Analysis of an Acoustic Guitar

I. Introduction

The acoustic guitar has been a common component in many genres of music for many years. Its versatile, rich tones and popularity with famous artists have allowed it to become one of the most common instruments played professionally, as well as played as a hobby. Although the construction of these instruments is fairly straightforward, many intricate details define the type of sound that they emit. Smaller models made of springier materials offer a brighter sound, while larger models specialize in rich low notes and a full, comforting sound. These guitars are loud enough to be played without amplification in small settings, but are often aided electronically for larger performances.

It is of interest to examine how the structure, and properties, of an acoustic guitar combine to result in the beautiful sounds that it produces. In the following pages, a Zager model ZAD-50CE electrified acoustic guitar will be analyzed in a variety of ways. These analyses include a full body modal analysis (both experimental and computer simulation), and a comparison of waveforms of various musical notes. Specifically, the waveform analysis will attempt to determine why the notes of a guitar have a slightly different sound and texture when the string is plucked at varying locations.

II. Instrument

As stated above, the subject of this analysis is a Zager ZAD-50CE. The company, established in 1969, produces guitars that are designed and made for optimum playability. Strings are laid close to the fretboard to allow for easier string fretting, and special bracing patterns enhance the sound that emanates from the body. Each guitar is hand finished to ensure the quality control of the processes.

The specific guitar that is to be analyzed was purchased in 2007, and is composed primarily of spruce and mahogany. The face plate of the guitar is spruce, while the back, sides, neck, and headstock are mahogany. 80/20 bronze wound strings are used for the low E to G strings, with B and high E being made of plain steel (strings refer to standard guitar tunings). Electric pickups are incorporated into the body, and are controlled through a plastic module embedded in the side of the guitar that doubles as an on-board tuner.

Figure 1 - Zager CAD-50CE

III. Wave Analysis

There is a significant difference in a note's sound when the string of a guitar is plucked at various locations along its length. Qualitatively, notes plucked near the bridge of the guitar seem to have a brighter, sharper sound when compared to the richer tone that comes from it being plucked over the soundhole. In this experiment, the harmonics that occur when the various strings are plucked in three locations will be compared. The harmonic composition of sounds is a large factor in determining its qualities. The test locations were over the soundhole (standard playing location), next to the bridge, and over the 6th fret (on the neck).

Procedure

A microphone was used to record the sounds that resulted from each string being plucked at all three locations. These sounds were captured and stored onto the Marantz PMD671 stereo 24-bit recorder available in the POM physics lab. This allowed the sounds to be converted into a digital waveform, and to be transferred onto a PC. An existing MATLAB script was then used to analyze the signal, as well as break down the harmonic structure of each note.

Results

Experimental

The focus of this analysis was on the first eight harmonics of each string. The relative amplitudes of each individual harmonic were plotted on the same chart for comparison purposes. Figure 2 shows the harmonic diagrams for each string of the acoustic guitar. It is interesting to note that each location has a fundamentally different structure. As an overall trend, notes plucked at the bridge have a stronger contribution from the higher harmonics (5-8) when compared to the soundhole distribution. It can be concluded that this increased presence of higher harmonics is primarily responsible for the “bright” sound that is heard when plucking by the bridge. The harmonic distribution trends from notes plucked on the neck are a little more difficult to discern. Overall, they appear to have a slightly more even distribution (over the range of harmonics) of contributions when compared to the soundhole distributions. It is postulated that this difference can be attributed to the amplification effect that the guitar body gives to the notes. The resonances of an acoustic guitar body tend to occur in the lower frequency ranges, which is where the lower harmonics reside. Therefore, notes over the soundhole should generally have a relatively stronger response to lower harmonics, which is apparent in the data.

Analytical

It is also possible to determine the relative strengths of each harmonic analytically using Fourier analysis. The physics lab has a piece of software that performs this analysis for a fixed-fixed string, for a given pluck location. This fixed-fixed string model is a greatly simplified version of a guitar string, so it is not expected that the results will match closely to the experimental data. However, the general trend should be approximately the same. The ends of a guitar string being played on a guitar body experience much more movement and flexibility than the rigidly anchored model being used in the software.

With that important caveat identified, the software was used to calculate the Fourier coefficients, which correspond to the measured harmonics of the strings. These coefficients were normalized and plotted to allow for a fair comparison between the analytical and experimental results. These plots can be seen in Figure 3. Upon inspection, it can be seen that the higher harmonics of notes plucked at the bridge are stronger than their counterparts on the soundhole harmonics graph. That is consistent with the experimental findings, and helps to confirm the conclusion that this is the main reason behind the bright tone heard from bridge plucking. The neck-plucked coefficients are a little more ambiguous, but it does appear that the harmonic strengths are slightly differently distributed from the soundhole results, as can also be seen in the experimental data.

