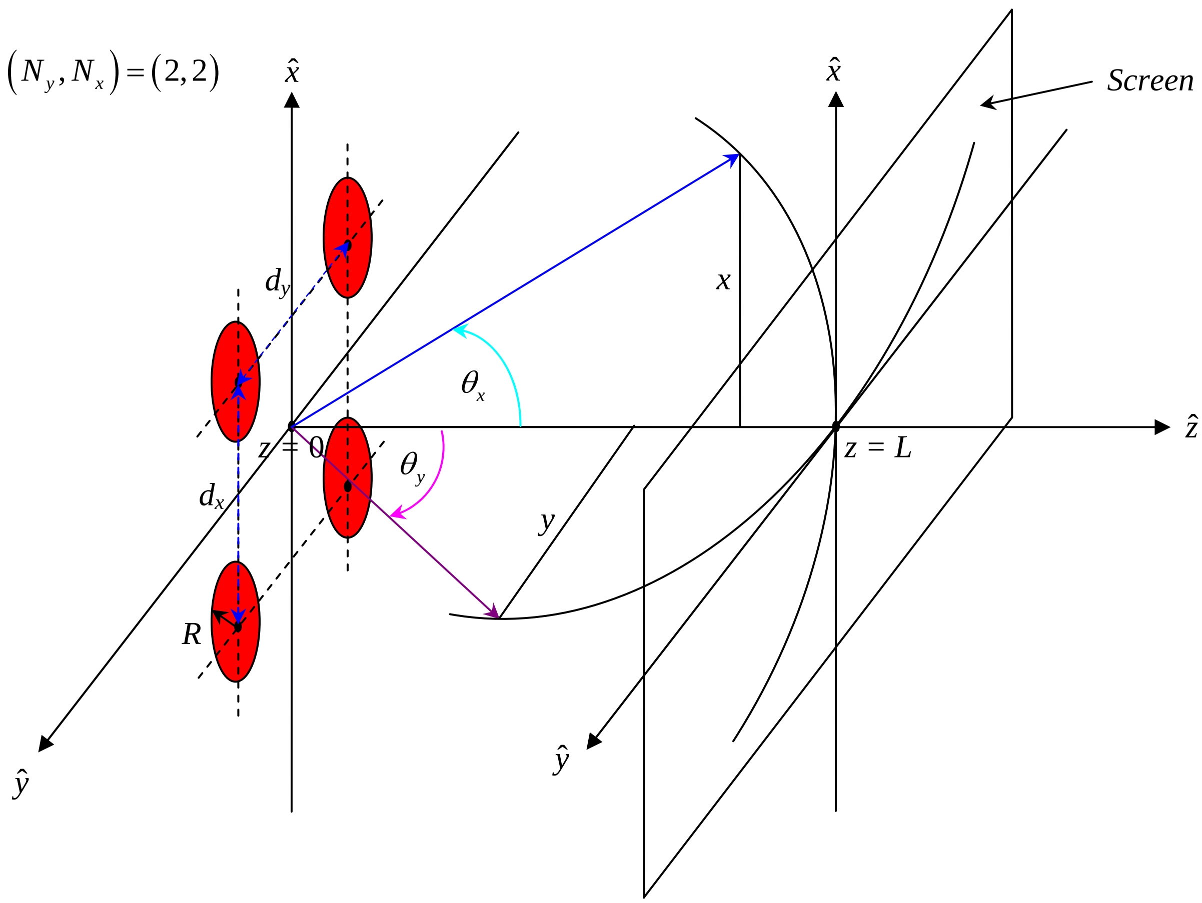
2-D Interference-Diffraction with N_y-N_x Circular Apertures

In this example, we show plots of the sound intensity *vs*. angle and observer/listener position (x_{screen}, y_{screen}) on a screen for the simplest theory of interference-diffraction pattern associated with a 2-D array of N_y-N_x identical circular apertures, each of radius R, with transverse separation distances d_y, $d_x > R$. This is an approximation to *e.g.* a 2-D array of N_y-N_x loudspeakers mounted on an infinite baffle, as shown in the figure below. The observer/listener is located far from the apertures, a perpendicular distance $L\ (m)$ away, such that the conditions $R < d_y, d_x \ll L$ **.and.** $\lambda \ll L$ both hold simultaneously, where $\lambda\ (m)$ is the wavelength of the sound – this is the so-called "far-field" limit.



The expression for interference-diffraction associated with a 2-D array of N_y-N_x identical circular apertures is given by the product of {independent} interference × diffraction factors:

$$I_{tot}(\theta) = I_o \underbrace{\left\{ \frac{\sin^2\left(N_x \delta_x(\theta_x)/2\right)}{\sin^2\left(\delta_x(\theta_x)/2\right)} \right\}}_{x-Interference} \cdot \underbrace{\left\{ \frac{\sin^2\left(N_y \delta_y(\theta_y)/2\right)}{\sin^2\left(\delta_y(\theta_y)/2\right)} \right\}}_{y-Interference} \cdot \underbrace{\left\{ \frac{2J_1(\rho(\theta))}{\rho(\theta)} \right\}^2}_{Diffraction}$$

$$\text{and } SIL(\theta) \equiv 10\log_{10}\left(I_{tot}(\theta)/I_{ref}\right)\ (dB)$$

where I_o $\left(Watts/m^2\right)$ is the maximum sound intensity associated with an individual circular aperture, the interference phases $\delta_y\left(\theta_y\right)=k\Delta L_y\left(\theta_y\right)=kd_y\sin\theta_y$ and $\delta_x\left(\theta_x\right)=k\Delta L_x\left(\theta_x\right)=kd_x\sin\theta_x$ $(radians)$, $k=2\pi/\lambda$ $(radians/m)$ is the wavenumber and $\Delta L_y\left(\theta_y\right)=d_y\sin\theta_y$ $\left(\Delta L_x\left(\theta_x\right)=d_x\sin\theta_x\right)$ (m) are the *y*-projected (*x*-projected) angle-dependent path length ***differences***, respectively between pairs of adjacent sound sources in the *y*- (*x*-) direction to the observer/listener, located far away from the sound sources. The diffraction phase $\rho(\theta)=kR\sin\theta=(2\pi/\lambda)R\sin\theta\,(radians)$ and $J_1\left(\rho(\theta)\right)$ is the ordinary Bessel function of the first kind, of order $\nu=1$. $I_{ref}\equiv10^{-12}$ $\left(Watts/m^2\right)$ is the reference sound intensity for the sound intensity level (SIL).

Interference minima – *i.e.* intensity zeroes (complete destructive interference) occur when the numerator factor $N\,\delta/2 = \pm\pi, \pm2\pi, \pm3\pi, \ldots = n\pi$, $n = \pm1, \pm2, \pm3, \ldots$ *except* when the denominator factor simultaneously has $\delta/2 = \pm\pi, \pm2\pi, \pm3\pi, \ldots = n\pi$, $n = \pm1, \pm2, \pm3, \ldots$ then have *global maxima* of the intensity, where $I_{tot} = N^2 I_o$.

From simple trigonometry, it is easy to show that the path length difference $\Delta L_y\left(\theta_y\right)=d_y\sin\theta_y$, where θ_y is the *y*-projected angle the observer/listener makes with respect to the normal, or forward axis of the array of N_y circular apertures, and similarly for $\Delta L_x\left(\theta_x\right)=d_x\sin\theta_x$.

Diffraction minima (intensity zeroes) occur at the non-trivial zeros of the Bessel function $J_1(\rho)$, which occur at ρ = 3.8317, 7.0156, 10.1735, 13.3237, 16.4706, 19.6159, …

The corresponding location of the observer's/listener's position y_{screen} on a screen located a perpendicular distance *L* away from the *N* sound sources is: $r_{screen}(\theta)=L\tan\theta$, or conversely: $\theta=\tan^{-1}\left(r_{screen}/L\right)$, where $r_{screen}=\sqrt{x_{screen}^2+y_{screen}^2}$ and $\theta_y=\tan^{-1}\left(y_{screen}/L\right)$, $\theta_x=\tan^{-1}\left(x_{screen}/L\right)$.

We coded up the above formulas using MATLAB to make plots of I_{tot} *vs.* θ and I_{tot} *vs.* $\left(x_{screen}, y_{screen}\right)$ *e.g.* for (N_y, N_x) = (1,1), (1,2), (2,1), (2,2), (2,4), (4,2), (4,4) for $d_y = d_x = 1.0$ m and $R = 1.0$ *m*, with the following parameter values: $I_o = 1\ Watt/m^2$, observer/listener distance (at $\theta=0$) of $L=10\,m$, the speed of propagation in free air/great-wide open: $v_{air}=343\,m/s$ and frequency of $f=1000\,Hz$, thus $\lambda=v_{air}/f=0.345\,m$.

In the following figures, note that the angular width of the central maxima decreases as the radius of the circular aperture increases, since the diffraction phase $\rho(\theta)=kR\sin\theta=(2\pi/\lambda)R\sin\theta$ $(radians)$ is linearly proportional to *R*. Note also that the angular width of the central maxima decreases linearly with increasing frequency *f*, since the diffraction phase increases linearly with frequency:

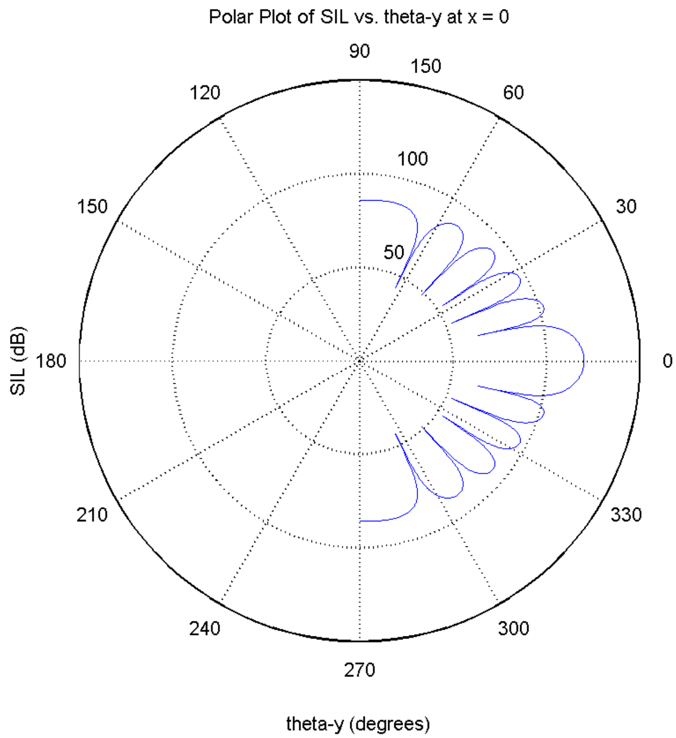
$$\rho(\theta)=kR\sin\theta=(2\pi/\lambda)R\sin\theta=\left(2\pi f/v_{air}\right)R\sin\theta\,(radians).$$

