The Point Acoustic Monopole

In this example, we show plots of the complex over-pressure, radial particle velocity, radial specific acoustic impedance, radial sound intensity, energy density, *etc*. *vs*. radial observer/listener position and also *vs*. frequency for fixed radial position associated with an isotropic sound source – the point acoustic monopole.

The time-domain and frequency-domain expressions for the complex over-pressure and (radial) particle velocity associated with a point acoustic monopole radiating sound at a single frequency $\omega = 2\pi f$ are:

$$\tilde{p}(r,t) = \frac{B_o}{r}\cdot e^{i(\omega t - kr)} = \tilde{p}(r,\omega)\cdot e^{i\omega t} \qquad i.e. \quad \tilde{p}(r,\omega) = \frac{B_o}{r}\cdot e^{-ikr}$$

$$\tilde{u}_{\mathrm{r}}(r,t) = \frac{1}{z_o}\frac{B_o}{r}\left[1-\frac{i}{kr}\right]\cdot e^{i(\omega t - kr)} = \tilde{u}_{\mathrm{r}}(r,\omega)\cdot e^{i\omega t} \quad i.e. \quad \tilde{u}_{\mathrm{r}}(r,\omega) = \frac{1}{z_o}\frac{B_o}{r}\left[1-\frac{i}{kr}\right]\cdot e^{-ikr}$$

where the characteristic longitudinal specific acoustic impedance of "free-air" (*aka* the "great wide open") is: $z_o \equiv \rho_o c_o \simeq 415\,\Omega_a\ (Rayls)$ with phase speed: $c_o = \omega/k \simeq 344\,m/s$ @ NTP.

The frequency-domain expressions for the complex radial specific acoustic impedance and radial sound intensity associated with a point acoustic monopole radiating sound at a single frequency are:

$$\tilde{z}_{\mathrm{r}}(r,t) \equiv \frac{\tilde{p}(r,t)}{\tilde{u}_{\mathrm{r}}(r,t)} = \frac{\tilde{p}(r,\omega)\cdot \cancel{e^{i\omega t}}}{\tilde{u}_{\mathrm{r}}(r,\omega)\cdot \cancel{e^{i\omega t}}} = \frac{\tilde{p}(r,\omega)}{\tilde{u}_{\mathrm{r}}(r,\omega)} \equiv \tilde{z}_{\mathrm{r}}(r,\omega)$$

$$= z_o \frac{1}{\left[1-i/kr\right]} = z_o \frac{\left[1+i/kr\right]}{\left[1+(1/kr)^2\right]}$$

and:

$$\tilde{I}_{\mathrm{r}}(r,\omega) \equiv \frac{1}{2}\,\tilde{p}(r,\omega)\cdot\tilde{u}_{\mathrm{r}}^{*}(r,\omega) = \frac{1}{2}\frac{1}{z_o}\frac{B_o^2}{r^2}\left[1+\frac{i}{kr}\right]$$

We coded up the above formulas using MATLAB to make plots of the complex over-pressure, radial particle velocity, radial specific acoustic impedance, radial acoustic intensity, energy density, *etc*. *vs*. radial observer/listener position and also *vs*. frequency for fixed radial position associated with an isotropic sound source – the point acoustic monopole.

The first set of plots (Figures 1-9) shows these complex quantities as a function of radial position for the following parameter values: $f = 300\,Hz$, $B_o = 1.0\,RMS\ Pa\text{-}m$.

The second set of plots (Figures 10-16) shows these same complex quantities as a function of frequency for the following parameter values: $r = 1.0\,m$, $B_o = 1.0\,RMS\ Pa\text{-}m$.

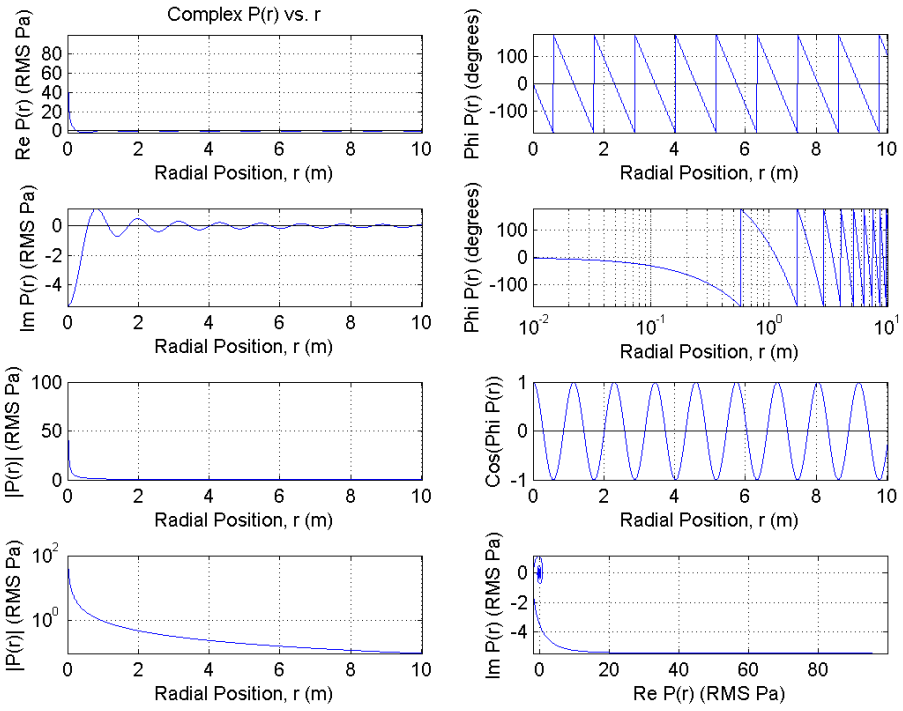

Figure 1. Complex over-pressure *vs*. radial distance for a point acoustic monopole.