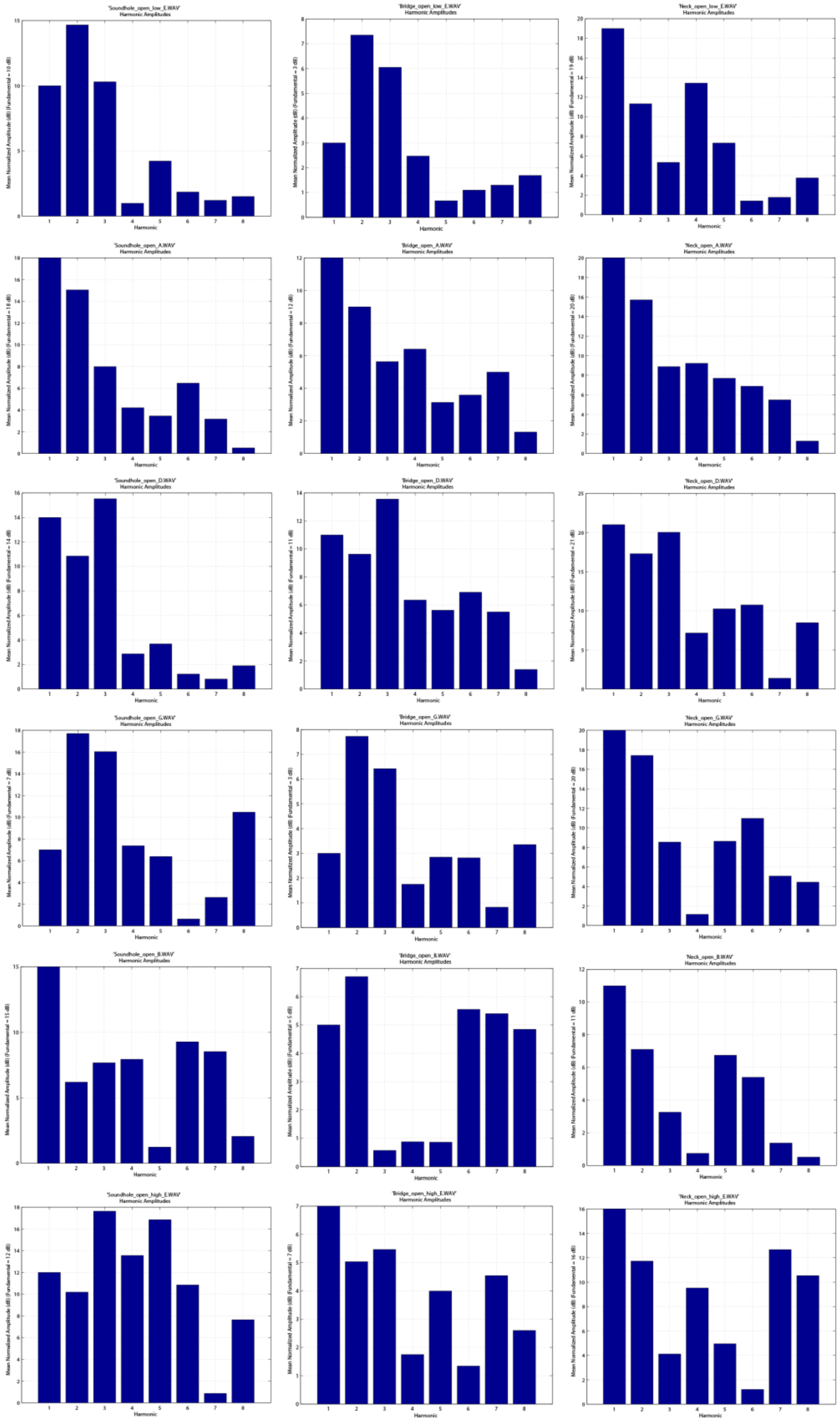

Figure 2 - Relative harmonic distributions for different plucking locations for each guitar string

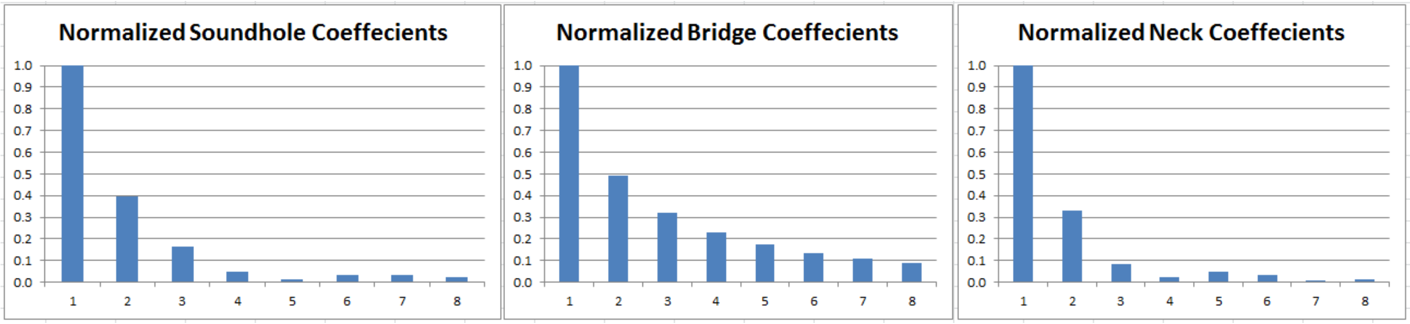

Figure 3 - Analytical Fourier coeffecients

IV. Modal Analysis

The second major type of analysis to be performed on the acoustic guitar is a full body modal analysis. This analysis will identify the natural vibrational modes of the guitar body, which is controlled by the properties of the materials and its structural shape. The natural resonances of the body help to amplify and improve the tones that are played using the strings, so these modal properties are integral to the performance of an acoustic guitar. A virtual modal simulation was carried out, as well as an experimental test of the body's resonant frequencies.

