A Study of Chaos
Chua's Circuit

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Physics 303
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Introduction:

Chaos is the idea that a system will produce random long-term results when the same initial conditions are applied to that system. It is common practice to study chaos with physical systems such as the pendulum and water drop experiments seen in our lab. However, circuits provide a very useful system for studying chaos as well as easy application to modern technology. Chua's circuit, presented by L. O. Chua with the Department of Electrical Engineers and Computer Sciences at the University of California in Berkeley, CA, is one of the most interesting examples of chaos in a circuit.

Theory:

Chua's circuit consists of an 18 mH inductor, 100 nF capacitor, a 10 nF capacitor and a nonlinear resistor all in parallel and a controlled resistor in series between the two capacitors (see fig. 1). There are several examples of nonlinear resistors used in the circuit, but they all have the same current to voltage function (see fig. 2). Although the given function is in terms of the nonlinear (Nr) resistor shown.

This circuit is commonly studied because it yields very stable attractors. In other words, although the actual results vary widely, the same attractors always appear. This is not the case for the pendulum or water drop experiments (See the water drop report done by Sarah Rose for this class fall 2000 for more detail). Chua's circuit has a very defined and consistent pattern of achieving its "maximum" chaos known as the double scroll. These patterns as well as the double scroll are illustrated in the attached figures.

For my experiment, I tried to analyze the frequencies at which the period of the circuit doubled tripled etc. To do this, I attached a frequency counter to channel one and channel two of the oscilloscope. The oscilloscope was set up to measure the voltage across the 100 nF capacitor (C1) versus the 10 nF capacitor (C2). The figures attached show the progression of the attractors are the images produced by this set up on the oscilloscope.

Also, by working with the physics 398 Electronic Musical Instruments class, I was able to study the effects the circuit had on an input by a guitar. The guitar input studied was being applied across either C1 or C2. The effects were similar and with controllable voltage and chaos across the capacitors, I was able to effect the sound to achieve various amplitudes and disturbances.

Experiment and Results:

With the circuit set up shown fig. 1, adjusting Nr from zero to maximum will distort the circuit output from a simple loop through the stages shown including period doubling, tripling, bifurcation and double scroll. At maximum, the circuit achieves a trapezoidal output with "looped" ends. The determinable frequencies are listed below. Unfortunately, the frequency counter only takes sample frequencies. So, the frequencies listed are a range that the voltage frequencies ranged over while in that state. Also, the frequency counter showed that the range of frequencies over the various states between normal period and the double scroll were the same. This is explained because the width and height of the plots are the same. However, at double scroll, the "width" shrinks to a near point across C1. Also, upon closer examination of the normal period, I found that the normal period was several loops that ran within a few hertz of each other (see fig 3). Also, the range across C1 and C2 varied because the capacitors differ by a factor of 10.

State	Frequency across C1
Normal period	2830 Hz
Period doubling	2780 Hz
Wild Bifurcation	450Hz-1000Hz
Double Scroll Random	500Hz-800Hz
Double Scroll Steady	300Hz-450Hz
Trapezoidal Loop	2946Hz
	Frequency across C2
Normal period	2780
Period doubling	2780
Wild Bifurcation’s	-
Double Scroll Random	2900
Double Scroll Steady	2900
Trapezoidal Loop	2940

Inserting a signal input across either capacitor kills the output unless there is a driving signal on the input. Then the circuit distorts the input in various ways (not all of which have been determined). By hooking an amplifier to the output, the guitar signal is distorted similarly to a distortion pedal. I tried to illustrate in fig. 4 the signal of a guitar string with Nr at double scroll as it appeared on the oscilloscope.

Analysis:

Although the results of this experiment show very little applicable data, I do feel that there is great potential for this kind of research. First, with more time, I had planned on using a frequency analyzer to pinpoint the various frequencies of the periods seen. Also, there is more study to be done with the sound possibilities of this circuit. For instance, there is an attached article that details how this circuit can be used to mimic the random vibrations seen in the frequencies produced by woodwind instruments. This would be used to achieve a more accurate reproduction than the synthesizers used in equipment such as keyboards and computer programs. Also, the potential for classroom use is infinite, as the results are more consistent than the results of the pendulum. Students can see the attractors and using different kinds of circuits and resistors maybe even discover a link between the kind of nonlinearity and the attractors seen. For instance, I hypothesize that the double scroll attractor is a result of the input output relation created by the Nr used here. Given an input, the resistor produces either a positive or negative voltage that is the run back across the capacitors. Creating a dual response for a single input. But, that is not in the scope of this course or this lab.