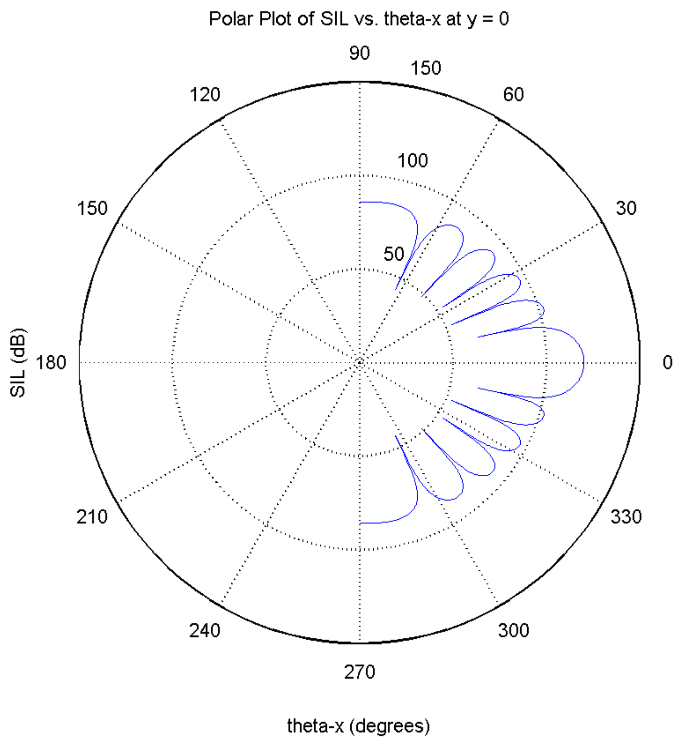
Note also that the angular width of the central maxima decreases as the number of circular apertures N increases. Note further that the number of maxima/minima increases linearly with increasing frequency f, since the y-, x-interference phase differences increase linearly with frequency:

$$\delta_y(\theta_y) = k\Delta L_y(\theta_y) = (2\pi/\lambda)\Delta L_y(\theta_y) = (2\pi f/v_{air})\Delta L_y(\theta_y) = (2\pi f/v_{air})d_y \sin\theta_y .$$

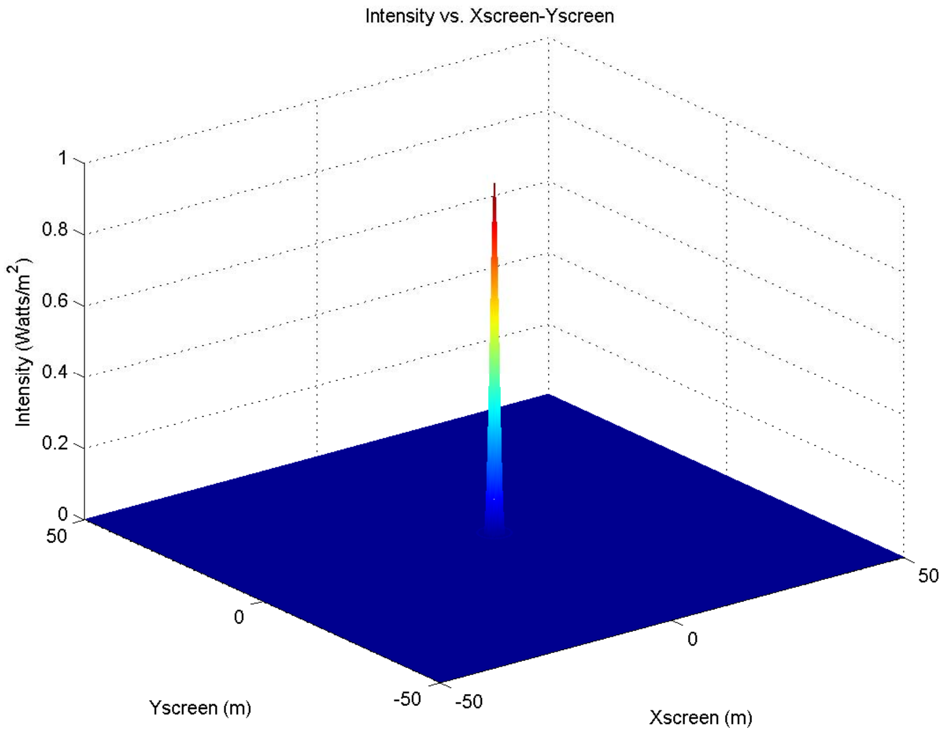
$$\delta_x(\theta_x) = k\Delta L_x(\theta_x) = (2\pi/\lambda)\Delta L_x(\theta_x) = (2\pi f/v_{air})\Delta L_x(\theta_x) = (2\pi f/v_{air})d_x \sin\theta_x$$

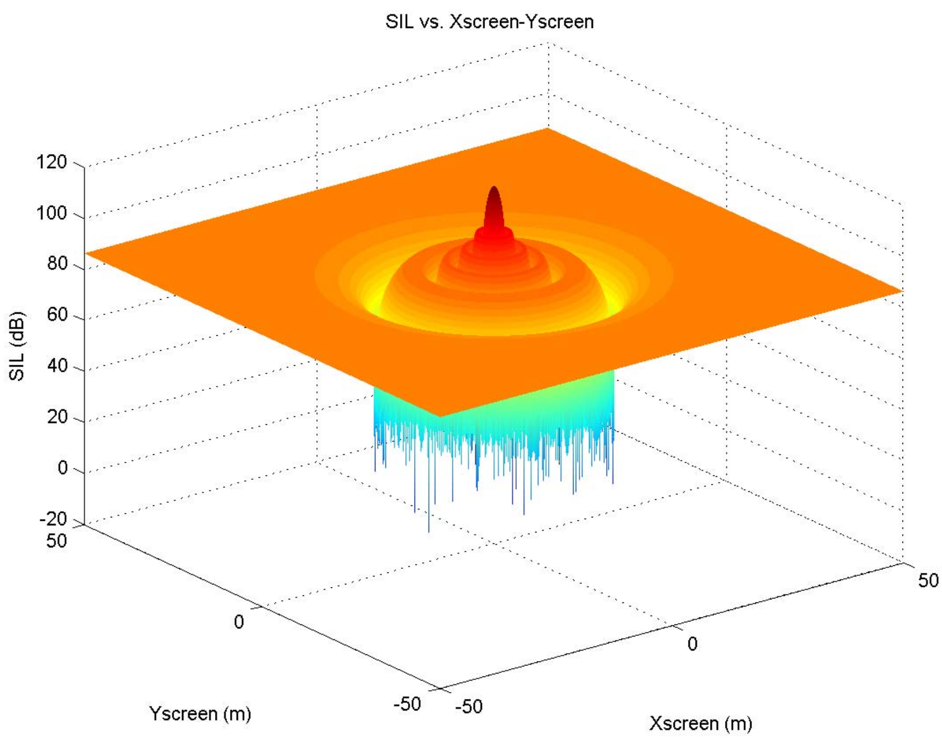
Polar plots of $SIL\ vs.\ \theta_y$ and $SIL\ vs.\ \theta_x$ for $(N_y, N_x) = (1,1)$ with $d_y = d_x = R = 1.0\,m$:



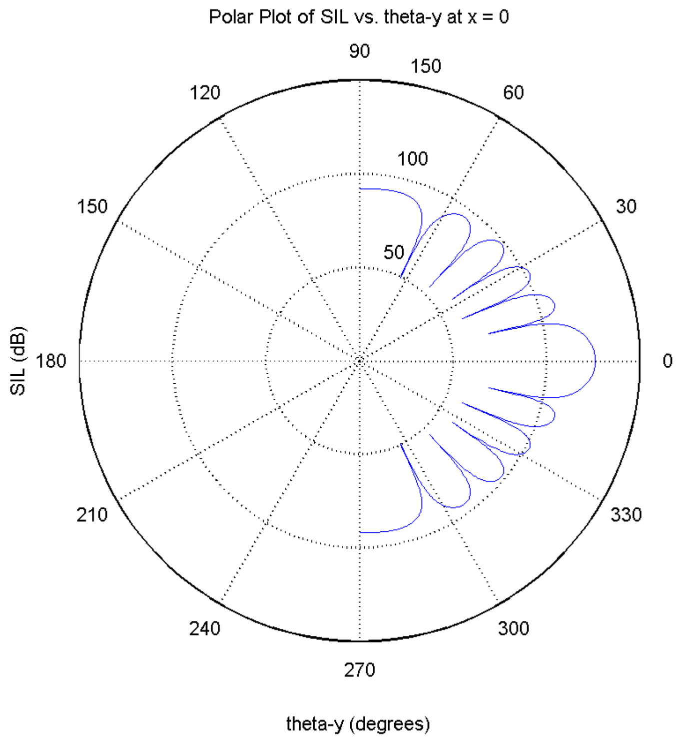


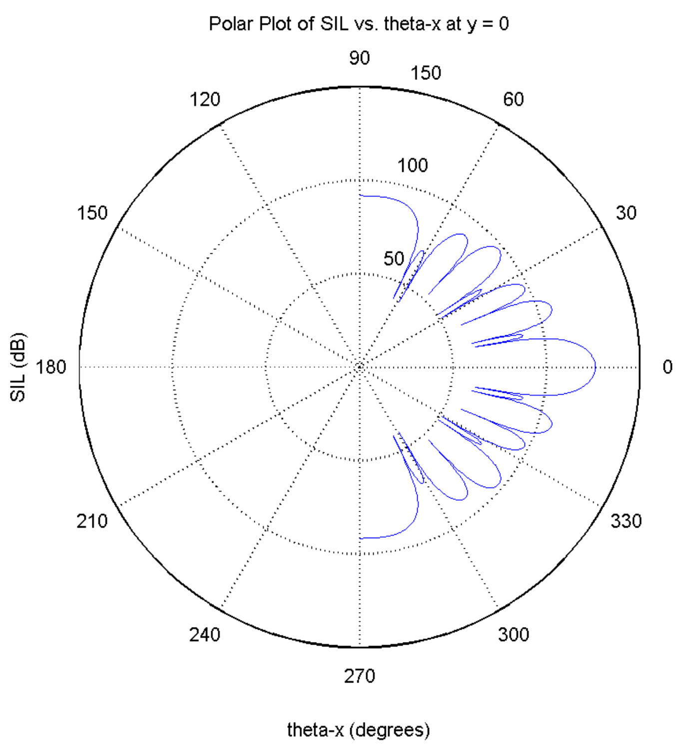
Plots of I_{tot} *vs.* (x_{scr}, y_{scr}) and *SIL vs.* (x_{scr}, y_{scr}) for $(N_y, N_x) = (1,1)$ with $d_y = d_x = R = 1.0\,m$:



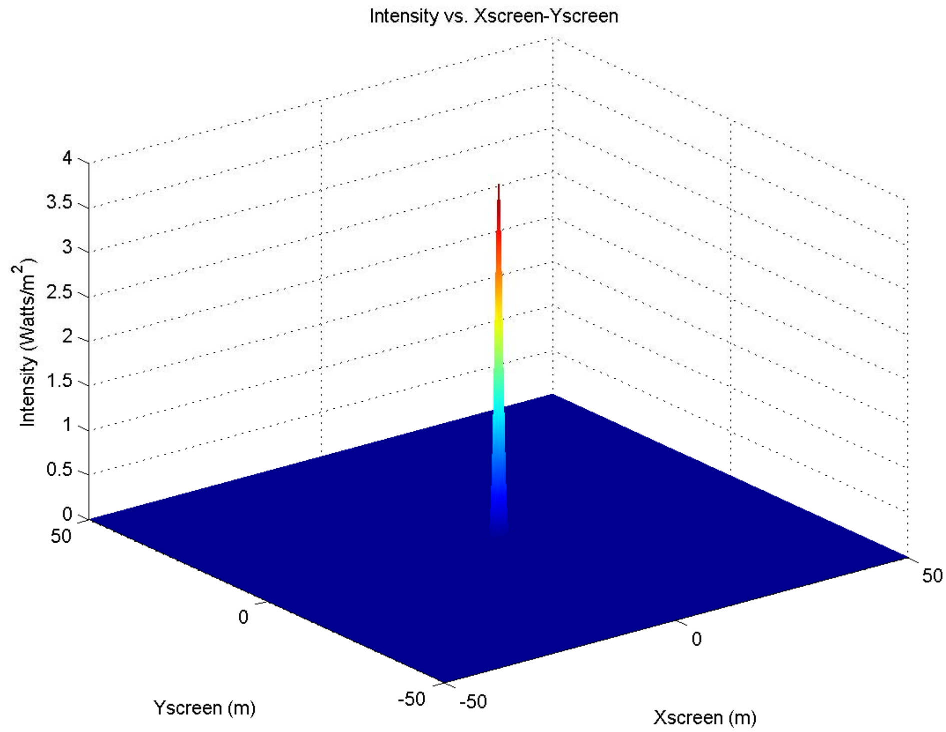


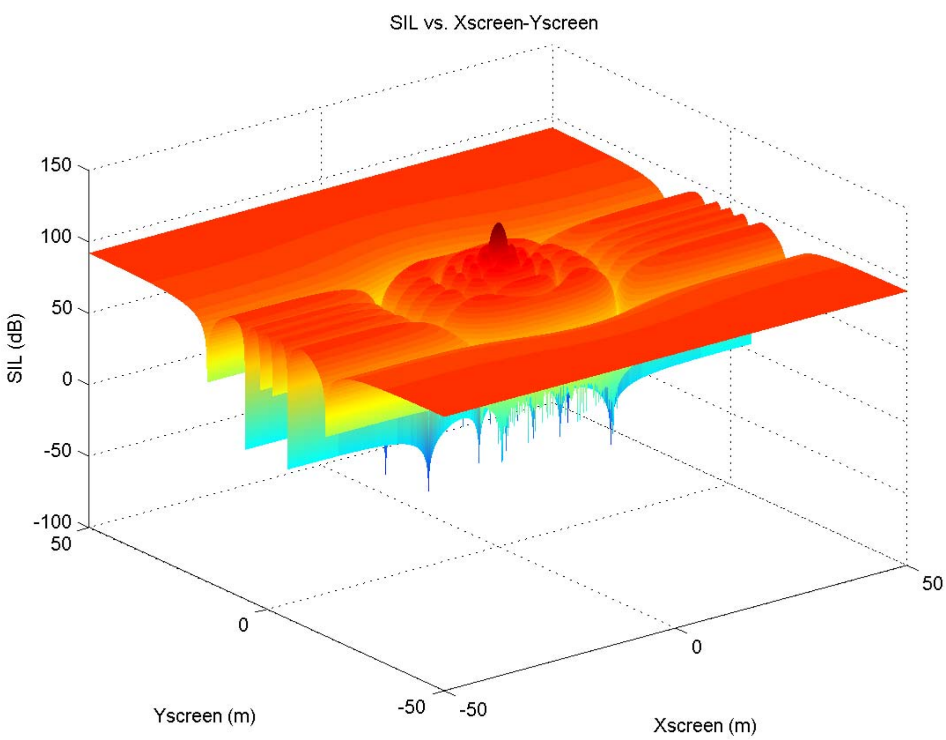
Polar plots of $SIL\ vs.\ \theta_y$ and $SIL\ vs.\ \theta_x$ for $\left(N_y, N_x\right) = (1,2)$ with $d_y = d_x = R = 1.0\,m$:



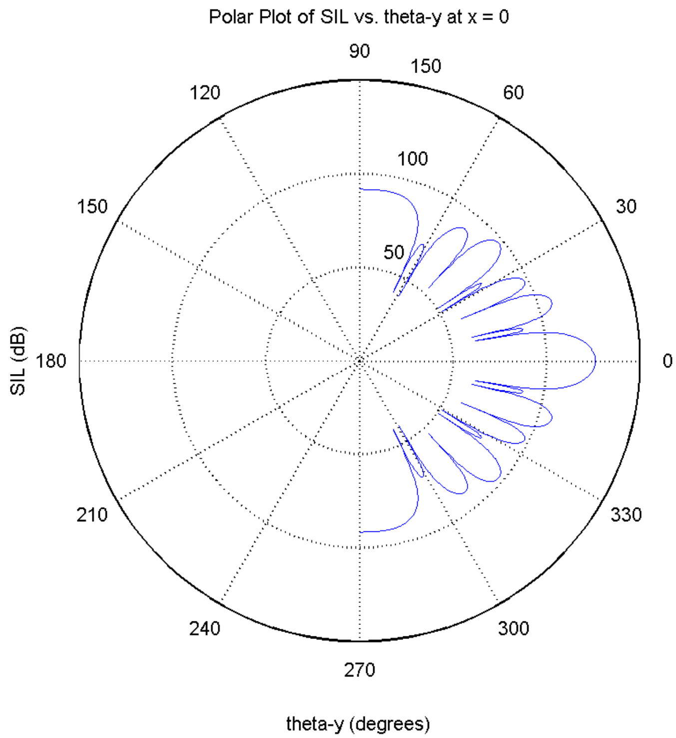


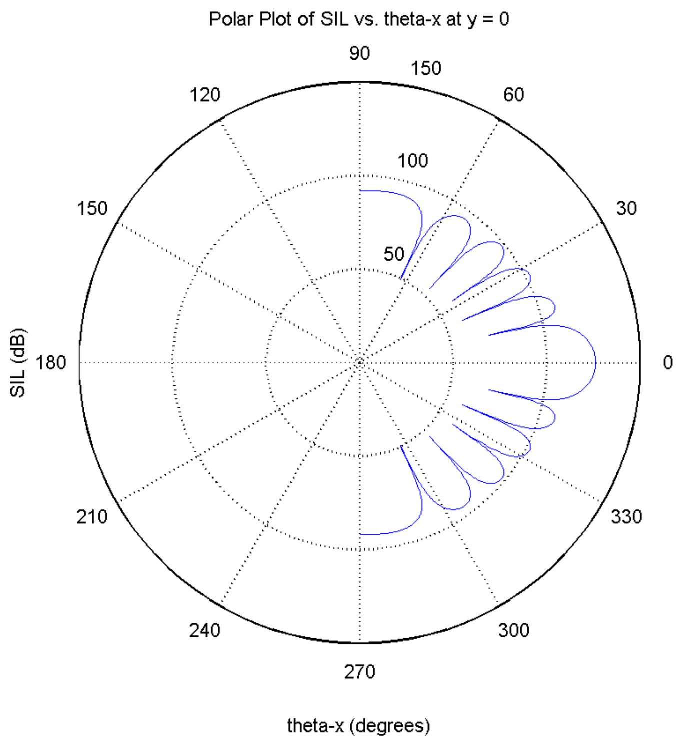
Plots of I_{tot} *vs.* (x_{scr}, y_{scr}) and *SIL vs.* (x_{scr}, y_{scr}) for $(N_y, N_x) = (1,2)$ with $d_y = d_x = R = 1.0\,m$:



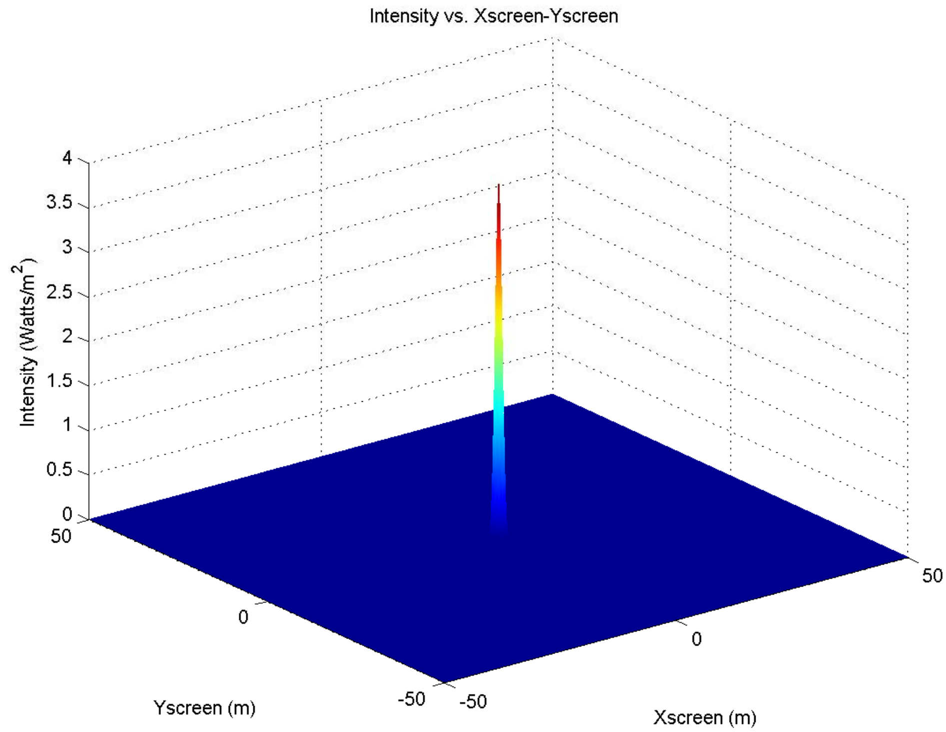


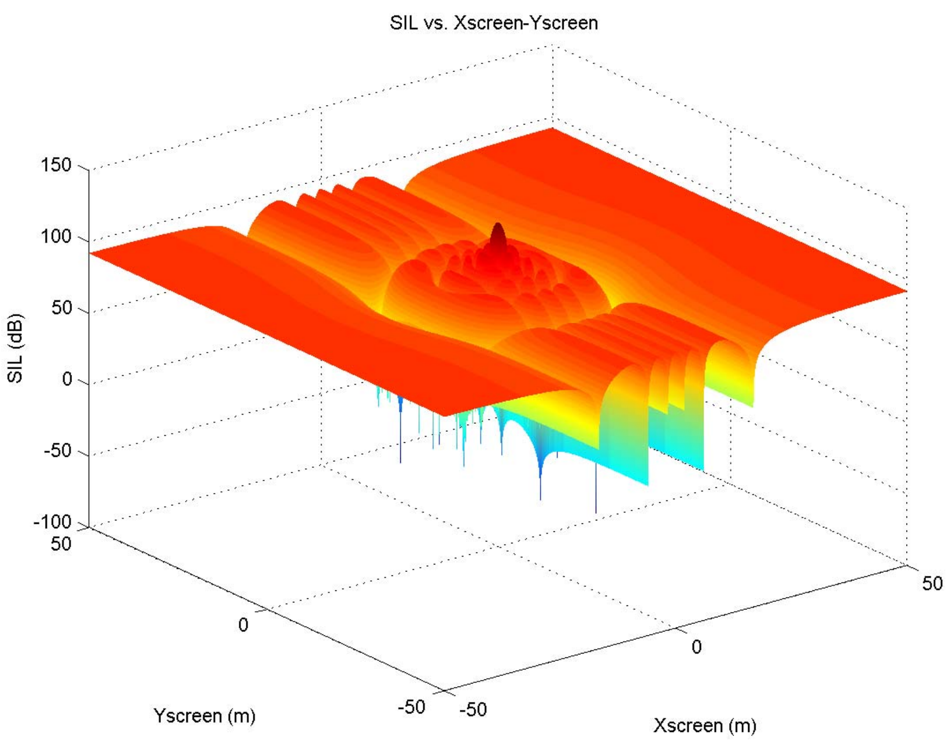
Polar plots of $SIL\ vs.\ \theta_y$ and $SIL\ vs.\ \theta_x$ for $\left(N_y, N_x\right) = \left(2,1\right)$ with $d_y = d_x = R = 1.0\,m$:



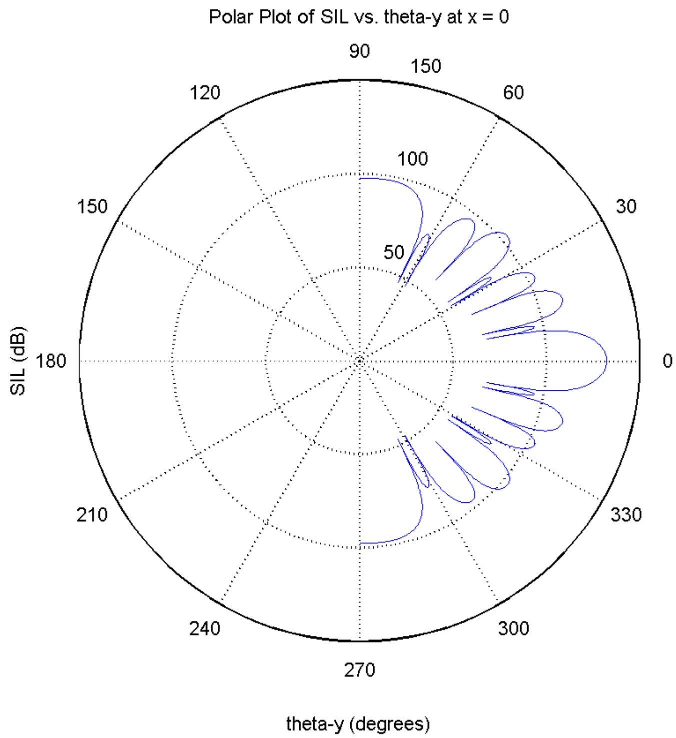


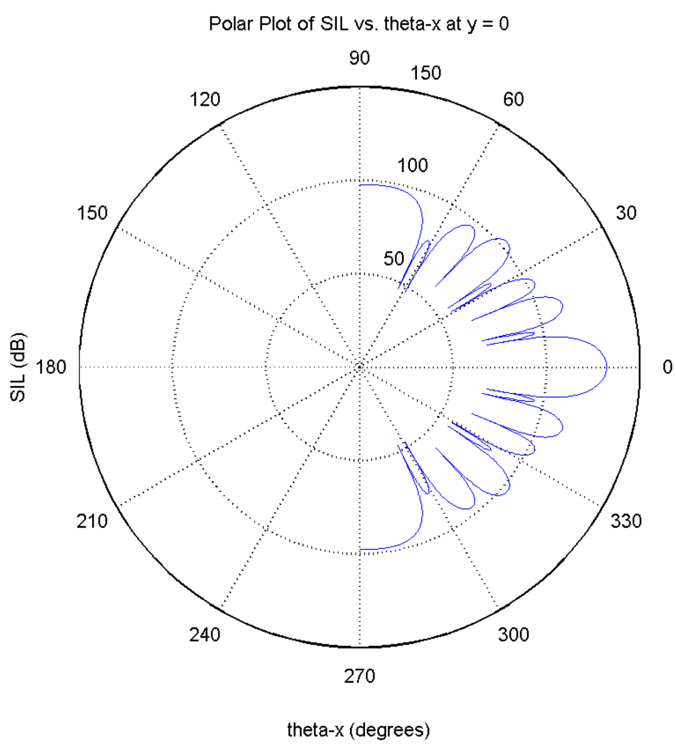
Plots of I_{tot} *vs.* $\left(x_{scr}, y_{scr}\right)$ and *SIL vs.* $\left(x_{scr}, y_{scr}\right)$ for $\left(N_y, N_x\right)=\left(2,1\right)$ with $d_y = d_x = R = 1.0\,m$:



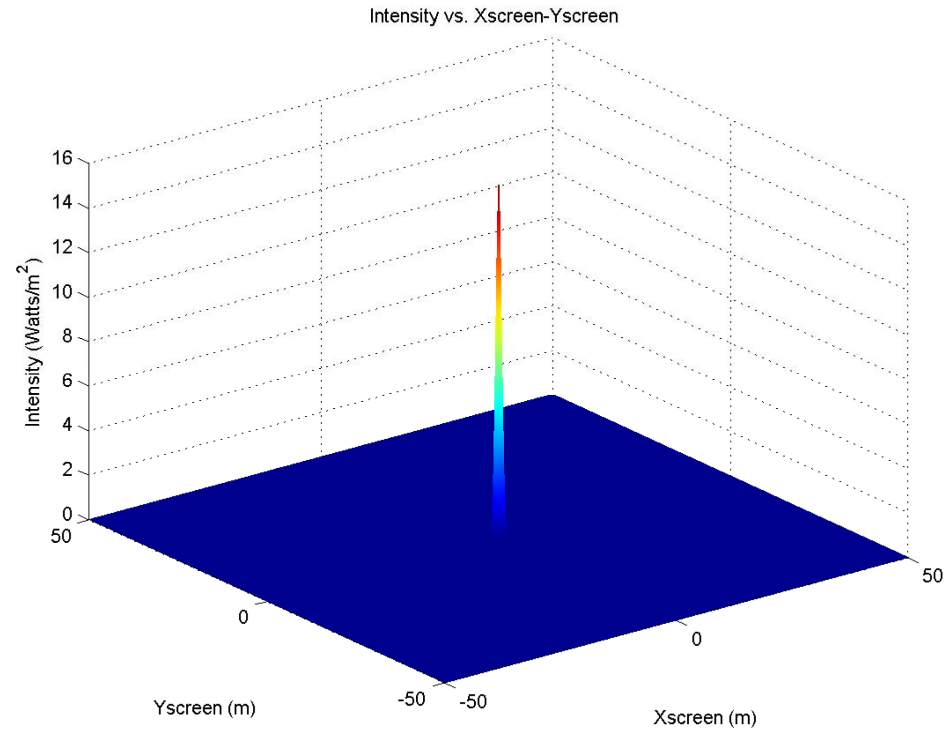


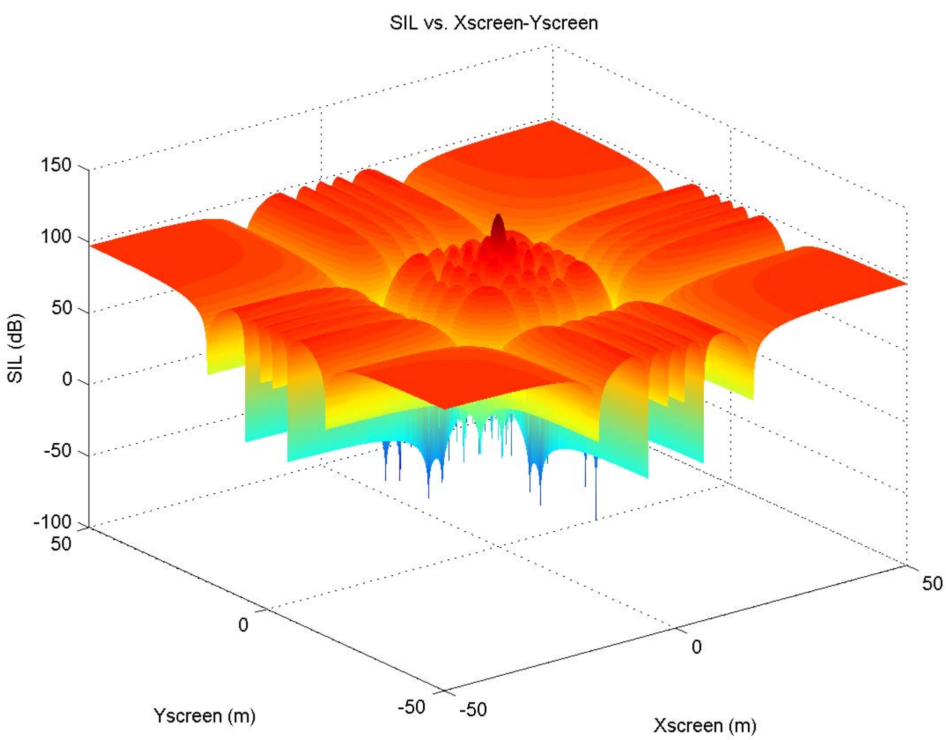
Polar plots of $SIL\ vs.\ \theta_y$ and $SIL\ vs.\ \theta_x$ for $\left(N_y, N_x\right) = \left(2, 2\right)$ with $d_y = d_x = R = 1.0\,m$:



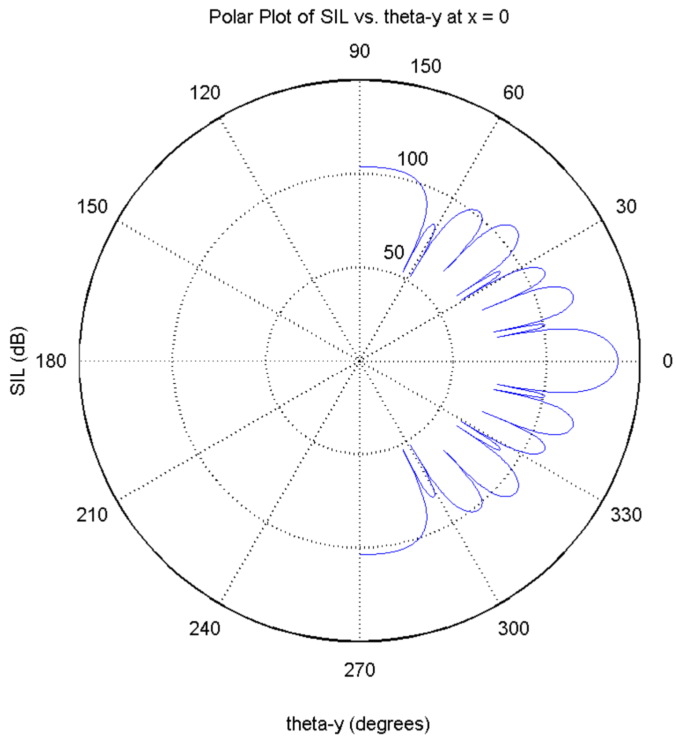


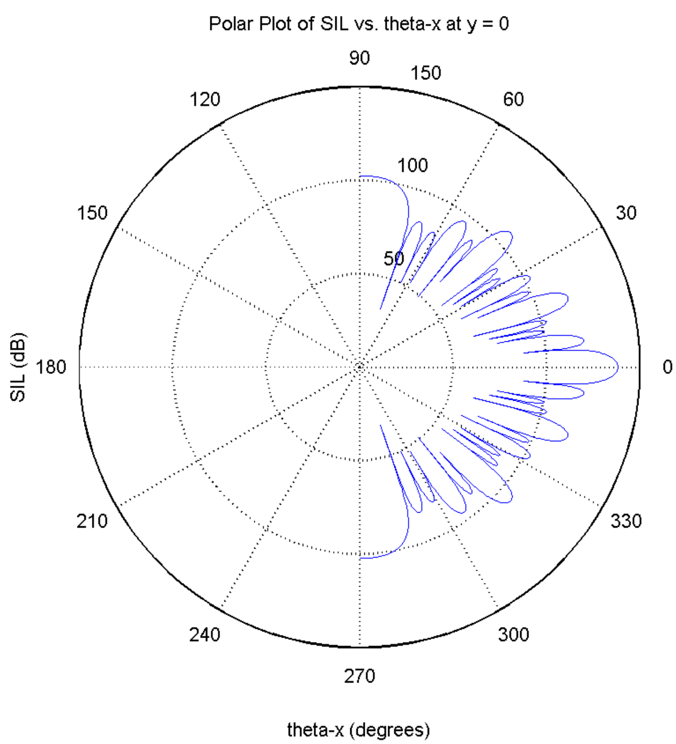
Plots of I_{tot} *vs.* (x_{scr}, y_{scr}) and *SIL vs.* (x_{scr}, y_{scr}) for $(N_y, N_x) = (2,2)$ with $d_y = d_x = R = 1.0\,m$:



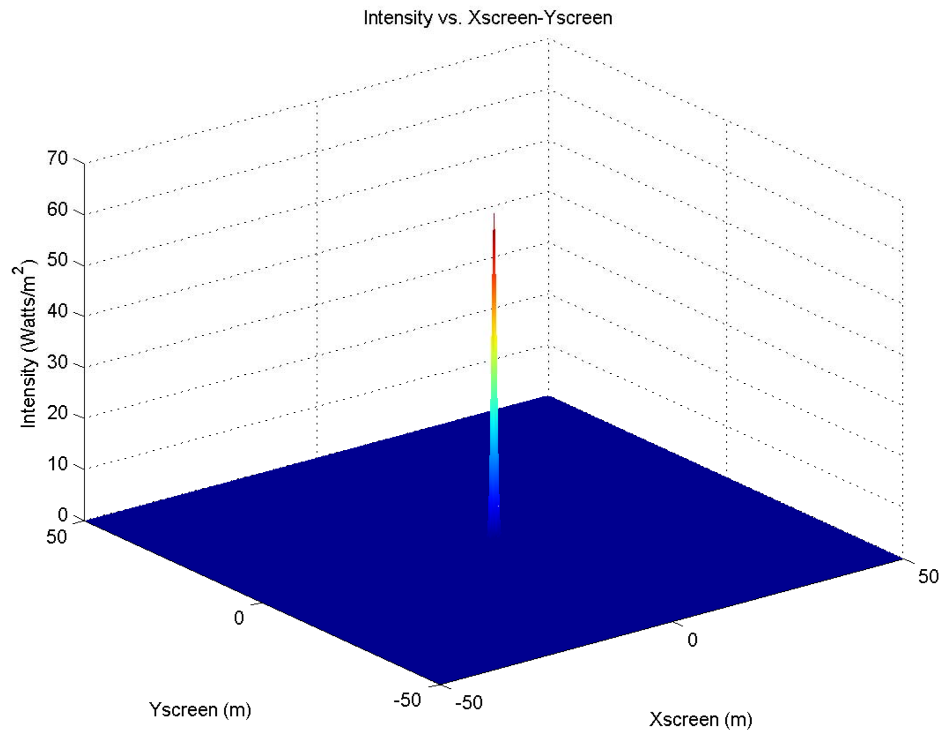


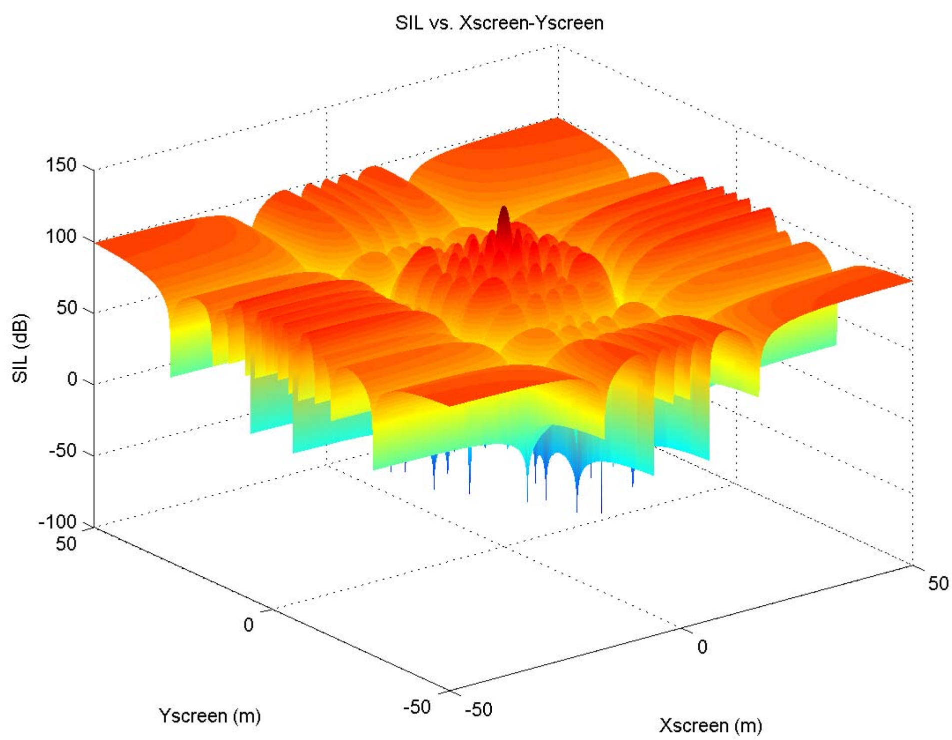
Polar plots of $SIL\ vs.\ \theta_y$ and $SIL\ vs.\ \theta_x$ for $(N_y, N_x) = (2,4)$ with $d_y = d_x = R = 1.0\,m$:



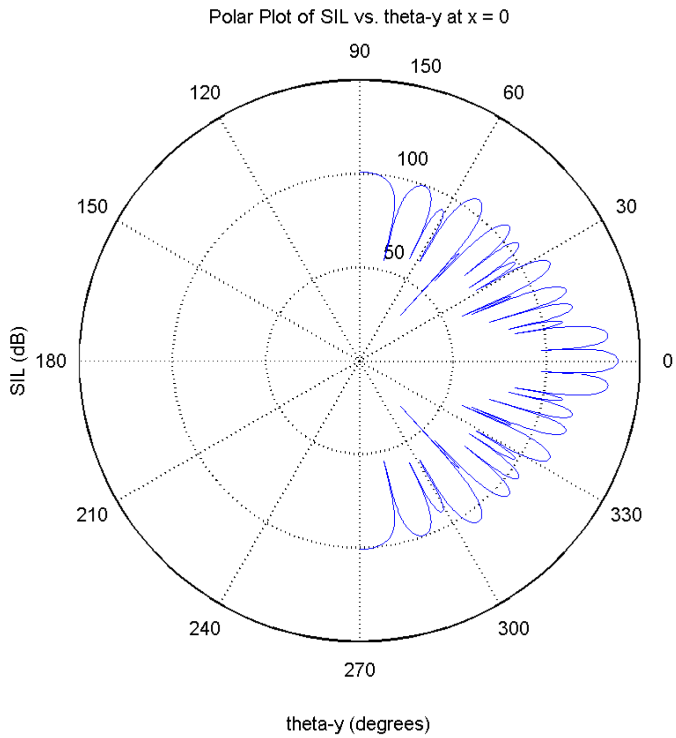


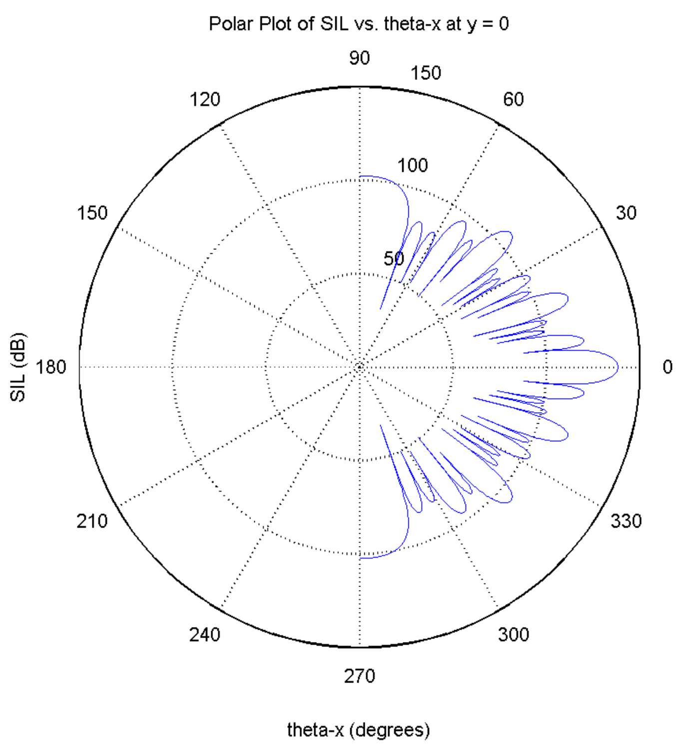
Plots of I_{tot} *vs.* (x_{scr}, y_{scr}) and *SIL vs.* (x_{scr}, y_{scr}) for $(N_y, N_x) = (2,4)$ with $d_y = d_x = R = 1.0\,m$:





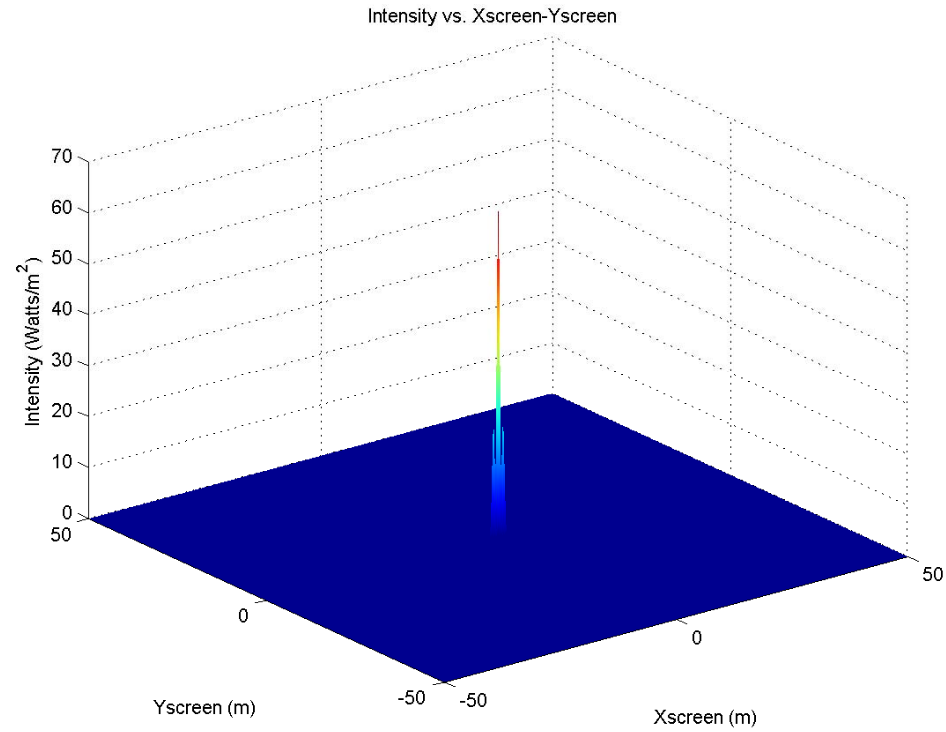
Polar plots of $SIL\ vs.\ \theta_y$ and $SIL\ vs.\ \theta_x$ for $\left(N_y, N_x\right) = (2,4)$ with $d_y = 3\,m, d_x = R = 1.0\,m$:

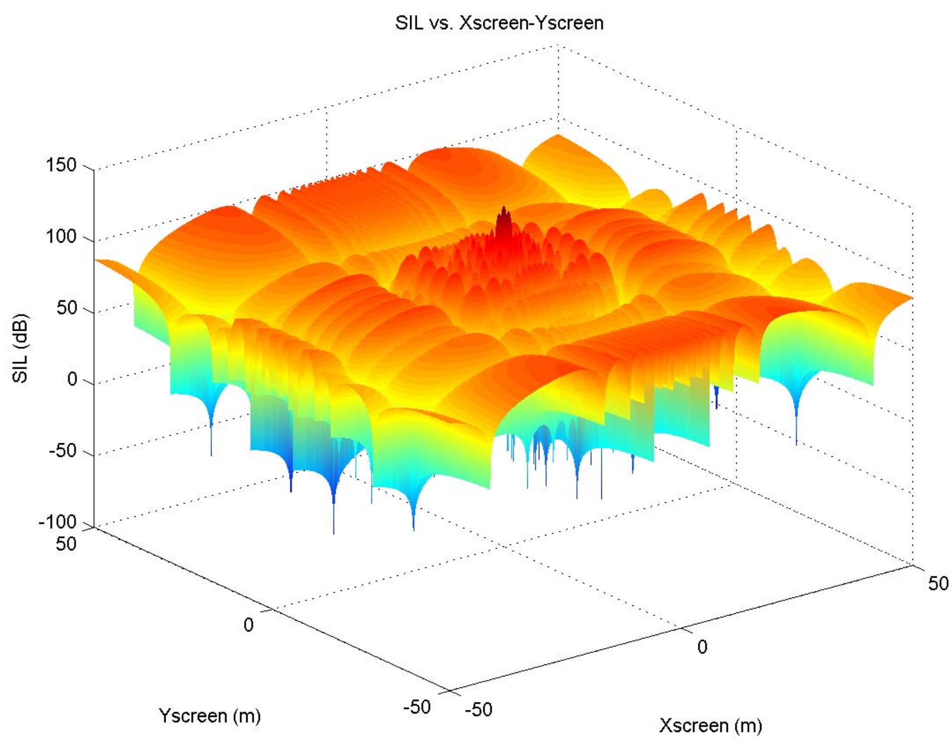




{n.b. This is a model for 2×4 PA columns}

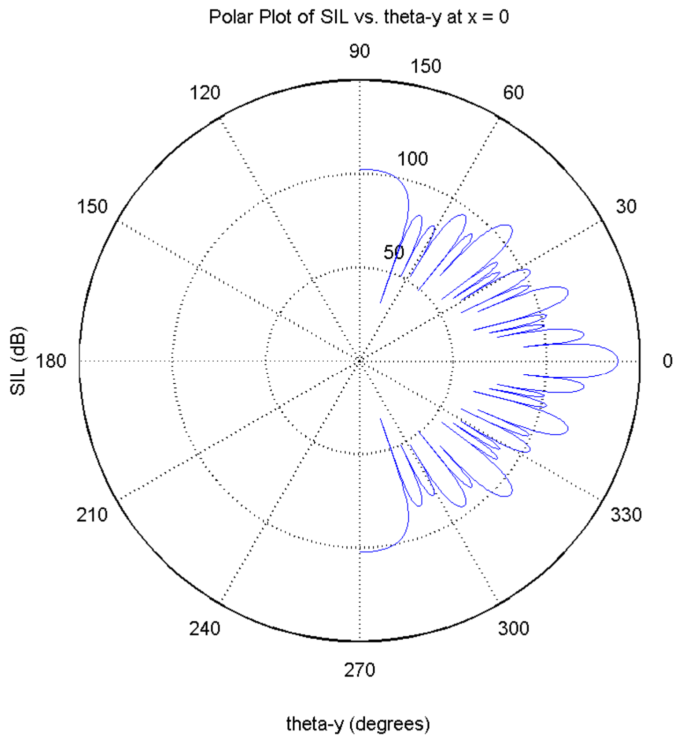
Plots of I_{tot} *vs.* (x_{scr}, y_{scr}) and *SIL vs.* (x_{scr}, y_{scr}) for $(N_y, N_x) = (2,4)$ with $d_y = 3\,m, d_x = R = 1.0\,m$:

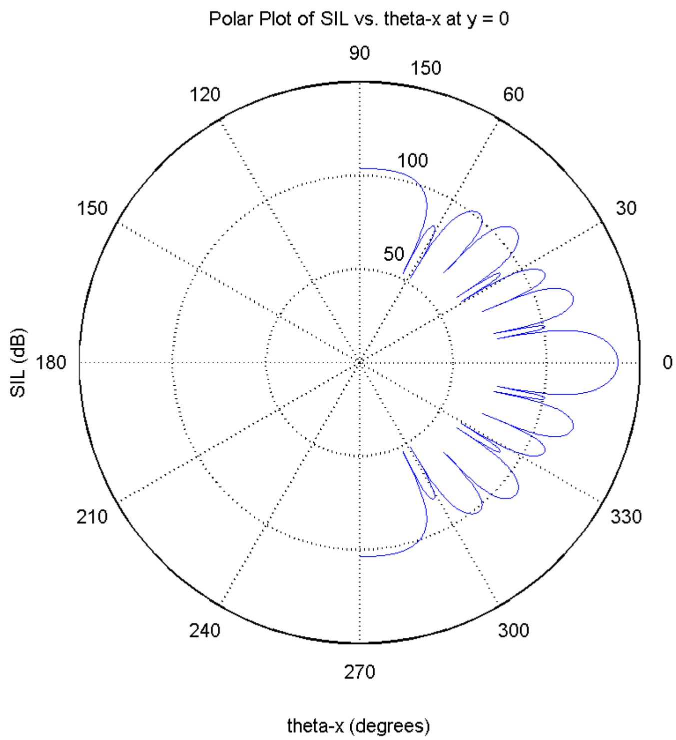




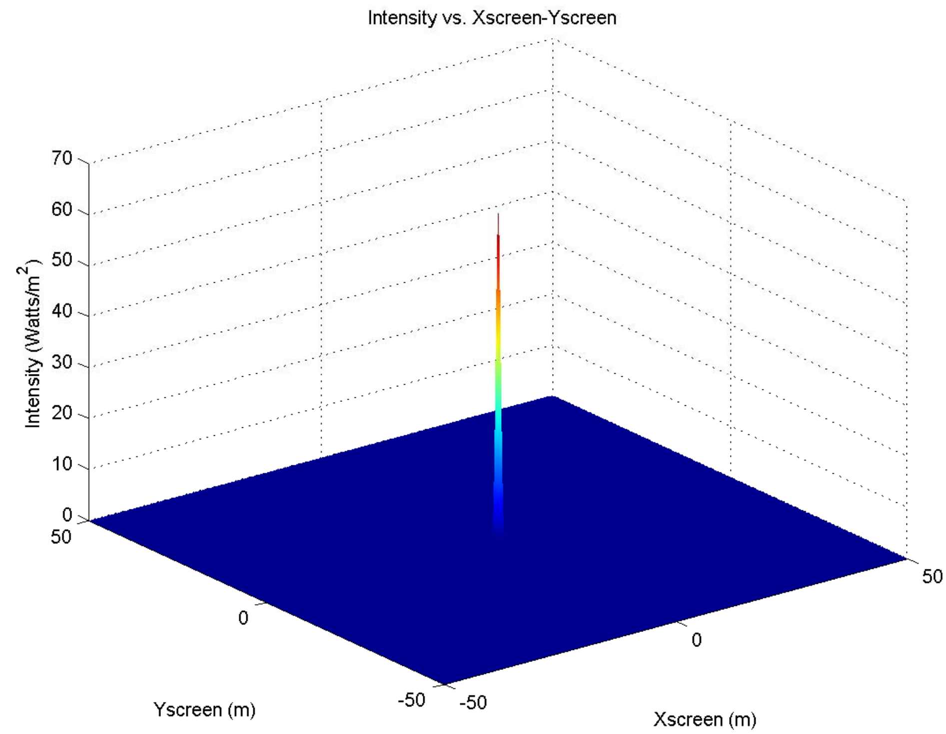
{n.b. This is a model for 2×4 PA columns}