Simulation

Model Setup

The virtual modal study was performed using PTC's Pro/Mechanica software module, which is a finite element solver integrated into the same platform as the Pro/Engineer CAD software. First, measurements of every aspect of the guitar were taken in order to accurately model the shape and structure within the CAD window. Image processing software was used to make an accurate curve of the faceplate/main body of the guitar, since it is very important to get that shape correct in order to run a meaningful simulation. Parts were modeled in such a way that the material properties of each piece could be controlled, thus modeling the structure as closely as possible. In the interest of simplification, however, the plastic electronics and tuner module were omitted, and all joints between pieces were assumed to be completely bonded. The CAD model can be seen in Figure 4.

As discussed in the introduction section, the face plate is made of spruce, and the rest of the wood components are mahogany. The anchors and tuning pegs are steel. The appropriate material properties were applied to each part, respectively. The main properties that are important for the purpose of modal analyses are the density, Young's Modulus, and Poisson ratio of the material, as they describe the spring-like response and the deformation characteristics of the materials. These are shown in Table 1. Tension forces were applied to the tuning pegs and the anchor points at the bridge to simulate the forces that tuned strings would apply at those locations. They were calculated as shown in Table 2.

Table 1

	Density [kg/m^3]	Young's Modulus [GPa]	Poisson's Ratio
Spruce	430	10.3	0.4
Mahogany	560	10.24	0.5
Steel	7900	200	0.3

Courtesy of http://www.conradlumberco.com/pdfs/ch4-Mechanical-Properties-of-Wood.pdf

Table 2

Material	String	rho [kg/m^3]	d_{string} [in]	d_{string} [m]	A_{string} [m^2]	L [m]	f [Hz]	T [N]
Steel	Hi-E	7900	0.011	2.79E-04	6.13E-08	0.65	330	89.14
Steel	B	7900	0.015	3.81E-04	1.14E-07	0.65	247	92.86
80/20 Bronze	G	8596	0.023	5.84E-04	2.68E-07	0.65	196	149.59
80/20 Bronze	D	8596	0.032	8.13E-04	5.19E-07	0.65	147	162.88
80/20 Bronze	A	8596	0.042	1.07E-03	8.94E-07	0.65	110	157.12
80/20 Bronze	Low-E	8596	0.052	1.32E-03	1.37E-06	0.65	82	133.84

$$T = \mu {v_L}^2 = \rho A_{string}(2Lf)^2 \quad [N]$$

$$where\ L\ is\ the\ length\ of\ the\ string, f\ is\ the\ frequency\ of\ the\ string, A\ is\ the\ corss\ sectional\ area, and\ \rho\ is\ the\ density\ of\ the\ string$$