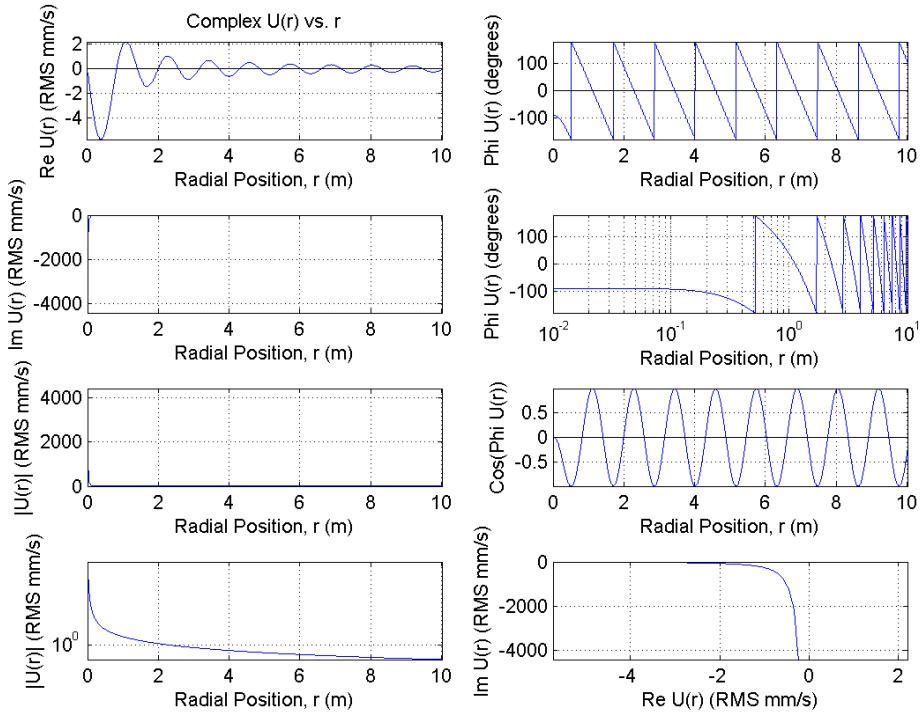

Figure 2. Complex radial particle velocity *vs*. radial distance for a point acoustic monopole.

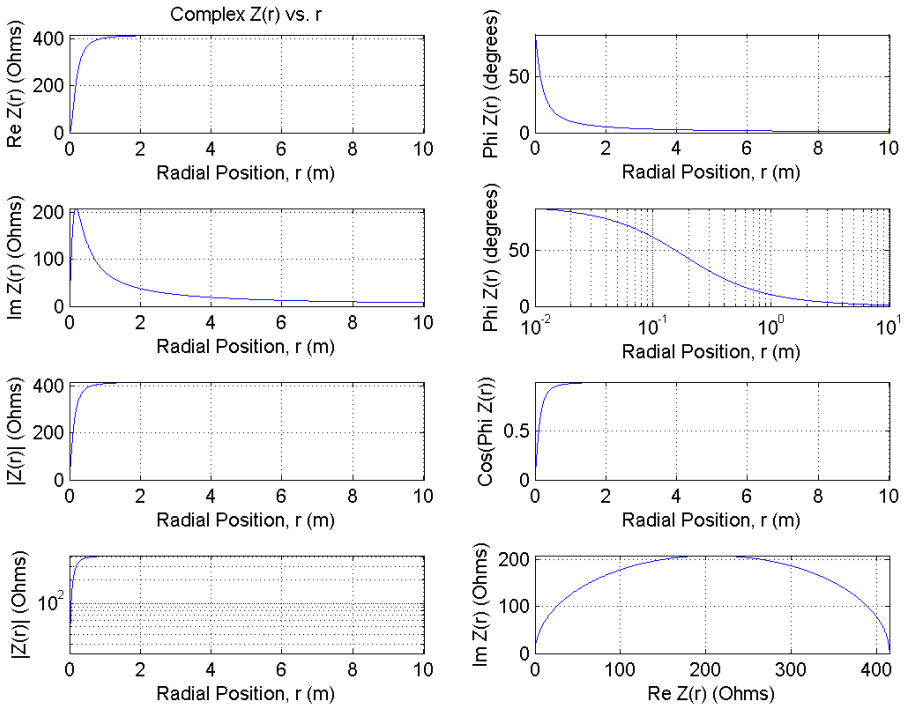

Figure 3. Complex specific acoustic impedance *vs*. radial distance for a point acoustic monopole.

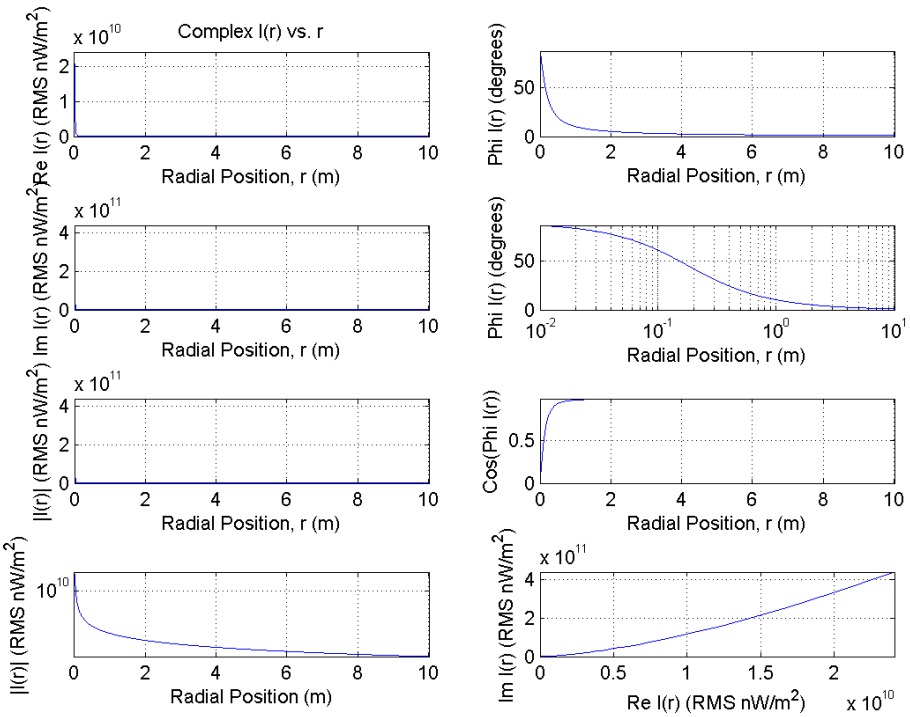

Figure 4. Complex radial acoustic intensity *vs*. radial distance for a point acoustic monopole.

