The Baffled Circular Piston

In this example, we show plots of the complex over-pressure, axial particle velocity, axial specific acoustic impedance, axial sound intensity, energy density, *etc*. *vs*. axial observer / listener *z*-position and also *vs*. frequency for fixed axial *z*-position associated with the ***on-axis*** $(r = z, \theta = 0)$ sound field from a baffled circular piston of radius *a*.

The time-domain and frequency-domain expressions for the complex over-pressure and axial particle velocity associated with a baffled circular piston radiating sound at a single frequency are:

$$\tilde{p}(r=z,t) = p_o\left[e^{-ikz} - e^{-ikz\sqrt{1+(a/z)^2}}\right]\cdot e^{i\omega t} = \tilde{p}(r=z,\omega)\cdot e^{i\omega t} \;\; i.e. \;\; \tilde{p}(r=z,\omega) = p_o\left[e^{-ikz} - e^{-ikz\sqrt{1+(a/z)^2}}\right]$$

$$\tilde{u}_z(r=z,t) = u_o\left[e^{-ikz} - \frac{1}{\sqrt{1+(a/z)^2}}e^{-ikz\sqrt{1+(a/z)^2}}\right]\cdot e^{i\omega t} \quad i.e. \;\; \tilde{u}_z(r=z,\omega) = u_o\left[e^{-ikz} - \frac{1}{\sqrt{1+(a/z)^2}}e^{-ikz\sqrt{1+(a/z)^2}}\right]$$

The frequency-domain expressions for the complex axial specific acoustic impedance and axial sound intensity associated with a baffled circular piston radiating sound at a single frequency are:

$$\tilde{z}_{a_z}(r=z,\omega) = \frac{\tilde{p}(r=z,t)}{\tilde{u}_z(r=z,t)} = \frac{\tilde{p}(r=z,\omega)\cdot \cancel{e^{i\omega t}}}{\tilde{u}_z(r=z,\omega)\cdot \cancel{e^{i\omega t}}} = \frac{\tilde{p}(r=z,\omega)}{\tilde{u}_z(r=z,\omega)} = z_o\frac{\left[1-e^{-ikz\left(\sqrt{1+(a/z)^2}-1\right)}\right]}{\left[1-\frac{1}{\sqrt{1+(a/z)^2}}e^{-ikz\left(\sqrt{1+(a/z)^2}-1\right)}\right]}$$

$$\tilde{I}_a^{\parallel}(r=z,\omega) = \tfrac{1}{2}\,\tilde{p}(r=z,\omega)\tilde{u}_z^*(r=z,\omega) = \tfrac{1}{2}\,p_o u_o\left[1-e^{-ikz\left(\sqrt{1+(a/z)^2}-1\right)}\right]\left[1-\frac{1}{\sqrt{1+(a/z)^2}}e^{+ikz\left(\sqrt{1+(a/z)^2}-1\right)}\right]$$

$$= \tfrac{1}{2}I_o\left[1-e^{-ikz\left(\sqrt{1+(a/z)^2}-1\right)}\right]\left[1-\frac{1}{\sqrt{1+(a/z)^2}}e^{+ikz\left(\sqrt{1+(a/z)^2}-1\right)}\right] = I_o^{rms}\left[1-e^{-ikz\left(\sqrt{1+(a/z)^2}-1\right)}\right]\left[1-\frac{1}{\sqrt{1+(a/z)^2}}e^{+ikz\left(\sqrt{1+(a/z)^2}-1\right)}\right]$$

We coded up the above formulas using MATLAB to make plots of the complex over-pressure, axial particle velocity, axial specific acoustic impedance, axial sound intensity, energy density, etc. *vs*. axial observer/listener *z*-position and also *vs*. frequency for fixed axial *z*-position associated with the on-axis sound radiated from a baffled circular piston of radius *a*.

The first set of plots (Figures 1-9) shows these complex quantities as a function of axial *z*-position for the following parameter values: $a = 60\,cm$, $f = 3\,KHz$, $p_o = 1.0\,RMS\;Pa$.

The second set of plots (Figures 10-18) shows these same complex quantities as a function of frequency for the following parameter values: $a = 60\,cm$, $z = 1.0\,m$, $p_o = 1.0\,RMS\;Pa$.

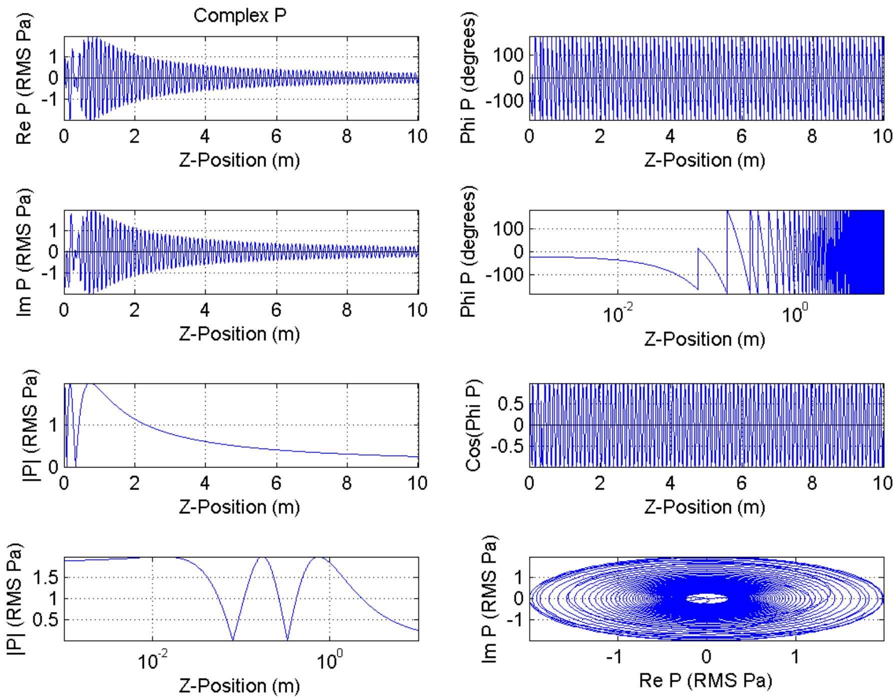

Figure 1. Complex over-pressure *vs*. axial distance for a baffled circular piston.

