Single Slit Diffraction in 1-Dimension – Simplest Theory

In this example, we show plots of the sound intensity *vs*. angle and observer/listener position y_{screen} on a screen for the simplest theory of Fraunhofer diffraction through an infinitely long, horizontal single slit of vertical width *a*. The observer/listener is located far from the slit, at a perpendicular distance $L\ (m)$ away, such that the conditions $a \ll L$ **.and.** $\lambda \ll L$ both hold simultaneously, where $\lambda\ (m)$ is the wavelength of the sound – this is the so-called "far-field" limit.

The expression for 1-D single-slit diffraction in this simplest theory is given by (see P406POM Lecture Notes P406POM_Lect3_Part2):

$$I_{tot}(\theta) = I_o \left\{ \frac{\sin\delta(\theta)}{\delta(\theta)} \right\}^2 \equiv I_o \operatorname{sinc}^2\delta \text{ and } SIL(\theta) \equiv 10\log_{10}\left(I_{tot}(\theta)/I_{ref}\right)\ (dB)$$

where $I_o\ \left(Watts/m^2\right)$ is the maximum sound intensity associated with the single slit, the phase $\delta(\theta) = (\pi a/\lambda)\sin\theta\ \ (radians)$. $I_{ref} \equiv 10^{-12}\ \left(Watts/m^2\right)$ is the reference sound intensity for the sound intensity level (SIL).

Diffraction <u>minima</u> (intensity zeroes) occur when $\delta = (\pi a/\lambda)\sin\theta = \pm\pi, \pm2\pi, \pm3\pi, \ldots = \pm m\pi$, $m = 1, 2, 3, \ldots$.

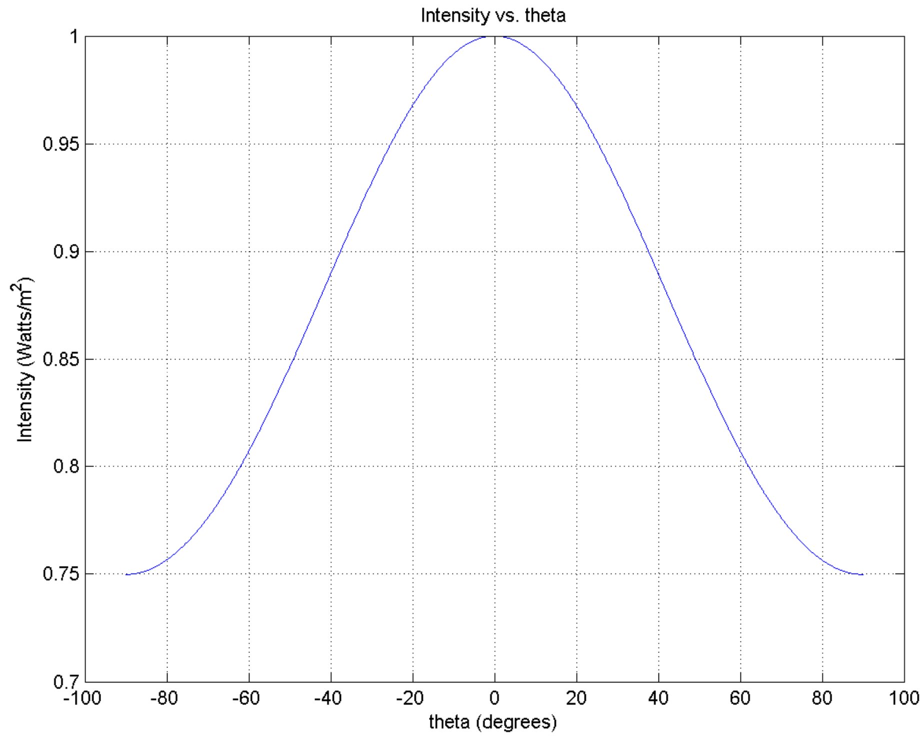
The corresponding location of the observer's/listener's position y_{screen} on a screen located a perpendicular distance L away from the sound source is: $y_{screen} = L\tan\theta$, or conversely: $\theta = \tan^{-1}\left(y_{screen}/L\right)$.