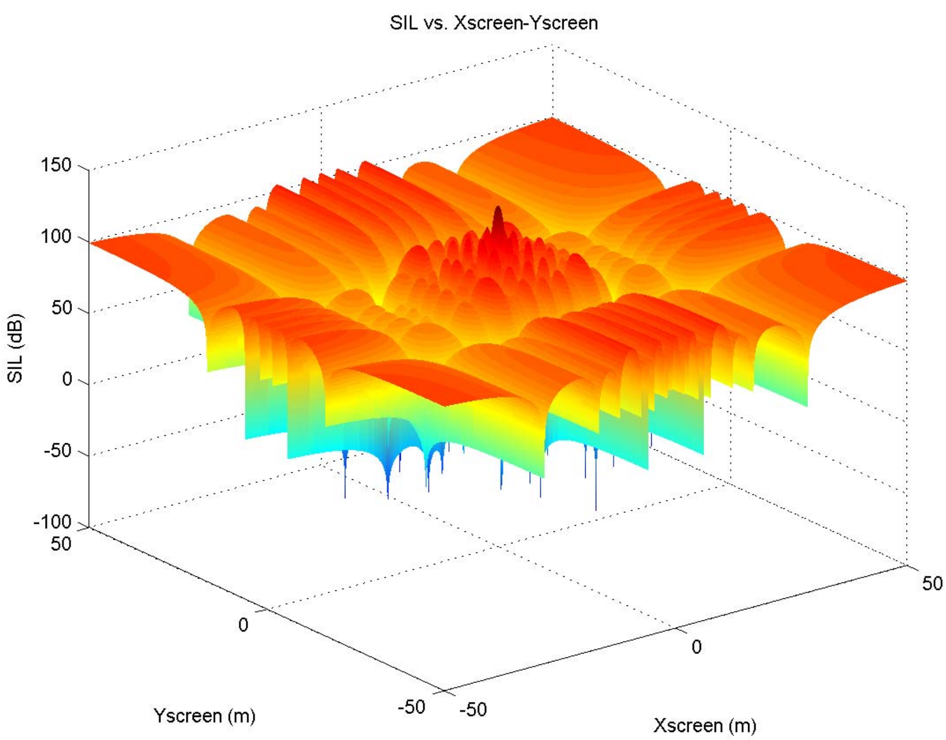
Polar plots of $SIL\ vs.\ \theta_y$ and $SIL\ vs.\ \theta_x$ for $(N_y, N_x) = (4, 2)$ with $d_y = d_x = R = 1.0\,m$:



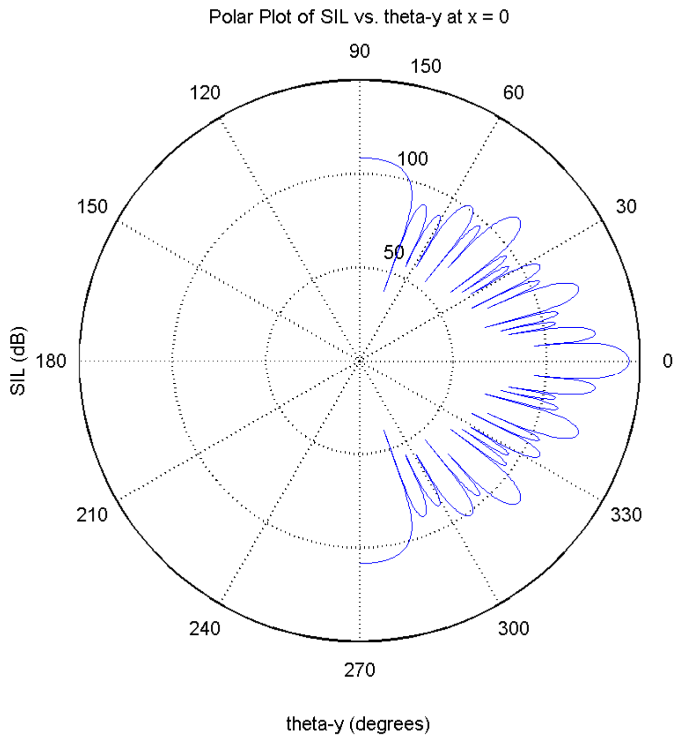


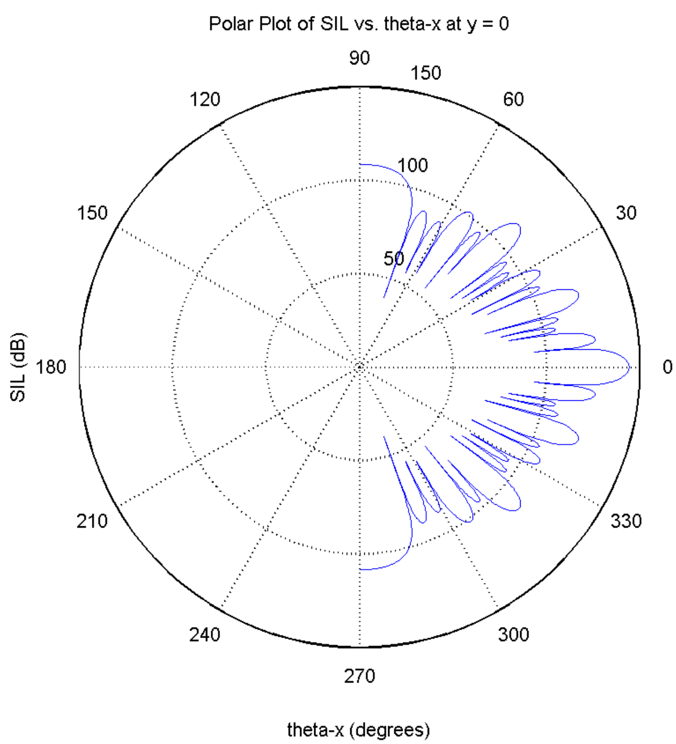
Plots of I_{tot} *vs.* (x_{scr}, y_{scr}) and *SIL vs.* (x_{scr}, y_{scr}) for $(N_y, N_x) = (4,2)$ with $d_y = d_x = R = 1.0\,m$:



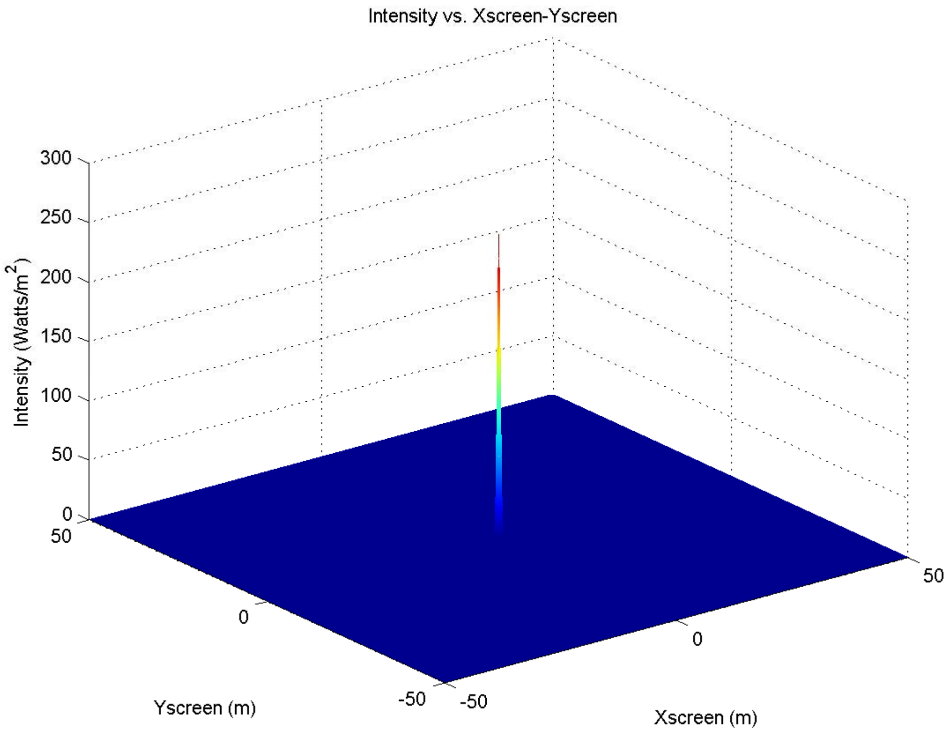


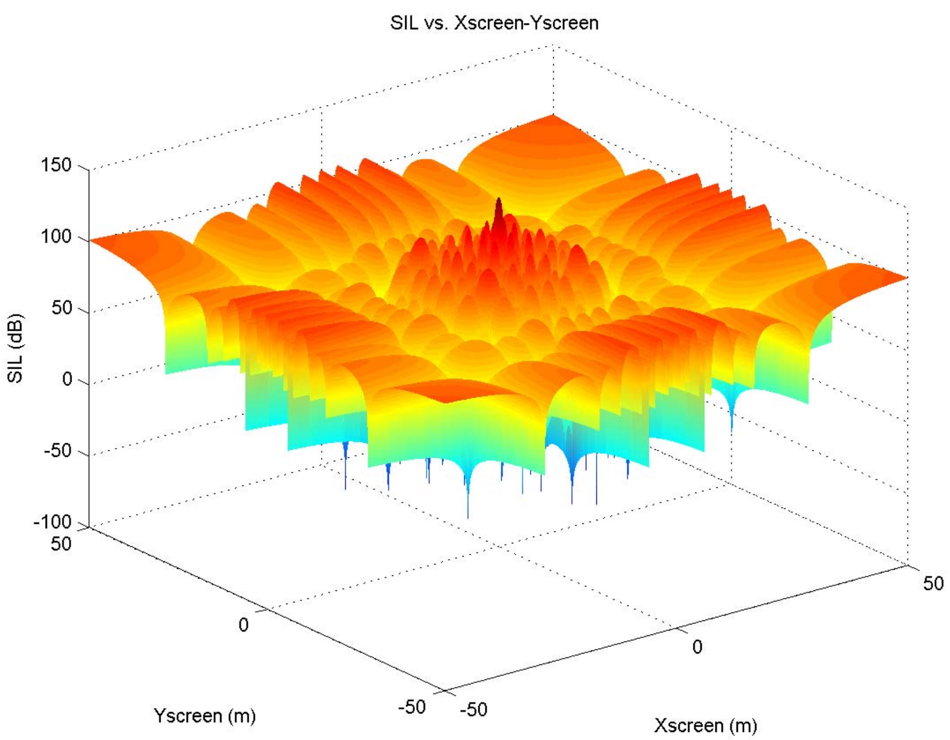
Polar plots of $SIL\ vs.\ \theta_y$ and $SIL\ vs.\ \theta_x$ for $(N_y, N_x) = (4,4)$ with $d_y = d_x = R = 1.0\, m$:





Plots of I_{tot} *vs.* (x_{scr}, y_{scr}) and *SIL vs.* (x_{scr}, y_{scr}) for $(N_y, N_x) = (4,4)$ with $d_y = d_x = R = 1.0\,m$:





Listing of the MATLAB code:

```
%===========================================================================
% Intf_Diffn_2D_Circ_Aperture_Thy.m
%
% 2-D interference-diffraction associated with N circular apertures
% - simplest theory - far-field/plane-wave approx!
% Sound waves assumed to be propagating in free air/great wide-open!
%
% Ny circular apertures distributed symmetrically along horizontal y-axis
% Nx circular apertures distributed symmetrically along  vertical  x-axis
% We use a right-handed x-y-z coordinate system.
%===========================================================================
%
% Written by Prof. Steven Errede  Last Updated: Feb. 11, 2011 09:10 hr
%
%===========================================================================
close all;
clear all;

single    thtxr(1800);
single    thtxd(1800);

single    thtyr(1800);
single    thtyd(1800);

single   Itotx1(1800);
single    SILx1(1800);

single   Itoty1(1800);
single    SILy1(1800);

single     xscr(2000);
single   Itotx2(2000);
single    SILx2(2000);

single     yscr(2000);
single   Itoty2(2000);
single    SILy2(2000);

single   Itotxy(2000,2000);
single LgItotxy(2000,2000);
single    SILxy(2000,2000);


% Specify the # of y,x circular apertures:
Nyapr  = 2; % 1; 2; 1; 2; 2; 4; 4; horizontal
Nxapr  = 4; % 1; 1; 2; 2; 4; 2; 4;  vertical

% Specify the numerical values of parameters:
Io     =    1.0;    % intensity from single slit (Watts/m^2)
Ir     = 1.0*10^-12;% reference sound intensity  (Watts/m^2)
Vair   =  343.0;    % speed of propagation of sound - free air (m/s)
freq   = 1000.0;    % frequency (Hz or cps)
lambda = Vair/freq; % wavelength (m)
Lobs   =   10.0;    % observer distance           (m) n.b. lambda << Lobs
Dx     =    1.0;    % x-distance between apertures(m) n.b.   Dx   << Lobs
Dy     =    3.0;    % y-distance between apertures(m) n.b.   Dy   << Lobs
Rapr   =    1.0;    % 0.6; 1.0; aperture radius   (m) n.b.   Rapr << Lobs

nu     =      1;    % order of bessel function of 1st kind (see below)

%====================================================================
% Calculate Itot, SIL vs. theta-y along horizontal y-axis @ x = 0:
%====================================================================
Thetad =  -90.0;    % angle theta of observer in degrees
dTheta =    0.1;    % step angle in degrees

for i = 1:1800;
     thtyd(i) = Thetad;            % angle theta of observer in degrees
```

```
     Thetar   = (pi/180.0)*Thetad; % angle theta of observer in radians
     thtyr(i) = Thetar;

     deltay  =   ((2.0*pi*Dy)/lambda)*sin(Thetar);  % int'f phase (radians)
     rho     = ((2.0*pi*Rapr)/lambda)*sin(Thetar);  % diffn phase (radians)

   Itoty1(i) = Io*(Nxapr)^2*(sin(Nyapr*deltay/2.0)/sin(deltay/2.0))^2
                 *((2.0*bessel(nu,rho))/rho)^2;
    SILy1(i) = 10.0*log10(Itoty1(i)/Ir); % Sound Intensity Level (dB)

     Thetad  = Thetad + dTheta;   % increment angle for next calculation
end

%===================================================================
% Calculate Itot, SIL vs. yscreen along horizontal y-axis @ x = 0:
%===================================================================
 y = -50.00; % starting position on screen (m)
dy =   0.05; % step size on screen (m);
for i = 1:2000;
     yscr(i) = y;            % position of observer on perp. screen (m)
     Thetar  = atan(y/Lobs); % angle theta of observer in radians

     deltay  =   ((2.0*pi*Dy)/lambda)*sin(Thetar); % int'f phase (radians)
     rho     = ((2.0*pi*Rapr)/lambda)*sin(Thetar); % diffn phase (radians)

   Itoty2(i) = Io*(Nxapr)^2*(sin(Nyapr*deltay/2.0)/sin(deltay/2.0))^2
                 *((2.0*bessel(nu,rho))/rho)^2;
    SILy2(i) = 10.0*log10(Itoty2(i)/Ir); % Sound Intensity Level (dB)

     y  = y + dy;   % increment screen position for next calculation
end

%======================================================================
% Calculate 1-D Itot, SIL vs. theta-x along horizontal x-axis @ y = 0:
%======================================================================
Thetad =  -90.0;    % angle theta of observer in degrees
dTheta =    0.1;    % step angle in degrees

for i = 1:1800;
     thtxd(i) = Thetad;            % angle theta of observer in degrees
     Thetar   = (pi/180.0)*Thetad; % angle theta of observer in radians
     thtxr(i) = Thetar;

     deltax  =   ((2.0*pi*Dx)/lambda)*sin(Thetar);  % int'f phase (radians)
     rho     = ((2.0*pi*Rapr)/lambda)*sin(Thetar);  % diffn phase (radians)

   Itotx1(i) = Io*(Nyapr)^2*(sin(Nxapr*deltax/2.0)/sin(deltax/2.0))^2
                 *((2.0*bessel(nu,rho))/rho)^2;
    SILx1(i) = 10.0*log10(Itotx1(i)/Ir); % Sound Intensity Level (dB)

     Thetad  = Thetad + dTheta;   % increment angle for next calculation
end

%======================================================================
% Calculate 1-D Itot, SIL vs. xscreen along horizontal x-axis @ y = 0:
%======================================================================
 x = -50.00; % starting position on screen (m)
dx =   0.05; % step size on screen (m);
for i = 1:2000;
     xscr(i) = x;            % position of observer on perp. screen (m)
     Thetar  = atan(x/Lobs); % angle theta of observer in radians

     deltax  =   ((2.0*pi*Dx)/lambda)*sin(Thetar); % int'f phase (radians)
     rho     = ((2.0*pi*Rapr)/lambda)*sin(Thetar); % diffn phase (radians)

   Itotx2(i) = Io*(Nyapr)^2*(sin(Nxapr*deltax/2.0)/sin(deltax/2.0))^2
                 *((2.0*bessel(nu,rho))/rho)^2;
    SILx2(i) = 10.0*log10(Itotx2(i)/Ir); % Sound Intensity Level (dB)

     x  = x + dx;   % increment screen position for next calculation
end
```

```
figure(01);
plot(thtyd,Itoty1,'b');
grid on;
xlabel('theta-y (degrees)');
ylabel('Intensity (Watts/m^{2})');
title('Intensity vs. theta-y at x = 0');

figure(02);
semilogy(thtyd,Itoty1,'b');
grid on;
xlabel('theta-y (degrees)');
ylabel('Intensity (Watts/m^{2})');
title('Log10 Intensity vs. theta-y at x = 0');

figure(03);
plot(thtyd,SILy1,'b');
grid on;
xlabel('theta-y (degrees)');
ylabel('SIL (dB)');
title('SIL vs. theta-y at x = 0');

figure(04);
polar(thtyr,SILy1,'b');
grid on;
xlabel('theta-y (degrees)');
ylabel('SIL (dB)');
title('Polar Plot of SIL vs. theta-y at x = 0');

figure(05);
plot(yscr,Itoty2,'b');
grid on;
xlabel('Yscreen (m)');
ylabel('Intensity (Watts/m^{2})');
title('Intensity vs. Yscreen at x = 0');

figure(06);
semilogy(yscr,Itoty2,'b');
grid on;
xlabel('Yscreen (m)');
ylabel('Intensity (Watts/m^{2})');
title('Log10 Intensity vs. Yscreen at x = 0');

figure(07);
plot(yscr,SILy2,'b');
grid on;
xlabel('Yscreen (m)');
ylabel('SIL (dB)');
title('SIL vs. Yscreen at x = 0');

figure(11);
plot(thtxd,Itotx1,'b');
grid on;
xlabel('theta-x (degrees)');
ylabel('Intensity (Watts/m^{2})');
title('Intensity vs. theta-x at y = 0');

figure(12);
semilogy(thtxd,Itotx1,'b');
grid on;
xlabel('theta-x (degrees)');
ylabel('Intensity (Watts/m^{2})');
title('Log10 Intensity vs. theta-x at y = 0');

figure(13);
plot(thtxd,SILx1,'b');
grid on;
xlabel('theta-x (degrees)');
ylabel('SIL (dB)');
title('SIL vs. theta-x at y = 0');
```

```
figure(14);
polar(thtxr,SILx1,'b');
grid on;
xlabel('theta-x (degrees)');
ylabel('SIL (dB)');
title('Polar Plot of SIL vs. theta-x at y = 0');

figure(15);
plot(xscr,Itotx2,'b');
grid on;
xlabel('Xscreen (m)');
ylabel('Intensity (Watts/m^{2})');
title('Intensity vs. Xscreen at y = 0');

figure(16);
semilogy(xscr,Itotx2,'b');
grid on;
xlabel('Xscreen (m)');
ylabel('Intensity (Watts/m^{2})');
title('Log10 Intensity vs. Xscreen at y = 0');

figure(17);
plot(xscr,SILx2,'b');
grid on;
xlabel('Xscreen (m)');
ylabel('SIL (dB)');
title('SIL vs. Xscreen at y = 0');

%==========================================================================
fprintf('\n Very CPU-intensive I(x,y) vs. x,y calcs - please be patient!! \n')
%==========================================================================

%==========================================
% Calculate 2-D Itot, SIL vs. x,y-screen:
%==========================================
 x = -50.00; % x-starting position on screen (m)
dx =   0.05; % x-step size on screen (m);
for j = 1:2000;
     xscr(j) = x; % x-position of observer on perp. screen (m)

     Thetax  = atan(x/Lobs);    % x-projected angle theta-x in radians
     deltax  = ((2.0*pi*Dx)/lambda)*sin(Thetax); % int'f phase (radians)

     y = -50.00; % y-starting position on screen (m)
    dy =   0.05; % y-step size on screen (m);
    for i = 1:2000;
         yscr(i) = y; % y-position of observer on perp. screen (m)
         rscr    = sqrt((xscr(j))^2 + (yscr(i))^2); % radial pos'n on perp. screen (m)
         Thetar  = atan(rscr/Lobs); % angle theta of observer in radians
         Thetay  = atan(y/Lobs);    % y-projected angle theta-y in radians
         deltay  =   ((2.0*pi*Dy)/lambda)*sin(Thetay); % int'f phase (radians)
         rho     = ((2.0*pi*Rapr)/lambda)*sin(Thetar); % diffn phase (radians)

     Itotxy(j,i) = Io*(sin(Nxapr*deltax/2.0)/sin(deltax/2.0))^2
                    *(sin(Nyapr*deltay/2.0)/sin(deltay/2.0))^2
                    *((2.0*bessel(nu,rho))/rho)^2; % total intensity (Watts/m^2)
   LgItotxy(j,i) = log10(Itotxy(j,i));     % log10 of total intensity (Watts/m^2)
      SILxy(j,i) = 10.0*log10(Itotxy(j,i)/Ir);      % Sound Intensity Level (dB)

         y  = y + dy;   % increment y-screen position for next calculation
    end
     x  = x + dx;   % increment x-screen position for next calculation
end

figure(21);
surf(xscr,yscr,Itotxy);
shading interp;
xlabel('Xscreen (m)');
ylabel('Yscreen (m)');
zlabel('Intensity (Watts/m^{2})');
title ('Intensity vs. Xscreen-Yscreen');
```

```
figure(22);
surf(xscr,yscr,LgItotxy);
shading interp;
xlabel('Xscreen (m)');
ylabel('Yscreen (m)');
zlabel('Log10(Intensity) (Watts/m^{2})');
title ('Log10(Intensity) vs. Xscreen-Yscreen');

figure(23);
surf(xscr,yscr,SILxy);
shading interp;
xlabel('Xscreen (m)');
ylabel('Yscreen (m)');
zlabel('SIL (dB)');
title ('SIL vs. Xscreen-Yscreen');

%========================================================================
beep;
fprintf('\n Calculation of intf-diffn for 2-D array of Nx, Ny circular apertures completed !!!
\n')
%========================================================================
```