Modal Analysis of a Square Aluminum Plate

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Abstract

The motivation behind this experiment was to analyze the modal vibrations of a metal plate. Originally, we started with the goal of creating Chladni plates for future use in this particular lab at UIUC. However, due to the ease of simply creating Chladni plates, it was decided that we would proceed to analyze the vibrations of these plates, as driven by a coil and set of magnets. This set-up, with the plate being supported in the middle of the plate and free at the edges, has been unexplored, theoretically speaking, before this experiment. Moreover, Chladni plates only use the physical displacement of the plate to create patterns in fine-grained sand; in comparison, by analyzing the plate vibrations via computers, we can manipulate and analyze the data with much more ease, along with getting more than just the physical image of where the nodes are.

Introduction

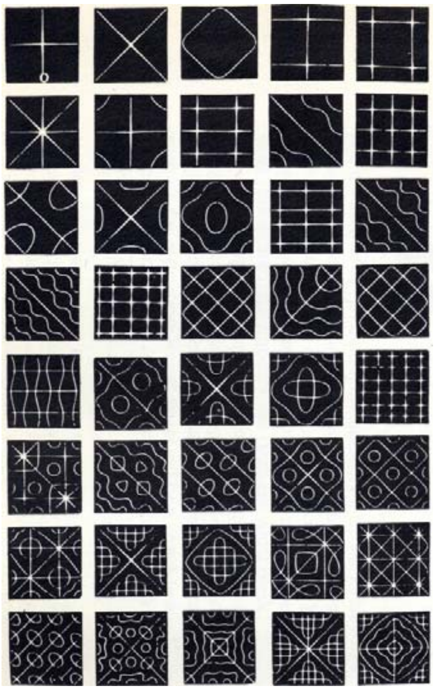

Figure 1: Some basic, and some more complex, Chladni Patterns

Ernest Chladni (1756-1827) is widely considered to be one of the fathers of acoustics. He did much research in both acoustics and meteorology, but he is most well known for his technique of demonstrating different modes of vibrations on a rigid surface (first published in 1787 in his book <u>Discoveries in the Theory of Sound</u>). By sprinkling sand onto a metal plate, Chladni could use a violin bow to vibrate the metal plate at resonant frequency, causing "Chladni patterns". These patterns were created by the fact that there are boundary lines on the plate where it was not vibrating (known as nodal lines). Due to lack of movement, the sand could settle there, while being pushed off of the vibrating areas, and observers could view the patterns of the nodes. Chladni's technique has been modified over the years, employing electronic signal generators instead of violin bows to produce a more accurate and more easily adjustable frequency. Furthermore, Chladni also came up with a simple equation to predict the nodal patterns that would be found on vibrating circular plates and other objects.

In this experiment, a simple Chladni plate was constructed and then analyzed. Although Chladni and many others have since come up with the mathematics behind different plates and set-ups, this set-up (supported in the center and free along all edges) has been unexplored thus far, making this experiment novel.

Materials and Methods

The set-up of this project was generally straightforward. The set-up included the aluminum plate apparatus, along with the sensors, the computer program and the device to drive the current.

Figure 2: Experimental Set-up

The most important part of the set-up, and the first part to be completed, was the aluminum plate apparatus. The aluminum plate was a square foot and was between an eighth and a sixteenth of an inch thick. It was supported by a metal rod and metal tripod underneath. It was fixed to the stand by a flat-head screw in the middle of the plate. The middle of the plate was simply found by drawing diagonals between the corners. The stand was made of scrap materials found in a storage room for UIUC physics laboratories. It was very important that when the plate was screwed into the rod that it was as close to flat on the top as possible, so as to not interfere with the sensors during the scanning. The hole in the middle of the plate was therefore angled outward slightly, so that the screw could fit snuggly into the hole.

