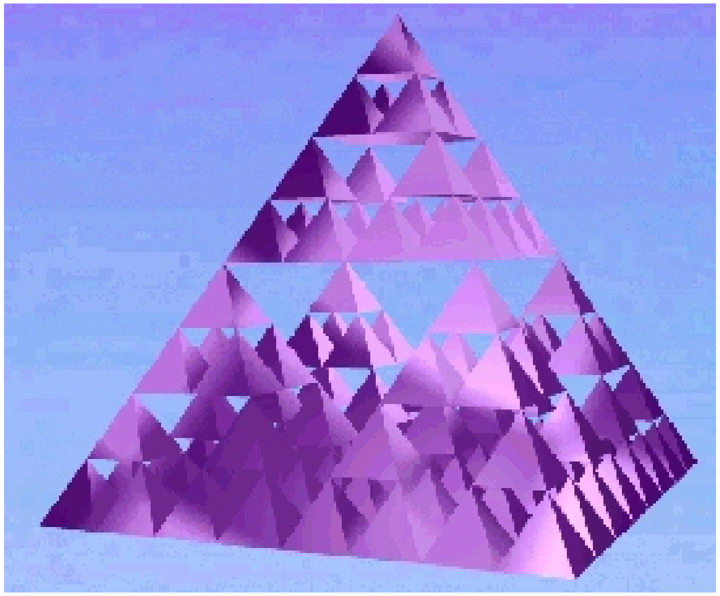
Initial Conditions:
Change of t = 0.01 and
the initial values
x0 = 2, y0 = 3 and z0 = 5

# Fractal Music



The Sierpinski Triangle
is a fractal structure with fractal dimension 1.584.

The area of a Sierpinski Triangle is ZERO!



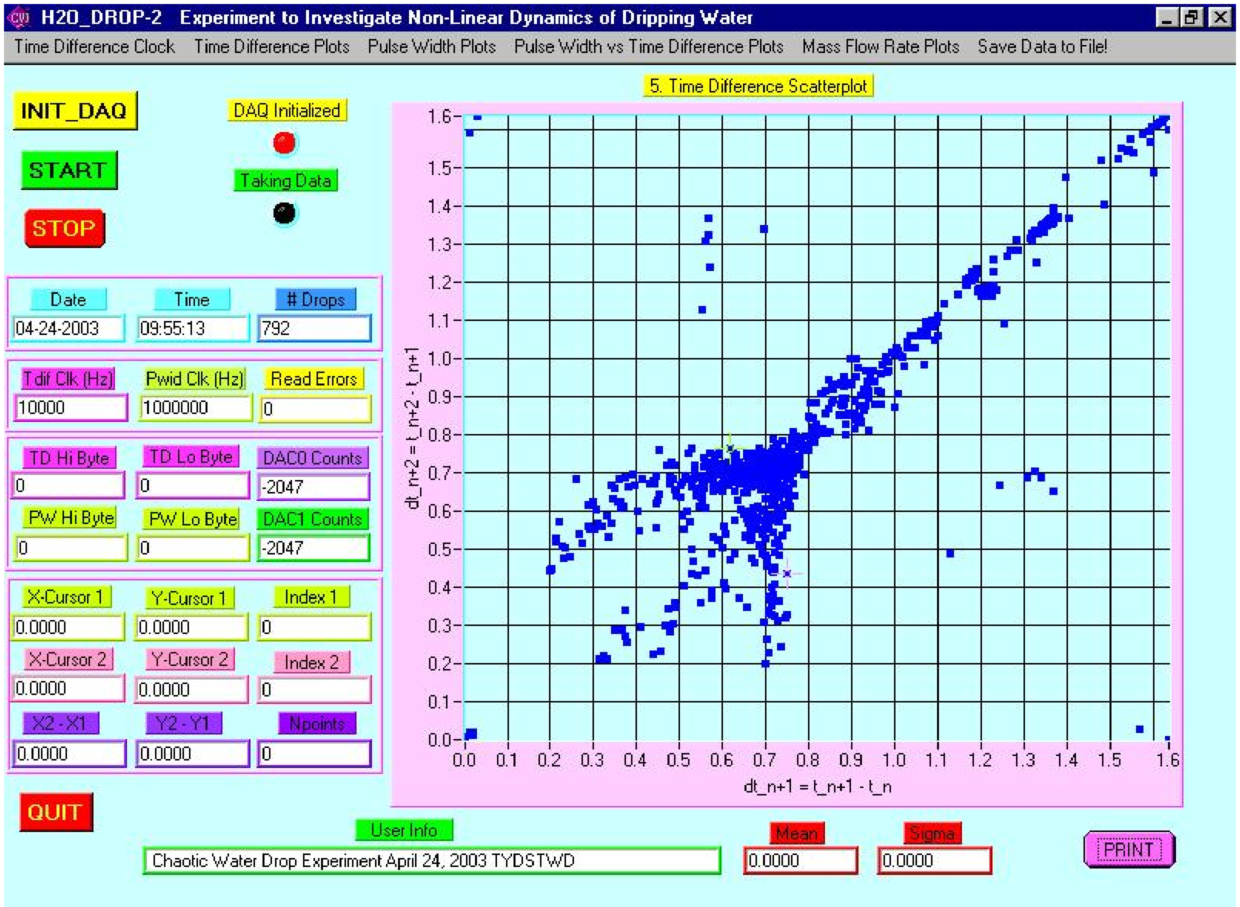
3-D Sierpinski Pyramid

Beethoven's Piano Sonata no. 15, op. 28, 3rd Movement (Scherzo) is a combination of binary and ternary units iterating on diminishing scales, similar to the Sierpinski Structure !!!

# Fractal Music in Nature – chaotic dripping of a leaky water faucet!

**Convert successive drop time differences and drop sizes to frequencies**

**Play back in real-time (online!) using FG – can *hear* the sound of chaotic dripping!**



## **Conclusions and Summary:**

- Music is an intimate, very important part of human culture
- Music is deeply ingrained in our daily lives - it's everywhere!
- Music constantly evolves with our culture - affected by many things
- Future: Develop new kinds of music...
- Future: Improve existing & develop totally new kinds of musical instruments...
- There's an immense amount of physics in music - much still to be learned !!!
- Huge amount of fun – combine physics & math with music – can hear/see/touch/feel/think!!

***MUSIC***

**Be a Part of It - Participate !!!**

**Enjoy It !!!**

**Support It !!!**

For additional info on Physics of Music at UIUC - see e.g.

**Physics 199 Physics of Music Web Page:**

**http://wug.physics.uiuc.edu/courses/phys199pom/**

**Physics 398 Physics of Electronic Musical Instruments Web Page:**

**http://wug.physics.uiuc.edu/courses/phys398emi/**