Sources:

The circuit I used was provided by the physics 398 EMI class as was all the source material which is photcopied and attached.

1. “Experimental Chaos Via Chua’s Circuit,” by Michael Peter Kennedy, Electronics Research Laboratory, University of California, Berkeley, CA 94720.

2. “The Double Scroll,” by Takashi Matsumoto, fellow, IEEE; Leon O. Chua, fellow, IEEE; and Motomasa Komuro.

3. “Examples of ‘Musical’ Sounds from the Chua Circuit,” **http://www.ccsr.uiuc.edu/people/gmk/papers/IEEE/ieee52da/section3_10.html**

4. Notes from Prof. Zhong of the EE department at UC-Berkeley.

My Thanks to Prof. Steve Errede at UIUC Department of Physics for the use of his lab, equipment and resources.

Chua's Circuit

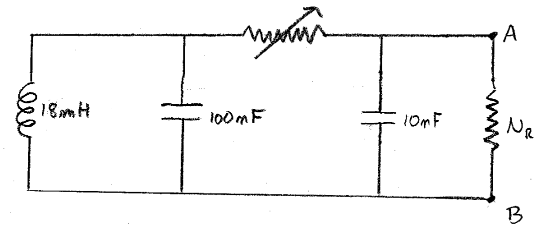
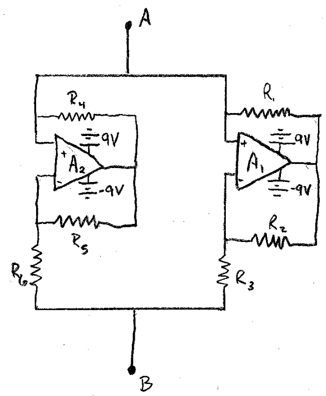
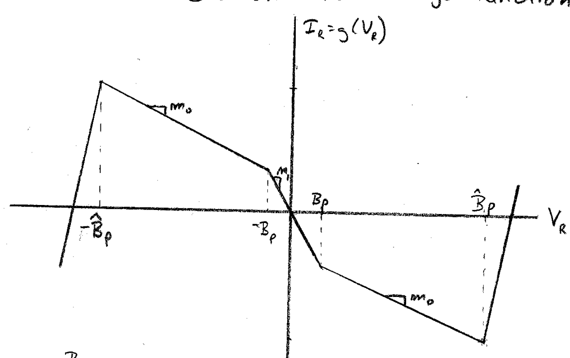


fig. 1



Example of N_R

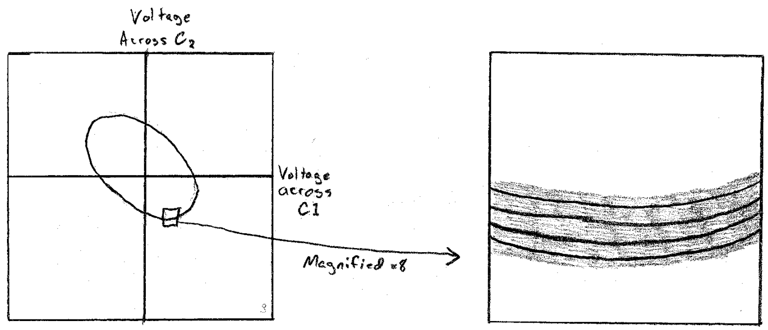
Current to Voltage function



$$B_p = \frac{R_6}{R_5+R_6}E \qquad m_0 = \frac{-R_2}{R_1 R_3} + \frac{1}{R_4}$$

$$\hat{B}_p = \frac{R_3}{R_2+R_3}E \qquad m_1 = \frac{-R_2}{R_1 R_3} - \frac{R_5}{R_4 R_6}$$

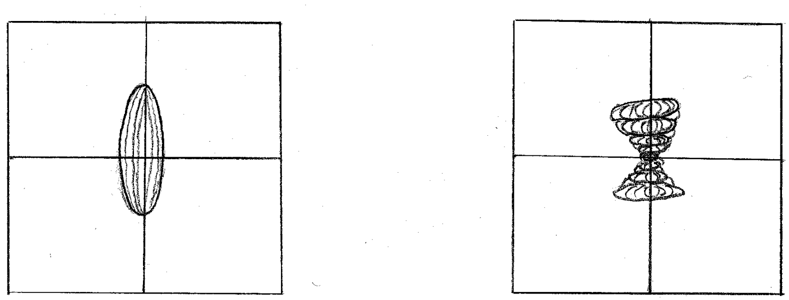
fig. 2



View of "Normal" Period on oscilloscope.