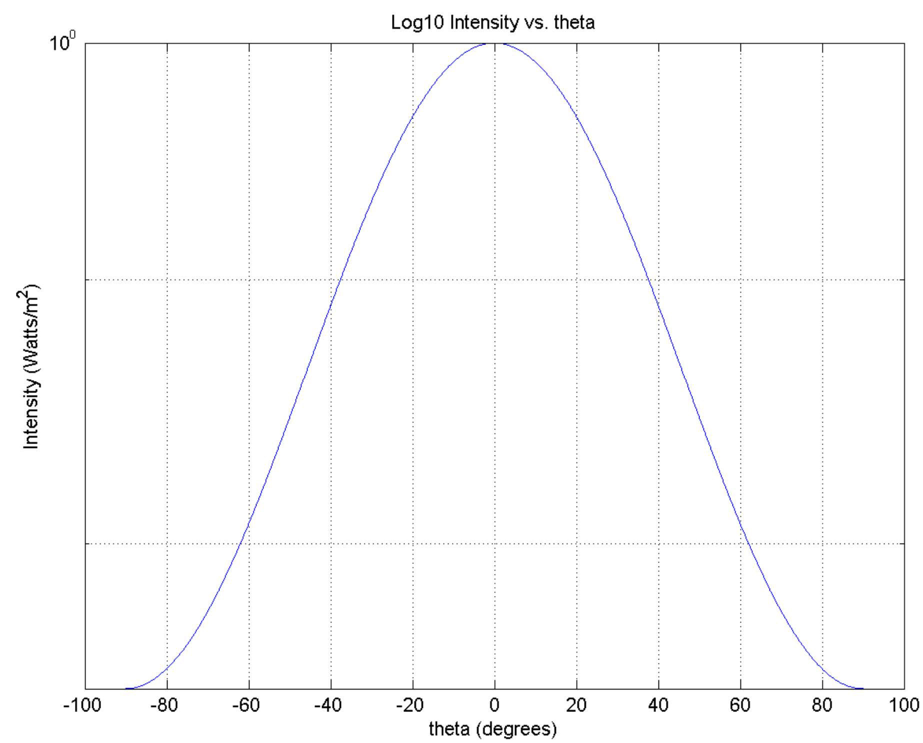
We coded up the above formulas using MATLAB to make plots of I_{tot} *vs.* θ and I_{tot} *vs.* y_{screen} *e.g.* for a = 0.1 and 1.0 *m*, with the following parameter values: $I_o = 1\ Watt/m^2$, observer/listener distance (at $\theta = 0$) of $L = 10\ m$, the speed of propagation in free air/great-wide open: $v_{air} = 343\ m/s$ and frequency of $f = 1000\ Hz$, thus $\lambda = v_{air}/f = 0.345\ m$.

In the following figures, note that the angular width of the central maxima decreases as the slit width *a* increases, since the phase difference $\delta(\theta) = (\pi a/\lambda)\sin\theta$ is linearly proportional to *a*. Note also that the angular width of the central maxima decreases linearly with increasing frequency *f*, since the phase difference increases linearly with frequency:

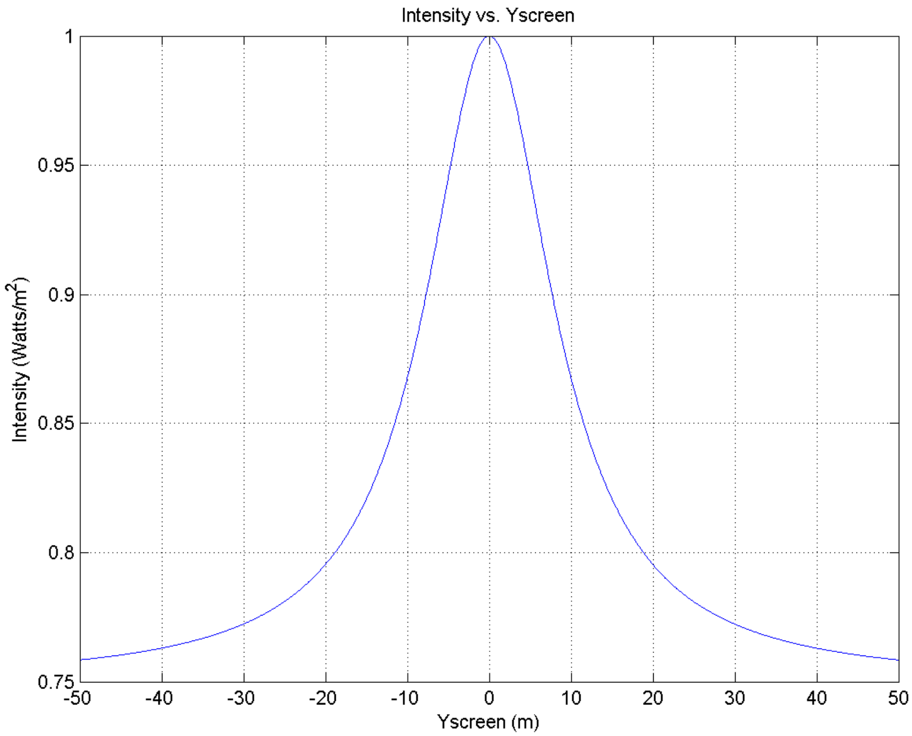
$$\delta(\theta) = (\pi a/\lambda)\sin\theta = (\pi a f/v_{air})\sin\theta\,.$$

