Fraunhofer Diffraction Through a Circular Aperture

In this example, we show plots of the sound intensity *vs*. angle and observer/listener position y_{screen} on a screen for the simplest theory of Fraunhofer diffraction through a circular aperture of radius R. This is an approximation to e.g. a loudspeaker mounted on an infinite baffle. The observer/listener is located far from the aperture, a perpendicular distance $L\ (m)$ away, such that the conditions $R \ll L$ **.and.** $\lambda \ll L$ both hold simultaneously, where $\lambda\ (m)$ is the wavelength of the sound – this is the so-called "far-field" limit.

The expression for Fraunhofer diffraction through a circular aperture is given by (see P406POM Lecture Notes P406POM_Lect3_Part2):

$$I_{tot}(\theta) = I_o \left\{ \frac{2J_1(\rho(\theta))}{\rho(\theta)} \right\}^2 \text{ and } SIL(\theta) \equiv 10\log_{10}\left(I_{tot}(\theta)/I_{ref}\right)\ (dB)$$

where $I_o\ \left(Watts/m^2\right)$ is the maximum sound intensity associated with the circular aperture, the phase $\rho(\theta) = kR\sin\theta = (2\pi/\lambda)R\sin\theta\,(radians)$ and $J_1(\rho(\theta))$ is the ordinary Bessel function of the first kind, of order $\nu = 1$. $I_{ref} \equiv 10^{-12}\ \left(Watts/m^2\right)$ is the reference sound intensity for the sound intensity level (SIL).

Diffraction <u>minima</u> (intensity zeroes) occur at the non-trivial zeros of the Bessel function $J_1(\rho)$, which occur at ρ = 3.8317, 7.0156, 10.1735, 13.3237, 16.4706, 19.6159, …

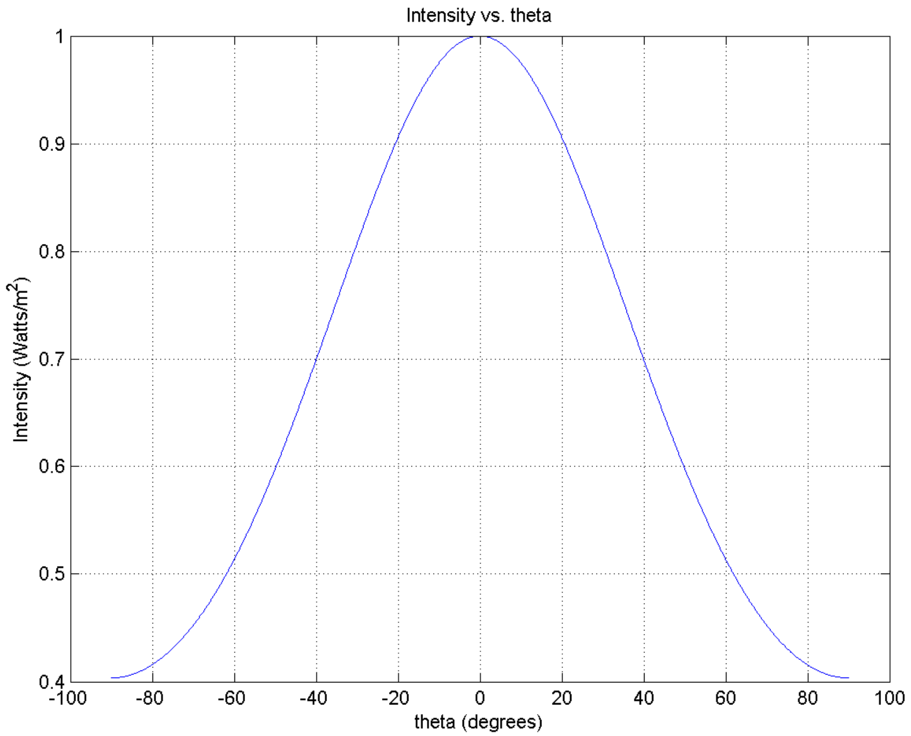
The corresponding location of the observer's/listener's position y_{screen} on a screen located a perpendicular distance L away from the N sound sources is: $r_{screen}(\theta) = L\tan\theta$, or conversely: $\theta = \tan^{-1}(r_{screen}/L)$, where $r_{screen} = \sqrt{x_{screen}^2 + y_{screen}^2}$.

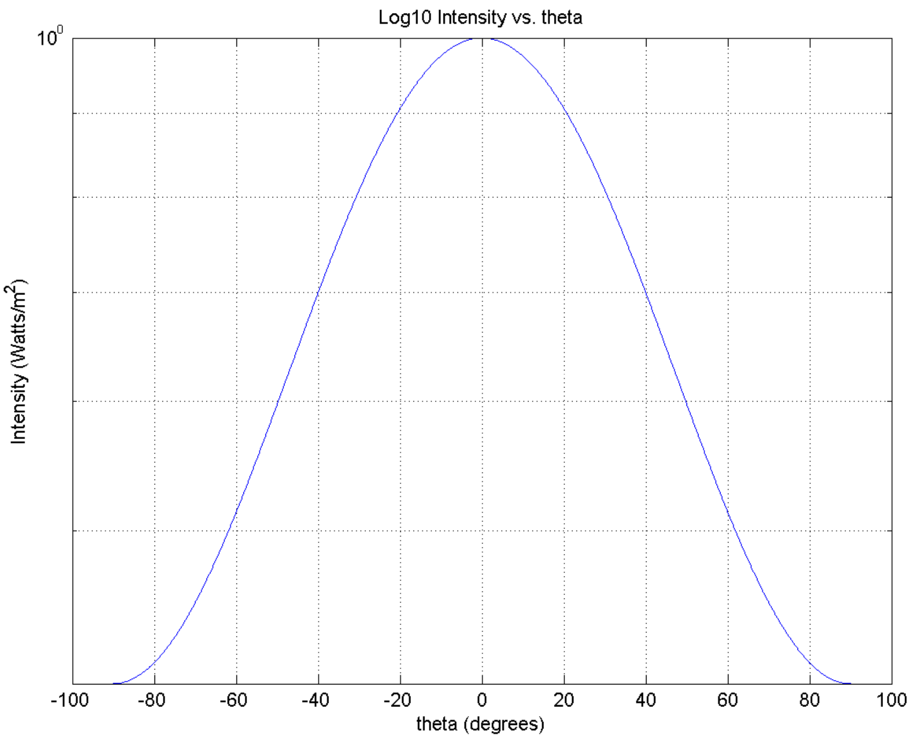
We coded up the above formulas using MATLAB to make plots of I_{tot} *vs.* θ and I_{tot} *vs.* y_{screen} *e.g.* for R = 0.1 and 1.0 m, with the following parameter values: $I_o = 1\ Watt/m^2$, observer/listener distance (at $\theta = 0$) of $L = 10\ m$, the speed of propagation in free air/great-wide open: $v_{air} = 343\ m/s$ and frequency of $f = 1000\ Hz$, thus $\lambda = v_{air}/f = 0.345\ m$.

