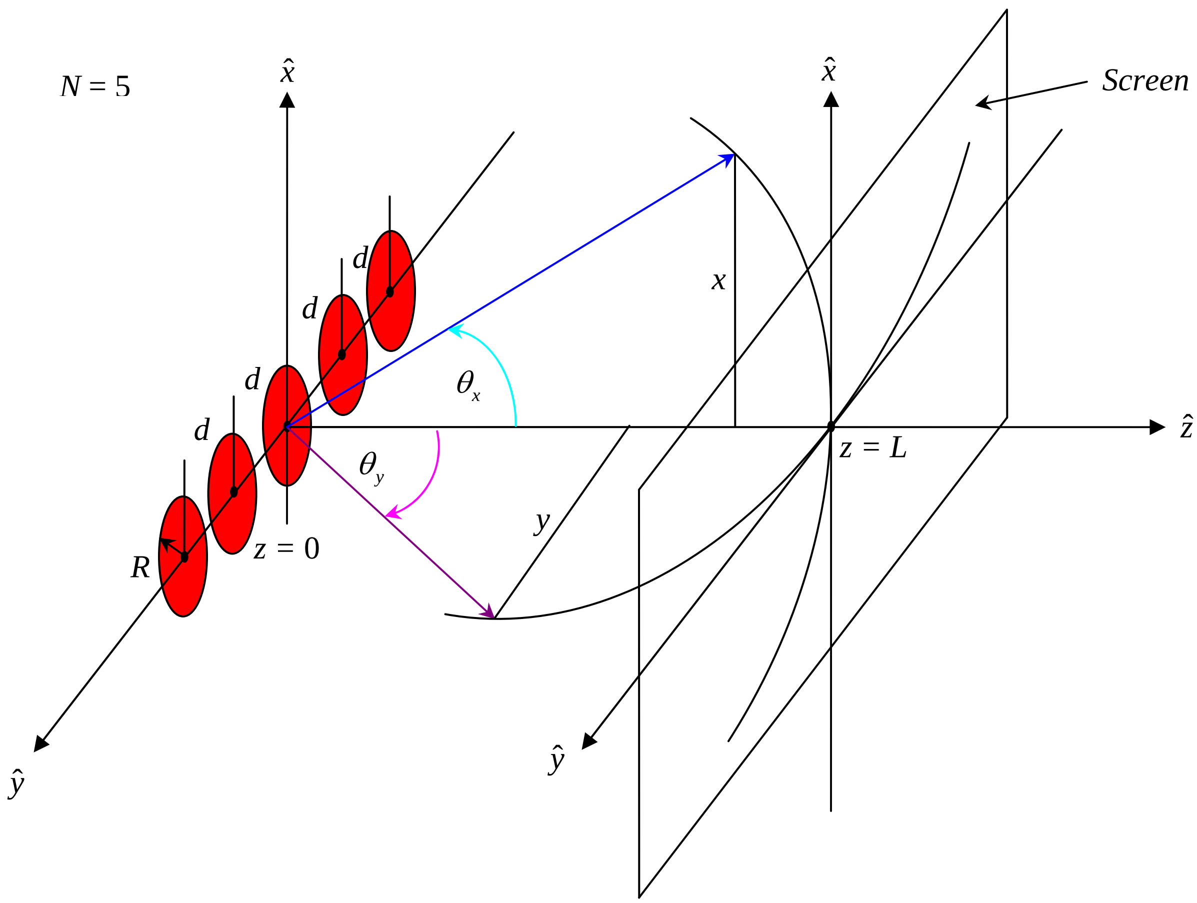
1-D Interference-Diffraction with *N* Circular Apertures

In this example, we show plots of the sound intensity *vs*. angle and observer/listener position (x_{screen}, y_{screen}) on a screen for the simplest theory of interference-diffraction pattern associated with a 1-D linear array of N identical circular apertures, each of radius R, with transverse separation distance $d > R$. This is an approximation to *e.g*. a linear array of N loudspeakers mounted on an infinite baffle, as shown in the figure below. The observer/listener is located far from the apertures, a perpendicular distance $L\ (m)$ away, such that the conditions $R < d \ll L$ **.and.** $\lambda \ll L$ both hold simultaneously, where $\lambda\ (m)$ is the wavelength of the sound – this is the so-called "far-field" limit.



The expression for interference-diffraction associated with a 1-D linear array of N circular aperture is given by the product of interference × diffraction factors:

$$I_{tot}(\theta) = I_o \underbrace{\left\{\frac{\sin^2(N\delta(\theta_y)/2)}{\sin^2(\delta(\theta_y)/2)}\right\}}_{Interference} \cdot \underbrace{\left\{\frac{2J_1(\rho(\theta))}{\rho(\theta)}\right\}^2}_{Diffraction} \text{ and } SIL(\theta) \equiv 10\log_{10}\left(I_{tot}(\theta)/I_{ref}\right)\ (dB)$$

where I_o $\left(Watts/m^2\right)$ is the maximum sound intensity associated with an individual circular aperture, the interference phase $\delta\left(\theta_y\right) = k\Delta L\left(\theta_y\right) = kd\sin\theta_y$ $\left(radians\right)$, $k = 2\pi/\lambda$ $\left(radians/m\right)$ is the wavenumber and $\Delta L\left(\theta_y\right) = d\sin\theta_y$ $\left(m\right)$ is the *y*-projected angle-dependent path length ***difference*** between pairs of adjacent sound sources to the observer/listener, located far away from the sound sources. The diffraction phase $\rho\left(\theta\right) = kR\sin\theta = \left(2\pi/\lambda\right)R\sin\theta\left(radians\right)$ and $J_1\left(\rho\left(\theta\right)\right)$ is the ordinary Bessel function of the first kind, of order $\nu = 1$.

$I_{ref} \equiv 10^{-12}$ $\left(Watts/m^2\right)$ is the reference sound intensity for the sound intensity level (SIL).

Interference minima – *i.e*. intensity zeroes (complete destructive interference) occur when the numerator factor $N\,\delta/2 = \pm\pi, \pm 2\pi, \pm 3\pi, \ldots = n\pi$, $n = \pm 1, \pm 2, \pm 3, \ldots$ *except* when the denominator factor simultaneously has $\delta/2 = \pm\pi, \pm 2\pi, \pm 3\pi, \ldots = n\pi$, $n = \pm 1, \pm 2, \pm 3, \ldots$ then have *global maxima* of the intensity, where $I_{tot} = N^2 I_o$.

From simple trigonometry, it is easy to show that the path length difference $\Delta L\left(\theta_y\right) = d\sin\theta_y$, where θ_y is the *y*-projected angle the observer/listener makes with respect to the normal, or forward axis of the array of *N* circular apertures.

Diffraction minima (intensity zeroes) occur at the non-trivial zeros of the Bessel function $J_1\left(\rho\right)$, which occur at ρ = 3.8317, 7.0156, 10.1735, 13.3237, 16.4706, 19.6159, …

The corresponding location of the observer's/listener's position y_{screen} on a screen located a perpendicular distance *L* away from the *N* sound sources is: $r_{screen}\left(\theta\right) = L\tan\theta$, or conversely: $\theta = \tan^{-1}\left(r_{screen}/L\right)$, where $r_{screen} = \sqrt{x_{screen}^2 + y_{screen}^2}$.

We coded up the above formulas using MATLAB to make plots of I_{tot} *vs.* θ and I_{tot} *vs.* y_{screen} *e.g*. for *N* = 1, 2, 4, 10 and *d* = 1.0 m and *R* = 1.0 *m*, with the following parameter values: $I_o = 1\ Watt/m^2$, observer/listener distance (at $\theta = 0$) of $L = 10\,m$, the speed of propagation in free air/great-wide open: $v_{air} = 343\,m/s$ and frequency of $f = 1000\,Hz$, thus $\lambda = v_{air}/f = 0.345\,m$.

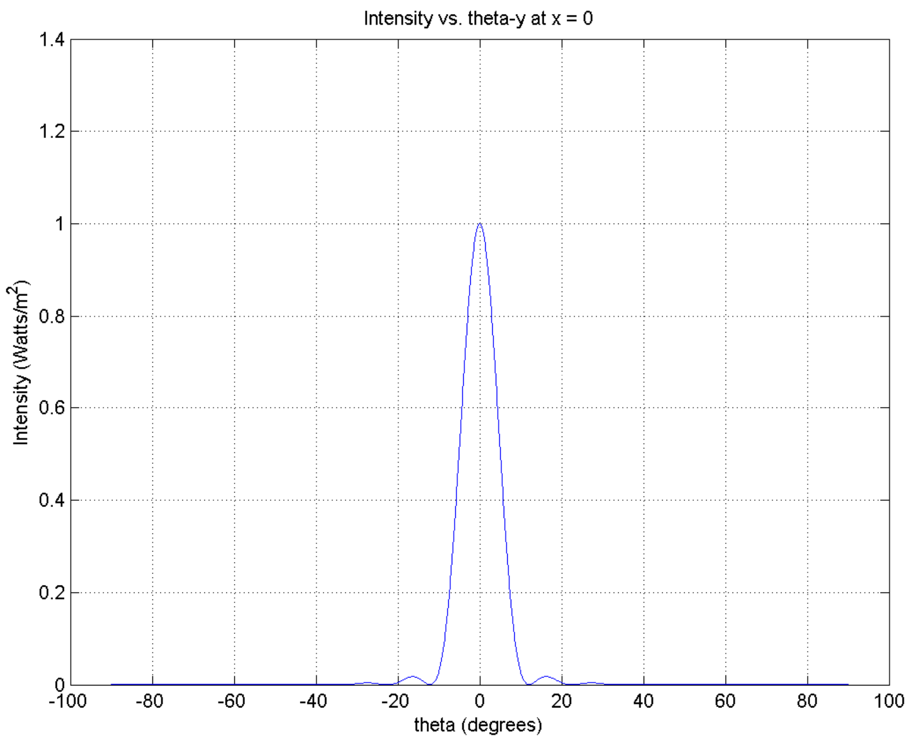
In the following figures, note that the angular width of the central maxima decreases as the radius of the circular aperture increases, since the diffraction phase $\rho\left(\theta\right) = kR\sin\theta = \left(2\pi/\lambda\right)R\sin\theta$ $\left(radians\right)$ is linearly proportional to *R*. Note also that the angular width of the central maxima decreases linearly with increasing frequency *f*, since the diffraction phase increases linearly with frequency:

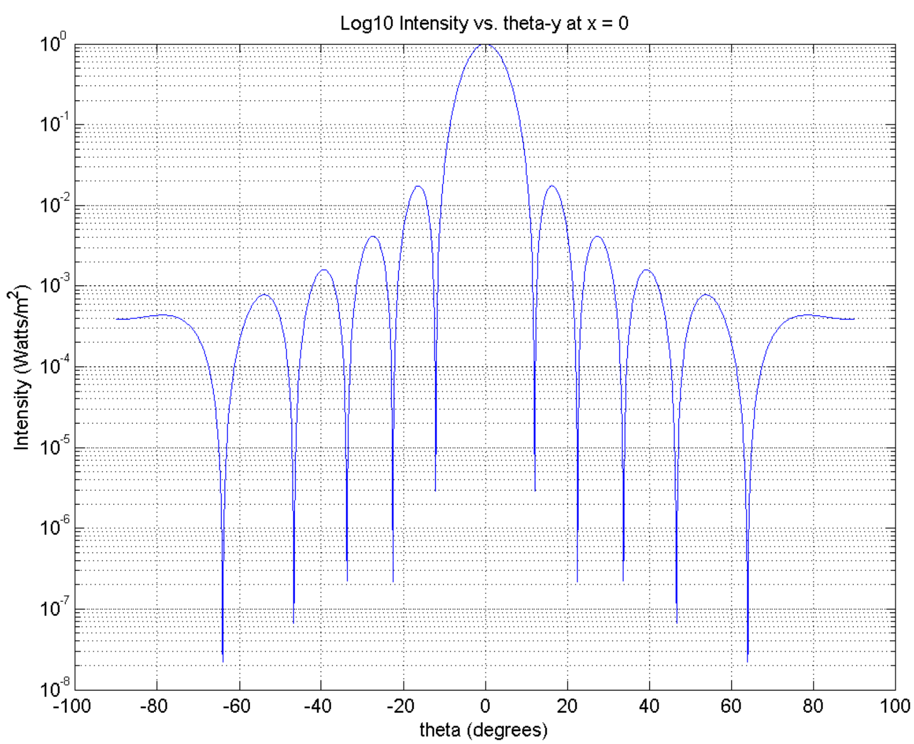
$$\rho\left(\theta\right) = kR\sin\theta = \left(2\pi/\lambda\right)R\sin\theta = \left(2\pi f/v_{air}\right)R\sin\theta\left(radians\right).$$