The plate was driven using a set of magnets and a coil underneath the plate. A pair of magnets was placed in the same place, one on top of the plate and one below. The coil underneath the plate then had an alternating current running through it. Taking advantage of the changing current, the induced magnetic field then drove the set of magnets. Therefore, it was fairly simple to control the frequency at which the plate was driven, by controlling the frequency of the alternating current. The frequency was adjusted using a electronic signal generator in the laboratory. There was only a small issue during the scans, due to the internal crystal (inside of the signal generator) that controls the frequency and its lack of sensitivity, therefore inhibiting our ability to get extremely accurate results. This was fixed by using an external device to control the frequency and this solution provided us with much more precise results. The magnets were placed at three different locations on the plates: at the corner, the center edge (meaning on the center axis, but on the edge of the plate), or the quarter edge (still on the edge of the plate, but halfway between the corner and the center axis of the plate).

There were two different sets of sensors for this experiment, one set above and on below. The sensor above is labeled in the date as ‘Scan’, or just ‘S’, while the one below was ‘Monitor’, or ‘M’.

The actual process of running the scans was extremely intensive and time consuming. Each of the scans took somewhere between 4 and 8 hours each. Fortunately, the supervising professor was gracious enough to run these scans for me. There were two different types of scans that were performed. The first was just a general scan, in which the plate was driven at a variety of frequencies, ranging from 0 Hz to 2000 Hz. The purpose o this type of scan was to find where the resonant frequencies occurred. When a resonant frequency was reached, there was a sharp peak, as seen in the graph below (the peaks are marked by a pinkish-purple open circle). A narrow peak meant that the data was must more accurate and precise. As can be seen, the peaks that were reached in the general scans were very promising. A computer program preformed the variance the frequency.

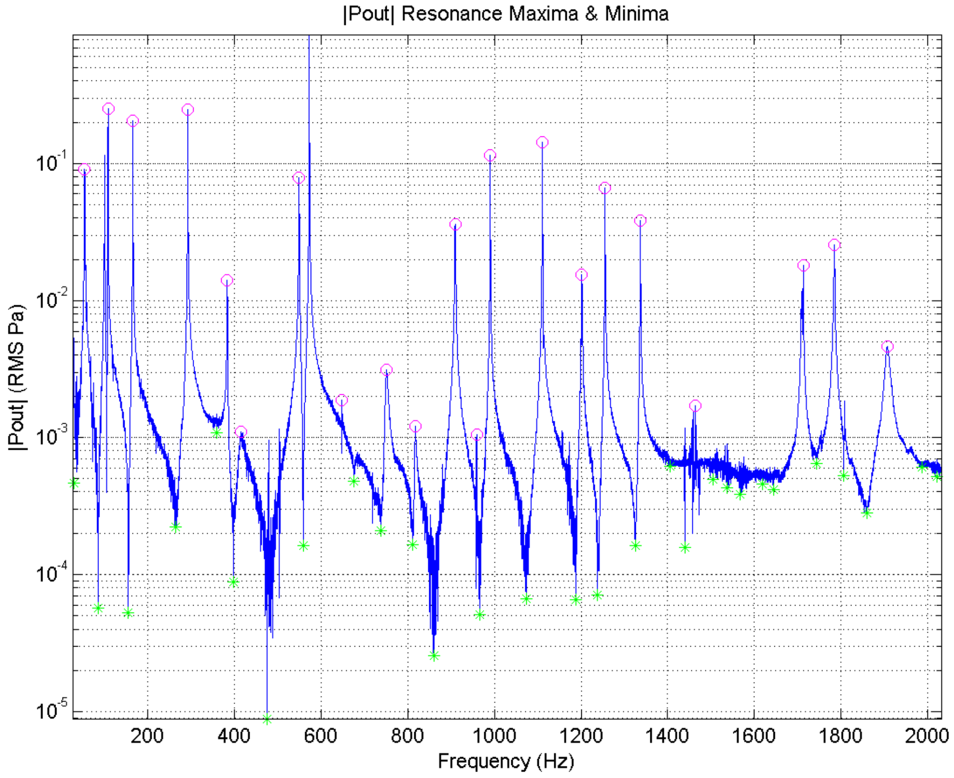

Figure 3: General Scan plot, showing resonant frequencies, being excited from quarter-edge.