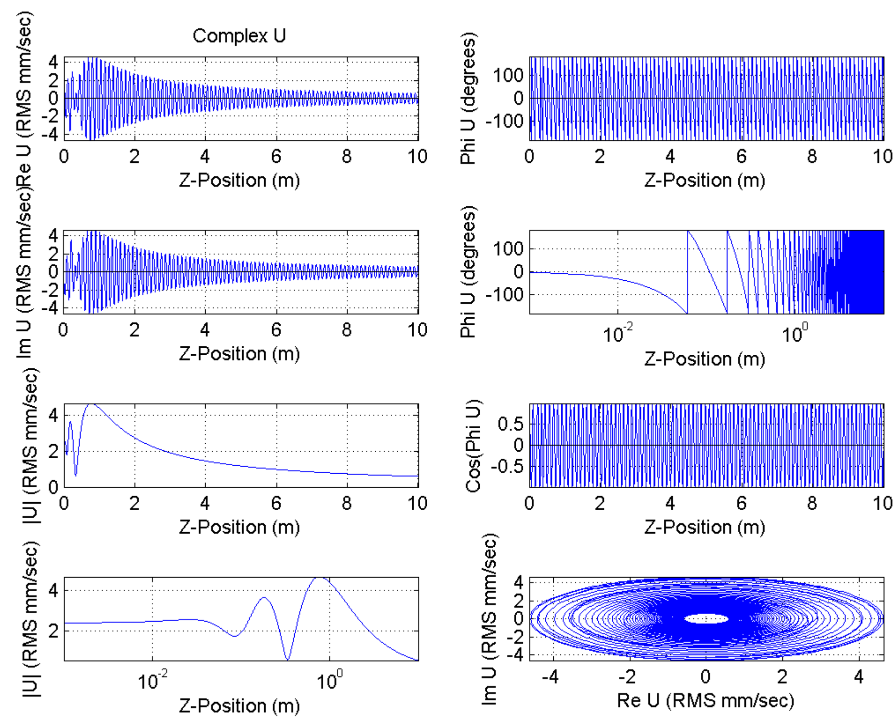

Figure 2. Complex axial particle velocity *vs*. axial distance for a baffled circular piston.

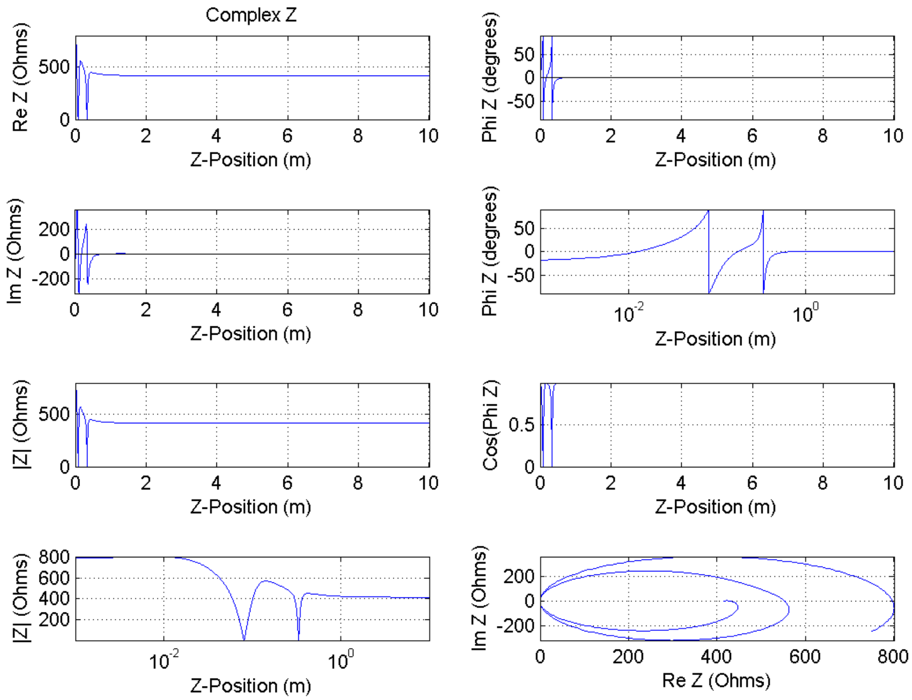

Figure 3. Complex axial specific acoustic impedance *vs*. axial distance for a baffled circular piston.

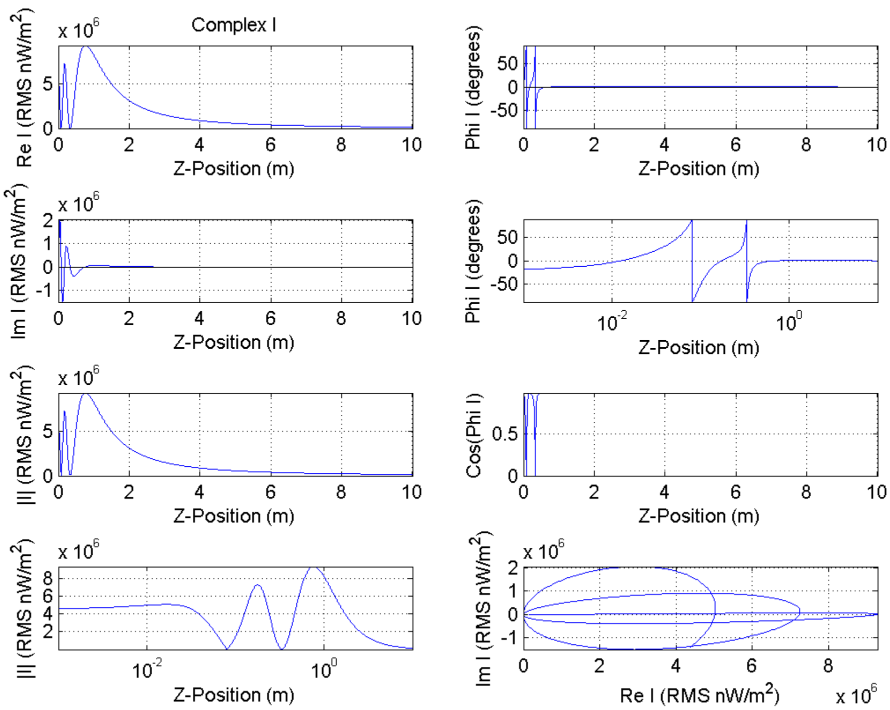

Figure 4. Complex axial acoustic intensity *vs*. axial distance for a baffled circular piston.