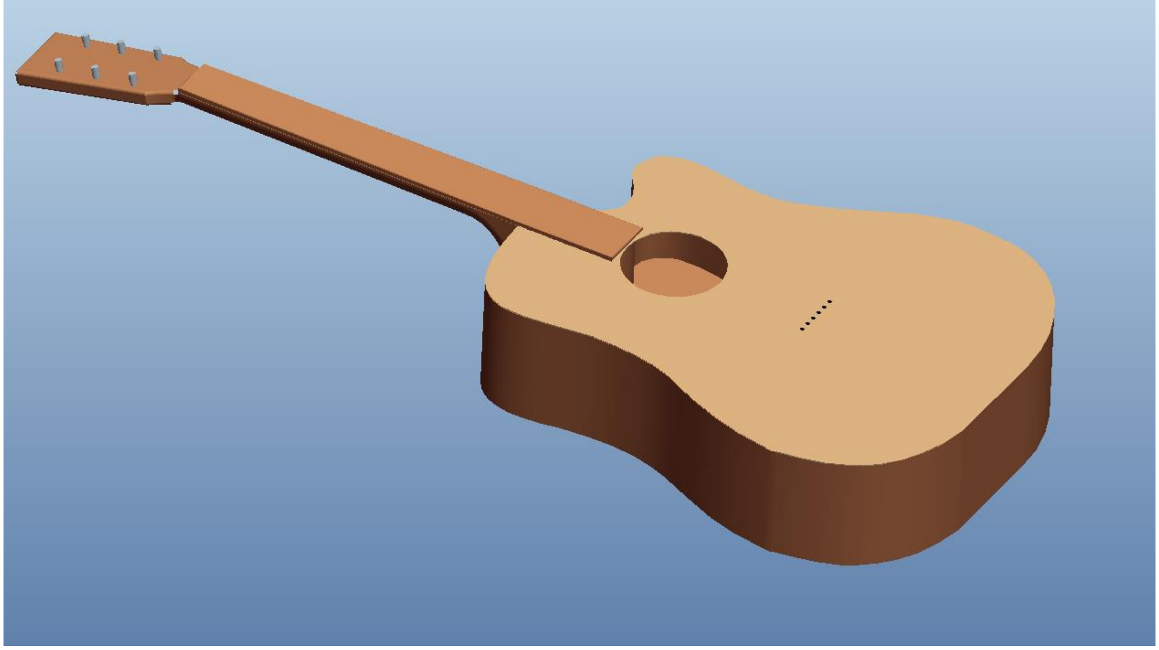

Figure 4 – CAD model of the acoustic guitar

Helmholtz Resonance

Since the CAD model was built using the exact dimensions of the guitar, the geometrical property calculators in Pro/Engineer could be used to find an accurate value of surface areas and volumes of the body. These measurements can be used to calculate the Helmholtz Resonant frequency and the Q factor of the guitar, since the hollow part of the body behaves very much like a classical Helmholtz resonator (albeit with a very large neck opening). The Q factor is a measure of how “sharp” the resonance peak is with respect to the frequency. High Q factors mean that the resonator has a very narrow range of frequencies that exhibit a resonant response. The Helmholtz frequencies and Q factor can be calculated by:

$$f_r = \frac{v}{2\pi}\sqrt{\frac{A_h}{V(h+1.7r)}}$$

$$f_r = \mathbf{123.2\ Hz}$$

$$Q = 2\pi\sqrt{V\left(\frac{(h+1.7r)}{A_h}\right)^3}$$

$$\boldsymbol{Q = 31.2}$$

$$where\ A_h\ is\ the\ area\ of\ the\ opening, V\ is\ the\ body\ volume, h\ is\ the\ thickness\ of\ the\ opening, and\ r\ is\ the\ radius\ of\ the\ opening$$

These values will later be compared to the results of the experimental modal analysis study. The collected data should show a clear, fairly sharp resonance response occurring around the calculated 123 Hz value.

Studies

Two modal studies were performed: one complete body analysis, and one that only included the face plate. For the face plate analysis, the outside edges of the plate were rigidly constrained in all directions in order to isolate the modes of the plate within the context of the guitar body system.

The full body modal analysis concluded that the first six modes would occur at 67.4 Hz, 105 Hz, 125.8 Hz, 147.3 Hz, 159.8 Hz, and 190 Hz.

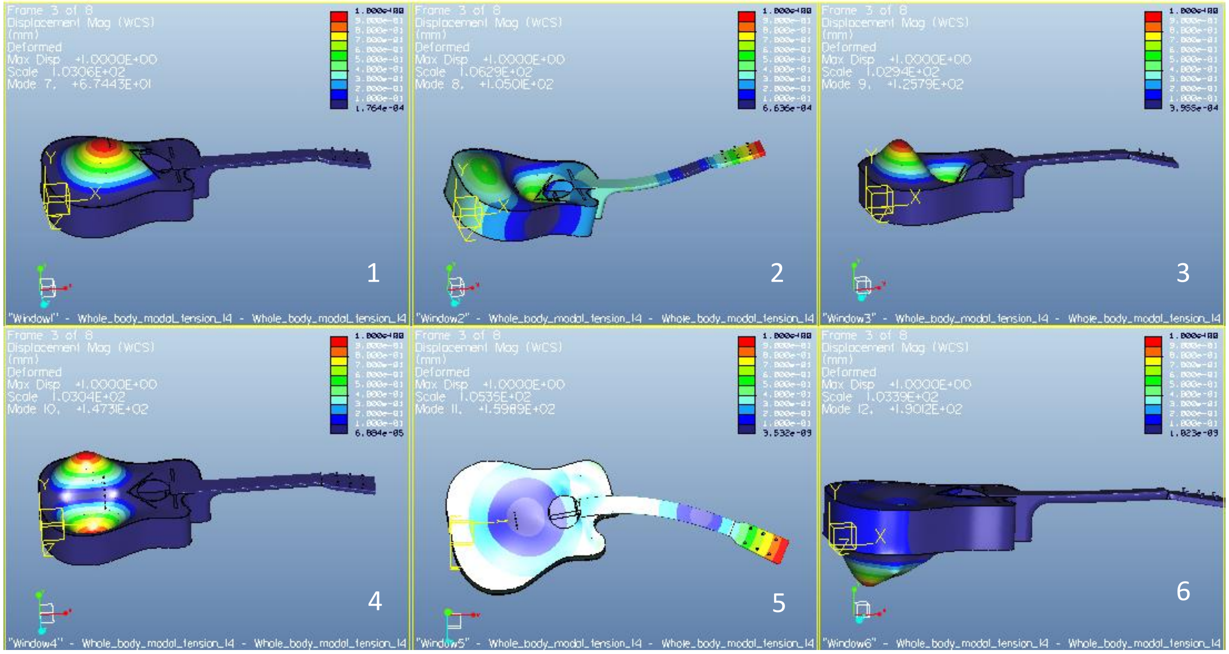

Figure 5 - Full body acoustic modes from Pro/Mechanica

The first eight modes of the isolated faceplate were found to be 63.5 Hz, 121 Hz, 143 Hz, 208.5 Hz, 238.6 Hz, 242Hz, 273Hz, and 302Hz.