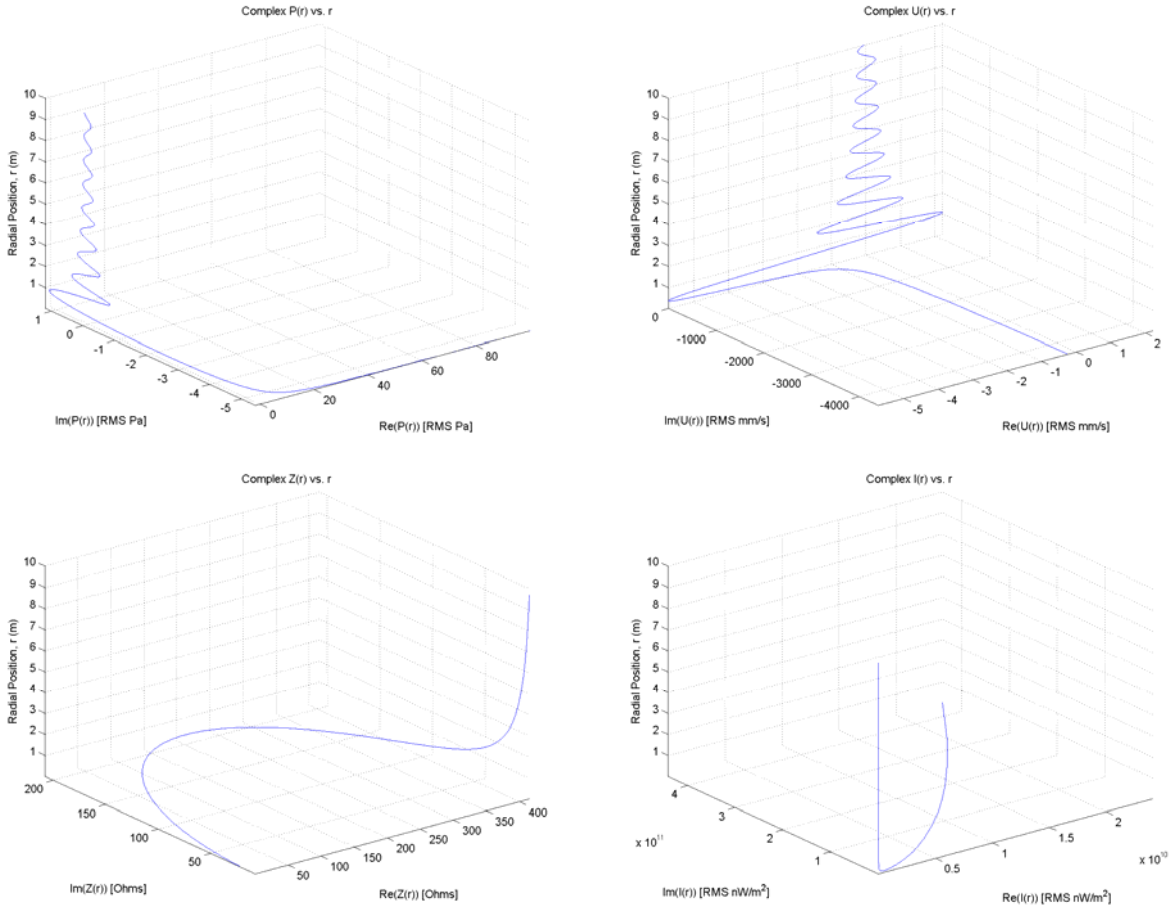

Figure 5. 3-D plots of the complex plane *vs*. radial distance for complex over-pressure, particle velocity, radial acoustic specific impedance and acoustic intensity for a point acoustic monopole.

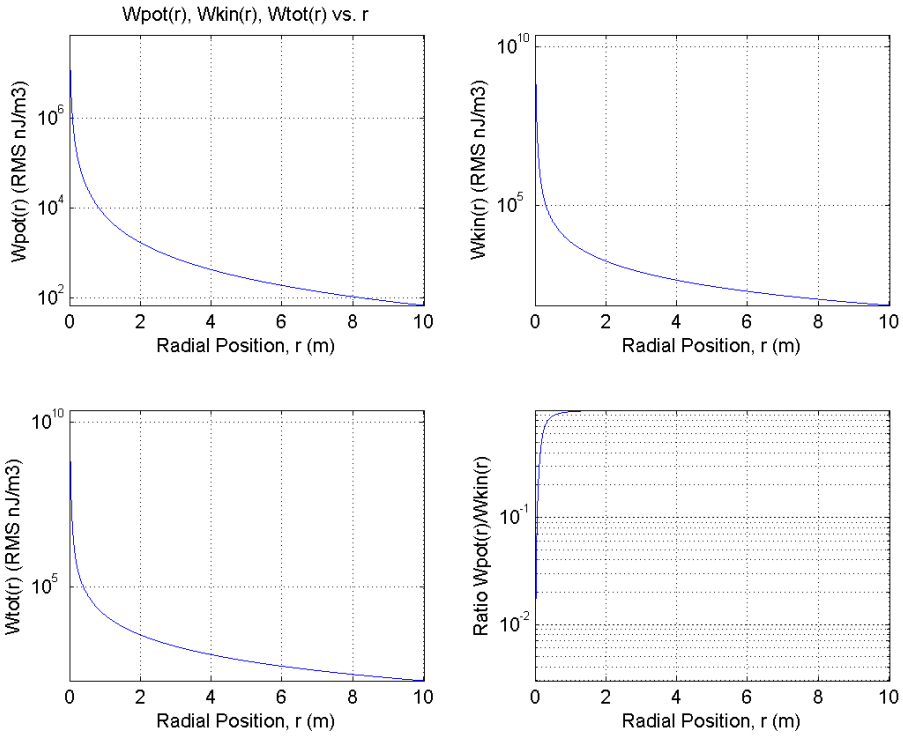

Figure 6. Potential, kinetic and total acoustic energy density *vs*. radial distance for a point acoustic monopole.