The second scan was a Mode-Locking Scan. The purpose of this scan was to stay as close as possible to the resonant frequency. To do this, the imaginary part of the frequency is adjusted so that the real part of the frequency stays as close to its maximum as possible (a more in-depth explanation of mode-locking is available in other texts, but exceeds the depth of this paper). As seen in the graph below, there is only slight variance in the frequency, between 548.12 Hz and 548.21 Hz. The variance is expected in a mode-lock scan, and the fact that for the resonant frequency of 548 Hz only varied by .09 Hz is extremely satisfying.

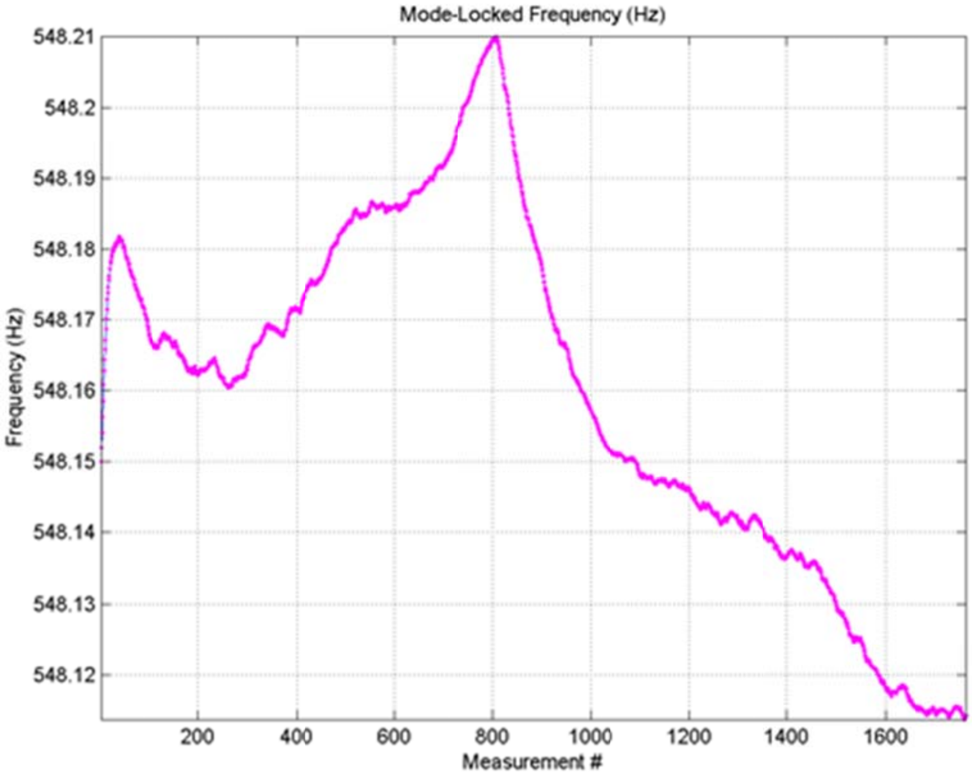

Figure 4: Mode-Locking plot around 548 Hz.

Results

Over the past few months, a tremendous amount of data has been collected and analyzed, producing hundreds of plots. On average, there were 160 plots for each scan that was run. Furthermore, there were eight separate scans in just the corner; the number eight was simply the number of resonant frequencies that were analyzed during the time period of this semester. Unfortunately, there is no way to properly discuss all of these graphs in this paper, so I have chosen some plots and graphs that I feel are the most important to explain and talk about. Some of the more interesting plots are below.

