Tensions of Guitar Strings

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Introduction

The object of this experiment was to determine the tensions of various types of guitar strings when tuned to the proper pitch. It was not necessary to actually place the strings on the guitar to make measurements. The tensions could be found from simple measurements of the mass and the length. The tension of a string could prove to be a very handy thing to know for future experiments.

Theory*

Traveling Wave Analysis
Guitar strings vibrate as standing waves, which are a result of a harmonic wave reflecting off of fixed ends. These standing waves can be expressed as two traveling waves, traveling in opposite directions:

$$y(x,t) = A \sin 2\mathbf{p}(x/? - t/\mathbf{t}) + A \sin 2\mathbf{p}(x/? + t/\mathbf{t})$$

where λ is the wavelength, τ is the period, x is the longitudinal displacement, t is the time, and y is the transverse displacement. We will just look at the term that moves to the right, i.e. the first term, since they are the same wave moving in opposite directions. We can rewrite the traveling wave as

$$y(x,t) = A \sin \frac{2\mathbf{p}}{\mathbf{l}}(x - \frac{\mathbf{l}}{\mathbf{t}}t)$$

We also know that a wave that moves to the right can be expressed as

$$y(x,t) = A \sin \frac{2\mathbf{p}}{\mathbf{l}}(x - V_x t)$$

Therefore, we see that

$$V_x = \frac{\mathbf{l}}{\mathbf{t}} = f\mathbf{l} \quad \text{(Eqn. 1)}$$

Wave Equation
On the other hand, we can analyze the motion of a string using the wave equation. Look at a segment of string that spans the region from x to x+Δx (Fig.1).
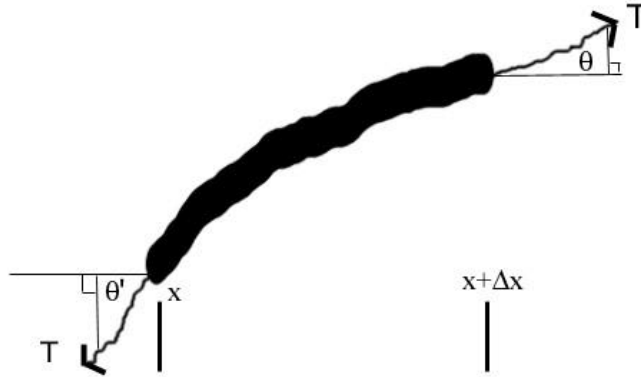
Fig.1

The force in the y direction is

$$F_y = T\sin\theta - T\sin\theta' \approx T\frac{\partial y}{\partial x}(x+\Delta x) - T\frac{\partial y}{\partial x}(x)$$

Using the definition of a double derivative

$$\frac{\partial y}{\partial x}(x+\Delta x) - \frac{\partial y}{\partial x}(x) = \Delta x\frac{\partial^2 y}{\partial x^2}$$

it is easily shown that

$$F_y \approx T\Delta x\frac{\partial^2 y}{\partial x^2}$$

Now, using Newton's 2nd law, we equate this force to the mass times the acceleration in the y direction

$$F_y \approx T\Delta x\frac{\partial^2 y}{\partial x^2} = ma_y = m\Delta x\frac{\partial^2 y}{\partial t^2}$$

where μ is the mass per unit length. The wave equation states

$$\frac{\partial^2 y}{\partial x^2} = \frac{1}{V_x^2}\frac{\partial^2 y}{\partial t^2}$$

Therefore

$$V_x = \sqrt{\frac{T}{m}} \quad \text{(Eqn. 2)}$$

Solving for Tension

The velocity in the x direction has now been solved via two different methods: a traveling wave analysis and the wave equation. If we equate the two expressions we have for the velocity

$$V_x = f\lambda = \sqrt{\frac{T}{m}}$$

and solve for the tension

$$T = mV_x^2 = m\lambda^2 f^2$$

The tension is now in terms of the mass per unit length, the frequency and the wavelength. All three of these quantities are easily solved for. The mass per unit length is the only variable that needs to be determined experimentally.   It is a straightforward measurement of the mass of a certain length of string divided by the length. The frequency is determined by which of the six strings on the guitar is being analyzed. The frequency for each guitar string is shown in figure 2.