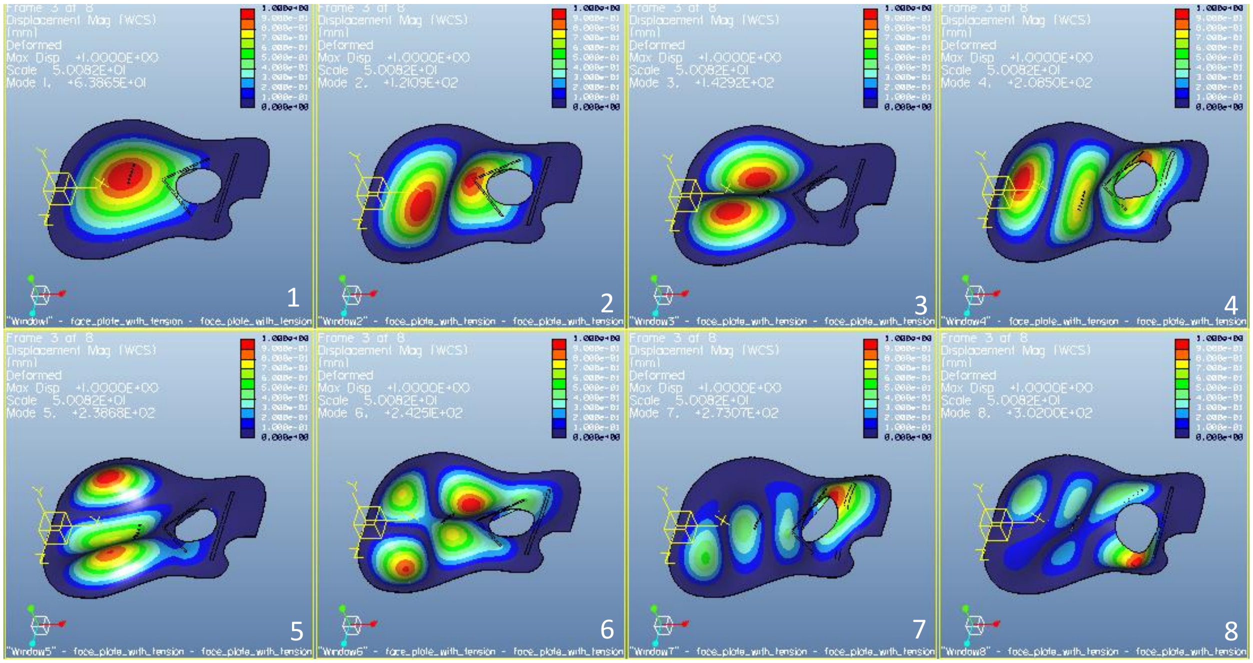

Figure 6 - Face plate acoustic modes from Pro/Mechanica

Experiment

Setup

In order to experimentally measure the body resonances of the acoustic guitar, it was tested over a range of frequencies. The guitar was first suspended in the air to allow it to vibrate freely and minimize the influence of outside forces. A driving frequency was applied near the bridge using a piezotransducer controlled by a function generator. A second piezotransducer was placed at six different locations on the face plate and headstock to measure the response of the body to the driving force. For each location, the frequency was swept from 29.5 Hz to 2000.5 Hz in 1 Hz steps. A diagram of the measurement locations can be seen in Figure 7, and the experimental setup can be seen in Figure 8. In addition to the piezotransducers, particle and pressure microphones were both positioned over the soundhole to measure the air movement caused by the driving vibrations. A preexisting MATLAB script was used to process the data.

Figure 7 - Measurement Locations

Figure 8 - Experimental Setup

Helmholtz

The Helmholtz frequency will be apparent from the particle microphone data, in the form of particle velocity. In order to compare the experimental value with the theoretical calculation, the data was processed to get the particle velocity distribution over the whole frequency sweep range. This can be seen graphically in Figure 9. The large peak in the graph corresponds to the Helmholtz frequency, and it appears to occur around the expected 123 Hz value.