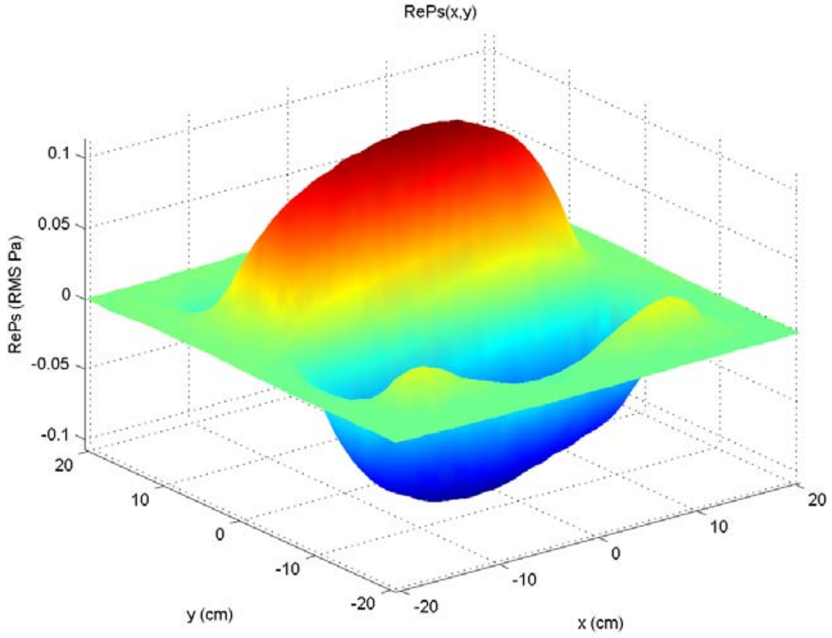

Figure 5: Real part of the Pressure, as collected from the Scanning sensor at 548 Hz.

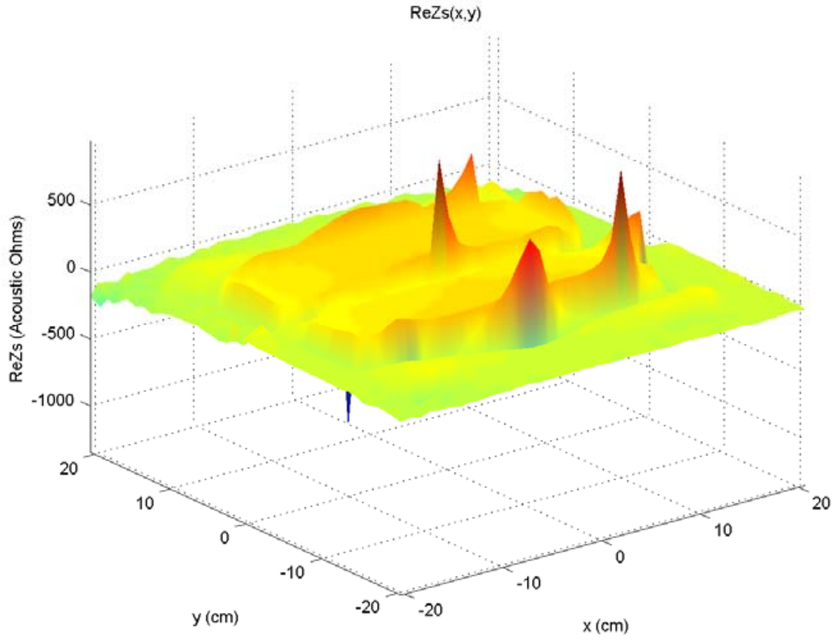

Figure 6: Real part of the Impedance, as collected from the Scanning sensor, at 548 Hz.