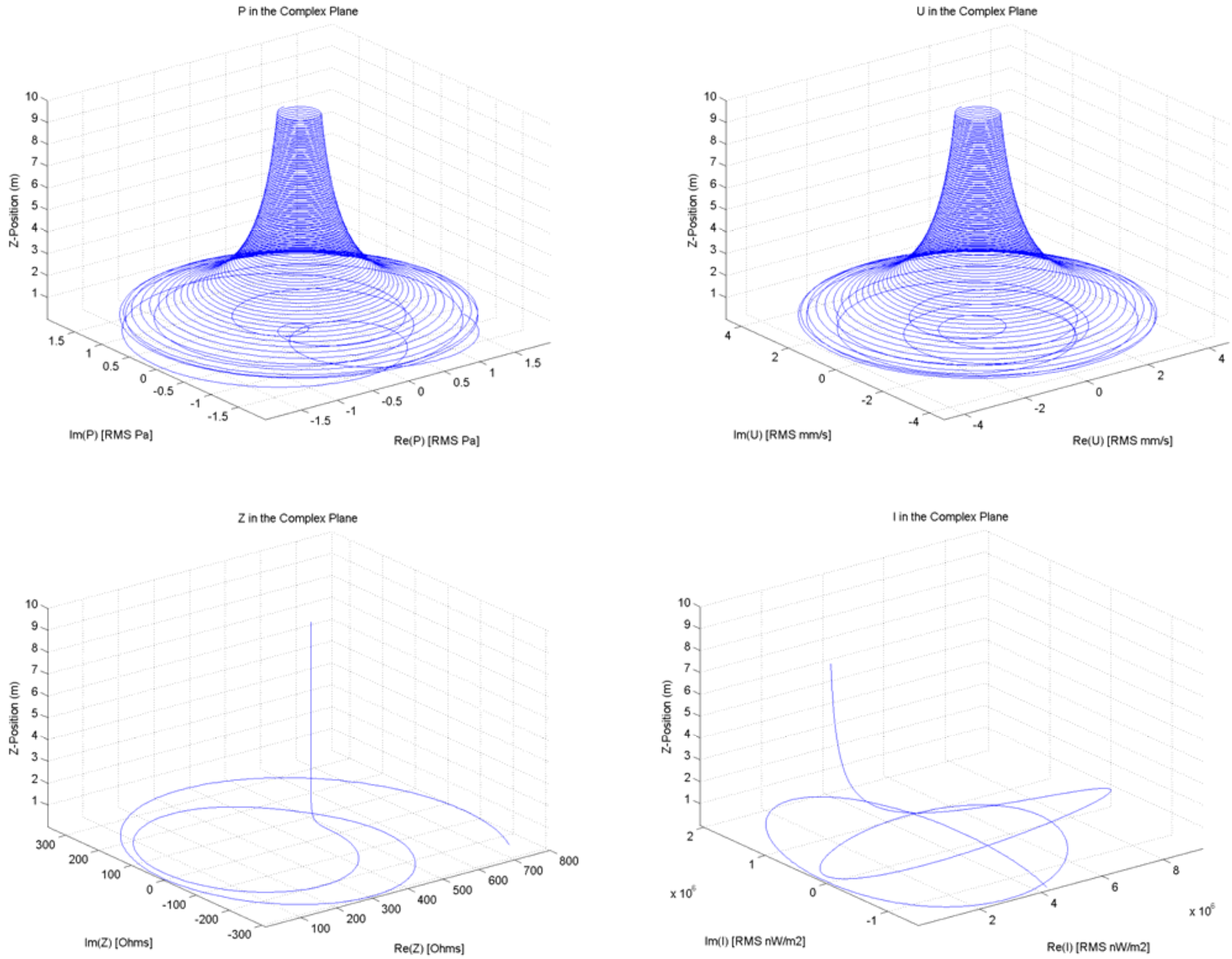

Figure 5. 3-D plots of the complex plane *vs*. axial distance for complex over-pressure, particle velocity, axial acoustic specific impedance and acoustic intensity for a baffled circular piston.

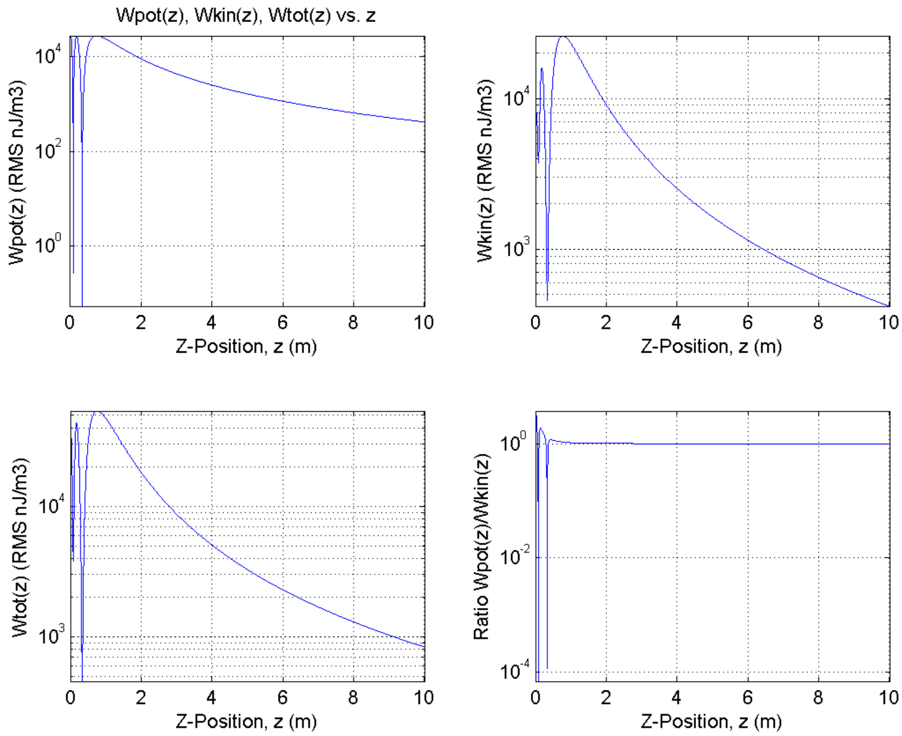

Figure 6. Potential, kinetic and total acoustic energy density *vs*. radial distance for a baffled circular piston.