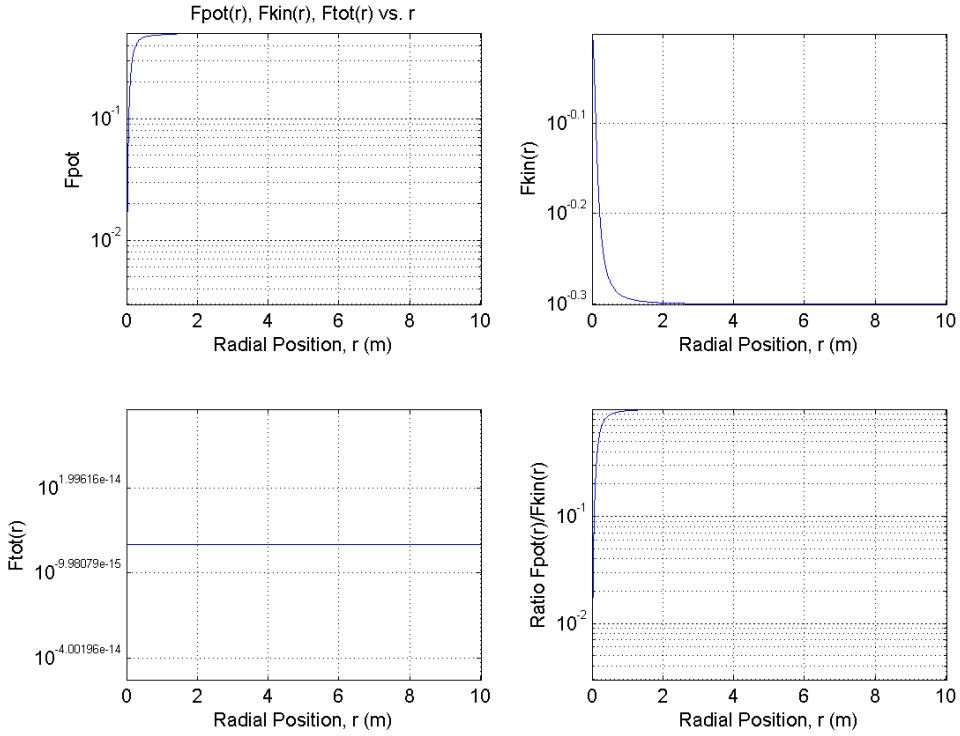

Figure 7. Potential, kinetic and total acoustic energy density fractions *vs*. radial distance for a point acoustic monopole.

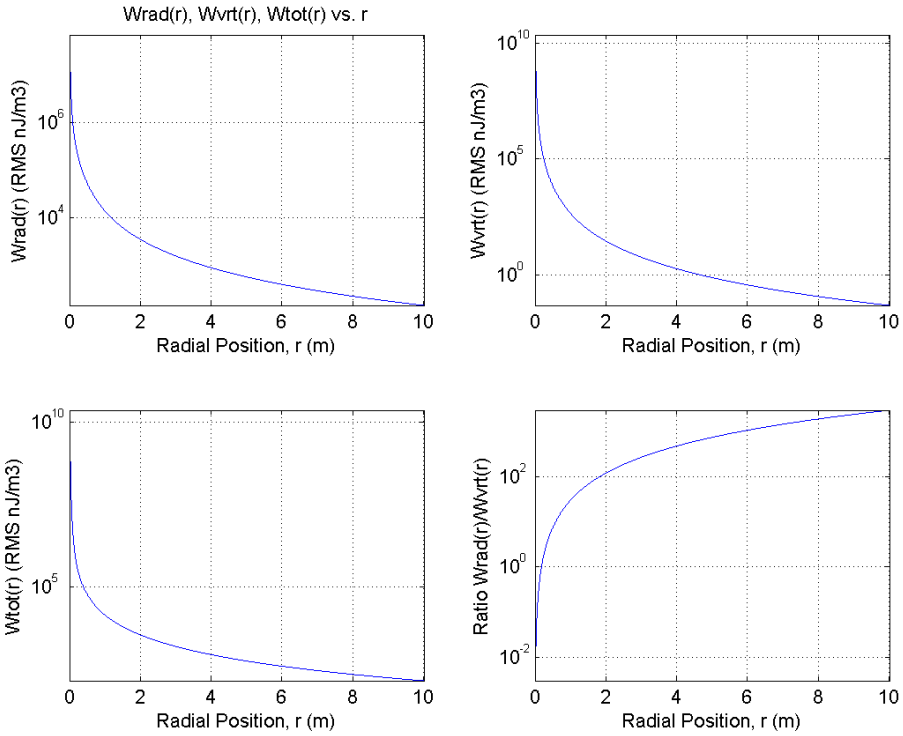

Figure 8. Propagating, non-propagating and total acoustic energy density *vs*. radial distance for a point acoustic monopole.