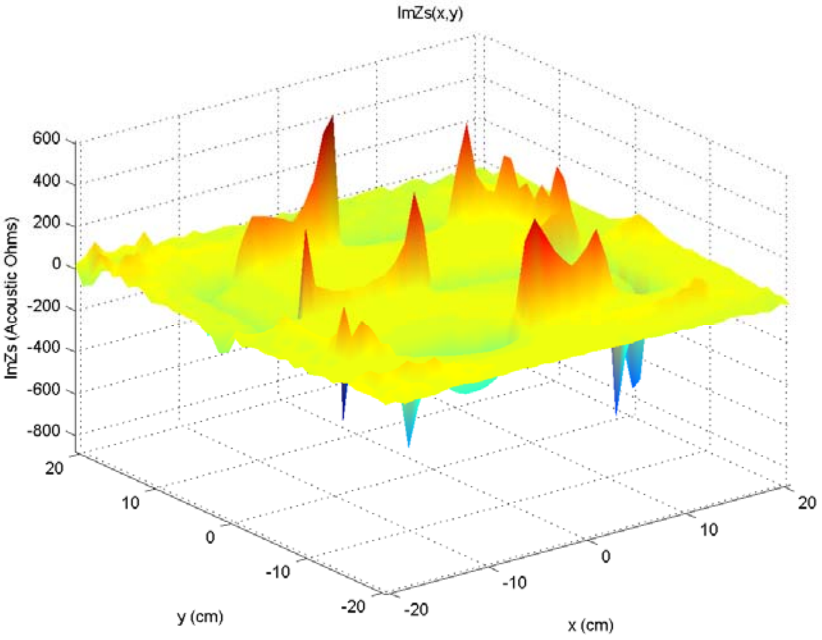

Figure 7: Imaginary part of the Impedance, as collected from the Scanning sensor, at 548 Hz.

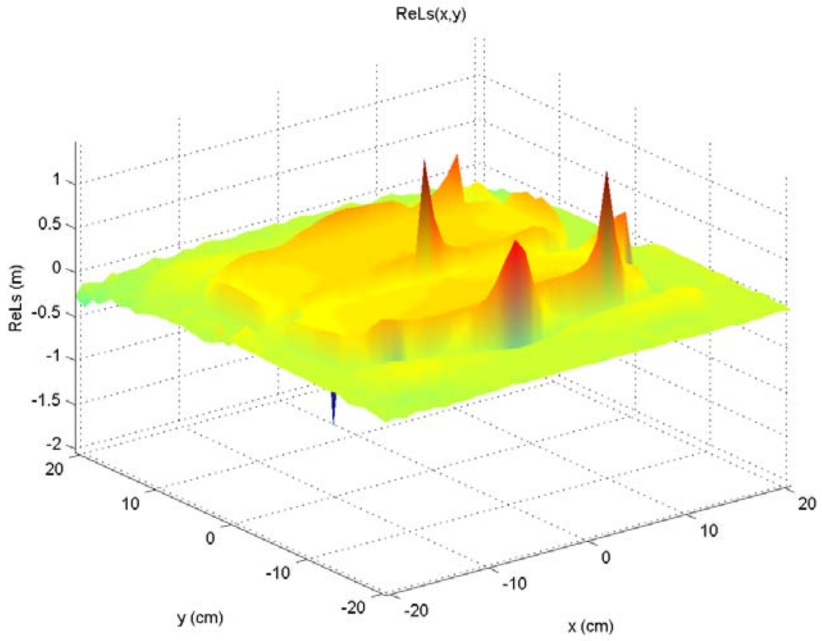

Figure 8: Real part of the wavelength ('L'=Lamdba), as collected from the Scanning sensor, at 548 Hz.

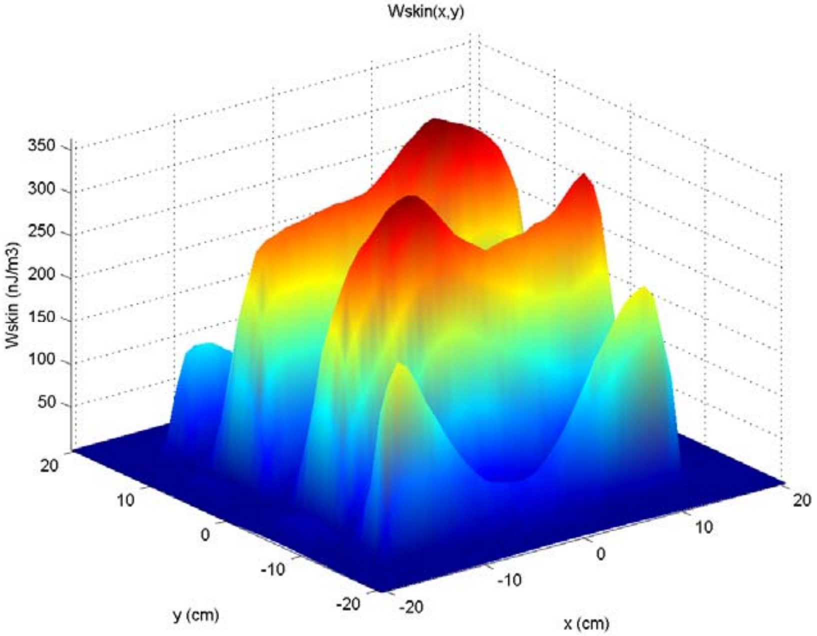

Figure 9: Kinetic Energy, as collected by the scanning sensor, at 548 Hz.