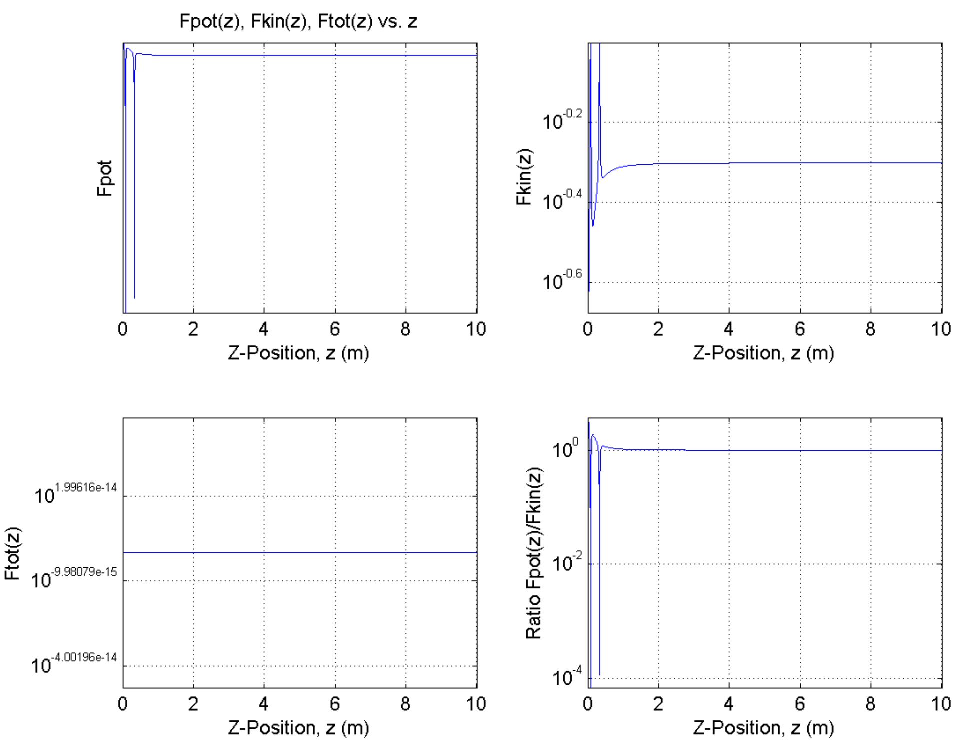

Figure 7. Potential, kinetic and total acoustic energy density fractions *vs*. radial distance for a baffled circular piston.

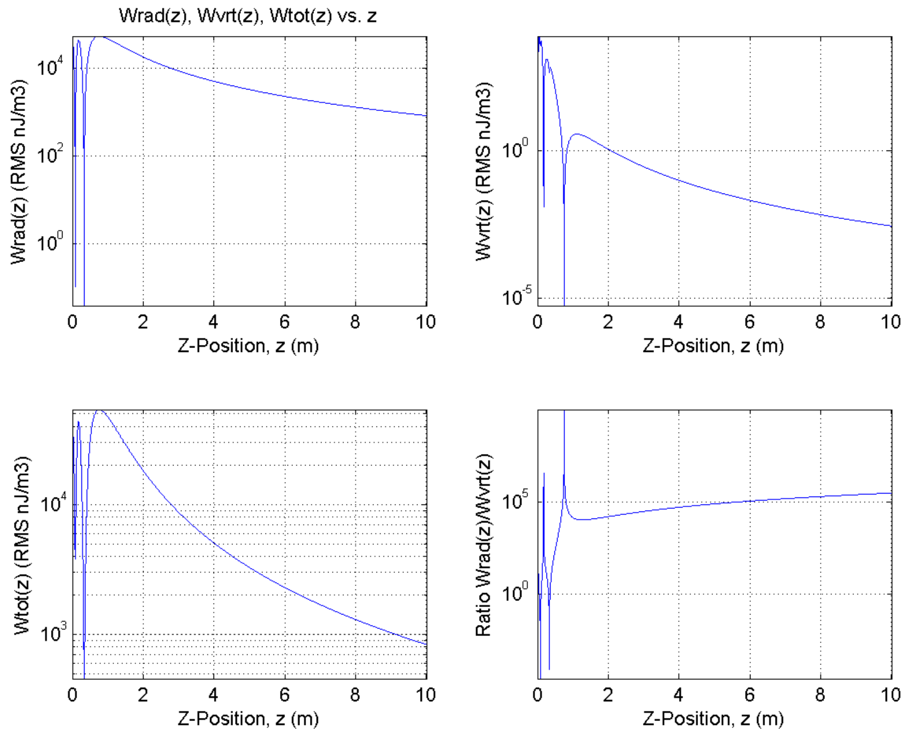

Figure 8. Propagating, non-propagating and total acoustic energy density *vs*. radial distance for a baffled circular piston.