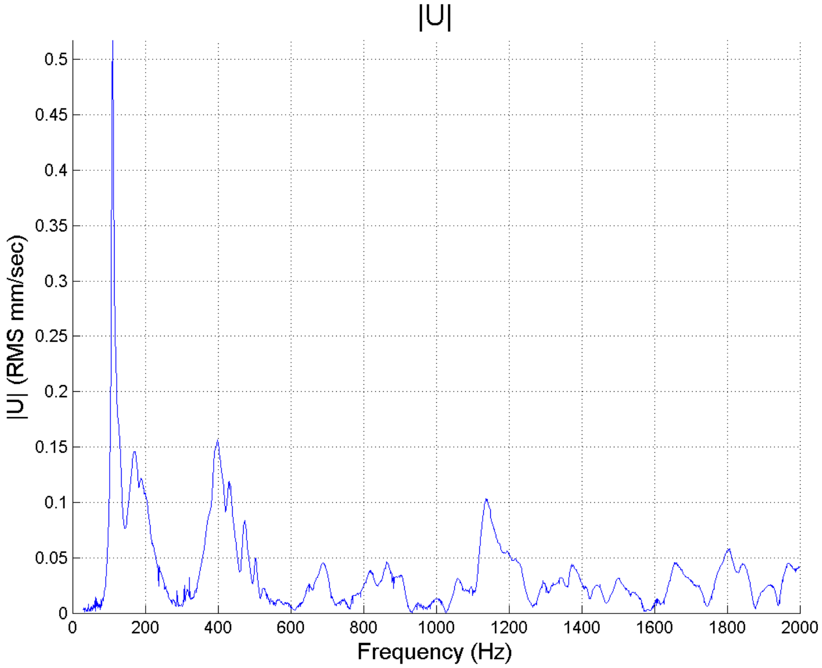

Figure 9 - Particle velocity over range of frequencies

Quantitatively, from direct result data calculation, the maximum occurs at a frequency of 110.5 Hz. Calculation from $Q = \frac{f}{FWHM}$ yields a Q factor of 10.05. While these values do not match the theoretical calculations exactly, they are reasonably in range given the simplifications and assumptions used. The internal electronics play a role in disrupting the air inside the body, and are most likely one of the major causes of the difference in values. They do not allow the body to behave as an ideal resonator.

Experimental vs. Simulation Comparison

In order to further compare analytical data with the data taken in this experiment, it was of interest to map the face plate displacement data onto a geometric representation of the plate. While the displacement magnitudes are different, by normalizing each set of data one can compare the deformed mode shapes from each data set. To accomplish this, a MATLAB script was created to pull numbers from the experimental data that corresponded to the frequencies of the modes calculated in the simulation. Phase data from the experiment was used to determine which direction the displacement was occurring in. These values were then plotted in a three dimensional surface to give a rough idea of what the guitar body was doing for these frequencies. The mappings of the displacements can be seen in Figures 10 and 11. These are specifically for the calculated face-plate-only modes.