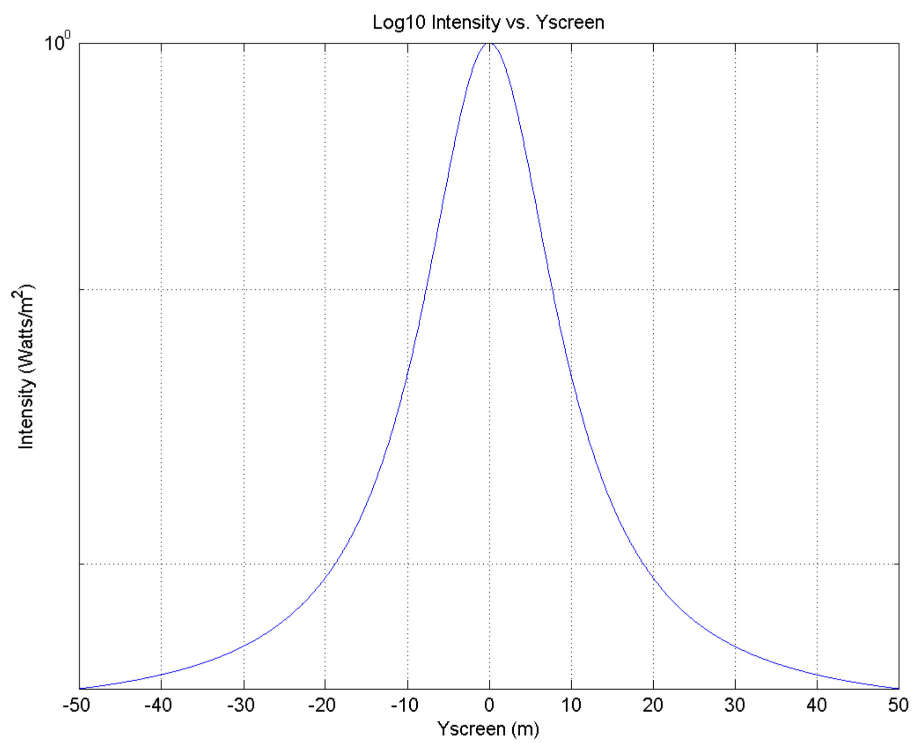
Linear and Semilog Plots of I_{tot} *vs.* θ for $a = 0.1\ m$:



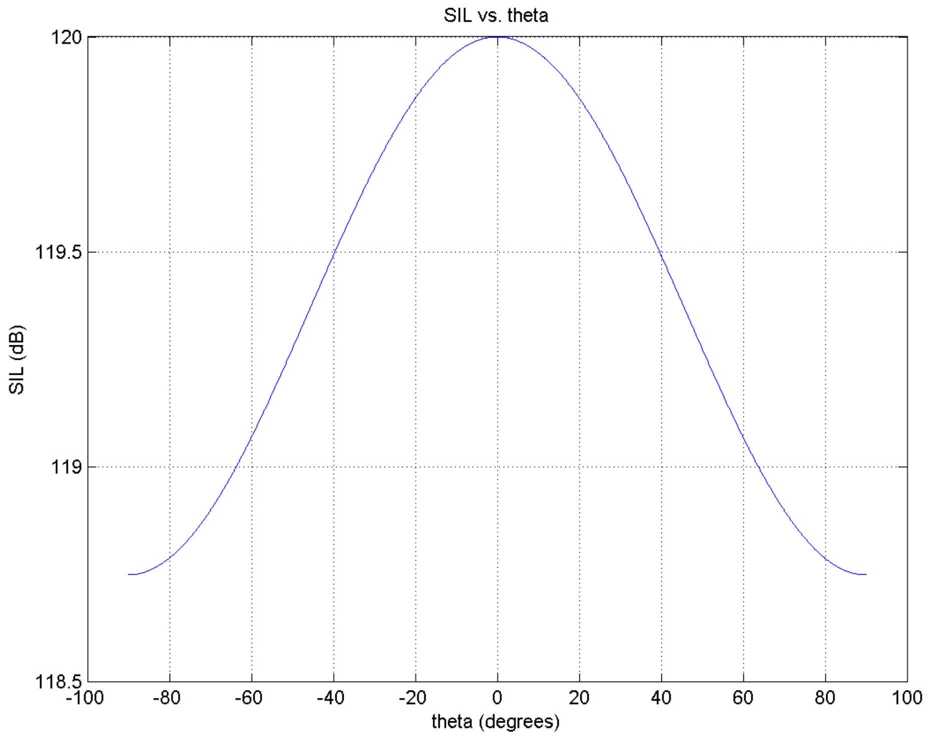


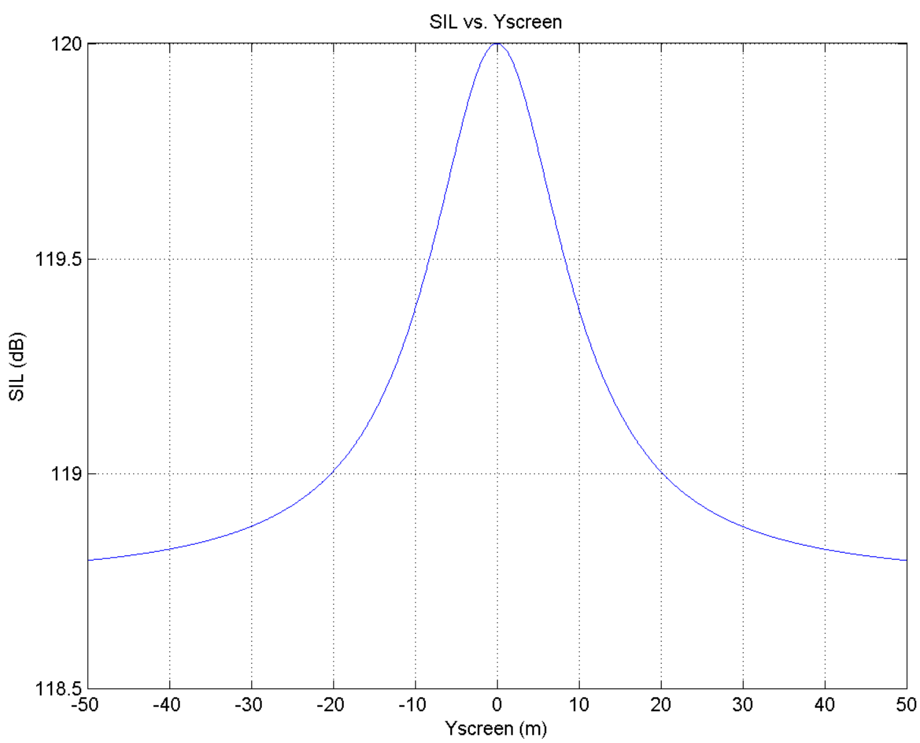
Linear and Semilog Plots of I_{tot} *vs.* y_{screen} for $a = 0.1\ m$:



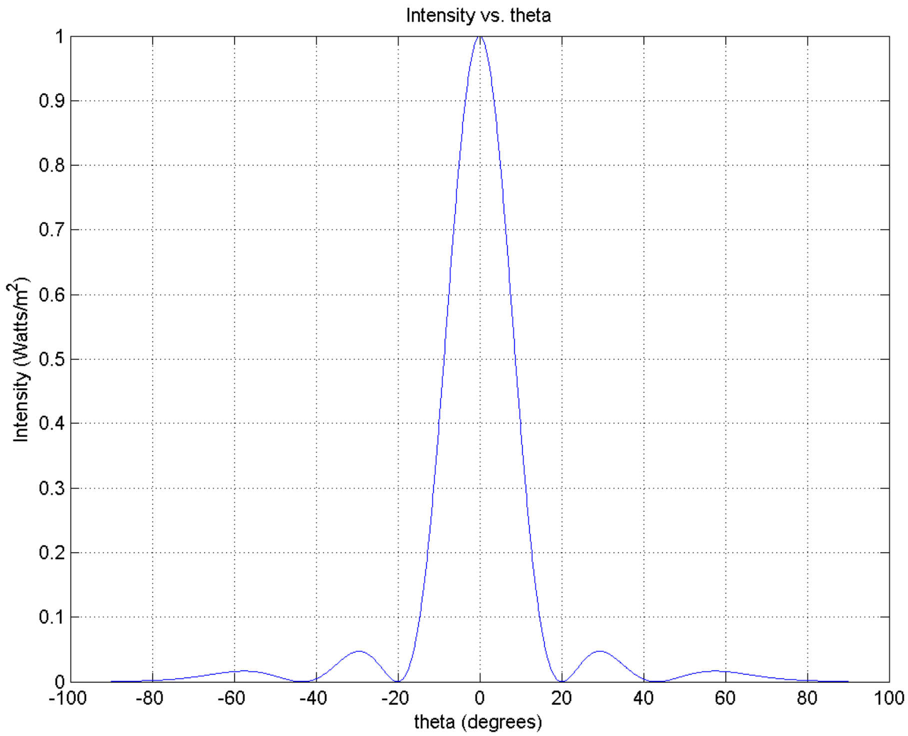


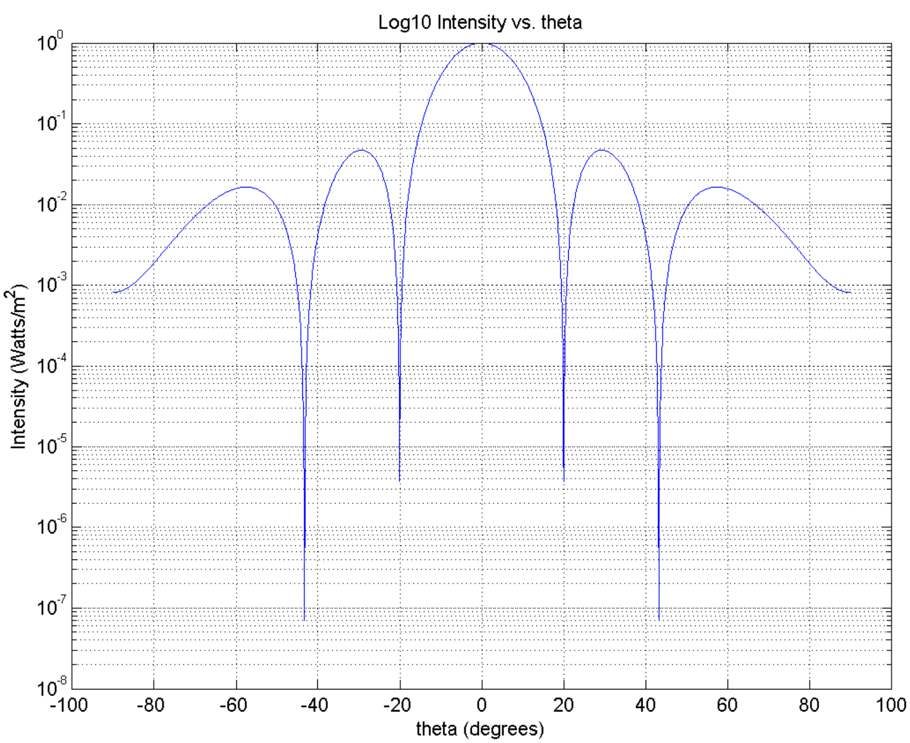
Plots of $SIL(\theta)\,vs.\ \theta$ and $SIL\ vs.\ y_{screen}$ for $a = 0.1\ m$:



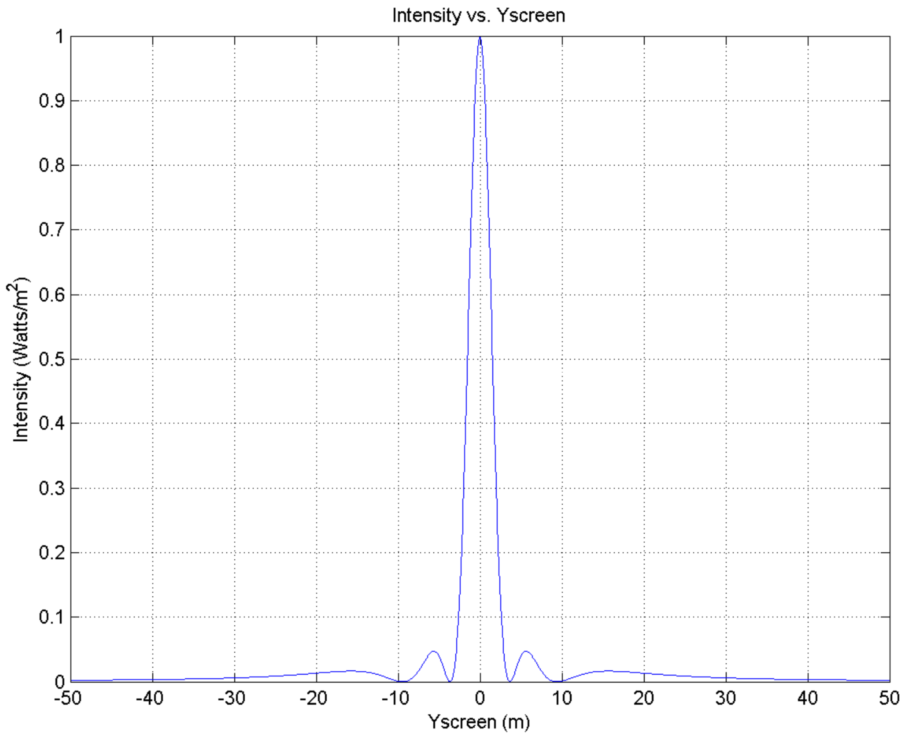


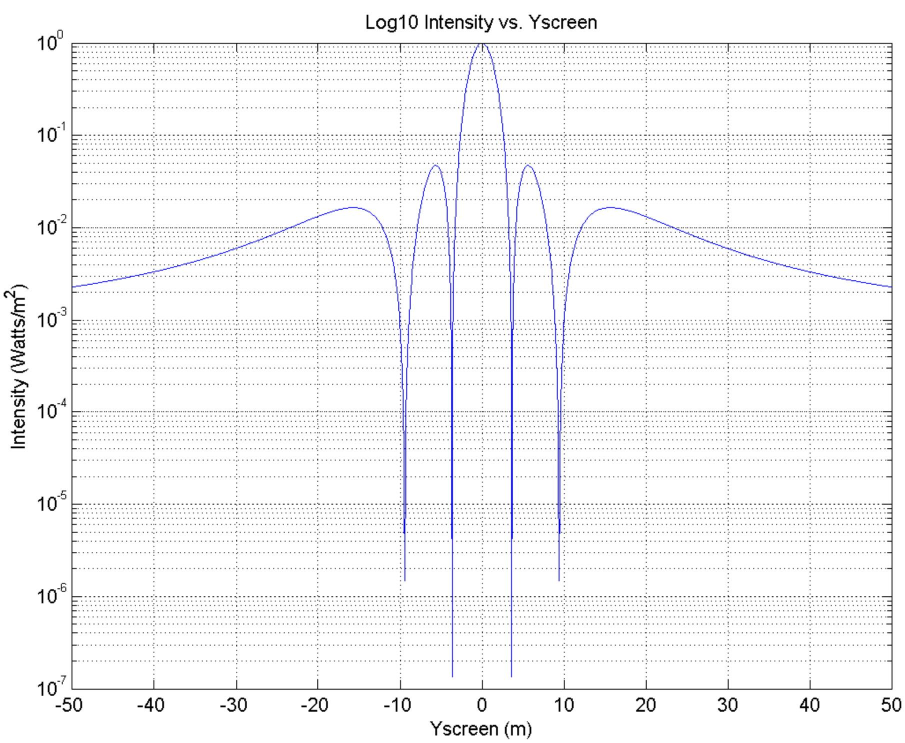
Linear and Semilog Plots of I_{tot} *vs.* θ for $a = 1.0\ m$:



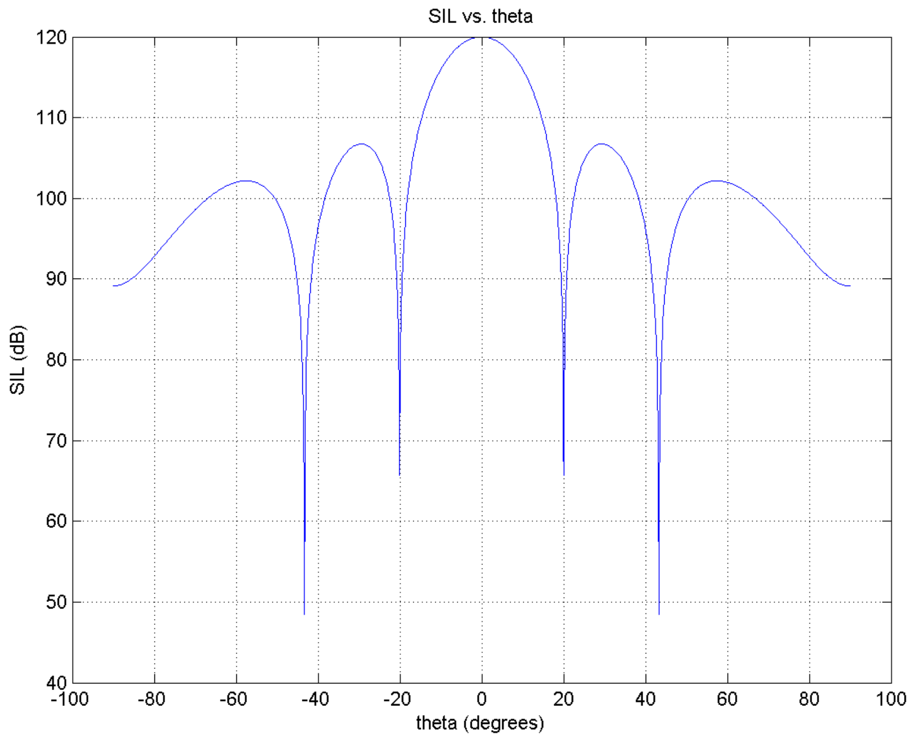


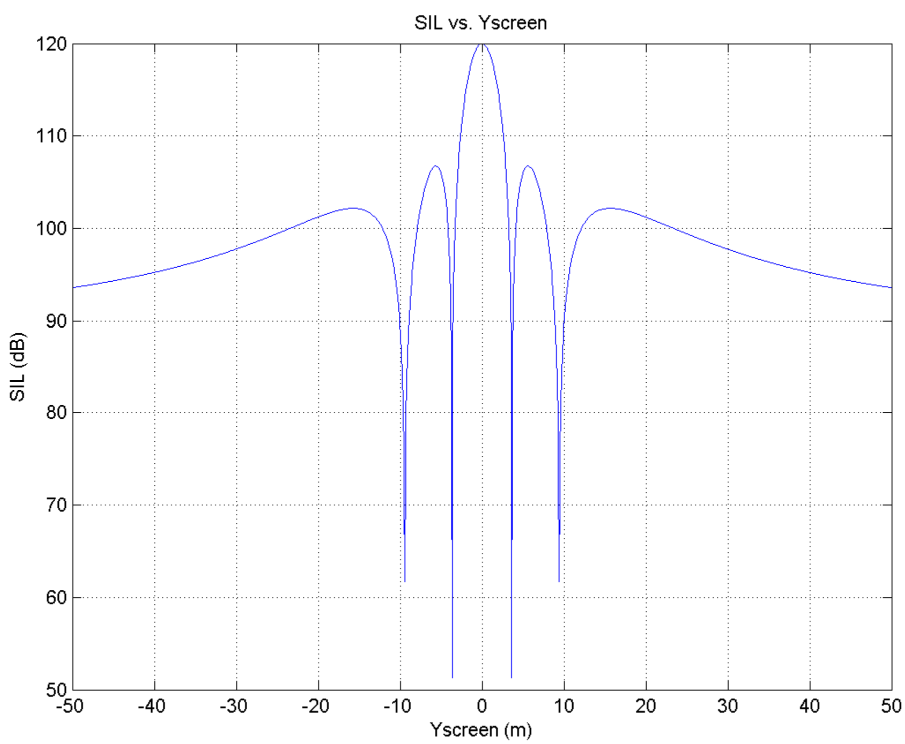
Linear and Semilog Plots of I_{tot} *vs.* y_{screen} for $a = 1.0\ m$:



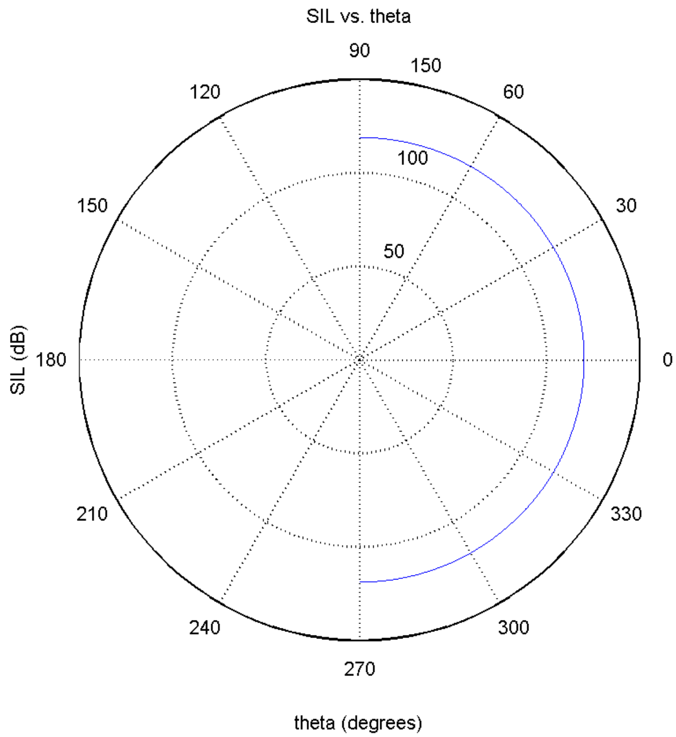


Plots of $SIL(\theta)\ vs.\ \theta$ and $SIL\ vs.\ y_{screen}$ for $a = 1.0\ m$:

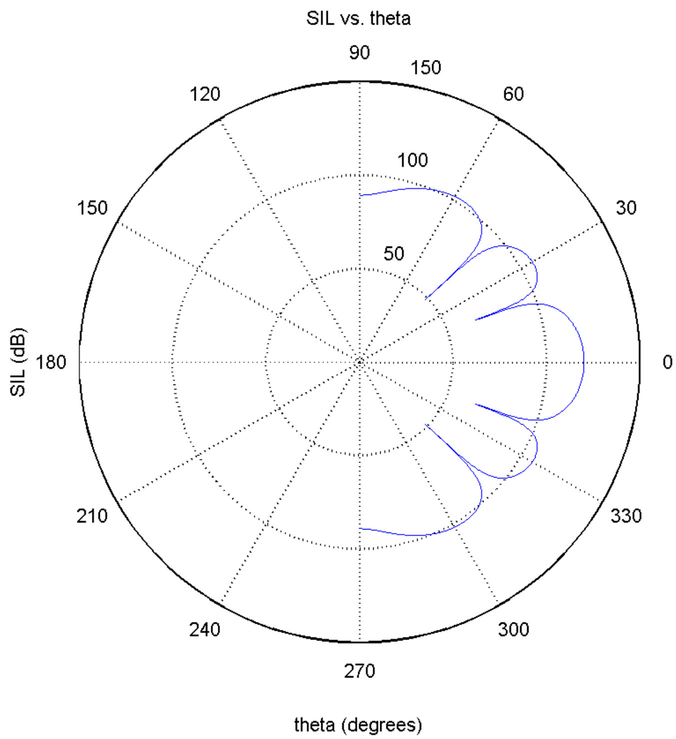




Polar plot of $SIL(\theta)\, vs.\ \theta$ for $a = 0.1\ m$:



Polar plot of $SIL(\theta)\, vs.\ \theta$ for $a = 1.0\ m$:



Listing of the MATLAB code:

```
%==========================================================================
% Sngl_Slit_Diffn_Thy_1D.m
%
% Single Slit Diffraction - simplest theory - far-field/plane-wave approx!
% Sound waves assumed to be propagating in free air/great wide-open!
%
%==========================================================================
%
% Written by Prof. Steven Errede  Last Updated: Feb. 7, 2011 11:15 hr
%
%==========================================================================
close all;
clear all;

single  thtr(1800);
single  thtd(1800);
single Itot1(1800);
single  SIL1(1800);

single  yscr(2000);
single Itot2(2000);
single  SIL2(2000);

% Specify numerical values of parameters:
Io     =    1.0;    % intensity from single slit (Watts/m^2)
Ir     = 1.0*10^-12;% reference sound intensity  (Watts/m^2)
Vair   =  343.0;    % speed of propagation of sound - free air (m/s)
freq   = 1000.0;    % frequency (Hz or cps)
lambda = Vair/freq; % wavelength (m)
Lobs   =   10.0;    % observer distance    (m) n.b. lambda << Lobs

% Specify slit width (m):
Wdth   =    0.1;    % 0.1; 1.0; slit width (m) n.b.   Wdth << Lobs

%====================================
% Calculate Itot vs. theta:
%====================================
Thetad =  -90.0;    % angle theta of observer in degrees
dTheta =    0.1;    % step angle in degrees
for i = 1:1800;
     thtd(i) = Thetad;            % angle theta of observer in degrees
     Thetar  = (pi/180.0)*Thetad; % angle theta of observer in radians
     thtr(i) = Thetar;

     delta   = ((pi*Wdth*sin(Thetar))/lambda); % phase (radians)

    Itot1(i) = Io*(sin(delta)/delta)^2; % total intensity (Watts/m^2)
     SIL1(i) = 10.0*log10(Itot1(i)/Ir); % Sound Intensity Level (dB)

     Thetad  = Thetad + dTheta;   % increment angle for next calculation
end

%====================================
% Calculate Itot vs. yscreen:
%====================================
 y = -50.00; % starting position on screen (m)
dy =   0.05; % step size on screen (m);
for i = 1:2000;
     yscr(i) = y;            % position of observer on perp. screen (m)
     Thetar  = atan(y/Lobs); % angle theta of observer in radians

     delta   = ((pi*Wdth*sin(Thetar))/lambda); % phase (radians)

    Itot2(i) = Io*(sin(delta)/delta)^2; % total intensity (Watts/m^2)
     SIL2(i) = 10.0*log10(Itot2(i)/Ir); % Sound Intensity Level (dB)
```

```
     y  = y + dy;   % increment screen position for next calculation
end


figure(01);
plot(thtd,Itot1,'b');
grid on;
xlabel('theta (degrees)');
ylabel('Intensity (Watts/m^{2})');
title('Intensity vs. theta');

figure(02);
semilogy(thtd,Itot1,'b');
grid on;
xlabel('theta (degrees)');
ylabel('Intensity (Watts/m^{2})');
title('Log10 Intensity vs. theta');

figure(03);
plot(thtd,SIL1,'b');
grid on;
xlabel('theta (degrees)');
ylabel('SIL (dB)');
title('SIL vs. theta');

figure(04);
polar(thtr,SIL1,'b');
grid on;
xlabel('theta (degrees)');
ylabel('SIL (dB)');
title('SIL vs. theta');


figure(11);
plot(yscr,Itot2,'b');
grid on;
xlabel('Yscreen (m)');
ylabel('Intensity (Watts/m^{2})');
title('Intensity vs. Yscreen');

figure(12);
semilogy(yscr,Itot2,'b');
grid on;
xlabel('Yscreen (m)');
ylabel('Intensity (Watts/m^{2})');
title('Log10 Intensity vs. Yscreen');

figure(13);
plot(yscr,SIL2,'b');
grid on;
xlabel('Yscreen (m)');
ylabel('SIL (dB)');
title('SIL vs. Yscreen');

%==========================================================================
beep;
fprintf('\n Single Slit Diffraction Calculation Completed !!! \n')
%==========================================================================
```