View of section of "Normal" Period on oscilloscope, magnified.

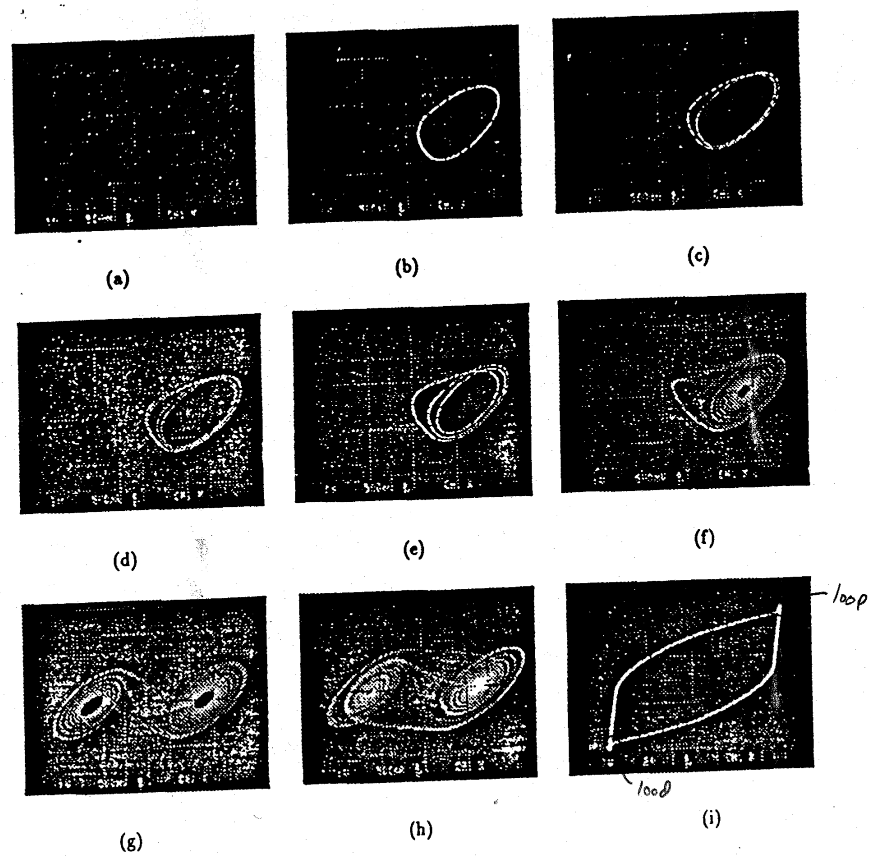
fig. 3



Estimated view of guitar pitch frequencies not connected to Chua's Circut.

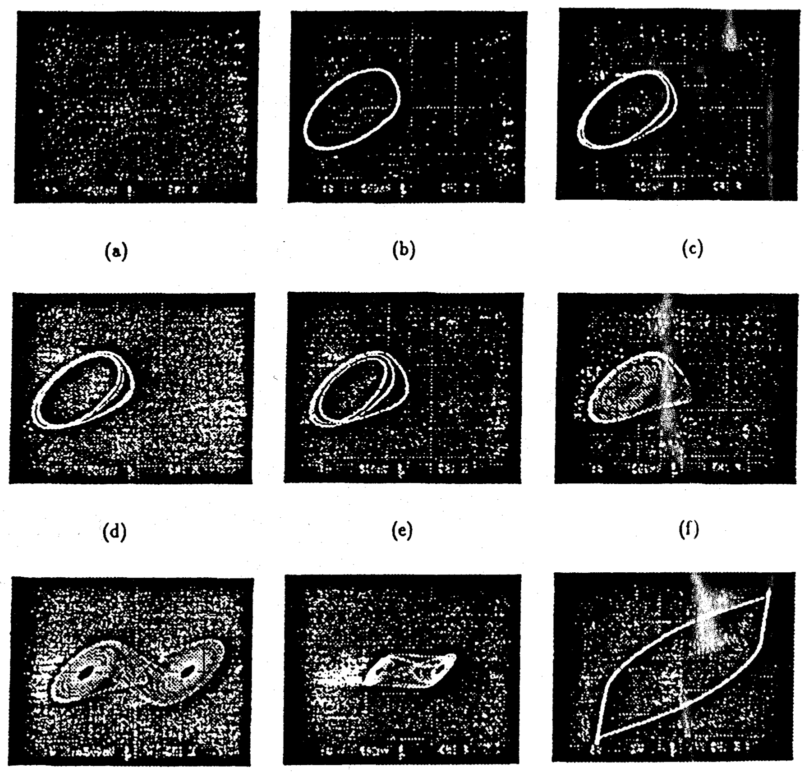
Approximate view of guitar pitch frequencies when connected to Chua's circuit set at double scroll chaos.