String	Low E	A	D	G	B	High E
Frequency (Hz)	82.41	110.00	146.83	196.00	246.94	329.63

Fig. 2

The wavelength of the fundamental (lowest mode of vibration) of a standing wave on a guitar string is determined by the so-called scale-length of the guitar, L_S. The wavelength of the fundamental is twice the length of the guitar, i.e. $\lambda = 2\,L_S$ (as shown in Fig. 3). For our experiment a guitar with a scale length of $L_S = 25\ \frac{1}{2}$ inches was used. Therefore, $\lambda = 2\,L_S = 51.0$ inches $= 1.2954$ meters.
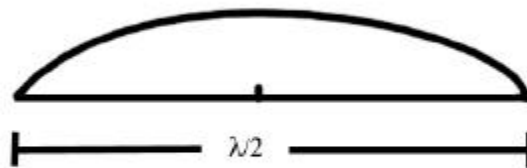
Fig. 3

Experimental Procedure

Four different types of strings were chosen for this experiment: Fender Nickel Plated Super 250 Regulars, Fender Nickel Plated Super 250 Lights, D'Addario Nickel Wound Regular Light Gauge EXL110's, and D'Addario Nickel Wound Super Light EXL120's. Both ends of all the strings used were cut off to ensure the strings homogeneity. For each set of six strings, the length was measured five times with a standard meter stick, the diameter was measured ten times with a digital caliper accurate to a tenth of a mil or an analog caliper that was less precise, and the mass was measured with a scale accurate to a thousandth of a gram. Vinyl gloves were worn the entire time to ensure no body oils came in contact with the nickel guitar strings. This not only aided in accurate mass measurements, but also will extend the lifetime of the strings. Body oils will make the metal strings deteriorate more quickly than uncontaminated strings.

Experimental Results

The results of the measurements are as follows:

Fender Super 250L's

String	Hi E (~.009)	B (~.011)	G (~.016)	D (~.024)	A (~.032)	Low E (~.042)
MASS (Kg)	3.32E-04	4.88E-04	1.07E-03	1.92E-03	3.18E-03	5.42E-03
DENSITY(Kg/m)	3.09E-04	4.77E-04	1.03E-03	1.62E-03	3.50E-03	5.78E-03
DENSITY(Kg/Vol)	7.97E+03	7.83E+03	7.99E+03	5.48E+03	6.75E+03	6.60E+03
FREQUENCY(Hz)	329.63	246.94	196.00	146.83	110.00	82.41
TENSION (N)	56.40	48.86	66.61	58.55	71.11	65.90

Fender Super 250R's

String	Hi E (~.010)	B (~.013)	G (~.017)	D (~.026)	A (~.036)	Low E (~.046)
MASS	3.32E-04	4.88E-04	1.07E-03	1.92E-03	3.18E-03	5.42E-03
DENSITY(Kg/m)	3.09E-04	4.77E-04	1.03E-03	1.62E-03	3.50E-03	5.78E-03
DENSITY(Vol)	7.97E+03	7.83E+03	7.99E+03	5.48E+03	6.75E+03	6.60E+03
FREQUENCY	329.63	246.94	196.00	146.83	110.00	82.41
TENSION	56.40	48.86	66.61	58.55	71.11	65.90

D'Addario EXL 120's

String	Hi E (~.009)	B (~.011)	G (~.016)	D (~.024)	A (~.032)	Low E (~.042)
MASS	3.26E-04	4.74E-04	1.09E-03	1.87E-03	4.22E-03	5.83E-03
DENSITY(Kg/m)	3.29E-04	4.78E-04	1.04E-03	1.91E-03	4.27E-03	5.90E-03
DENSITY(Vol)	7.94E+03	7.82E+03	8.05E+03	6.65E+03	6.90E+03	6.64E+03
FREQUENCY	329.63	246.94	196.00	146.83	110.00	82.41
TENSION	59.93	48.87	67.35	69.06	86.61	67.29

D'Addario EXL 110's

String	Hi E (~.010)	B (~.013)	G (~.017)	D (~.026)	A (~.036)	Low E (~.046)
MASS	3.85E-04	6.94E-04	1.19E-03	2.19E-03	4.18E-03	6.69E-03
DENSITY(Kg/m)	3.89E-04	6.66E-04	1.14E-03	2.23E-03	4.21E-03	6.79E-03
DENSITY(Vol)	7.69E+03	7.95E+03	8.22E+03	6.93E+03	6.61E+03	6.54E+03
FREQUENCY	329.63	246.94	196.00	146.83	110.00	82.41
TENSION	70.87	68.15	73.62	80.77	85.57	77.41