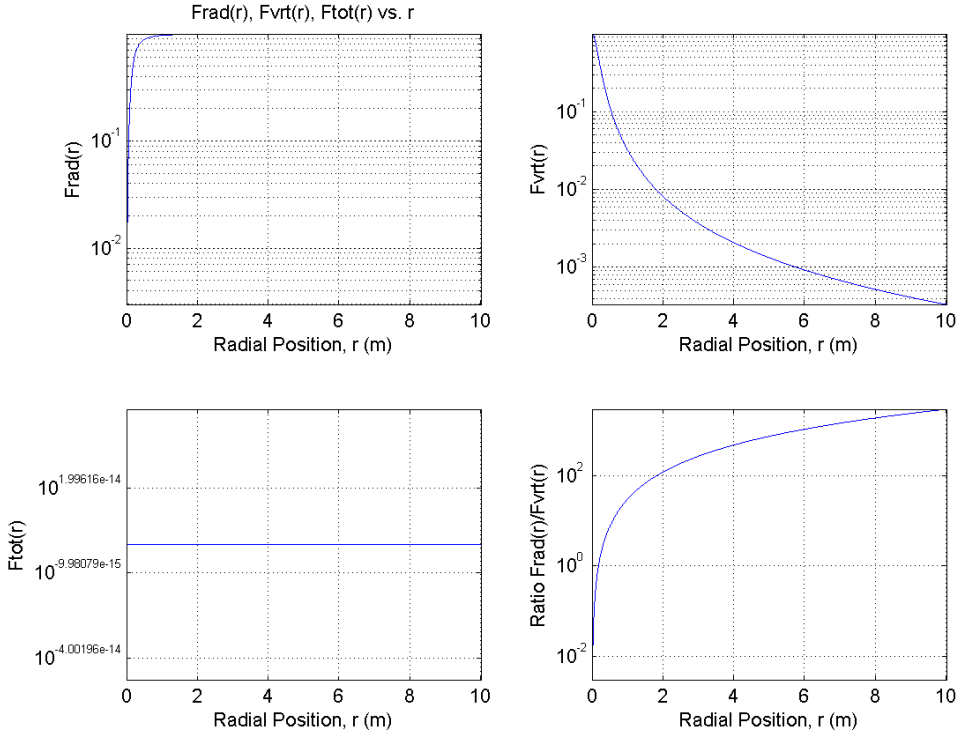

Figure 9. Propagating, non-propagating and total acoustic energy density fractions *vs*. radial distance for a point acoustic monopole.

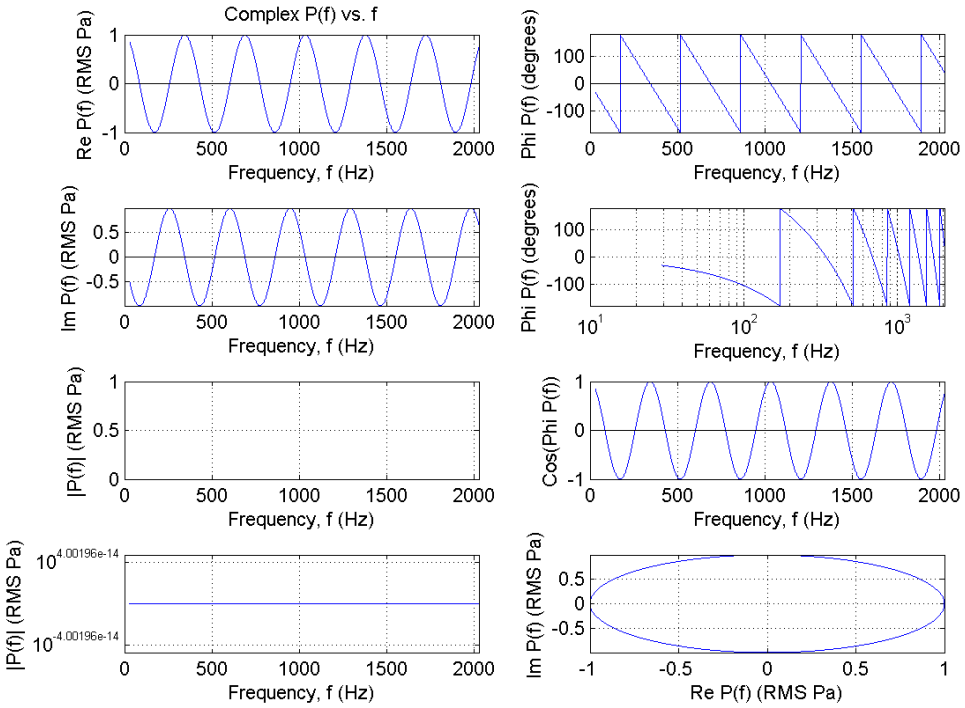

Figure 10. Complex over-pressure *vs*. frequency for a point acoustic monopole.

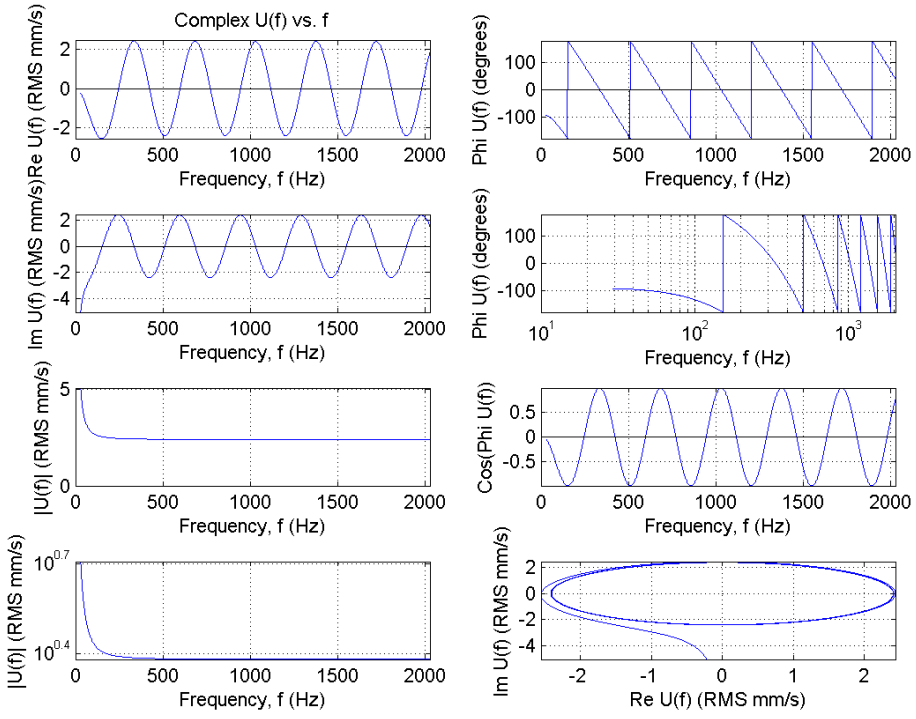

Figure 11. Complex radial particle velocity *vs*. frequency for a point acoustic monopole.