fig. 4



These images are taken from "Experimental Chaos via Chua's Circuit." They show the progression from the "normal" period to the looped trapezoidal (maybe hysterisis).

(a) Nr is off so no current flows
(b) Normal loop
(c) Period Doubling
(d) + (e) Increasing periods appear
(f) Max. Periods before bifurcation starts
(g) Steady double scroll
(h) Max. Chaos of double scroll
(i) Final "periodic" state



(a) (b) (c) (d) (e) (f) (g) (h) (i)

These images show that although the progression is slightly different, the same attractors appear in the same order. The progression through the attractors varies slightly due to the resonances of the inductor, capacitor, capacitor sequence. Sometive the voltages is mostly neg., sometimes mostly pos.

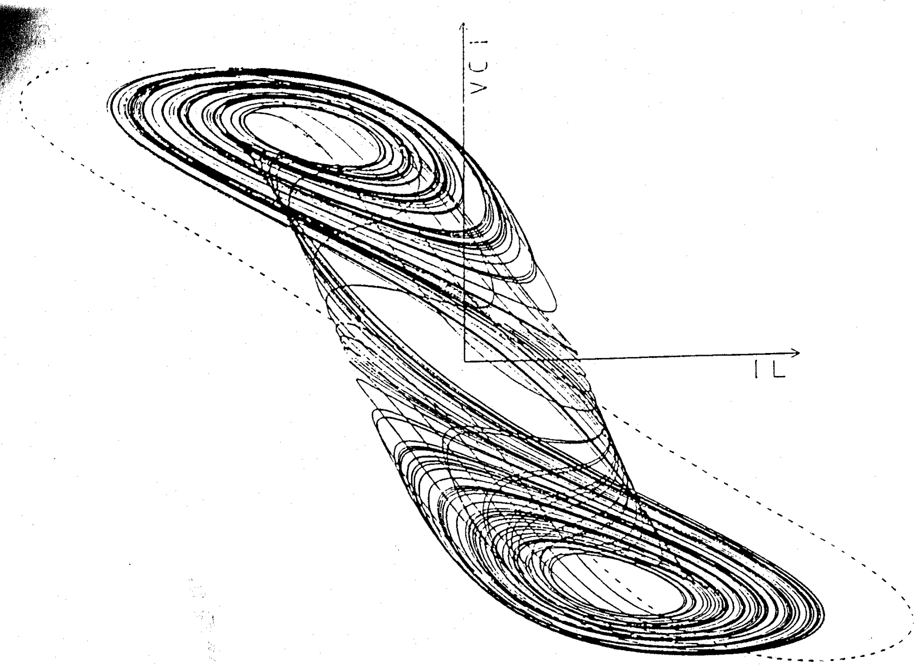


Fig. 8. The chaotic attractor and the saddle-type periodic orbit with the smooth resistor constitutive relation (1.5). Initial conditions: $v_{C_1}(0) = 1.47147$, $v_{C_2}(0) = 0.83242$, $i_L(0) = 2.23418$ for the attractor, and $v_{C_1}(0) = 9.998048$, $v_{C_2}(0) = 1.980972$, $i_L(0) = -10.908448$ for the saddle-type periodic orbit with period 4.49. The length of each arrow is 11.

These images are taken from "the Double Scroll" essay and show a graphical analysis of the double scroll. As can be seen, the period loops are a steady supperposition of the wild bifurcations of the "f" graph on the previous two pages.