Note also that the angular width of the central maxima decreases as the number of circular apertures *N* increases. Note further that the number of maxima/minima increases linearly with increasing frequency *f*, since the interference phase difference increases linearly with frequency:

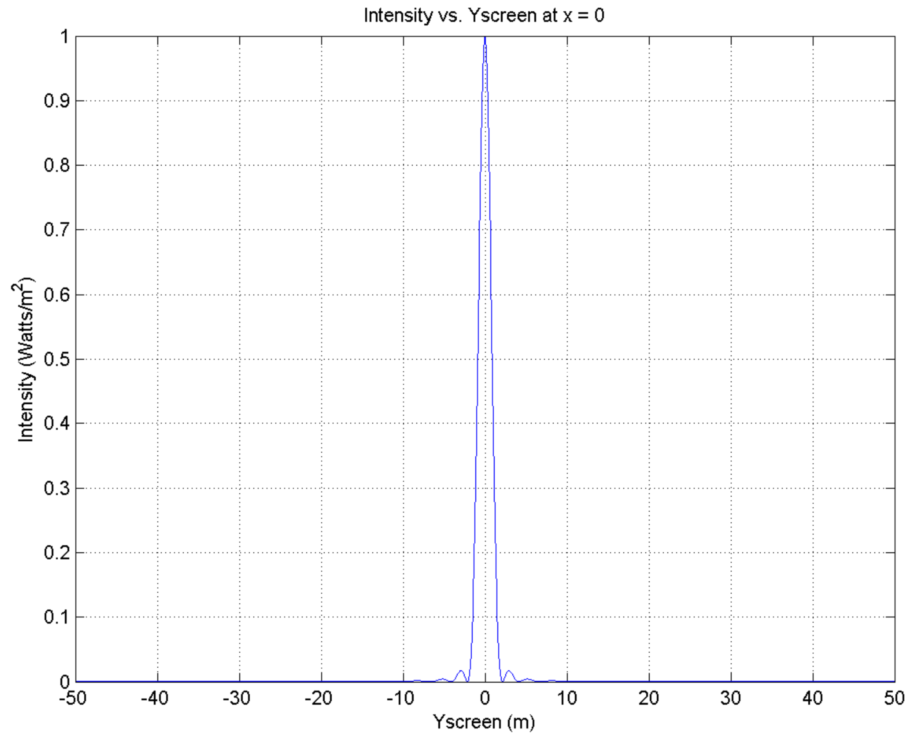
$$\delta\left(\theta_y\right) = k\Delta L\left(\theta_y\right) = \left(2\pi/\lambda\right)\Delta L\left(\theta_y\right) = \left(2\pi f/v_{air}\right)\Delta L\left(\theta_y\right) = \left(2\pi f/v_{air}\right)d\sin\theta_y .$$

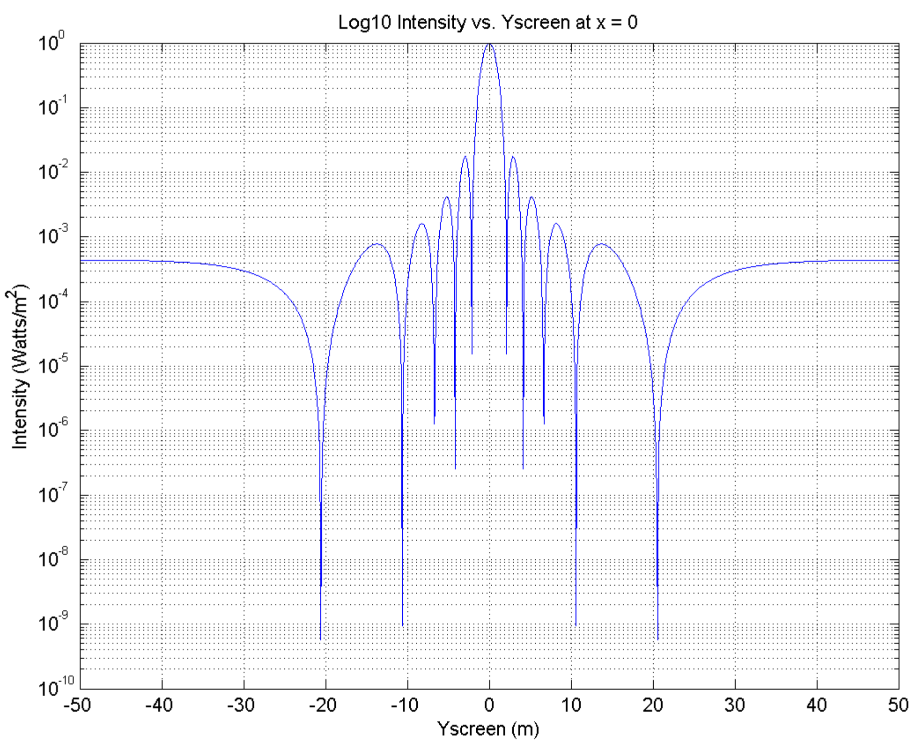
Linear and Semilog Plots of I_{tot} *vs.* θ_y for $N = 1$, $R = d = 1.0\ m$:



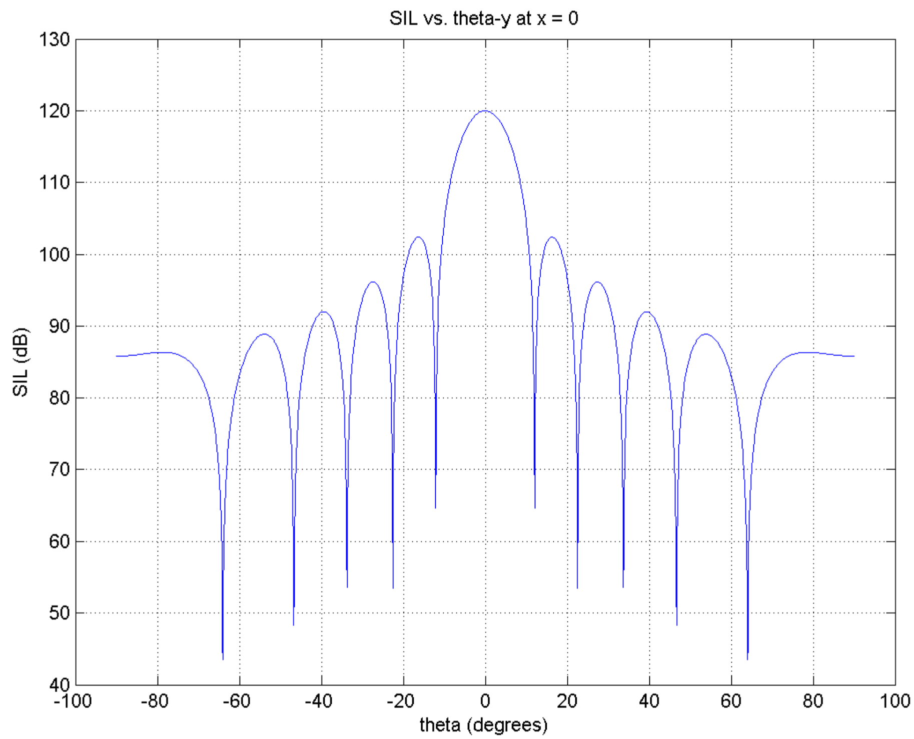


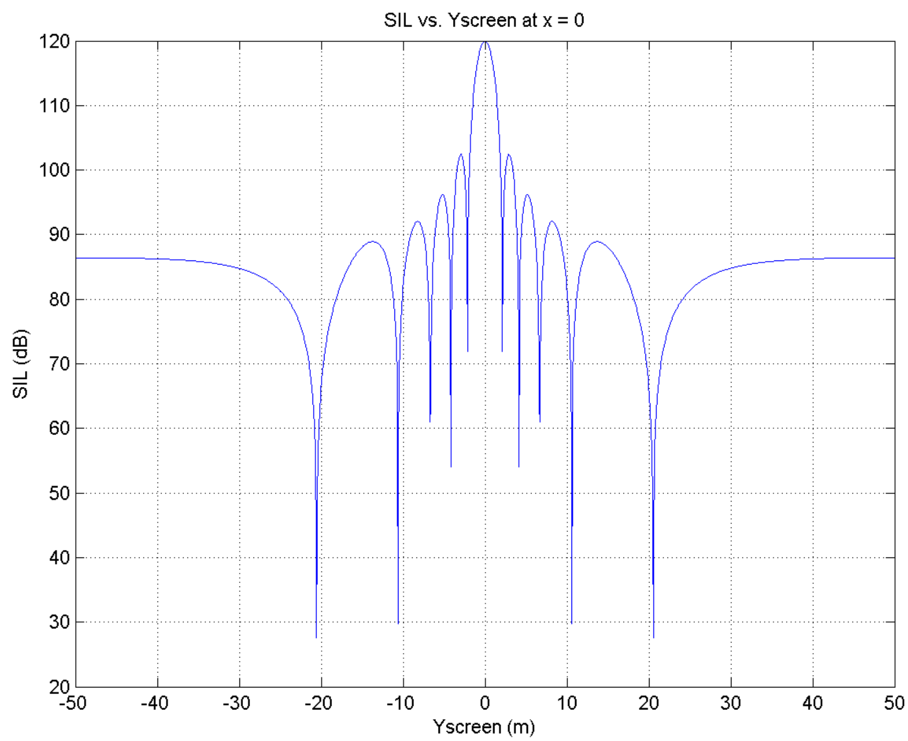
Linear and Semilog Plots of I_{tot} *vs.* y_{screen} for $N = 1$, $R = d = 1.0\ m$:



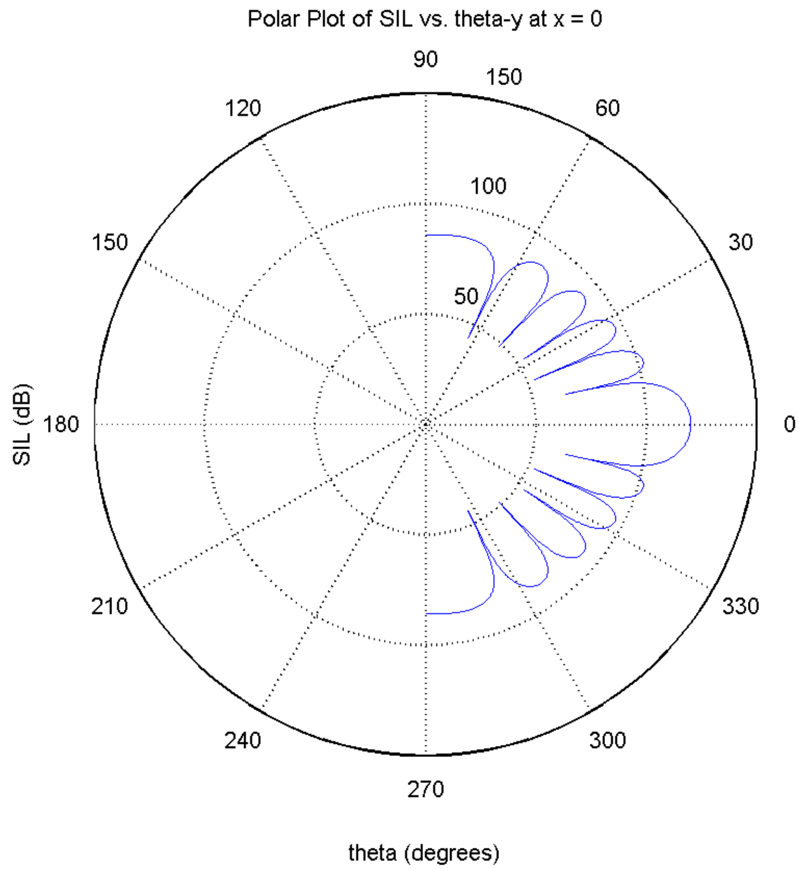


Plots of SIL $vs.$ θ_y and SIL $vs.$ y_{screen} for $N = 1$, $R = d = 1.0\ m$:

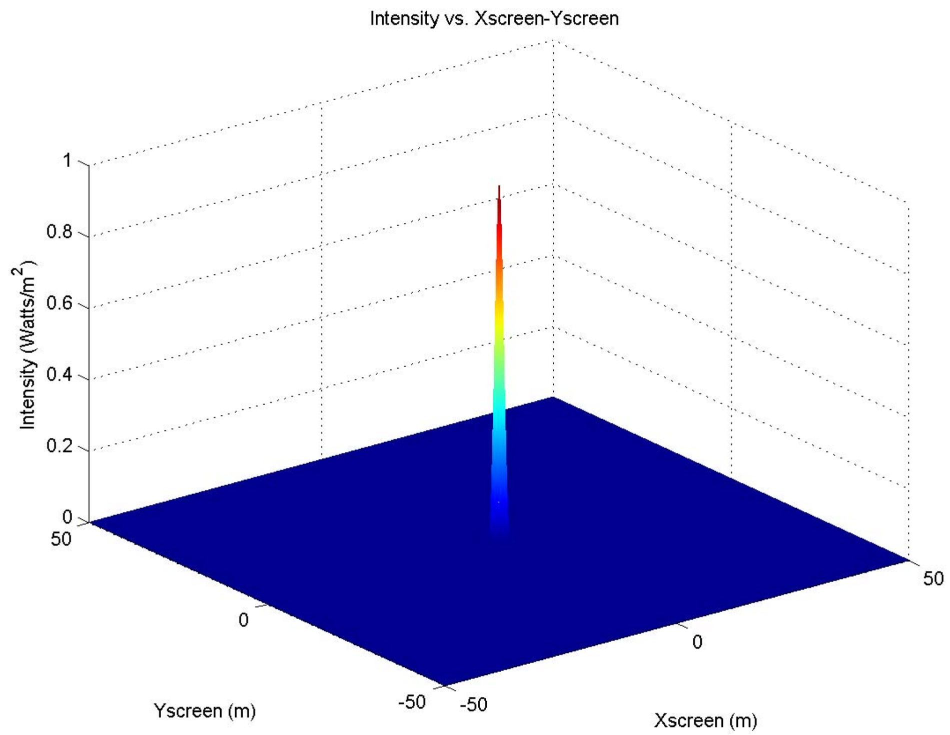


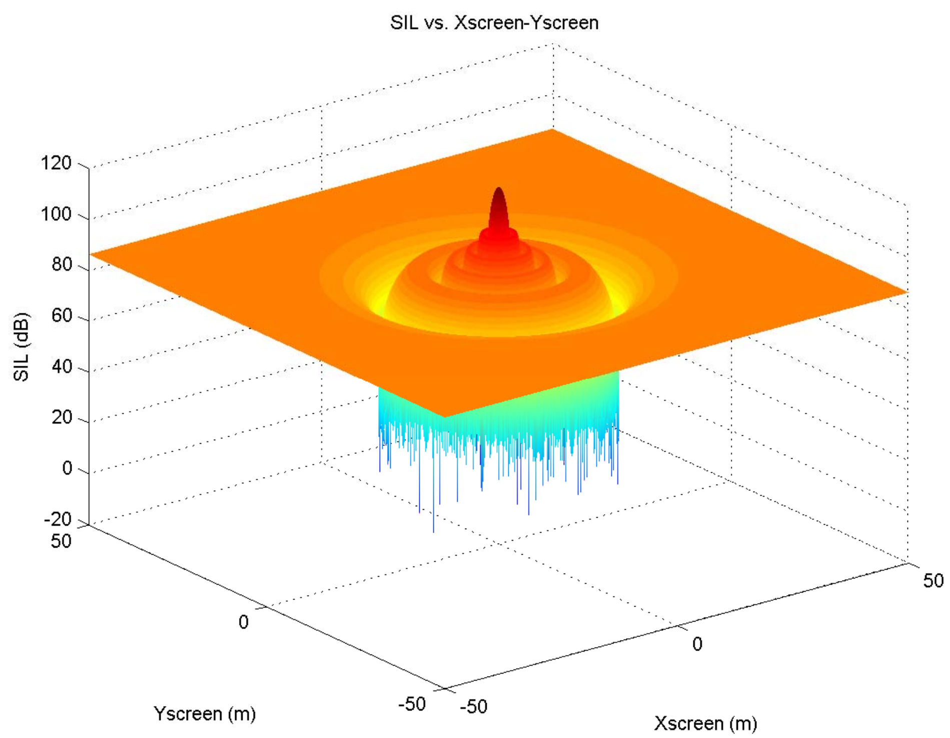


Polar plot of SIL *vs.* θ_y for $N = 1$, $R = d = 1.0\ m$:

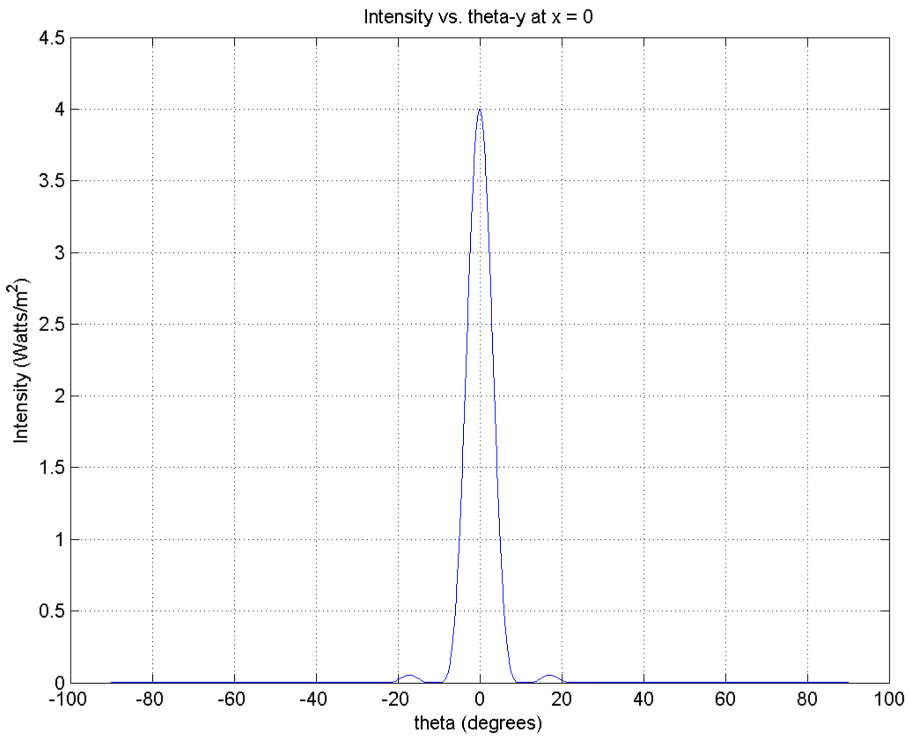


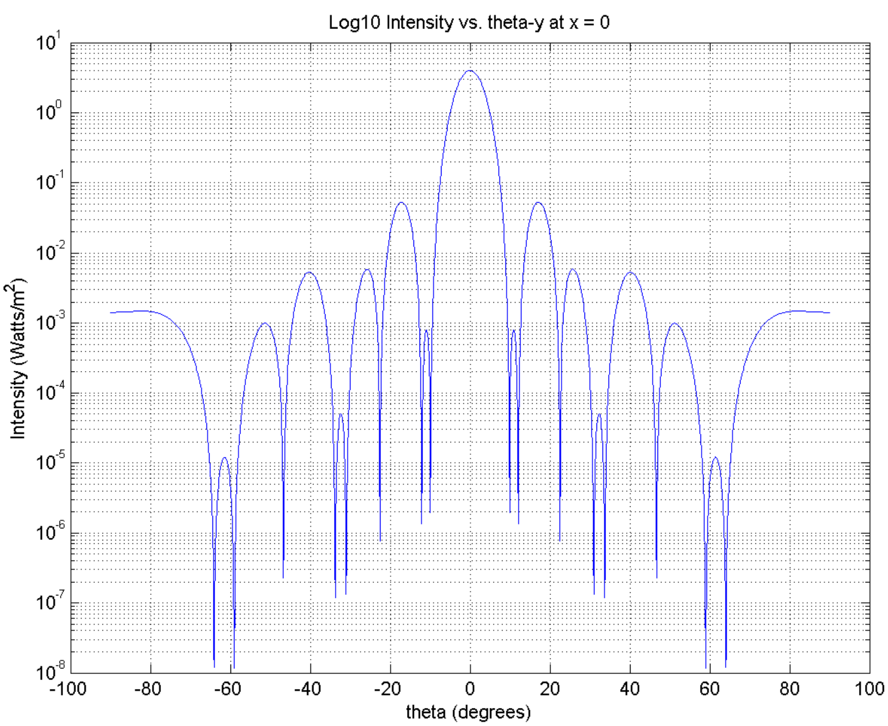
Plots of $I_{tot}(x_{scr}, y_{scr})$ *vs.* (x_{scr}, y_{scr}) and $SIL(x_{scr}, y_{scr})$ *vs.* (x_{scr}, y_{scr}) for $N = 1$, $R = d = 1.0$ m:



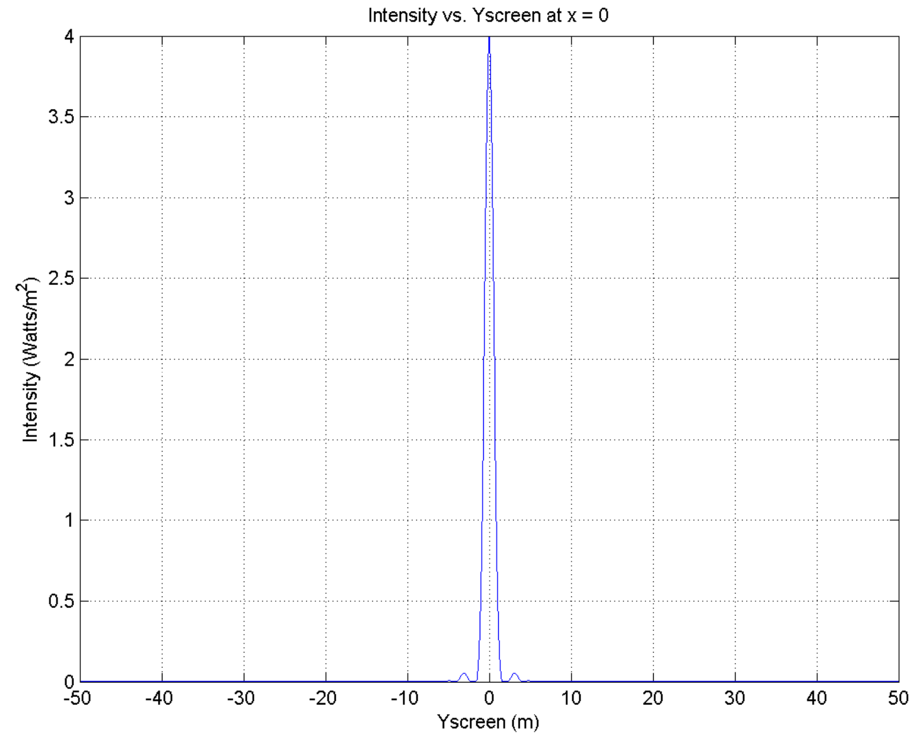


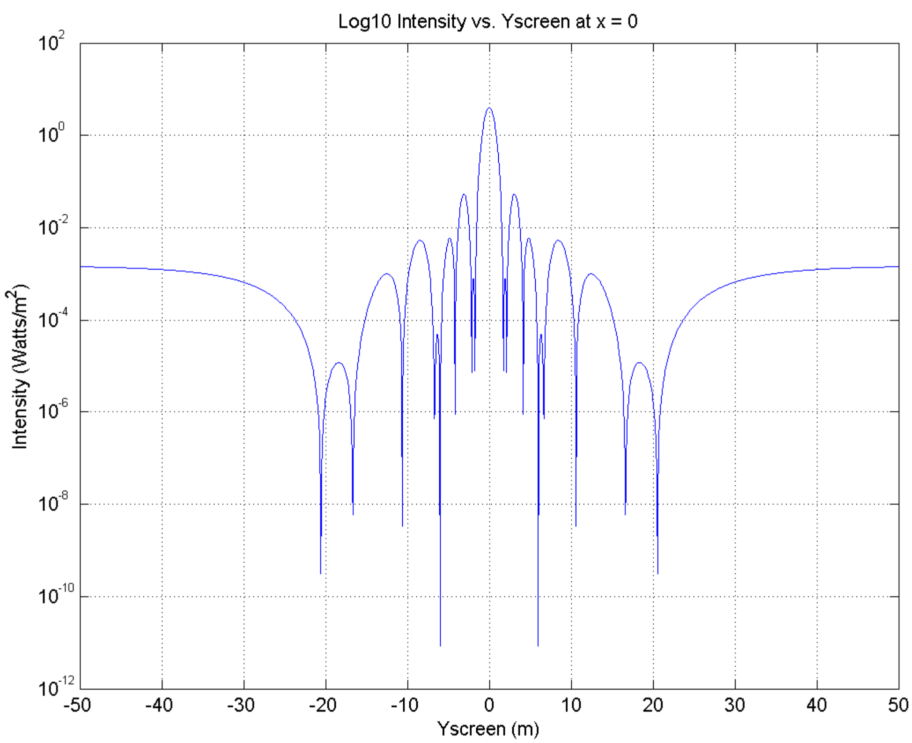
Linear and Semilog Plots of I_{tot} *vs.* θ_y for $N = 2$, $R = d = 1.0\ m$:



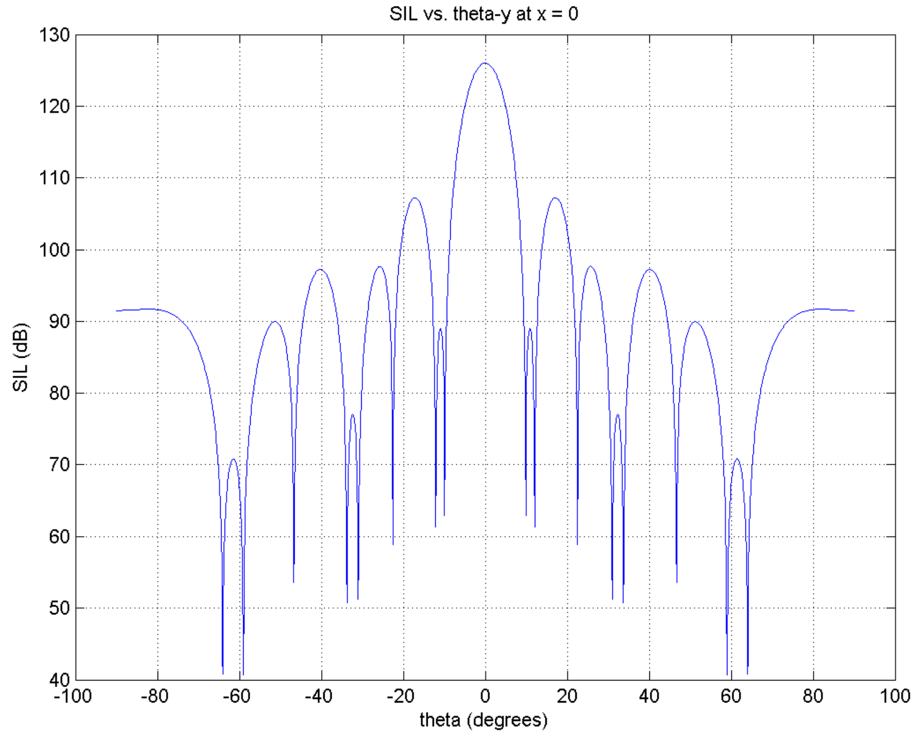


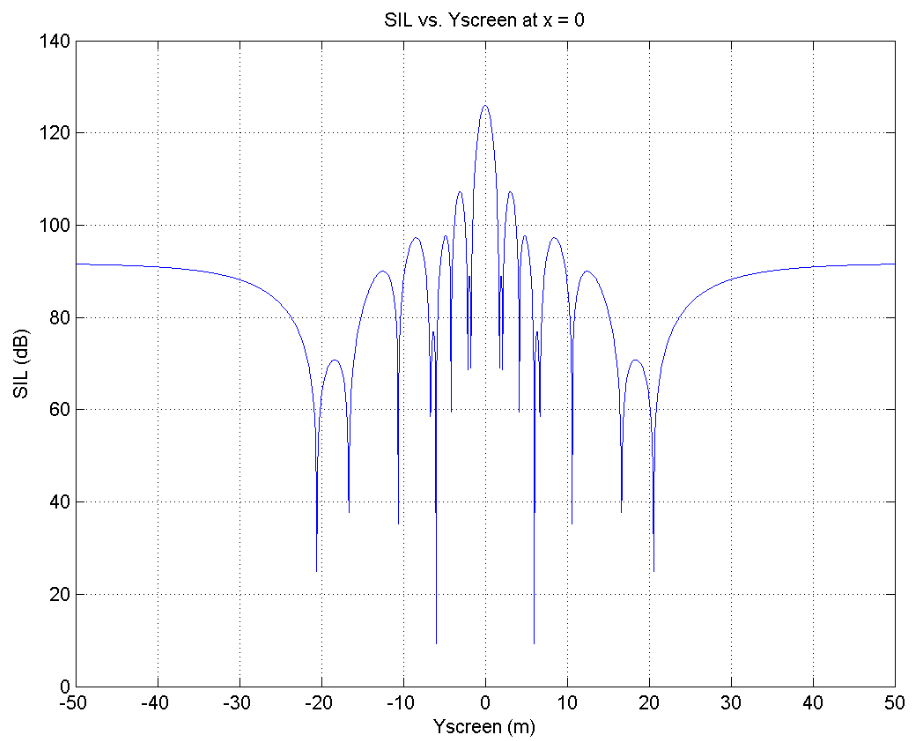
Linear and Semilog Plots of I_{tot} *vs.* y_{screen} for $N = 2$, $R = d = 1.0\ m$:



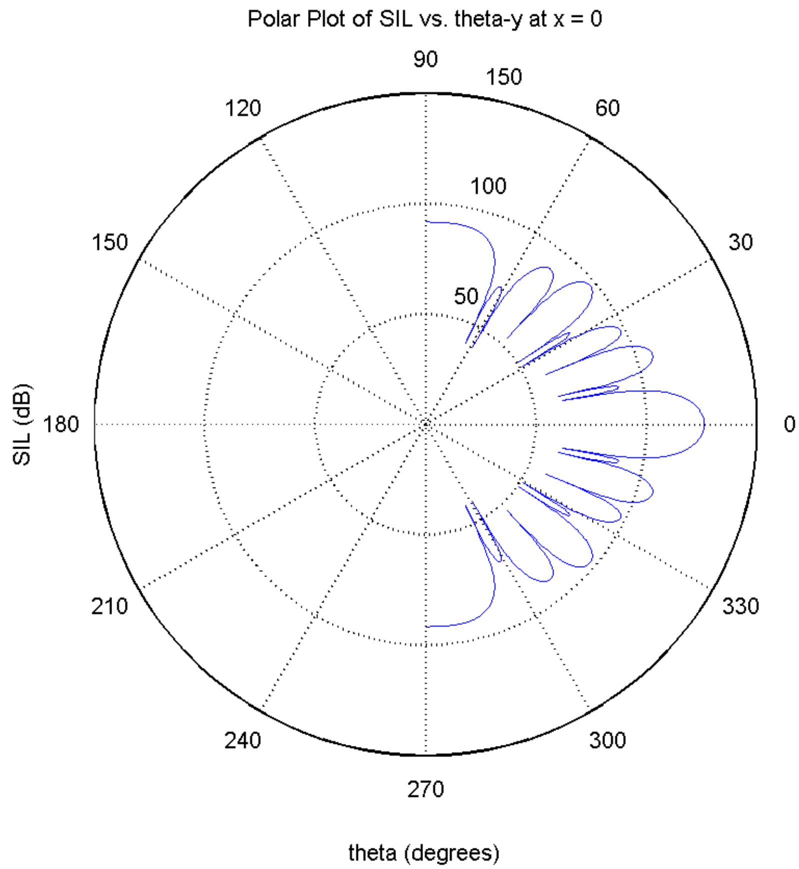


Plots of SIL $vs.$ θ_y and SIL $vs.$ y_{screen} for N = 2, R = d = 1.0 m:

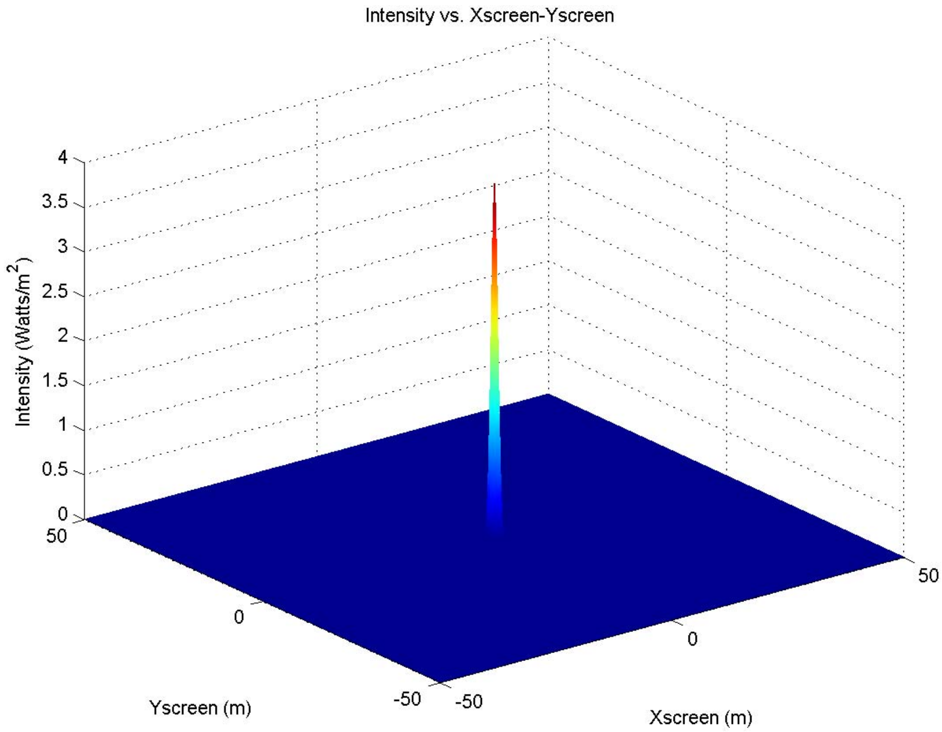


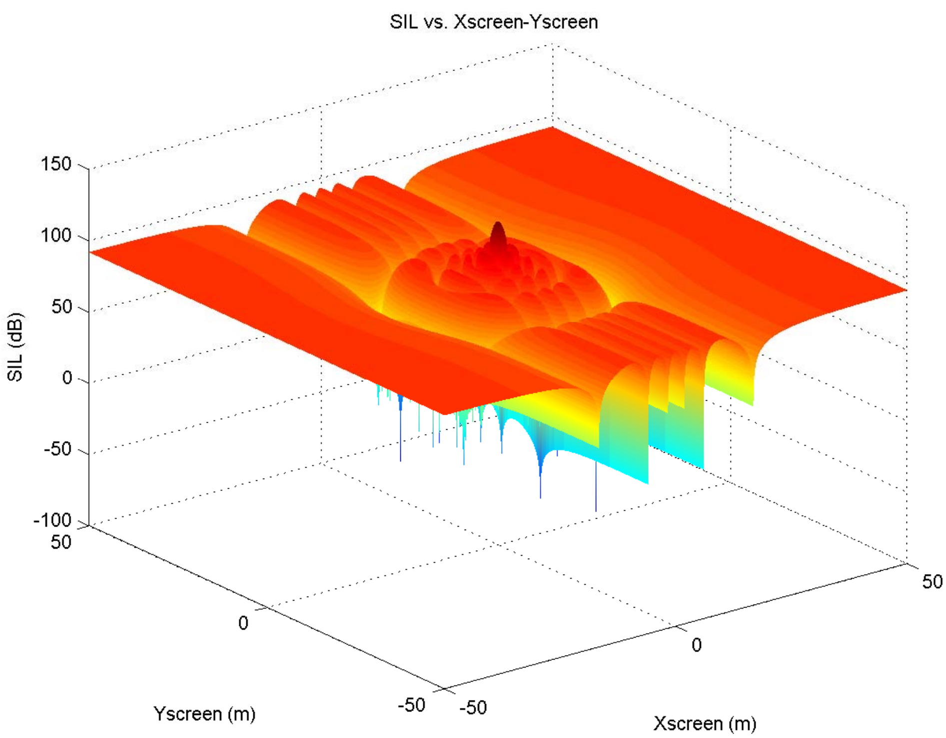


Polar plot of SIL *vs.* θ_y for $N = 2$, $R = d = 1.0\ m$:

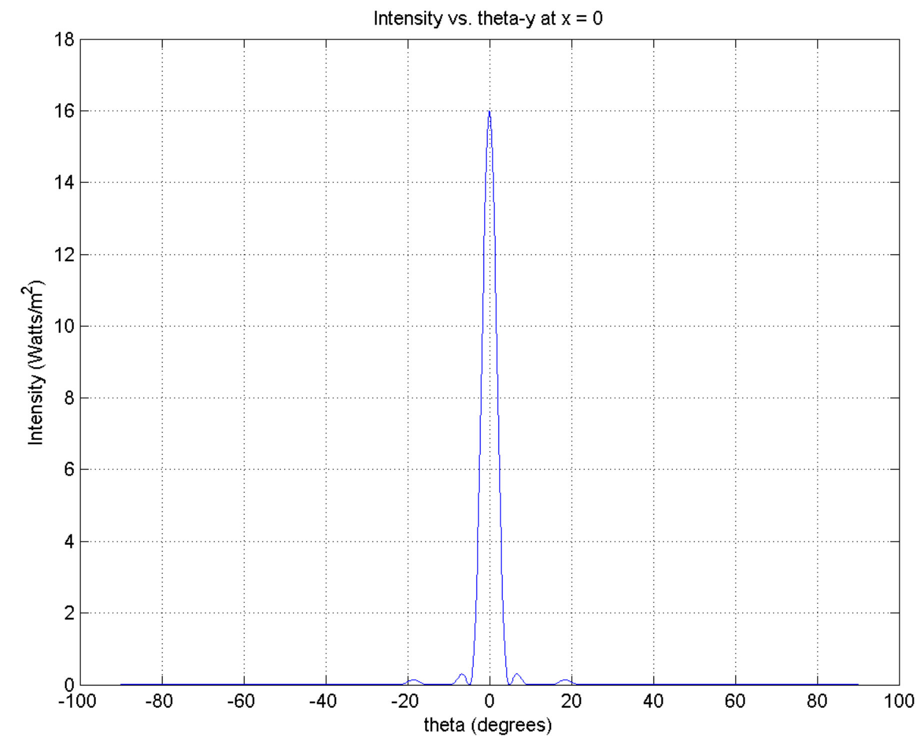


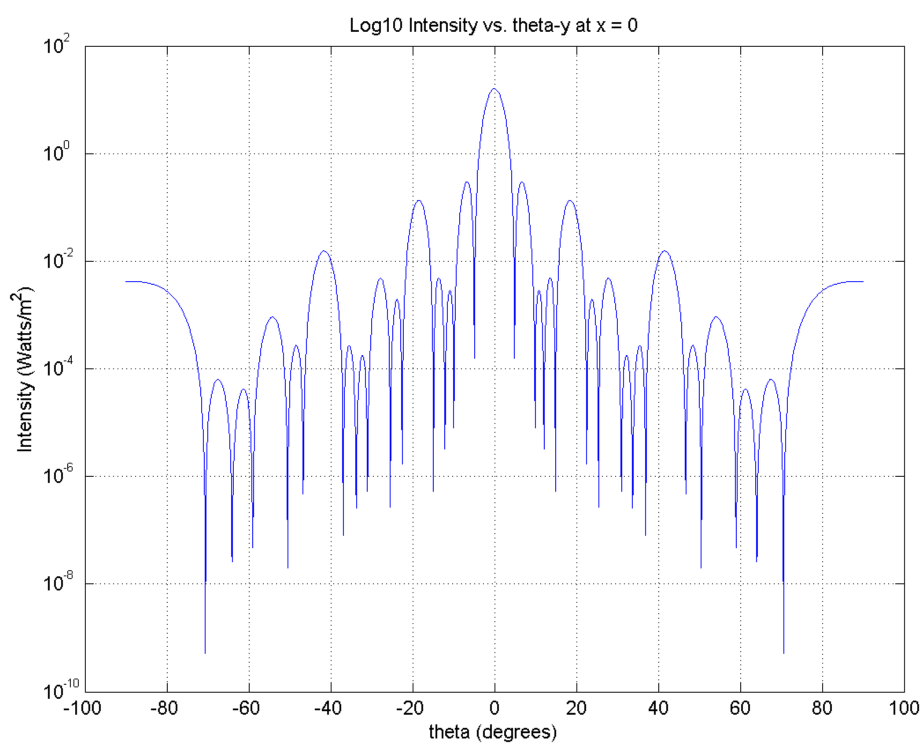
Linear and Semilog Plots of $I_{tot}\left(x_{scr}, y_{scr}\right) vs. \left(x_{scr}, y_{scr}\right)$ for $N = 2$, $R = d = 1.0\ m$:



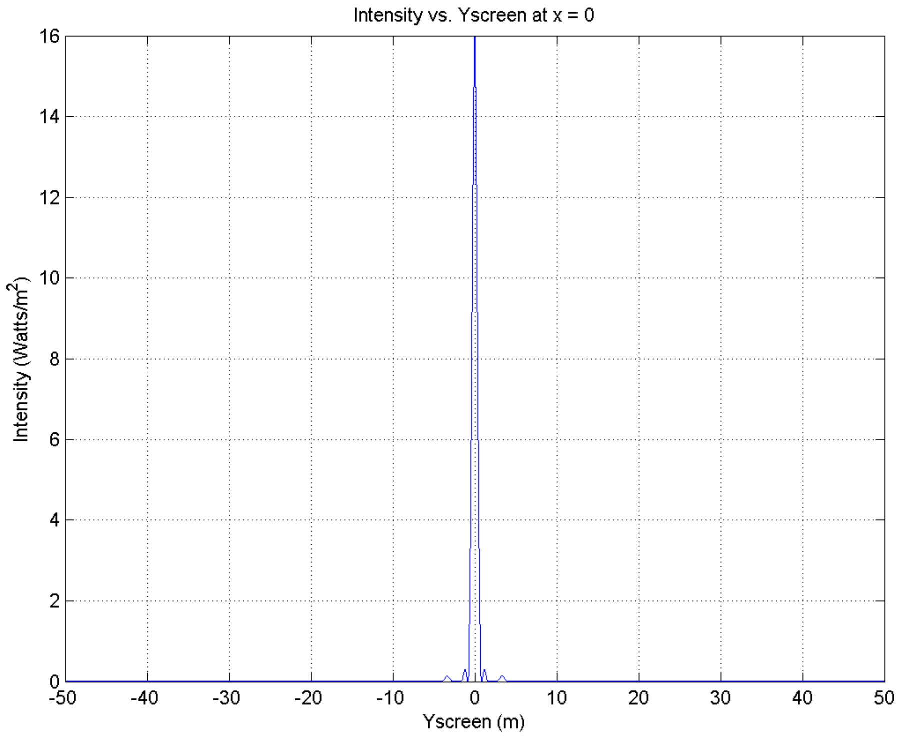


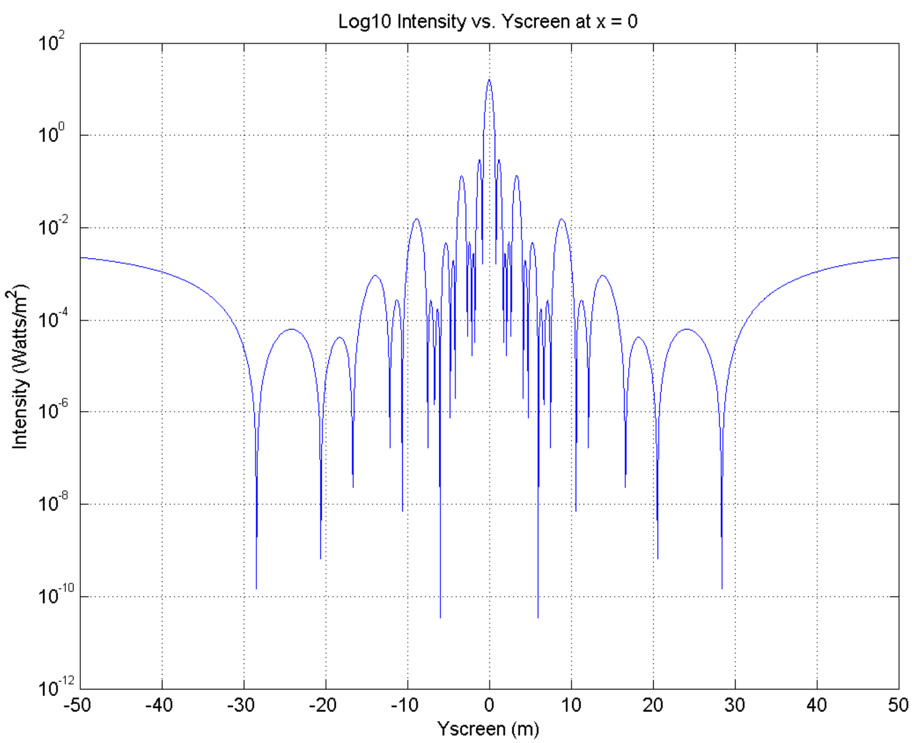
Linear and Semilog Plots of I_{tot} *vs.* θ_y for $N = 4$, $R = d = 1.0\ m$:



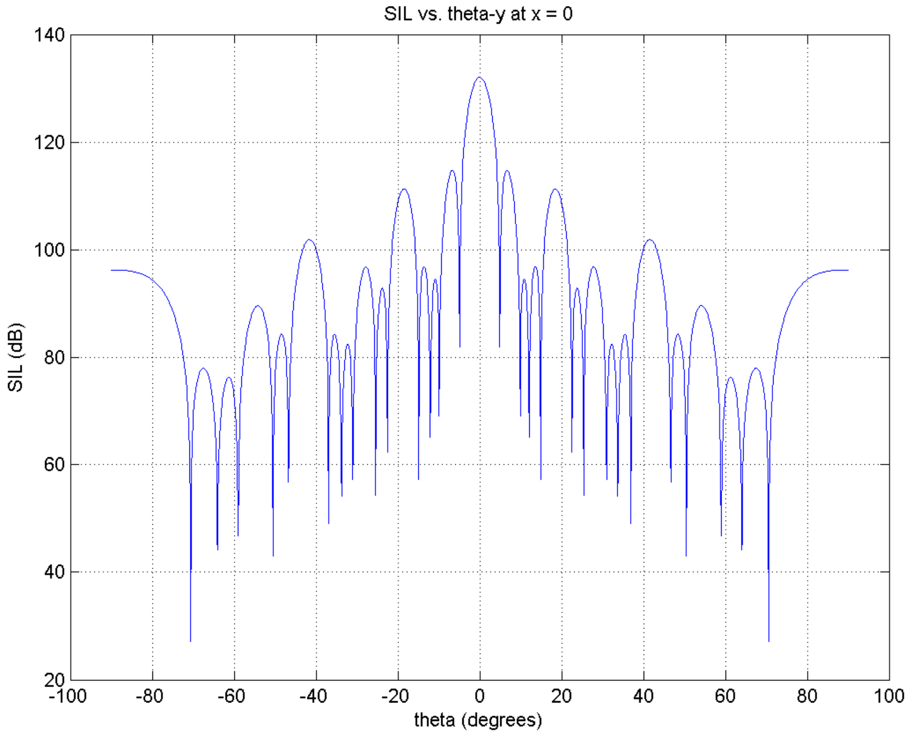


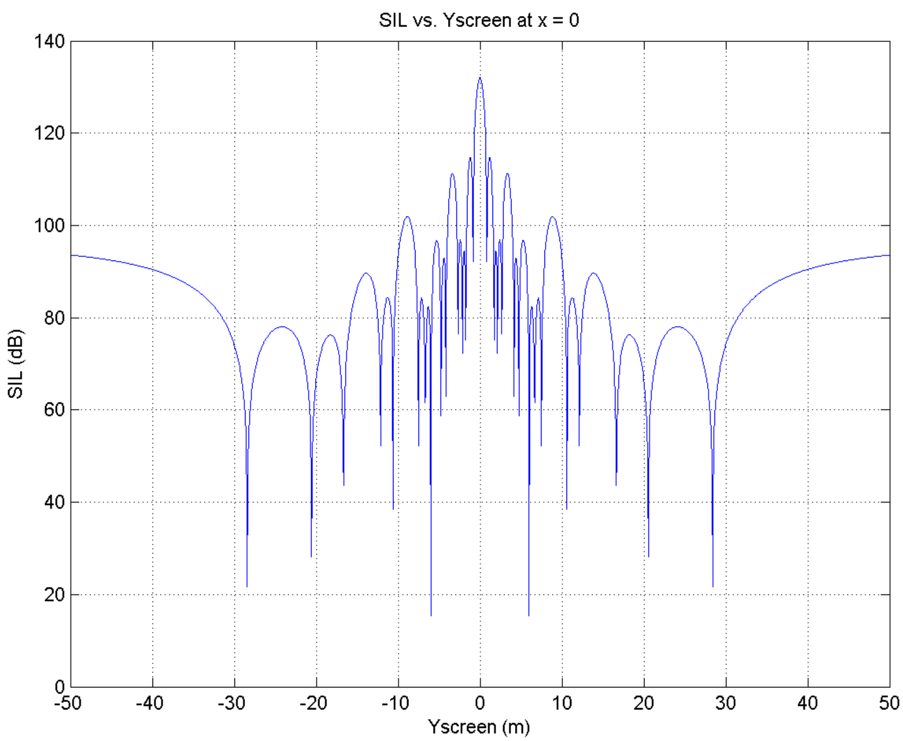
Linear and Semilog Plots of I_{tot} *vs.* y_{screen} for $N = 4$, $R = d = 1.0\ m$:



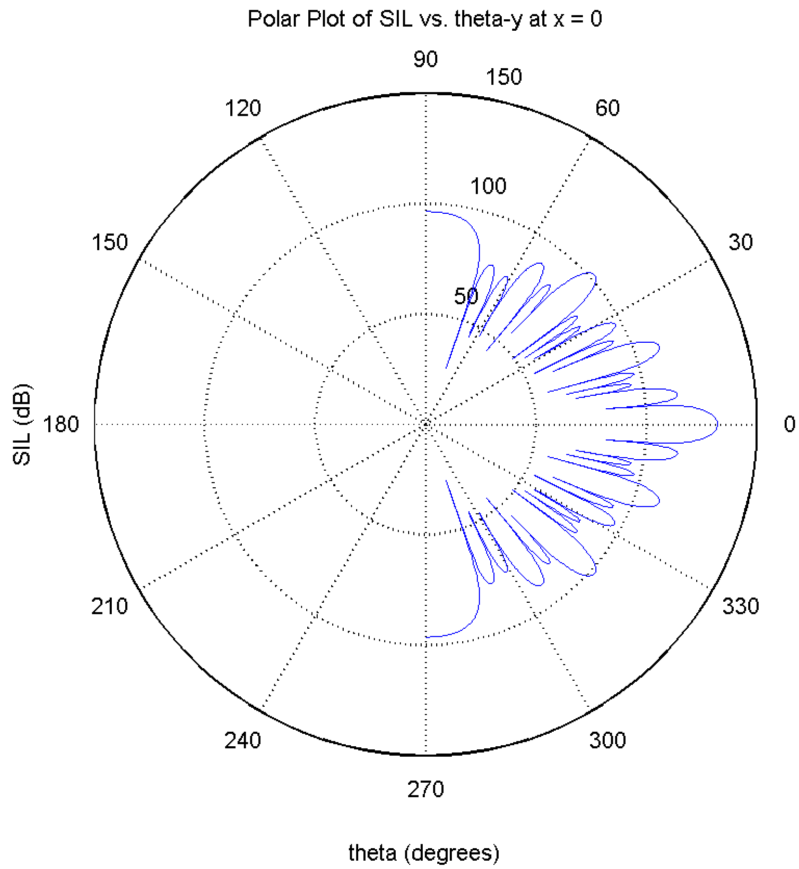


Plots of $SIL\ vs.\ \theta_y$ and $SIL\ vs.\ y_{screen}$ for $N = 4$, $R = d = 1.0\ m$:

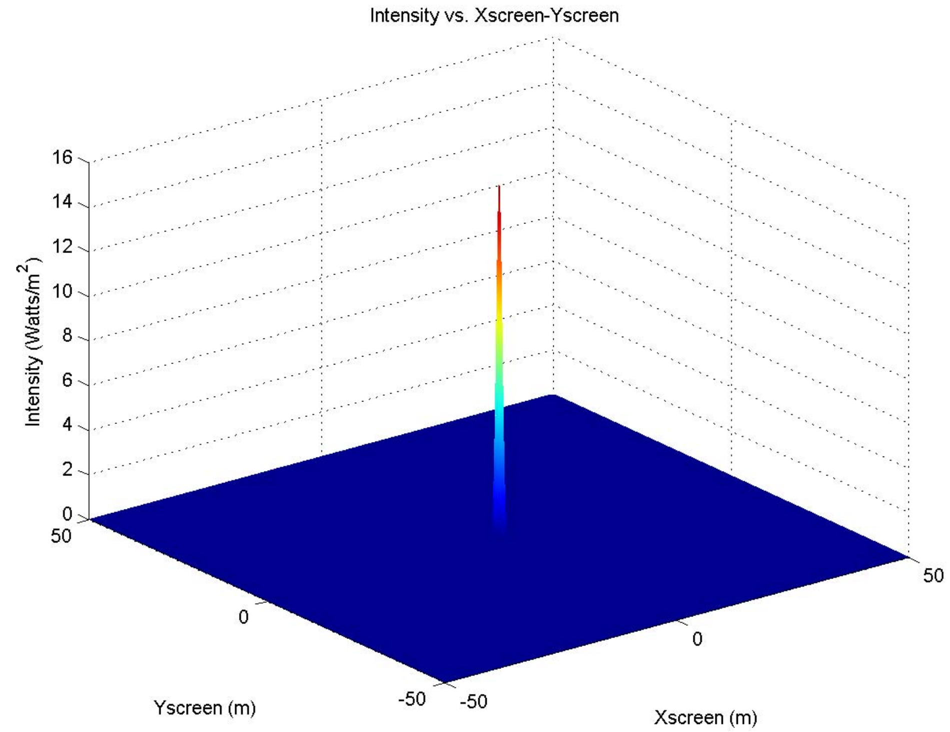


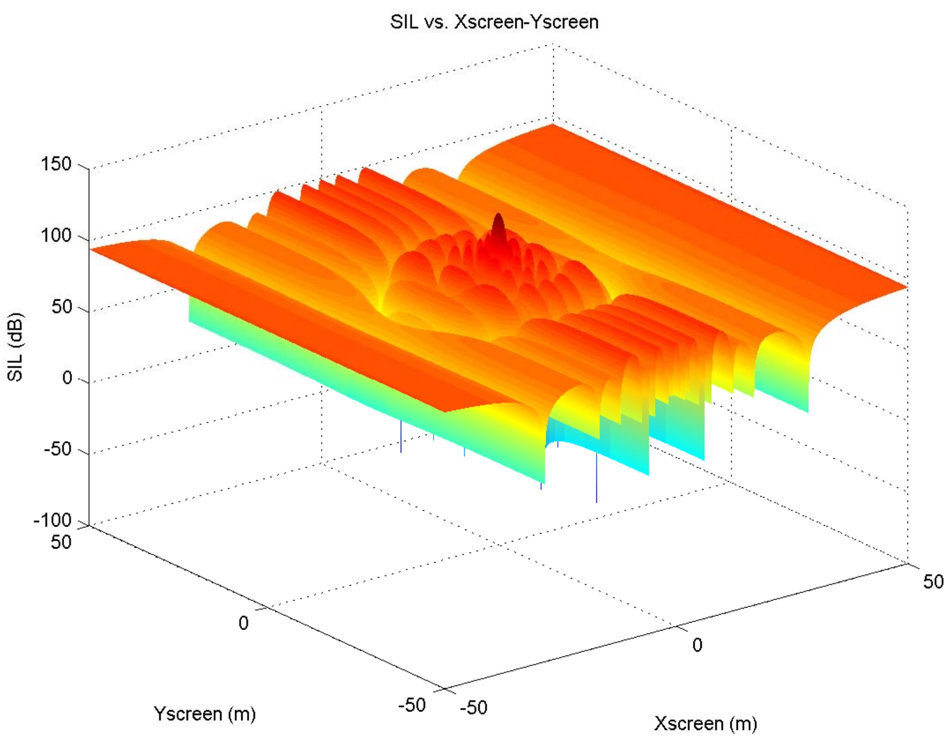


Polar plot of *SIL vs.* θ_y for $N = 4$, $R = d = 1.0\ m$:

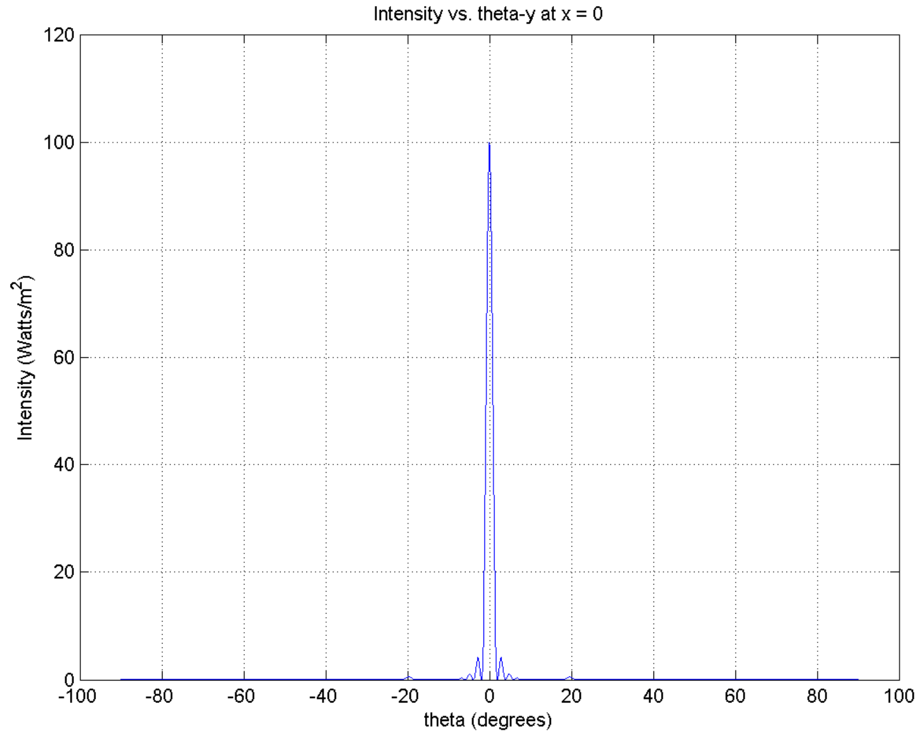


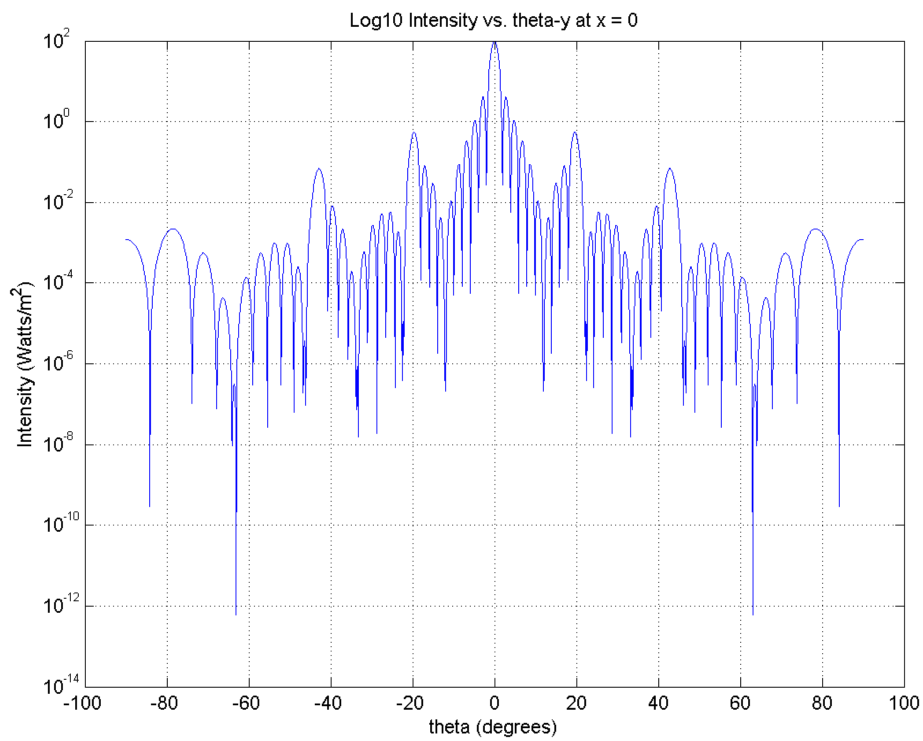
Linear and Semilog Plots of $I_{tot}\left(x_{scr}, y_{scr}\right)$ *vs.* $\left(x_{scr}, y_{scr}\right)$ for $N = 4$, $R = d = 1.0\ m$:



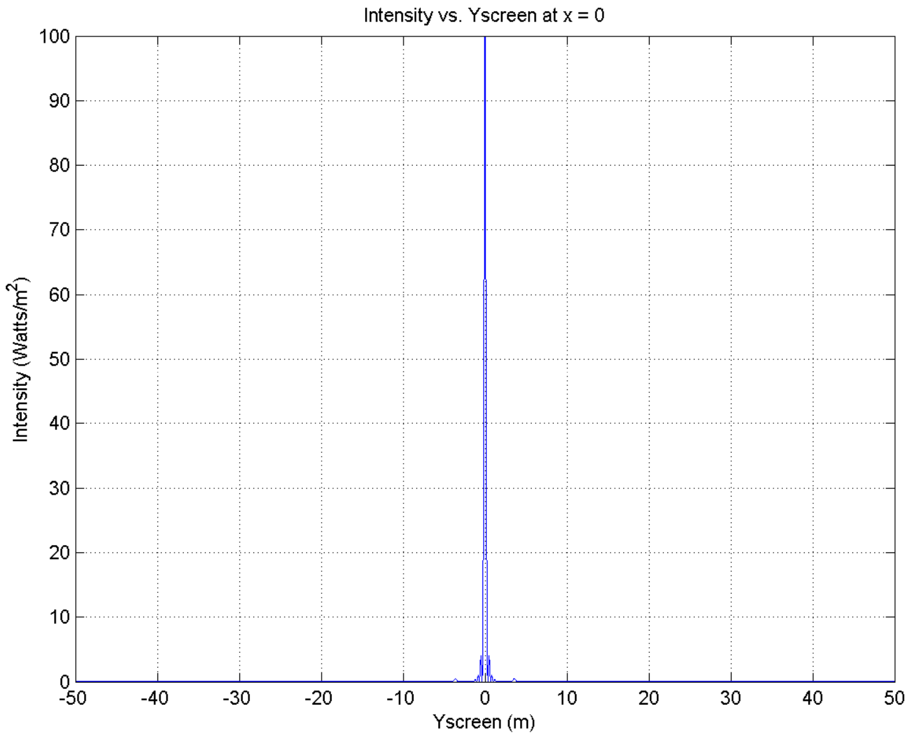


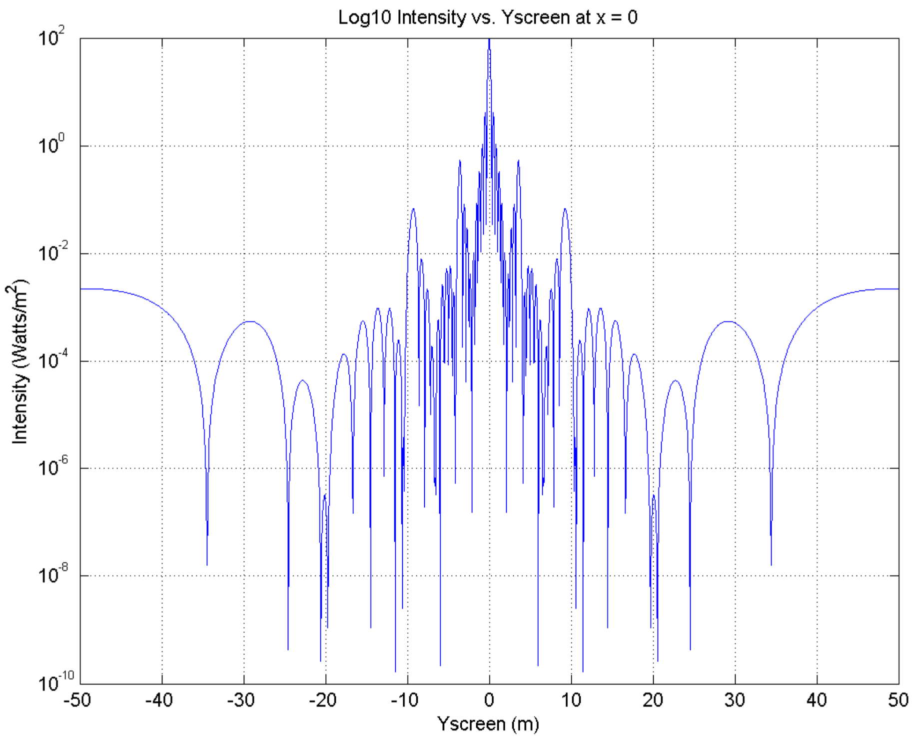
Linear and Semilog Plots of I_{tot} *vs.* θ_y for $N = 10$, $R = d = 1.0\ m$:



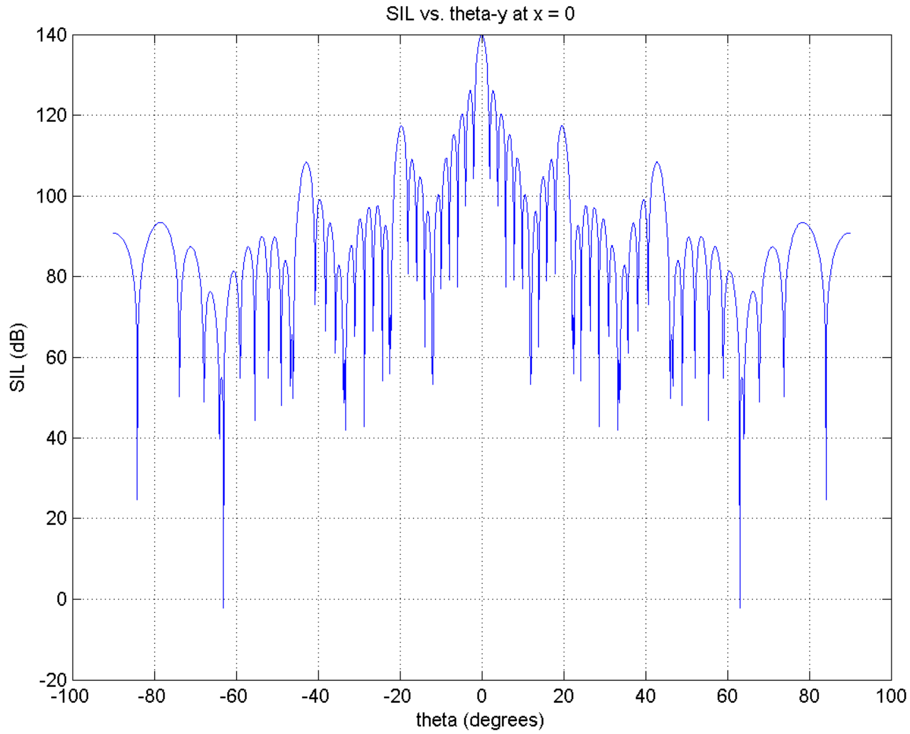


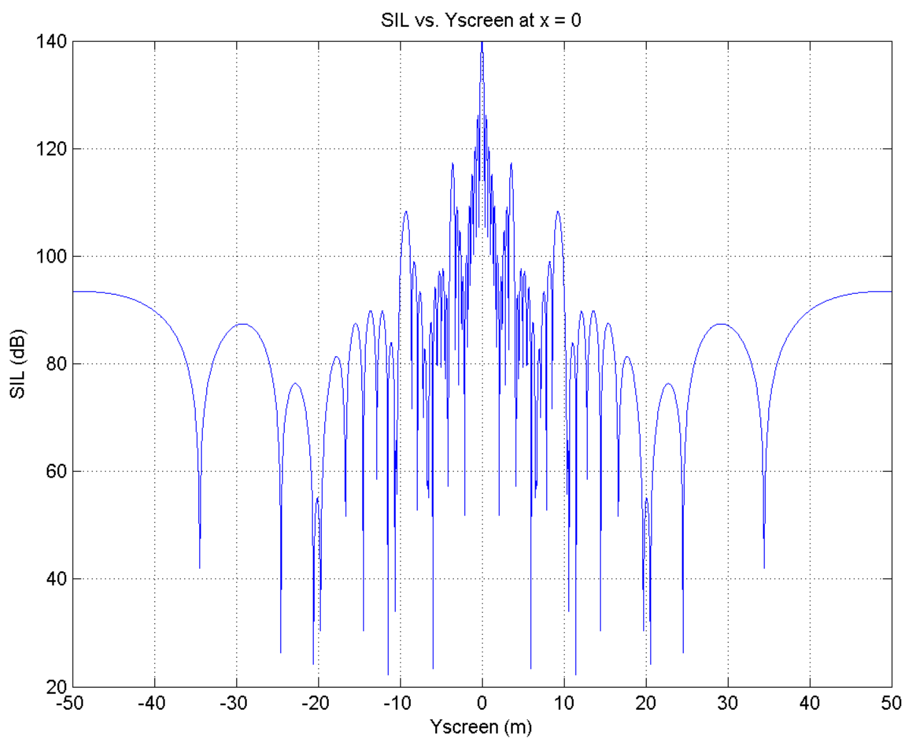
Linear and Semilog Plots of I_{tot} *vs.* y_{screen} for $N = 10$, $R = d = 1.0$ m:



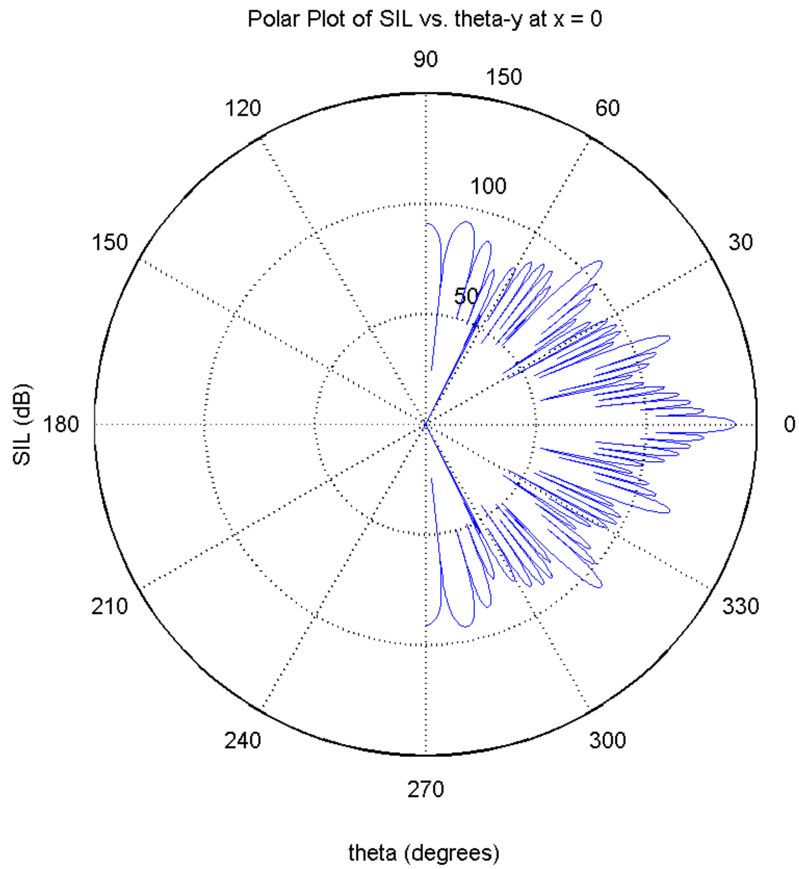


Plots of SIL *vs.* θ_y and SIL *vs.* y_{screen} for N = 10, R = d = 1.0 m:

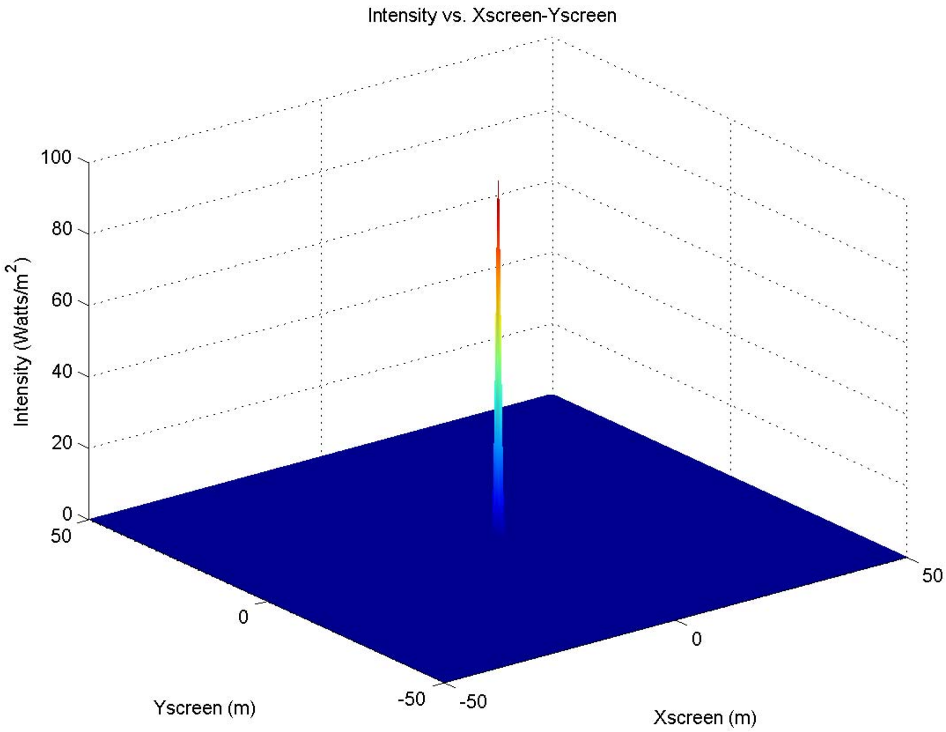


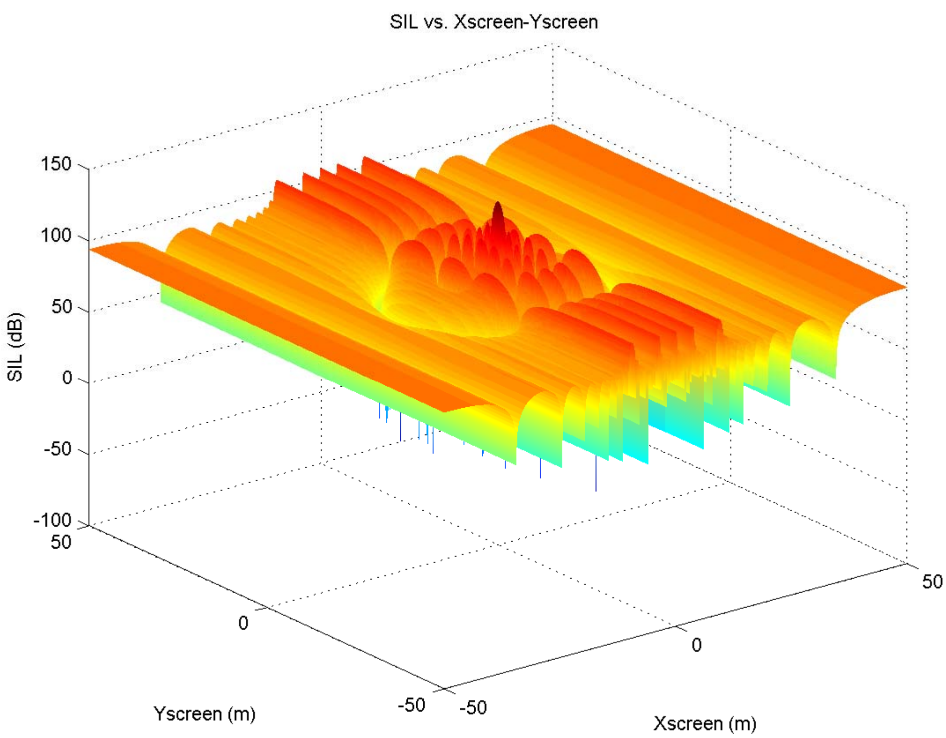


Polar plot of SIL *vs.* θ_y for $N = 10$, $R = d = 1.0\ m$:



Linear and Semilog Plots of $I_{tot}\left(x_{scr}, y_{scr}\right) vs. \left(x_{scr}, y_{scr}\right)$ for $N = 10$, $R = d = 1.0\ m$:





Listing of the MATLAB code:

```
%==========================================================================
% Intf_Diffn_1D_Circ_Aperture_Thy.m
%
% 1-D interference-diffraction associated with N circular apertures
% - simplest theory - far-field/plane-wave approx!
% Sound waves assumed to be propagating in free air/great wide-open!
%
% N circular apertures distributed symmetrically along horizontal y-axis
% x-axis is vertical. x-y-z right-handed coordinate system.
%==========================================================================
%
% Written by Prof. Steven Errede  Last Updated: Feb. 8, 2011 10:30 hr
%
%==========================================================================
close all;
clear all;

single     thtr(1800);
single     thtd(1800);
single    Itot1(1800);
single     SIL1(1800);

single     yscr(2000);
single    Itot2(2000);
single     SIL2(2000);

single   Itotxy(2000,2000);
single LgItotxy(2000,2000);
single    SILxy(2000,2000);


% Specify the # of apertures:
Napr   = 10; % 1; 2; 4; 10;

% Specify the numerical values of parameters:
Io     =    1.0;    % intensity from single slit (Watts/m^2)
Ir     = 1.0*10^-12;% reference sound intensity  (Watts/m^2)
Vair   =  343.0;    % speed of propagation of sound - free air (m/s)
freq   = 1000.0;    % frequency (Hz or cps)
lambda = Vair/freq; % wavelength (m)
Lobs   =   10.0;    % observer distance         (m) n.b. lambda << Lobs
Dsrc   =    1.0;    % distance between apertures(m) n.b.   Dsrc << Lobs
Rapr   =    1.0;    % 0.6; 1.0; aperture radius (m) n.b.   Rapr << Lobs

nu     =      1;    % order of bessel function of 1st kind (see below)

%===================================================================
% Calculate Itot, SIL vs. theta-y along horizontal y-axis @ x = 0:
%===================================================================
Thetad =  -90.0;    % angle theta of observer in degrees
dTheta =    0.1;    % step angle in degrees

for i = 1:1800;
     thtd(i) = Thetad;            % angle theta of observer in degrees
     Thetar  = (pi/180.0)*Thetad; % angle theta of observer in radians
     thtr(i) = Thetar;

     delta   = ((2.0*pi*Dsrc)/lambda)*sin(Thetar);  % int'f phase (radians)
     rho     = ((2.0*pi*Rapr)/lambda)*sin(Thetar); % diffn phase (radians)

    Itot1(i) = Io*(sin(Napr*delta/2.0)/sin(delta/2.0))^2*((2.0*bessel(nu,rho))/rho)^2;
     SIL1(i) = 10.0*log10(Itot1(i)/Ir); % Sound Intensity Level (dB)

     Thetad  = Thetad + dTheta;   % increment angle for next calculation
end
```

```
%==================================================================
% Calculate Itot, SIL vs. yscreen along horizontal y-axis @ x = 0:
%==================================================================
 y = -50.00; % starting position on screen (m)
dy =   0.05; % step size on screen (m);
for i = 1:2000;
     yscr(i) = y;            % position of observer on perp. screen (m)
     Thetar  = atan(y/Lobs); % angle theta of observer in radians

     delta   = ((2.0*pi*Dsrc)/lambda)*sin(Thetar); % int'f phase (radians)
     rho     = ((2.0*pi*Rapr)/lambda)*sin(Thetar); % diffn phase (radians)

    Itot2(i) = Io*(sin(Napr*delta/2.0)/sin(delta/2.0))^2*((2.0*bessel(nu,rho))/rho)^2;
     SIL2(i) = 10.0*log10(Itot2(i)/Ir); % Sound Intensity Level (dB)

     y  = y + dy;   % increment screen position for next calculation
end

figure(01);
plot(thtd,Itot1,'b');
grid on;
xlabel('theta (degrees)');
ylabel('Intensity (Watts/m^{2})');
title('Intensity vs. theta-y at x = 0');

figure(02);
semilogy(thtd,Itot1,'b');
grid on;
xlabel('theta (degrees)');
ylabel('Intensity (Watts/m^{2})');
title('Log10 Intensity vs. theta-y at x = 0');

figure(03);
plot(thtd,SIL1,'b');
grid on;
xlabel('theta (degrees)');
ylabel('SIL (dB)');
title('SIL vs. theta-y at x = 0');

figure(04);
polar(thtr,SIL1,'b');
grid on;
xlabel('theta (degrees)');
ylabel('SIL (dB)');
title('Polar Plot of SIL vs. theta-y at x = 0');


figure(11);
plot(yscr,Itot2,'b');
grid on;
xlabel('Yscreen (m)');
ylabel('Intensity (Watts/m^{2})');
title('Intensity vs. Yscreen at x = 0');

figure(12);
semilogy(yscr,Itot2,'b');
grid on;
xlabel('Yscreen (m)');
ylabel('Intensity (Watts/m^{2})');
title('Log10 Intensity vs. Yscreen at x = 0');

figure(13);
plot(yscr,SIL2,'b');
grid on;
xlabel('Yscreen (m)');
ylabel('SIL (dB)');
title('SIL vs. Yscreen at x = 0');
```

```
%===========================================================================
fprintf('\n Very CPU-intensive I(x,y) vs. x,y calcs - please be patient!! \n')
%===========================================================================

%=========================================
% Calculate 2D Itot, SIL vs. x,y-screen:
%=========================================
 x = -50.00; % x-starting position on screen (m)
dx =   0.05; % x-step size on screen (m);
for j = 1:2000;
     xscr(j) = x; % x-position of observer on perp. screen (m)

     y = -50.00; % y-starting position on screen (m)
    dy =   0.05; % y-step size on screen (m);
    for i = 1:2000;
         yscr(i) = y; % y-position of observer on perp. screen (m)
         rscr    = sqrt((xscr(j))^2 + (yscr(i))^2); % radial pos'n on perp. screen (m)
         Thetar  = atan(rscr/Lobs); % angle theta of observer in radians
         Thetay  = atan(y/Lobs);    % y-projected angle theta-y in radians
         delta   = ((2.0*pi*Dsrc)/lambda)*sin(Thetay); % int'f phase (radians)
         rho     = ((2.0*pi*Rapr)/lambda)*sin(Thetar); % diffn phase (radians)

     Itotxy(j,i) = Io*(sin(Napr*delta/2.0)/sin(delta/2.0))^2*((2.0*bessel(nu,rho))/rho)^2;
   LgItotxy(j,i) = log10(Itotxy(j,i));     % log10 of total intensity (Watts/m^2)
      SILxy(j,i) = 10.0*log10(Itotxy(j,i)/Ir);      % Sound Intensity Level (dB)

         y  = y + dy;   % increment y-screen position for next calculation
    end
     x  = x + dx;   % increment x-screen position for next calculation
end

figure(21);
surf(xscr,yscr,Itotxy);
shading interp;
xlabel('Xscreen (m)');
ylabel('Yscreen (m)');
zlabel('Intensity (Watts/m^{2})');
title ('Intensity vs. Xscreen-Yscreen');

figure(22);
surf(xscr,yscr,LgItotxy);
shading interp;
xlabel('Xscreen (m)');
ylabel('Yscreen (m)');
zlabel('Log10(Intensity) (Watts/m^{2})');
title ('Log10(Intensity) vs. Xscreen-Yscreen');

figure(23);
surf(xscr,yscr,SILxy);
shading interp;
xlabel('Xscreen (m)');
ylabel('Yscreen (m)');
zlabel('SIL (dB)');
title ('SIL vs. Xscreen-Yscreen');

%===========================================================================
beep;
fprintf('\n Calculation of intf-diffn thru 1-D array of N circular apertures completed !!! \n')
%===========================================================================
```