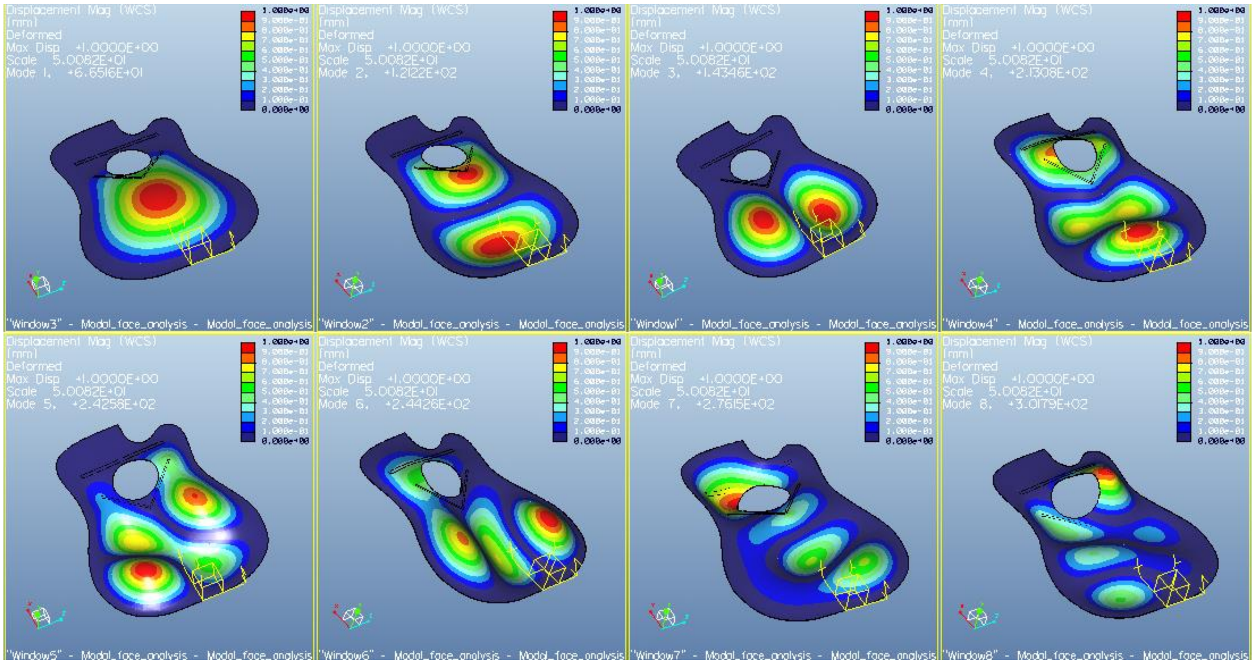

Figure 10 - Simulation results of face deformation

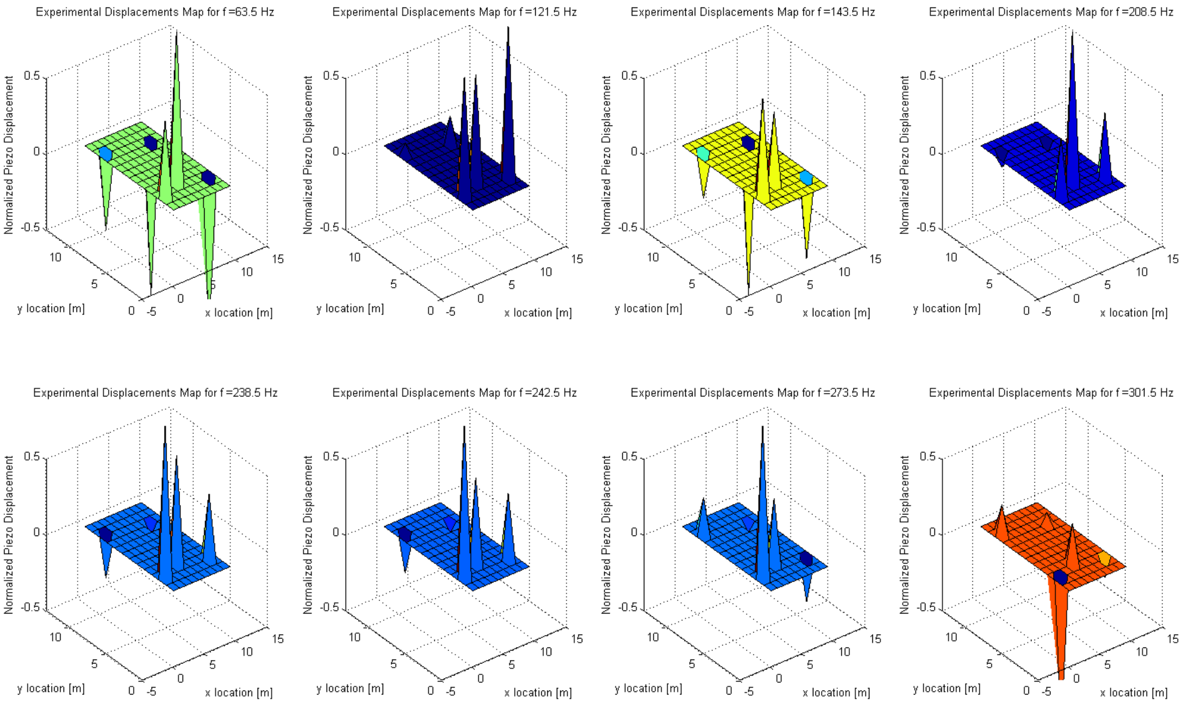

Figure 11 - Experimental results of face deformation

There were mixed results with these mappings. While the overall trends seem to match fairly well, one must look very closely at each plot. The measurements taken in the experiment seemed to, at many times, occur in smaller deformation areas/"dead" zones, which will skew the inferred shapes. However, the data does seem to promise to be on the right track. If the experiment were to be repeated, using more locations to measure the results at, the results would start to match much better. Five data points proved to be insufficient to match the harmonic shapes, but they do seem to echo the simulation trends. For completeness, the results for the full body mode data are shown in Figures 12 and 13 (shown on the following page). It is of some note that the fifth mode of the full guitar body was a deformation in the x-z plane, while the other deformations all seem to be primarily in the y direction.

