N-Slit Interference in 1-Dimension – Simplest Theory

In this example, we show plots of the sound intensity *vs*. angle and observer/listener position y_{screen} on a screen for the simplest theory of *N*-slit interference – where the *N* sound sources, all in phase with each other, are assumed to be comprised of infinitely long, parallel slits of infinitely narrow width – *i.e*. infinitesimally thin slits, each separated from each other by a distance *d*. The observer/listener is located far from the sound sources, a perpendicular distance $L\ (m)$ away, and such that the conditions $d \ll L$ **.and.** $\lambda \ll L$ both hold simultaneously, where $\lambda\ (m)$ is the wavelength of the sound – this is the so-called "far-field" limit.

The expression for 1-D *N*-slit interference in this simplest theory is given by (see P406POM Lecture Notes P406POM_Lect3_Part2):

$$I_{tot}(\theta) = I_o \left\{ \frac{\sin^2(N\delta(\theta)/2)}{\sin^2(\delta(\theta)/2)} \right\} \ (Watts/m^2) \text{ and } SIL(\theta) \equiv 10\log_{10}\left(I_{tot}(\theta)/I_{ref}\right)\ (dB)$$

where $I_o\ (Watts/m^2)$ is the sound intensity associated with a single slit, the phase $\delta(\theta) = k\Delta L(\theta)$ $(radians)$, $k = 2\pi/\lambda\ (radians/m)$ is the wavenumber and $\Delta L(\theta)\ (m)$ is the angle-dependent path length ***difference*** between adjacent sound sources to the observer/listener, located far away from the sound sources. $I_{ref} \equiv 10^{-12}\ (Watts/m^2)$ is the reference for the sound intensity level (SIL).

Minima – *i.e*. intensity zeroes (complete destructive interference) occur when the numerator factor $N\,\delta/2 = \pm\pi, \pm 2\pi, \pm 3\pi, \ldots = n\pi$, $n = \pm 1, \pm 2, \pm 3, \ldots$ *except* when the denominator factor simultaneously has $\delta/2 = \pm\pi, \pm 2\pi, \pm 3\pi, \ldots = n\pi$, $n = \pm 1, \pm 2, \pm 3, \ldots$ then have *global maxima* of the intensity, where $I_{tot} = N^2 I_o$.

From simple trigonometry, it is easy to show that the path length difference $\Delta L(\theta) = d\sin\theta$, where θ is the angle the observer/listener makes with respect to the normal, or forward axis of the array of *N* slits.

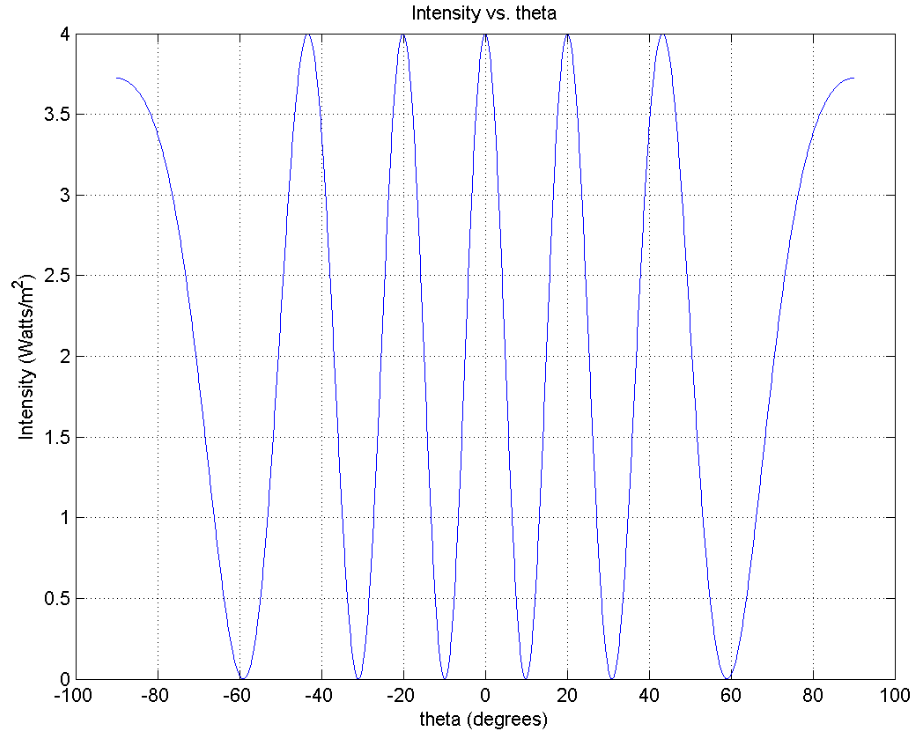
The corresponding location of the observer's position y_{screen} on a screen located a perpendicular distance *L* away from the *N* sound sources is: $y_{screen} = L\tan\theta$, or conversely: $\theta = \tan^{-1}(y_{screen}/L)$.

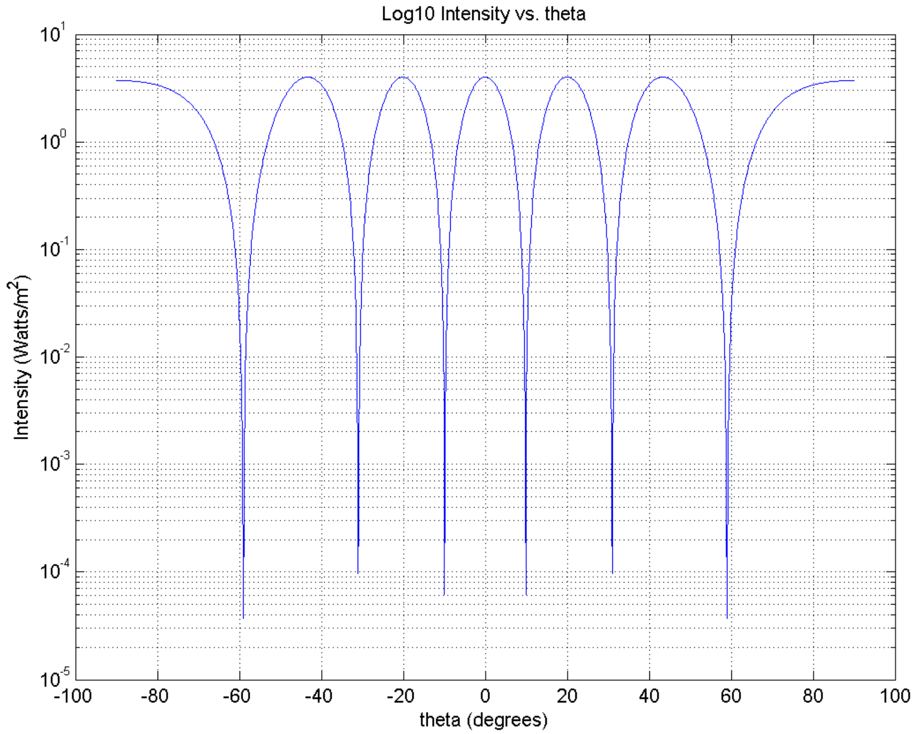
We coded up the above formulas using MATLAB to make plots of I_{tot} *vs.* θ and I_{tot} *vs.* y_{screen} *e.g*. for *N* = 2, 5 and 10 slits, with the following parameter values: $I_o = 1\ Watt/m^2$, slit separation distance(s) of $d = 1\,m$, observer/listener distance (at $\theta = 0$) of $L = 10\,m$, the speed of propagation in free air/great-wide open: $v_{air} = 343\,m/s$ and frequency of $f = 1000\,Hz$, thus $\lambda = v_{air}/f = 0.345\,m$.