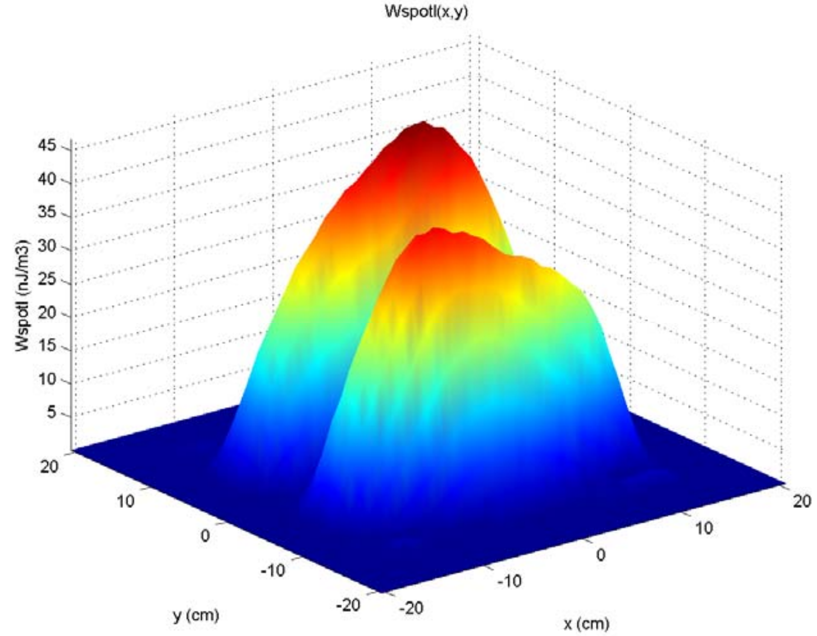

Figure 10: Potential Energy, as collected by the scanning sensor, at 548 Hz.

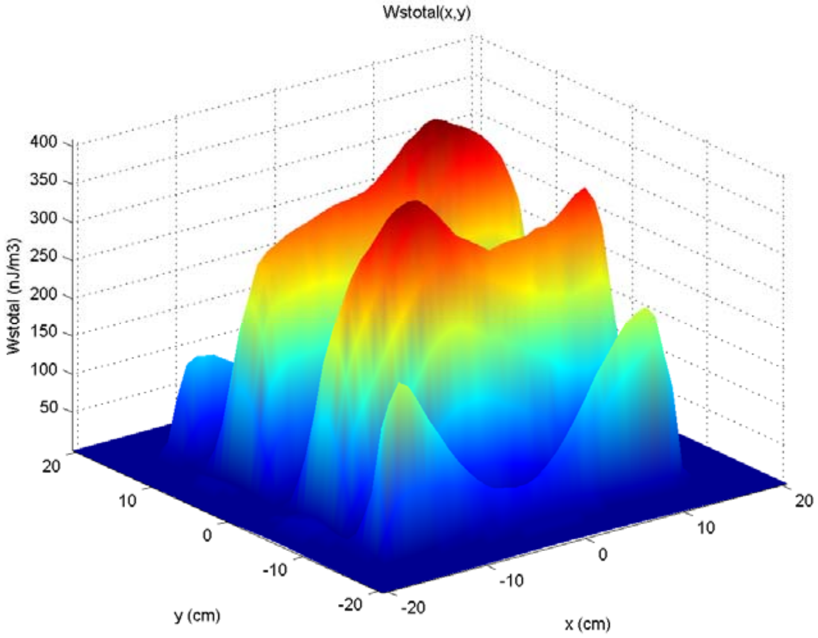

Figure 11: Total Energy, as collected by the scanning sensor, at 548 Hz.