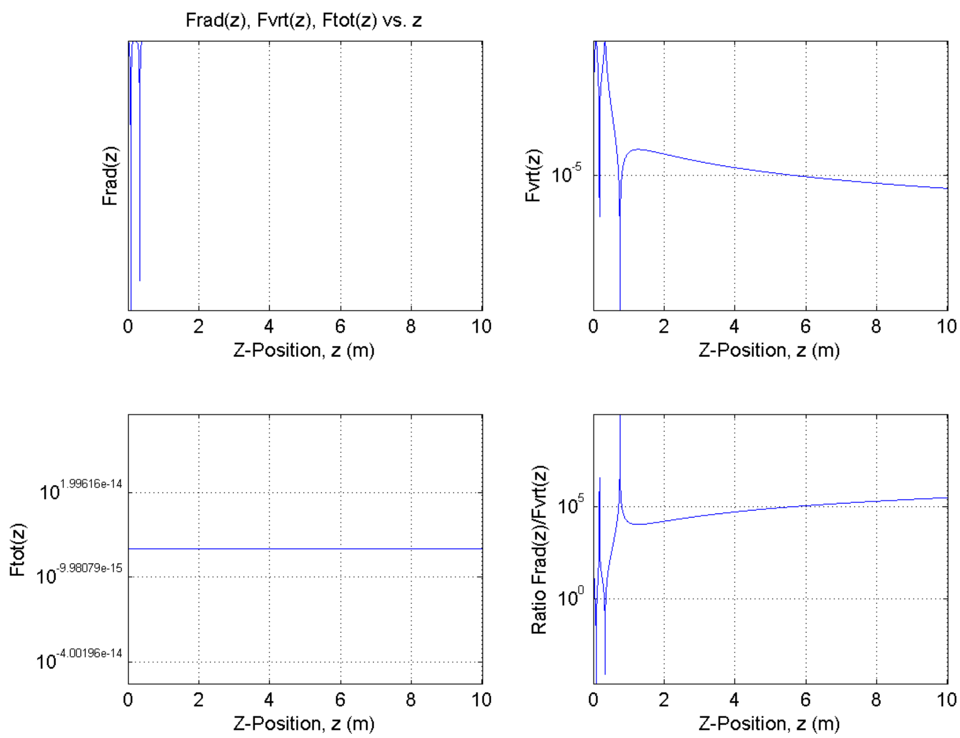

Figure 9. Propagating, non-propagating and total acoustic energy density fractions *vs*. radial distance for a baffled circular piston.

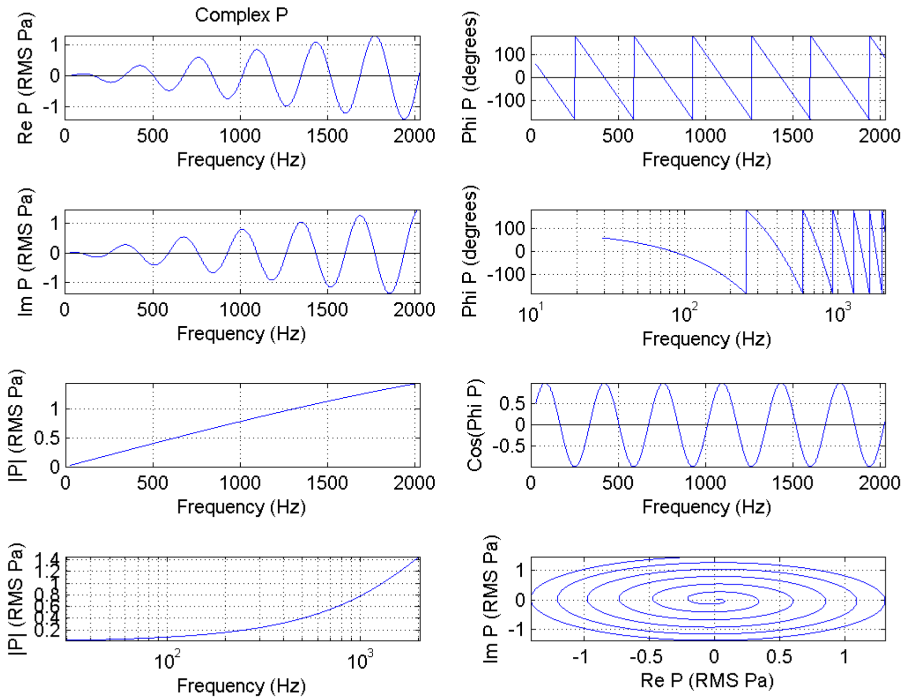

Figure 10. Complex over-pressure *vs*. frequency for a baffled circular piston.

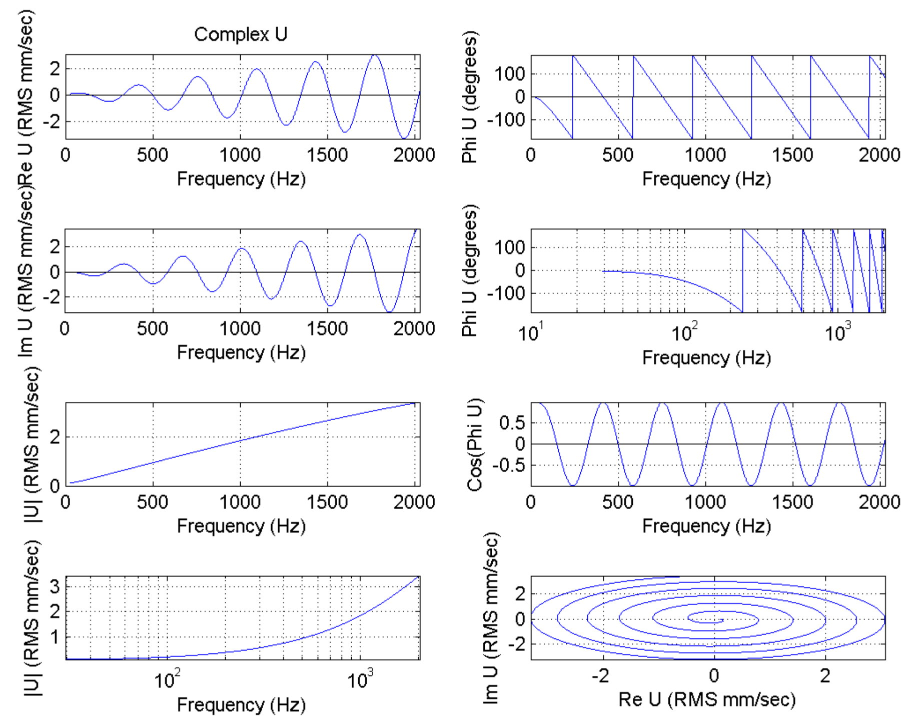

Figure 11. Complex axial particle velocity *vs*. frequency for a baffled circular piston.