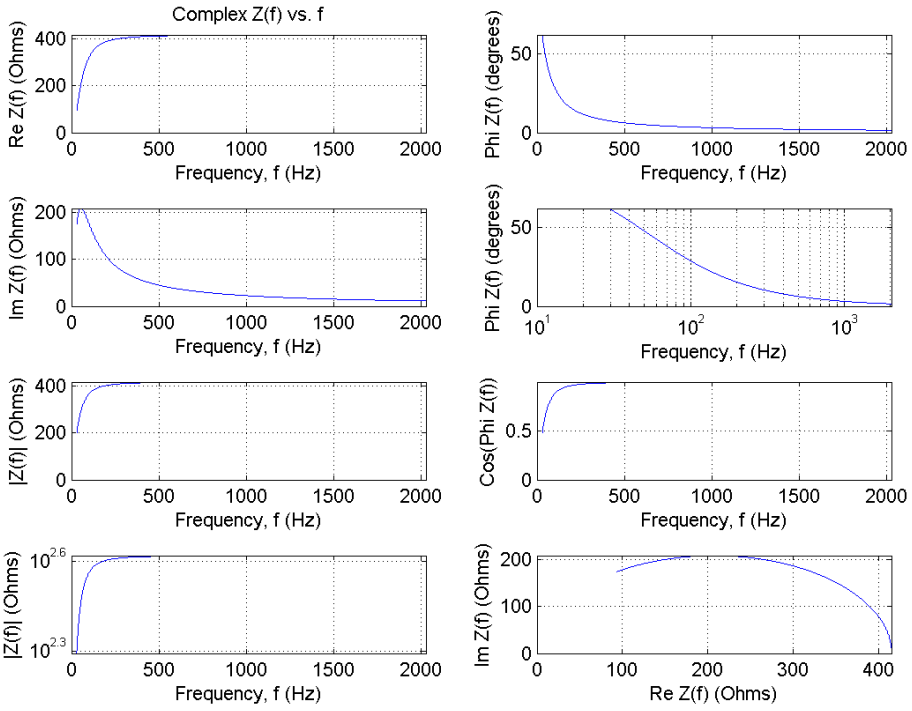

Figure 12. Complex specific acoustic impedance *vs*. frequency for a point acoustic monopole.

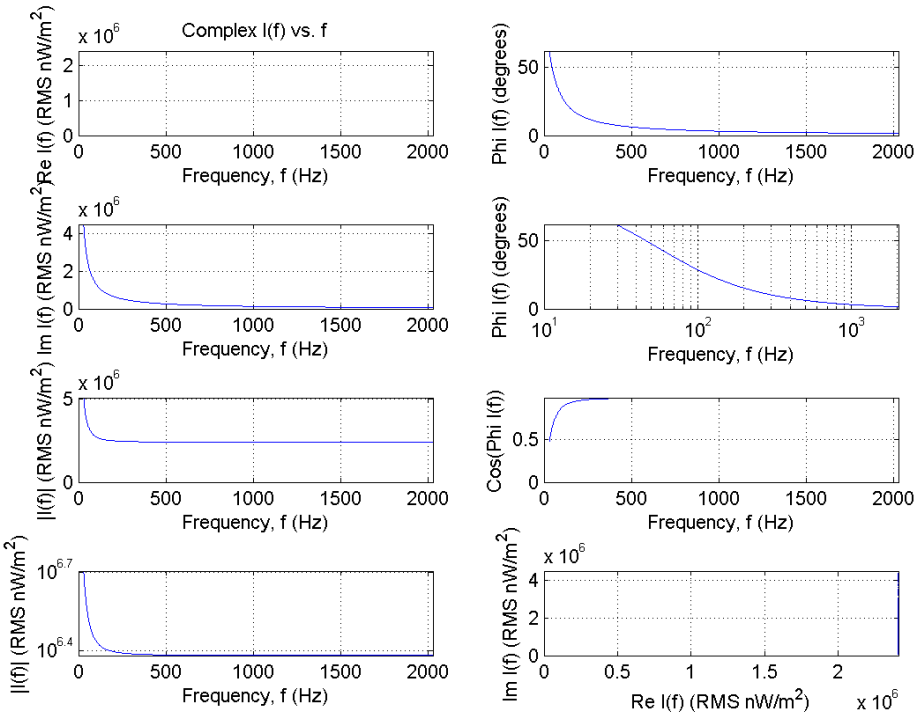

Figure 13. Complex radial acoustic intensity *vs*. frequency for a point acoustic monopole.