In the following figures, note that the angular width of the central maxima decreases as the radius of the circular aperture increases, since the phase $\rho = kR\sin\theta = (2\pi/\lambda)R\sin\theta\,(radians)$ is linearly proportional to R. Note also that the angular width of the central maxima decreases linearly with increasing frequency f, since the phase increases linearly with frequency:

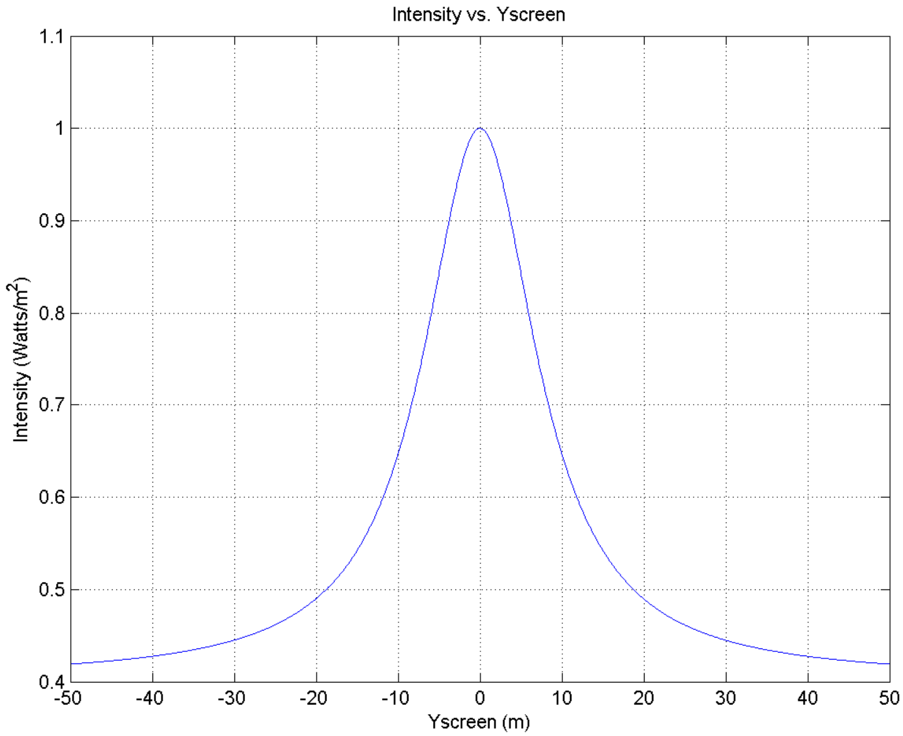
$$\rho(\theta) = kR\sin\theta = (2\pi/\lambda)R\sin\theta = (2\pi f/v_{air})R\sin\theta\,(radians).$$

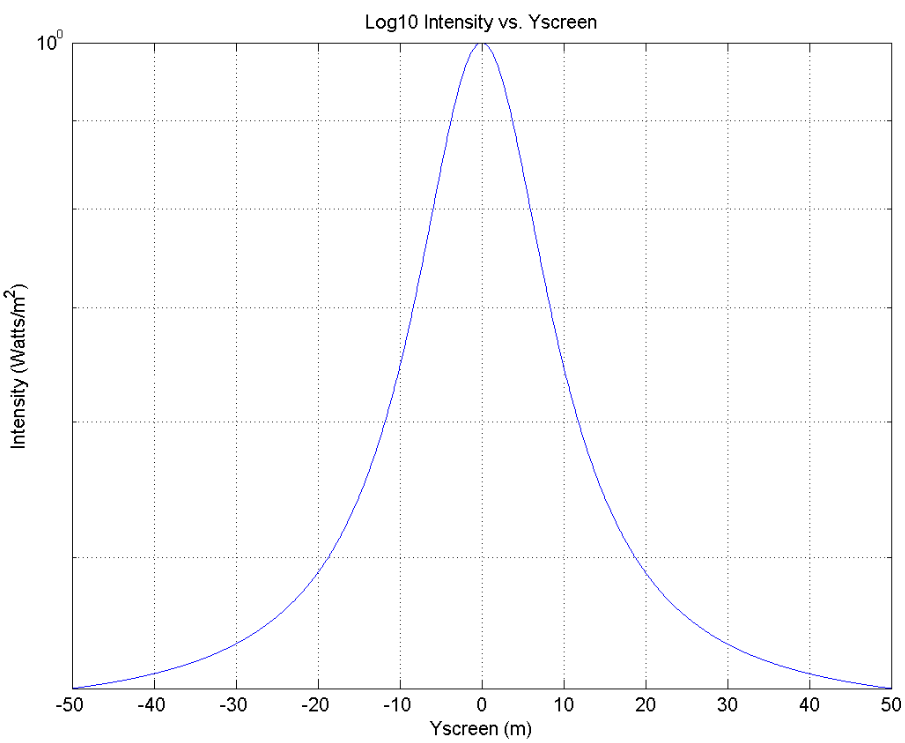
Linear and Semilog Plots of I_{tot} *vs.* θ for $R = 0.1\ m$:



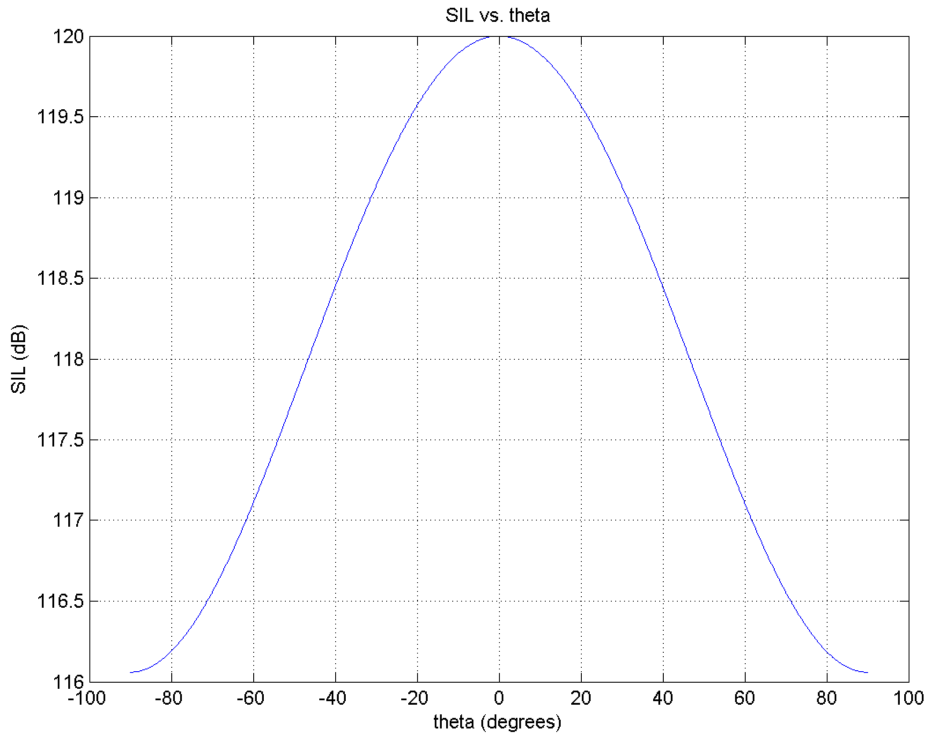


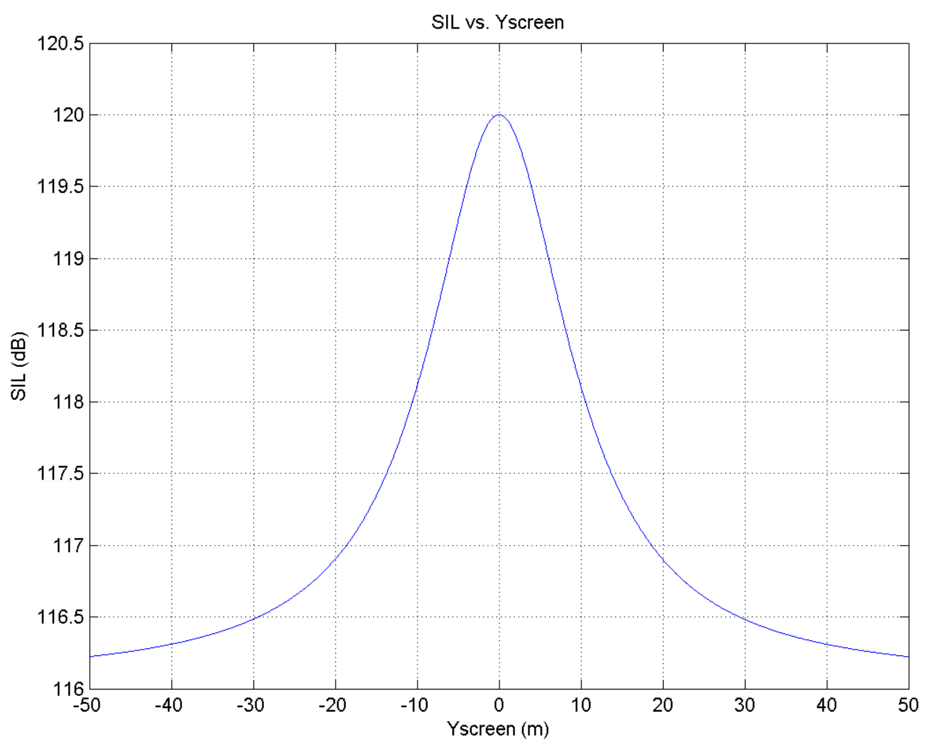
Linear and Semilog Plots of I_{tot} *vs.* y_{screen} for $R = 0.1\ m$:



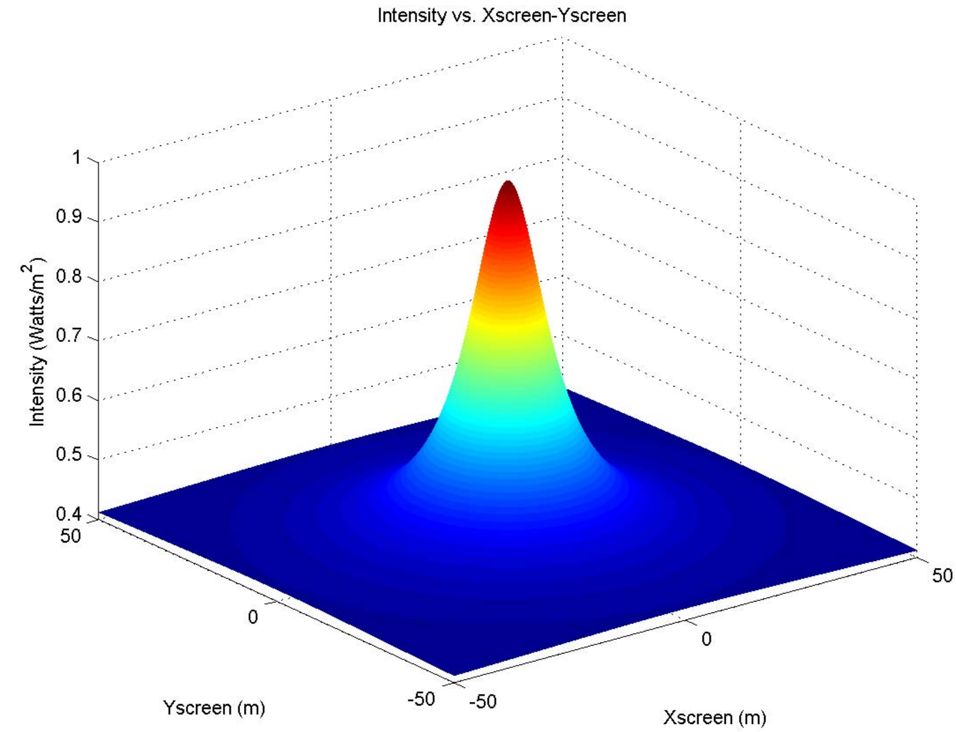


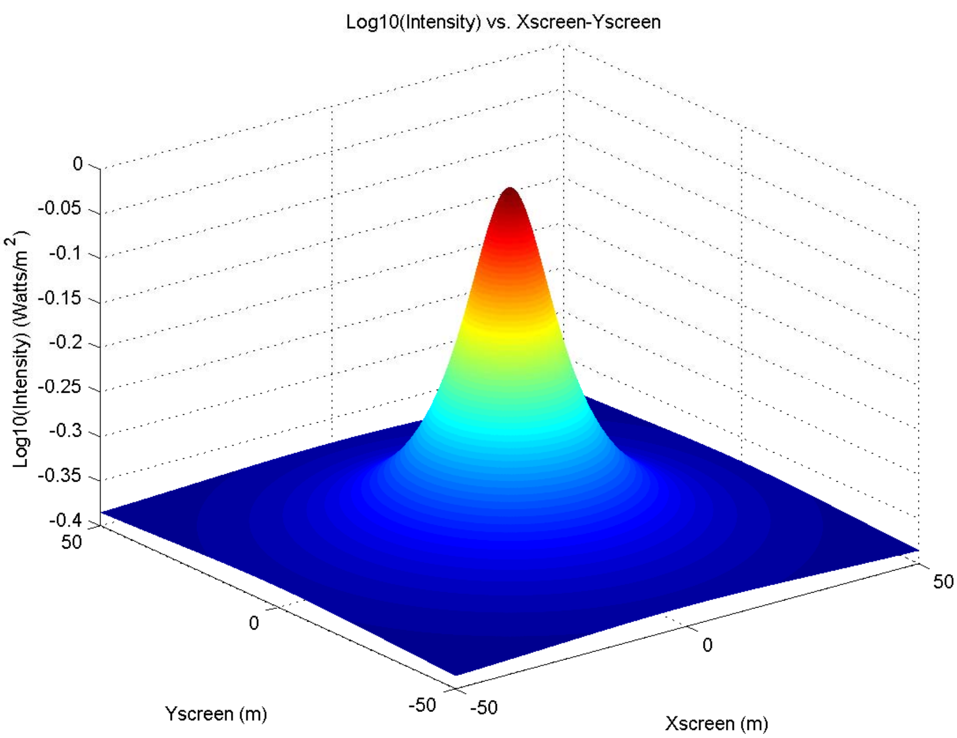
Plots of SIL $vs.$ θ and SIL $vs.$ y_{screen} for $R = 0.1$ m:



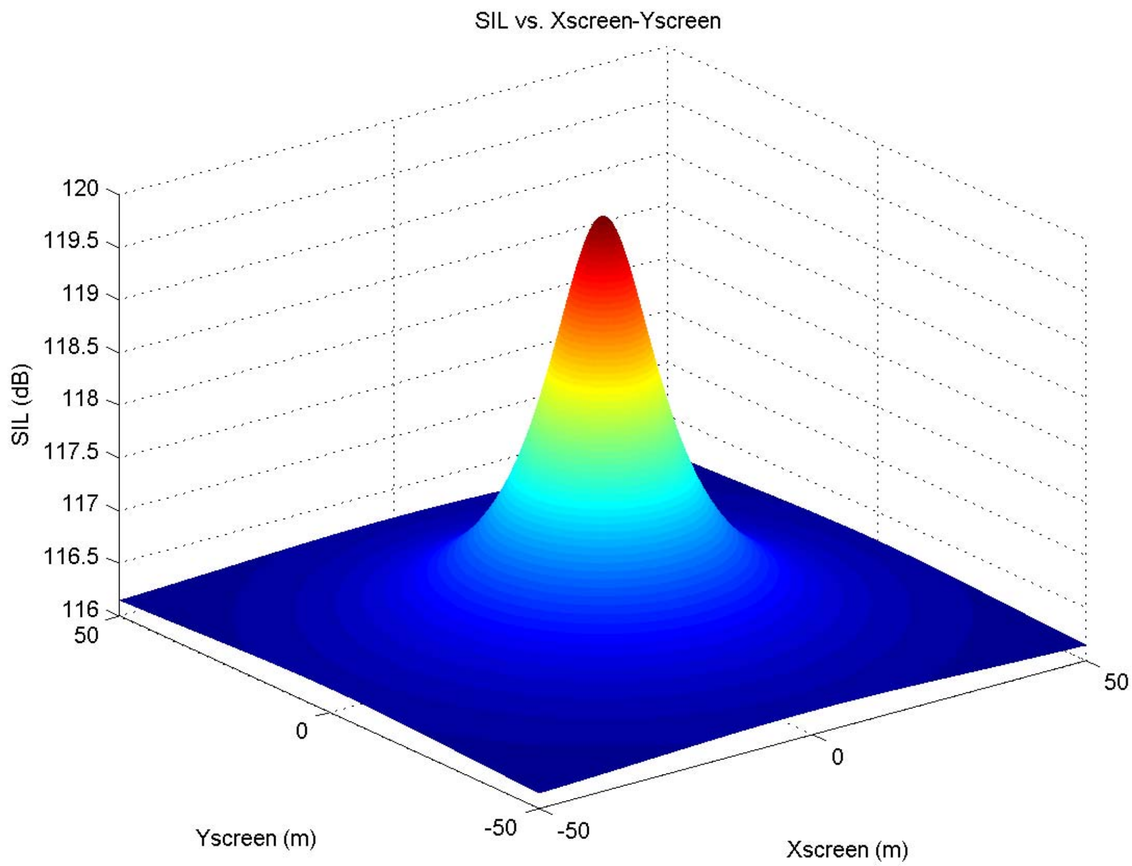


Linear and Semilog Plots of $I_{tot}\left(x_{scr}, y_{scr}\right) vs. \left(x_{scr}, y_{scr}\right)$ for $R = 0.1\ m$:

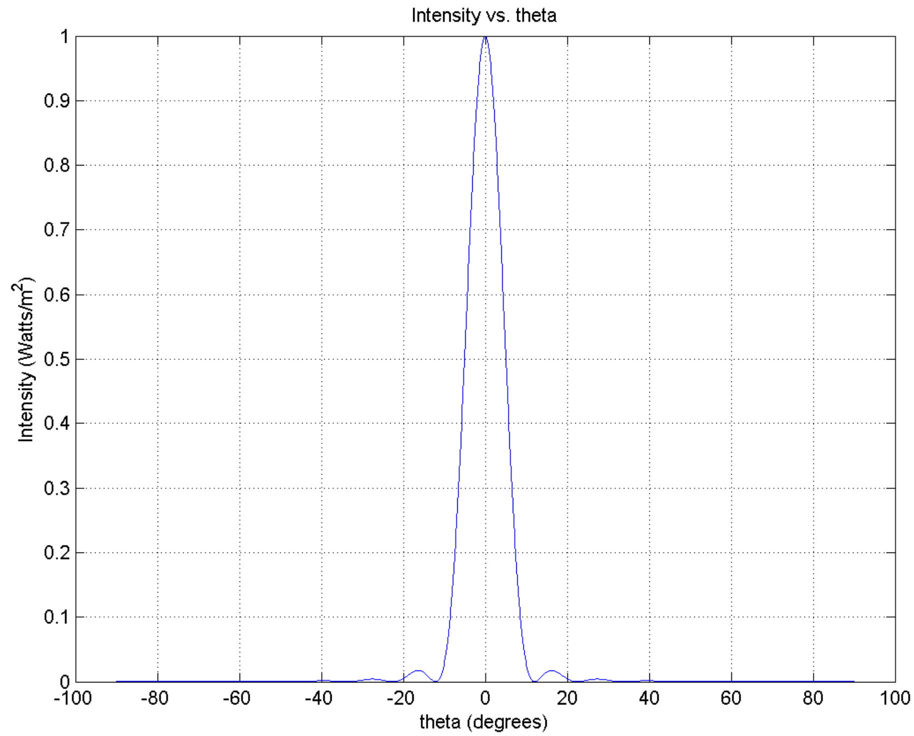


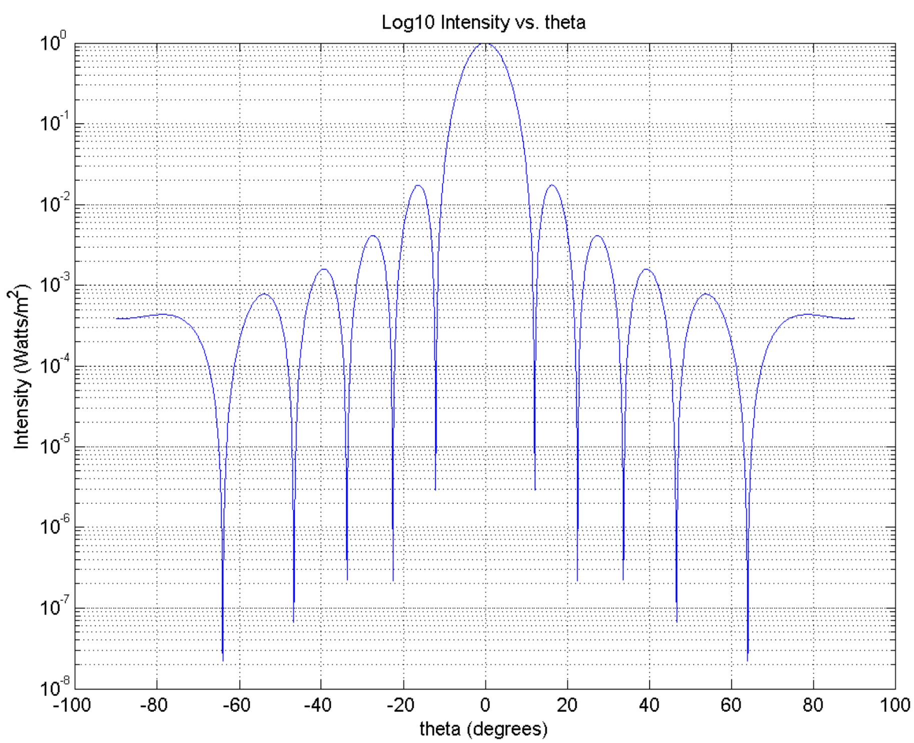


Plots of $SIL(x_{scr}, y_{scr})$ *vs.* (x_{scr}, y_{scr}) for R = 0.1 *m*:

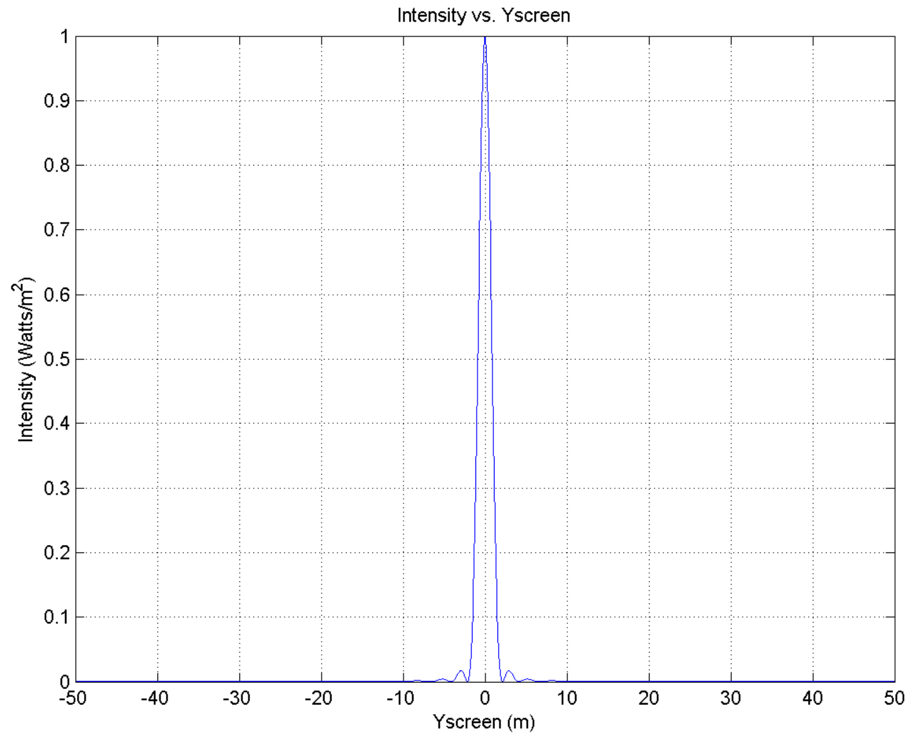


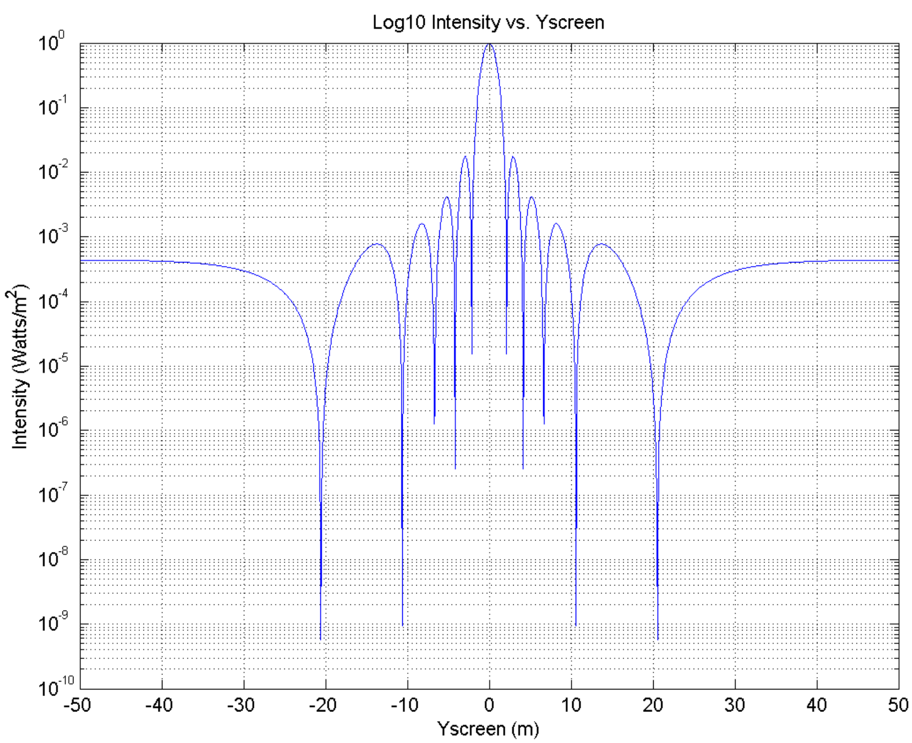
Linear and Semilog Plots of I_{tot} *vs.* θ for $R = 1.0\ m$:



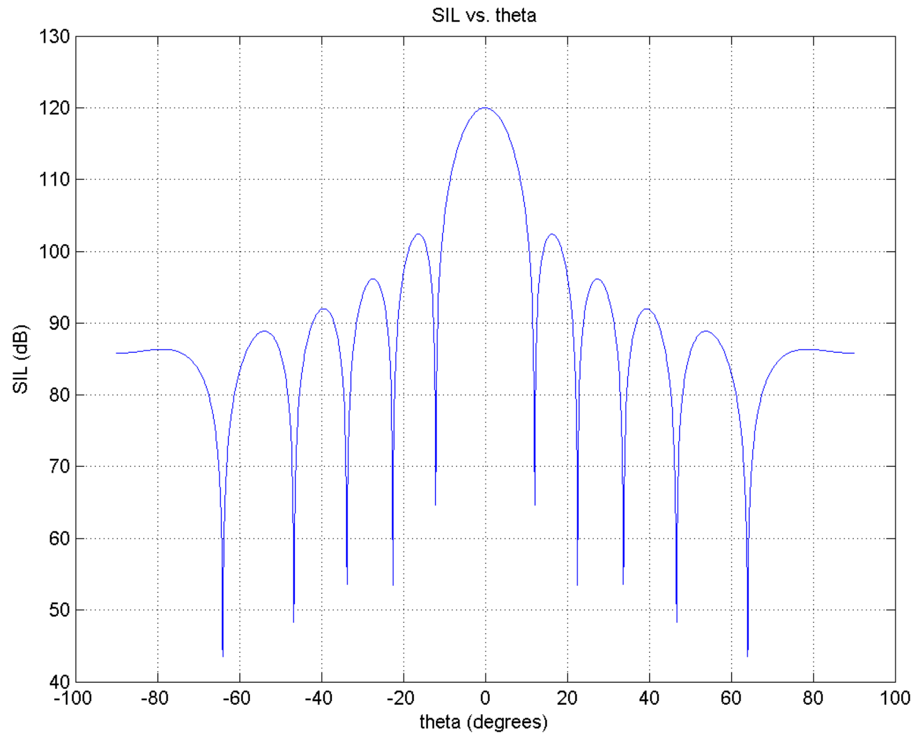


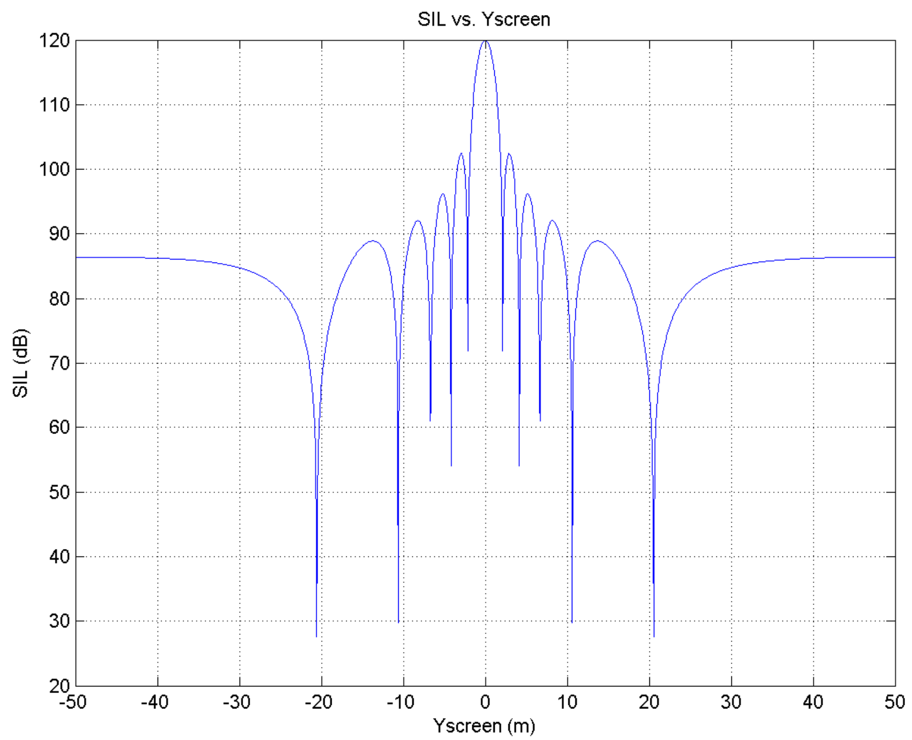
Linear and Semilog Plots of I_{tot} *vs.* y_{screen} for $R = 1.0\ m$:



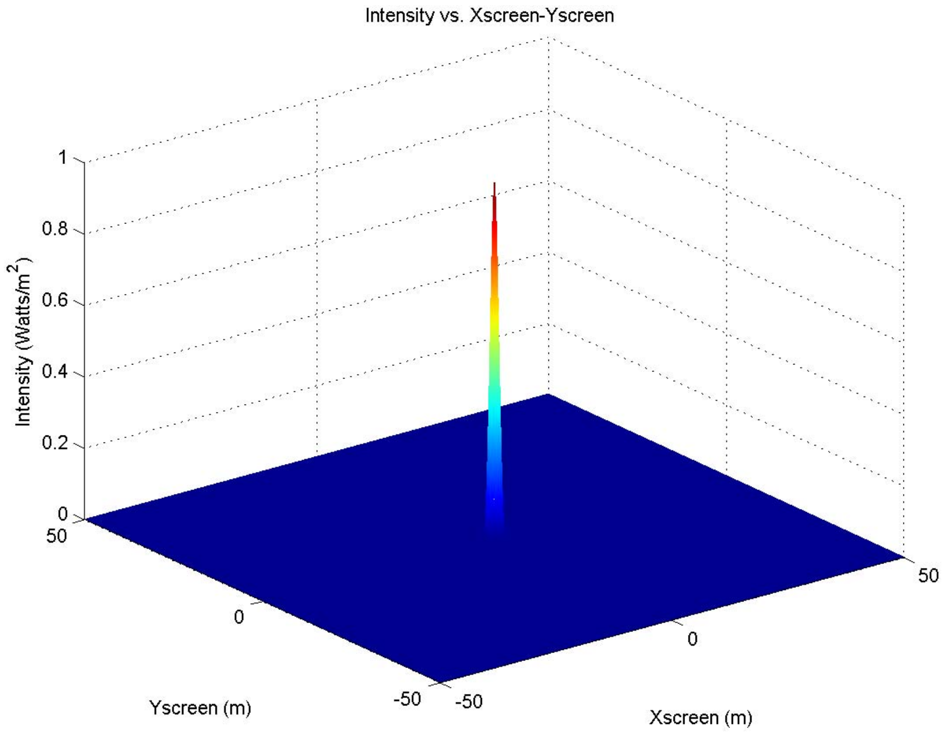


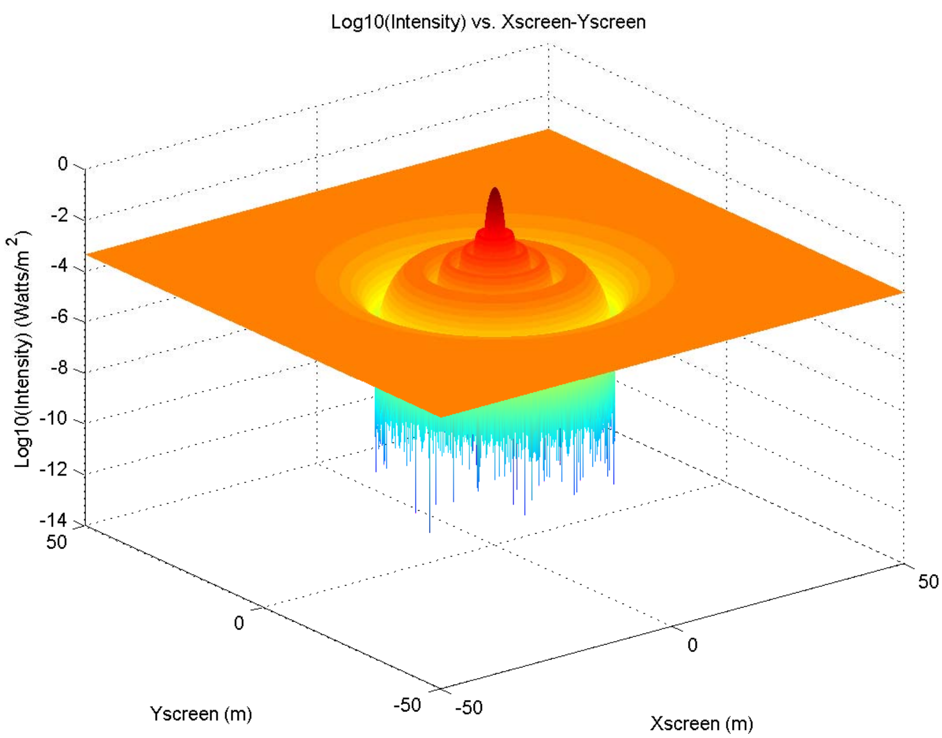
Plots of SIL *vs.* θ and SIL *vs.* y_{screen} for $R = 1.0\ m$:



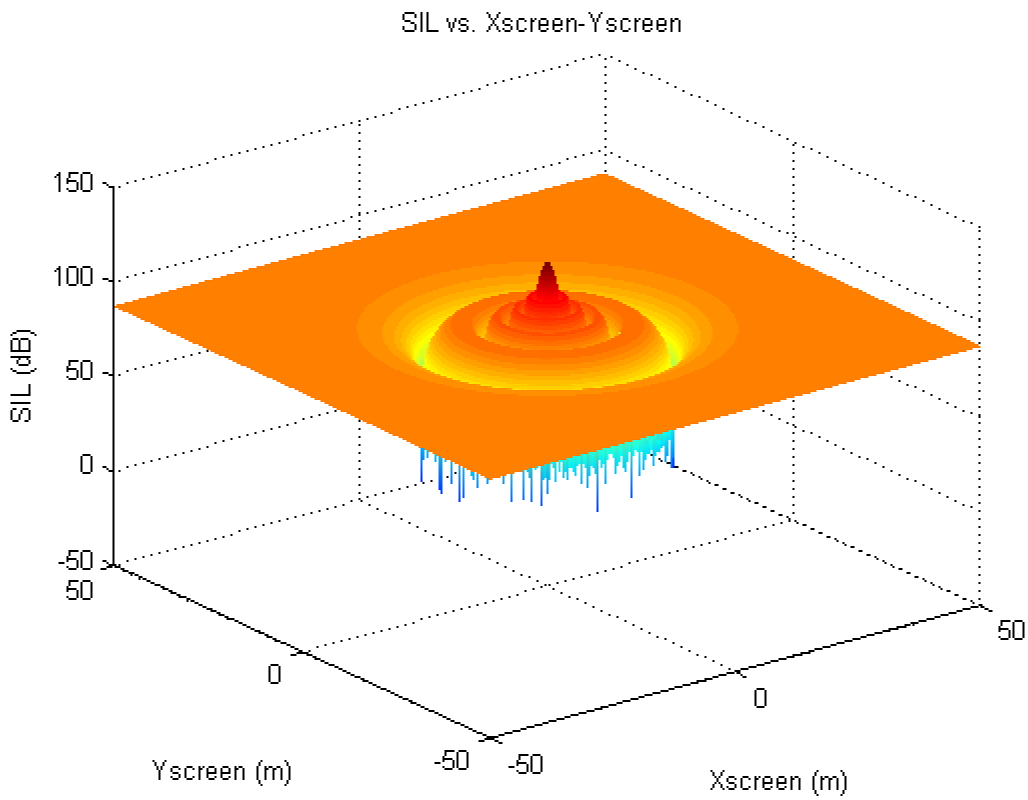


Linear and Semilog Plots of $I_{tot}\left(x_{scr}, y_{scr}\right) vs. \left(x_{scr}, y_{scr}\right)$ for $R = 1.0\ m$:

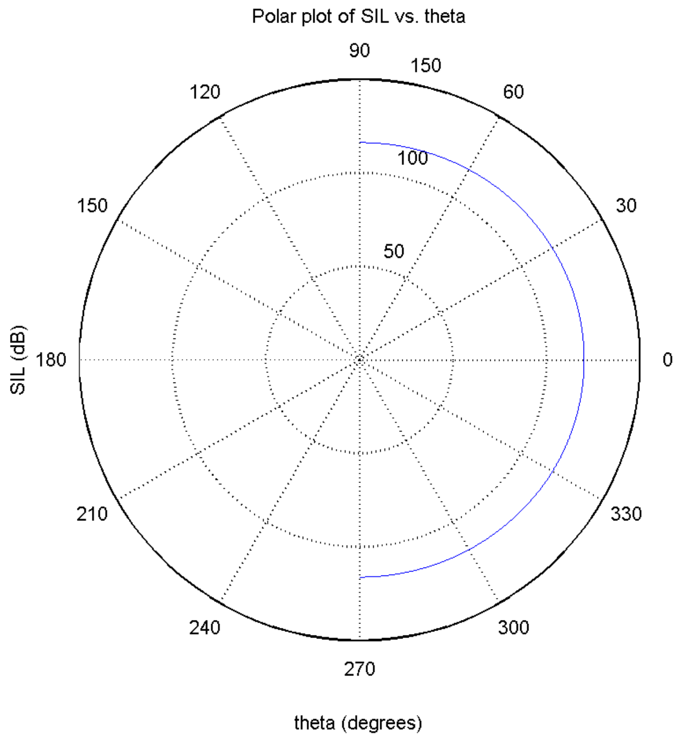




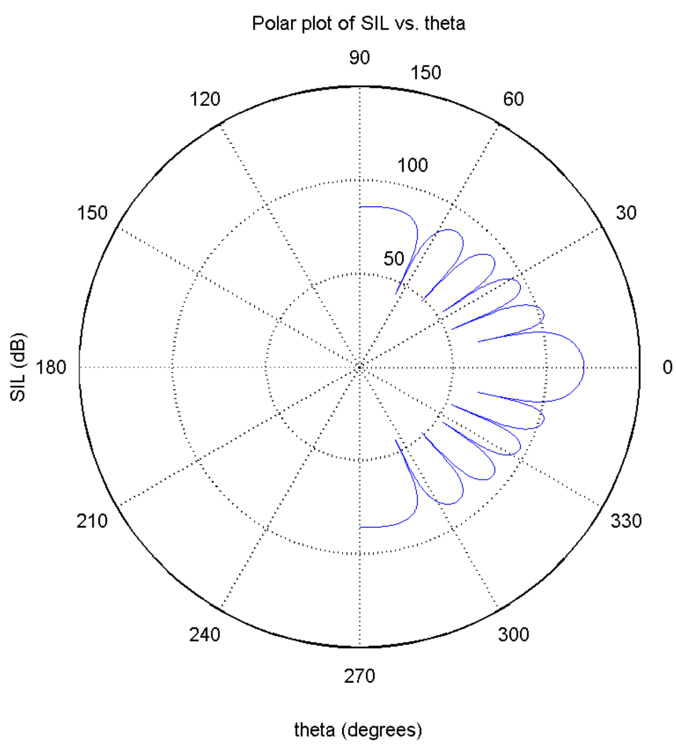
Plots of $SIL(x_{scr}, y_{scr})$ *vs.* (x_{scr}, y_{scr}) for $R = 1.0\ m$:



Polar plot of $SIL(\theta)\, vs.\ \theta$ for $R = 0.1\ m$:



Polar plot of $SIL(\theta)\, vs.\ \theta$ for $R = 1.0\ m$:



Listing of the MATLAB code:

```
%===========================================================================
% Diffn_Circ_Aperture_Thy.m
%
% Fraunhofer diffraction through a circular aperture - simplest theory
% - far-field/plane-wave approx!
% Sound waves assumed to be propagating in free air/great wide-open!
%
%===========================================================================
%
% Written by Prof. Steven Errede  Last Updated: Feb. 7, 2011 12:25 hr
%
%===========================================================================
close all;
clear all;

single     thtr(1800);
single     thtd(1800);
single    Itot1(1800);
single     SIL1(1800);

single     yscr(2000);
single    Itot2(2000);
single     SIL2(2000);

single   Itotxy(2000,2000);
single LgItotxy(2000,2000);
single    SILxy(2000,2000);


% Specify numerical values of parameters:
Io     =    1.0;    % intensity from single slit (Watts/m^2)
Ir     = 1.0*10^-12;% reference sound intensity  (Watts/m^2)
Vair   =  343.0;    % speed of propagation of sound - free air (m/s)
freq   = 1000.0;    % frequency (Hz or cps)
lambda = Vair/freq; % wavelength (m)
Lobs   =   10.0;    % observer distance         (m) n.b. lambda << Lobs

% Specify the aperture radius (m):
Rapr   =    1.0;    % 0.1; 1.0; aperture radius (m) n.b.   Rapr << Lobs

nu     =      1;    % order of bessel function of 1st kind (see below)

%====================================
% Calculate Itot vs. theta:
%====================================
Thetad =  -90.0;    % angle theta of observer in degrees
dTheta =    0.1;    % step angle in degrees

for i = 1:1800;
     thtd(i) = Thetad;            % angle theta of observer in degrees
     Thetar  = (pi/180.0)*Thetad; % angle theta of observer in radians
     thtr(i) = Thetar;

     rho     = ((2.0*pi*Rapr)/lambda)*sin(Thetar); % phase (radians)

    Itot1(i) = Io*((2.0*bessel(nu,rho))/rho)^2; % total intensity (Watts/m^2)
     SIL1(i) = 10.0*log10(Itot1(i)/Ir); % Sound Intensity Level (dB)

     Thetad  = Thetad + dTheta;   % increment angle for next calculation
end

figure(01);
plot(thtd,Itot1,'b');
grid on;
xlabel('theta (degrees)');
ylabel('Intensity (Watts/m^{2})');
title('Intensity vs. theta');
```

```
figure(02);
semilogy(thtd,Itot1,'b');
grid on;
xlabel('theta (degrees)');
ylabel('Intensity (Watts/m^{2})');
title('Log10 Intensity vs. theta');

figure(03);
plot(thtd,SIL1,'b');
grid on;
xlabel('theta (degrees)');
ylabel('SIL (dB)');
title('SIL vs. theta');

figure(04);
polar(thtr,SIL1,'b');
grid on;
xlabel('theta (degrees)');
ylabel('SIL (dB)');
title('Polar plot of SIL vs. theta');

%====================================
% Calculate Itot vs. yscreen:
%====================================
 y = -50.00; % starting position on screen (m)
dy =   0.05; % step size on screen (m);
for i = 1:2000;
     yscr(i) = y;            % position of observer on perp. screen (m)
     Thetar  = atan(y/Lobs); % angle theta of observer in radians

     rho     = ((2.0*pi*Rapr)/lambda)*sin(Thetar); % phase (radians)

    Itot2(i) = Io*((2.0*bessel(nu,rho))/rho)^2; % total intensity (Watts/m^2)
     SIL2(i) = 10.0*log10(Itot2(i)/Ir); % Sound Intensity Level (dB)

     y  = y + dy;   % increment screen position for next calculation
end

figure(11);
plot(yscr,Itot2,'b');
grid on;
xlabel('Yscreen (m)');
ylabel('Intensity (Watts/m^{2})');
title('Intensity vs. Yscreen');

figure(12);
semilogy(yscr,Itot2,'b');
grid on;
xlabel('Yscreen (m)');
ylabel('Intensity (Watts/m^{2})');
title('Log10 Intensity vs. Yscreen');

figure(13);
plot(yscr,SIL2,'b');
grid on;
xlabel('Yscreen (m)');
ylabel('SIL (dB)');
title('SIL vs. Yscreen');

%===========================================================================
beep;
fprintf('\n Very CPU-intensive I(x,y) vs. x,y calcs - please be patient!! \n')
%===========================================================================

%=========================================
% Calculate 2D Itot vs. x,y-screen:
%=========================================
 x = -50.00; % x-starting position on screen (m)
dx =   0.05; % x-step size on screen (m);
for j = 1:2000;
     xscr(j) = x; % x-position of observer on perp. screen (m)
```

```
     y = -50.00; % y-starting position on screen (m)
    dy =   0.05; % y-step size on screen (m);
    for i = 1:2000;
         yscr(i) = y; % y-position of observer on perp. screen (m)
         rscr    = sqrt((xscr(j))^2 + (yscr(i))^2); % radial pos'n on perp. screen (m)
         Thetar  = atan(rscr/Lobs); % angle theta of observer in radians

         rho     = ((2.0*pi*Rapr)/lambda)*sin(Thetar); % phase (radians)

     Itotxy(j,i) = Io*((2.0*bessel(nu,rho))/rho)^2; % total intensity (Watts/m^2)
   LgItotxy(j,i) = log10(Itotxy(j,i));     % log10 of total intensity (Watts/m^2)
      SILxy(j,i) = 10.0*log10(Itotxy(j,i)/Ir);      % Sound Intensity Level (dB)

         y  = y + dy;   % increment y-screen position for next calculation
    end
     x  = x + dx;   % increment x-screen position for next calculation
end

figure(21);
surf(xscr,yscr,Itotxy);
shading interp;
xlabel('Xscreen (m)');
ylabel('Yscreen (m)');
zlabel('Intensity (Watts/m^{2})');
title ('Intensity vs. Xscreen-Yscreen');

figure(22);
surf(xscr,yscr,LgItotxy);
shading interp;
xlabel('Xscreen (m)');
ylabel('Yscreen (m)');
zlabel('Log10(Intensity) (Watts/m^{2})');
title ('Log10(Intensity) vs. Xscreen-Yscreen');

figure(23);
surf(xscr,yscr,SILxy);
shading interp;
xlabel('Xscreen (m)');
ylabel('Yscreen (m)');
zlabel('SIL (dB)');
title ('SIL vs. Xscreen-Yscreen');

%==========================================================================
beep;
fprintf('\n Calculation of diffraction through circular aperture completed !!! \n')
%==========================================================================
```