The following is a graph showing the six tensions for each set of guitar strings. It makes it easier to see the correlations that exist.
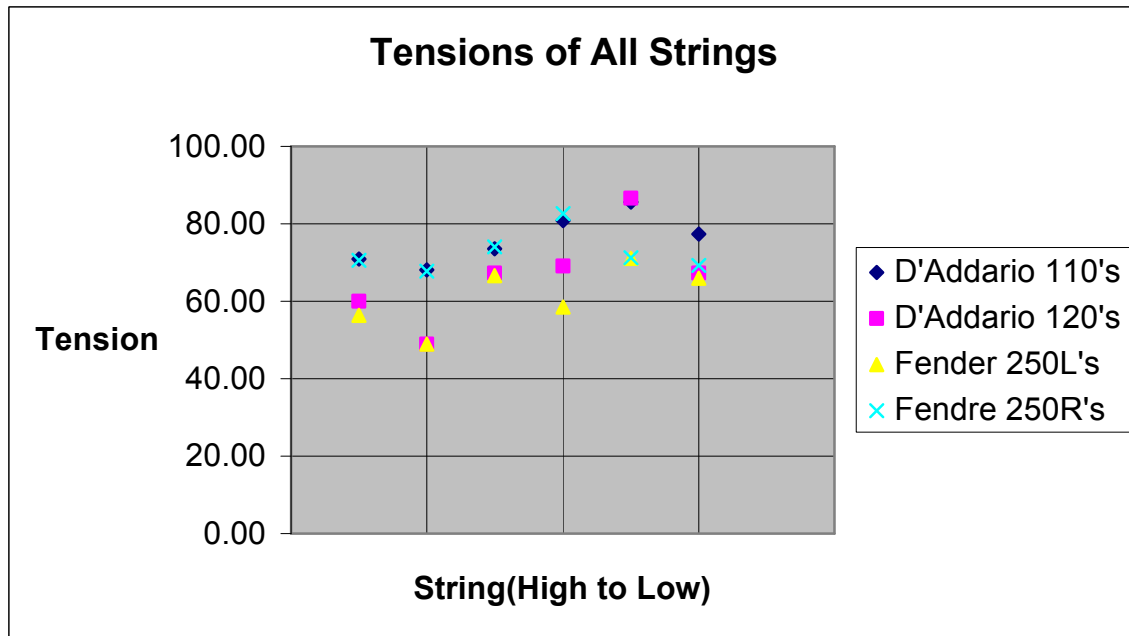
Tensions of All Strings

Tension

String(High to Low)

Legend: ♦ D'Addario 110's ■ D'Addario 120's ▲ Fender 250L's ✕ Fendre 250R's

Fig. 4

The average value for each string was:

String	High E	B	G	D	A	Low E
AVE T	64.43	58.43	70.42	72.77	78.62	69.97

Average Tension

Tension (N)
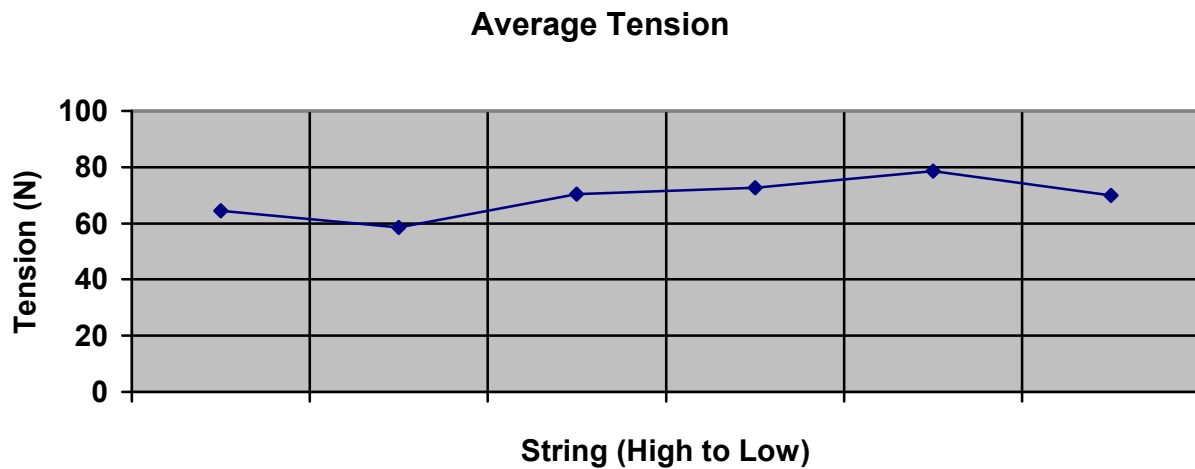
String (High to Low)

Fig. 5

Conclusions

As is seen in Figure 5, the average tension for each string is essentially between 60 and 80 Newtons. However, more specifically, one can see a correlation between two sets of strings in Figure 4. The D'Addario 110's seem to match up well with the Fender 250R's and the D'Addario 120's match up with the Fender 250L's. This makes sense because each of those pairs has the same diameter as its partner.

An interesting side note is that the A string of the D'Addario EXL 120's was said to have a diameter of 32 mils. This was quite wrong, and the diameter of the A string was actually ~36 mils.

Knowing the tensions of the strings on a guitar is very useful. These figures can be used for many other different experiments. One key problem that would be interesting to investigate would be the detuning of the guitar as the player frets higher and higher notes. It is a well known fact that as the strings are fretted more, they become out of tune with each other, while when played open, they are in tune. Guitar manufacturers attempted to fix this ailment by having the length of the strings be adjustable near the pickup. However, this only partially solves the problem. Perhaps a solution to the problem would be to find a set of strings that have equal tension. This may improve the intonation between strings.

The tension of the strings could also be used in experiments studying the damping of strings or in experiments studying how the string deforms when it has been stretched across the guitar for some period of time. The tension is a quantity that is in general a very useful thing to know, and can be applied to many other topics. This experiment is just the foundation for what could be many very interesting studies.

*Acknowledgements