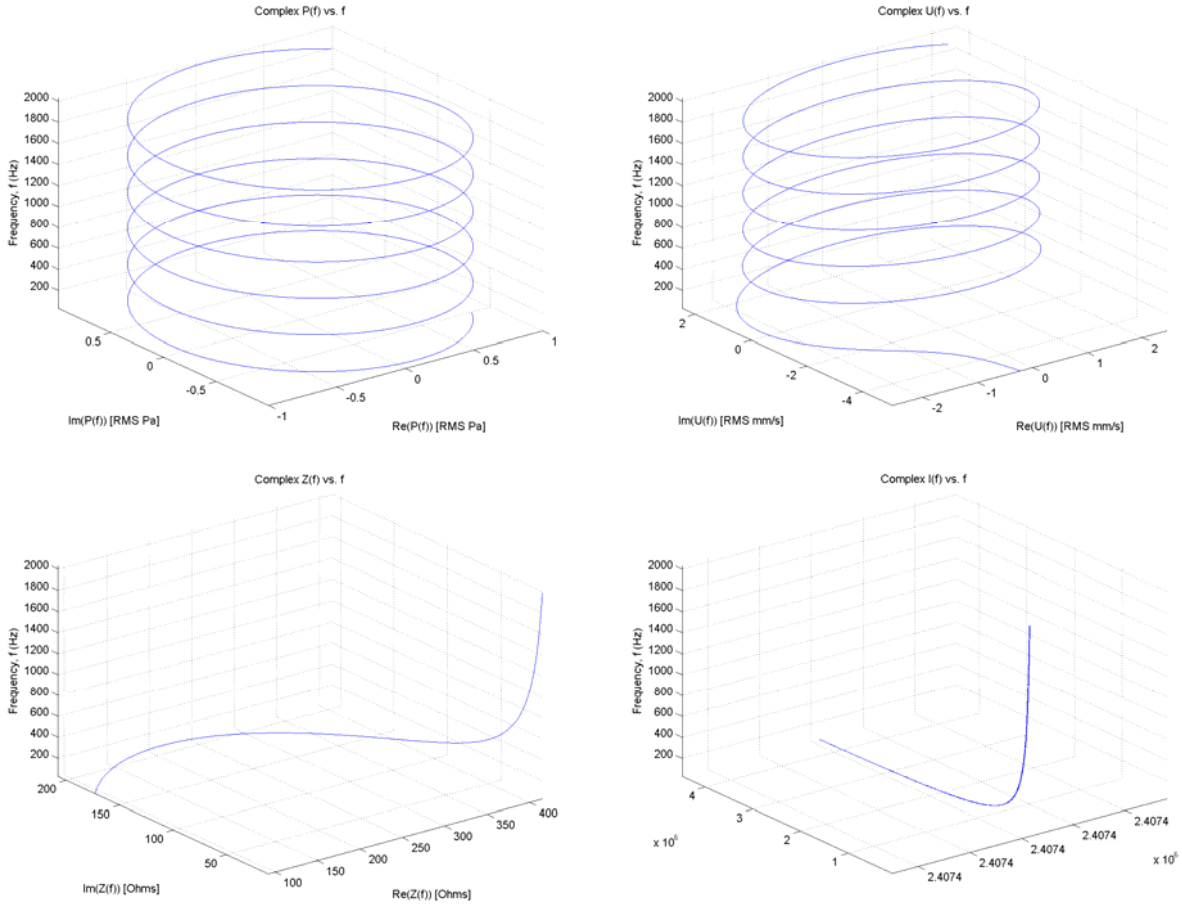

Figure 14. 3-D plots of the complex plane *vs*. frequency for complex over-pressure, particle velocity, radial acoustic specific impedance and acoustic intensity for a point acoustic monopole.

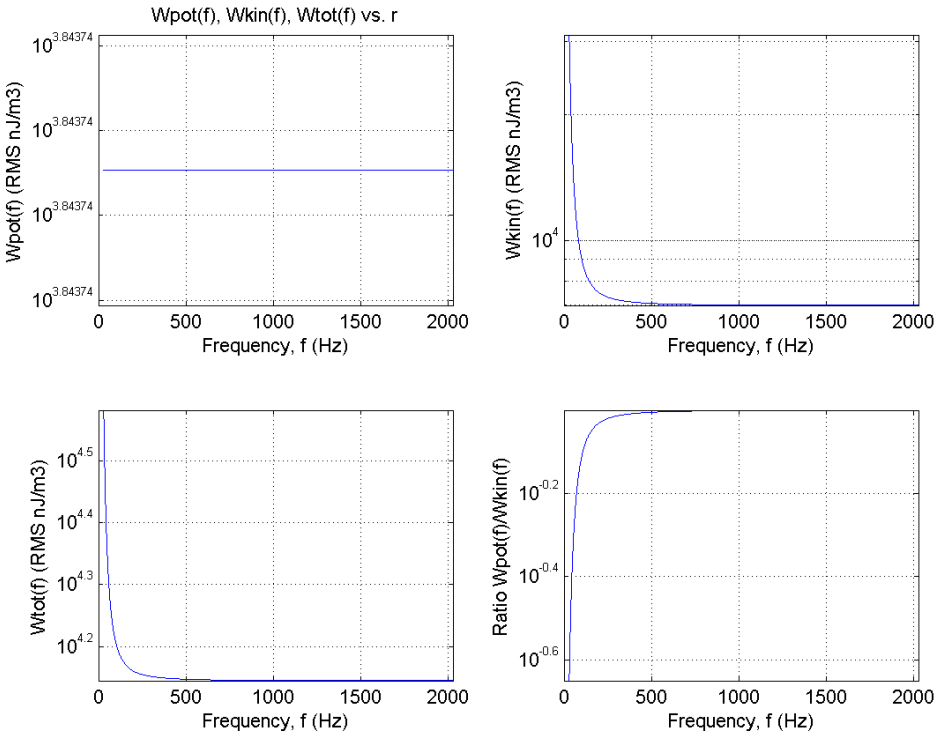

Figure 15. Potential, kinetic and total acoustic energy density *vs*. frequency for a point acoustic monopole.