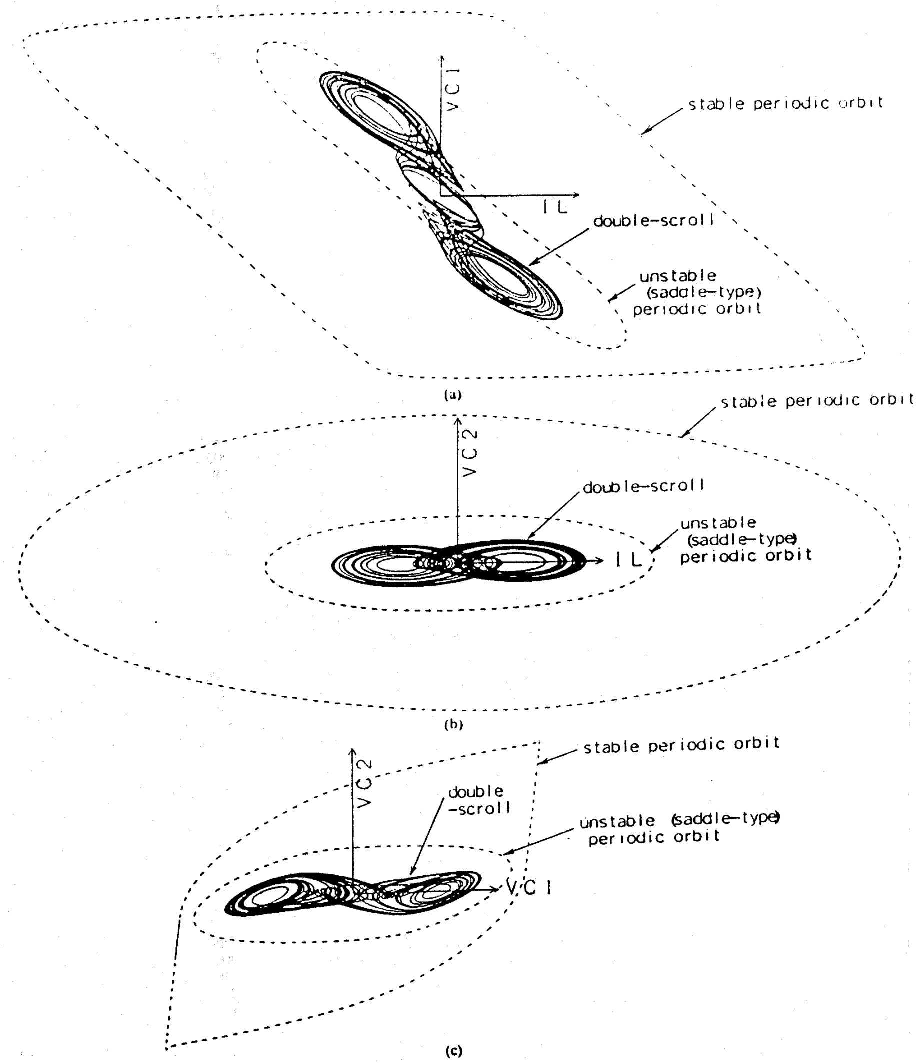


Fig. 6. The large stable limit cycle with the chaotic attractor and the saddle-type periodic orbit. (a) Projection onto the (i_L, v_{C_1})-plane. (b) Projection onto the (i_L, v_{C_2})-plane. (c) Projection onto the (v_{C_1}, v_{C_2})-plane. Initial conditions for the large stable limit cycle: $v_{C_1}(0) = -3.08832$, $v_{C_2}(0) = -1.0423$, $i_L(0) = 6.93155$ with period 2.87. The lenght of each arrow is 2.5.

(ii) $0.5 \leq 1/C_2 \leq 1.08$, when $1/C_1 = 9$, $1/L = 7$ and $G = 0.7$ are fixed,

(iii) $5.7 \leq 1/L \leq 7.13$, when $1/C_1 = 9$, $1/C_2 = 1$ and $G = 0.7$ are fixed, and

(iv) $0.68 \leq G \leq 0.76$, when $1/C_1 = 9$, $1/C_2 = 1$ and $1/L = 7$ are fixed.