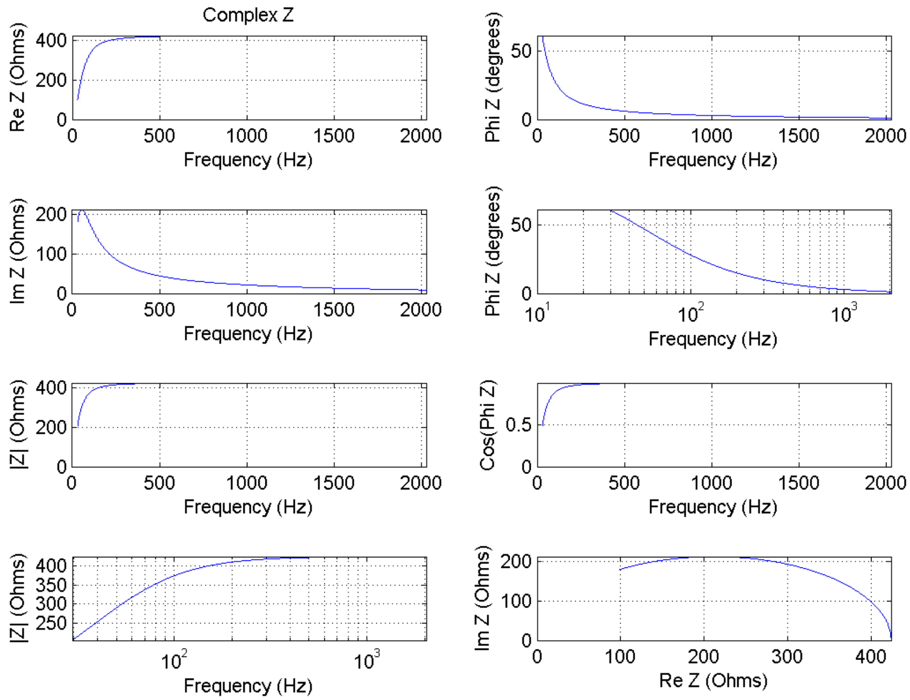

Figure 12. Complex axial specific acoustic impedance *vs*. frequency for a baffled circular piston.

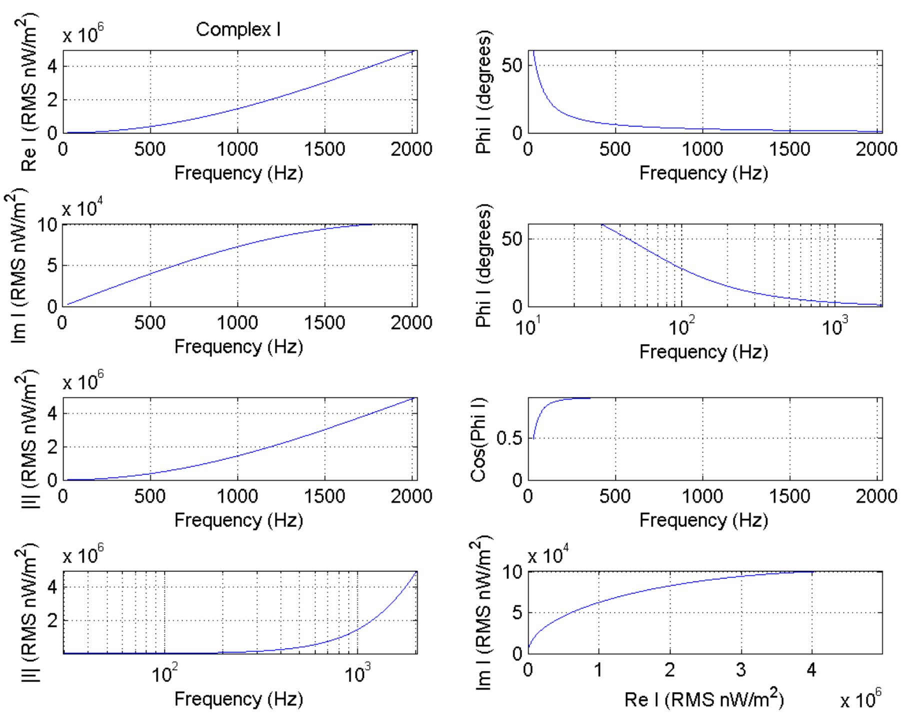

Figure 13. Complex axial acoustic intensity *vs*. frequency for a baffled circular piston.