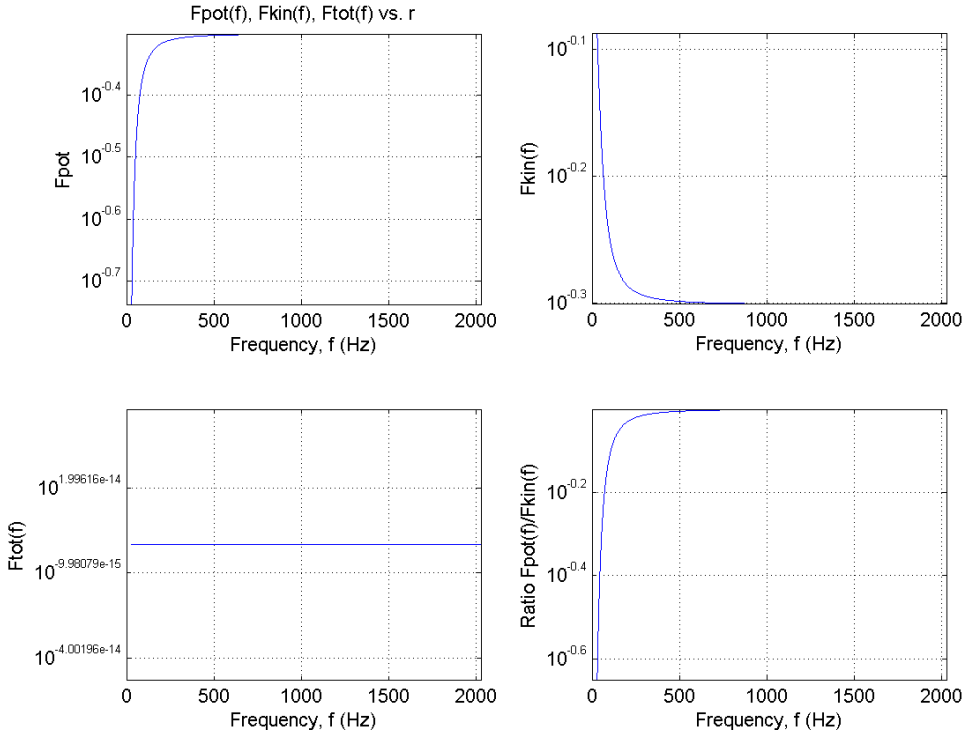

Figure 16. Potential, kinetic and total acoustic energy density fractions *vs*. frequency for a point acoustic monopole.

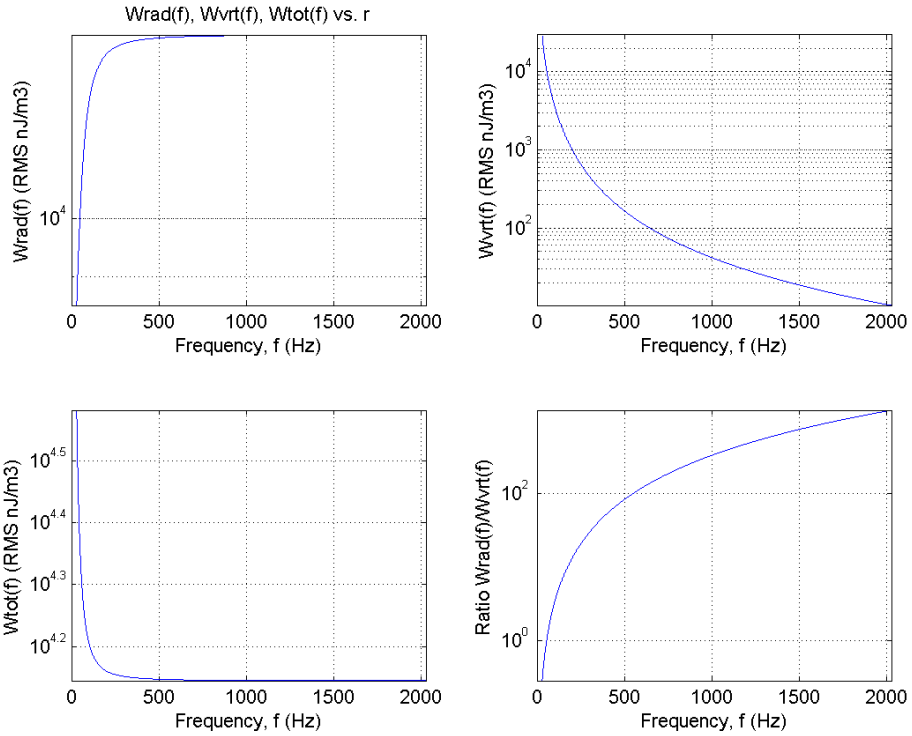

Figure 17. Propagating, non-propagating and total acoustic energy density *vs*. frequency for a point acoustic monopole.

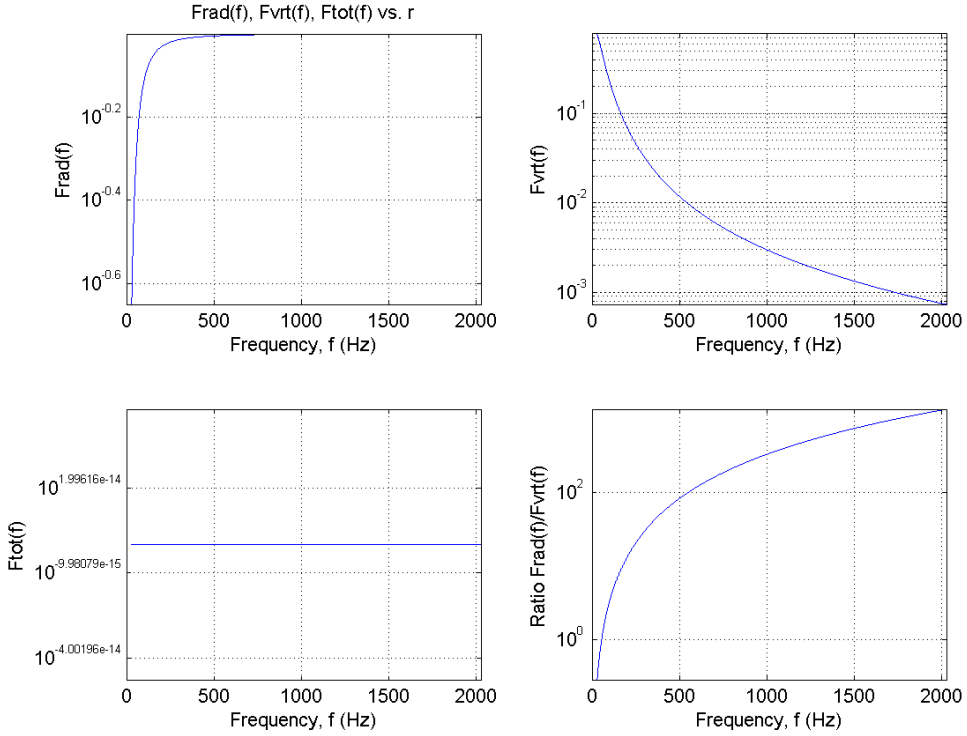

Figure 18. Propagating, non-propagating and total acoustic energy density fractions *vs*. frequency for a point acoustic monopole.