Since there are two attractors in Fig. 6 (the c[illegible] attractor and the periodic attractor), one naturally wo[illegible] what boundary separates the domain of attraction [illegible] chaotic attractor and the domain of attraction fo[illegible] periodic attractor. Similarly, in Fig. 2, one wonders [illegible] distinguishes those initial states that are attracted

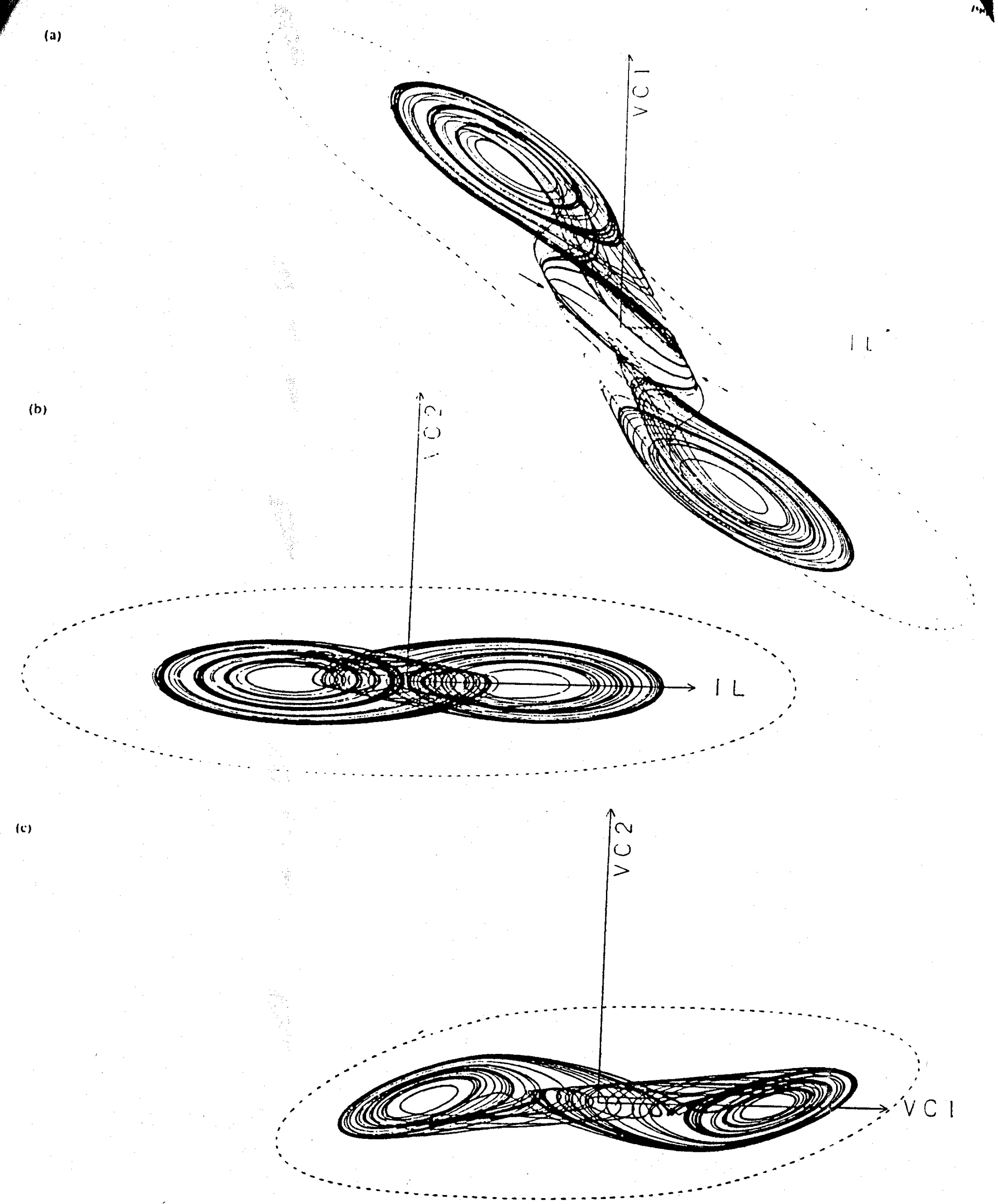


Fig. 2. The chaotic attractor and the saddle-type periodic orbit. (a) Projection onto the (i_L, v_{C_1})-plane. (b) Projection onto the (i_L, v_{C_2})-plane. (c) Projection onto the (v_{C_1}, v_{C_2})-plane. Runge–Kutta was iterated 10000 times with step size 0.04. Initial conditions: $v_{C_1}(0) = 0.15264$, $v_{C_2}(0) = -0.02281$, $i_L(0) = 0.38127$ for the attractor and $v_{C_1}(0) = 2.532735$, $v_{C_2}(0) = 1.285458 \times 10^{-3}$, $i_L(0) = -3.367482$ for the saddle-type periodic orbit with period 3.54793. The length of each arrow is 2.5.