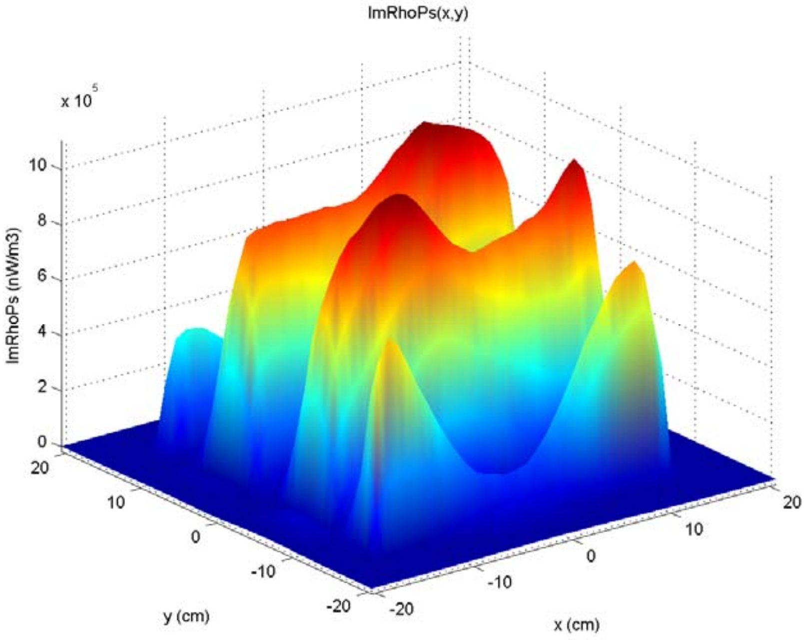

Figure 12: Imaginary part of the power density, as collected by the scanning sensor, at 548 Hz.

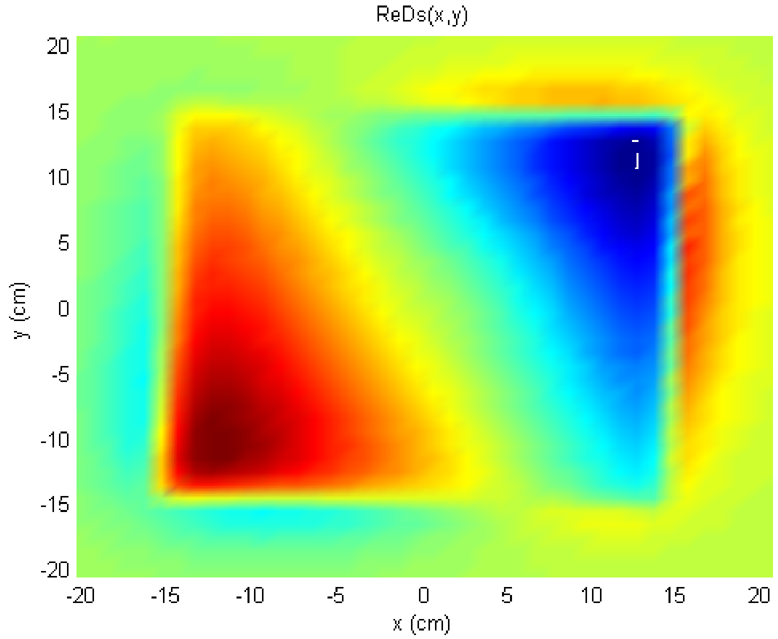

Figure 13: Depiction of the physical displacement of the plate, at 55 Hz, excited at the corner.