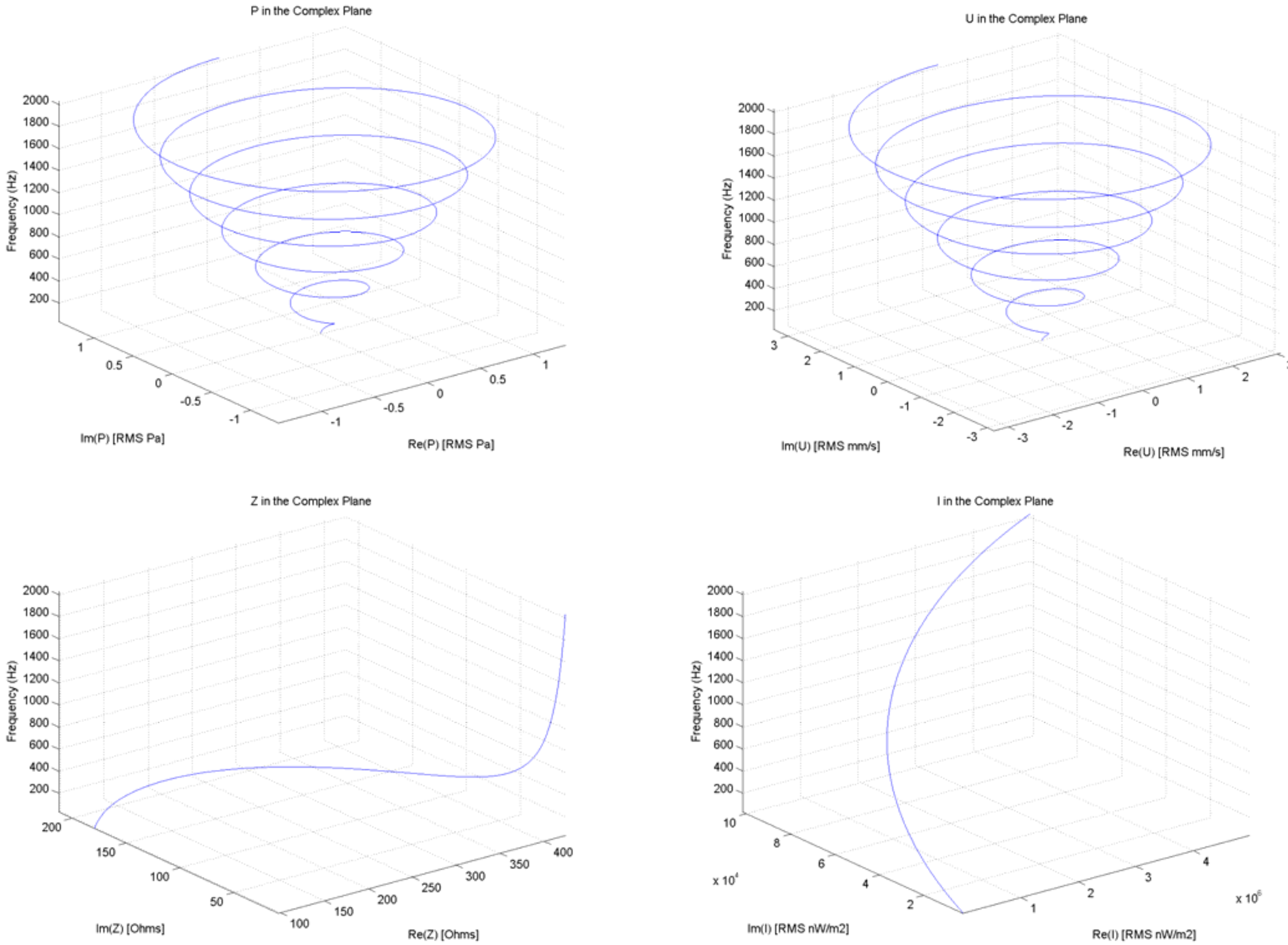

Figure 14. 3-D plots of the complex plane *vs*. frequency for complex over-pressure, particle velocity, axial acoustic specific impedance and acoustic intensity for a baffled circular piston.

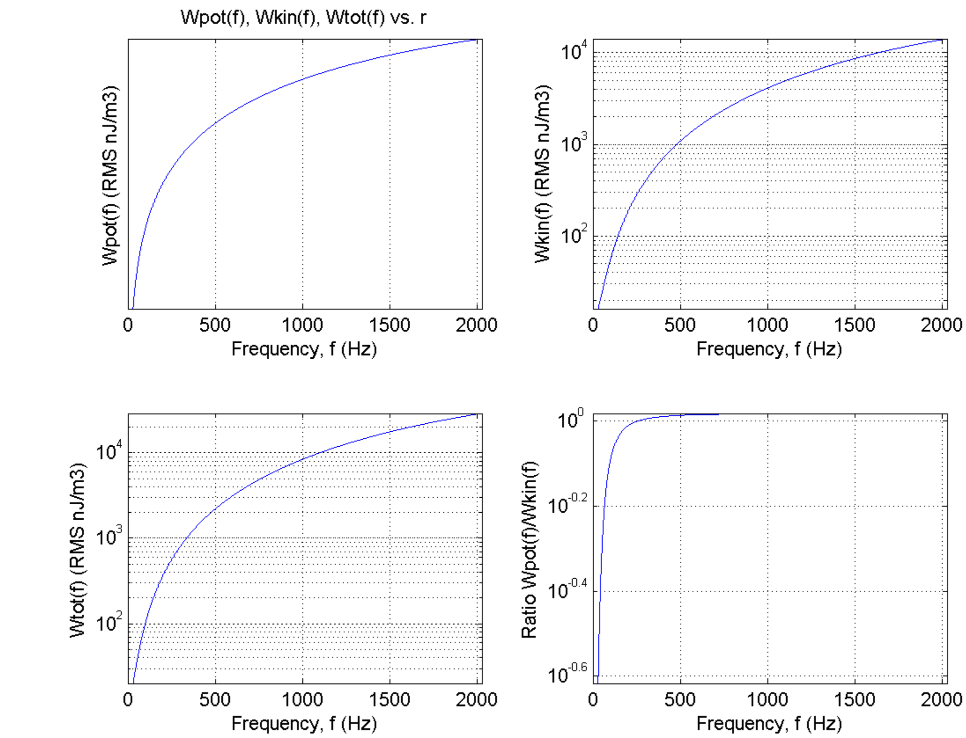

Figure 15. Potential, kinetic and total acoustic energy density *vs*. frequency for a baffled circular piston.