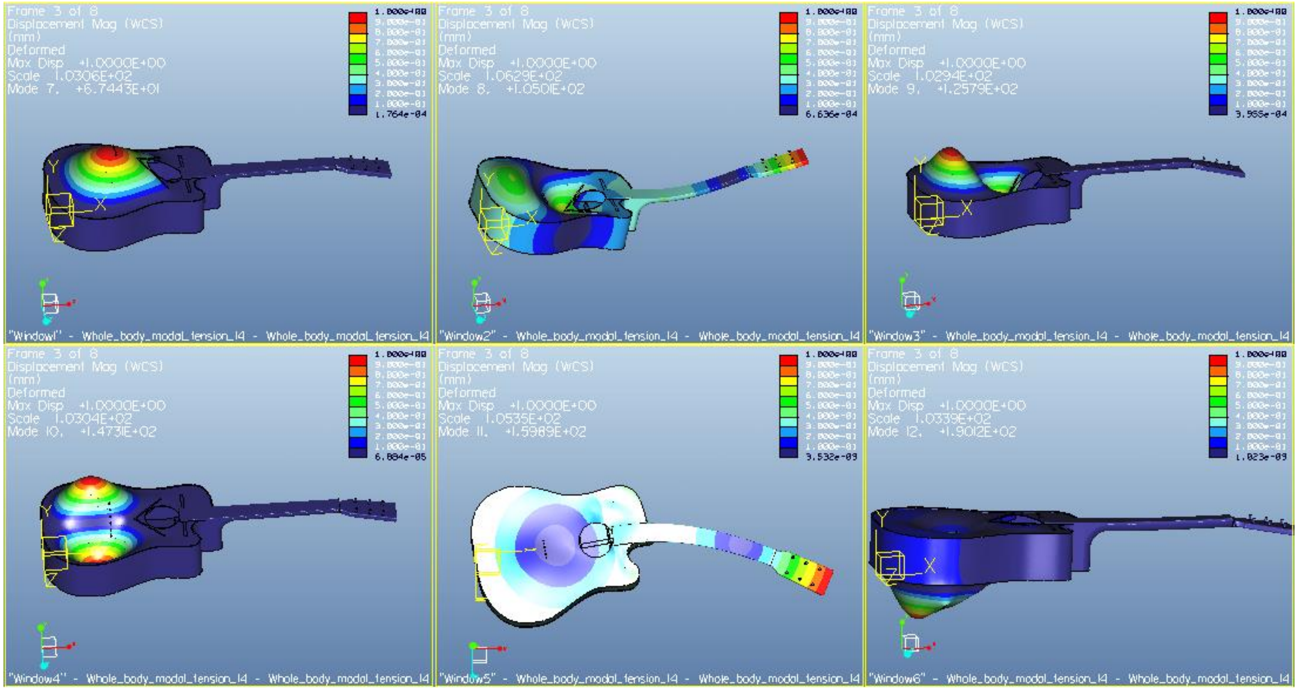

Figure 12 – Simulation results of body deformation

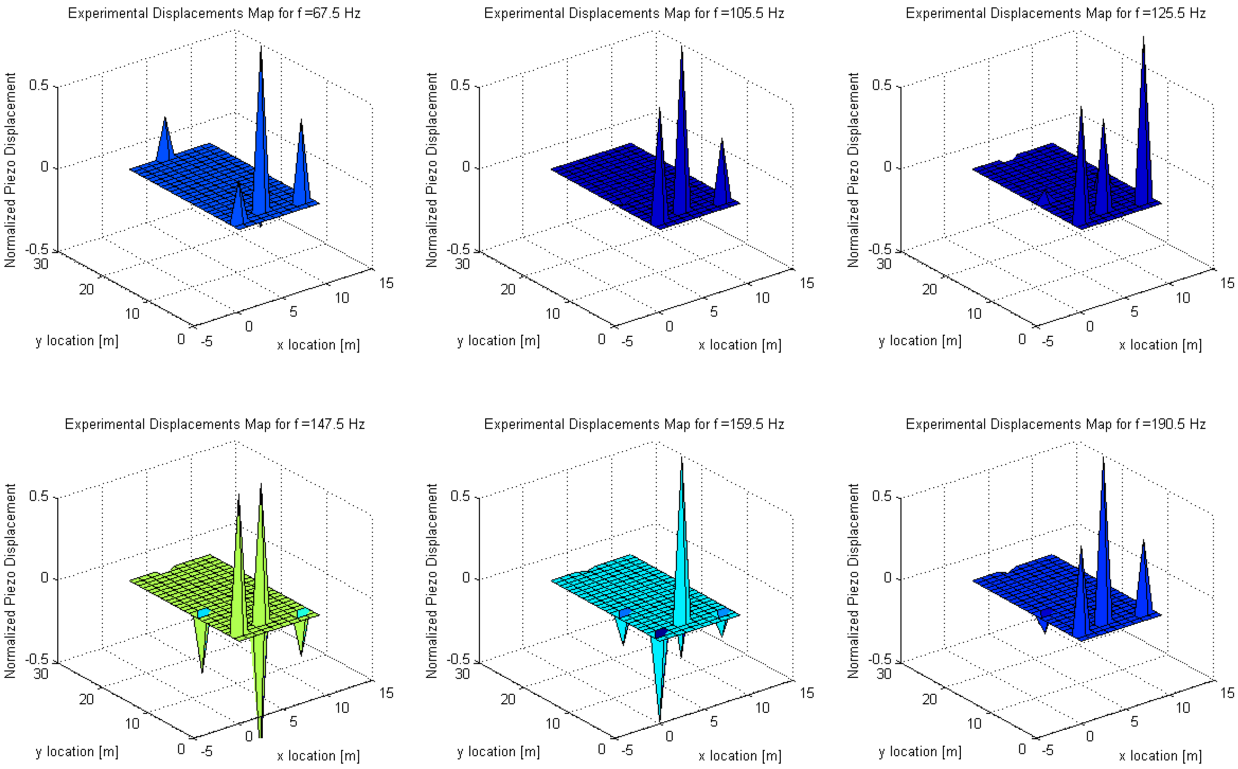

Figure 13 - Experimental results of body deformation

V. Summary

The acoustic guitar is deceivingly a very complicated structure, with many aspects of the design contributing to its unique sound. Harmonic analysis of different pluck locations showed that the brighter notes is caused by a stronger presence of higher harmonics. This is why notes plucked at the bridge have a very different sound than ones plucked over the soundhole. Simulation data gave some interesting results and insight as to how the guitar body is moving at various frequencies (displacements were on a very small scale, of course). Experimental data also allowed a comparison between the ideal calculations and the real world response of an actual instrument. Simplifications had to be made, as is the case with any form of modeling/analysis. Other work can be done with a project such as this. For example different bracing patterns would be interesting to investigate. Different shapes and/or string materials would add another layer of complexity to the overall sound. Attack and sustain tendencies of the notes could also be discovered using the data collected for the harmonic analysis. More measurement locations could be added to the modal analysis for a more accurate experimental displacement mapping. Overall, there is much to learn from this popular instrument.

VI. Acknowledgements

Special thanks goes to Professor Errede for assisting with the whole project process, and for diligently keeping the full body experiment going through various computer problems and data collecting hiccups. Also, the use of his pre-written MATLAB scripts was a great advantage when it came to processing the large amount of data taken in the experiments. Also, thanks to the course TAs, Michael Hopkins and Cody Jones, for their assistance.