In the following figures, note that the angular width of the maxima decreases as the number of slits *N* increases. Note also that the number of maxima/minima increases linearly with increasing frequency *f*, since the phase difference increases linearly with frequency:

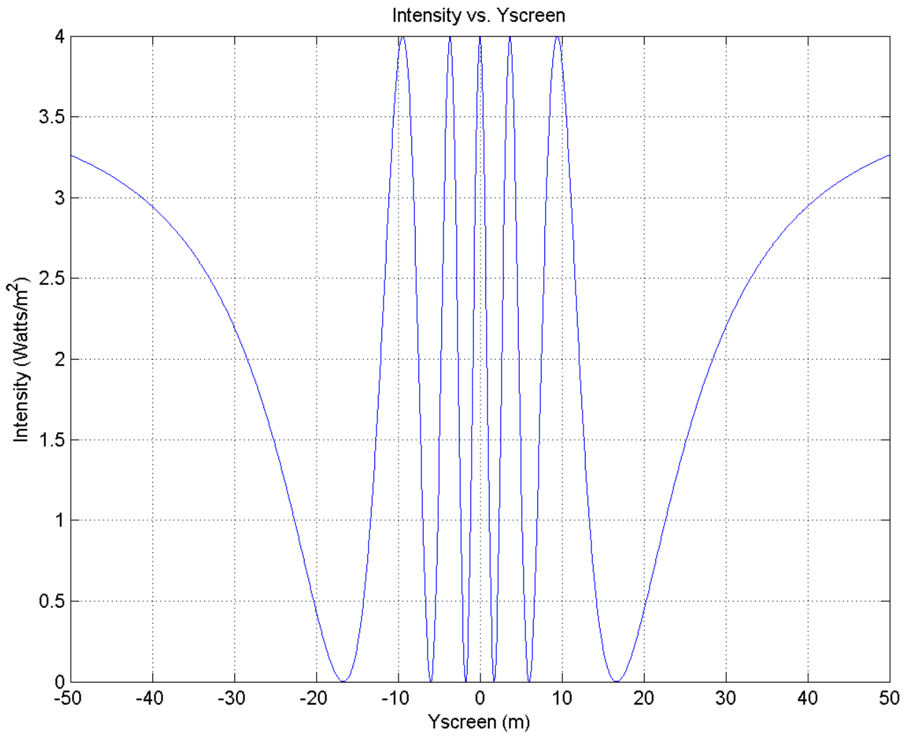
$$\delta(\theta) = k\Delta L(\theta) = (2\pi/\lambda)\Delta L(\theta) = (2\pi f/v_{air})\Delta L(\theta) = (2\pi f/v_{air})\,d\sin\theta\,.$$

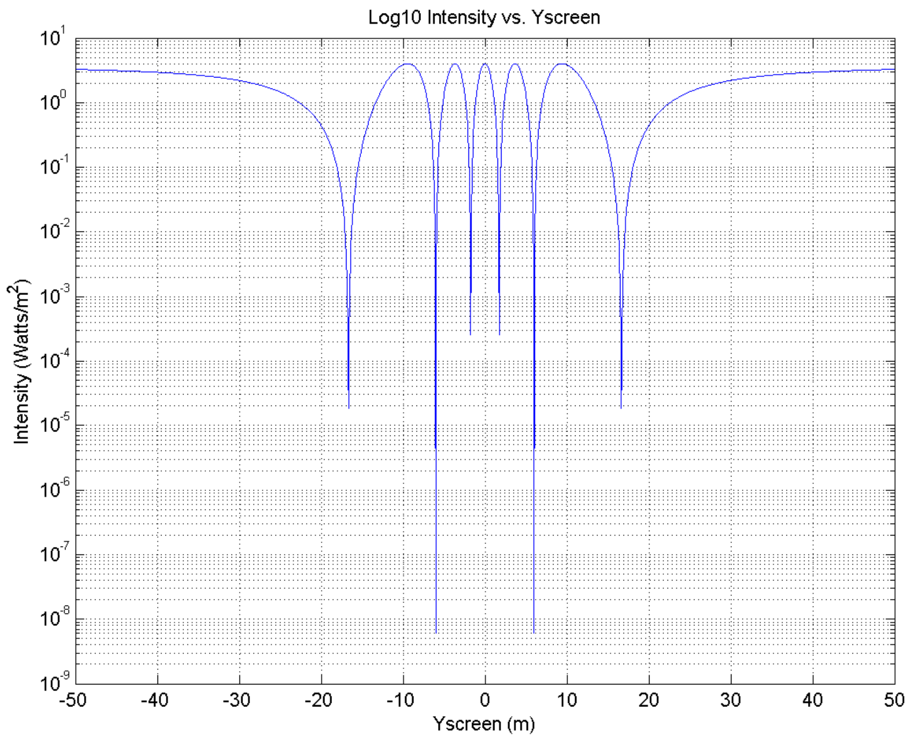
Plots of I_{tot} *vs.* θ for $N = 2$ slits:



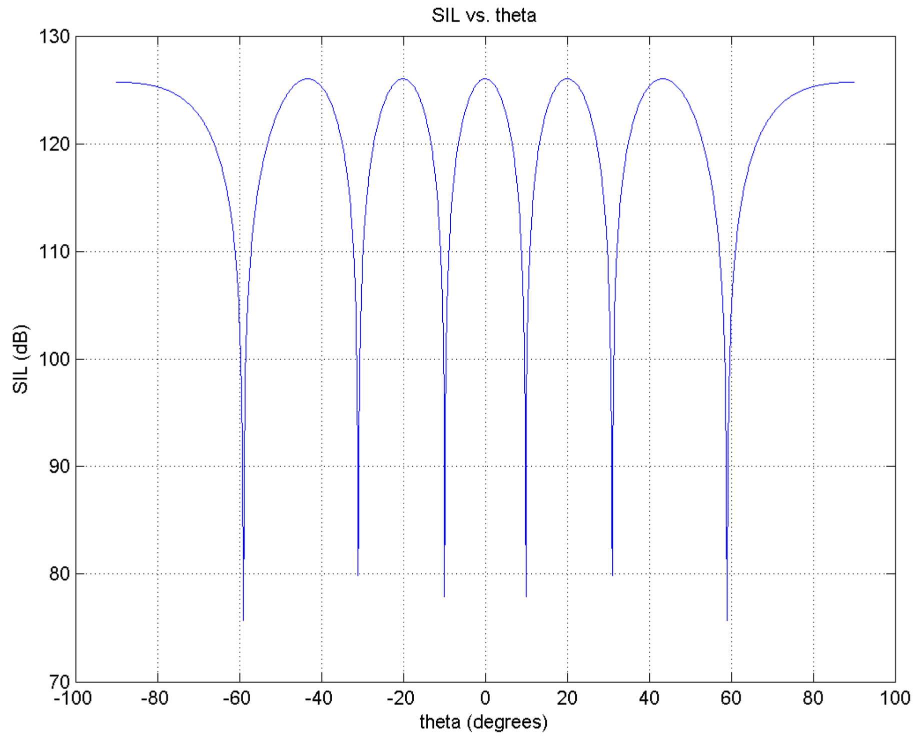


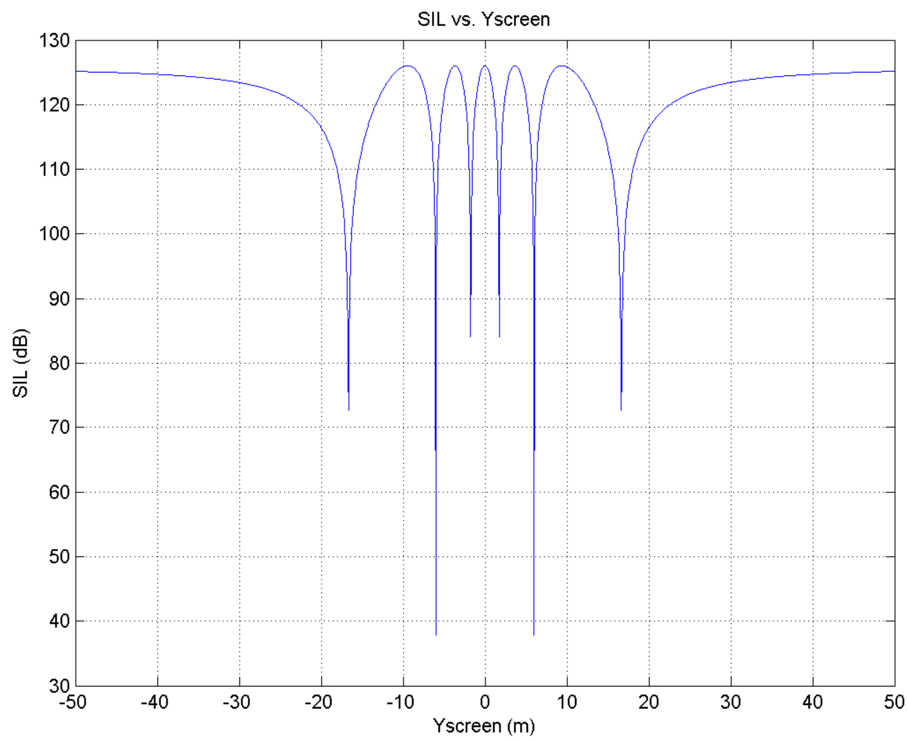
Plots of I_{tot} *vs.* y_{screen} for $N = 2$ slits:



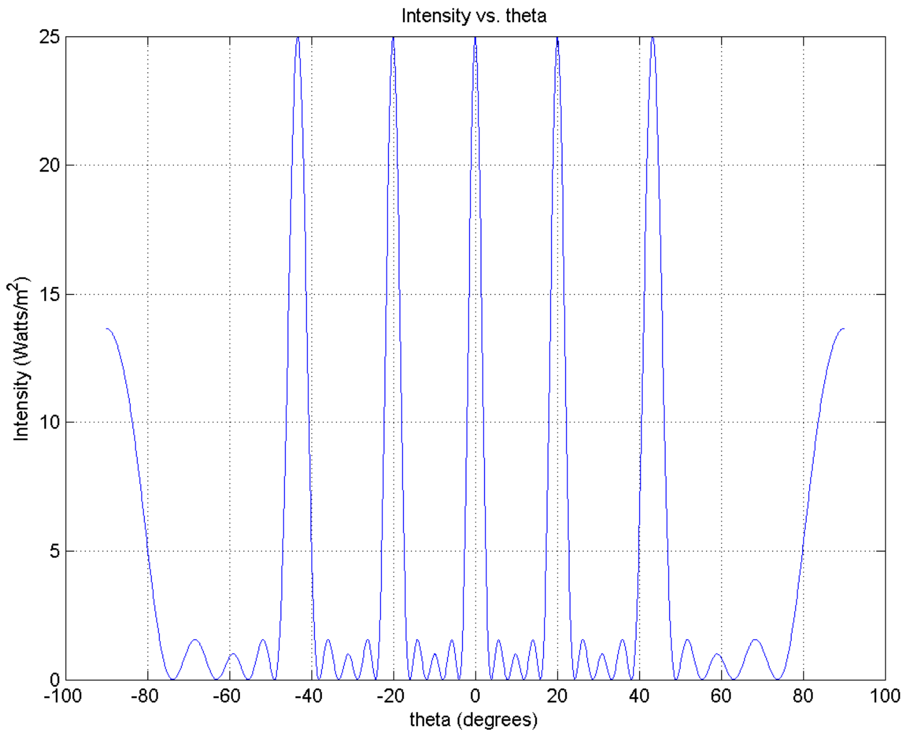


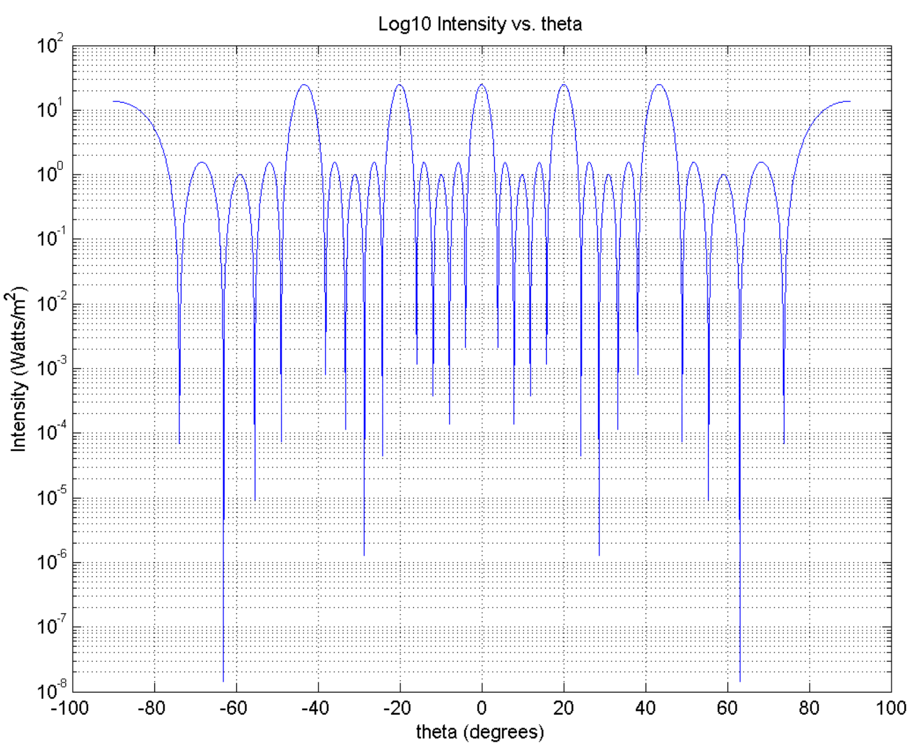
Plots of SIL *vs.* θ and SIL *vs.* y_{screen} for N = 2 slits:



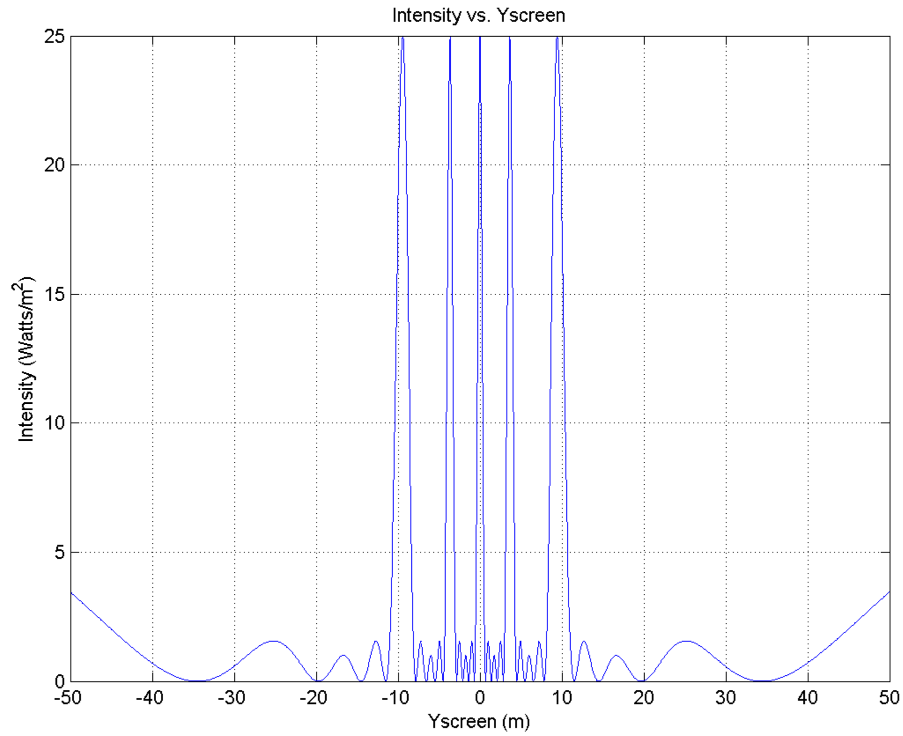


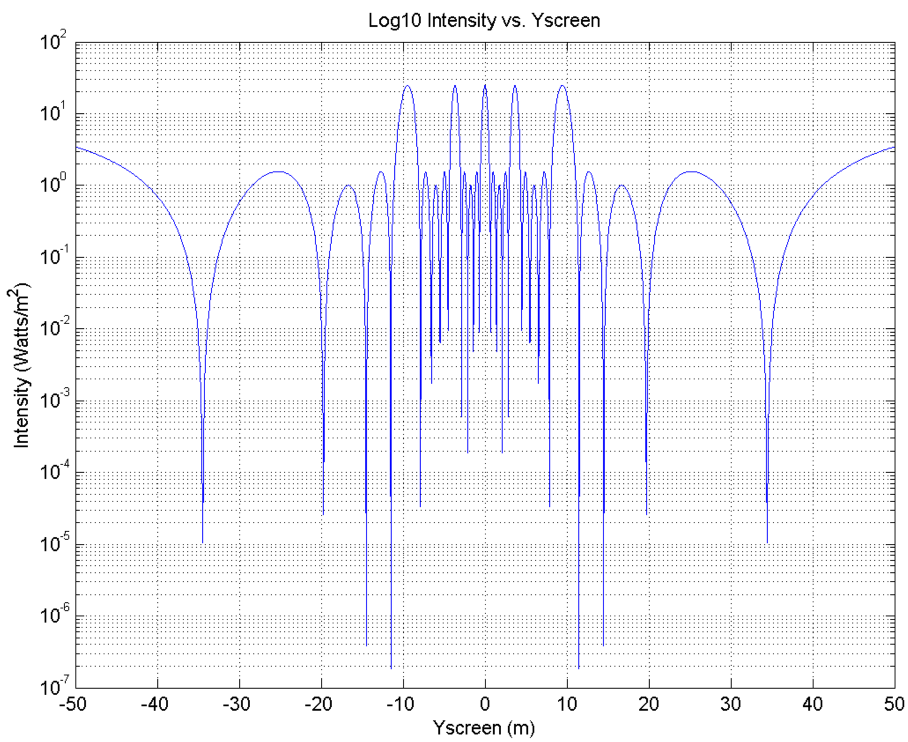
Plots of I_{tot} *vs.* θ for $N = 5$ slits:



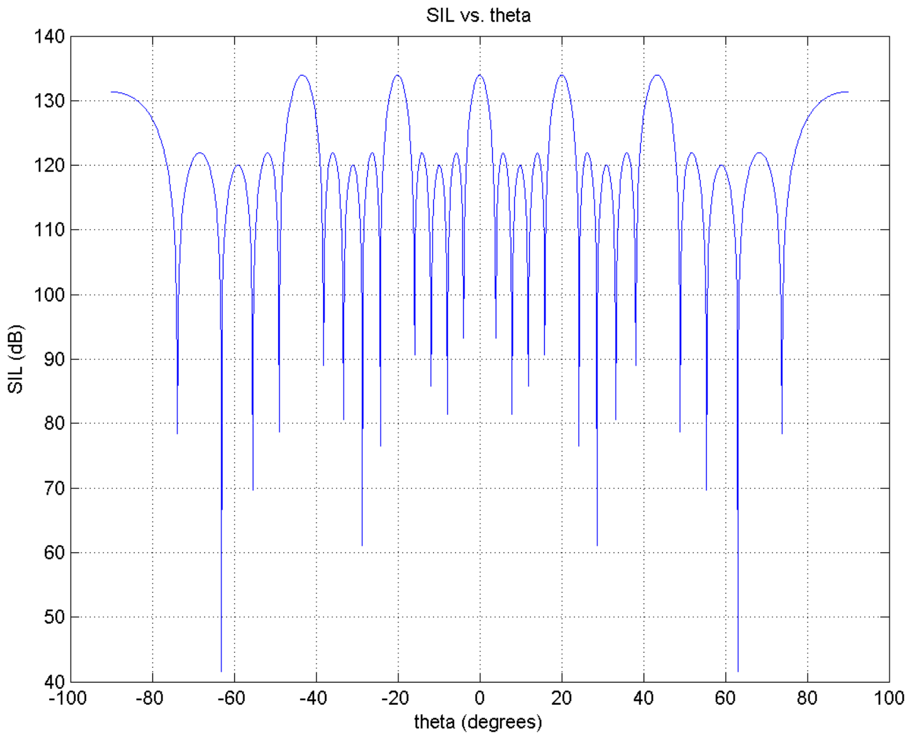


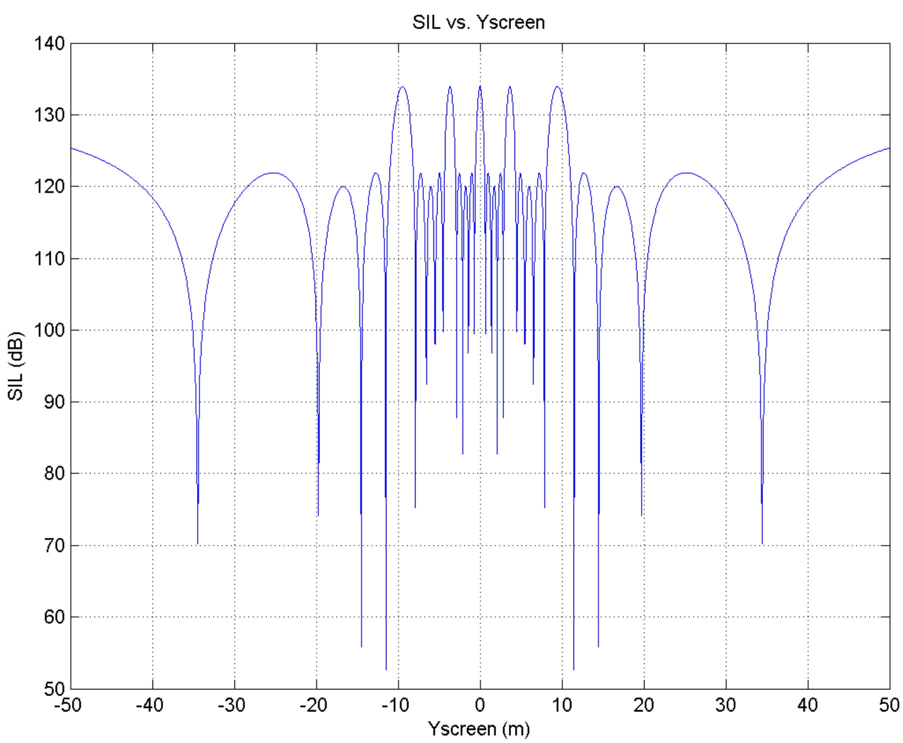
Plots of I_{tot} *vs.* θ and I_{tot} *vs.* y_{screen} for $N = 5$ slits:



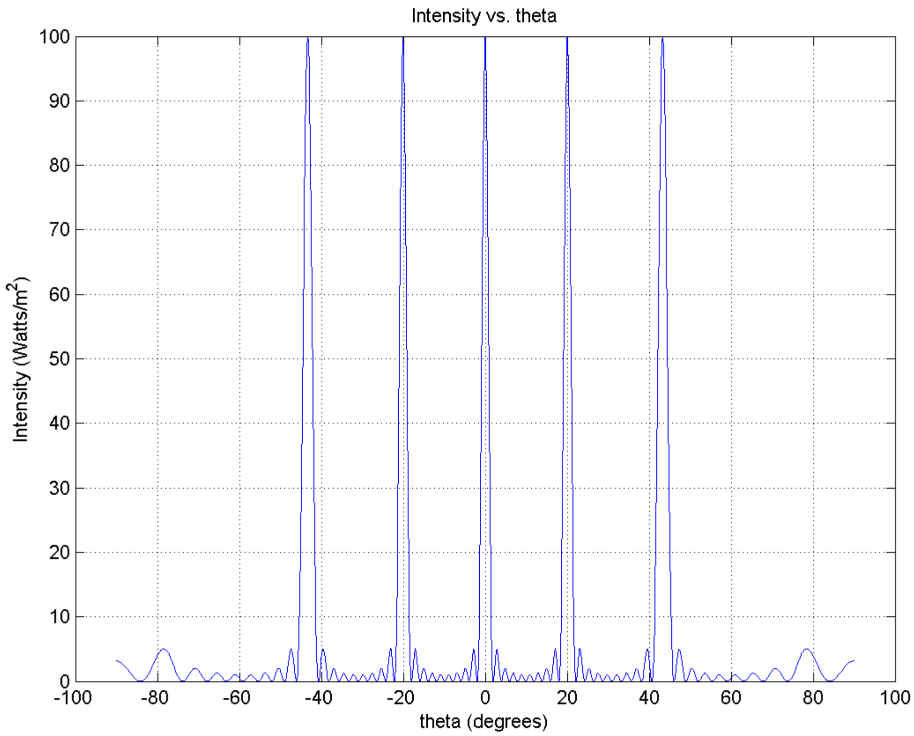


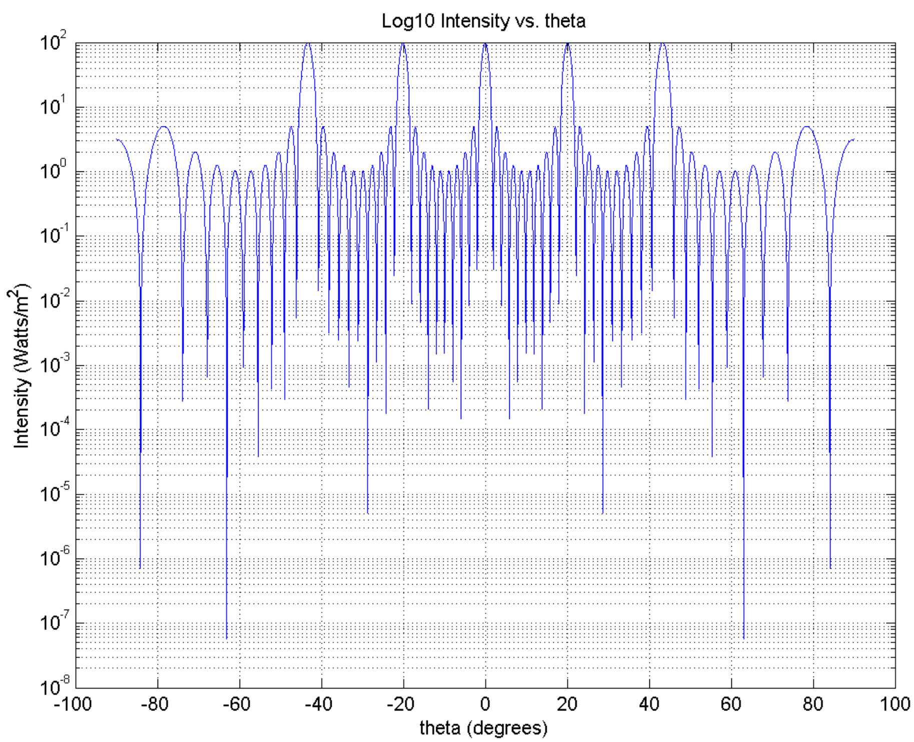
Plots of *SIL vs.* θ and *SIL vs.* y_{screen} for $N = 5$ slits:



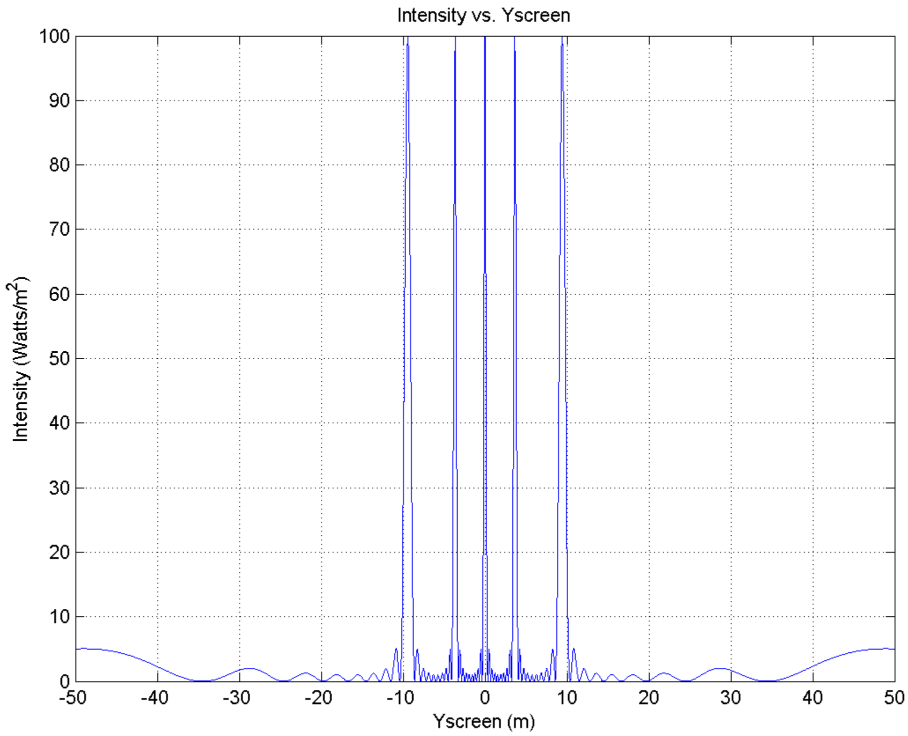


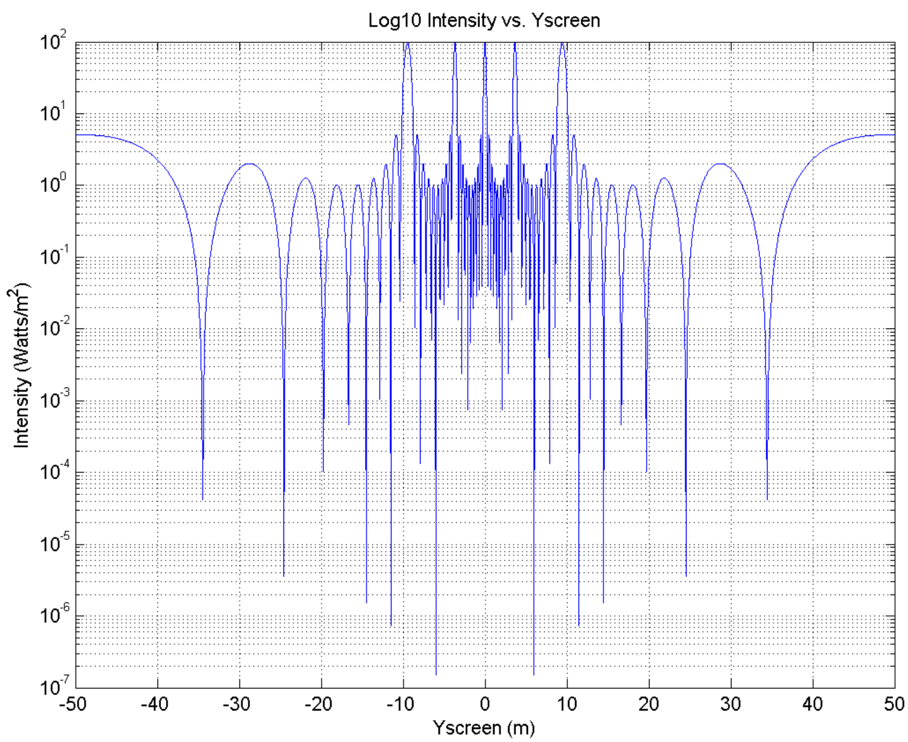
Plots of I_{tot} *vs.* θ for $N = 10$ slits:



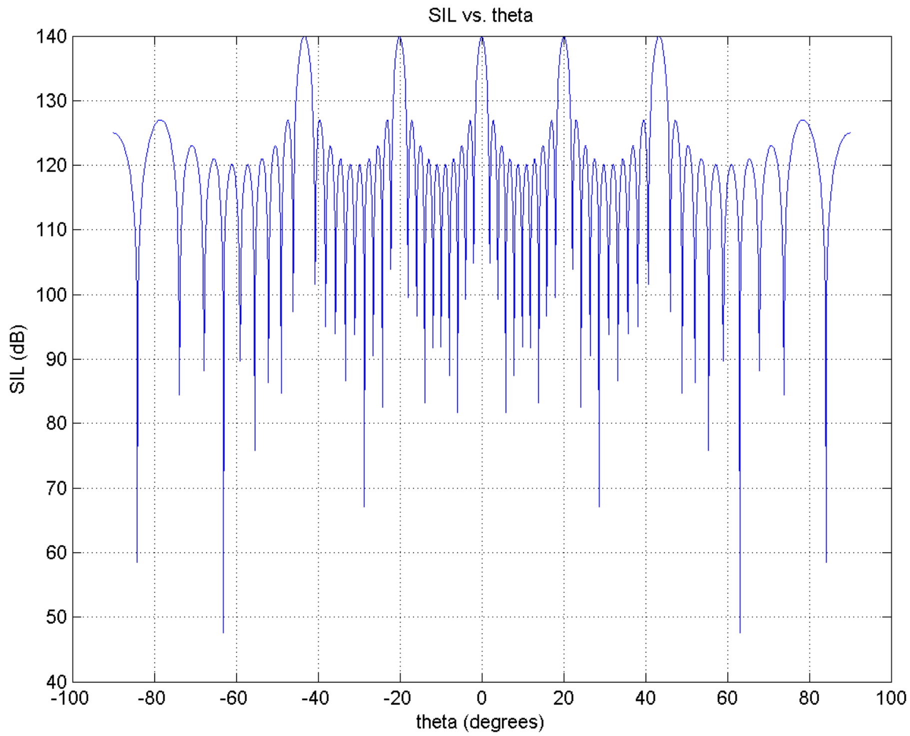


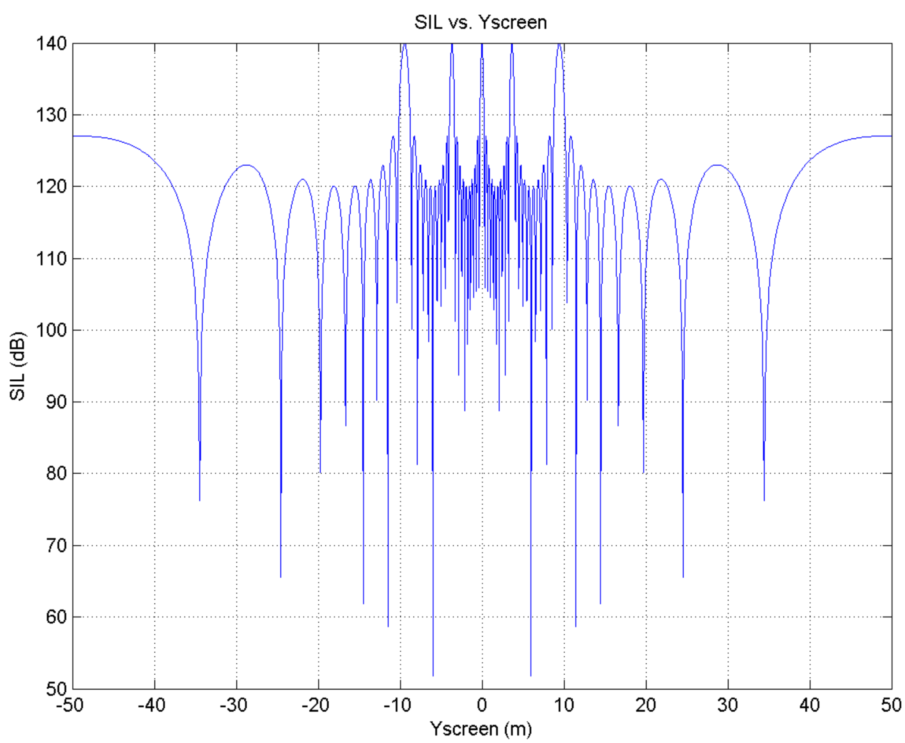
Plots of I_{tot} *vs.* y_{screen} for N = 10 slits:



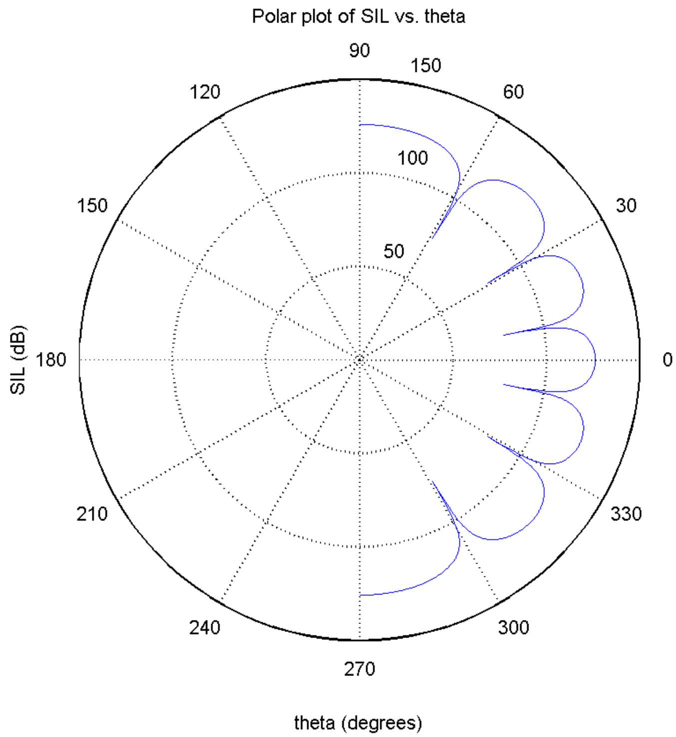


Plots of SIL *vs.* θ and SIL *vs.* y_{screen} for N = 10 slits:

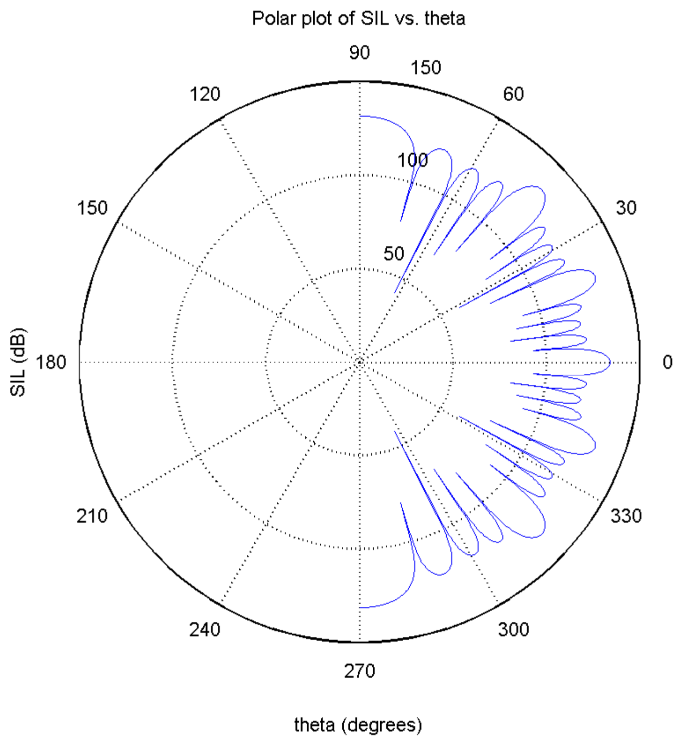




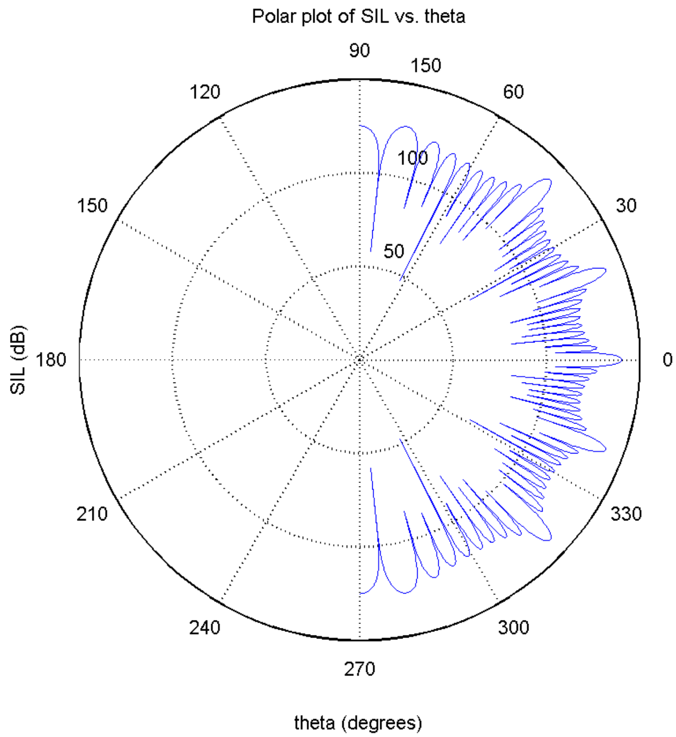
Polar plot of $SIL(\theta)\, vs.\ \theta$ for $N = 2$ slits:



Polar plot of $SIL(\theta)\, vs.\ \theta$ for $N = 5$ slits:



Polar plot of $SIL(\theta)\,vs.\ \theta$ for $N = 10$ slits:



Listing of the MATLAB code:

```
%=========================================================================
% 1D_Nslit_Intf_Smpl_Thy.m
%
% 1D N-slit interference - simplest theory - far-field/plane-wave approx!
% sound waves assumed to be propagating in free air/great wide-open!
% each "slit" treated simplistically as point source - no spatial extent!
%
%=========================================================================
%
% Written by Prof. Steven Errede  Last Updated: Feb. 7, 2011 10:55 hr
%
%=========================================================================
close all;
clear all;

single  thtr(1800);
single  thtd(1800);
single Itot1(1800);
single  SIL1(1800);

single  yscr(2000);
single Itot2(2000);
single  SIL2(2000);

% Specify # of slits (n.b. Nslits = 1 => no interference):
Nslits =  2; % 2; 5; 10;

% Specify other needed parameters:
Io     =    1.0;    % intensity from single slit (Watts/m^2)
Ir     = 1.0*10^-12;% reference sound intensity  (Watts/m^2)
Vair   =  343.0;    % speed of propagation of sound - free air (m/s)
freq   = 1000.0;    % frequency (Hz or cps)
lambda = Vair/freq; % wavelength (m)
Lobs   =   10.0;    % perp. distance observer from slits (m) n.b. lambda << Lobs
Dsrc   =    1.0;    % transv. distance between slits     (m) n.b.   Dsrc << Lobs

%====================================
% Calculate Itot, SIL vs. theta:
%====================================
Thetad =  -90.0;    % angle theta of observer in degrees
dTheta =    0.1;    %  step angle in degrees
for i = 1:1800;
     thtd(i) = Thetad;            % angle theta of observer in degrees
     Thetar  = (pi/180.0)*Thetad; % angle theta of observer in radians
     thtr(i) = Thetar;

     delta   = (2.0*pi/lambda)*Dsrc*sin(Thetar); % resultant Nslit phase diff (radians)

    Itot1(i) = Io*(sin(Nslits*delta/2.0)/sin(delta/2.0))^2; % total intensity (Watts/m^2)
     SIL1(i) = 10.0*log10(Itot1(i)/Ir); % Sound Intensity Level (dB)

     Thetad  = Thetad + dTheta;   % increment angle for next calculation
end

%====================================
% Calculate Itot, SIL vs. yscreen:
%====================================
 y = -50.00; % starting position on screen (m)
dy =   0.05; % step size on screen (m);
for i = 1:2000;
     yscr(i) = y;            % position of observer on perp. screen (m)
     Thetar  = atan(y/Lobs); % angle theta of observer in radians

     delta   = (2.0*pi/lambda)*Dsrc*sin(Thetar); % resultant Nslit phase diff (radians)

    Itot2(i) = Io*(sin(Nslits*delta/2.0)/sin(delta/2.0))^2; % total intensity (Watts/m^2)
     SIL2(i) = 10.0*log10(Itot2(i)/Ir); % Sound Intensity Level (dB)
```

```
     y  = y + dy;   % increment screen position for next calculation
end


figure(01);
plot(thtd,Itot1,'b');
grid on;
xlabel('theta (degrees)');
ylabel('Intensity (Watts/m^{2})');
title('Intensity vs. theta');

figure(02);
semilogy(thtd,Itot1,'b');
grid on;
xlabel('theta (degrees)');
ylabel('Intensity (Watts/m^{2})');
title('Log10 Intensity vs. theta');

figure(03);
plot(thtd,SIL1,'b');
grid on;
xlabel('theta (degrees)');
ylabel('SIL (dB)');
title('SIL vs. theta');

figure(04);
polar(thtr,SIL1,'b');
grid on;
xlabel('theta (degrees)');
ylabel('SIL (dB)');
title('Polar plot of SIL vs. theta');


figure(11);
plot(yscr,Itot2,'b');
grid on;
xlabel('Yscreen (m)');
ylabel('Intensity (Watts/m^{2})');
title('Intensity vs. Yscreen');

figure(12);
semilogy(yscr,Itot2,'b');
grid on;
xlabel('Yscreen (m)');
ylabel('Intensity (Watts/m^{2})');
title('Log10 Intensity vs. Yscreen');

figure(13);
plot(yscr,SIL2,'b');
grid on;
xlabel('Yscreen (m)');
ylabel('SIL (dB)');
title('SIL vs. Yscreen');

%===========================================================================
beep;
fprintf('\n 1-D Nslit interference simple thy calculation completed !!! \n')
%===========================================================================
```