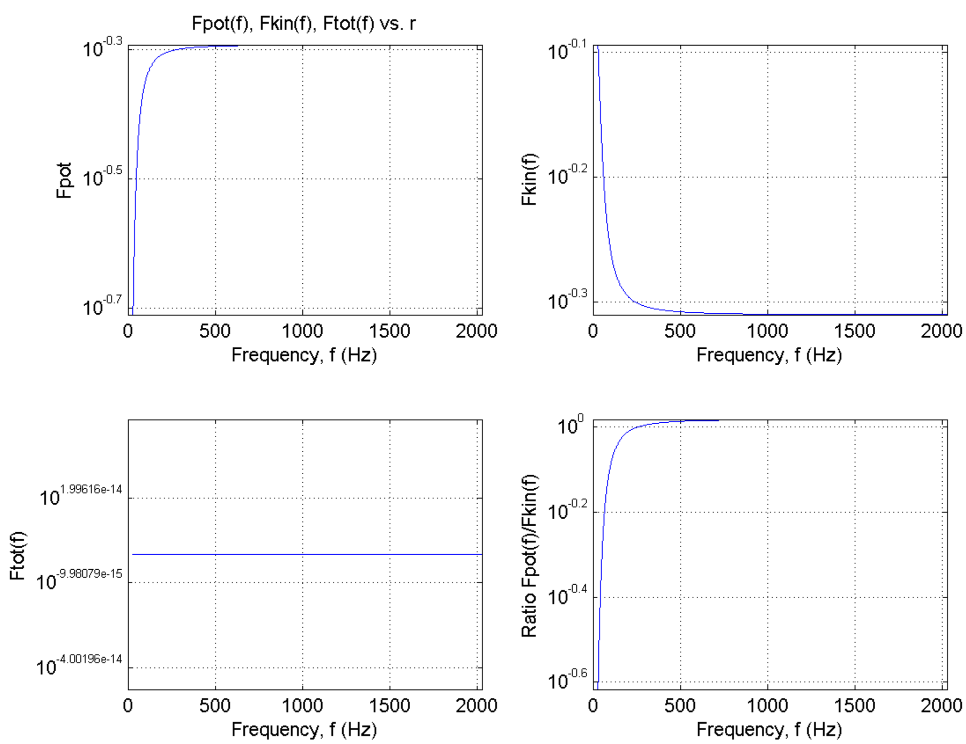

Figure 16. Potential, kinetic and total acoustic energy density fractions *vs*. frequency for a baffled circular piston.

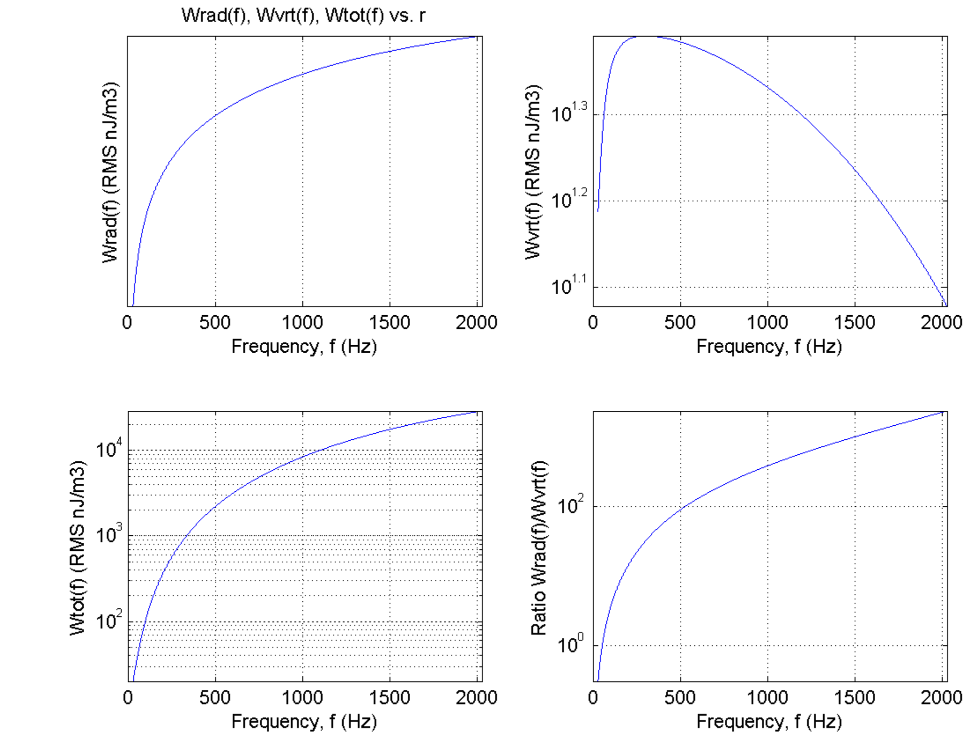

Figure 17. Propagating, non-propagating and total acoustic energy density *vs*. frequency for a baffled circular piston.

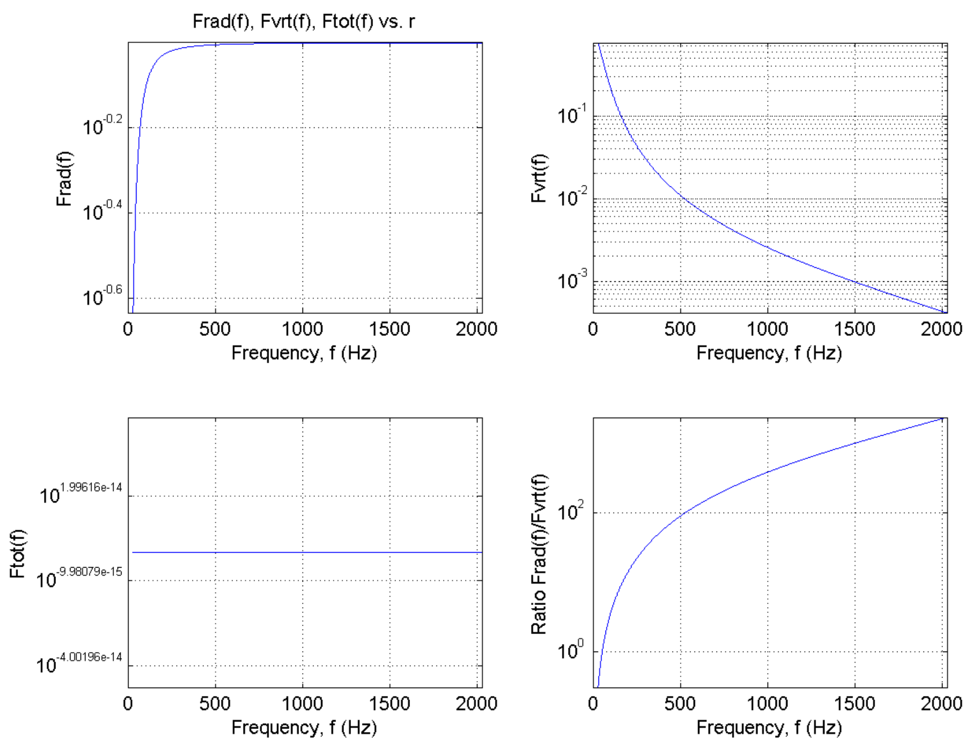

Figure 18. Propagating, non-propagating and total acoustic energy density fractions *vs*. frequency for a baffled circular piston.