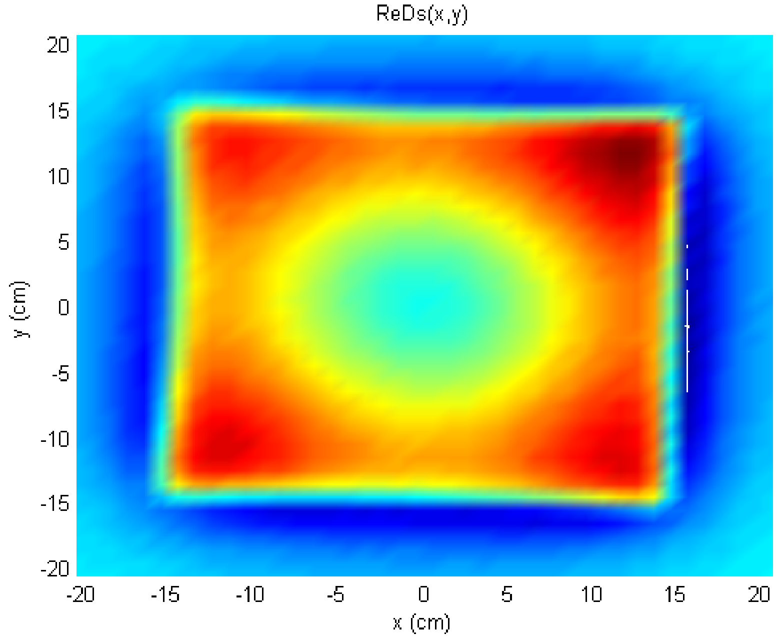

Figure 14: Depiction of the physical displacement of the plate, at 101 Hz, excited at the corner.

Discussion

The sheer amount of data and plots given after analysis was overwhelming. Information included plots for impedance, energy, displacement, acceleration, pressure, and particle velocity. For ease and brevity, most of the plots shown are from the set-up with the magnet at the corner and at the resonant frequency of 548 Hz. Many of the plots are self-explanatory, so this section will only discuss non-obvious, interesting aspects.

One of the most interesting aspects is that the plot of the real part of the impedance and the plot of the real part of the wavelength are identical. Similarly, the likeness between the kinetic energy, the total energy, and the imaginary part of the power density is striking. Unfortunately, at this time, I do not know how to explain these two sets of similarities, but I would hope that they would be explored in any furtherance of this experiment. Also, it should be noted that only the imaginary part of the power density is shown above. This is due to the fact, that mathematically, the real part of the power density actually equals zero.

One of the biggest issues for running the scans was the set-up being bumped while the scans were running. There were extraneous vibrations that affected the mode-lock and general scans, caused by both construction in the laboratory building and by janitors in the building coming into the laboratory. This could be mostly fixed by placing the whole set-up into an enclosed box, where no extra vibrations or change in air pressure could affect it. Also, there are environmental issues that can affect what the resonant frequency is, such as room temperature. Any slight change in room temperature affects the resonant frequency, and that is seen in the variation of the frequency in the mode-locking scans.

In addition, there is a great amount of potential in the theoretical portion of this experiment. Although there have been calculations regarding modal analysis of square plates, there have not been previous calculations of this set-up, where the edges are free and the plate is supported only in the center of the plate.

Conclusions

Generally speaking, this experiment was a great success, and the future of this project is extremely bright and promising. There are many more experiments that could be done using the Chladni plate constructed here.

One of the more obvious steps to be taken is to take a step back and use some type of fine-grained salt to create physical patterns. This was tried once or twice this semester, but the problem arose of the grains being too large, therefore being too heavy to be moved by the vibrations of the plate. Adding sand has been attempted with other flat surfaces, such as a snare drum head, but static electricity between the drumhead the sand prevented the patterns from forming. Attempting to use sand to create physical nodal lines would be the next logical step for this project.

Additionally, we recorded the sound of the vibrations made by the plate and analyzed those; however, the analysis never went any further. This would also be a good way to go in the future. It would be very interesting to see the resonant frequencies produced by a mallet hit and compare those to the resonant frequencies of the driven vibrations.

Lastly, there is so much more to do in the computational field of this set-up. The supervising professor has already produced some working equations for this set-up (which was previously thought to be too difficult to solve) and those equations could be used in the future.

Overall, this project has a lot of potential for future experimentation over the next few years, especially because of the novelty of the unsupported edges. There was a vast amount of new information produced in only one semester and the amount that could be produced in the upcoming semesters are very promising.